THE UNIVERSITY OF MICHIGAN RESEARCH INSTITUTE
ANN ARBOR, MICHIGAN

Quarterly Report

ATMOSPHERIC PHENOMENA AT HIGH ALTITUDES

(February 1, 1958, to April 30, 1958)

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UMRI Project 2387

DEPARTMENT OF THE ARMY, PROJECT NO. 3-17-02-001
METEOROLOGICAL BRANCH, SIGNAL CORPS PROJECT NO. 1052A
CONTRACT NO. DA-36-039-SC-64659

September 1958

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ABSTRACT

DOVAP data reduction has been completed on two of the ten Ft. Churchill grenade experiments and partially completed on four others. Work has started on the DOVAP and ballistic camera tracking system for the Guam program of grenade experiments.

The leak-rate calculations for air-sample bottles have been revised.

The initial phase of the study on shock-wave propagation has been summarized in a report which is attached hereto as an appendix.

A report, by V. C. Liu, entitled "On Pitot Pressure in an Almost-Free-Molecule Flow—A Physical Theory for Rarefied Gas Flows," has been accepted for publication in the *Journal of Aeronautical Sciences*.

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1. INTRODUCTION

This is the twelfth in a series of quarterly reports on Contract No. DA-36-039-SC-64659. The purposes of the contract are:

a. to adapt the rocket-grenade experiment for use in the Arctic during the International Geophysical Year;

b. to participate in the preparation and firing of the IGY rocket-grenade experiments;

c. to collect and analyze upper-air samples; and

d. to engage in the general investigation of problems relating to upper-air research.

2. GRENADE EXPERIMENT

2.1 FORT CHURCHILL DATA REDUCTION

Considerable progress on DOVAP data reduction for the Fort Churchill grenade experiments was made during this quarterly work period. The status of the DOVAP data-reduction work as of the end of this period was as follows:

SM1.01 - Complete
SM1.02 - Complete
SM1.03 - Counting 30% complete
SM1.04 - Counting, editing, and initial spin corrections made, initial computer run unsatisfactory
SM1.05 - Counting completed
SM1.06 - Not started
SM1.07 - Not started
SM1.08 - Not started
SM1.09 - Not started
SM2.10 - Counting completed

The existence of "anomalous" spin effects on SM1.02 and SM1.04 has complicated the data-reduction process. Theoretical analysis by personnel of the antenna laboratory at Stanford Research Institute[1] has shown that these anomalous effects can be obtained as a result of "imperfect" tuning of the DOVAP missile antenna system, leading to the existence of an axial dipole radiation mode in addition to the normal transverse small loop mode. Under these conditions, anomalous spin effects can be obtained for certain geometrical relations

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[1] C. W. Steele and T. Morita, "Pattern Characteristics of DOVAP Missile Antennas," Technical Report 5, SRI Project 898, Stanford Research Institute, Menlo Park, California, Aug., 1957.

between the ground station and missile antenna system. Usually such effects occur at all stations other than launch during the first portion of the rocket flight. These effects have been encountered on SM1.02 and SM1.04, and have been corrected for on SM1.02.

Of the two flights for which the DOVAP data reduction is complete, SM1.01 has been reported on in the last quarterly report. For SM1.02, as for SM1.01, three sets of trajectory tabulations have been made:

1. Trajectory coordinates, at half-second intervals (range time), with respect to launcher.
2. Grenade Aerobee coordinates, at the time of each explosion, with respect to launcher.
3. Grenade Aerobee coordinates, at the time of each explosion, with respect to the center microphone at Twin Lakes.

The data of (3) above are reproduced in Table I. The quality of the data can be judged from the value $\bar{\sigma}_u$ to be attached to each data point. From the data of Table II, it can be seen that the $\bar{\sigma}_u$ to be attached to the position data for grenades Nos. 3, 7, and 18, at altitudes of approximately 20, 30, and 60 miles, respectively, are 2.9, 3.1, and 5.1 ft, respectively. The criteria for judging Fort Churchill DOVAP data-reduction results, as outlined by W. Dean of BRL, are:

σ_u = 5 ft - "good" (normal)
σ_u = 8 ft - "probably O.K."
σ_u = 15 ft - "something wrong"

According to these criteria, the results for SM1.02 are "good."

A further check on the accuracy of these data will be obtained by comparison with the results of ballistic camera determination of grenade-explosion positions for this flight. These data have not yet been obtained from BRL, but should be available soon.

It should be noted that the data in Table I give the position of the DOVAP missile antenna system. The position of each grenade explosion can be obtained by adding the relative distance between the DOVAP antenna system and the grenade explosion. This correction, which will be explained further in a later report, affects the absolute position only slightly. Grenade layers are affected even less. For all practical purposes, temperature and wind data can be calculated from the above data without loss of accuracy.

2.2 GUAM PROGRAM

All the initial work is underway. Discussions with U.S.A.S.R.D.L. have established that The University of Michigan is to be responsible for the DOVAP

TABLE I

SM1.02 GRENADE AEROBEE POSITION AT TIME OF EACH GRENADE EXPLOSION*

Grenade No.	Range Time	x(+N)	σ_x	y(+w)	σ_y	z(+up)	σ_z
1	39.0358	34678.56	5.00	5888.48	5.41	85947.95	0.73
2-1		- 1086.87	2.52	741.54	2.72	11357.47	0.24
2	42.0298	33591.69	7.52	6630.02	8.13	97305.42	0.96
3-2		- 1122.23	0.47	767.50	0.52	11452.67	0.11
3	45.1531	32469.47	7.98	7397.51	8.64	108731.09	0.90
4-3		- 1160.00	0.38	801.58	0.41	11526.59	0.15
4	48.4166	31309.47	7.60	8199.09	8.24	120257.68	0.78
5-4		- 1174.44	0.04	815.50	0.05	11319.44	0.10
5	51.7342	30135.04	7.63	9014.59	8.27	131577.12	0.72
6-5		- 1241.36	1.01	866.48	1.09	11563.88	0.10
6	55.2460	28893.68	8.64	9881.07	9.36	143140.99	0.77
7-6		- 1265.36	1.12	885.96	1.21	11389.81	0.15
7	58.8352	27628.32	7.52	10767.02	8.14	154530.80	0.65
8-7		- 1271.77	1.39	893.15	1.51	11011.69	0.15
8	62.4407	26356.55	6.13	11660.18	6.64	165542.49	0.51
9-8		- 1340.32	1.54	953.08	1.67	11219.30	0.13
9	66.2662	25016.23	7.67	12613.25	8.30	176761.78	0.64
10-9		- 1413.36	2.34	1001.33	2.53	11302.20	0.21
10	70.2945	23602.87	10.01	13614.58	10.83	188063.98	0.83
11-10		- 1483.25	2.69	1061.75	2.93	11299.14	0.23
11	74.5227	22119.62	7.31	14676.34	7.90	199363.12	0.61
12-11		- 1580.39	0.75	1121.17	0.82	11402.59	0.07
12	79.0256	20539.24	6.57	15797.51	7.08	210765.71	0.56
13-12		- 1651.85	5.59	1181.39	6.08	11219.17	0.51
13	83.7303	18887.39	12.16	16978.90	13.16	221984.88	1.06
15-13		- 3706.31	3.76	2692.95	4.11	22698.51	0.30
15	94.3385	15181.08	8.40	19671.84	9.05	244683.38	0.79
16-15		- 2112.07	7.18	1516.04	7.71	11318.68	0.73
16	100.3767	13069.02	15.58	21187.08	16.77	256002.06	1.52
17-16		- 2411.70	5.40	1762.46	5.95	11483.22	0.49
17	107.2573	10657.31	10.18	22950.34	10.82	267485.28	1.03
18-17		- 2723.43	16.97	1960.67	18.39	11179.80	1.91
18	115.0386	7933.88	27.15	24911.01	29.20	278665.08	2.93

*Referred to a right Cartesian coordinate system having its origin at the center microphone at Twin Lakes.

TABLE II

CALCULATION OF $\overline{\sigma}_u$ FOR SM1.02 DOVAP DATA*

Grenade No.	σ_x (ft)	$\frac{\sigma_x}{\sigma_u}$	σ_u (ft)	σ_y (ft)	$\frac{\sigma_y}{\sigma_u}$	σ_u (ft)	σ_z (ft)	$\frac{\sigma_z}{\sigma_u}$	σ_u (ft)	$\overline{\sigma}_u$ (ft)
3	8.0	2.2	3.6	8.6	2.4	3.6	0.9	0.6	1.5	2.9
7	7.5	3.0	2.5	8.1	3.3	2.5	0.65	0.5	1.3	3.1
18	27.2	5.7	4.8	29.2	6.1	4.8	2.9	0.5	6.0	5.1

*Values of $\frac{\sigma_x}{\sigma_u}$, $\frac{\sigma_y}{\sigma_u}$, $\frac{\sigma_z}{\sigma_u}$ were taken from graphs prepared by W. Dean for the Fort Churchill range.

equipment, telemetering, transponders, missile antennas, and ballistic cameras.

A survey indicates that about $60,000 will be needed to purchase equipment not furnished by U.S.A.S.R.D.L. Purchase has been initiated.

The equipment will be housed in two vans which were furnished by U.S.A.S.R.D.L. It is planned to shock-mount all gear in the vans for shipment to Guam.

The DOVAP transmitter has arrived and preliminary checkout indicates that the unit is extremely unstable. Responsibility for the transmitter has been assigned to a crew of two people.

DOVAP receivers are also at hand and have also proven unstable. Malfunctions in the I.F. stages have been corrected and the units are now ready for noise-ratio, sensitivity, and other performance checks.

Personnel have been assigned for all operations except for transponder and rocket-antenna installation and checkout. However, it is believed that these assignments will be made in the near future.

3. AIR-SAMPLING EXPERIMENT

A recheck of the leakage rates of several upper-atmosphere bottles was performed, and a series of bottles from earlier flights was checked for leakage. Table III gives the results of all leakage determinations to date. The method of leak-checking was that described in Report 2387-32-P.

The leak rate of the standard leak (C.E.C. calibrated standard glass leak) is constantly being reduced due to helium loss. Our original bottle-leak-rate calculations, shown in column 3 of the chart, were based on the data printed on the standard leak. A series of tests was devised to establish the present leak rate of the standard leak and its rate of decay. These corrections were applied to the rates of column 3 and appear in column 4 as the "new" rates.

Probable errors were computed for the results of the analysis of three upper-atmosphere bottles. These appear in Table IV.

A paper entitled "Analysis of Helium and Neon from the Upper Atmosphere" was started.

Studies were made of equipment which might be used to analyze upper-atmosphere samples for argon. Work on the subject was stopped when it was decided to discontinue work on the problem of bottle sampling.

TABLE III

RESULTS OF LEAKAGE DETERMINATIONS

Bottle No.	I He leak rate found 10^{-15} cc NTP/sec	II He leak rate required 10^{-15} cc NTP/sec	III Old	IV New
B-15	46	17	2.7	0.69
B-10	2.1	7.9	0.26	0.067
C-23-B	.028	1.5	0.02	0.005
E-2	2.8	5.8	0.48	0.12
B-25	109	33 (1) 66 (2)	3.3 1.65	0.85 0.42
B-8	.021	72	0.0003	0.00008
C-1	< .028	7.25	< 0.004	0.001
B-9	2.0	136 (1) 242 (3)	0.014 0.008	0.004 0.003
C-11	28,000,000	8.4	3,400,000	850,000
B-6	0.058	32	0.001	0.0003
B-15	29	16	1.78	0.45

I He leak rate found on leak detector, using leak rate of standard glass leak as noted on its label.

II He leak rate required for the excess of helium over ground air found in the sample.

III I ÷ II

IV I ÷ II, with I corrected for loss of helium in standard glass leak.

(1) From analysis of upper air.

(2) From analysis of ground air in bottle.

(3) From leakage test on analyzer.

TABLE IV

PROBABLE ERRORS FOR ANALYSES OF THREE
UPPER-ATMOSPHERE BOTTLES

Bottle No.	Probable Error	
	He	Ne
B-15	0.58%	0.49%
B-10	1.4 %	0.86%
C-23-B	0.13%	0.78%

4. SHOCK WAVES FROM EXPLOSIONS

The initial phase of the study on this topic has been summarized in an informal report, entitled "The Determination of Upper-Air Densities from the Measured Characteristics of Shock Waves due to Grenade Explosions," which is attached as an appendix to this quarterly report.

The general study of this phenomenon is continuing with the study of the theory of propagation of spherical shocks from:

a. Chemical explosions
b. Point source of energy
c. High-pressure spheres

The study is being made to see whether an accurate determination of upper-air density or pressure can be made from the measured rate of travel of a spherical shock.

5. RAREFIED GAS DYNAMICS RESEARCH

During this period (February-April, 1958), the analysis of the pitot pressure in almost-free-molecule flow has been completed. A report, entitled "On Pitot Pressure in an Almost-Free-Molecule Flow—A Physical Theory for Rarefied Gas Flows," was submitted and accepted for publication in the Journal of Aeronautical Sciences. A summary of this paper is given below.

A physical theory of pitot pressure in the transition-flow regime, i.e., the moderately rarefied gas-flow region, is proposed. The ratio λ/b (the mean-free path to the radius of the cavity opening) is assumed to be of the order unity or larger. A general formula for the perturbation to the pitot pressure in the free-molecule flow is given. This perturbation is attributed to the intermolecular collisions, which are neglected on the basis of the free-molecule hypothesis. The expected rate of collision is calculated for rigid spheres, using the classical kinetic theory.

Although this is intended as an approximate theory, the theoretical results check surprisingly well with the limited experimental data that are available. The present theory shows that the ratio Re/M (Reynolds number to Mach number) is the governing parameter for determining the intermolecular collision effect on pitot pressure in the transition-flow regime.

6. LABORATORIES VISITED

U. S. Army Signal Engineering Laboratories

7. ACKNOWLEDGMENTS

We are indebted to the Meteorological Branch of the U. S. Army Signal Engineering Laboratories for continued collaboration and support.

APPENDIX

THE DETERMINATION OF UPPER-AIR DENSITIES FROM THE MEASURED CHARACTERISTICS OF SHOCK WAVES DUE TO GRENADE EXPLOSIONS

INTRODUCTION

A series of ten firings of the Aerobee rocket-grenade experiment for upper-air temperatures and winds has recently been completed at Fort Churchill as a part of the U. S. IGY rocket program.

In this experiment, grenades carried in the rocket are exploded at intervals along the up-leg of the rocket's trajectory. The time of each explosion is determined by ground or missile-borne flash detectors, and the position of each explosion is determined by an optical or electronic tracking system. These data, plus the arrival time (at an array of microphones on the ground) of the sound wave from each explosion, are used to calculate the average temperature and average wind velocity in the layers between consecutive grenades.

In this series of experiments, the electronic tracking system DOVAP was used to track the rocket. Examination of the DOVAP data obtained showed that each grenade explosion produced a disturbance in the operation of the DOVAP system. Some evidence concerning the nature of this disturbance is contained in the data record for each of the ten flights.

A possible explanation of the disturbance of the DOVAP is:

a) an initial scattering or reflection of the electromagnetic radiation of the DOVAP system at the surface of the expanding shock wave from the explosion, followed by
b) a detuning or "shorting out" of the DOVAP antenna system when the shock wave has traveled that far.

Study of the theory of propagation of the shock wave from an explosion indicated that ambient density of the air in which the grenades were exploded could be calculated from an equation relating the ambient density to the

a) DOVAP carrier frequency,
b) explosion-antenna system distance,
c) intervals measured from the disturbance in the DOVAP record, and
d) the energy released by the explosion.

Although some of the data were not known very accurately, it was thought desirable to check the above explanation for the disturbance of the DOVAP system by calculating ambient density for the sixteen grenades on the first flight of the experiment, and to compare these calculated values with known data on upper-air density. This was done and the results seem to agree well with known values of upper-air density, although the equation used was an approximate one and the inaccuracies of the data lead to considerable scatter in the density-vs.-altitude curve plotted from the data.

The results are good enough to warrant an investigation of the possible use of this phenomenon for accurate measurements of upper-air density.

DESCRIPTION OF THE DOVAP SYSTEM

The DOVAP tracking system (DOppler Velocity And Position) is a continuous-wave electronic tracking system. A ground transmitter transmits a 38-mc signal to the rocket and to at least three receiving stations, which are preferably located below the general region of the probable rocket trajectory. The signal is received by the DOVAP "transponder" in the rocket. The frequency of this signal is doubled, and the resulting 76-mc signal is retransmitted to the receiving stations, where the 38-mc signal from the ground transmitter is also received. Its frequency is doubled and compared with the 76-mc signal received from the rocket. The two frequencies differ because of the Doppler effect upon the signal received and transmitted by the rocket. This difference or Doppler frequency is recorded on magnetic tape. For a given receiving station i, each Doppler cycle indicates an increase of one wavelength in the distance u_i, where

$$u_i = r + r_i \quad ,$$

and

r = the distance, transmitter to missile,
r_i = the distance, missile to receiver i.

At any time t, each u_i defines an ellipsoid of revolution having as foci the transmitter and receiver i. The position of the missile is the intersection of three such ellipsoids of revolution.

It is the Doppler frequency records for the grenade experiments at Fort Churchill which show the disturbances due to the grenade explosions.

THE DISTURBANCE OF THE DOVAP CYCLE-COUNT RECORDS

Figure 1 shows an enlargement of a section of a Doppler cycle-count record containing a disturbance due to a grenade explosion. The data recorded on magnetic tape are "played back" and recorded on 35-mm film by cameras which photograph a line-up of cathode-ray oscillograph tubes. Figure 1 is an enlargement of a portion of such a 35-mm film record. The five cycle-count records D, $L_{L.H.}$, $L_{R.H.}$, T.L., and M were obtained at five different ground stations. The set of closely spaced dashes between $L_{L.H.}$ and $L_{R.H.}$ is a recording of the phase difference between the Doppler cycles on these two records. The dots appearing at regular intervals between D and $L_{L.H.}$, $L_{R.H.}$, and T.L., and T.L. and M are 0.01-second time markers.

In Fig. 1, time increases from right to left. The first time marker at the right indicates 41.73-sec range time. The time of explosion, as obtained from various flash-detector records was 41.744 to 41.745 sec, with an accuracy of something less than ± 0.001 sec. At this region, the cycle-count records show a number of small oscillations superimposed upon the regular Doppler cycles. The starting point of these small oscillations is at 41.7437 sec.

DESCRIPTION OF GRENADE EJECTION AND DETONATION

Figure 2 is a diagram indicating the method of ejection of the grenades. Each grenade is ejected through the rocket nose cone and travels forward with respect to the rocket, unwinding a lanyard as it goes. When the lanyard is completely unwound, a pin is pulled from the detonator, and the grenade is exploded. If the operation goes as planned, the grenade will be detonated so that the center of the explosion is about 12 ft in front of the rocket nose-cone tip. The DOVAP antenna system is located back on the rocket body. It consists of a receiving and a transmitting antenna, each of which is made up of two half loops mounted on opposite sides of the rocket and fed out of phase to produce the radiation pattern of a small loop antenna. The center of the antenna system lies at a point on the rocket body, 22.7 ft from the normal position of the center of the explosion.

Ground-test photographs and measurement of the time intervals between grenade ejection and detonation on the rocket flights indicate that the grenades do not always travel the normal distance before exploding. Apparently, sometimes the lanyard does not unwind completely before pulling the pin of the detonator.

POSSIBLE PHYSICAL PROCESS BY WHICH THE DISTURBANCE IS PRODUCED

Figure 3 is a diagram showing the Aerobee rocket and the manner in which an idealized spherical shock wave from the grenade explosion would expand as a function of time. The spherical shock wave is shown at distances of $n \cdot 3.328$ ft for values of $n = 1, 2, 3, 4, 5, 6, 7$. The total time of travel to each of these distances is indicated.

The grenade is shown in the position it would normally occupy at the time of detonation. The fact that there are shock waves from the grenade (before explosion) and from the conical nose tip of the rocket is also indicated.

Figure 4 illustrates the assumed scattering of the DOVAP radiation from the shock wave. Note that there is a direct 38-mc wave from the transmitter to the missile and a direct 76-mc wave from the missile to the receiver. Additional 38-mc and/or 76-mc scattered waves are shown.

Each of these waves will suffer Doppler effects. Assume that the velocity of the missile with respect to ground is v, and that the velocity of shock wave with respect to the missile is u; then we have the following radiated frequencies.

Transmitted from x, f_0

Received at Shock Wave

$$f^{I} = f_0\left(1 + \frac{u-v}{c}\right) \tag{1}$$

Received at Missile

$$f^{II} = f_0\left(1 - \frac{v}{c}\right), \qquad f^{III} = f_0\left(1 + \frac{u-v}{c}\right)\left(1 + \frac{u}{c}\right) \tag{2}$$

Transmitted from Missile

$$2\,f^{II}, \qquad 2\,f^{III} \tag{3}$$

Received at Shock Wave

$$f^{IV} = 2\,f_0\left(1 - \frac{v}{c}\right)\left(1 + \frac{u}{c}\right), \qquad f^{V} = 2\,f_0\left(1 + \frac{u-v}{c}\right)\left(1 + \frac{u}{c}\right)^2 \tag{4}$$

Received at the Ground

$$\left.\begin{aligned} f^{VI} &= 2\,f_0\left(1 - \frac{v}{c}\right)^2, \qquad f^{VII} = 2\,f_0\left(1 + \frac{u-v}{c}\right)\left(1 + \frac{u}{c}\right)\left(1 - \frac{v}{c}\right), \\ f^{VIII} &= 2\,f_0\left(1 - \frac{v}{c}\right)\left(1 + \frac{u}{c}\right)\left(1 + \frac{u-v}{c}\right), \qquad f^{IX} = 2\,f_0\left(1 + \frac{u-v}{c}\right)^2\left(1 + \frac{u}{c}\right)^2 \end{aligned}\right\} \tag{5}$$

Now

$$\left.\begin{aligned} f^{VI} \approx 2 f_o \left(1 - \frac{2v}{c}\right), \quad f^{VII} = f^{VIII} \approx 2 f_o \left[1 + 2\left(\frac{u-v}{c}\right)\right] \\ f^{IX} = 2 f_o \left[1 + 2\left(\frac{2u-v}{c}\right)\right] \end{aligned}\right\} \quad (6)$$

At the ground, the difference frequencies between 2 f_o and f^{VI}, f^{VII}, f^{VIII}, and f^{IX} are taken. These difference frequencies are:

$$D = 2 f_o \cdot \frac{2v}{c}, \quad D' = 2 f_o \cdot \frac{2(u-v)}{c}, \quad D'' = 2 f_o \left(\frac{4u-2v}{c}\right) \quad (7)$$

First,

$$2v = D \cdot \frac{c}{2 f_o} = D \cdot \lambda , \quad (8)$$

where

$$\lambda = \frac{c}{2 f_o} \text{ is taken as the "Doppler" wavelength.}$$

For each D cycle, the sum of the distances $r + r_i$ changes by a distance λ; or the distance of the missile with respect to the ground changes by a distance $\lambda/2$.

Second,

$$u-v = D' \frac{c}{2 f_o} \frac{1}{2} = D' \frac{\lambda}{2} , \quad (9)$$

and

$$u = D' \frac{\lambda}{2} + v .$$

For each D' cycle, the distance of the shock wave relative to the ground will have changed by $\lambda/2$, or the distance of the shock wave relative to the missile will have changed by $\lambda/2 + v \cdot T_1$ where T_1 is the period of that cycle of the frequency D'.

Third,

$$4u - 2v = D'' \cdot \frac{c}{2 f_o} = D''\lambda , \quad (10)$$

and

$$u = D'' \frac{\lambda}{4} + \frac{v}{2} .$$

For each D" cycle, the distance of the shock wave relative to the missile will have changed by $\lambda/4 + v/2 \cdot T_2$ where T_2 is the period of the frequency D".

THEORETICAL RELATIONS RELATING UPPER-AIR AMBIENT DENSITY TO THE SHOCK-WAVE PROPAGATION

Taylor has shown that when energy is released in a highly concentrated form, a spherical shock wave is propagated outward and that the distance traveled, R, is related to the time, t, by the equation

$$R = S(\gamma)\ t^{2/5}\ E^{1/5}\ \rho_0^{-1/5} \quad , \tag{11}$$

where $S(\gamma)$ is an unknown function of γ, the ratio of specific heats of the gas; E is the energy released; and ρ is the ambient density of the air. Solving for ρ_0, we can write

$$\rho_0 = \frac{[S(\gamma)]^5\ E \cdot t^2}{R^5} \quad . \tag{12}$$

Brode has obtained the solutions of differential equations describing spherical blasts in air by means of numerical integration. He has obtained solutions for explosions originating from

a) an instantaneous release of energy at a point,
b) isothermal spheres at high pressures, and
c) a detonation of a bare sphere of TNT.

The solutions are expressed in nondimensional form in terms of the initial total energy W, and initial pre-shock ambient conditions p_0, ρ_0, T_0, and E_0 (pressure, density, temperature, and internal energy of the ambient gas).

The numerical solutions for the differential equations are given in the form of graphs for the shock propagation as a function of time. One set of graphs for the solution of type (a) above shows dimensionless shock pressure $\pi = p/p_0$ as a function of dimensionless distance $\lambda = R/\alpha$, where $\alpha = (W/p_0)^3$. The shock wave is shown at many positions λ. The dimensionless time $\tau = tc_0/\alpha$ (where c_0 is the velocity of sound propagation) is given for each of these shock-wave positions. Thus it is possible to plot a curve of λ vs. τ from the data supplied by this set of graphs.

A plot of log λ vs. log τ is a straight line for $3 \cdot 10^{-2} < \lambda < 4 \cdot 10^{-1}$ and $2 \cdot 10^{-4} < \tau < 5 \cdot 10^{-2}$. For this region of λ and T we can write

$$\lambda = k\ \tau^n \quad . \tag{13}$$

The values of k and n can be determined from the log λ vs. log τ plot.

n is found to be equal to 0.4. Therefore we have:

$$\lambda = \frac{r}{\left(\frac{W}{p_0}\right)^{1/3}} = k\left[\frac{t \cdot c_0}{\left(\frac{W}{p_0}\right)^{1/3}}\right]^{2/5} = k\,\frac{t^{2/5} \cdot c_0^{2/5}}{\left(\frac{W}{p_0}\right)^{2/15}} \; ; \qquad (14)$$

then

$$\frac{r^5}{\left(\frac{W}{p_0}\right)^{5/3}} = k^5\,\frac{t^2 \cdot c_0^2}{\left(\frac{W}{p_0}\right)^{2/3}} \; ,$$

and

$$\frac{p_0}{W} = \frac{k^5 \cdot t^2 \cdot c_0^2}{r^5} \; , \qquad (15)$$

but

$$p_0 = \rho_0 R T_0 \qquad (16)$$

$$c_0^2 = \gamma R T_0 \; , \qquad (17)$$

where R is the gas constant, and γ is the ratio of specific heats of the gas. Thus,

$$\rho_0 = k^5\,\frac{\gamma \cdot W \cdot t^2}{r^5} \; . \qquad (18)$$

From the log γ - log τ curve we find $k^5 = 0.625$. Taking $\gamma = 1.404$, we find

$$\rho_0 = 0.878\,\frac{W \cdot t^2}{r^5} \; . \qquad (19$$

DENSITY DATA CALCULATED FROM RECORDS FOR SM1.01

Calculations of upper-air densities were made for the grenade explosions of the first of the ten Aerobee rockets at Fort Churchill. The very idealized picture of the explosion characteristics expressed by Eq. (19) was used. The value of W was taken to be $7.44 \cdot 10^{13}$ ergs. Values of r were taken in two ways.

1) $r = n(\lambda/4) + (v/2)T$ for any values of n for which the time interval could be measured.
2) The distance from the estimated point of explosion to the center line of the DOVAP antenna system.

Sample calculations are given in Table AI, and the results are plotted in Fig. 5. Also plotted in Fig. 5 are balloon-sonde data obtained within hours of rocket flight, and preliminary density data obtained in a flight of the sphere experiment at approximately the same latitude within one day of the flight of SM1.01 at Fort Churchill.

CONCLUSIONS

The technique of determining upper-air densities through the measurement of the characteristics of shock waves from grenades would be a valuable and desirable development of the present grenade-rocket experiment for upper-air temperatures and pressures.

The initial results calculated from the disturbance of the DOVAP cycle-count data on SM1.01 check well enough with known values of upper-air density to warrant further evaluation of the possibility of developing this technique.

It is proposed that the evaluation of this possibility, which entails

1) the continued investigation and study of the explosion phenomenon, the shock-wave propagation, and the interaction of electromagnetic radiation with the high-velocity shock front, and
2) the design, construction, and operation of experiments necessary to test out these theories,

may be used to satisfy the requirements for a Ph.D. thesis in Instrumentation Engineering.

TABLE AI

SAMPLE CALCULATIONS

Grenade No.	n	$\frac{n\lambda}{4}$ cm	T 10^{-3} sec	$\frac{v}{2}$ 10^{4} cm/sec	$\frac{vT}{2}$ cm	r cm	T^2	r^5	$\frac{T^2}{r^5}$	ρ	Altitude, km
1	2	202.8	0.43	4.91	21.1	223.9	$1.85\ 10^{-7}$	$5.52\ 10^{11}$	$3.35\ 10^{-19}$	$2.19\ 10^{-5}$	
	3	304.2	0.90	4.91	44.1	348.3	$8.1\ 10^{-7}$	$5.18\ 10^{12}$	$1.56\ 10^{-19}$	$1.02\ 10^{-5}$	
	Antenna	546.	4.30	--	--	546.	$1.85\ 10^{-6}$	$4.84\ 10^{13}$	$3.82\ 10^{-19}$	$2.51\ 10^{-5}$	
	Avg									$1.91\ 10^{-5}$	25.2
2	2	202.8	0.35	4.67	16.3	219.1	$12.25\ 10^{-8}$	$4.95\ 10^{11}$	$2.48\ 10^{-19}$	$1.62\ 10^{-5}$	
	3	304.2	0.78	4.67	36.4	340.6	$6.1\ 10^{-7}$	$4.49\ 10^{12}$	$1.36\ 10^{-19}$	$0.89\ 10^{-5}$	
	4	405.6	1.28	4.67	59.8	465.4	$1.64\ 10^{-6}$	$1.73\ 10^{13}$	$9.48\ 10^{-20}$	$0.62\ 10^{-5}$	
	Antenna	625.	5.08	--	--	625.	$2.58\ 10^{-7}$	$9.55\ 10^{13}$	$2.7\ 10^{-19}$	$1.77\ 10^{-5}$	
	Avg									$1.23\ 10^{-5}$	28.4

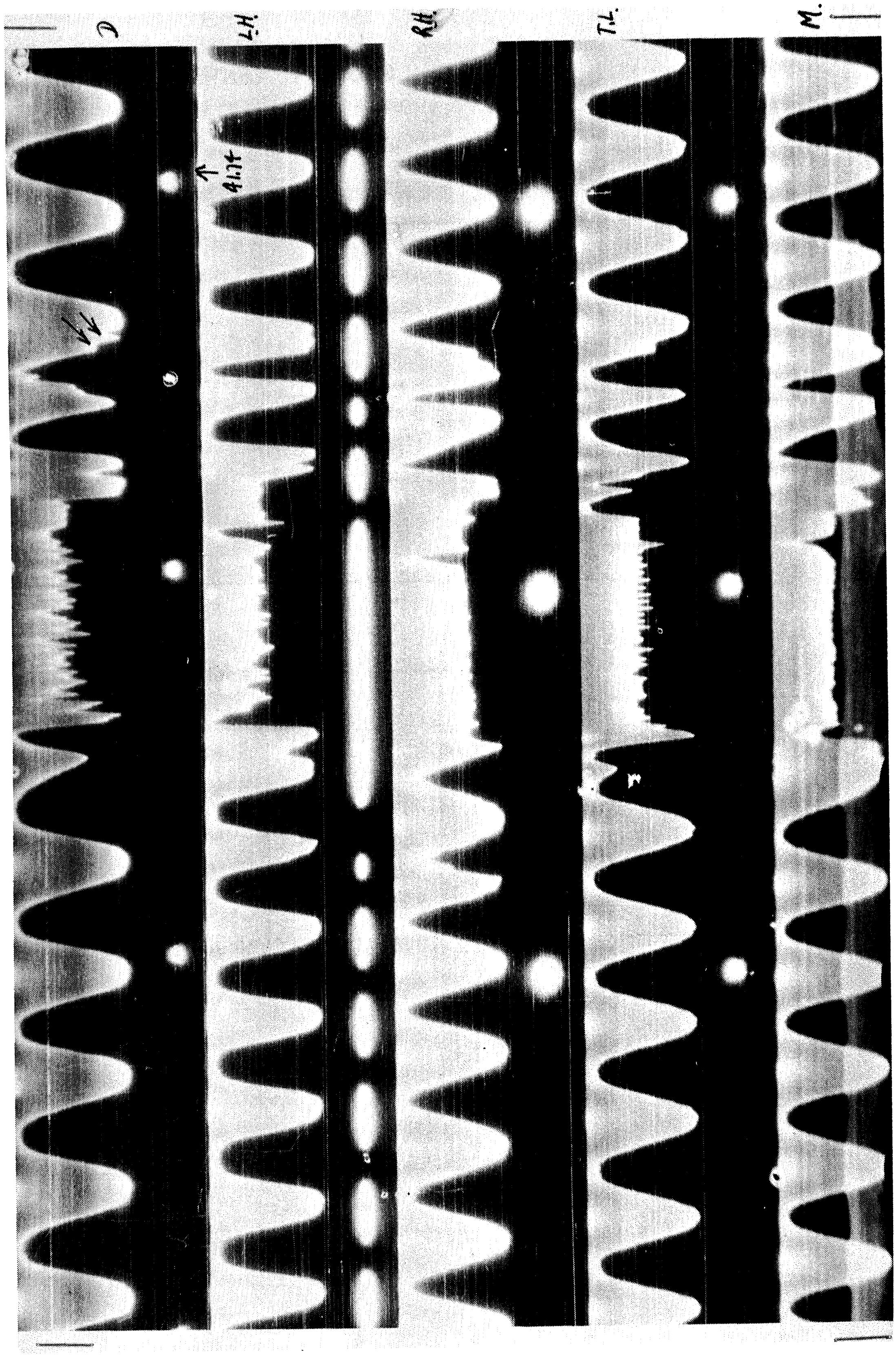

Fig. 1. Disturbance of DOVAP cycle-count record.

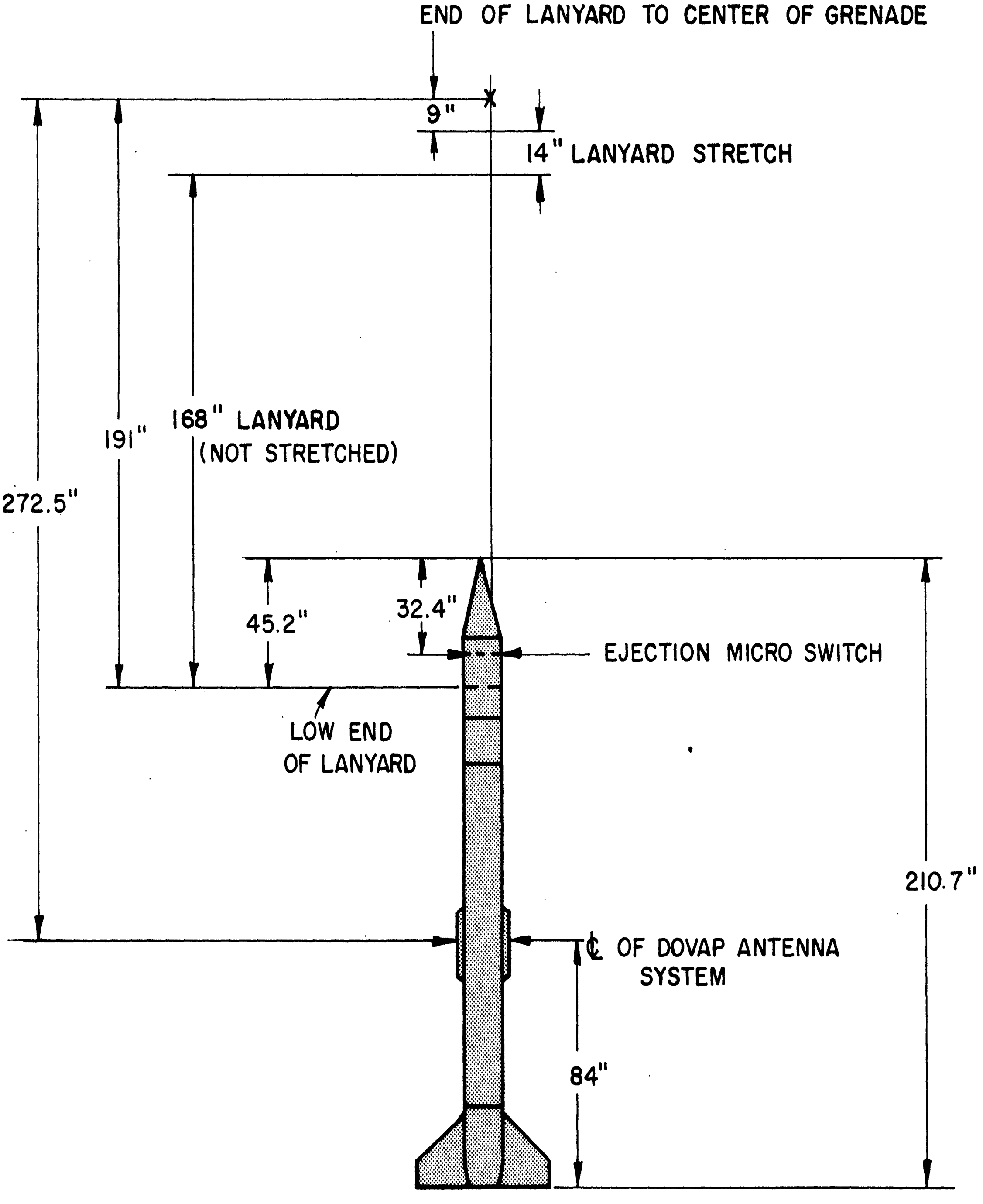

Fig. 2. Diagram indicating method of grenade ejection.

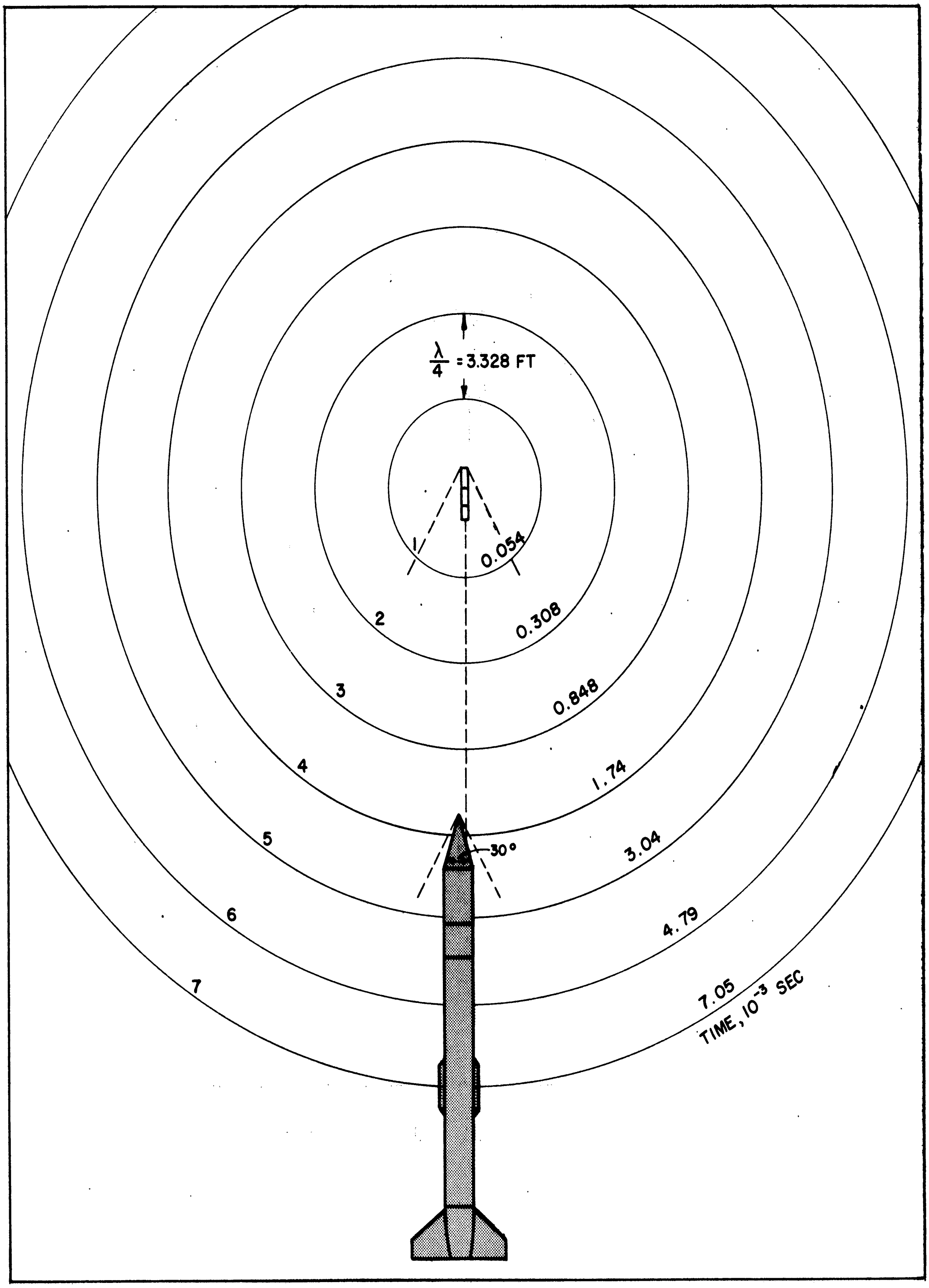

Fig. 3. Expansion of spherical shock from grenade explosion.

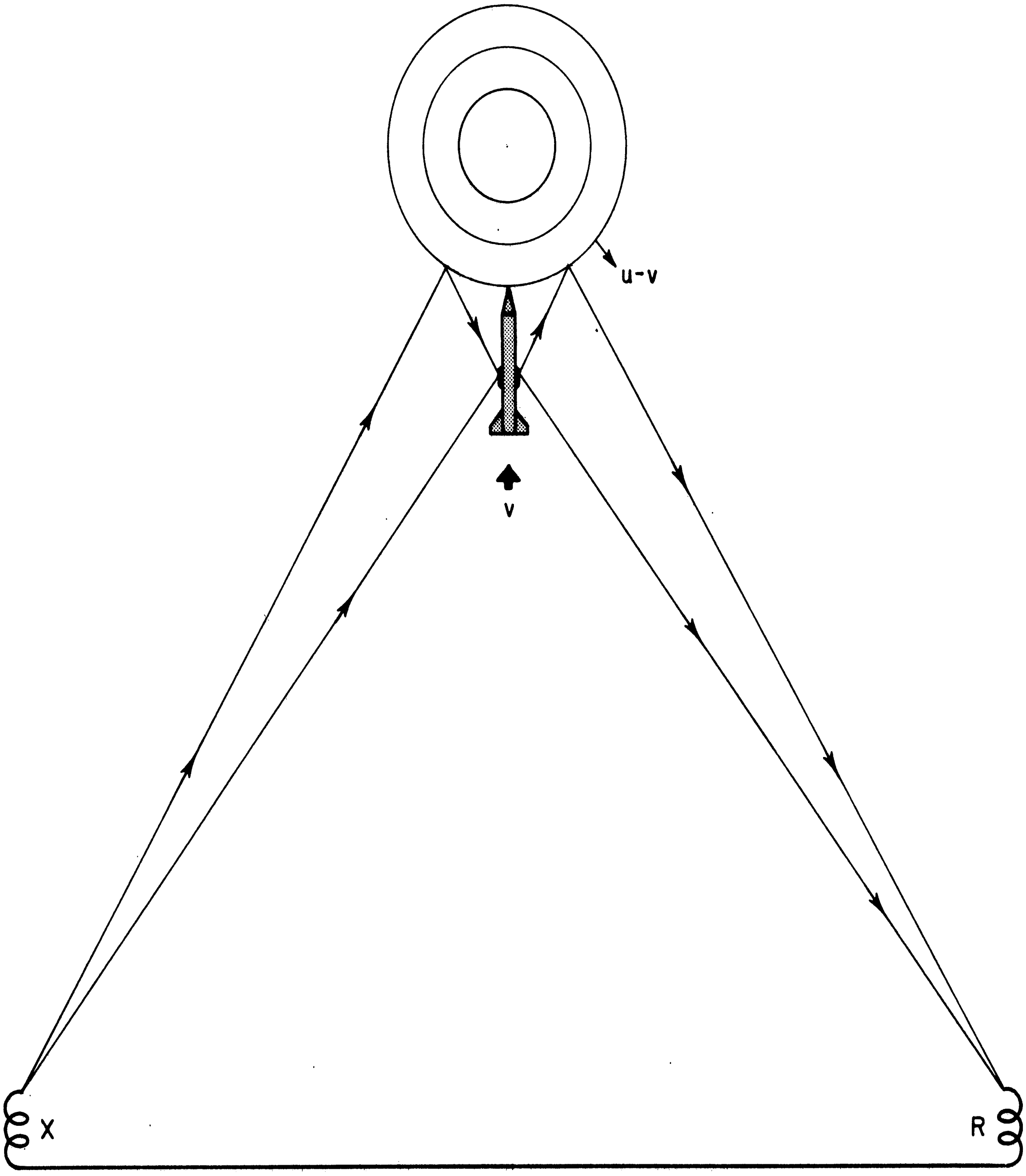

Fig. 4. Scattering of DOVAP radiation from shock wave.

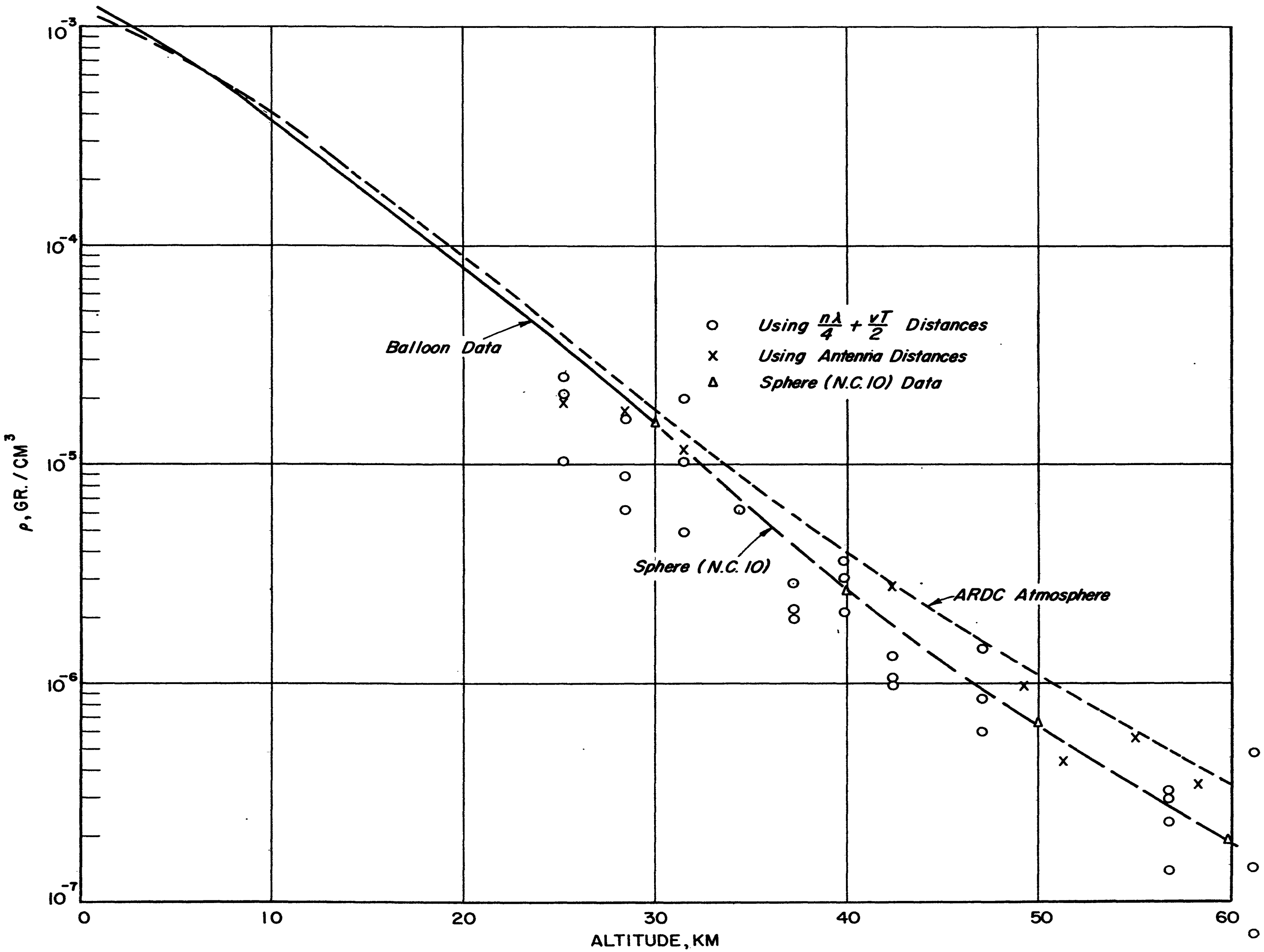

Fig. 5. SM1.01 density data. Calculated from shock-propagation theory (point source).