

THE UNIVERSITY OF MICHIGAN

7455-5-Q

TECHNICAL REPORT ECOM-01378-6

OCTOBER 1966

Missile Plume Radiation Characteristics

Fifth Quarterly Report

1 August to 31 October 1966

Report No. 5

Contract No. DA 28-043 AMC-01378(E)

Prepared by
M.L. Barasch, J.J. LaRue and C-M Chu
The University of Michigan
Department of Electrical Engineering
Radiation Laboratory
Ann Arbor

For
U.S. Army Electronics Research and Development Activity
White Sands Missile Range
New Mexico

~~1111~~
UMR 0209

THE UNIVERSITY OF MICHIGAN

7455-5-Q

TABLE OF CONTENTS

ABSTRACT	v
I. GENERAL SUMMARY OF RESEARCH STATUS	1
1.1 Statement of Problem	1
1.2 Method of Attack and Division of Problem into Subsidiary Areas	1
1.3 Status of Subsidiary Research Areas	2
II. COMPUTATIONS ON CONTINUUM MECHANISMS INVOLVING FREE ELECTRONS	4
2.1 Volumetric Spectral Density of Power from Electron- Ion Interactions	4
2.2 Effective Absorption Coefficient in the Field of Neutrals	5
2.3 Effective Absorption Coefficient in the Field of Ions	7
2.4 Spectral Density of Volumetric Power from Electron- Ion Bremsstrahlung	8
III. FORMULATION FOR CO LINE AND MODIFICATION OF PREVIOUS CONCLUSIONS ON WATER VAPOR LINE	11
3.1 Introduction	11
3.2 Effective Absorption Coefficient at Line Center	12
3.3 The Spectral Density, $ds/d\omega$, of Volumetric Power from Spontaneous Emission Near Line Center	13
3.4 The Matrix Elements and Population Factors	15
3.5 Correction of Previous Results for Water Vapor	17
REFERENCES	19

THE UNIVERSITY OF MICHIGAN

7455-5-Q

ABSTRACT

This is the fifth quarterly report on Contract DA 28-043 AMC-01378(E), the objective of which is prediction of the intensity of millimeter wave radiation from the exhausts of certain missiles during boost phase. The status of the research is summarized. Preliminary calculations which will be used in evaluating those contributions to power generated and effective absorption coefficients which arise from physical mechanisms associated with free electrons are reported. A formulation required for computation of the intensity at line peak arising from the 115.3 GHz, rotational transition of CO is presented. A consideration which modifies results previously given for the 183.3 GHz line of H₂O also appears here.

THE UNIVERSITY OF MICHIGAN

7455-5-Q

I

GENERAL SUMMARY OF RESEARCH STATUS

1.1 Statement of Problem

The scope of work covers the study of specific missile fuel configurations to determine their theoretical electromagnetic characteristics in the 30-300 GHz frequency range. The radiation characteristics of interest occur during the boost phase for each configuration.

The missile fuel configurations to be studied are: (a) Pershing booster, (b) Nike Hercules booster, (c) Nike Zeus booster, (d) Nike X booster, (e) Honest John, and (f) Sergeant.

The results required from this study include, but are not limited to, the following:

- (a) Spectral distribution of radiation within the 30-300 GHz region.
- (b) Type of spectra involved and the mechanism causing same.
- (c) Estimate of power levels involved.
- (d) Changes in the characteristics during boost phase due to altitude effects, acceleration effects, velocity effects, etc.
- (e) Comparison of theoretical results with known experimental or field measurements available to the Contractor during the contract period.

1.2 Method of Attack and Division of Problem into Subsidiary Areas

Since the goal of this study is essentially a characterization of the missile exhausts as sources of radiative intensity, it was felt that a reasonable

THE UNIVERSITY OF MICHIGAN

7455-5-Q

procedure for accomplishing this task would be to investigate first the manner in which radiative intensity depends on the source. Therefore, a formal solution to the equation of radiative transfer was necessary. This solution was found to contain two parameters characterizing the source, the effective absorption coefficient α and the spectral density S of volumetric power production. It was clearly in order to evaluate and list the mechanisms contributing to S and α next, so that by combining the contributions from all mechanisms, a first-order approximation to the radiative intensity can be predicted. To account for the possible coupling between various mechanisms would, in our judgement, render the problem intractable. As a consequence of the analysis described above, we have investigated the following subsidiary areas:

- (1) The formal solution of the equation of radiative transfer.
- (2) Continuum radiation and absorption associated with free electrons.
- (3) Thermal continuum radiation processes associated with solid particles.
- (4) Processes associated with discrete line spectra.
- (5) Preliminary acquisition of data on missile exhausts.
- (6) Examination of available field measurements of radiation.

1.3 Status of Subsidiary Research Areas.

(1) and (2): No further effort will be, or has been, devoted to areas (1) and (2), except for computations based on the theories pertaining to (2), some of which are presented in Chapter II.

(3): After examination of the literature and contact with other workers whose reports we have received and analyzed, we must conclude that, at this time, no consistent set of theoretical or experimental values for the complex

THE UNIVERSITY OF MICHIGAN

7455 -5-Q

index of refraction and the size distribution of solid carbon and aluminum particles in the exhausts is available. Most work on the index of refraction has been at optical and near-infrared frequencies, from which extrapolation is difficult.

(4): The formulation for the 115.3 GHz line of CO is presented in Chapter III of this report. As for H₂O, some expressions have been given previously, in Quarterly Report 7455-4-Q. However, an expression for power rather than spectral density of power, was presented here. In the present report, we indicate in Chapter III how the conversion may be performed. A technical report (7455-1-T) on this 183.3 GHz line of H₂O is in the final stages of preparation and will be submitted in the near future.

(5): There is little left to do in this area, except for acquiring more information on Sprint.

(6): No further effort in this area is in order.

II

COMPUTATIONS ON CONTINUUM MECHANISMS
INVOLVING FREE ELECTRONS

2.1 Volumetric Spectral Density of Power Production from Electron-Neutral Interactions

By this heading we mean the power per unit volume per unit frequency interval resulting from scattering of electrons by neutral molecules. For this quantity we may employ, after minor regularization and correction of notation, equation (2.36) of Barasch et al, (1962). This equation will be rewritten in the form

$$\frac{ds}{d\omega} = \frac{2}{3} \left(\frac{e^2}{\hbar c} \right) Y^3 \hbar c n_e \sum_{\text{species}} n_{N_i} \sigma_i(Y). \quad (2.1)$$

Here $\left(\frac{8kT}{\pi mc^2} \right)^{1/2}$, the mean thermal electron velocity divided by the velocity of light.

$\frac{e^2}{\hbar c}$: is the "fine structure constant" of physics, $[137.0373]^{-1}$

c : is the velocity of light, $2.998 \cdot 10^{10}$ cm/sec

\hbar : is $1/2\pi \cdot$ Planck's constant, or $1/2\pi \cdot 6.625 \cdot 10^{-27}$ erg sec

m : the electron mass, is $9.108 \cdot 10^{-28}$ g.

$k = 1.38 \cdot 10^{-16}$ erg/degree K

n_{N_i} : is the abundance of the i th molecular species, and n_e the electron density

$\sigma_i(Y)$: is the momentum transfer cross section of the i th species with electrons. which is a function of \bar{v}_e or cY .

Values of $\sigma_i(Y)$ for species expected to be important to us may be found in

THE UNIVERSITY OF MICHIGAN

7455-5-Q

Table IX of Smoot and Underwood (1966). We list some in Table I.

Intending to supply values of T , n_e , and n_{N_1} subsequently for the individual missiles, we have prepared a tabulation of the values of the quantity which, when multiplied by $n_e \sum_i n_{N_1} \sigma_i$ expressed in cgs units, will yield $ds/d\omega$ in units $[\text{cm}^3 \text{ sec}]^{-1}$ /radian per sec, as a function of T . Its temperature dependence, as is evident from the Y^3 factor in equation (2.1), is $T^{3/2}$. With this table, the eventual computations of emergent power will be easier and faster. Values were obtained for $T = [1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 7.5, 8, 8.5, 9, 10,] \cdot 10^{-3} \text{ } ^\circ\text{K}$, a set which was expected to cover most of the range of interest. The result is given in Table II, which represents

$$\frac{2}{3} \left(\frac{e^2}{\hbar c} \right) Y^3 \hbar c \equiv F(T) .$$

Alternatively, the results may be summarized by writing

$$\frac{ds}{d\omega} = 1.099 \cdot 10^9 T^{3/2} n_e \sum_i n_{N_1} \sigma_i(Y)$$

erg $[\text{cm}^3 \text{ sec}]^{-1}$ /radian per sec, with n_e , n_{N_1} , and $\sigma_i(Y)$ expressed in cgs units.

2.2 Effective Absorption Coefficient in the Fields of Neutrals

This quantity is discussed in Olte et al (1962), the results of which may be written

TABLE I
Collision Cross of Electrons with Molecules

Species	$\sigma(\text{cm}^2)$	$T(^{\circ}\text{K})$
CO	$2.08 \cdot 10^{-23} \bar{v}_e + 2.46 \cdot 10^{-16}$	$< 2.5 \cdot 10^4$
CO ₂	$4.7 \cdot 10^{-8} (\bar{v}_e)^{-1}$	$< 10^4$
H ₂ O	$5.9 (\bar{v}_e)^{-2}$	$< 10^4$
HCl	$1.85 (\bar{v}_e)^{-2}$	$< 10^4$

TABLE II
F(T), As defined in Section 2.1

$T \cdot 10^{-3} \text{ } ^{\circ}\text{K}$	1	1.5	2	2.5	3
F(T)	$3.48 \cdot 10^{13}$	$6.38 \cdot 10^{13}$	$9.83 \cdot 10^{13}$	$1.37 \cdot 10^{14}$	$1.80 \cdot 10^{14}$
$T \cdot 10^{-3} \text{ } ^{\circ}\text{K}$	3.5	4	4.5	5	6
F(T)	$2.28 \cdot 10^{14}$	$2.78 \cdot 10^{14}$	$3.32 \cdot 10^{14}$	$3.99 \cdot 10^{14}$	$5.11 \cdot 10^{14}$
$T \cdot 10^{-3} \text{ } ^{\circ}\text{K}$	7	8	8.5	9	10
F(T)	$6.44 \cdot 10^{14}$	$7.86 \cdot 10^{14}$	$8.61 \cdot 10^{14}$	$9.38 \cdot 10^{14}$	$1.10 \cdot 10^{15}$

THE UNIVERSITY OF MICHIGAN

7455-5-Q

$$\alpha(\omega) = \frac{16\sqrt{2}}{3} Y n_e \pi \left(\frac{c}{\omega}\right)^2 r_o \sum_{\text{species}} n_{N_i} F_i(Y), \quad (2.2)$$

in which we have introduced r_o , the "classical electron radius", the value of which is $2.82 \cdot 10^{-13}$ cm. All other symbols retain the definitions of Section 2.1. We have computed the numerical factors in advance, so that

$$\alpha(\omega) = 5.97 \cdot 10^9 \text{ cm}^{-1} \frac{n_e Y}{\omega^2} \sum_{\text{species}} n_{N_i} \sigma_i(Y) \quad (2.3)$$

where n_e , n_{N_i} , $\sigma_i(Y)$, and ω are to be given in cgs units and Y of course is dimensionless. ω is the angular or radian frequency, in units of radians per sec.

2.3 Effective Absorption Coefficient in the Fields of Ions

We base our computations on equations (4.48) and (4.49) of Olte et al, (1962). After much manipulation, we have obtained the following result for this quantity:

$$\alpha(\omega) = \frac{\pi^3}{3} n_e^2 r_o^3 \left(\frac{c}{\omega}\right)^2 Y^{-3} \ln \left[\frac{24\sqrt{2}}{\pi} Y^3 (n_e r_o^3)^{-1/2} \right]. \quad (2.4)$$

We have extracted the factor $n_e^{(2)} (c/\omega)^2$ for subsequent multiplication. If these quantities are employed in cgs units, they will yield $\alpha(\omega)$ in units of cm^{-1} after multiplication by the quantity we have computed as a function of n_e and T . This quantity, which we shall denote by $I(n_e, T)$ is recorded

2.4 Spectral Density of Volumetric Power Production from Electron-Ion Bremsstrahlung

We have found that relatively complex expressions derived in this laboratory to be in satisfactory agreement with the simpler forms given by Bekefi and Brown (1961). Accordingly, computations will be based on equations 12 and 14 of their paper. We have already obtained values of the Gaunt factor G for $\omega_{\rho} \ll \omega$, as given in their equation (1), and here in equation (2.5), which we have obtained from (14).

$$G = \frac{3}{\pi} \ln \left[\frac{4\pi\epsilon_0}{e^2} m \left(\frac{3kT}{m} \right)^{3/2} \omega^{-1} \right] \quad (2.5)$$

Values of ω corresponding to 30-300 GHz and the temperature range previously employed were inserted. The resulting values of G , which is a pure numeric, are given in Table IV. This Gaunt factor is to be inserted into equation (12) of Bekefi and Brown, which we reproduce here as equation (2.6), after some modification to allow for neutrality and the absence of multiple ionization.

($N_e = N_{ion}$ and $Z = 1$).

$$dS = 1.09 \cdot 10^{-51} N_e^2 T^{-1/2} G d\omega \quad (2.6)$$

The power calculated from (2.6) is in units of watts / meter³ if N_e is given in meter⁻³ and T in °K.

TABLE III
Values of Factor Appearing in Absorption Coefficient
for Electron-Ion Free-Free Transitions

$10^{-3}T(^{\circ}K)$	n_e (cm^{-3}) 10^4	10^5	10^6	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
1.0	$2.00 \cdot 10^{-34}$	$1.86 \cdot 10^{-34}$	$1.72 \cdot 10^{-34}$	$1.58 \cdot 10^{-34}$	$1.44 \cdot 10^{-34}$	$1.30 \cdot 10^{-34}$	$1.16 \cdot 10^{-34}$	$1.01 \cdot 10^{-34}$	$8.73 \cdot 10^{-35}$
1.5	$1.13 \cdot 10^{-34}$	$1.05 \cdot 10^{-34}$	$9.77 \cdot 10^{-35}$	$9.00 \cdot 10^{-35}$	$8.23 \cdot 10^{-35}$	$7.46 \cdot 10^{-35}$	$6.70 \cdot 10^{-35}$	$5.93 \cdot 10^{-35}$	$5.16 \cdot 10^{-35}$
2.0	$7.53 \cdot 10^{-35}$	$7.03 \cdot 10^{-35}$	$6.53 \cdot 10^{-35}$	$6.03 \cdot 10^{-35}$	$5.53 \cdot 10^{-35}$	$5.03 \cdot 10^{-35}$	$4.54 \cdot 10^{-35}$	$4.04 \cdot 10^{-35}$	$3.54 \cdot 10^{-35}$
2.5	$5.50 \cdot 10^{-35}$	$5.13 \cdot 10^{-35}$	$4.78 \cdot 10^{-35}$	$4.42 \cdot 10^{-35}$	$4.06 \cdot 10^{-35}$	$3.71 \cdot 10^{-35}$	$3.35 \cdot 10^{-35}$	$3.00 \cdot 10^{-35}$	$2.63 \cdot 10^{-35}$
3.0	$4.24 \cdot 10^{-35}$	$3.97 \cdot 10^{-35}$	$3.70 \cdot 10^{-35}$	$3.42 \cdot 10^{-35}$	$3.16 \cdot 10^{-35}$	$2.88 \cdot 10^{-35}$	$2.61 \cdot 10^{-35}$	$2.34 \cdot 10^{-35}$	$2.07 \cdot 10^{-35}$
3.5	$3.41 \cdot 10^{-35}$	$3.19 \cdot 10^{-35}$	$2.98 \cdot 10^{-35}$	$2.76 \cdot 10^{-35}$	$2.55 \cdot 10^{-35}$	$2.33 \cdot 10^{-35}$	$2.12 \cdot 10^{-35}$	$1.90 \cdot 10^{-35}$	$1.69 \cdot 10^{-35}$
4.0	$2.82 \cdot 10^{-35}$	$2.64 \cdot 10^{-35}$	$2.47 \cdot 10^{-35}$	$2.29 \cdot 10^{-35}$	$2.12 \cdot 10^{-35}$	$1.94 \cdot 10^{-35}$	$1.76 \cdot 10^{-35}$	$1.59 \cdot 10^{-35}$	$1.41 \cdot 10^{-35}$
4.5	$2.39 \cdot 10^{-35}$	$2.24 \cdot 10^{-35}$	$2.09 \cdot 10^{-35}$	$1.94 \cdot 10^{-35}$	$1.80 \cdot 10^{-35}$	$1.65 \cdot 10^{-35}$	$1.50 \cdot 10^{-35}$	$1.35 \cdot 10^{-35}$	$1.20 \cdot 10^{-35}$
5.0	$2.06 \cdot 10^{-35}$	$1.93 \cdot 10^{-35}$	$1.80 \cdot 10^{-35}$	$1.68 \cdot 10^{-35}$	$1.55 \cdot 10^{-35}$	$1.42 \cdot 10^{-35}$	$1.30 \cdot 10^{-35}$	$1.17 \cdot 10^{-35}$	$1.05 \cdot 10^{-35}$
6.0	$1.59 \cdot 10^{-35}$	$1.49 \cdot 10^{-35}$	$1.39 \cdot 10^{-35}$	$1.30 \cdot 10^{-35}$	$1.20 \cdot 10^{-35}$	$1.12 \cdot 10^{-35}$	$1.01 \cdot 10^{-35}$	$9.14 \cdot 10^{-36}$	$8.18 \cdot 10^{-36}$
7.0	$1.27 \cdot 10^{-35}$	$1.20 \cdot 10^{-35}$	$1.12 \cdot 10^{-35}$	$1.05 \cdot 10^{-35}$	$9.69 \cdot 10^{-36}$	$8.93 \cdot 10^{-36}$	$8.17 \cdot 10^{-36}$	$7.41 \cdot 10^{-36}$	$6.65 \cdot 10^{-36}$
7.5	$1.16 \cdot 10^{-35}$	$1.09 \cdot 10^{-35}$	$1.02 \cdot 10^{-35}$	$9.49 \cdot 10^{-36}$	$8.80 \cdot 10^{-36}$	$8.12 \cdot 10^{-36}$	$7.43 \cdot 10^{-36}$	$6.74 \cdot 10^{-36}$	$6.05 \cdot 10^{-36}$
8.0	$1.05 \cdot 10^{-35}$	$9.91 \cdot 10^{-36}$	$9.29 \cdot 10^{-36}$	$8.67 \cdot 10^{-36}$	$8.04 \cdot 10^{-36}$	$7.42 \cdot 10^{-36}$	$6.80 \cdot 10^{-36}$	$6.17 \cdot 10^{-36}$	$5.55 \cdot 10^{-36}$
8.5	$9.67 \cdot 10^{-36}$	$9.10 \cdot 10^{-36}$	$8.53 \cdot 10^{-36}$	$7.96 \cdot 10^{-36}$	$7.39 \cdot 10^{-36}$	$6.82 \cdot 10^{-36}$	$6.25 \cdot 10^{-36}$	$5.68 \cdot 10^{-36}$	$5.11 \cdot 10^{-36}$
9.0	$8.91 \cdot 10^{-36}$	$8.39 \cdot 10^{-36}$	$7.87 \cdot 10^{-36}$	$7.34 \cdot 10^{-36}$	$6.82 \cdot 10^{-36}$	$6.30 \cdot 10^{-36}$	$5.78 \cdot 10^{-36}$	$5.25 \cdot 10^{-36}$	$4.73 \cdot 10^{-36}$
10.0	$7.67 \cdot 10^{-36}$	$7.22 \cdot 10^{-36}$	$6.78 \cdot 10^{-36}$	$6.33 \cdot 10^{-36}$	$5.88 \cdot 10^{-36}$	$5.44 \cdot 10^{-36}$	$4.99 \cdot 10^{-36}$	$4.55 \cdot 10^{-36}$	$4.10 \cdot 10^{-36}$

TABLE IV
Gaunt Factor in Electron-Ion Bremsstrahlung

ν (GHz)	30	60	120	150	200	250	300
1000	2.93	2.55	2.17	2.04	1.88	1.76	1.66
1500	3.27	2.88	2.50	2.38	2.22	2.10	2.00
2000	3.50	3.12	2.74	2.62	2.46	2.33	2.23
2500	3.69	3.31	3.92	2.80	2.64	2.52	2.42
3000	3.84	3.46	3.08	2.95	2.79	2.67	2.57
3500	3.97	3.58	3.20	3.08	2.92	2.80	2.70
4000	4.08	3.69	3.31	3.19	3.03	2.91	3.81
4500	4.17	3.79	3.41	3.29	3.13	3.01	2.90
5000	4.26	3.88	3.50	3.37	3.26	3.09	2.99
6000	4.41	4.03	3.65	3.52	3.37	3.24	3.14
7000	4.54	4.16	3.78	3.65	3.49	3.37	3.27
7500	4.60	4.21	3.83	3.71	3.55	3.43	3.33
8000	4.65	4.27	3.89	3.76	3.60	3.48	3.38
8500	4.70	4.32	3.94	3.81	3.65	3.53	3.43
9000	4.75	4.37	3.98	3.86	3.70	3.58	3.48
10000	4.83	4.45	4.07	3.95	3.79	3.67	3.56

III

FORMULATION FOR CO LINE AND MODIFICATION OF PREVIOUS
CONCLUSIONS ON WATER VAPOR LINE

3.1 Introduction

The $J = 1 \rightarrow 0$ rotational transition of CO occurs at 115.3 GHz, as observed by Gilliam, Johnson and Gordy (1950), whose work is the source for the listing in the tabulation of Kisliuk and Townes (1952). Since the frequency is within the range of interest, we shall consider expressions governing the radiative intensity due to this line. This intensity depends on both the effective absorption coefficient, α' , and the spectral density of volumetric power production by spontaneous emission, $ds/d\omega$. These quantities are both dependent on line shape and line width. Although no information on line shape and width appears to be available in the literature, a simple estimate indicates that pressure broadening should be the dominant effect. In such a situation, expressions for the effective absorption coefficient at line center are well known (see, for example, equation 1.52) of Townes and Schawlow, 1952), and will be employed. As for the spectral density of volumetric power production, we find it necessary to derive an expression for its value at line center, considering the effect of line-broadening. In this chapter, the expressions for α' and $ds/d\omega$ will be given, together with the derivation of the latter, and values of the molecular constants of CO which enter the expressions. Since it is not obvious that the maximum intensity is radiated at the nominal or center frequency of the line, we may digress here to support this conclusion and thus justify the evaluation of $ds/d\omega$ and α' at this frequency for the purposes of this contract. It may be recalled that equations (2.13) and (2.14) of The Third Quarterly Report 7455-3-Q indicated expressions for spectral power intercepted per unit receiver area which were written as

$$\frac{P_{\omega}}{A} = \frac{S}{\alpha} \iint d\theta d\phi f(\theta, \phi) [1 - \exp(-\alpha d_{\Omega})] \quad . \quad (3.1)$$

Replacing the symbol S used in Reports 7455-2-Q and 7455-3-Q for the spectral density by the more appropriate notation $ds/d\omega$, (3.1) becomes

$$\frac{P_{\omega}}{A} = \frac{ds/d\omega}{\alpha} \iint d\theta d\phi f(\theta, \phi) [1 - \exp(-\alpha d_{\Omega})] \quad . \quad (3.2)$$

Since $ds/d\omega$ and α have the same line-shape factor, their ratio is frequency-independent. Now, the factor $1 - \exp(-\alpha d_{\Omega})$ inside the integral increases with increasing α , and is therefore a maximum at line-center, which is thus the frequency of maximum P_{ω}/A . With this preamble, we embark on the task of supplying expressions for $ds/d\omega$ and α .

3.2 Effective Absorption Coefficient at Line Center

This quantity is given by equation (1.52) of Townes and Schawlow (1952), under the approximation $h\nu_0/kT \ll 1$ which is valid for us at temperatures of 1000°K or higher, by the following expression:

$$\alpha' = \frac{8\pi^2 N f_L |\mu|^2 \nu_0^2}{3ckT \Delta\nu} \quad . \quad (3.3)$$

In (3.3) the symbols have the following significance, and will be discussed in more detail in a later section.

- ν_0 the center frequency, 115.3 GHz,
- $|\mu|$ the dipole moment matrix element for the transition,
- N the number of CO molecules present per unit volume,
- f_L the fraction of these molecules occupying the lower or originating state for this absorption transition,
- c, k the usual physical constants,
- T gas temperature, assumed identical with rotational and vibrational temperatures.
- $\Delta\nu$ the half-width at half maximum of the line.

3.3 The Spectral Density, $d\dot{s}/d\omega$, of Volumetric Power from Spontaneous Emission Near Line Center

By this phrase we mean the power per unit volume and per unit angular frequency interval originating in spontaneous downward transitions between molecular states. First of all, let us consider the dominant cause of line broadening, in order to derive a line-shape factor. For Doppler broadening, as stated in equation (3.94) of Aller (1963), one has

$$\frac{\Delta\lambda}{\lambda} = 7.16 \cdot 10^{-7} \left[\frac{T}{\mu} \right]^{1/2} \tag{3.4}$$

where μ is the molecular weight, which is 28 for the most abundant isotropic continuation, $C^{12}O^{16}$. Now

$$\left| \frac{\Delta\nu}{\nu} \right| = \left| \frac{\Delta(c/\lambda)}{c/\lambda} \right| = \frac{c\lambda^2 \Delta\lambda}{c\lambda} = \frac{\Delta\lambda}{\lambda} = 7.16 \cdot 10^{-7} \left[\frac{T}{\mu} \right]^{1/2}, \quad (3.5)$$

so that the Doppler half-width for this line of CO for T of the order of 1000°K is approximately 1 MHz. As for collision broadening, the line width does not seem to appear in the literature, and so it is justified to adopt the procedure of Kisliuk and Townes (1952), according to which $\Delta\nu$ is taken as 25 MHz/mm of Hg in the absence of more specific information. At pressures of only 10^{-1} atm., this estimate is equivalent to a value of over 1 GHz for $\Delta\nu$. We conclude, therefore, that it is a good approximation to consider the line shape as determined by collision broadening alone. In such a case, the line shape is of the form (Aller, 1963, eq. 7.30)

$$\frac{ds}{d\nu} = \frac{A}{2\pi^2} \frac{1}{(\nu+\nu_0)^2 + (\Delta\nu)^2} \quad (3.6)$$

in which A is to be determined by equating the total power in the line to that which would be predicted in the absence of broadening. That is,

$$\frac{A}{2\pi^2} \int_{-\infty}^{\infty} \frac{d\nu}{(\nu-\nu_0)^2 + (\Delta\nu)^2} = N f_U h \nu_0 P \quad (3.7)$$

where f_U is the population fraction in the upper or originating state and P the probability per second per molecule for spontaneous emission by a molecule in the originating (J=1) state. In general, for dipole transitions such as this one, P is given (Schiff, 1949, p. 255) by

$$P = \frac{4}{3} \frac{\omega^3}{\hbar c^3} |\mu|^2, \quad (3.8)$$

where $|\mu|$ is the dipole moment matrix element for the upward transition. The integration indicated in (3.7) is elementary, and we find

$$A = N \frac{\Delta\nu}{\pi} f_U h\nu_0 P, \quad (3.9)$$

whence

$$\frac{ds}{d\nu} = \frac{\Delta\nu}{\pi} N f_U h\nu_0 P \frac{1}{(\nu - \nu_0)^2 + (\Delta\nu)^2} \quad (3.10)$$

and near the line center, where $\nu \rightarrow \nu_0$, we obtain the spectral density

$$\frac{ds}{d\nu} = \frac{N}{\pi \Delta\nu} f_U h\nu_0 P. \quad (3.11)$$

3.4 The Matrix Elements and Population Factors

In (3.3), the squared dipole moment matrix element for $J=0 \rightarrow 1$ may be obtained from Townes and Schawlow (1933, eq. 1.76): its value is

$$|\mu|^2 = \mu^2 \frac{J+1}{2J+1}$$

where $J = 0$ and μ is the static dipole moment, or in other words, for absorption we have

$$|\mu|^2 = \mu^2. \quad (3.12)$$

However, for emission, as indicated by the reference just cited, we have

$$|\mu|^2 = \mu^2 \frac{J+1}{2J+1}$$

with $J = 1$, or

$$|\mu|^2 = 2/3 \mu^2 \tag{3.13}$$

The value given by (3.13) is to be employed in (3.8) and the subsequent equations in which P appears, and which therefore involve the value of $|\mu|^2$ for emission.

The population factors f_U and f_L are given, to a good approximation, by the product of a vibrational and rotational factor. The rotational occupation probability factor will be considered first. It is derived from the following equation:

$$f_J^R = (2J+1) hB/kT \exp \left[-J(J+1) hB/kT \right], \tag{3.14}$$

where B is the molecular rotational constant. For f_U we take $J = 1$ and f_L corresponds to $J = 0$, so that

$$\begin{aligned} f_U^R &= 3hB/kT \exp \left[-2 hB/kT \right] \\ f_L^R &= hB/kT . \end{aligned} \tag{3.15}$$

These dipole moment matrix elements and rotational occupation probabilities apply strictly to a rigid rotator, in that vibrational state structure has been ignored. A consideration of the vibrational population factors does indicate

THE UNIVERSITY OF MICHIGAN
7455-5-Q

an appreciable occupation of the lower excited vibrational levels as well as the ground one, at temperatures we expect to encounter. However, the vibration-rotation interaction may be calculated as quite small, so that the values given by equations (3.13) and (3.15) should apply fairly well for the transition we are interested in, giving the f and $|\mu|^2$ values with accuracy sufficient for our purposes. The molecular constants still to be defined have the following values for $C^{12}O^{16}$ (taken from Townes and Schawlow):

$$\mu = 10^{-19} \text{ esu cm.}$$

$$B = 57,897.5 \text{ MHz.}$$

3.5 Correction of Previous Results for Water Vapor

In the last quarterly report on this contract (7455-4-Q), the value was furnished for " $S(T, P_W)$ ", the source intensity, in equation (2.2) of that report. No error has been discovered in that result, but it is now realized that a spectral density rather than S itself is the quantity of interest. As may be seen from equation (3.11) of this report, a simple division by $\pi \Delta \nu$ will convert from S to spectral density. The line-width for H_2O was given in (2.3) of 7455-4-Q by

$$\Delta \bar{\nu} = 0.1 P \sqrt{\frac{273}{T}} \text{ cm}^{-1},$$

which corresponds to

$$\Delta \nu = 3 P \sqrt{\frac{273}{T}} \text{ GHz} . \tag{3.16}$$

THE UNIVERSITY OF MICHIGAN

7455-5-Q

Thus, we recommend that ds/dv near the line center for the 183 GHz line of H_2O be obtained by dividing the value of (2.2) from 7455-4-Q by $3\pi P \sqrt{\frac{273}{T}}$ GHz, where P is the pressure in atmosphere, and T is expressed in $^{\circ}K$.

REFERENCES

- Aller, L. H. (1963), Astrophysics - The Atmospheres of the Sun and Stars, 2nd Ed., Ronald Press Company, New York.
- Barasch, M. L. et al (1962), "Radiation Mechanisms in the Exhaust of Rockets and Ion Propulsion Systems," The University of Michigan Radiation Laboratory Report 3572-2-F, AD 328 887. SECRET
- Bekefi, G. and S. C. Brown (1961), "Emission of Radio-Frequency Waves from Plasmas," Am. J. Phys., 29, No. 7, 404.
- Gilliam, O. R., C. M. Johnson and W. Gordy (1950), "2- to 3-mm Spectroscopy," Phys. Rev., 78, 140.
- Kisliuk, P. and C. H. Townes (1952), "Molecular Microwave Spectra Tables," NBS Circular No. 518.
- Olte, A. et al (1962), "An Investigation of the Absorption of Electromagnetic Radiation by a Plasma of Sub-critical Density," The University of Michigan Radiation Laboratory Report 4386-2-F, AD 274 014. UNCLASSIFIED.
- Schiff, L. I. (1949), Quantum Mechanics, McGraw-Hill Book Company, New York.
- Smoot, L. D. and D. L. Underwood (1966), "Prediction of Microwave Attenuation Characteristics of Rocket Exhausts," J. Spacecraft, 3, No. 3, 302.
- Townes, C. H. and A. L. Schawlow (1955), Microwave Spectroscopy, McGraw-Hill Book Company, New York.

UNIVERSITY OF MICHIGAN



3 9015 02227 1475