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PORTFOLIO DIVERSIFICATION, LONG-TERM MARKETABLE SECURITIES AND PORTFOLIO SELECTION MODELS

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Introduction

The analysis of portfolio diversification among common stocks has received considerable attention in the past. Recent research has broadened the spectrum of financial assets to include other security classes such as bonds and preferred stocks. The findings suggest there are gains to be had by the investor when the set of potential investments is expanded beyond common stocks to include these and other financial assets.

The general framework employed in analyzing diversification among securities involves the mean-variance theory of portfolio selection described by Markowitz (1952). Assuming asset returns are stochastic, his theory postulated that rational investors should select a portfolio from the set of all portfolio (named the efficient set) which offered minimum risk (measured by variance) for varying levels of expected return.

Observation of the securities comprising the efficient set indicates which financial assets possess attributes making them potentially worthwhile components of an optimally diversified portfolio. This paper will be concerned with forming an efficient set from four security classes--common stocks, preferred stocks, corporate bonds, and U.S. government bonds, denoted CS, PS, CB, and GB, respectively. The first objective will be to derive and analyze an efficient set from a sample of these securities in order to determine which securities have potential benefits for diversification.

The derivation of efficient sets as proposed by Markowitz requires inputs on the expected return vector and a matrix representing the covariance of returns on the assets. As this requires a considerable amount of data when analyzing a large number of assets, various models have been proposed by Sharpe (1963) and Cohen and Pogue (1967) to describe security returns which reduce the data needed. These models will generate an efficient set identical to that of Markowitz, provided their assumptions hold.

William F. Sharpe proceeded to derive the efficient set by use of a diagonal model, also known as the market or single index model, hereafter denoted the SI model. Kalman J. Cohen and Jerry E. Pogue derived two different types of multiple index models, reasoning that they would require less rigorous assumptions than the SI model. While these multiple index models required more data than the SI model, they still reduced the amount of input substantially when compared with the Markowitz model.

The second objective of this paper will be to analyze and compare these index models when considering common stocks, preferred stocks, corporate bonds, and U.S. government bonds. Attempts will be made to determine which model least violates its assumptions and to ascertain if this model can be used as a reasonable approximation to the Markowitz model.

The paper is divided into four parts. Part I contains a review of the literature. Part II presents the methodology used, followed by the observations made in Part III. Last, Part IV presents the conclusions and implications of this research.

Review of the Literature

Markowitz model

The Markowitz portfolio selection model was developed as an extension of the expected utility model. The expected utility model was concerned with how an individual could maximize his or her utility from consumption over time in a world of uncertainty.

Each individual must decide how to allocate his or her wealth[2] between consumption and investment for every time period into the future. The Markowitz model determines the optimal investments for the individual for a single time period by considering the probability distributions, along with their relationship to each other, of the rates of return for all possible investments. It is based on the following five assumptions:

1. Individuals act to maximize one-period expected utility.
2. Individuals prefer higher expected returns for a given level of risk.
3. Individuals prefer lower levels of risk for a given level of expected return.
4. Capital markets are perfect.
5. Individuals base decisions solely on the expected return and variance of possible investment portfolios.

There are two conditions (each sufficient but not necessary, as shown in Tsiang [1972]) that enable the investor to choose his portfolio only on the basis of its expected return and variance. First, if security returns are normally distributed, they can be completely described by two parameters--mean and variance. Second, if investors behave as if they had quadratic utility functions, it can be shown[3] that they will choose between alternative portfolios on the basis of mean and variance of return. Assuming one of these conditions is valid, finding the optimal portfolio involves determining the efficient set, and choosing from the efficient set that portfolio which maximizes the investor's utility.

The variance of a portfolio, Var (R_p), is $X^T CX$ where X is an N by 1 vector representing the proportions of the investor's funds that are to be placed in each of the N securities, and C is an N by N matrix representing the covariance of returns between the N securities. The expected return of the portfolio, $E(R_p)$, is $X^T\ E(R)$ where E(R) is the vector representing the expected returns of the N securities.

Deriving the efficient set involves solving the following problem for various E*, where E* represents a given level of expected return:

$$\begin{aligned} &\text{Minimize Var } (R_p) = X^T CX \\ &\text{Subject to } E\ (R_p) = X^T E(R) = E^* \qquad (1) \\ &X^T K = 1 \\ &X \geq 0. \end{aligned}$$

Here K is an N by 1 vector with all its elements equal to one. The problem therefore involves solving a quadratic objective function with linear constraints. As inputs, the investor must generate $(N^2 + 3N)/2$ inputs describing the securities expected returns, variances, and covariances.

Single index model

Because of the number of statistics and the computer time needed to solve equation (1), William F. Sharpe (1963) constructed a theoretical framework of analysis which reduced both the number of statistics and the computer time needed. Called the single index model, it assumes that the rates of return on securities are related only through their common relationships with some basic underlying factor. The return on any security is presumed to be determined solely by random factors and this basic underlying factor. Stated algebraically,

$$R_i = A_i + B_i I + C_i \tag{2}$$

where A_i and B_i are parameters associated with security i, C_i is a random variable with mean zero and variance Q_i^2, and I is the value of the basic underlying factor, called the index. By taking the expected value of the equation, the expected rate of return on security i is

$$E(R_i) = A_i + B_i E(I). \tag{3}$$

Furthermore, the rate of return on the index I can be described by

$$I = A_{N+1} + C_{N+1} \tag{4}$$

where $E(I) = A_{N+1}$ and $E(C_{N+1}) = 0$ with $E(C_{N+1}^2) = Q_{N+1}^2$. Given N securities are being considered for investment, all portfolios will have

$$\begin{aligned}\text{Mean} = E(R_p) &= \sum_{i=1}^{N} X_i E(R_i) = \sum_{i=1}^{N} X_i [A_i + B_i E(I)] \\ &= \sum_{i=1}^{N} X_i (A_i + B_i A_{N+1}) \\ &= \sum_{i=1}^{N+1} X_i A_i = X^T A \text{ where } X_{N+1} = \sum_{i=1}^{N} X_i B_i; \end{aligned} \tag{5}$$

$$\text{Variance} = \text{Var}(R_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \text{Cov}(R_i R_j) = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j [B_i B_j Q_{N+1}^2 + E(C_i C_j)]$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j B_i B_j \; Q_{N+1}^2 + \sum_{i=1}^{N} X_i^2 \; Q_i^2$$

$$= X^T S_{N+1} \; X \tag{6}$$

where S_{N+1} is a diagonal matrix with diagonal element (i,i) being equal to Q_i^2 and element (N+1,N+1) being equal to Q_{N+1}^2. In order to use these formulas, Sharpe made two more assumptions:

1. $E(C_{N+1}C_i) = 0$ for all i.
2. $E(C_iC_j) = 0$ for all i and j (except when i = j).

In summary, to derive the efficient set from Sharpe's SI model, the investor needs the following statistics:

1. Expected value of the index I (one statistic),
2. Variance of the index I (one statistic),
3. The values A_i, B_i, and Q_i^2 for each security (3N statistics).

Therefore, Sharpe has reduced the number of statistics necessary for deriving the efficient set from $(N^2 + 3N)/2$ to 3N+2.

Multiple index models

Given the extreme differences between the Sharpe and Markowitz approaches, it would seem wise to consider an intermediate approach, i.e., a multiple index model. This would reduce the computer time and statistics needed by the Markowitz model (but the reduction would not be as great as that resulting from Sharpe's approach) while requiring assumptions less questionable than Sharpe's. Two such multiple index models were developed by Cohen and Pogue (hereafter denoted CP).[4]

The first multiple index model has been called the covariance form, and will be denoted the MIC model. The initial assumption is that all N securities in the sample can be placed in one of M classes, with the return R_i on each security i being linearly related to the level of an index J_{j_i} in the class N_j to which it belongs.[5] Stated algebraically,

$$R_i = A_i + B_i J_{j_i} + C_i \{i | i \varepsilon N_j\}. \tag{7}$$

Here A_i and B_i are parameters associated with security i, and C_i is a random variable with mean zero and variance Q_i^2. The M class indices are related by their covariances with each other, represented by a covariance matrix S_M. Each index has a rate of return described by the equation

$$J_j = A_{N+j} + C_{N+j} \tag{8}$$

with $E(J_j) = A_{N+j}$, $E(C_{N+j}) = 0$, and $E(C_{N+j}^2) = Q_{N+j}^2$.

In addition, the following assumptions are made:

$E(C_i C_k) = 0$ for any two securities i and k.

$E(C_i C_{N+k}) = 0$ for any security i and index N+k.

From this model all portfolios will have

$$\text{Mean} = E(R_p) = \sum_{i=1}^{N} X_i E(R_i) = \sum_{j=1}^{M} \sum_{\{i|i\varepsilon N_j\}} X_i [A_i + B_i E(J_j)]$$

$$= \sum_{j=1}^{M} \sum_{\{i|i\varepsilon N_j\}} X_i A_i + \sum_{j=1}^{M} \sum_{\{i|i\varepsilon N_j\}} X_i B_i A_{N+j}$$

$$= \sum_{i=1}^{N+M} X_i A_i = X^T A \text{ where } X_{N+j} = \sum_{\{i|i\varepsilon N_j\}} X_i B_i; \tag{9}$$

$$\text{Variance} = \text{Var}(R_p) = \sum_{i=1}^{N} \sum_{k=1}^{N} X_i X_k \text{Cov}(R_i R_k)$$

$$= \sum_{i=1}^{N} \sum_{k=1}^{N} X_i X_k [B_i B_k E(C_{N_i} C_{N_k}) + E(C_i C_k)]$$

$$= \sum_{i=1}^{N} X_i^2 Q_i^2 + \sum_{i=1}^{N} \sum_{k=1}^{N} X_i X_k B_i B_k E(C_{N_i} C_{N_k})$$

$$= X^T S_{N+M} X \qquad \text{where } S_{N+M} = \begin{bmatrix} S_N & 0 \\ 0 & S_M \end{bmatrix} \tag{10}$$

where S_N is a diagonal matrix with element (i,i) being equal to Q_i^2 and S_M is as previously described.

The second multiple index model has been called the diagonal form, and will be denoted the MID model. This model, similar to the MIC model, has the initial assumption that all N securities in the sample can be placed in one of M classes, with the return R_i on each security i being linearly related to the level of an index J_{j_i} in the class N_j to which it belongs. Where it differs from the covariance form lies in an additional assumption, stating that each class index J_j is itself linearly related to an overall index I. Stated algebraically,

$$R_i = A_i + B_i J_{j_i} + C_i \qquad \{i | i \varepsilon N_j\} \tag{11}$$

$$J_j = A_{N+j} + B_{N+j} I + C_{N+j} \qquad j = 1 \ldots M \tag{12}$$

$$I = A_{N+M+1} + C_{N+M+1}. \tag{13}$$

Here A_i and B_i are parameters associated with security i, A_{N+j} and B_{N+j} are parameters associated with index J_j, C_i is a random variable with mean zero and variance Q_i^2, C_{N+j} is a random variable with mean zero and variance Q_{N+j}^2, and C_{N+M+1} is a random variable with mean zero and variance Q_{N+M+1}^2. In addition, the following assumptions are made:

$E(C_i C_k) = 0$ for any two securities i and k.

$E(C_{N+j} C_{N+q}) = 0$ for any two indices N+j and N+q.

$E(C_{N+j} C_i) = 0$ for any index N+j and security i.

$E(C_{N+M+1} C_i) = 0$ for index I and any security i.

$E(C_{N+M+1} C_{N+j}) = 0$ for index I and any index N+j.

From this model all portfolios will have

$$\text{Mean} = E(R_p) = \sum_{i=1}^{N} X_i E(R_i) = \sum_{j=1}^{M} \sum_{\{i | i \varepsilon N_j\}} X_i [A_i + B_i E(J_j)]$$

$$= \sum_{j=1}^{M} \sum_{\{i | i \varepsilon N_j\}} X_i A_i + \sum_{j=1}^{M} \sum_{\{i | i \varepsilon N_j\}} X_i B_i A_{N+j} +$$

$$\sum_{j=1}^{M} \sum_{\{i | i \varepsilon N_j\}} X_i B_i B_{N+j} A_{N+M+1}$$

$$= \sum_{i=1}^{N+M+1} X_i A_i = X^T A \tag{14}$$

where $X_{N+j} = \sum_{\{i|i\epsilon N_j\}} X_i B_i$ and $X_{N+M+1} = \sum_{j=1}^{M} X_{N+j} B_{N+j}$;

$$\text{Variance} = \text{Var}(R_p) = \sum_{i=1}^{N} \sum_{k=1}^{M} X_i X_k \text{Cov}(R_i R_k)$$

$$= \sum_{i=1}^{N} \sum_{k=1}^{N} X_i X_k [B_i B_k B_{N+j_i} B_{N+j_k} Q^2_{N+M+1} +$$

$$B_i B_k E(C_{N+j_i} C_{N+j_k}) + E(C_i C_k)]$$

$$= \sum_{i=1}^{N} X_i^2 Q_i^2 + \sum_{j=1}^{M} X^2_{N+j} Q^2_{N+j} + X^2_{N+M+1} Q^2_{N+M+1}$$

$$= X^T S_{N+M+1} X \text{ where } S_{N+M+1} = \begin{bmatrix} S_N & 0 & 0 \\ 0 & S_M & 0 \\ 0 & 0 & Q^2_{N+M+1} \end{bmatrix} \qquad (15)$$

where S_N and S_M are diagonal matrices with elements (i,i) and (N+j,N+j) being equal to Q_i^2 and Q^2_{N+j}, respectively. Since S_{N+M+1} is a diagonal matrix (whereas S_{N+M} in the MIC model isn't), this model's name is appropriate.

A summary of the features, assumptions, and statistics for each of the four models is included in Appendix A.

Validity of the Index Models

Since Sharpe's SI model rests on several crucial assumptions it can be questioned empirically. As CP have stated,

> In tying the variability of security yield only to a general market index, some of the important relationships among securities--originally expressed in the Markowitz formulation as independently determined covariances between each pair of securities--may be lost. It is hence possible that the original single index model does not generate a truly efficient set of portfolios.[6]

Within a sample of common stocks, multivariate analysis by King (1966) has shown the existence of industry effects in stock price movements. The implication is that the assumptions necessary for the use of the SI model are unrealistic (i.e., the existence of industry effects in addition to general market effects rebukes the assumption of only general market effects).[7] In a later study, Meyers (1973) refutes King's results and concludes that the assumptions are closer to reality than King had implied. This gives empirical support to Blume's (1971) contention that the SI model represents a good first approximation to the Markowitz model when restricted to common stocks.

In their study, CP pointed out that the SI model's efficient set tended to dominate[8] those of the multiple index models over a wide range of expected returns on a sample of common stocks. Their study broke the sample down into industries, then proceeded to compare the single and multiple index models previously mentioned. Their conclusion that "...the structuring of the models to include a number of indexes has not had as major an effect on reducing the covariability among yield residuals for the universe of common stocks considered as might have been expected"[9] was qualified, however. Continuing, the authors stated that they felt this "...to be the result of dealing with strictly common stock universes, in which industries tend to be strongly interrelated and amenable to single index type of assumptions."[10] They concluded that if a broader universe of securities had been considered (e.g., adding bonds and preferred stocks to the common stocks already selected for the sample), the more accurate representation of security returns permitted by the multiple index models in comparison with the SI model would have been apparent. The implication is that their results would have been reversed.

Wallingford (1967) noted that it would seem disconcerting if multiple index models did worse than the SI model in approximating the Markowitz model's efficient set while requiring less restrictive assumptions to be made. Observing that CP had this result occur with a sample of common stocks, he performed a similar study which yielded conclusions contradicting those of CP. Using only two indices and 20 common stocks, six of which were in one industry (CP's results were identical for both samples of 75 and 150 common stocks, chosen from a universe of 543 common stocks broken into ten industries), he found that the two-index model outperformed the SI model. Wallingford attempts to explain the discrepancy by stating

> The reason for this difference may lie in the considerable reduction in sample size employed in our research and the resulting omission of maximum investment constraints upon the model. Furthermore, our indices were computed from the sample itself, whereas Cohen and Pogue computed their indices from a large population. All of these factors might tend to reduce the correlation between the indices which we used, thereby improving the relative performance of the two-index model.[11]

In another test, Wallingford replaced the six industry stocks with preferred stocks and recomputed his indices accordingly. After deriving the efficient frontiers, Wallingford concluded again that the two-index model dominated the SI model. The two-index model used in both tests was of the covariance form.

Extensions of the Models

There has been a considerable amount of research performed analyzing the potential benefits of supplementing a common stock portfolio with securities from other investment media. Herzog (1964) analyzed ex post yields for common stocks, preferred stocks, and corporate bonds from 1929 through 1962. Soldofsky and Miller (1969) observed ex post returns for fifteen classes of securities (drawn from CS, PS, CB, and GB samples) and then proceeded to analyze the risk-return tradeoff suggested in the data. McEnally (1972 and 1973) rebutted the analysis of Soldofsky and Miller and proceeded to examine the risk characteristics of nineteen classes of long-term marketable securities. In exploring the implications of these characteristics for diversification, McEnally found that government bonds provided significant benefits to the investor when added to a common stock portfolio and were more useful than corporate bonds in diversifying a common stock portfolio.

Fisher and Weil (1971) noted that bonds have a low average return along with a disproportionately lower dispersion in comparison with common stocks. They suggest that this makes bonds a worthwhile component in the construction of a portfolio. Norgaard (1974) argues the opposite, however, stating that the risk-reward tradeoff isn't adequate to justify the addition of corporate bonds to a common stock portfolio. In support of Fisher and Weil, Sarnat (1974) has shown that the percentage of funds invested in CB and GB rises monotonically as the expected return is decreased along the efficient set. It should be noted that his sample consisted of seven securities, two of which were bonds.[12]

Robichek, Cohn, and Pringle (1972) observed ex post returns for twelve alternative investment media from 1949 to 1969 (included were samples of CS, CB, and GB). The authors directly addressed the implications of the data for portfolio construction. After noting the low correlation coefficients between the rates of return for the various media, they derived efficient sets and concluded that "...enlarging the universe of investment alternatives may offer benefits for portfolio construction in terms of improving the risk/return opportunities along the efficient frontier."[13]

The first attempt at using multiple index models to describe returns of securities from different investment media was performed by Wallingford (1967). As previously stated, Wallingford found that the MIC model dominated the SI model in approximating the Markowitz model's efficient set when applied to a sample of fourteen common stocks and six preferred stocks. McEnally (1973), in comparing the correlation coefficients among the indices implied by the MID model with those coefficients computed directly from his data, concluded that the MID model poorly estimated the relationships between the security indices. An implication of his research is that the MID model will poorly approximate the Markowitz model.

II. Methodology

Choosing the sample

This study will use quarterly rates of return for the period January 1963 through December 1972. For the sample chosen, rates of return for security i during period t were calculated as

$$R_{it} = \frac{(P_{it} - P_{it-1} + CE_{it})}{P_{it-1}} \quad . \tag{16}$$

Here P_{it} denotes the price of security i at time t and CE_{it} denotes all cash and cash equivalents received during period t on security i. Thus for each security in the sample, forty observations of quarterly rates of return were calculated from which the necessary statistics and parameters for the various models were calculated.

A sample of fifty-three common stocks was chosen at random from those listed on the CRSP tapes during the period January 1963 through December 1972.[14] The following formula was used to adjust the monthly rates of return to quarterly rates of return:

$$R_{it_Q} = \frac{P_{it-1}R_{it} + P_{it}\ R_{it+1} + P_{it+1}\ R_{it+2}}{P_{it-1}} \quad . \tag{17}$$

Here P_{it} and R_{it} indicates the price and rate of return of security i at time t, respectively. Fisher's (1966) Combination Investment Performance Index was calculated monthly from the following formula:

$$CIPI_{t_m} = .56 \sum_{i=1}^{N} \frac{R_{it}}{h} + .44 \left(\prod_{i=1}^{h} R_{it} \right)^{1/h} . \qquad (18)$$

Here h is the number of securities contained on the tape for month t. From the monthly values of the index, the following formula was used to get quarterly returns:

$$CIPI_{t_Q} = (1 + CIPI_{t_M})\,(1 + CIPI_{t+1_M})\,(1 + CIPI_{t+2_M}) - 1 . \qquad (19)$$

This index was felt to adequately reflect the market movements of the New York Stock Exchange, and will be denoted CSI, for common stock index.

A sample of twenty-two preferred stocks was chosen at random from those listed on the NYSE during the period January 1963 through December 1972.[15] The data source used was the ISL Daily Stock Price Record. Convertible or participating preferred stocks were not considered. Equation (16) was used to calculate the quarterly rates of return. As an index representing the market in preferred stocks the Standard and Poor's "Preferred Stock Index" (found in their Trade & Securities Statistics) was used and recalculated as a rate of return by using equation (16) and the characteristics of the index as provided by Standard and Poor.[16]

For a sample of corporate bonds, the series of twelve corporate bond indices provided by Standard and Poor's Trade and Securities

Statistics were used.[17] It was felt that each of these indices depicted the return expected from a portfolio of bonds having the same characteristics as each index. These indices will hereafter be referred to as corporate bonds. These twelve bonds differ according to default risk, with the rates of return reflecting differing types of this risk. However, interest rate risk must also be reflected in the rates of return, as shown by Soldofsky (1970) and Sarnat (1974). Since Standard and Poor provides the yield to maturity and suggests a 4 percent coupon, twenty-year life as providing a reasonable description of the bonds, rates of return were calculated quarterly using this information.[18] As an index representing the market in corporate bonds, an arithmetic average of the twelve bonds' quarterly rates of return was used.[19]

For a sample of government bonds, the series of three government bond indices provided by Standard and Poor's Trade and Securities Statistics were used.[20] Once again, it was felt that each of these indices depicted the return expected from a portfolio of bonds having the same characteristics as each index. The yields to maturity were converted to rates of return in a manner analogous to that done for the corporate bonds. As an index representing the market in government bonds, an equally weighted arithmetic average of the three bonds' quarterly rates of return was used.

For an overall index (necessary for the SI and MID model) an equally weighted arithmetic average of the four indices used was computed and is denoted EWFSOI (for equally weighted-four-security, overall index). A market-weighted arithmetic average was also calculated, where the weights were derived from the New York Stock Exchange Factbook. It is denoted MWFSOI, for market weighted, four-security, overall index. Fisher's index (CSI) was also used as an overall index. Last, an overall index derived by taking an arithmetic average of the common stock and preferred stock indices was computed and denoted TSOI (for two-security overall index). The purpose of the latter index will be explained shortly. The following assumptions, patterned after those of CP, were made:

1. The expected return for each security was the arithmetic average of the quarterly returns observed from 1963-72, i.e., $E(R_i) = \sum_{t=1}^{40} \frac{R_{it}}{40}$.

2. The expected value for each index was the average of the levels obtained from 1963-72.

3. Similar assumptions were made regarding variability and covariability of security returns and index levels, i.e.,

$$\mathrm{Var}(R_i) = \sum_{t=1}^{40} \frac{[R_{it} - E(R_i)]^2}{39} ;$$

$$\mathrm{Cov}(R_i R_k) = \sum_{t=1}^{40} \frac{[R_{it} - E(R_i)]\,[R_{kt} - E(R_k)]}{39} .$$

4. The expected values of the parameters of the index models were equal to the values developed from the 1963-72 period using least squares regression techniques. Hence simple

extrapolations of past returns were used as expectations of future returns. This procedure was not expected to introduce any bias into the data.

A caveat is in order regarding this procedure for generating ex ante forecasts. As CP have stated, "in an operational situation we would definitely not advocate any method of forming expectations which is based strictly on historical data."[21] Their calculations were justified by stating that the concern of the study was only a part of the portfolio analysis process. That statement is also applicable here.

The procedure described for computing the parameters of the index models will result in the use of E(R) vectors which are equal under all four models. The Markowitz model has the arithmetic averages of each security as the elements of its E(R) vector. The value of the indices used in the index models is also their arithmetic value. Since the regression line must pass through the point where the means of the independent (index rate of return) and dependent (security rate of return) variables lie and the forecasted value of the index rate of return is its mean, it follows that the forecasted value of the security's rate of return is its mean.[22] Thus the E(R) vectors used for all four models will be identical. The only place where the models could fail to generate identical efficient sets is in the implied covariance matrices. These may differ as a result of the violation of the assumptions to varying degrees.

Once the necessary statistics were generated, the efficient sets were generated (1) for the combined CS, PS, CB, and GB sample (hereafter called the combined sample) by the Markowitz model, (2) for the combined sample by the MIC model, (3) for the combined sample by the MID model using the MWFSOI, EWFSOI, and CSI as overall indices, (4) for the combined sample by the SI model using the MWFSOI, EWFSOI, and CSI as overall indices, (5) for the CS and PS sample (hereafter called the stock sample) by the Markowitz model, (6) for the stock sample by the MIC model, and (7) for the stock sample by the SI model using the TSOI as an overall index.

When comparing efficient sets, the solutions from the index models (the solutions are the weights of the X_i variables for varying levels of expected return) were applied to the Markowitz model's covariance matrix, as CP and Wallingford had done. The efficient sets' solutions must be either equal to or inferior to the Markowitz model's efficient sets, as the latter's efficient set is the optimal solution given its covariance matrix. Hence no solution can be superior to it, and the index models can do no better than duplicate it, provided their assumptions are valid.

To supplement these single and multiple index tests, the multivariate technique of principal component analysis was used to analyze the ninety securities' quarterly rates of return.[23] Principal component analysis can be used to see if (1) the returns within each security class (CS, PS, CB, GB) share a large common element of variance and (2) the common elements of each security class are largely independent from those of the other security classes. If so, two results would be expected. First, the efficient set computed from a sample of CS, PS, CB, and GB would be expected to dominate the efficient set derived from the subsample of CS. This is due to the greater opportunity for risk reduction through diversification held by the larger sample. Second, multiple index models would be expected to outperform the SI model, as they would be able to more realistically describe security rates of return.

III. Observations

Choosing an overall index

The SI and MID models require the use of an overall index. Three overall indices were used to derive efficient sets for the sample of ninety securities for each of the two models. The three overall indices used were the CSI, EWFSOI, and MWFSOI. The results are displayed in Table 1.

In comparing the efficient sets generated using the SI model and the three overall indices, two observations can be made. First, the market-weighted index (MWFSOI) appears to produce the efficient set which is the best approximation to the efficient set of the Markowitz model. Second, all three overall indices produce nearly identical efficient sets.

In comparing the efficient sets derived using the MID model and the three overall indices, the same observations are apparent as with the SI model. As the MWFSOI was superior under both the MID and SI models, further comparisons of the models will involve the use of this particular overall index only.

Ninety securities efficient sets

The efficient sets for the sample of ninety securities consisting of CS, PS, CB, and GB calculated under the four models--the Markowitz, SI, MIC, and MID models--are presented along with the efficient set for the sample of fifty-three common stocks, in Table 2.

Several observations should be made. First, the multiple index models performed almost identically. There appears to be no significant difference between them. Second, the multiple index models outperformed the SI model. Third, all the index models seemed to do an excellent job of representing security returns and in duplicating the Markowitz model's efficient set. Fourth, by including PS, CB, and GB in the sample (in addition to CS), the efficient set improved its position for low values of expected return.

Table 3 presents the percentage of funds invested in both bonds and, more generally, fixed income securities (PS, CB, and GB) for various levels of E*. Note that in column 7 the percentage of funds invested in fixed income securities rises along the efficient set as the value of the portfolio's expected return falls. This feature also holds for just the bonds and is primarily because of the presence of GB in the sample, as demonstrated in columns (4) and (5).

Comparison with Wallingford's results

In Table 4 the efficient sets for the samples of CS and PS only is presented. Since the MIC model outperformed the SI model (i.e., the results were consistent with Wallingford's), no additional tests were made. However, once again, both models appear to do an excellent job of representing security returns and in duplicating the Markowitz model's efficient set.

Principal component analysis

Table 5 shows the percentage of variance attributable to the first three principal components for the various covariance matrices (the first component is the general market element), and Table 6 shows the correlation between the first principal components. The value of R for the .95 and .99 levels of significance are .3160 and .4076, respectively. There appears to be strong market elements in the PS, CB, and GB samples, particularly in the last two. The correlations of the market factors are all significant at the .95 level, and all but one are significant at the .99 level. It is possible that by utilizing these market elements the index model could accurately represent the returns on these securities. This was verified by (1) the efficient sets derived, where it was shown that all three index models did an excellent job of approximating the Markowitz model's efficient set, and (2) Table 8 where it was shown that the indices accurately corresponded to the market element. Hence the first result of the principal component analysis lends support to the results obtained when using index models to approximate the Markowitz model over four security classes. The existence of a large market element for the securities within each particular class and the excellent job done by the index models (especially the MID and MIC models) in approximating the Markowitz model are consistent with each other.

The second major result of the principal component analysis indicated that, as shown in Tables 6 and 7, the correlations of both the first principal components and indices were significant for all the security classes at the .95 level. The lowest correlations were between the various fixed income security classes and common stocks. Table 3 displays the percentage of funds in each of these classes along the Markowitz model's efficient set as the expected return decreases. Note that the fixed income securities do enter the efficient set in increasingly large proportions as the expected return falls. Also, the government bond sample seems to dominate the composition of the efficient set at the lower levels of return. This is consistent with the low correlation its first principal component had with the common stock sample's first component.

Fisher and Weils belief that holding bonds in a well-diversified portfolio would increase the portfolio's return for low levels of risk is supported by the results displayed in Table 3. This table displays the fact that bonds generally do play an increasingly large part in the composition of the efficient set as its expected return is decreased. Sarnat's findings are therefore confirmed.

TABLE 1

A Comparison of the Indices:
Efficient Sets for 90 Securities
(CS, PS, CB, GB)

Portfolio Number	Rate of Return	Mark. Model	SI Model			MID Model		
			EWFSOI	MWFSOI	CSI	EWFSOI	MWFSOI	CSI
1	.0991	.0166	.0166	.0166	.0166	.0166	.0166	.0166
2	.0892	.0109	.0115	.0110	.0110	.0110	.0110	.0110
3	.0793	.0070	.0073	.0070	.0070	.0070	.0070	.0070
4	.0694	.0048	.0048	.0048	.0048	.0048	.0048	.0048
5	.0595	.0035	.0035	.0035	.0035	.0035	.0035	.0035
6	.0495	.0023	.0025	.0025	.0025	.0025	.0024	.0024
7	.0396	.0015	.0216	.0016	.0016	.0016	.0016	.0016
8	.0297	.0009	.0010	.0010	.0010	.0010	.0009	.0010
9	.0198	.0005	.0006	.0006	.0006	.0006	.0006	.0006
10	.099	.0003	.0004	.0005	.0005	.0004	.0004	.0004
11	Minimum Variance	.0003 (E*=.0054)	.0004 (E*=.0044)	.0005 (E*=.0036)	.0006 (E*=.0032)	.0003 (E*=.0050)	.0003 (E*=.0046)	.0004 (E*=.0049)

TABLE 2

A Comparison of the Models:
Efficient Sets for 90 Securities
(CS, PS, CB, GB)

Portfolio Number	Rate of Return	Mark. Model 53CS	Mark. Model 90Sec	MIC Model	MID Model MWFSOI	SI Model MWFSOI
1	.0991	.0166	.0166	.0166	.0166	.0166
2	.0892	.0109	.0109	.0110	.0110	.0110
3	.0793	.0070	.0070	.0070	.0070	.0070
4	.0694	.0048	.0048	.0048	.0048	.0048
5	.0595	.0035	.0034	.0035	.0035	.0035
6	.0495	.0027	.0023	.0024	.0024	.0025
7	.0396	.0022	.0015	.0016	.0016	.0016
8	.0297	.0019	.0009	.0010	.0009	.0010
9	.0198	.0018 (E*=.0210)	.0005	.0006	.0006	.0006
10	.0099		.0003	.0004	.0004	.0005
11	Minimum Variance		.0003 (E*=.0054)	.0004 (E*=.0061)	.0003 (E*=.0046)	.0005 (E*=.0036)

TABLE 3

The Composition of the Markowitz
Model's Efficient Set for 90 Securities
(CS, PS, CB, GB)

(1) Portfolio Number	(2) Rate of Return	(3) Percentage in CB	(4) Percentage in GB	(5)=(3)+(4) Percentage in B	(6) Percentage in PS	(7)=(5)+(6) Percentage in FIS
1	.0991	–	–	–	–	–
2	.0892	–	–	–	–	–
3	.0793	–	–	–	–	–
4	.0694	–	–	–	–	–
5	.0595	–	–	–	12.85	12.85
6	.0495	–	–	–	29.80	29.80
7	.0396	–	14.53	14.53	31.18	45.71
8	.0297	–	37.55	37.55	23.48	61.03
9	.0198	–	57.23	57.23	19.74	76.97
10	.0099	–	70.31	70.31	22.16	92.47
11	.0054	21.35	68.14	80.49	16.78	97.27

TABLE 4

Wallingford's Test I:
Efficient Sets for 75 Securities

Portfolio Number	Rate of Return	Markowitz Model	MIC Model	SI Model
1	.0991	.0166	.0166	.0166
2	.0892	.0109	.0110	.0112
3	.0793	.0070	.0070	.0071
4	.0694	.0048	.0048	.0048
5	.0595	.0034	.0035	.0035
6	.0495	.0023	.0024	.0025
7	.0396	.0015	.0016	.0017
8	.0297	.0010	.0011	.0011
9	.0198	.0008	.0010	.0009
10	.0099	.0007	.0009	.0009
11	Minimum Variance	.0007 (E*=.0093)	.0010 (E*=.0018)	.0009 (E*=.0064)

TABLE 5

The Percentage of Variance
Explained by Principal Components

Security Classes in Covariance Matrix	Percentage of Variance		
	1st PC	2nd PC	3rd PC
53 Common Stocks	38.88	8.29	8.01
22 Preferred Stocks	51.92	19.02	6.30
12 Corporate Bonds	80.65	7.04	3.67
3 U.S. Government Bonds	94.19	4.58	1.23
15 Bonds (CB & GB)	76.86	6.79	5.93
37 Fixed Income Securities (PS+CB+GB)	55.03	16.07	5.43

TABLE 6

Correlations of
First Principal Components

	CS	PS	CB	GB	B	FIS
CS	1.0000					
PS	.6609	1.0000				
CB	.5387	.9141	1.0000			
GB	.3423	.7326	.7308	1.0000		
B	.5272	.9191	.9940	.8009	1.0000	
FIS	.6351	.9942	.9502	.7645	.9558	1.000

TABLE 7

Correlations of Indices

CSI	1.0000			
PSI	.6520	1.0000		
CBI	.5250	.8578	1.0000	
GBI	.3370	.7404	.7219	1.000
	CSI	PSI	CBI	GBI

TABLE 8

Correlations of Indices and
First Principal Components

CSI	.9609	.6091	.5318	.3465
PSI	.6494	.8702	.9057	.7310
CBI	.5314	.8595	.9995	.7283
GBI	.3348	.7417	.7245	.9997
	PC CS	PC PS	PC CB	PC GB

Since all the correlations in Table 6 are significant, it isn't surprising that all the index models did a good job of approximating the Markowitz model. These high correlations indicate that a single index model would work almost as well as a multiple index model in approximating the Markowitz model, since they suggest that the market elements for each security class are similar. This is what did occur, with the multiple index models being slightly superior.

Support for these conclusions is contained in Tables 7 and 8. After noting the correlations between the first components, displayed in Table 6, the indices used to represent the market element for each of the four security classes were correlated. They displayed a correlation matrix similar to the matrix correlating the first components (Table 6). This result led to the first components being correlated with the indices used. The results, displayed in Table 8, show that the first components and the indices were highly correlated when matched according to security class, i.e., along the diagonal of the correlation matrix. Therefore it can be concluded that the indices used accurately represent the market element as computed by principal component analysis, and conclusions drawn by having the first components represent the market are valid when replaced by the proper indices.

The correlation coefficients between the first principal component of all securities and (1) EWFSOI and (2) MWFSOI were .8844 and .9617, respectively. Noting the high figure for the latter, it wasn't surprising that the SI and MID models performed better using the MWFSOI.

In summary, the results of the principal component analysis are consistent with the results obtained in deriving the efficient sets under the various models and superindices. These results indicate that there are significant market elements within each security class, that the indices used accurately represent these market elements, and that diversification across all security classes (as opposed to limiting diversification to common stocks) is potentially beneficial.

IV. Conclusion

Portfolio theory was developed as a means to help the investor in his or her selection of assets. The initial method (called the Markowitz model) involved determination of the expected return and variance on each asset, along with its covariance with all other assets under consideration. For N assets, this involved determining the values of $\frac{(N^2 + N)}{N}$ statistics. From these statistics the efficient set of investments was formed, from which the investor chose his or her portfolio.

Index models were developed to reduce the number of statistics necessary in the generation of efficient sets. These models are each based on a set of assumptions which may or may not hold in the real world.

This study expanded the previous work done in exploring the feasibility of using index models to approximate the Markowitz model by considering various fixed income securities (preferred stocks, corporate bonds, and U.S. government bonds, in particular) in addition to common stocks.

It can be concluded that (1) the multiple index models do a better job of approximating the Markowitz model than the single index model does, (2) the two types of multiple index models are equivalent in their ability to reproduce the efficient set found by the Markowitz model, and (3) all the index models do a good job of representing the Markowitz model. These results are consistent with those found in the previous study performed by Wallingford, and apply to a sample consisting of more than one security class.

Expansion of the number of securities being considered for investment to include fixed income securities was shown to enhance the opportunities open to the investor by producing a more attractive efficient set--an efficient set with equal or lower levels of variance for all levels of expected return. This was most pronounced for lower levels of return, where the presence of fixed income securities was increasingly obvious. Fixed income securities would be expected to have lower returns and lower risks than common stocks (due to such things as a higher claim on the earnings of the firm), and this feature resulted in their presence at the lower levels of return of the efficient set.

The opportunity for beneficial diversification across security classes is enhanced by the securities within each class having low correlations with securities in other classes. After considering four security classes (common stock, preferred stock, corporate bonds, and U.S. government bonds), this feature was found to occur, especially when supplementing common stocks with government bonds, and it supports the result reported in the previous paragraph. Although low, the correlations were still significant. Strong market effects within each class of fixed income securities suggested that market movements can be used to explain security returns, and hence that the index models can adequately describe them. Therefore the results found in the derivation of efficient sets for the index models were consistent with these results.

In summary, the inclusion of certain fixed income securities in the selection of assets is beneficial to the investor by enhancing the location of the efficient set, particularly at low levels of return. Multiple index models may be used to derive the efficient set, since they accurately represent the Markowitz model. However, the improvement over the single index model of Sharpe is not significant.

FOOTNOTES

1. The model developed by Sharpe actually was first mentioned by Markowitz (1959).

2. Here wealth is defined as being equal to the total current market value of a person's resources.

3. See William H. Jean, The Analytical Theory of Finance (New York: Holt, Rinehart, and Winston, Inc., 1970), p. 76, for a demonstration of this property of quadratic utility functions.

4. There has been research done regarding other types of multiple index models of the form:

$$R_i = A_i + {}_1B_iI_1 + {}_2B_iI_2 + \dots + {}_kB_iI_k + C_i$$

These models differ from those being analyzed in this study in that they have two or more independent variables explaining a given security's rate of return. For examples, see William F. Sharpe, Portfolio Theory and Capital Markets, (New York: McGraw-Hill, Inc., 1970); Edwin J. Elton and Martin J. Gruber, "Estimating the Dependence Structure of Share Prices-Implications for Portfolio Selection," Journal of Finance 28 (Dec. 1973): 1203-32; James L. Farrell, Jr., "Analyzing the Covariation of Returns to Determine Homogenous Stock Groupings," and Donald R. Lessard, "World, National, and Industry Factors in Equity Returns," Journal of Finance 29 (May 1974): 379-91.

5. In these models each security i is linearly related to some class index j. The class index that security i is related to is denoted J_{j_i}. Since several securities (e.g., i,j,k) may be related to the same index j, there is a set of subscripts associated with each index (e.g., $J_{j_i} = j_{j_j} = j_{j_k}$). Hence, since there are M class indices $(J_1 \ldots J_M)$, each one has a set of securities associated with it, denoted N_j. Thus the set of securities associated with class index J_j can be denoted $\{i|i\varepsilon N_j\}$. It could also be denoted as $\{i|j_i\varepsilon j\}$, where j is the given class index and i is the subscript denoting security i.

6. Kalman J. Cohen and Jerry E. Pogue, "An Empirical Evaluation of Alternative Portfolio Selection Models," Journal of Business 40 (Apr. 1967): 166-93.

7. James L. Farrell, Jr., "Analyzing Covariation of Returns to Determine Homogeneous Stock Groupings," Journal of Business 47 (Apr. 1974): 186-207, using cluster analysis found that common stocks' variance of returns was due to four factors. They included (1) the market, (2) the industry, (3) the company, and (4) a system of classification corresponding to growth, stable, cyclical, and oil stocks.

8. An efficient set is said to dominate or be superior to another efficient set when its variance is less than the variance of the latter for some level or levels of expected return, and never has a larger level of variance. The efficient sets are said to be identical if the variances are equal for the two sets at all levels of expected return.

9. Cohen and Pogue, "An Empirical Evaluation of Alternative Portfolio Selection Models," p. 176.

10. Ibid., p. 178.

11. Buckner A. Wallingford, "A Survey and Comparison of Portfolio Selection Models," Journal of Financial and Quantitative Analysis 2 (June 1967): 104. A resolution of the discrepancy between these two studies has been provided by Gordon John Alexander, "Portfolio Selection Models: A Theoretical and Empirical Investigation," Ph.D. dissertation, The University of Michigan, 1975. Alexander's conclusion was that the SI model was superior to the multiple index models when market-based indices were used. This conclusion, however, was not valid when the indices used were based on the sample itself.

12. Marshall Sarnat, "Capital Market Imperfections and the Composition of Optimal Portfolios," Journal of Finance 29 (Sept. 1974): 1241-53, derived an efficient set from the Markowitz model on seven indices representing seven types of securities. Included in the seven indices were a corporate bond index and a government bond index.

13. Alexander A. Robichek, Richard A. Cohn, and John J. Pringle, "Returns an Alternative Investment Media and Implications for Portfolio Construction," Journal of Business 45 (July 1972): 434.

14. A listing of these securities is found in Appendix B. The random number table in Paul Games and George R. Karle, Elementary Statistics, (New York: McGraw-Hill, Inc., 1967), was used for all random selections.

CRSP is an acronym for The Center for Research in Security Prices, located at the University of Chicago. They have calculated and placed on magnetic tape the monthly rates of return and monthly closing prices for all common stocks listed on the New York Stock Exchange from the beginning of 1926 up through the end of 1972. Barr Rosenberg and Michel Houghlet, "Error Rates in CRSP and Compustat Data Bases and Their Implications," Journal of Finance 29 (Sept. 1974): 1303-10 have analyzed the accuracy of the CRSP tapes and concluded that it is a data base of unusually high quality.

15. A listing of these securities is found in Appendix C. The sample size (75) and design (53 common, 22 preferred stocks) were patterned to limit the differences between Wallingford's and this study's samples to size only. Since he had six out of twenty securities as preferred stocks, the same ratio was maintained in this study. Since seventy-five securities was the size of CP's sample, it was duplicated in this sample.

16. The prices used were provided by Standard and Poor, and were the conversion of the yield given by the index assuming a $100 par 7 percent coupon security. The CE used was $1.75, equal to one quarter of $7, the coupon on the security.

17. A listing of these securities is found in Appendix D.

18. Bond prices for the beginning and the end of the period were calculated by using the yield to maturity provided by Standard and Poor's and a bond table for 4 percent, twenty-year bonds. The CE received each quarter was calculated as 1 percent, equal to one-fourth of 4 percent. From the information equation (2) was used to get a rate of return. Although Standard and Poor provided the 4 percent twenty-year bond conversion of yields for only the AAA Industrial Index, the same procedure was followed for the remaining eleven corporate bond indices.

The difficulty of choosing a sample of bonds vis-à-vis these twelve indices is that ex post yields could not be used to generate expected returns, variances, and covariances. This is due to the fixed maturity date of the bonds, resulting in a changing term to maturity as the bonds' rates of return were calculated during the sample period. In order to properly compare ex post returns, a constant term to maturity must be present. Since there were few perpetual bonds available, the switch to the indices which do have a fairly constant term-to-maturity was necessitated. The source of error in this method is that change in default risk measured by the bonds' ratings, is neglected. However, it is believed that the term-to-maturity problem is more serious than the bond default problem.

19. The composite Standard and Poor's indices are calculated as an equally weighted arithmetic average. Hence this procedure was used here as well as in computing the government bond index.

20. A listing of these securities is found in Appendix E. The same comments made in Footnote 19 regarding the term-to-maturity problem hold with the government bonds. As there is no default problem with government bonds, there aren't any serious errors created by this approach.

21. Cohen and Pogue, "An Empirical Evaluation of Alternative Portfolio Selection Models," p. 175.

22. For a demonstration of this feature of least squares regression, see Ronald J. Wonnacutt and Thomas H. Wonnacutt, Econometrics, (New York: John Wiley and Sons, Inc., 1970), p. 8.

23. Principal component analysis seems appropriate, since the objective is to see how much of the variation in a multivariate system can be explained by one variable--the market element. In a study analagous to this one, Lessard stated that "the principal component solution is a particularly appropriate test for...the existence of a strong market factor." "International Portfolio Diversification: A Multivariate Analysis for a Group of Latin American Countries," p. 622.

APPENDIX A

Four Portfolio Selection Models

	Markowitz Model	SI Model	MIC Model	MID Model
Rate of return on security i	R_i	$A_i + B_i I + C_i$	$A_i + B_i J_{j_i} + C_i$	$A_i + B_i J_{j_i} + C_i$
Rate of return on class index j	NA	NA	$J_j = A_{N+j} + C_{N+j}$	$A_{N+j} + B_{N+j} I + C_{N+j}$
Rate of return on overall index	NA	$A_{N+1} + C_{N+1}$	NA	$A_{N+M+1} + C_{N+M+1}$
Covariance between securities i and k	$Cov(R_i R_k)$	$B_i B_k Q^2_{N+1} + E(C_i C_k)$	$B_i B_k E(C_{N+j_i} C_{N+j_k})$ $+ E(C_i C_k)$	$B_i B_k B_{N+j_i} B_{N+j_k} Q^2_{N+M+1}$ $+ B_i B_k E(C_{N+j_i} C_{N+j_k})$ $+ E(C_i C_k)$
For portfolios	NA	$X_{N+1} = \sum_{i=1}^{N} X_i B_i$	$S_{N+j} = \sum_{\{i \mid i \epsilon N_j\}} X_i B_i$	$X_{N+j} = \sum_{\{i \mid i \epsilon N_j\}} X_i B_i$ $X_{N+M+1} = \sum_{j=1}^{M} X_{N+j} B_{N+j}$

APPENDIX A--Continued

	Markowitz Model	SI Model	MIC Model	MID Model*
Portfolio Return = $E(R_p)$	$\sum_{i=1}^{N} X_i E(R_i)$	$\sum_{i=1}^{N+1} X_i A_i$	$\sum_{i=1}^{N+M} X_i A_i$	$\sum_{i=1}^{N+M+1} X_i A_i$
Portfolio Variance = $Var(R_p)$	$X^T CX$	$X^T S_{N+1} X$	$X^T S_{N+M} X$	$X^T S_{N+M+1} X$
Assumptions	NA	$E(C_i) = 0$	$E(C_i) = 0$	$E(C_{N+j} C_{N+i}) = 0$**
		$E(C_i C_j) = 0$**	$E(C_i C_j) = 0$**	$E(C_{N+M+1}) = 0$
		$E(C_i^2) = Q_i^2$	$E(C_i^2) = Q_i^2$	$E(C_{N+M+1} C_i) = 0$
		$E(C_{N+1}) = 0$	$E(C_{N+j}) = 0$	$E(C_{N+M+1} C_{N+j}) = 0$
		$E(C_{N+1} C_i) = 0$	$E(C_{N+j} C_i) = 0$	$E(C_{N+M+1}^2) = Q_{N+M+1}^2$
		$E(C_{N+1}^2) = Q_{N+1}^2$	$E(C_{N+j}^2) = Q_{N+j}^2$	

NA = not applicable

*In addition to the assumptions listed for the MID Model, all those listed for the MIC Model are also applicable.

** = Relationship holds when $i \neq j$.

APPENDIX B

Common Stock Sample

1. Amerada Hess Corporation
2. American Cyanamid Company
3. Ancorp National Services, Inc.
4. Atlas Corporation
5. Borden, Inc.
6. Briggs and Stratton Corporation
7. California Financial Corporation
8. Ceco Corporation
9. Cluett, Peabody, and Company, Inc.
10. Coca-Cola Company
11. Columbus and Southern Ohio Electric Company
12. Crown Cork and Seal Company, Inc.
13. Cutler Hammer, Inc.
14. Dayton Power and Light Company
15. Empire District Electric Company
16. Fischbach and Moore, Inc.
17. Gardner-Denver Company
18. General Portland, Inc.
19. Hammermill Paper Company
20. Helme Products, Inc.
21. Hoffman Electronics Corporation
22. Interco Inc.
23. International Minerals and Chemical Corporation
24. Jewel Company
25. Johns-Manville Corporation
26. Kansas City Southern Industries, Inc.
27. Keystone Consolidated Industries, Inc.
28. Kroehler Manufacturing Company
29. Lehigh Portland Cement Company
30. Lionel Corporation

31. Mississippi River Corporation
32. Monarch Machine Tool Company
33. Motorola, Inc.
34. Munsingwear, Inc.
35. National Steel Corporation
36. Northern States Power Corporation
37. Pacific Lighting Corporation
38. Phelps-Dodge Corporation
39. Puget Sound Power and Light Company
40. Quaker State Oil Refining Company
41. Ralston Purina Company
42. Roper Corporation
43. Royal Crown Cola Company
44. St. Joseph Light and Power Company
45. Sears, Roebuck and Company
46. Smith, Kline, and French Laboratories
47. Sparton Corporation
48. Sun Chemical Corporation
49. Trane Company
50. United Park City Mines Company
51. Universal Oil Products Company
52. Washington Water Power Company

APPENDIX C

Preferred Stock Sample

1. American Water Works, Inc. 5%
2. Carrier Corporation 4.5%
3. Continental Copper and Steel Industries, Inc. 5%
4. DuPont De Nemours Company $3.50
5. Duquesne Light Company 3.75%
6. Illinois Power Company 4.08%
7. Kansas City Power and Light Company 4.35%
8. Koppers Inc. 4%
9. Long Island Lighting Company 4.25%
10. Niagara Mohawk Power Corporation 3.60%
11. Northern Natural Gas Company 5.60%
12. Northern States Power Company $4.10
13. Pittsburgh, Fort Wayne, and Chicago Railway Company 7%
14. Pittsburgh, Youngstown, and Ashtabula Railway Company 7%
15. Public Service Company of Indiana 4.32%
16. Public Service Electric and Gas Company 5.28%
17. Reynold-Metals Company 4.75%
18. South Carolina Electric and Gas Company 5%
19. Southern Railway Company 5%
20. Union Electric Company $4.50
21. Virginia Electric and Power Company $4.20
22. West Penn Power Company 4.50%

APPENDIX D

Corporate Bond Sample

1. AAA Industrials
2. AA Industrials
3. A Industrials
4. BBB Industrials
5. AAA Railroads
6. AA Railroads
7. A Railroads
8. BBB Railroads
9. AAA Public Utilities
10. AA Public Utilities
11. A Public Utilities
12. BBB Public Utilities

Appendix E

U.S. Government Bond Sample

1. Short-Term Maturities

   3 percent
   3-1/2 years

2. Intermediate-Term Maturities

   3 percent
   7-1/2 years

3. Long-Term Maturities

   3 percent
   15 years

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