THE GENERATION OF THREE-DIMENSIONAL CONTOUR MAPS BY WAVEFRONT RECONSTRUCTION

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This letter contains experimental verification of two methods for obtaining three-dimensional images having constant altitude contours on their surfaces. Such images are obtained by wavefront reconstruction with the modification that the illumination consists of two sources in one case, and a single two frequency source in the other.

In a letter [1] published previously, the authors indicated a method for obtaining three-dimensional images containing constant range contours. In that paper attention was called to the similarity between a high-resolution radar system and holography. The method described was that of making a hologram in light containing two discrete frequencies and reconstructing in light of a single frequency. The range interval between countours was stated to be $\lambda_1^2/2\Delta\lambda_1$ where λ_1 was the mean frequency and $\Delta\lambda_1$ was the frequency separation.

PROPAGATION VECTOR OF THE REFERENCE SOURCE

OBJECT

HOLOGRAM

$\Delta\gamma$

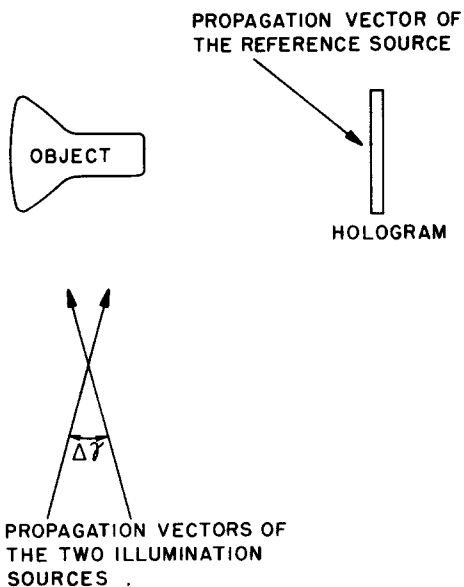PROPAGATION VECTORS OF THE TWO ILLUMINATION SOURCES .
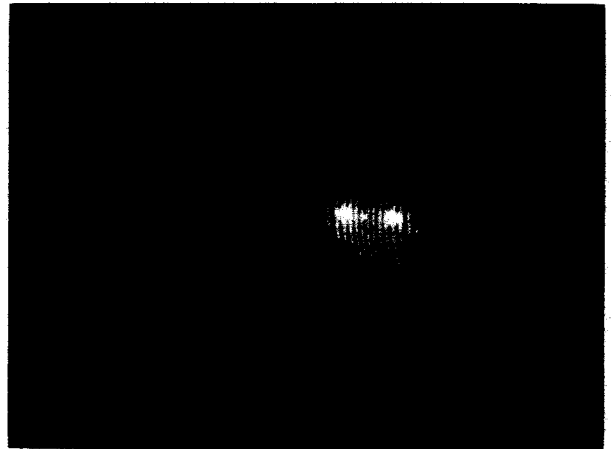
Fig. 1.

Fig. 2.

We now wish to report a second method, and to show experimental results obtained for both methods. These results were reported at the Spring meeting of the Optical Society of America at Washington, D.C. [2].

The new method for generating contoured images is to make the hologram with two spatially separated illuminating sources of the same frequency. The hologram is made in the conventional manner but at a definite position as indicated in fig. 1. The analysis shows that the range interval between contours is $\lambda_1/2 \sin \Delta\gamma$ where $\Delta\gamma$ is the angle between the propagation vectors of the two illuminating beams. Fig. 2 is a photograph of the images so produced. The object is a flood lamp. The range or depth interval between fringes in this case is 2.2 mm. The hologram was made
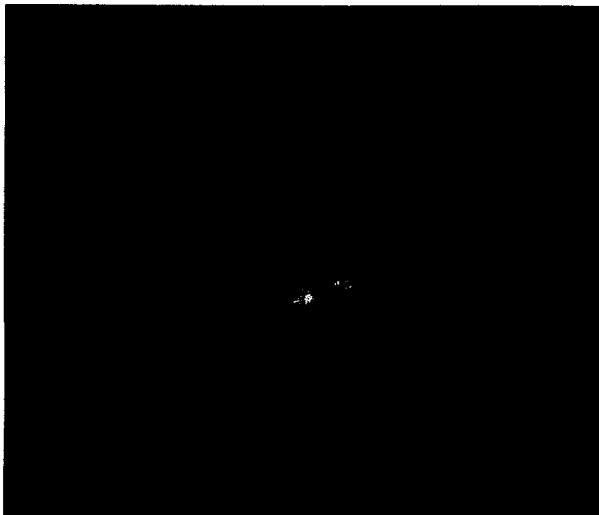
Fig. 3.

as a double exposure with the illumination source moved between exposures.

An experiment designed to demonstrate the two frequency method was performed with an argon laser using two frequencies separated by about 65 Å. The object was a sphere three inches in diameter. Fig. 3 shows a photograph of the image of the surface of the sphere. Since the range interval for this frequency separation is very small, 0.02 mm, considerable magnification is necessary. This photograph demonstrates, as expected, that the fringes become closer together as the curvature of the sphere, as seen from the observer's position, increases.

References
1. K. Haines and B. P. Hildebrand, Physics Letters 19 (1965) 10.
2. B. P. Hildebrand and K. A. Haines, Paper WB17, Fiftieth Anniversary Meeting, Optical Society of America, 16 March 1966.

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THE COMPLETE ANALYTIC EXTENSION OF THE REISSNER-NORDSTRÖM METRIC IN THE SPECIAL CASE $e^2 = m^2$

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A method originally developed for the Kerr solution is adapted in order to obtain the complete analytic continuation of the Reissner-Nordström metric in the special case $e^2 = m^2$. This case is the exterior field of a static spherical charged dust cloud.

The purpose of this note is to show how the complete analytic extension of the Reissner-Nordström metric for $e^2 = m^2$ may be obtained by slightly adapting a method developed for the Kerr solution [1].

The null form of the Reissner-Nordström metric is

$$ds^2 = -(1 - 2m/r + e^2/r^2)\, du^2 + 2du dr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) . \quad (1)$$

By introducing w such that:

$$u + w = F(r) \quad (2)$$

where $dF/dr = 2(1 - 2m/r + e^2/r^2)^{-1}$, we obtain

the double null form

$$ds^2 = (1 - 2m/r + e^2/r^2)\, du dw + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) . \quad (3)$$

In order that r be specified by (2) when $e^2 \leqslant m^2$ we must state whether it is in a region of type I $(r > r_+)$, type II $(r_+ > r > r_-)$, or type III $(r_- > r > 0)$, where $r_+ = m \pm \sqrt{m^2 - e^2}$. Both forms are incomplete when $e^2 \leqslant m^2$.

When $e^2 = m^2$, a complete manifold may be built out of overlapping patches of the form (1), transforming from one to the other via the form (3) using the symmetry of u and w, in the manner described in sect. 5 of ref. 1.