

SELECTING TENANTS IN A SHOPPING MALL

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As a developer of premium regional shopping malls, Homart Development Company is responsible for the design, construction, and leasing of new centers. Early in the development process, accommodations for the large department store tenants are negotiated and arranged. Once the department stores are situated and a general mall floorplan is known, the developer must decide how to lease the many smaller store spaces which complete the mall. The types, sizes, and locations of these smaller tenants play an important role in determining the financial success of a center. This problem of deciding a center's "tenant mix" was formulated as a nonlinear integer program and solved using a linear approximation. Validation results show the method yields a potential 10 to 26 percent improvement in the present worth of a center.

Homart Development Company is one of the largest commercial land developers in the United States. Details about the firm may be found in Bean, Noon, and Salton [1986]. The firm currently owns or is developing 31 regional shopping centers, 18 major office buildings and is involved in land development of properties which total over 1000 acres.

Homart's shopping center development business involves identifying opportunities for development, feasibility analysis, obtaining the necessary governmental approvals, overseeing design and construction, leasing and then managing the properties as long term investments. Regional shopping centers contain approximately one million square feet of selling area and represent investments of \$60 million and more. Profits from shopping center development are a major component of Homart's total profits.

Shopping Center Development

In developing a new center the first step is to evaluate feasibility considering factors such as marketing demographics, building costs, and expected revenues. Next Homart produces a tentative floor plan or "footprint" for the mall, outlining its size, shape, and spaces for large department stores. Leasing agreements are reached with the two or three major department stores that will act as "anchor stores" for the mall. The anchor stores largely determine the character of the center and provide visibility needed to attract customers. They are able to negotiate highly favorable occupancy agreements, typically either paying low rent or receiving other concessions. Homart's profits are made primarily from the rent paid by the non-anchor tenants, the smaller stores which lease space along the aisles of the mall. Since the durations of the original lease agreements are typically five to fifteen years, the initial decision of allocating space to potential tenants, called the tenant mix, is crucial to the success of the investment.

The Tenant Mix Problem

The tenant mix describes the desired stores in the mall by their size, general location, and type of merchandise or service provided. For example, the tenant mix might specify two small jewelry stores in a central section of the mall, and a medium-sized shoe store and large restaurant in one of the side

aisles. In the past, Homart developed a plan for tenant mix using rules of thumb developed over years of experience in mall development. The leasing agents would use this plan as a basis for negotiation with potential tenants.

Although Homart's tenant mixes had been successful in the past, there was no way of knowing if they could be improved. The concept of an "optimal" tenant mix had not been formulated. Homart's staff could estimate the profitability of any particular prospective tenant, given its size and general location in the mall. The hard part was to choose a mix of tenants, along with their sizes and locations, that would together comprise a mall attractive to consumers and profitable for the retailers and Homart. We were asked in 1984 to develop a mathematical model of the tenant mix problem.

Two features of the tenant mix problem lend complexity to its formulation. One is the presence of "interaction effects" between stores of the same type; that is, the dependence of one store's sales on the number of other stores similar to it in the mall. Consumers are drawn to a mall by a balance of variety and homogeneity of merchants. A customer with a long and varied list of items to buy will be attracted to a mall with many kinds of stores. One desiring a large selection of a particular item may be attracted to a mall containing several stores stocking that item. For example, if a mall contains several shoe stores, it will be perceived as a place to shop for shoes. A tenant mix for a regional mall is designed to attract both customers. As the number of stores in a category, e.g., shoes, increases, the return for each store initially grows. At some point this gain diminishes or even becomes negative as the stores begin to compete. To use interaction effects to their maximum advantage, the tenant mix should contain "enough" of each type of tenant but not "too many."

The determination of Homart's revenue from the mall also adds complexity. Their revenue consists of rental income and proceeds from the eventual sale of the mall (the problem of determining when to sell a mall is discussed in Bean, Noon, and Salton [1986]). The rent charged each year is the maximum of a base figure and a specified percentage of the tenant's sales. The base rent per square foot depends on the type of store and its general location in the mall. The percentage of sales, or "overage rent", also depends on the store's type. Base rents are typically collected for the first several years of the mall's

operation, until the mall is established and retailers' sales grow. Then, overage rents determine income. The final sale value is proportional to the developer's rent revenue in the last year of ownership. This value is seen as an indication of this tenant mix's profitability. Thus, Homart's total revenue from mall development can be expressed as the sum of contributions to it by each store in the mall.

Due to the rent structure, Homart's income from each tenant is the sum of a deterministic amount (the base rent) and a stochastic amount (the amount of overage rent that exceeds base rent). During the study horizon, the stochastic part is typically small compared to the deterministic part. Therefore, although the income depends on tenants' uncertain future sales, typically only a small portion of the total is uncertain.

The Mathematical Model

Literature on the tenant mix problem is sparse. Seagle [1967] presents a linear programming model for the tenant mix problem. His model finds the allocation of square feet to each tenant class that maximizes the total present value of the mall. The model has constraints on the use of space, investment, and other resources of the developer. Jensen [1980] approaches the tenant mix problem with a mixed-integer programming model which solves for both space allocation and numbers of tenants selected within a tenant class. Our model expands on these efforts by solving for the number of tenants in each class, along with a rough description of each tenant's size and location in the mall. It also considers interaction effects among tenants of the same type.

We formulated the tenant mix problem, including interaction effects, as a nonlinear integer program. The mathematical formulation is contained in the Appendix. The model assumes that tenants are classified into categories according to the type of merchandise or service they provide. We also assume that for each store type the store sizes can be discretized. For example, a small jewelry store contains about 700 square feet and a large one contains about 2200 square feet. To provide a third descriptor of potential tenants, the mall is divided into areas according to differences in the value of the space. A central walkway between two major stores sees more traffic than that in a side aisle leading to the parking lot. For this reason, its tenants can expect more sales, and space there commands higher rent than space along the side aisle.

With these type, size, and location classes defined, the primary decision variable, x_{ijk} , is the number of stores of type i and size k in location class j . To account for interaction effects, we also define the binary decision variable y_{il} to equal one if there are exactly l stores of type i in the mall, and zero otherwise.

Constraints are designed to reflect both physical realities and desirable qualities for the tenant mix. They are:

1. Upper bound on the total square feet available for leasing in each location class.
2. Upper and lower bounds on the total square feet allocated to each type of tenant.
3. Upper and lower bounds on the number of stores of each type.
4. Upper bound on the total number of stores in the smallest size class.
5. Upper bound on the amount of money spent for interior finishing allowances.
6. Structural constraints governing the form of the y_{il} variables and linking them with the x_{ijk} variables.

For the objective function, we calculate the present worth of the tenant mix to Homart over some finite time horizon. The components of the income stream from a given store are illustrated in Figure 1. The only cost associated with each store is an interior finishing allowance, which consists of a single payment at time zero. From then on, the model assumes that the store makes a single payment of rent at the end of each year. The final year's rent is multiplied by a constant to obtain the store's contribution to the sale value.

Interaction effects bring nonlinearity into the objective function. Homart's Market Research Group can estimate a store's sales over the study horizon given its type, size, location class, and the number of stores of its type in the mall. These revenue figures, together with rental rates depending on the store's characteristics, determine the rental income Homart will receive. Then a present worth coefficient, PW_{ijkl} , is calculated to be the total contribution to present worth of a store of type i , location class j , and size k if it is one of l stores of type i in the mall. The objective function is then formed by summing

the product of PW_{ijkl} , y_{il} , and x_{ijk} over all values of i , j , k , and l . Thus, the y_{il} variables select which present worth coefficient to use for the stores described by (i, j, k) . See the Appendix for further detail.

Solution Procedure

The formulation as a nonlinear integer program, with twenty store types, three size classes, and three space subdivisions results in 180 integer (X) variables and over a hundred binary (Y) variables. A linear integer program of this size would be difficult and time-consuming to solve, but the problem's nonlinearity makes it intractable. For this reason, we made two approximations, as depicted in Figure 2.

The first was to replace the nonlinear objective function with a linear approximation. Instead of calculating interaction effects exactly, we calculated a new set of coefficients, R_{il} , to be the average marginal revenue added by bringing in the l^{th} store of type i . We also defined new variables S_{il} to equal one if there are *at least* l stores of type i and zero otherwise. Then the sum over all i and l of the product of R_{il} and S_{il} approximates the total income due to interaction effects. The linear objective function is formed by adding to this value the sum over all i , j , and k of the product of PW_{ijkl} and x_{ijk} .

The second approximation was to relax the integrality constraints. We believed this would work for two reasons. First, many of the constraints were soft. For example, the upper bound on the number of small stores could be increased by one with no practical effect. Second, the integrality requirements were soft. Though a small jewelry store is defined to contain 700 square feet, a jewelry store with ten percent more area would still be considered small. Thus, 1.1 small jewelry stores are nearly equivalent to one small jewelry store.

The solution procedure for the approximate problem has three stages. The raw data are converted into MPS input format and objective function calculated by a matrix generator. This file is then used as input for the MINOS linear programming code (see Murtagh and Saunders [1983]). A postprocessor extracts an integer solution from the linear programming solution using simple heuristic methods. Fractional values close to integers are rounded to the nearest integer. Then for each type i , the remaining fractional parts are added together and divided into integral values. For example, half of a small store and half of a large

store together may be approximately equal in area to one medium store. The heuristic makes as many of these adjustments as possible without violating constraints, and the remaining fractional variables are simply truncated.

Implementation/Validation

The model was developed at The University of Michigan during the summer of 1984. Once the formulation was agreed upon and the necessary inputs identified, the matrix generator, the heuristic postprocessor, and a simple reporting module were coded for testing. A mall in its early stages of development was chosen for a pilot test. As expected, the initial runs resulted in a number of model and data adjustments. The output solutions appeared reasonable enough to Homart management to warrant further model development and validation. During the summer of 1985, the code was moved to the Homart headquarters in Chicago for validation and testing.

The matrix generator, post-processor and report modules were installed on a personal computer at the Homart headquarters. The MINOS code at The University of Michigan was retained to solve the linear program. Model operation and data collection were streamlined by noting that much of the data for the pilot mall applied to any new Homart center. The data common to all malls were incorporated into the matrix generator. The remaining input data were condensed into fewer parameters describing demographics and physical characteristics for a specific mall.

Homart's leasing executives were initially skeptical about the model's ability to predict the financial impact of a tenant mix since the final rental rates charged are the result of negotiation. To gain their confidence in the optimization process, we had to show that rents were calculated accurately in the objective function. We also wanted to be able to compare alternate tenant mixes for the same mall. An additional module, named the Evaluator, was written to address these concerns. The input for the Evaluator is a list of the stores in the mall described by type, location class, and area in square feet, together with the model's input data for the mall. Some parameters in the model input data are specified according to the small, medium, and large size class for each type. Since the actual store sizes deviate

from those specified, the Evaluator interpolates or extrapolates according to size to find the appropriate parameters for each store. It calculates the present worth of the tenant mix according to the true nonlinear objective.

Three relatively new malls were chosen to test the model’s validity. For the first test, we used the Evaluator to compare the actual post-negotiation base rents with those calculated by the model. The estimated average base rent deviated at most 0.3 percent from the actual for these three malls. The first two columns of Table 1 display the actual and estimated amounts. When the figures were compared for each tenant category, the deviation remained small. These results demonstrated that even though the base rents are negotiable, the amounts realized can be accurately predicted with a fair level of confidence. Since the optimization objective uses the same rental prediction, we were now confident in the formulation.

The second test was to compare the optimal tenant mix found by the model for each of the three malls with the respective one that had been implemented. This time, we used the Evaluator to compare present worths. The aggregate results are displayed in Table 1.

Mall	Actual Base Rent	Estimated Base Rent	Current Mix PW	Model Mix PW	Percent Improvement
Center B	13.26	13.22	133.44	162.66	21.9
Center S	17.42	17.46	175.88	221.89	26.2
Center D	20.12	20.10	215.21	237.58	10.4

Table 1. Validation and Potential Improvement for Test Malls.

(All rent and present worth figures in dollars per square foot.)

These results demonstrate the potential for improvement offered by the model. The optimization solution offers a starting point for targeting lease prospects. Perhaps more importantly, in cases where the model’s tenant mix differs from the traditional leasing norm, it gives quantitative explanation of the deviation.

The optimization model has been used on five mall projects to date. Four projects were for new centers and one for a mall expansion. The Evaluator output has been enhanced to provide detailed reports by tenant, tenant category, size, and location. In addition, a table displaying constraint shadow prices can now be extracted from the optimization model for a better understanding of the quantitative impacts

of design or policy constraints. The importance of this information was clearly demonstrated during the review of the pilot test results. For the mall under study, the model suggested including a small drugstore into the tenant mix. The Vice President of Leasing explained that small drugstore tenants are difficult to obtain in that market and therefore should be constrained out of the problem. After incorporating the constraint and re-running the model, the shadow prices indicated that the rewards for including the small drugstore tenant were considerably higher than expected and far outweighed the expense of obtaining such a tenant. The plan was revised to include a small drugstore tenant and the shadow price used as a first estimate of the value to Homart of enticing a firm to enter the mall.

Conclusion

The tenant mix model is a powerful tool for enhancing Homart's mall planning and leasing activities. Determining and implementing the ideal mall mix obviously consists of more than calculating the mathematically optimal allocation of space among store categories. It entails building a mix in which each tenant provides a unique service or product. The model is not intended to substitute for decision making in the field or at the executive level of Homart. Rather, it is a tool for generating and updating merchandising plans throughout the leasing process and for establishing financial standards against which the performance of centers can be measured.

This system has the flexibility to accommodate the unique structural constraints and market characteristics of each mall it evaluates. It can be adapted for help with decisions concerning the development of a new mall, the operation of an existing Homart mall or the acquisition of an established mall. The model is currently being expanded to handle the task of optimally remerchandising a mall or leasing an expansion.

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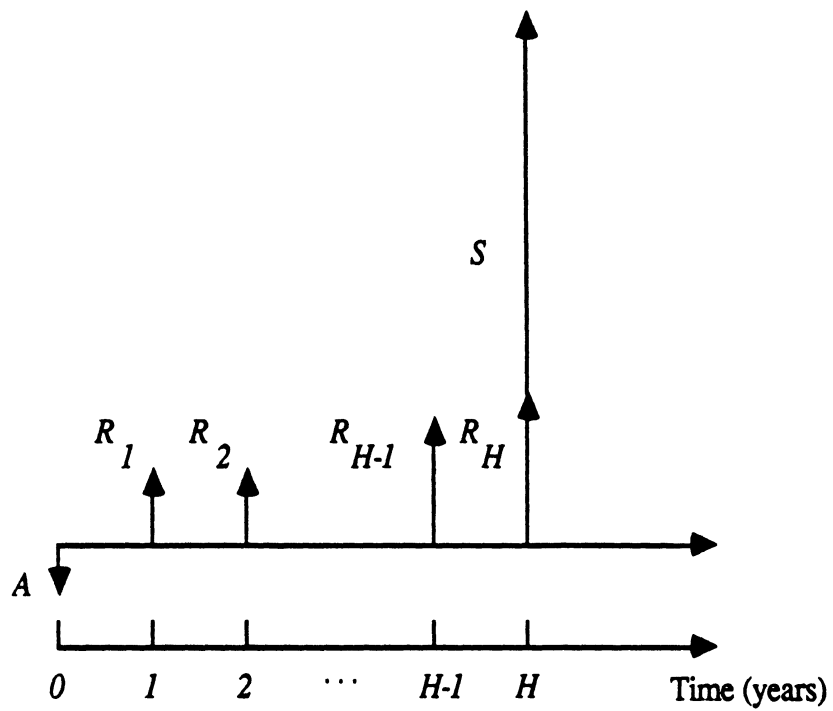
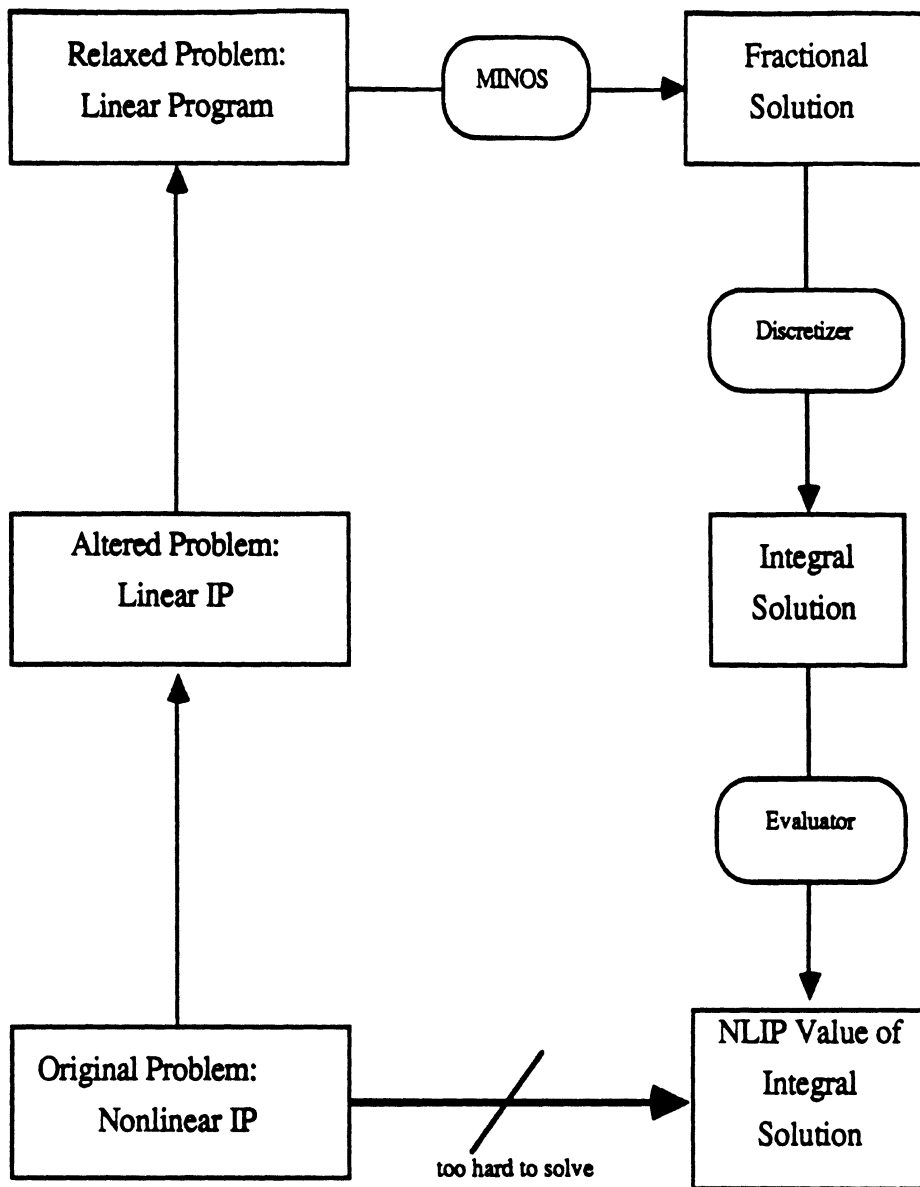


Figure 1: Cash Flows From a Single Tenant With Study Horizon H.
A = Finishing Allowance, R_i = Rent in Year i, S = Sale Value.

FIGURE 2: Solution Approach



Appendix

Define the following parameters:

- I = the number of store types
- J = the number of location classes
- K = the number of store size classes
- A_{ik} = amount of area required for a store of type i and size k
- G_j = total amount of square feet (gross leaseable area) available in location class j
- f_i = least amount of square feet available for type i
- F_i = largest amount of square feet available for type i
- L_i = amount of finishing allowance given to a tenant of type i per square foot leased
- B = tenant allowance budget
- M_i = maximum number of tenants of type i
- m_i = minimum number of tenants of type i
- N_S = maximum number of small stores

The decision variables are:

- x_{ijk} = number of tenants of type i and size k in location class j , where $k = 1$ for the smallest size class
- y_{il} = binary variable set to 1 if there are exactly l stores of type i (nonlinear objective function)
- S_{il} = binary variable set to 1 if there are at least l stores of type i (linear objective function)

The objective function coefficients are:

- PW_{ijkl} = present worth of a store described by (i, j, k) if it is one of l stores of type i
- R_{il} = average marginal revenue added by including the l^{th} store of type i

The nonlinear objective function is:

$$\text{Max} \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=m_i}^{M_i} PW_{ijkl} y_{il} x_{ijk}$$

The linear objective function is:

$$\text{Max} \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K PW_{ijk1} x_{ijk} + \sum_{i=1}^I \sum_{l=m_i}^{M_i} R_{il} S_{il}$$

Subject to the constraints:

$$\sum_{i=1}^I \sum_{k=1}^K A_{ik} x_{ijk} \leq G_j, \quad j = 1, \dots, J$$

$$f_i \leq \sum_{j=1}^J \sum_{k=1}^K A_{ik} x_{ijk} \leq F_i, \quad i = 1, \dots, I$$

$$m_i \leq \sum_{j=1}^J \sum_{k=1}^K x_{ijk} \leq M_i, \quad i = 1, \dots, I$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ij1} \leq N_S$$

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K L_i A_{ik} x_{ijk} \leq B$$

$$x_{ijk} \geq 0, \text{ integer}, \quad i = 1, \dots, I, \quad j = 1, \dots, J, \quad k = 1, \dots, K.$$

In the case of the nonlinear objective function, add:

$$\sum_{l=m_i}^{M_i} l y_{il} = \sum_{j=1}^J \sum_{k=1}^K x_{ijk}, \quad i = 1, \dots, I$$

$$\sum_{l=m_i}^{M_i} y_{il} = 1, \quad i = 1, \dots, I$$

$$y_{il} \in \{0, 1\}, \quad i = 1, \dots, I, \quad l = m_i, \dots, M_i.$$

In the case of the linear objective function, add:

$$\sum_{l=m_i}^{M_i} S_{il} = \sum_{j=1}^J \sum_{k=1}^K x_{ijk}, \quad i = 1, \dots, I$$

$$S_{il} \geq S_{i,l+1}, \quad i = 1, \dots, I, \quad l = m_i, \dots, M_i - 1$$

$$S_{il} \in \{0, 1\}, \quad i = 1, \dots, I, \quad l = m_i, \dots, M_i.$$