ASSET DIVESTITURE
AT HOMART DEVELOPMENT CO.

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ABSTRACT

For Homart Development Co. one of the most important strategic issues is scheduling divestiture of shopping malls and office buildings. In the 1986 strategic plan 170 assets were analyzed for possible divestiture over ten years. The problem is complicated by requirements imposed by their parent corporation, Sears, Roebuck and Co., that Homart meet a minimum aggregate return on equity each year. With these constraints all divestiture decisions are linked. An integer programming model designed especially for problems of this structure was developed and coded. Following the first run of the model additional profit of $40 million was achieved relative to the performance of the traditional techniques for solving the problem. Further, since the problems addressed by this model were strategic, results of the model changed the implemented policies at Homart's highest level of decision-making. In particular, policies toward sales of assets at a loss and the relationships between shopping malls and office buildings were changed as a direct result of the model. This model is now institutionalized as a core element of a six month strategic planning cycle.

Homart Development Co. is one of the largest commercial land developers in the United States. The firm currently owns or is developing 31 regional shopping centers, 18 major office buildings and is involved in land development of properties which total over 1000 acres. Homart is a consolidated wholly owned subsidiary of Sears, Roebuck and Company and occupies its own offices in Chicago, Illinois. Were the firm independent, Homart's assets would qualify it to rank in the Fortune 500.

Homart's business involves the full cycle of real estate operations. This cycle involves identifying opportunities for development, feasibility analysis, obtaining the necessary governmental approvals, overseeing design and construction, leasing and then managing the properties as long term investments. The properties currently involved in this cycle are of three types. Suburban office buildings are typically in the 200,000 to 400,000 square foot range and represent individual investments of between $20 to $40 million. Regional shopping centers are approximately one million square feet of selling area and represent investments of $60 million and up. Multi-use projects represent combinations of office, retail shopping, hotels and occasionally light industry. The total cost of multi-use projects runs into the hundreds of millions.

Homart maintains an unusually strong financial position which is predicated on its own corporate resources. Therefore, the corporation is not usually financially constrained in its investment decisions. Additionally, Homart restricts its development activity to premium properties suitable for institutional investment. Since the supply of such properties is not extensive, there are few situations in which development decisions arise. Due to these conditions, Homart is able to evaluate each project independently using traditional financial analysis based on discounted cash flow techniques.

Once a project reaches a fully leased economic steady state, the property moves to an investment status. During this period, the risk, management resources required, and the incremental return are all reduced. Since financial resources are not a constraint, there is no need to divest in order to free capital to pursue other development opportunities. Rather, independent divestiture decisions are made on the basis of economic performance.

Homart maintains an arm's length relationship to its parent organization, in effect, functioning as an independent strategic business unit. Although it is a member of the Coldwell Banker Real Estate Group, it pays full commissions for all brokerage it receives. In those cases where Sears assumes a position in a Homart shopping center, Homart engages in the same negotiation process, requires the same legal documentation, and receives payment in the same manner as from other department stores. This posture permits Homart to be continuously evaluated as an independent business entity subject to the competitive constraints of its market.

Homart is a consolidated subsidiary of Sears. This position requires that both the assets and liabilities of Homart be combined with those of its parent and be represented on the parent firm's financial statements. This does not effect the strategic business unit character since Homart does not materially distort the balance sheet of its much larger parent. Dividend payments are made to the parent in the form of a check rather than throûgh an inter–company transfer. Further, following general business practice, not all earnings are paid out in the form of dividends with some retained for reinvestment in the business.

Return on Equity Requirements

While certain types of divestiture decisions can be made independently, others demand that the entire portfolio be considered simultaneously when arriving at a divestiture decision. One such case arises as the result of the nature of Sears' equity. As a publicly held corporation, Sears must remain keenly aware of the impact which Wall Street can have on the value of its outstanding stock. One facet of this evaluation rests on the reliability and consistency of book earnings.

While Homart's financial performance is consolidated with other divisions of Sears, its significance is sufficient to require that it be broken out and reported in summary form in the footnotes to the financial statement. Interested financial analysts have access to the independent performance of the subsidiary. Since a portion of Homart's earnings is associated with the divestiture of assets, analysts can interpret the earnings as being of an "extraordinary" character and thus not expected to reoccur in the future. If this determination is made, the analyst or investor would not weight this contribution as heavily in estimates of future earnings expectations (Masch [1986]). This decision would, in turn, dilute the value assigned to the stock since that value is a function of future earnings expectations. Thus, the regularity of earnings has a direct and material influence on shareholder wealth.

In an effort to maintain an appropriate valuation of Homart's earnings, Sears has prescribed a lower bound on the return on equity which the subsidiary must achieve each year. By demonstrating a consistency in earnings year after year, the discount which is applied to "extraordinary" gains is minimized, the value of the stock improved, and immediate shareholder wealth augmented.

The imposition of these requirements significantly alters the resulting divestiture schedules and return flows. Figure 1 shows the return requirements, return flow of the optimal solution with the requirements imposed, and return flow of the optimal solution ignoring the return requirements. The imposition of the return requirements achieves its goal of leveling the return flow. The impact section will discuss the economic cost of this policy.

It is significant to note that return on equity is an accounting concept and does not necessarily coincide with the management scientist's definition of "economic" maximization. The accounting concept of profit takes as its base the initial cost of the asset less any depreciation or amortization which has occurred over the period it was held — whether or not that decline in value has actually been realized. Two assets which have been purchased and sold for the same amounts can have radically different book profit effects. A simple illustration of the effect is obtained from a firm which buys and holds a parcel of land which is not subject to depreciation and a building which is depreciated. Both are held for 10 years with no interim earnings occurring on either. This example yields the discrepant result in Table 1.

Reports of accounting profits are required by both Wall Street Analysts and the Internal

Revenue Service. However, the firm accepts as an operating goal the maximization of the net present value of the cash value of the firm at the end of any given planning horizon. Thus, the strategic plan attempts to satisfy the return on equity requirement in a manner that minimizes any adverse effect on the long term net present value of the firm. The net effect is to reconcile the short and long term interests of the firm.

Operating earnings from rents, fees and other activities of the firm are sometimes insufficient to meet the return on equity requirement prescribed by the parent. In these cases the subsidiary must consider selling a property. The sale of an operating property diminishes the operating income and thus can influence the need for other such actions in the future. The identification of a property to be divested must consider its future earnings effect. In addition, knowledge that a property is to be sold provides opportunities for "grooming" which can influence the value received from the sale. Thus there are multiple incentives for planning an optimal divestiture strategy well into the future.

In the past, divestiture planning was accomplished by hand with participation extending as high as an Executive Vice President, and involving the significant participation of the financial planning staffs. The size of the portfolio of mature properties and the relatively short 5 year planning horizon, permitted the identification of feasible solutions. However, optimality was clearly at question even from the perspective of superficial examination. The problem became more complicated as the planning horizon extended to 10 years and less mature properties became an increasingly significant proportion of the portfolio. A less mature property typically yields less book profit. Thus, multiple sales may be required to achieve a given level of return. The potential combinations of sales quickly extend beyond the ability of traditional "what if" approaches to test even a small proportion of the universe of possibilities. In 1985, for example, the problem involved 170 assets which were available to be scheduled for sale over the next 10 years.

The complexity of the effort is perhaps best illustrated by an example. An initial divestiture schedule was constructed and evaluated for book return. If it did not meet the requirements in some year an asset sale would be moved to cover that year. If alterations of this type did not produce a feasible schedule, assets would be moved, not to cover a requirement, but to free another asset to be moved to cover a requirement. By use of such two level switches, at best, feasibility was achieved. Optimality would require exploration of three, four or even higher levels of switches. Due to the large sums of money involved in each divestiture and the long time periods affected by the divestiture, the cost of deviation from optimality easily runs into tens of millions of dollars.

The traditional approach had another important failing. Long range strategic planning requires a variety of behavioral assumptions for items which can affect the economic performance. Because the optimality of sales could not be assured, it was impossible to run the models under varying economic and policy scenarios and be assured that the effects observed were attributable to those changes. An unknown portion of the effect was always attributable to variations in planned sales strategies.

In December of 1982, a project was initiated to determine the feasibility of a solution to the problem using mathematical optimization techniques. Sufficient theoretical and empirical evidence was assembled that a full scale development of an Asset Sale Model commenced in the fall of 1983.

Model Specification

The discussion above led to the following assumptions for a model.

1) All asset potential value profiles from both continued operation and sale are assumed to be deterministic and known.
2) No assets need to be sold to raise capital for more attractive investments. Acquisition of assets is dealt with outside of this model.
3) During each year, a specified dollar return on equity must be met. This return can be met by operating income from the assets and capital gains from selling the assets. Any excess return from one year cannot be carried to the next for purposes of meeting return on equity requirements. These return requirements include both an annual base return requirement, and any one time cash demands from Sears.

The first assumption allows a deterministic analysis. Though the actual problem is stochastic, this assumption was considered acceptable for several reasons. First, commercial real estate transactions are reasonably predictable since sale prices are largely based on the rent flows and leases run from seven to fifteen years. Second, the current hand constructed analysis assumed deterministic data. Hence, that data was available and its use represented no deterioration relative to current practice. Further, resolving the combinatorial aspects of the problem was considered more important than the stochastic aspects. Solution of the stochastic combinatorial problem was considered intractable. Instead, the model is used for sensitivity analysis on the random effects.

The second assumption allowed us to concentrate on divestiture without linking acquisitions to each decision. It implies that the company has a sufficient cash pool from operations to develop desirable projects. Were the company funding constrained, assets could be divested to provide resources for an attractive acquisition opportunity. Acquisitions were included in the model only in that new acquisitions during the study period were included as prototype or "average" assets. For example, a prototype asset developed two years from now is considered for divestiture near the end of a ten year study.

The third assumption drives the model. Without it the problem decomposes to traditional financial analysis. With the return on equity requirement, a proposal to sell an asset has financial impacts on all years of the model; before, during, and after the sale.

A simplified example of the effects of an asset sale will illustrate the process which must be simulated in the Asset Sale Model. Prior to sale a property contributes to both book and cash income through the tenant's rent payments, net of operating and other expenses. The cash is invested to earn interest, an investment which generates both book and cash income. This book income, in addition to the income earned from the rent, works toward meeting the firm's return on equity requirements for the year. By agreement with the parent, a specified portion of the book earnings is declared as a dividend and is paid to the parent in cash in the period in which it is realized. The total book earnings, less the dividends, increase the equity base and, consequently, increase absolute dollars necessary to meet the percent return on equity requirement.

At the point of sale, the process is the same as described above, though with much larger sums of money. Book and cash income are generated, dividends are paid, excess cash is invested and the equity base is adjusted. It is desired that the return on equity be as close as possible to the lower bound. Any amounts over that will increase the equity base and establish a higher and potentially unattainable target for the succeeding year.

Due to the complexity of the transactions and the need to simultaneously maintain both cash and book accounting records, the objective function in the model is an approximation. A Corporate Financial Model evaluates the output of the Asset Sale Model as a final validity check. The details of this evaluation are discussed in the implementation section.

The divestiture decisions were viewed as an assignment of one of several choices to each asset. In a T year problem, the asset could be sold in year one, sold in year two, or any year up to year T, or could be held through the study. If for some policy reason a subset of these alternatives is not acceptable, they can simply be deleted from the list. This allows implicit implementation of requirements such as a lower bound on the years an asset is held.

This interpretation of the problem allows us to formulate it as a multiple choice integer program. The multiple choice integer program is a zero–one problem where the variables are separated into sets so that each variable is in exactly one set. For the asset divestiture problem we let $x_{is} = 1$ if asset i is sold in year s, and 0 otherwise. All variables associated with a particular asset are grouped together in the same set. The model requires that any feasible solution assign the value one to exactly one variable in each set. This requirement is satisfied for the asset divestiture problem since an asset can be sold in at most one year. The Appendix details the full formulation of the problem.

Solution Methodology

The multiple choice integer program has been discussed often in the recent literature (see Bean

[1984], Sweeney and Murphy [1981], Martin [1980], Martin, Sweeney and Doherty [1982]). There have also been several successful applications of this structure to a wide variety of problems such as catalogue space planning (Johnson, Zoltners and Sinha [1979]), sales resource allocation (Zoltners and Sinha [1980]), and production scheduling (Bean, Birge, Mittenthal and Noon [1984]).

The solution technique used in this application is that described in Bean [1984]. This is a branch-and-bound approach using a branching technique specially designed for multiple choice problems. The branch-and-bound tree used for this problem has the characteristic that going forward from any node corresponds to fixing all of the variables associated with one asset; one at 1 and the others at 0. In a general binary tree only one variable would be fixed at a time. Conceptually, this approach reduces the problem from $N(T+1)$ decisions to N decisions. In the general tree there are $2^{N(T+1)}$ branches, whereas, the tree employed by this technique has only $(T+1)^N$ branches. Hence, when the problem is solved with this algorithm the number of potential solutions to be investigated increases exponentially in the number of assets, but only polynomially in the study horizon.

This basic procedure is enhanced two ways. The variable reduction techniques of Sweeney and Murphy [1981] are used to eliminate variables from consideration a priori. Reduced costs are computed from the optimal solution to the linear programming relaxation of the integer program. For any two variables corresponding to the same asset, if the difference in the reduced costs is wider than some critical value, the variable with the higher reduced costs (lower profit) is eliminated from the problem. If the critical value used is not less than the difference between the optimal values of the integer program and the linear programming relaxation, then this variable elimination cannot eliminate an optimal solution. Hence, there is no loss and the problem can be made substantially smaller.

Since the optimal value of the integer program is not known a priori, this result is implemented heuristically by estimating the gap between the optimal values of the integer program and the linear programming relaxation. If an error is made it is recognized and a bound on the possible loss reported. If the bound is unacceptably large a new critical value can be entered and the problem solved again. In practice the critical value is simple enough to estimate that it virtually never causes error.

For the resulting smaller problem a Lagrangian version is solved that greatly speeds up the computation times (see Fisher [1981] or [1985]). The result of this Lagrangian approach is a heuristic algorithm that produces very little error. In divestiture problems the upper bound on error is on the order of one half of a percent.

The Lagrangian problem solved is not a Lagrangian relaxation, but a Lagrangian alteration of the objective coefficients. A penalty is added to the objective for violating the return constraints.

While in the more common Lagrangian relaxation these constraints would then be relaxed, in Lagrangian alteration the feasible region is kept in tact. This Lagrangian alteration significantly improves the computational efficiency of the algorithm in the following manner. The branch-and-bound routine chooses nodes by highest profit. This directs the search procedure toward divestitures with high profit, regardless of their effect on the return constraints. As mentioned earlier, the constrained solutions differ significantly from the unconstrained. Hence, the algorithm would waste a great deal of time chasing infeasible solutions. After doing a Lagrangian alteration, the new "profits" reflect both profits and contribution toward feasibility. Near optimal solutions are then discovered very early in the process.

Though altering the objective coefficients produces a slightly different problem resulting in the small errors mentioned above, the system is quite good at reporting bounds of the error. Since no relaxation is done, the solution to the altered problem is feasible in the problem of interest.

The Study Horizon

As with any dynamic optimization problem, the study horizon must be chosen carefully to avoid contamination of early decisions by end-of-study effects. These difficulties can arise because the model believes that the world is coming to an end at the study horizon. This problem is partially alleviated by adding salvage values to the objective coefficients for variables representing the choice not to sell some asset during the study. These values, however, are simply approximations of the actual salvage values which would take into consideration the return constraints for years beyond the chosen horizon.

It is impossible to eliminate the end-of-study problem completely. However, if these effects can be confined to decisions far in the future the immediate decisions can be implemented without loss. The model will be rerun with updated data before the future decisions are implemented. Results from Bean and Smith [1984] can be used to show that, for almost all interest rates, there exists a sufficiently distant horizon that planning to that time or beyond will eliminate end-of-study effects from at least the first year's divestiture schedule. The length of the horizon is a function of specific problem structure and data. For the Asset Sale Model the horizon is typically three years.

This structural result can be used algorithmically to solve real problems. A study horizon is chosen that frees the first year of error. A problem is solved using that horizon. The first year's schedule is fixed at the first year optimal solution and the rest of the solution ignored. The process rolls forward one year and repeats, constructing the second year solution. This technique is well studied in production planning (see Baker [1977]).

The difficulty in such a procedure is finding a horizon that is long enough to eliminate error and short enough to result in a computationally tractable problem. We approached this problem by solving ever longer problems and observing the changes in the first year's schedule. It was observed that the schedule settled down very quickly. Table 2 presents the results of this study.

We chose a rolling horizon of four years. Hence, to solve the desired ten year horizon we solved six four year problems. The last problem solved covered years six through ten and all schedules were kept. Though this potentially led to higher error in the last few schedules, error ten years in the future was considered permissible.

Validation

Any model of a real process involves assumptions and approximations. The model must be validated to determine if these assumptions and approximations are close enough to reality to allow us to draw conclusions about the real system from the model results. A common approach to validation is to run the model on historical data and compare the model results with actual outcomes. For this model such historical data was not readily available since the traditional process did not use some of the data required by the model. The missing data could have been generated, but such a process was deemed prohibitively expensive.

In lieu of statistical validation, a small but representative subset of the assets was selected for algorithmic development and model validation. The results of the model were reviewed and approved by the appropriate experts.

We also sought to validate the capability of the code to solve problems of realistic size. Integer programs are notorious for computational performance that varies greatly with small changes in the data. We felt it necessary to test the sensitivity of computation for this class of problems varying data and problem size.

Assets were added to create problems ranging from 10 to 25 assets. For each problem size a set of five problems was constructed by randomly perturbing the data of the actual problem. Table 3 shows CPU times in seconds on the Amdahl 5860 at The University of Michigan.

Note that computation times for these small problems decreased as the number of assets increased. This occurred since the duality gap, the difference in the optimal values for the integer program and its linear programming relaxation, is large. Hence, the linear program gives very poor bounds and branch-and-bound becomes very inefficient. Duality gaps in integer programs often occur because tight constraints can have slack (this cannot happen in a linear program). This occurred in the Asset Sale Model because an asset is typically sold to make up a shortfall in returns. If there are few assets the return goal may be exceeded significantly resulting in slack. As more assets are added, greater choice results in smaller excess returns and better bounding. Hence the algorithm takes fewer iterations and less time to solve a larger problem.

Eventually, as the problems continue to grow larger, computation times again increase. However, by this time excess returns are small. Computation does not increase nearly as fast as its exponential bound would suggest. We concluded that the full 170 asset problem could be attacked with the appropriate preprocessing and decomposition.

Implementation

Following this extensive testing the model was used for the 1986–95 strategic plan. The implementation phase was carried out during the summer of 1985 at Homart.

The incumbent system consisted of two modules: an Asset Evaluation Model that predicted the cash flows, book flows, and potential proceeds for all divestiture alternatives; and a large Corporate Financial Model. These modules had been used for prior strategic planning cycles. Part of the input to the Corporate Financial Model was the divestiture schedule that, to date, had been constructed by hand. Our task was to insert the Asset Sale Model, the third module, into the system to provide this crucial input. The information flows between the three modules are depicted in Figure 2.

The Corporate Financial Model served two purposes. It included all the intricate financial structure of the corporation in detail beyond the scope of the Asset Sale Model. Hence, divestiture schedules were run through the Corporate Financial Model to explode out the financial detail. Also, the Corporate Financial Model served as the last check on the Asset Sale Model. If its evaluation of a divestiture schedule differed greatly from the approximations of the Asset Sale Model, the quality of the divestiture schedule would be questioned. If they agreed, then the objective approximation in the Asset Sale Model would be externally validated.

In the early runs there were substantial discrepancies caused by incompatibilities in data definitions and assumptions. For example, the Asset Sale Model initially assumed that selling a shopping mall involved the mall only. At the same time the Corporate Financial Model assumed it implied sale of the mall and all peripheral land around the mall. In another case one model assumed that taxes are paid annually, while the other assumed quarterly payments. Roughly a dozen such differences were reconciled during the implementation process.

While the testing was done on an Amdahl 5860 at The University of Michigan, the system was to be run on the Sperry–Univac 1100 72/H2 at Homart in Chicago. Since this computer is much smaller than the Amdahl, we were unsure how the computational experience would transfer. Computation times were substantially longer on the Sperry. However, greater computing time was also available.

The first run was set to execute at night to evaluate computation. The operating system in use was not accustomed to jobs of this type (massive). It was trained that if a job took a long time it must be very important and should be given ever larger portions of the computing power and core. By morning the system believed that this was the most important job that had ever been run.

As secretaries arrived for work they did not understand that this would not be a day like any other. As the job crunched into the morning the operating system dedicated nearly all computing resources to it. There was no word processing, no electronic mail, and no accounting at Homart. The operators had watched the job devour their machine but, never having experienced such a monster, were unable to kill it. One finally called a vice-president and exclaimed, "its at 800k and still growing!"

After much streamlining we were able to run the model without disrupting other computer operations. Currently, each four year problem takes on the order of one hour to solve. For a system used twice a year, with the potential of saving millions of dollars at each use, this was considered satisfactory.

Finally, in August 1985, the three modules merged into a strategic planning system. The output was carefully analyzed by the financial staff and the corporate officers. With few changes, the output became a major component of the 1986–95 strategic plan. The executive vice-president of Homart has stated the intent to use the model twice a year for strategic monitoring as well as planning purposes. Personnel have been trained to operate the system. Operation of the system has been written into performance goals. The Asset Sale Model has been institutionalized at Homart.

Impact

The impact of this model on Homart has been profound on several levels. On a technical level, the divestiture problem had grown to the point that quality of the schedule was not as great an issue as the ability to construct any feasible schedule at all. This was exacerbated by a mandate from Sears to extend Homart's strategic plan from the historic five year horizon, to a ten year horizon. Fortunately, the model had been under development for two years at this time. The Asset Sale Model provided feasible solutions in a timely manner.

Direct savings from improved schedules are difficult to estimate since a hand constructed schedule for 1986–95 was never created. Further, since no ten year horizon problem had ever been solved comparisons with past solutions were not valid. To estimate improvements we altered the model to mimic the traditional procedure. The cash net present value objective function was discarded. The branch-and-bound search was directed to choose scenarios that helped cover the return requirements. If a particular pattern led to a divestiture schedule that did not cover the returns, the procedure backtracked and tried some other combination. This is roughly the process used in the traditional method. From this analysis the increased profit was estimated at $26 million, before taxes.

An intangible impact of the project was increased communication between departments of the company involved with this process. The model served as a focus of communication that forced implicit assumptions into the open and allowed them to be analyzed. It defined the set of important data so that data refinement could be limited to areas where it had an impact. Even if the output of the model had not been used, this alone would have improved the quality of the resultant plan.

Another unusual gain on a technical level was the integration of accounting and economic measures within a single context. These represent two distinct, though reconcilable, methods of measuring value. In this sense the asset divestiture problem can be viewed as a multi-criteria problem.

Of much greater importance was the model's impact on a strategic policy level. On this level, the Asset Sale Model's contribution again turned on near optimality and speed of response. For example, for the first time, estimates are available of the lost profit due to imposition of the return on equity requirements. The model was run with and without the return requirements. On a percentage basis, the loss from requiring stable returns was on the order of 1.9% of the value of the portfolio. Given the substantial improvement in income flow depicted in Figure 1, the decision to impose such constraints does not seem unreasonable.

This relatively small financial impact does not imply that these constraints posed a simple problem for Homart. To obtain these results the divestiture schedules were changed substantially. In no year do the divestiture schedules agree between the constrained and unconstrained models. Less sophisticated techniques may have resulted in large errors. Not only could there have been a large monetary loss, but the loss traceable to the return requirement would remain inaccessible to analysis. By quantifying the costs, the policy decision can be continuously reevaluated under changing economic and capital market conditions.

The strategic impact of this model can be measured by changes in Homart's implemented policies. Many of these impacts were unforeseen at the beginning of the project. They have occurred because of the ability to do "what-if" analyses on policies that had never been evaluated before.

For example, Homart has always operated under the policy that projects were not to be divested at a loss. Homart was strong enough to keep them forever (they do not have many losers) on the theory that improvement could be realized some time in the future. The true cost of this policy had never been evaluated since there had been no efficient way to evaluate the alternatives of holding versus selling. The Asset Sale Model was easily modified to make this comparison. The model was run with and without this policy and the results were startling. The present value cost of this policy over ten years was in the order of $14 million. Hence, the total tangible savings from the first run of the model were $26 million from improved divestiture schedules, plus $14 million from this change in policy, totalling $40 million, before tax, over ten years.

A more subtle change in the strategic process is the perception of the divestiture problem as a portfolio problem rather than a collection of individual assets. Assets can now be discussed, not only in terms of their individual financial characteristics, but in terms of their effect on returns relative to the remainder of the portfolio. In particular, there is a new understanding of the roles of shopping malls and office buildings in the Homart portfolio. Shopping malls are nearly always kept longer than office buildings. Several future divestitures have been changed due to this improved understanding.

The increased schedule construction speed afforded by this model has led to a significant restructuring of the strategic planning process at Homart. In the past, divestiture schedules were constructed every three years with the strategic plan. In the third year of this cycle, assets were divested based on decisions made without recent portfolio information. Homart is now committed to reconstruct schedules every six months. No estimate on the savings due to this change in structure will be available for several years. However, it can be conservatively evaluated in the tens of millions of dollars.

Another impact of this model is the change in attitudes of senior management. Real estate is an entrepreneurial business which has had little use for management science techniques at any level. This model has reached the highest levels of the corporation. Its output has been reviewed personally by the president and vice–presidents and has been discussed at the board of directors level. The success of the project has shed an entirely new light on the contributions available through the use of quantitative techniques. Senior management believes that management science can have an impact at the highest levels of their business and are open to future projects. In fact, in its strategic plan, the corporation has a stated strategy of exploiting its quantitative capabilities as a means of achieving a competitive advantage.

As evidence of this new direction, projects are currently underway to develop an integer programming based system for determining the optimal mix of merchants for new or reconstructed malls. While this project is in the prototype stage, test runs have resulted in several substantive changes in current mall plans. Full implementation has been accepted as a goal for the 1986 calendar year.

On another front, dynamic programming is being used to solve stochastic office building sizing problems. As has been reported for the last few years, there is a glut in office space in many markets. This project seeks to improve the techniques used to size and sequence office development to avoid these problems and maximize the returns on the investments made.

Transportability

The integer programming approach to divestiture described here could be used on many similar problems. The key to its transportability is the acceptance of general linear constraints. No two real problems are identical. Hence, to be transportable a model must be able to handle the side constraints that vary from problem to problem. Unlike most special structure discrete optimization models, the multiple choice integer program can incorporate any linear inequality.

Conclusion

Scheduling asset divestiture with return on equity requirements is an important strategic problem at Homart Development Co. We have addressed this problem with an integer programming based rolling horizon procedure that has had substantial impact on the strategic policies of this corporation. Due to the strategic nature of the problem it is difficult to measure this impact in dollar terms. We have estimated the monetary effect of two aspects of this impact at $40 million. This figure is dwarfed by the impact of the model on strategic policies. In particular, a stringent policy not to sell assets at a loss has been analyzed and revoked. This is resulting in substantial changes in the schedule of assets to be divested in the next few years. The relationship between shopping malls and office buildings is now better understood. Several divestiture plans will be changed due to this new insight.

The strategic planning process itself has been greatly affected. In the past divestiture planning was done only every third year. With the model it is now institutionalized as a bi-annual process. Improved speed and accuracy allows sensitivity analysis on economic factors. This project has succeeded where few have at impacting decisions made in a corporate boardroom. This combined with savings over $40 million have made this project a success from the point of view of the field of management science and of the corporation.

APPENDIX : Model Details

Notation

The parameters i and s index an asset sale choice. The parameter t is the running index of monetary flows.

Parameters

1) c_{is}: net cash present value profit from asset i for the scenario where it is sold in year s

2) $a_{is}(t)$: contribution to book returns in year t for the scenario of selling asset i in year s

3) N: number of assets considered for divestiture

4) T: number of years in the study horizon

Decision Variables

1) x_{is}: has the value 1 if asset i sold in year s, 0 otherwise

Note that the model allows for separate profiles of return, or so called book value, and actual cash. Typically the return constraints deal with the book values whereas the profit calculations are done with the cash figures.

Formulation

The definition of the decision variables above describes much of the modeling approach taken here. For each asset held at the beginning of the study there are $T+1$ possible decisions. The asset can be sold in year $1, 2, \ldots, T$ or, as choice $T+1$, can be held through the end of the study. An asset may be sold during the study for either of two reasons: it is economic to do so, in which case traditional analysis would have indicated the same divestiture strategy; or to satisfy return constraints.

The model can be stated mathematically as follows:

$$\text{maximize} \sum_{s=1}^{T+1} \sum_{i=1}^{N} c_{is} x_{is}$$

$$\text{subject to :} \sum_{s=1}^{T+1} \sum_{i=1}^{N} a_{is}(t) x_{is} \geq R(t) \quad \text{for } t = 1, 2, \ldots, T$$

$$\sum_{s=1}^{T+1} x_{is} = 1 \quad \text{for } i = 1, 2, \ldots, N$$

$$x_{is} \epsilon \{0, 1\} \text{ for all } i, s.$$

The first set of constraints are the yearly return requirements. The second set insure that exactly one of the viable options is chosen for each asset.

When Lagrangian alteration is performed on the problem above the term

$$\sum_{t=1}^{T} \lambda(t)[\sum_{s=1}^{T+1} \sum_{i=1}^{N} a_{is}(t) x_{is} - R(t)]$$

is added to the objective. Also, the new variables, $\lambda(t)$, are restricted to be non-negative.

REFERENCES

Baker, K. [1977], "An Experimental Study of the Effectiveness of Rolling Schedules in Production Planning," **Decision Sciences**, Vol. 8, pp. 19–27.

Bean, J. [1984],"A Lagrangian Algorithm for the Multiple Choice Integer Program," **Operations Research**, Vol. 32, pp. 1185–1193.

Bean, J., J. Birge, J. Mittenthal, C. Noon [1984], "An Adaptive Approach to Real-Time Scheduling," Technical Report 84-18, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109.

Bean, J. and R. Smith [1984],"Conditions for the Existence of Planning Horizons," **Mathematics of Operations Research**, Vol. 9, pp. 391–401.

Fisher, M. [1981],"Lagrangian Relaxation Method for Solving Integer Programming Problems," **Management Science**, Vol. 27, pp. 1–18.

Fisher, M. [1985], "An Applications Oriented Guide to Lagrangian Relaxation," **Interfaces**, Vol. 15, pp. 10–21.

Johnson, M., A. Zoltners and P. Sinha [1979],"An Allocation Model for Catalogue Space Planning," **Management Science**, Vol. 25, pp. 117–129.

Martin, K. [1980],"Branching Strategies for Integer Programming with Special Ordered Sets," Presented at ORSA/TIMS National Meeting, Colorado Springs.

Martin, K., D. Sweeney, and M. Doherty [1982],"The Reduced Cost Branch-and-Bound Algorithm for Mixed Integer Programming," Working Paper, Graduate School of Business, University of Chicago.

Masch, V. [1986], private communication.

Sweeney, D. and R. Murphy [1981],"Branch-and-Bound Methods for Multi-Item Scheduling," **Operations Research**, Vol. 29, pp. 853–864.

Zoltners, A. and P. Sinha [1980],"Models for Sales Resource Allocation," **Management Science**, Vol. 26, pp. 242–260.

TABLE 1: Accounting Values

	Land	Building
Purchase Price	$1,000	$1,000
Depreciation	0	(500)
Book Value	1,000	500
Sale Price	2,000	2,000
Book Profit	$1,000	$1,500

TABLE 2: Effects of Horizon on First Year Divestiture

1 year	2 year	3 year	4 year	5 year	6 year
A	A	A	A	A	A
	B	B	B	B	B
	C	C	C	C	C
		D	D	D	D
	E				
F					
G					

Each column lists assets chosen for first year divestiture using the horizon for that column.

TABLE 3: CPU Seconds on Amdahl 5860

	Fixed Horizon	
	4 Years	5 Years
10 Assets	.670	2.577
15 Assets	1.580	.932
20 Assets	.144	.856
25 Assets	.488	.562

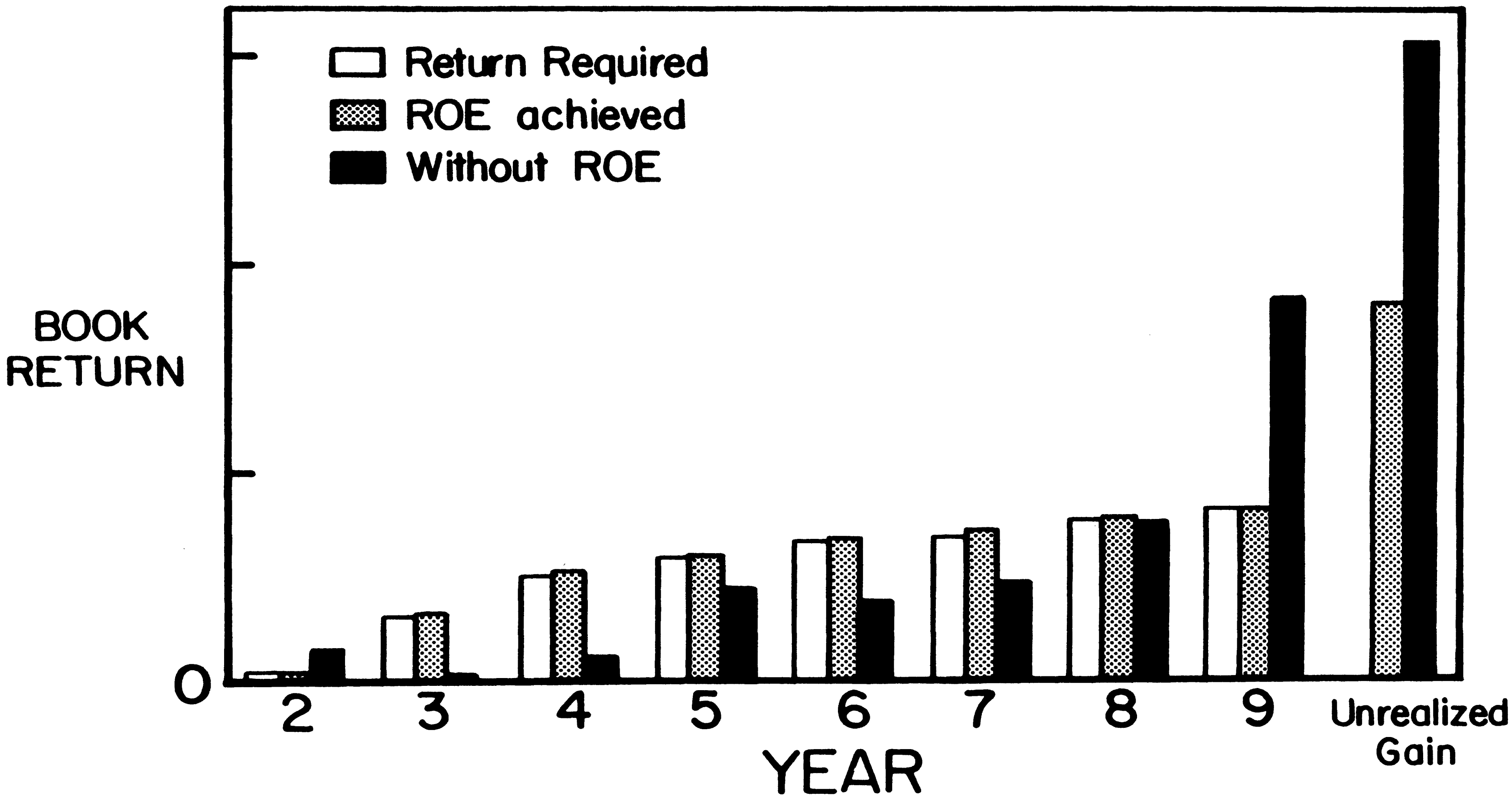
Return Required
ROE achieved
Without ROE
BOOK
RETURN
0
2
3
4
5
6
7
8
9
Unrealized
Gain
YEAR

FIGURE 2: Strategic Information Flow