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A MACROSCOPIC YIELD CRITERION FOR CRYSTALLINE POLYMERS

RAM S. RAGHAVA and ROBERT M. CADDELL
Department of Mechanical Engineering,
University of Michigan, Ann Arbor, Michigan, U.S.A.

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Summary—Yield studies including uniaxial tension, uniaxial compression and biaxial stress states (developed with internally pressurized thin wall tubes) were conducted with high density polyethylene. The experimental results are compared with a predicted yield locus based upon a pressure-modified von Mises criterion. Agreement was quite reasonable although a slight degree of anisotropy was noted in the test material. Since this same yield criterion has earlier been shown to provide excellent agreement with glassy amorphous polymers it appears unnecessary to employ different criteria for different polymers if one is concerned with macroscopic yielding.

NOTATION

$\sigma_1, \sigma_2, \sigma_3^*$	principal stresses
C	absolute value of compressive yield stress at atmospheric pressure
T	absolute value of tensile yield stress at atmospheric pressure
R_1	axial stress normalized with respect to T
R_2	circumferential (hoop) stress normalized with respect to T

INTRODUCTION

A YIELD criterion for isotropic, glassy amorphous polymers was proposed recently[1] and is, in essence, a pressure-modified von Mises-type which accounts for differences in tensile and compressive yield stress. The contributions by Whitney and Andrews,[2] Sternstein and Ongchin[3] and Bauwens[4] were noted[1] and comparisons of these efforts were discussed in two different works.[1,5] For the purposes of this paper, such comparisons are unnecessary, but from all of the aforementioned studies there is unanimous agreement that the magnitude of the mean normal stress or the "hydrostatic component" of the applied stress state does influence the macroscopic yield behavior of glassy amorphous polymers.

To test the credence of a proposed yield criterion, it is the usual practice to obtain an adequate number of experimental points which are compared with a predicted yield locus as plotted in two-dimensional stress space. It should be realized, however, that the maximum *range* of mean normal stress that is developed in such investigations is relatively small. This can be overcome by determining the uniaxial tensile and/or compressive yield stress under ever-increasing fluid pressure such that the range of mean normal stress is vastly increased. These points have been discussed recently by Caddell *et al.*[6]

One might question whether a single yield criterion is applicable for all types of isotropic polymers, with the principal concern being directed towards the

* True stresses are implicit throughout this paper.

influence of crystallinity. Bowden and Jukes[7] have, for example, proposed three different criteria for use with various polymers that "deform" differently. As there has been relatively little information published in regard to the macroscopic yield behavior of crystalline polymers, this study was conducted to determine if the criterion suggested by Raghava[1] would be applicable to crystalline as well as amorphous polymeric solids.

Certainly, the yield behavior of polymers is affected by strain rate,[8-10] temperature[3,10,11] and anisotropy as caused by orientation effects,[12-14] and these parameters must be fully explored if one is interested in deducing the overall yield behavior of a particular material. However, since the intention of the work reported in this paper was to explore the potential applicability of a particular yield criterion, no attempt has been made to determine what modifications in the criterion might be needed to extend its usefulness by including the added affects of the aforementioned parameters. This is in keeping with other publications[2-4] in which a similar goal was pursued.

SUGGESTED YIELD CRITERION

The form of criterion which has proved quite acceptable for glassy amorphous polymers is,

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+2(\sigma_1+\sigma_2+\sigma_3)(C-T) = 2CT, \tag{1}$$

where $\sigma_1, \sigma_2, \sigma_3$ are principal stresses while C and T are the *absolute values* of compressive and tensile yield stress, respectively, as measured under atmospheric conditions of pressure.

Note that the influence of the mean normal stress, σ_m, enters through the term that sums the three principal stresses; additionally, if the tensile and compressive yield stresses are equal, equation (1) reduces to the standard form of the von Mises criterion.

Since the type of studies discussed in this paper reduce to uniaxial or biaxial stress states, equation (1) is rewritten for the case where $\sigma_3 = 0$. In essence this simplified equation can then be transformed into a plot of a yield locus in σ_1, σ_2 space. By performing a number of experiments along different loading paths, the validity of the proposed criterion can then be determined. For the biaxial or plane stress case, equation (1) becomes

$$\sigma_1^2+\sigma_2^2+(\sigma_1+\sigma_2)(C-T)-\sigma_1\sigma_2 = CT. \tag{2}$$

A normalized form of (2) has been found useful; this results by defining two normalizing factors,

$$R_1 = \sigma_1/T \quad \text{and} \quad R_2 = \sigma_2/T.$$

Application of R_1 and R_2 into equation (2) results in the following form,

$$R_1^2+R_2^2-R_1R_2+(R_1+R_2)[(C/T)-1] = C/T. \tag{3}$$

EXPERIMENTAL PROCEDURE

Commercial rods of high density polyethylene were obtained in the form of 1·5 in. dia. solid rounds and were used in the "as-received" condition. As it was possible that variations might exist among these rods, they were coded 1, 2 and 3 for future reference. All tests were conducted at a temperature of about 25°C on an Instron Testing Machine whose crosshead speed was constant at 0·05 cm/min.

(a) *Uniaxial tension test*

From each bar, tensile specimens were produced with the longitudinal axis of the specimen being parallel to the axis of the original bar. Specimens had a uniform gage section of 4 in. length with a cross section that was 0·300 × 0·300 in. square. The overall length was 6 in. with the ends of the specimens being threaded for adaptation to grips on a 500-kg Instron Machine (Model TM-SM). Specimen elongation was measured with an Instron Strain Gage Extensometer (Type G51-11M). This had a 1-in. gage length and an allowable uniform extension of 10 per cent. The load was sensed by a standard Instron load cell whose calibration was checked periodically with dead weights. A record of load–elongation was obtained on the Instron chart recorder, with the output of the extensometer used to drive the chart. Contraction in the lateral directions during a test was, in general, measured simultaneously using flat micrometers. Comparative contraction measurements were also obtained using two Instron Transverse Strain Sensors (Type G57-12M). Agreement with micrometer values was so close that most of these data were obtained with micrometers because of simplicity. These measurements were used to evaluate true stress and also to check on possible anisotropic behavior of the material. By converting the longitudinal and lateral measurements to axial and lateral strains, Poisson's ratio was calculated. This was used in certain analyses of experimental results obtained from thin wall tube tests. Calibration of the extensometer and transverse sensors was performed with the aid of the table-actuating mechanism from a toolmaker's microscope; this calibrating device was accurate to the order of 10^{-4} in.

(b) *Compression tests*

Direct compression tests were conducted on specimens machined in both the axial and radial directions from the bar stock. The specimens were of $\frac{1}{2} \times \frac{1}{2}$ in. square cross-section and $\frac{3}{4}$ in. high. They were compressed between two hardened and ground platens, and to minimize frictional effects, molybdenum disulphide grease was used as a lubricant at the platen–specimen interface. Load was recorded on a gear-driven chart, while the change in the height of specimens was deduced by knowing chart speed and the Instron cross-head speed. Because it was necessary to make corrections for the machine stiffness during the change of specimen height, the combined stiffness of the compression load cell and the Instron was earlier determined by compressing the cross-head against the load cell.

Lateral dimensional changes were found with the aid of an Instron Transverse Strain Sensor which was mounted across two parallel faces. It was assumed that changes in each lateral dimension were reasonably similar and spot checks with micrometers indicated this to be a good assumption. Calibration of the sensor was discussed previously and during an actual test, the output of the sensor was recorded on a two-channel Sanborn Recorder (Model 152-100A).

(c) *Thin-wall tube tests*

Thin-wall tubes having an external diameter of 0·890 in. and wall thickness of 0·040 in. were machined from the bar stock. The overall length of the tubes was 6 in. with the test section length being $2\frac{1}{2}$ in. To ensure concentricity and to hold close tolerances on the wall thickness, care had to be exerted during machining. The sequence of machining was as follows: (a) first, a $\frac{3}{4}$ in. dia. hole was drilled; (b) this was then enlarged to $\frac{25}{32}$ in. dia.; (c) subsequently, two reamers of $\frac{51}{64}$ in. and $\frac{13}{16}$ in. dia. were used to finish the hole to the required dimensions. The tube was then mounted on a mandrel for the finish machining of the outer diameter. In this way excessive twisting of or distortion of the tube was avoided. The cutting fluid was hydraulic oil which was also eventually used as the fluid for providing internal pressure with the tubes. The particular cross-section of the tube was chosen so that the load capacity of the Instron machine was not exceeded during any of these tests.

The tubes were loaded under either tension or compression by Instron crosshead movement and the internal pressure was produced by a hydraulic pump. In order to achieve radial loading (the ratio of axial stress to tangential stress being constant), the ratio of load to pressure was predetermined for a particular value of the stress ratio. As the axial load increased, the pressure was also increased by continued actuation of a hand pump. Since the cross-head speed was quite low, sufficient time was available for increasing the pressure to a predetermined value, thereby closely approaching a true radial loading path. Concern might be expressed in regard to a variation in strain rate in the circumferential direction between one tube test and another. In terms of experimental limitations any potential effects were minimized by using the lowest possible cross-head speed concurrent with a pressure increase that kept pace with the change in axial load in order to maintain as close to a constant stress ratio as possible. Earlier studies[8, 15] support our feelings that any slight differences in strain rates encountered in this study had a truly minimal effect on the findings.

Various "constant stress ratio" tests were conducted in the tension–tension and tension–compression quadrants (i.e. first and fourth) of the yield locus. In addition, tests were performed with the equivalent of an "open-ended" tube; this provided a value of the tangential or hoop yield stress. Details of this test, as well as a complete description of the general tube tests, are fully described by Raghava.[5]

Axial deformation (either extension or contraction) was measured with the same extensometer used for tension testing, while tangential strains were evaluated by measuring the external diameter of the tube with micrometers. Some effort was expended to measure the actual thickness of the tubes during loading by using an inductance pickup as discussed elsewhere.[15] However, it was found that the changes in tube thickness were only a few thousandths of an inch. Such small changes could not have been used in the computation of stresses and strain with sufficient reliability. Instead it was found expedient to use the principle of volume constancy to determine both the thickness strain and the instantaneous thickness. In order to check the validity of this assumption, tension tests were conducted on tubes without using internal pressure and true stress values were found by using areas based upon constancy of volume. The true stress–strain curve obtained in this way was identical within experimental error to the true stress–strain curve determined with a standard solid tensile specimen.

RESULTS

Fig. 1 shows typical tensile true stress–true strain results, the implication being that the behavior of the three individual rods was not identical. Additionally, the difference between the two curves identified with bar number 3 indicates a degree of anisotropy.

Values of "yield stress' were based upon the use of a 0·003 offset as illustrated in Fig. 2. It might be noted that many more test points were plotted in this low strain region than are shown on Fig. 1.

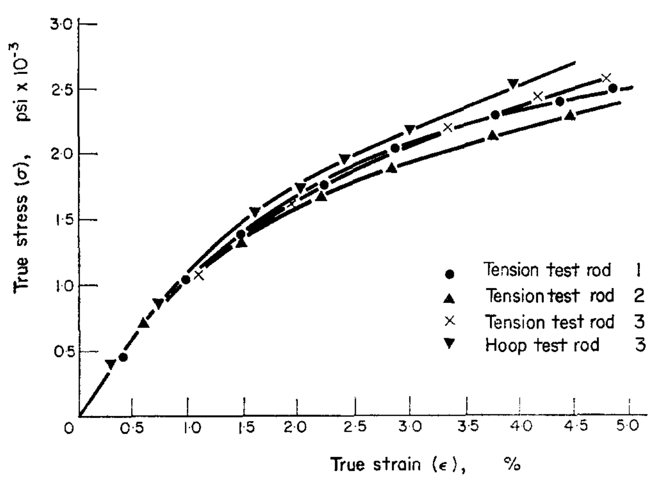

FIG. 1. Tensile true stress–true strain curves for the three rods of polyethylene.

Fig. 3 shows the direct compression results for the three bars; note that these compression tests were conducted in the axial directions of the original bars. Compressive yield stress was also determined using the 0·3 per cent offset. It should be noted here that compression tests conducted in the radial direction of the bars did not duplicate the

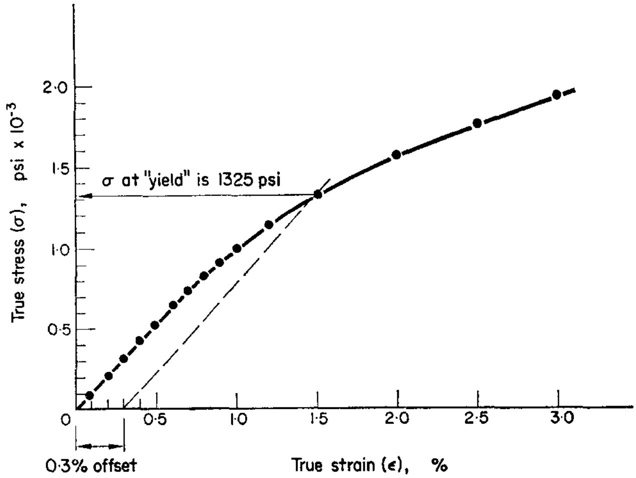

FIG. 2. Illustration of "yield stress" based upon a 0·3 per cent offset (standard tensile test for rod 1).

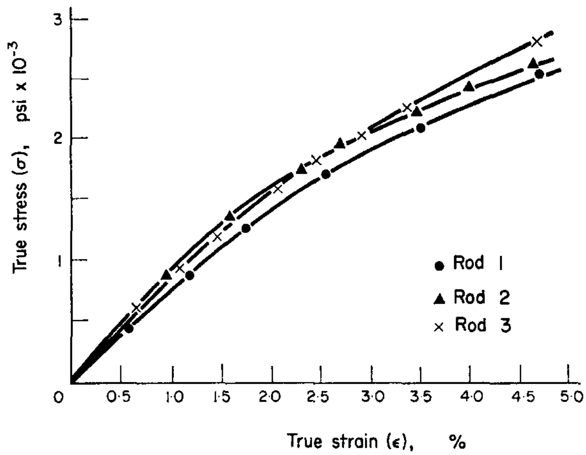

FIG. 3. Compressive true stress–true strain curves for the three rods of polyethylene.

plots shown in Fig. 3, thus there was little question that these three bars exhibited a degree of anisotropy. More will be said of this later on. With the tube tests, individual curves of hoop stress (σ_2) and axial stress (σ_1) were plotted against the "effective strain". The details regarding the determination of this strain function are discussed elsewhere[1, 5] and Fig. 4 shows typical results; this test was for a loading path whose stress ratio (σ_1/σ_2) was $-2{\cdot}25$. In effect, the hoop stress (σ_2) was tensile while (σ_1) was developed through a combination of internal pressure and axial *compression*. From Fig. 4 it can be seen that two values may be determined at yielding using the 0·3 per cent offset and that the ratio of these is just about $-2{\cdot}25$. Thus the interrelationship between σ_1 at yield, σ_2 at yield, and the loading path must be maintained. Because there was usually a little less scatter of points with the higher stress level curve (whether σ_1 or σ_2), that "yield stress" was determined from the offset and the corresponding yield stress for the lower curve was calculated using the known stress ratio. Table 1 contains the numerical and normalized values of stresses at yielding for the tests conducted in this study. Because of the variations in C and T found among the bars, the normalized stress values make a most sensible form for plotting. The ratio of C/T varied somewhat among the three bars but rather than attempt to plot a theoretical yield locus for the extreme values, an intermediate value of 1·3 was selected. Using this number in equation (3), a number of combinations of R_1 and R_2 were found and the locus of such points is shown as the solid line in Fig. 5. Actual test data, as tabulated in Table 1, are also shown in this figure.

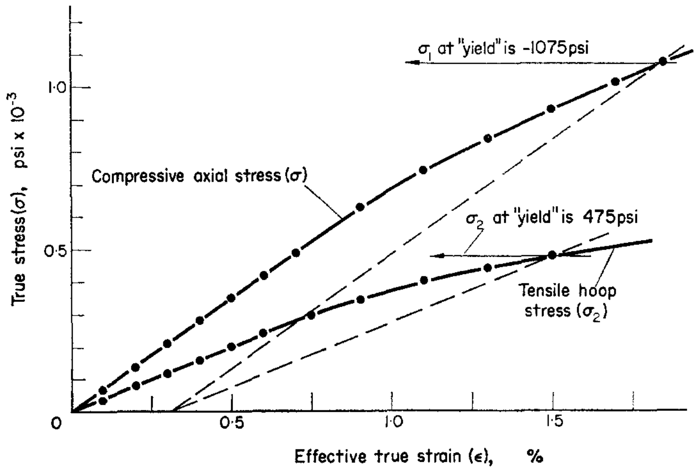

FIG. 4. Typical plot of biaxial stresses (σ_1 and σ_2) vs true effective strain, $\bar{\varepsilon}$, for a loading path whose stress ratio (σ_1/σ_2) was about $-2{\cdot}25$. The use of 0·3 per cent offset for each curve is indicated; rod 2 material.

TABLE 1. ACTUAL AND NORMALIZED VALUES OF YIELD STRESS OF POLYETHYLENE SUBJECTED TO DIFFERENT STRESS STATES

Rod No.	σ_1 (psi) (axial)	σ_2 (psi) (tangential)	$R_1 = \sigma_1/T$	$R_2 = \sigma_2/T$
1	1325*	0	1·0	0
	−1695†	0	−1·28	0
	1325	650	1·0	0·5
	1200	1200	0·905	0·905
2	1275*	0	1·0	0
	−1675†	0	−1·32	0
	−800	800	−0·63	0·63
	−550	1025	−0·43	0·80
	−1075	475	−0·84	0·37
3	1325*	0	1·0	0
	−1695†	0	−1·28	0
	0	1400	0	1·05
	−1525	425	−1·15	0·32
	−350	1060	−0·265	0·80
	740	1500	0·56	1·13
	425	1350	0·32	1·02

* These values are used as T for each rod.

† These values, in absolute form, are used as C for each rod, thus $|-1695| = 1695$ etc.

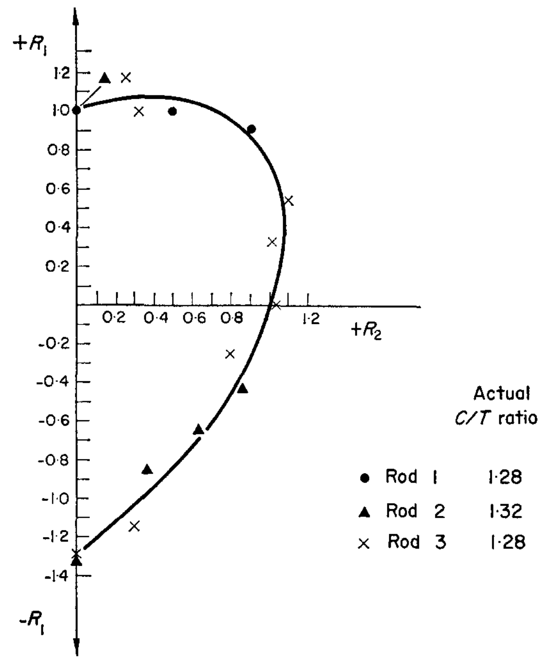

FIG. 5. Comparison of a predicted normalized yield locus based upon equation (3) for a C/T ratio of 1·3 with experimental results for specimens made from three rods of polyethylene.

DISCUSSION ON DEFINITION OF "YIELD STRESS"

Various methods are used to define the yield stress of polymers and all are, in essence, *arbitrary*. Some authors[2, 4, 7, 10, 12, 16] have used the maximum load as the yield load. One can find both the nominal stress and true stress associated with this maximum load being called the "yield stress" in such papers. Certainly, this difference in definition makes little sense and should be avoided. With ductile *metals* the same phenomenon of a maximum load occurring at the onset of tensile instability (or "necking") is encountered; historically, the tensile strength or ultimate strength is associated with this point on the load–extension curve. Thus if one uses the nominal stress at ultimate load as the yield stress, what should be defined as the tensile strength? The key point here is that for engineers who are interested in both metals and polymers, this situation is both confusing and misleading. In addition, with polymers such as polystyrene that do not exhibit the type of "load drop" discussed above, or with its removal by cold working,[17, 18] another definition of yield stress must be concocted. This has led others[7, 10, 12] to use an "extrapolation" technique.

To the authors of this paper it seems more sensible, at least in terms of consistency, to employ one definition of yield stress that is applicable to all polymers we have thus far studied. This is why the traditional "offset method" has been used here. It is certainly open to debate as to what percent offset is most reasonable, but it may be of help to note that we have used values from 0·3 to 0·9 per cent in this study and have found equivalent correlation between a predicted yield locus and our normalized experimental values. Thus we have selected 0·3 per cent offset for purposes of presentation; similar correlations, using this same definition of yield stress, have been published earlier[1] using glassy amorphous polymers as test materials.

CONCLUSIONS

It would seem from Fig. 5 that the modified von Mises criterion, as expressed in its most general form by equation (1), provides reasonable predictions in regard to the macroscopic yield behavior of high density polyethylene. That this same criterion has been found acceptable where glassy amorphous polymers are used makes it seem unnecessary to use different yield criteria for polymers of varying structural conditions. These observations are correct if the various materials are reasonably isotropic.

There was a degree of anisotropy noted in these three test bars, first by a difference in compressive behavior when tested in the axial and radial directions (which showed greater variation than seen in Fig. 3), and secondly in the difference in axial and hoop tensile yield stress as can be seen for rod 3 in Table 1 and in Fig. 3. It is suggested that this condition could contribute to the slight discrepancy between theory and experiment as seen in Fig. 5; the added effect of anisotropy is being studied currently by the authors.

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REFERENCES

1. R. RAGHAVA, R. M. CADDELL and G. S. Y. YEH, *J. Mat. Sci.* **8**, 225 (1973).
2. W. WHITNEY and R. D. ANDREWS, *J. Polymer. Sci.* (C) **16**, 2981 (1967).
3. S. STERNSTEIN and L. ONGCHIN, *A.C.S. Polymer Prep.* **10**, 1117 (1969).
4. J. C. BAUWENS, *J. Polymer Sci.* (A-2) 8, 893 (1970).
5. R. RAGHAVA, Ph.D. dissertation, University of Michigan (1972).
6. R. M. CADDELL, R. S. RAGHAVA and A. G. ATKINS *Mat. Sci. Engnr*, accepted for publication.
7. P. B. BOWDEN and J. A. JUKES, *J. Mat. Sci.*, **7**, 52 (1972).
8. R. M. CADDELL, T. BATES, JR. and G. S. Y. YEH, *Mat. Sci. Engr* **9**, 223 (1972).
9. J. G. WILLIAMS and H. FORD, *J. mech. Engr Sci.* **6**, 405 (1964).
10. I. M. WARD, *Pol. Sci. Symp.* No. 32, p. 195. Wiley, New York (1971).
11. D. SARDAR, S. V. RADCLIFFE and E. BAER, *Poly. Eng. Sci.* **8**, 290 (1968).
12. F. F. RAWSON and J. G. RIDER, *J. Polymer Sci.* (C) **33**, 87 (1971).
13. D. SHINOZAKI and G. W. GROVES, *J. Mat. Sci.* **8**, 71 (1973).
14. R. M. CADDELL, R. RAGHAVA and A. G. ATKINS, *J. Mat. Sci.*, accepted for publication.
15. R. RAGHAVA, R. M. CADDELL, L. BUEGE and A. G. ATKINS, *J. Macromolec. Sci.* (B6) **4**, 655 (1972).
16. A. W. CHRISTIANSEN, E. BAER and S. V. RADCLIFFE, *Phil. Mag.* **24**, 451 (1971).
17. J. G. WILLIAMS and H. FORD, *J. mech. Engr Sci.* **9**, 362 (1967).
18. L. J. BROUTMAN and S. KALPAKJIAN, *Soc. Plast. Engr J.* **25**, 46 (1969).