THE UNIVERSITY OF MICHIGAN

7455-4-Q

TECHNICAL REPORT ECOM-01378-4 JULY 1966

Missile Plume Radiation Characteristics

Fourth Quarterly Report

1 April to 31 July 1966

Report No. 4

Contract No. DA 28-043 AMC-01378(E)

Prepared by
M. L. Barasch, J. J. LaRue, C-M Chu and K. Nishina
The University of Michigan
Department of Electrical Engineering
Radiation Laboratory
Ann Arbor

For

U. S. Army Electronics Research and Development Activity
White Sands Missile Range
New Mexico

enm
UMR0208

TABLE OF CONTENTS

ABSTRACT

This is the fourth quarterly report on Contract DA 28-043 AMC-01378(E), the objective of which is the prediction of intensity of millimeter wave radiation from the exhausts of certain missiles during boost phase. Contributions to the absorption coefficient and volumetric power production function from the mechanisms of Bremsstrahlung in the fields of neutrals, and from thermal radiation by small solid particles, with their inverse absorption processes, are treated. Some results for the 183.3 GHz line corresponding to the transition $3_{1,3} \leftrightarrow 2_{2,0}$ of water vapor are presented. A summary of the work statement, general plan of attack, and status of the research is given.

I
GENERAL SUMMARY OF RESEARCH STATUS

1.1 Statement of Problem

The scope of work covers the study of specific missile fuel configurations to determine their theoretical electromagnetic characteristics in the 30-300 GHz frequency range. The radiation characteristics of interest occur during the boost phase for each configuration.

The missile fuel configurations to be studied are: (a) Pershing booster, (b) Nike Hercules booster, (c) Nike Zeus booster, (d) Nike X booster, (e) Honest John, and (f) Sergeant.

The results required from this study include, but are not limited to, the following:

(a) Spectral distribution of radiation within the 30-300 GHz region.
(b) Type of spectra involved and the mechanism causing same.
(c) Estimate of power levels involved.
(d) Changes in the characteristics during boost phase due to altitude effects, acceleration effects, velocity effects, etc.
(e) Comparison of theoretical results with known experimental or field measurements available to the Contractor during the contract period.

1.2 Method of Attack and Division of Problem into Subsidiary Areas

Since the goal of this study is essentially a characterization of the missile exhausts as sources of radiative intensity, it was felt that a reasonable procedure for accomplishing this task would be to investigate first the manner in which radiative intensity depends on the nature of the source, in other words the formal solution of the radiative transfer equation. This solution was found to depend on the parameters of the source through two quantities, the effective absorption coefficient α and the volumetric power production function S. It was therefore in order to enumerate and evaluate the contributions, from various physical mechanisms within the exhaust, to these two quantities. Both continuum and line spectra are of interest. Variation between different missiles, or with altitude, is most conveniently investigated after the problems discussed above have been solved, or computations performed for a fixed altitude, or for a certain missile, since then the physical parameters of major importance (e.g. these could be electron density, abundance of solid particles, temperature, pressure, abundance of certain molecules) would have been selected from the list of all possible parameters. Furthermore, the numerical solution of the radiative transfer equation may then be derived by scaling the solution at a fixed altitude or for a given missile according to the formal dependence of the solution on physical parameters, as determined by the physical mechanisms and the quantities appearing in the expressions for S and α corresponding to the most important mechanisms. For this reason, the research to this date has been concentrated on the sections (a), (b) and (c) of the work statement. Specifically, the following subsidiary areas were investigated:

(1) The formal solution of the equation of radiative transfer.

(2) Processes involving emission and absorption of radiation in the continuum.

(3) Processes associated with absorption and emission lines.

(4) Preliminary acquisition of data on missile exhausts.

(5) Examination of available field measurements.

1.3 Status of Subsidiary Areas

(1) The formal solution to the equation of radiative transfer has been obtained, and has been presented in report 7455-3-Q.

(2) Bremsstrahlung in the field of ions was reported on in 7455-3-Q. Bremsstrahlung with neutrals is discussed in Section IV of the present report, where expressions for S and α are given. Values of the parameters appearing in these expressions will be obtained for subsequent computation. The theoretical analysis for thermal radiation from small solid particles is given in section III of the present report. Values

of abundance, size distribution and electrical constants for whatever particles are actually present will be necessary for computations.

(3) The analysis of the line at 183.3 GHz which corresponds to the transition $3_{1,3} \leftrightarrow 2_{2,0}$ of the water molecule has been completed. Some numerical results appear in Section II and it is planned to submit a draft of the entire theoretical investigation to the sponsor for approval of publication as a separate technical report. The analysis for the CO line at 115.3 GHz arising from the rotational transition $J=0 \leftrightarrow 1$ continues to be deferred, since it will be comparatively simple.

(4) Much of the data acquisition has been completed, and contacts which are expected to fill in some remaining gaps have been initiated by mail.

(5) The actual comparison with experimental data will be possible only after computations have been completed. However, a combination of propellant information with information on the sponsor's field measurements has been employed as a guide to theoretical investigations, in the sense that it is a rough indication of what radiation generating mechanisms may be important. With this comparison to lend direction to the theoretical model, it is hoped that agreement between our model and the field measurements will be satisfactory.

II
INTENSITY OF THE 183.3 GHz LINE OF WATER VAPOR

2.1 Introduction and Brief Summary of Work

The existence of an absorption line at 183.3 GHz for water vapor was first reported by King and Gordy (1954). They attribute it to the rotational transition $3_{1,3} \leftrightarrow 2_{2,0}$ of this asymmetrical rotator molecule, on the basis of frequency and absorption coefficient calculated by King et al (1947). Because of the complexity of matrix elements for such a molecule, and a slight inconsistency, large enough to affect our computations, in values reported for the constants of this molecule, it was found necessary to reperform the work of King et al, in order to select a consistent set of parameters, confirm the resolution of certain ambiguities in the notation for the energy levels and wave functions of this type of molecule, and to establish that the molecule is sufficiently closer to the prolate limit than the oblate limit so that certain perturbation and interpolation procedures were justified. This preliminary investigation, which was lengthy and laborious, having been done, a calculation of the absorption coefficient at line peak, and the power per unit volume radiated by spontaneous emission at this frequency were performed in the dipole approximation, which is admissible here since the transition turns out to be allowed. (There had been some question in our minds regarding this point previously.) If the sponsor, to whom a rough draft will be submitted, approves, the details of this investigation will be published as a separate technical report. Some results will be presented below.

2.2 Values of S and α at Line Peak Frequency

The peak effective absorption coefficient in units of cm^{-1} is given as a function of absolute temperature T, and the ratio P_w/P_t of partial pressure of water to total pressure of gas, which is essentially the fractional abundance of water in the gas, by equation (2.1):

$$\alpha\left(T, \frac{P_w}{P_t}\right) = 6838 \cdot 10^{-6} \frac{P_w}{P_t} \left(\frac{287.7}{T}\right)^3 \frac{e^{-\frac{W_{2_{2,0}}}{kT}}}{0.5064} \tag{2.1}$$

Here $W_{2_{2,0}} = hc \cdot 136.15\,cm^{-1}$. By describing α as "effective", we mean that it has been corrected for stimulated emission, and is thus the correct quantity to use in the radiative transfer equation, the one in terms of which the solution has been given in a previous quarterly report on this contract.

The volumetric source intensity at line peak arising from spontaneous emission is given in units of erg sec^{-1}cm^{-3} by

$$S(T, P_w) = 5.87 \cdot 10^{-4} \left(\frac{287.7}{T}\right)^{5/2} \frac{e^{-\frac{W_{3_{1,3}}}{kT}}}{0.4908} P_w \ . \tag{2.2}$$

Here P_w, the partial pressure of water vapor, is to be measured in atmospheres. We have also the value

$$W_{3_{1,3}} = hc \cdot 142.30\,cm^{-1} \ .$$

A quantity of some interest may be the line width. This seems to be determined mainly by collision broadening, and is given in terms of wave-number spread $\Delta\bar{\nu}$ by

$$\Delta\bar{\nu} = 0.1\,cm^{-1} P_t \sqrt{\frac{273}{T}} \ , \tag{2.3}$$

in which P_t is to be expressed in units of atmospheres.

III
THE THERMAL CONTINUUM RADIATED BY AN ENSEMBLE OF SMALL SOLID PARTICLES

3.1 Expression of S and α in Terms of Cross Sections

Since the exhausts of many rocket propellants contain solid particles (carbon or alumina), the radiation from such particles must be investigated under the present contract. Although we have not yet obtained values of parameters such as abundance, size distribution, and index of refraction, we considered it appropriate to construct the theoretical framework into which these values will be inserted in order to obtain numerical solutions of the radiation transfer equation.

First of all, we may relate the source function S and coefficient of an ensemble containing n particles per unit volume, the total cross section and absorption cross section of which are denoted by σ_t and σ_a, are given respectively by equations (3.1) and (3.2), which follow:

$$\frac{dS}{d\nu} = nI_o(T,\nu)\sigma_a(\nu) \qquad \text{(see Stull and Plass, 1960)} \qquad (3.1)$$

$$\alpha = n\sigma_t(\nu) \ . \qquad (3.2)$$

Here $I_o(T,\nu)$ is the black body intensity per sterad in frequency interval $d\nu$, for temperature T.

3.2 Cross Sections in the Rayleigh Limit

The next requirement is that the cross sections σ_t and σ_a be expressed in terms of particle size, wavelength, and index of refraction of the material. These quantities are conveniently combined into the "electrical size parameter", ρ, which is defined by

$$\rho = k_2 a = 2\pi a/\lambda \qquad (3.3)$$

in which a is the particle radius, λ the wavelength in the particle, that is,

$$\rho = k_2 a = \frac{k_1 a}{N} = \frac{2\pi a}{N\lambda_o} \tag{3.4}$$

where k_2 and k_1 are the propagation constants within the particle and in free space, N is the complex index of refraction of the substance forming the particle, and λ_o is the free-space wavelength.

For the wavelengths we deal with, it is expected that ρ will be very small (in the absence of specific information on particle size distribution, we may be guided by the literature on flames, in which a, for soot, is roughly 10^3 Å as indicated by many experimenters (Erickson et al, 1964). It is also established (Fein, 1966) that particles of aluminum oxides in the exhausts are small enough for the Rayleigh limit to be valid, unless the index of refraction exhibits anomalous behavior. This possibility is being checked on; meanwhile it is appropriate to cite here the expressions for σ_t and σ_a in the Rayleigh limit. They are, respectively:

$$\sigma_t \cong \frac{6\pi}{k_2^2} \mathrm{Re}(b_1^r) , \tag{3.5}$$

and

$$\sigma_a \cong \frac{6\pi}{k_2^2} \left| b_1^r \right|^2 , \tag{3.5}$$

in which

$$b_1^r = - \frac{2i}{3} \left(\frac{N^2 - 1}{N^2 + 3} \right) \rho^3 \tag{3.6}$$

IV
BREMSSTRAHLUNG AND FREE-FREE ABSORPTION IN THE FIELD OF NEUTRALS

This effect gives rise to a continuous radiation background and its inverse process, free-free absorption in the field of neutrals, must be considered among contributions to the absorption coefficient. Although the expressions for S and α derived in previous reports (Barasch et al, 1962) are cumbersome and contain a parameter, $\sigma(0)$, difficult to evaluate for the various species of neutrals, it is possible to simplify them both by introducing asymptotic forms for the modified Hankel functions and by expressing $\sigma(0)$ in terms of collision frequencies between electrons and neutrals. These simplifications are discussed in Olte et al (1962). The resulting expressions for the effective absorption coefficient and the spectral density of volumetric power production are defined respectively by:

$$\alpha = \frac{4\sqrt{2}}{3} \frac{\nu_c}{c} (\omega_p/\omega)^2 \tag{4.1}$$

and

$$\frac{dS}{d(\hbar\omega)} = \frac{16 n_e}{3\pi} \frac{e^2}{\hbar c} \nu_c \left(\frac{\theta}{\mu}\right) e^{-\frac{\hbar\omega}{2\theta}} \quad . \tag{4.2}$$

In these two equations, the symbols have the following meanings: e, m, $\hbar$, c are the charge and mass of the electron, $\frac{1}{2\pi}$ · Planck's constant, and the velocity of light. ν_c is the collision frequency between electrons and neutrals, which will probably be obtained by reference to the experimental literature, θ is kT_e, the electron temperature in energy units, μ the electron rest energy mc^2, ω_p the plasma frequency and ω the angular frequency of the radiometric observation, n_e the electron density. Since ν_c is proportional to the density n_N of neutrals and ω_p^2 to n_e, both α and $dS/d(\hbar\omega)$ are proportional to $n_N n_e$, as they should be. Their temperature dependence is not strong in this range of parameters, and the frequency dependence of α is inverse square, as evident from (4.1). For the purpose of evaluating ν_c, an approximation not worse than the spread of values of experimental collision cross sections is obtained by neglecting the velocity variation of the cross sections σ_c, and assuming them to be valid at the average thermal electron velocity v_e, and also writing the relation between ν_c and σ_c in the form

$$\nu_c = \overline{v}_e \sigma_c \quad , \tag{4.3}$$

where

$$\overline{v}_e = \left(\frac{8\theta}{\pi m}\right)^{1/2} \quad . \tag{4.4}$$

The values of σ_c to be used will not be given here, but will be determined when needed for computations. It should, however, be noted here that (4.4) introduces another implicit variation with T_e into (4.1) and (4.2).

REFERENCES

Barasch, M. L., Z. Akcasu, E. G Fontheim, S. N. Samaddar and H. Weil (1962), "Radiation Mechanisms in the Exhaust of Rockets and Ion Propulsion Systems," The University of Michigan Radiation Laboratory Report No. 3572-2-F, AD 328 887. SECRET

Erickson, Williams and Hottel (1964), "Light Scattering Measurements on Soot in a Benzene-Air Flame," Combustion and Flame, 8, p. 127.

Fein, H. L. (1966), "Predicting AlO_2 Particle Size Distributions," AIAA J., 4, p. 92.

King, Hainer and Cross (1947), "Expected Microwave Absorption Coefficients of Water and Related Molecules," Phys. Rev., 71, p. 433.

King and Gordy (1954, "One-to-Two Millimeter Spectroscopy," Phys. Rev., 93, p. 407.

Olte, A., M. L. Barasch, E. G. Fontheim, A. E. Hines, J. S King and E. K. Miller (1962), "An Investigation of the Absorption of Electromagnetic Radiation by a Plasma of Sub-critical Density," The University of Michigan Radiation Laboratory Report No. 4386-2-F, AD 274 014. UNCLASSIFIED

Stull, V. R. and G. N. Plass (1960), "Emissivity of Dispersed Carbon Particles," J. Opt. Soc. Amer., 50, p. 121.