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Missile Plume Radiation Characteristics

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Prepared by
M. L. Barasch, J. J. LaRue and C-M Chu
The University of Michigan
Department of Electrical Engineering
Radiation Laboratory
Ann Arbor

For
U.S. Army Electronics Research and Development Activity
White Sands Missile Range
New Mexico

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ABSTRACT

This is the third quarterly report on Contract DA 28-043 AMC-01378(E), the objective of which is the prediction of intensity of millimeter wave radiation from the exhausts of certain rockets during boost phase. A formal solution for the radiative transfer equation is presented corresponding to a source which includes the features believed essential to the situations of interest here. Some expressions relating to the generation and absorption of power in the continuum are given here. A discussion of the plan of attack and of the progress in the subsidiary areas of research into which the problem has been divided is given.

I

APPROACH EMPLOYED AND SUMMARY OF PROGRESS

1.1 Statement of Problem

This scope of work covers the study of specific missile fuel configurations to determine their theoretical electromagnetic characteristics in the 30 - 300 Gcs frequency range. The radiation characteristics of interest occur during the boost phase for each configuration.

The missile fuel configurations to be studied are; a) Pershing booster, b) Nike Hercules booster, c) Nike Zeus booster, d) Nike X booster, e) Honest John, and f) Sergeant.

The results required from this study include, but are not limited to, the following:

a) Spectral distribution of radiation within the 30 - 300 Gcs region.
b) Type of spectra involved and the mechanism causing same.
c) Estimate of power levels involved.
d) Changes in the characteristics during boost phase due to altitude effects, acceleration effects, velocity effects, etc.
e) Comparison of theoretical results with known experimental or field measurements available to the Contractor during the contract period.

1.2 Method of Attack

To solve a problem of this type it is necessary, first of all, to obtain the form of the pertinent solution to the radiative transfer equation. This solution will be a function of frequency through the source function and absorption coefficient of the exhaust, and will be a function of aspect in a manner depending on the exhaust geometry. Only when this solution is at hand can further progress be made.

To serve as inputs to this solution, the source function and absorption coefficient within the exhaust are required. In general, these will contain contributions from both continuous-spectra processes and line processes. The lines may be regarded as merely superposed on the continuum at the correct frequency, since they are not coherent with it. The continuous and discrete spectrum mechanisms have therefore been treated separately.

The question of altitude dependence of the radiation should be answered by combination of the formal solution to the radiative transfer equation and the expressions corresponding to radiation and absorption mechanisms. That is one, possible source of altitude variation is aspect dependence of the formal solution. Another is altitude sensitivity of the solution through the source function and absorption coefficient, which do involve atomic and molecular densities, electron densities, and temperatures within the exhaust. These are, in principle, functions of ambient atmospheric conditions against which the exhaust expands. It is true, however, that the altitude region of interest is one over which the ambient conditions do not vary sharply.

It has not been possible to consider inhomogeneity of the exhaust, nor to obtain or generate information on the variation of exhaust composition with altitude due to changes in chemical reaction rates. Neither has the exact shape of the exhaust been determined. It is our decision that the interests of the sponsor have been best served by attempting to formulate a radiation model which is not based on such dubious and conjectural information. If the radiation model thus furnished agrees sufficiently well with experimental evidence available to the sponsor, time will not have been wasted in constructing elaborate theory on insecure foundations, but will have been spent on trying to do a respectable engineering type of approximation which includes those features of the problem expected to be most significant.

If, on the other hand, satisfactory agreement with theory is not obtained, the interests of the sponsor would be best served by supporting a small pilot study to decide which of the features omitted here accounts for the discrepancies, and whether theoretical apparatus of sufficient power exists to incorporate them into a radiation model. A related question is whether experimental determination of some quantities would be necessary and possible, in order to permit numerical application of an enhanced theory. Answering these questions would aid the sponsor in determining the optimum course for obtaining a closely-fitted theoretical model, and understanding which features of each vehicle cause it to radiate in its characteristic manner.

1.3 Status of Subsidiary Task Areas

a) Solution of the Radiative Transfer Equation

This has been completed, subject to certain simplifying assumptions believed to be reasonable for us. The solution is ready for use, and is presented in Chapter II of this report.

b) Continuum Radiation

The expressions for the source function arising from Bremsstrahlung of electrons on ions, and for the free-free absorption coefficient corresponding to the process inverse to this Bremsstrahlung have been obtained from the literature, and the criteria for their validity here checked. They appear in Chapter III of this report.

c) Line Radiation

For CO, there is no difficulty in writing down source function and absorption coefficient pertaining to the rotational spectra. This has been given in a previous report. The H_2O molecule, which seems to present theoretical difficulties, is being given renewed attention, but no results for it have been obtained yet. At this date, we have reason to question some of the statements in the literature regarding the H_2O line at 183 Gcs. Thus, some of the fundamental theoretical work on it appearing in previous papers must be re-investigated before using it as a basis for radiation model predictions.

II

SOLUTION OF THE RADIATIVE TRANSFER EQUATION

2.1 Formal Solution Neglecting Scattering

In the absence of scattering, which is not expected to introduce significant modifications in the specific intensity outside the source for situations of interest to us, the transport equation satisfied by the specific intensity, i, takes the form

$$\hat{\Omega}\cdot\nabla i(\vec{r},\hat{\Omega})+\alpha i(\vec{r},\hat{\Omega})=S(\vec{r},\hat{\Omega}) \quad . \tag{2.1}$$

Here α is the effective absorption coefficient and S the source or production function, i.e. the spectral density of power per unit volume produced within the source. All quantities are understood to be evaluated at the same frequency, and the frequency dependence of i is to be obtained. The frequency dependence of α and S will depend on the mechanisms operating within the source to generate and absorb radiation.

Now let us consider a bounded volume of radiating and absorbing systems as shown in Fig. 2-1.

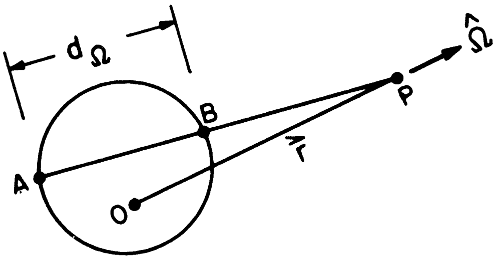

FIGURE 2-1

Since we expect eventually to specialize to a cylinder, all figures will relate to a cylindrical system. The specific intensity in the direction $\hat{\Omega}$ at any exterior point $P(\vec{r})$ will be obtained by integrating Eq. (2.1) at fixed $\hat{\Omega}$. In Fig. 2-1, O denotes an arbitrary origin. As is well known, for any direction $\hat{\Omega}$ in which the intensity is appreciable, the extension along $-\hat{\Omega}$ from P must intersect the source. The points of intersection are denoted by A and B; their separation by d_Ω.

If one measures distance along the direction $\hat{\Omega}$ from point A and designates it x_Ω, then (2.1) becomes

$$\frac{d}{dx_\Omega} i(x_\Omega, \hat{\Omega}) + \alpha(x_\Omega, \hat{\Omega}) i(x_\Omega, \hat{\Omega}) = S(x_\Omega, \hat{\Omega}) \ . \tag{2.2}$$

This is the first-order equation with the obvious boundary condition that no radiation be incident on the cylinder from outside, or

$$i(0, \hat{\Omega}) = 0 \ . \tag{2.3}$$

The standard procedure for solution of such a problem can be shown to yield

$$i(x_\Omega, \hat{\Omega}) = e^{-\int_0^{x_\Omega} \alpha(x''_\Omega, \hat{\Omega}) dx''_\Omega} \int_0^{x_{\hat{\Omega}}} S(x'_\Omega, \hat{\Omega})\, e^{\int_0^{x_\Omega} \alpha(x''_\Omega, \hat{\Omega}) dx''_\Omega} dx'_\Omega \tag{2.4}$$

Since S and α vanish for $x_\Omega > AB$, (2.4) may be reduced, for exterior points, to

$$i(\vec{r}, \hat{\Omega}) = \int_0^{d_\Omega} S(x'_\Omega, \hat{\Omega})\, e^{-\int_{x'_\Omega}^{d_\Omega} \alpha(x''_\Omega, \hat{\Omega}) dx''_\Omega} dx'_\Omega \ . \tag{2.5}$$

2.2 Radiation from Isotropic, Homogeneous Source

For the isotropic and homogeneous source, which we shall deal with, (2.5) takes the form

$$i(\vec{r},\hat{\Omega}) = S\int_0^{d_\Omega} e^{-\alpha(d_\Omega - x'_\Omega)}\, dx'_\Omega = S/\alpha(1-e^{-\alpha d_\Omega}) \ . \tag{2.6}$$

This solution exhibits correct limiting forms, since for the 'thin source', we have

$$\alpha d_\Omega \ll 1, \quad \text{and} \quad i(\vec{r},\hat{\Omega}) \cong S d_\Omega (1-\frac{1}{2}\alpha d_\Omega) \ , \tag{2.7}$$

while for the thick source, $\alpha d_\Omega \gg 1$ and

$$i(\vec{r},\hat{\Omega}) \cong S/\alpha, \quad \text{as for a black body.} \tag{2.8}$$

For more exact computations geometry enters the problem. In (2.6) the quantity d_Ω is to be evaluated explicitly. It should be noted that up to this point, no specialization to a cylindrical source has actually been made, and all work is completely general.

2.3 The Finite Cylinder as a Source

Let us consider a cylinder of length L and radius a, as shown in Figs. 2-2.

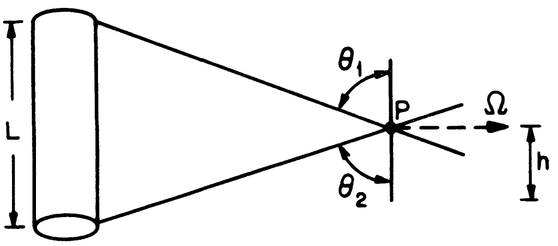

FIG. 2-2a: Side View

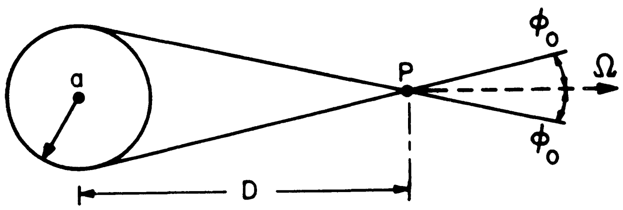

FIG. 2-2b: Top View

If the purpose of the investigation is to obtain i at point P a distance D from the axis of the cylinder and at axial distance h from one end, and if we define $\hat{\Omega}$ by the angles θ and ϕ in a set of local spherical coordinates centered at P, it is evident that

$$i = 0, \quad |\phi| > \phi_o \cong \sin^{-1}\frac{a}{D} \text{ or } \theta < \theta_2, \text{ or } \theta > \pi - \theta_1 \quad . \tag{2.9}$$

Exact calculation of θ_1 and θ_2 depends on the 'end configuration' of the cylinder. In practical cases, these configurations are not well-defined. However since we are interested in far-field applications, it is satisfactory to write

$$\theta_1 = \tan^{-1}\frac{L-h}{D} \quad , \qquad \theta_2 = \tan^{-1}\frac{h}{D} \tag{2.10}$$

For any $\hat{\Omega}$ defined by θ and ϕ such that L is not trivally zero, geometrical considerations yield the result

$$d_\Omega = 2\sqrt{a^2 - D^2\sin^2\phi}\,\csc\theta \quad . \tag{2.11}$$

Thus we have

$$i_P(\theta, \phi) = S/\alpha\left(1 - e^{-2\alpha\csc\theta\sqrt{a^2 - D^2\sin^2\phi}}\right) \tag{2.12}$$

and may write an expression for the spectral density of power intercepted per unit area by a receiver oriented normal to the axial distance D (Fig. 2-2b),

$$\frac{\text{Power}}{\text{Area}} \cong \frac{S}{\alpha}\int_{\theta_2}^{\pi-\theta_1}\sin^2\theta d\theta\int_{-\phi_o}^{\phi_o}\cos\phi d\phi\left(1 - e^{-2\alpha\csc\theta\sqrt{a^2 - D^2\sin^2\phi}}\right) \tag{2.13}$$

For an angular receiver oriented toward the source, then, one may write

$$\frac{\text{Power}}{\text{Effective Area}} \cong \frac{S}{\alpha} \int_{\theta_2}^{\pi-\theta_1} \sin\theta d\theta \int_{-\phi_o}^{\phi_o} d\phi \left(1 - e^{-2\alpha \csc\theta \sqrt{a^2 - D^2 \sin^2\phi}} \right) \tag{2.14}$$

Numerical evaluation of these integrals should not be difficult.

III

SOURCE FUNCTION AND ABSORPTION COEFFICIENT IN THE CONTINUUM

3.1 Bremsstrahlung Power from Electron-Ion Encounters

a) Contribution of the Faster Electrons to Source Function

Since different expressions are required for the faster and slower electrons, the resulting source functions being additive, we list the expressions separately. For the faster electrons, Equations (5.7) and (5.8) of a previous Laboratory report [+] are to be employed. The result, after minor corrections of notation, is that the power per unit volume in frequency interval $d\omega$ (or spectral density of source function) generated by the mechanism is given by

$$P d\omega = n_e n_i \frac{16}{3} e^6 \frac{4\pi}{m^4 c^5} \left(\frac{m}{2\pi kT}\right)^{3/2} d\omega I_2 \qquad (3.1)$$

where

$$I_2 = \mu kT \left[e^{-1/2} \left\{ \ell n \frac{\delta}{2} - \frac{3}{2} \epsilon \right\} - Ei\left(-\frac{1}{2}\right) \right] \qquad (3.2)$$

In (3.1), the symbols e and m are the charge and mass of the electron. T is temperature, c and k are the physical constants usually denoted by these letters. In (3.2) μ is mc^2, the rest energy of the electrons, e is the Naperi an base, δ is 4kT/K, where K is the photon energy $h\nu$ or $\hbar\omega \cdot Ei(-x)$ is the exponential integral, a tabulated function. $\epsilon = \gamma^2 kT/\mu K$, where $\gamma = \hbar c/\lambda$, in which λ is the Debye length $(kT/4\pi n_e e^2)^{1/2}$. n_e and n_i are the volume density of electrons and ions respectively. Single ionization, rather than multiple, has been assumed. At the temperatures we expect, this assumption is valid.

[+] Barasch, M.L. (August 1960), "Studies in Radar Cross Sections XLII: Microwave Bremsstrahlung From a Cool Plasma," The University of Michigan Radiation Laboratory Report 2764-3-T, AD 245070. UNCLASSIFIED.

b) Contribution of the Slower Electrons to Source Function

This will be obtained from Eq. (3.28) of Report 2764-3-T$^+$. This reads

$$Pd\omega = \frac{16}{3}\,\frac{n_i n_e e^6 d\omega}{m^4 c^5}\,4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} I_3 \ , \qquad (3.3)$$

in which

$$I_3 = \int_{\sqrt{2K\mu}}^{\sqrt{2kT\mu}} PdPe^{-P^2/2\mu kT}\left\{\sqrt{\left(\frac{\epsilon}{\lambda}\right)^2 + \left(\frac{\mu\epsilon\omega}{Pc}\right)^2}\; K_o(\sqrt{\ })K_1(\sqrt{\ }) - \frac{1}{2}\left[\left(\frac{\epsilon}{\lambda}\right)^2 + 2\left(\frac{\mu\epsilon\omega}{Pc}\right)^2\right]\left[K_1^2(\sqrt{\ }) - K_0^2(\sqrt{\ })\right]\right\} , \qquad (3.4)$$

In I_3 the functions K_0 and K_1 are 'modified Hankel functions' ϵ denotes the De-Broglie wavelength, $\epsilon = \hbar/mv_o$. Since $P = mv_o c$, ϵ is a function of the variable of integration rather than constant. λ is still the Debye distance, and all symbols previously defined retain their original significance. The symbol $\sqrt{\ }$ used in the argument of the K function is an abbreviation for that long radical which is written out in detail in the first place it appears inside the brackets. I_3 will require numerical evaluation, since no closed analytical form has been obtained for it.

c) Free-free Absorption by Faster Electrons in the Fields of Ions

It follows from the Report 2764-3-T (op. cit.) that this contribution is given formally by

$$\alpha_{FF}(\omega) = n_e n_i\left(\frac{16}{3}\,\alpha r_o^2 \pi^2\right)\left(\frac{c}{\omega}\right)^3 \int_0^\infty \frac{4\mu}{\sqrt{\pi}}\,\frac{P_2 dP_2}{s^3}\, e^{-P_2^2/s^2}\, F(\omega, P_1, P_2)$$

(3.5)

The new symbols here are r_o, the classical electron radius $= e^2/\mu$, and $s = (2\mu kT)^{1/2}$, while as usual α denotes the fine-structure constant. Now for the faster electrons the Born approximation is to be employed, and eventually one is able to write for this contribution,

$$\alpha_{FF}(\omega) = n_e n_i \left(\frac{16}{3}\alpha r_o^2 \pi^2\right)\left(\frac{c}{\omega}\right)^3 \frac{4\mu}{\sqrt{\pi}} I_2 \quad , \tag{3.6}$$

where the integral I_2 has been evaluated previously (Eq. 3.2) .

d) Free-free Absorption by Slower Electrons in the Fields of Ions

To obtain this contribution to the continuum absorption, one starts with (3.5). However, in this case, numerical evaluation is required with the special valid form

$$F = \left\{ \sqrt{\left(\frac{\epsilon}{\lambda}\right)^2 + \left(\frac{\mu\omega\epsilon}{Pc}\right)^2}\, K_o(\quad) K_1(\quad) - \frac{1}{2}\left[\left(\frac{\epsilon}{\lambda}\right)^2 + 2\left(\frac{\mu\omega\epsilon}{Pc}\right)^2\right]\left[K_1^2(\quad) - K_0^2(\quad)\right] \right\} . \tag{3.7}$$

Numerical evaluation is required.

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13. ABSTRACT

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