PERIPHERALITY AND πN SCATTERING*

M. ROSS
Westfield College, London N.W. 3 and University of Michigan, Ann Arbor, Michigan 48104, USA†

Received 23 December 1971
Revised manuscript received 3 January 1972

πN amplitudes, as far as they have been deduced, are used with an absorbed Reggeon exchange model to obtain amplitudes at larger $-t$ with specified phases. Thus, satisfactory peripheral amplitudes are found. The phase behavior of the diffraction amplitude is discussed.

On the basis of small $-t$ πN data at 5-6 GeV/c and below, amplitude analyses have been made [1]. The availability of amplitudes changes the task of the phenomologist. The question we shall examine is whether these πN amplitudes, as far in $-t$ as they have been determined, can be consistent with peripheral amplitudes. A strongly absorbed Reggeon exchange model is used in a natural way to construct the latter amplitudes.

Consider

$$\pi^- p \to \pi^- p, \quad \pi^+ p \to \pi^+ p, \quad \pi^- p \to \pi^- n \qquad (1,2,3)$$

and the measurements

$$\frac{d\sigma}{dt}(1) - \frac{d\sigma}{dt}(2), \quad P(1) - P(2), \quad P(3). \qquad (4,5,6)$$

There are four amplitudes $A_{In}(t)$ where $I = 0, 1$ is the isospin exchange and $n = 0, 1$ indicates s-channel helicity non-flip and flip respectively. The phases of A_{In} are δ_{In}

All the amplitudes aside from a common phase have been determined by Halzen and Michael at 6 GeV/c for $-t < 0.65$. From dispersion relations some information on the phase δ_{00} has been obtained:

$$\delta_{00}(t=0) \approx \tfrac{1}{2}\pi + 0.2,$$

$$\frac{d\sigma_{00}}{dt}(t=0) \approx 1.3 \pm 0.5, \qquad (7)$$

$$\delta_{00} = \tfrac{1}{2}\pi \text{ at some } t \ (0.8 \lesssim -t \lesssim 1.0).$$

The third condition is not very compelling. Inspection of fig.1 show these amplitudes roughly in this t range. The crucial result implied by

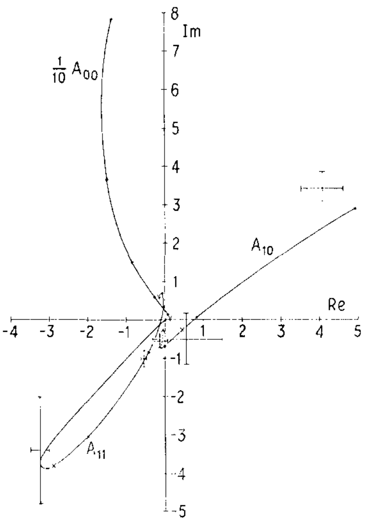

Fig.1. Argand diagram for the amplitudes calculated as discussed in text. The dots are at $-t$ = 0, 0.2, 0.4, 0.6, 0.8 on each locus. Points with errors were deduced from experiment by Halzen and Michael (ref.[1]) and are for A_{10} at $-t$ = 0, 0.25 (marked by an × on theory curve) and 0.5, and for A_{11} at $-t$ = 0.125 (marked by × on theory curve) and 0.375. (At 0.6 the point is essentially at the origin.)

new charge exchange polarization measurement [2] is that the amplitudes move clodkwise by the origin with $-t$.

Absorption models have predicted counter-clockwise motion in $-t$ of Reggeon exchange amplitudes in the complex plane [3]. This direction of motion depends on the sign of the small "orthogonal" part of the amplitude near the minimum in $|A|$. Counter-clockwise behaviour is predicted unambiguously by the absorption formula (partial wage j)

$$A(j) = B(j)\, S(j) \qquad (8)$$

* Supported in part by the U. S. Atomic Energy Commission.
† Permanent address.

where the amplitude is the product of the Born (Regge pole) amplitude and real transmission factor S. Similarly a band of single level resonances* implies counter-clockwise behaviour. We take A_{00} as inferred from Phillips and Ringland, Höhler and Jakob and the assumption that A_{01} is small and write

$$A00 \propto i\,(\exp\,(0.25i)\,\exp\,\{(3.5 - 1.7i)t\} + 0.15\,\exp\,\{(1.3 + 1.5i)t\} - 0.3i\,\exp\,(0.3t)), \quad (9)$$

(there is leeway in certain of these parameters) and we take the absorption factor in (8) to be

$$S = 1 + i\lambda A_{00}/\mathrm{IM}A_{00}(b = 0). \quad (10)$$

The particular curves for A_{1n} shown in fig.1 were obtained with $\lambda_{\text{non flip}} = \lambda_{\text{flip}} = 1.0$ and a ρ Reggeon "choosing simplicity" (no wrong signature zero) with $\alpha' = 1.0$, linear trajectory through the ρ, and $s_0 = 0.3$. The result shown is a sample; it is snesitive to A_{00}. No careful parameter search has been made. Similar results are easily obtained with other pole amplitudes, different absorption prescriptions (10), and different A_{00} at larger $-t$[4]. In the present calculation the large $-t$ term introduced in (9) is an exercise to make the A_{1n} more peripheral.

It is of interest that with these pahses the cross-over in (4) occurs at substantially smaller $-t$ than the minimum in $|A_{10}|$ and that the dip in the elastic polarization difference (5) is not quite a double zero but involves a change in sign with polarization rising to roughly 0.05, see fig. 2. The behavior of $\sin(\delta_{00} - \delta_{11})$ is due in part to the motion of δ_{00}.

The phase of S introduced here may apply at low energy only. If so, our discussion is closely related to the calculation of Hong Tuan et al.[5] † where a successful description is obtained with a a large ρf cut. In this case, (6) moves to negative values for $0.2 \lesssim -t \lesssim 0.5$ at high energy. In this case we are uncertain as to the high energy form of A_{00}. In particular, the imaginary part could change sign near $t = -0.7$ rather than the amplitude moving into the first quadrant as shown in fig.1. The double zero in (5) near $t = -0.6$ follows in either case, but if Im A_{00} changes sign there will be a second cross over,

* Resonance concentrated in a band in j near $j = kr$ with r some suitable radius.

† If strong absorption, e.g., ref. [7] is taken seriously the variation in the real part of A_{00} cannot be mainly in the f, since from the latter we expect a rapidly increasing real part near $-t = 0.2$. I would like to thank G. Kane for pointing this out.

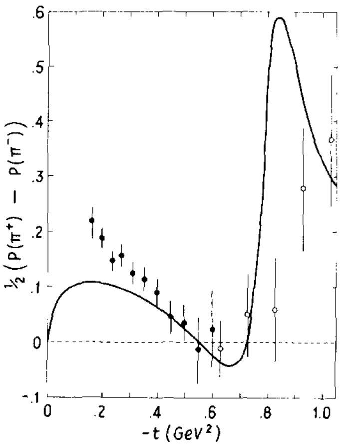

Fig.2. The difference in elastic polarization, exhibiting the approximate double zero. The sharp peaking near 0.85 is parameter sensitive and not characteristic of the model. The data are from ref. [12].

i.e. in (4), at $-t \approx 0.7$ for which there is no evidence at present. It seems more palatable that the phase of A_{00} used here continues to be valid at higher energies.

If the part of A_{00} which is leading in s has rapidly increasing phase with $-t$ at small $-t$ and rapidly decreasing phase with $-t$ at moderate $-t$, we should find connections with s dependence, and with spatial range dependence. If the s dependence enters through power law and log s, and phase through $\ln(s/s_0) - \frac{1}{2}i\pi$, we see that shrinkage (a decreasing $\alpha_{\text{eff}}(t)$ with $-t$), should be associated with increasing phase, and vice versa. However the relative role of power and log dependencies at a given s is unknown, so that this relation is not necessarily valid at a given s. It is tempting to speculate that at large range there is an expansion in range with s, resulting in shrinkage for small $-t$. This is the customary behavior associated with Reggeon exchange and with multiperipheral processes. However it is likely that the main diffractive process yielding most of the total cross section are associated with shorter range. Here the multiperipheral ideas are invalid[6]. There could be correlations in particle production leading to shrinkage in range, and to expansion in $-t$ at some s.

Let us conclude with remarks on the general status of absorbed Reggeon exchange. A controversial question has been the structure of the pole amplitudes (i.e. Born terms); especially do they have nonsense-wrong-signature and possibly wrong-signature zeros? Second, is the strength of absorption standard? Increasing experimental evidence for fairly strong absorption in $n = 0$ amplitudes [7] leads us to believe that the form of the Born terms may not be a crucial question at this time (i.e. the Born terms details are obscured by the strength of absorption and the lack of an accurate absorption model). The questions facing absorbed Reggeon exchange are now being changed in emphasis into:

a) Are all the Reggeon exchange amplitudes peripheral? * Where extensive data are available, as in πN elastic, it is becoming possible to approach this question deductively.

b) Is there an efficient quantitative absorption model (i.e. with very few parameters)?

On point (a), if Reggeon exchange and absorption are physical processes, as in low energy nuclear physics, we expect the amplitudes to be peripheral. More accurately, since particle-particle scattering is probably not very opaque at $b = 0$, we should demand peripherality of the amplitude projected onto the line with phase δ on which the low $-t$ amplitude is concentrated, i.e. $|A_{1n}(t)| \cos(\delta_{1n} - \bar{\delta}_{1n})$ is peripheral. The small orthogonal part need not be. It is of interest that qualitative peripherality of $n = 0, 1$ amplitudes can be distinguished from non-peripherality rather easily: as a zeroth approximation peripheralism implies an oscillating amplitude $J_n(b\sqrt{-t})$, $b_0 \approx 1\,\text{fm}$, moving on a line of roughly fixed phase [7]. In more detail, we expect some precession through positive phase with increasing $-t$ due to the motion of the underlying Regge pole, and there may be a small amplitude orthogonal to the zeroth approximation which we have seen can be on the clockwise side of the origin. To see inspection if an amplitude is peripheral, examine it in the complex plane: check that it passes near the origin near the first zero of $J_n(b_0\sqrt{-t}$, $b_0 \approx 1\,\text{fm}$, and examine the projection on an axis with phase $\bar{\delta}$ along which the small $-t$ part of the amplitude tends to lie, e.g., roughly 35^0 and 50^0 for $n = 0$ and 1 ρ exchange. It is peripheral if the average $\int \mathrm{d}t A(t)$ is small, e.g., if

*Strong absorption means that the weighted amplitude $b|M(b)|$ is peaked for $b \approx 5$ and is an order of magnitude smaller than the input pole for small b say $b \approx 1$. But we need a concept without reference to the input pole amplitude. For $n = 0$ we expect the magnitude of the unweighted pole amplitude to decrease close to an order of magnitude from $b = 1$ to $b = 5$, so let me define a strongly peripheral amplitude, $b|M(b)|$, as peaked at $b \approx 5$ and an order of magnitude smaller at $b = 1$. For $n = 1$ this condition is not very powerful; it is more difficult to decide if an empirically determined $n = 1$ amplitude is peripheral. For $n > 1$ any reasonable amplitude is strongly peripheral and no further test of peripherality is in practice possible.

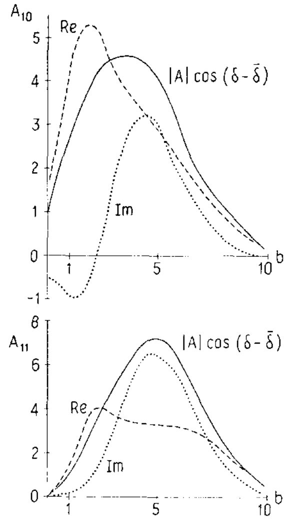

Fig.3. Impact parameter distributions for the amplitudes $|A_{1n}|\cos(\delta_{1n} - \bar{\delta}_{1n})$ with $\bar{\delta}_{10} = 35^0$ and $\bar{\delta}_{11} = 50^0 + \pi$. Re$A_{1n}$ and ImA_{1n} are also shown. The units of b are GeV^{-1}.

the integration involves considerable cancellation. Fortunately, the magnitude of high energy amplitudes falls so rapidly with $-t$ that the first two maxima tell the whole story. The $A_{1n}(b)$ amplitudes corresponding to the $A_{1n}(t)$ in fig.1 are shown in fig.3 and are seen to be peripheral. This picture is to be contrasted with two others: certain analyticity arguments may suggest that real parts are non-peripheral [8], as argued by Harari and Henzi [9]. Another possibility is that only $n = 0$ amplitudes are strongly peripheral, as suggested by Cohen-Tannoudji [10]. The data can be analyzed in ways to make any of these three possibilities consistent with experiment at present.

On point (b) we note that the advantage of the strong absorption model for Reggeon exchange in which all amplitudes are strongly peripheral, has been that it is a general model applying to all two-body reactions including those of higher spin [11]. Two detailed modifications of this absorption prescription have been proposed: that of this letter and a radius parameterization [7]. It is not clear that the excessively large number of parameters of this model would be removed.

I would like to thank Drs. G. Kane, K. Moriarty, G. Ringland and D. P. Roy for helpful comments, and especially Dr. Brian Hartly for calculations. I would like to thank Imperial College and Westfield College for their hospitality and assistance. I would like to thank R. Kelly for in-

formation on a very interesting amplitude analysis (in progress) by R. Cutokosky and himself.

References

[1] R. J. N. Phillips and G. A. Ringland, Nucl. Phys., to be published:
V. Barger and R. Phillips, Phys. Rev. 187 (1969) 2210;
F. Halzen and C. Michael, Phys. Lett. 36B (1971) 367;
G. Höhler and H. P. Jakob, preprint.

[2] P. Bonamy et al., Saclay-DESY-Orsay-College de France collaboration, reported by O. Guisan in High energy phenomenology, ed. J. Tran Thanh Van, Orsay, 1971.

[3] R. Arnold, Phys. Rev. 153 (1967) 1523;
F. Henyey, G. Kane, J. Pumplin and M. Ross, Phys. Rev. 182 (1969) 1579;
G. Cohen-Tannoudji, A. Morel and H. Navelet, Nuovo Cimento 48A (1967) 1975;
J. P. Holden and D.C. Robertson, Phys. Rev. D4 (1971) 233;
S. Kogitz and R.K. Logan, preprint.

[4] Related calculations have been done by Alexander Martin and Paul Stevens, preprint.

[5] R. Hong Tuan, J. Kaplan, G. Sanguinetti, Saclay preprint.

[6] M. Ross, Symp. on High energy interactions, Argonne National Laboratory, November 1970;
M. Kugler, RHEL preprint.

[7] M. Ross, F. Henyey and G. Kane, Nucl. Phys. B23 (1970) 269.

[8] G. A. Ringland and D. P. Roy, RHEL preprint, state they have empirical evidence for non-peripheral ReA_{10}. This result is reversed if a more peripheral amplitude ReA_{11} (as in fig.1) is employed in their more definitive case rather than a non-peripheral amplitude like the pole with nonsense-wrong-signature zero.

[9] H. Harari, Phys. Rev. Letters 26 (1970) 1400;
R. Henzi, preprint, and Nuovo Cimento 52A (1967) 772.

[10] G. Cohen-Tannoudji, private communication.

[11] G. Kane, F. Henyey, D. Richards, M. Ross and G. Williamson, Phys. Rev. Letters 25 (1970) 1519;
Henyey et al., ref.[3].

[12] Data at small $-t$ taken from M. Barghini et al. Phys. Lett. B24 (1967) 77 at 6.0 GeV/c and at larger $-t$ taken from R. Esterling et al., Phys. Rev. Lett. 21 (1968) 1419 at 5.15 GeV/c.

* * * * *