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January 1979

The Relevance of the Lease-or-Buy Decision

Working Paper No. 161

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October 1978

I am grateful for helpful comments received from Timothy Nantell, David Brophy, and especially James Scott and Enrique Arzac, who both served on my dissertation committee. Remaining errors are my responsibility alone.

The Relevance of the Lease-or-Buy Decision

Introduction

Recently, three articles, [8], [11], and [15], which appeared in this journal argued that in the absence of market imperfections the leasing decision is irrelevant in competitive capital markets. In retrospect, this result is not surprising since it is well known ([10], [12]) that a firm can affect its market value only by investing in profitable assets; the mode of acquisition of those assets should not matter. However, Scott [16] has shown that when interest payments are tax deductible and costly bankruptcy is possible, the capital structure decision is relevant; moreover, in comparing secured debt with leases [17], Scott implied that the leasing decision could be relevant under similar circumstances. This paper models the leasing decision, explicitly allowing for tax deductible interest payments and costly bankruptcy. It will be shown that under these circumstances it can easily matter whether a firm leases an asset from a lessor or buys it from an equipment manufacturer and finances at least part of the acquisition by issuing debt. Furthermore, by allowing for different degrees of liquidity among acquired assets, it is argued that differentiated forms of financing are relevant: i.e., it is optimal for the firm to maintain outstanding amounts of debt and leases simultaneously.

Arguments for the optimality of financing decisions which appeal to the impact of bankruptcy costs ([e.g., [15], [5], [6]) have come under attack recently. In an empirical study, Warner

[19] found that the bankruptcy costs for a sample of 11 railroads amounted to only 5.3 percent of the market value of those firms' securities. Citing this result, Miller [9] goes on to argue that if bankruptcy costs were significant we would expect to see firms issuing income bonds, a security whose tenure in the market is nearly extinct today. Both of these arguments are specious: Warner's work measures only the direct costs of bankruptcy, i.e., payments to lawyers and administrators, but it ignores the more important indirect costs, such as the impact of bankruptcy on the firm's sales and investment policy (for a discussion of these indirect costs, see Baxter [1]). Moreover, even if the total costs of bankruptcy were small, so long as they are positive they will affect firm decision making at the margin. As for income bonds, to attribute their lack of popularity to the unimportance of bankruptcy costs is illogical: there are undoubtedly other, more cogent explanations of this phenomenon, such as the information costs associated with issuing a security as complex as an income bond.

A theoretical argument for the insignificance of bankruptcy costs was presented by Haugen and Senbet [2]; they suggest that the unsecured bondholders who bear the costs of bankruptcy can avoid these costs by forming a new firm, issuing shares, and buying out the equity holders, or in other words by reorganizing the firm. While they improve upon Warner's work by considering the indirect costs of bankruptcy, there are conceptual difficulties in their discussion. First, even if the costs of reorganization are less than the liquidation costs incurred in a bankruptcy proceeding, they may still bring about an optimal capital structure so long as they are positive; in fact, the financing decision of a firm facing the prospect of the cheaper fate of a bankruptcy or a reorganization was modelled by Scott [16]. Secondly, the paper fails to consider the equilibrium determination of market prices insofar as it is unlikely that the unsecured bondholders can purchase the equity at its current price since the prospect of reorganization changes its value. The problem is not unlike the problem of the take-over bid analyzed by Leland [7], in which the acquirer must pay a price in excess of the current price specifically because the equity would be worth more under the new management.

As these considerations indicate, it has not been established that bankruptcy costs are unimportant. Consequently, the analysis in this paper will proceed under the assumption that it is possible for firms to go bankrupt and that in this contingency the firm incurs positive costs.

The Model

This paper uses a single period framework to examine the behavior of a firm which has the option at the beginning of the period of acquiring a capital asset with which it can produce marketable output, which it can sell at the end of the period to earn a net operating profit. This profit, denoted $\tilde{R}$, is random because of the product's uncertain demand, although its probability density function is known. If the firm acquires the asset, it can do so by buying it and financing at least part of the purchase by issuing debt, or it can lease the asset from a lessor. Regardless of the mode of acquisition, financial markets are perfect (i.e., there are no transactions costs in trading or issuing securities, information is costless and fully available, and securities are infinitely divisible) and competitive, so that all agents are price takers and suppliers of capital (e.g.,bondholders and lessors) can earn no monopoly profit.

In the case of the market for real assets, however, the secondary market is imperfect, so that distress sales of assets incur transactions costs. This feature is important in the event that the firm goes bankrupt at the end of the period (i.e.,$\tilde{R}$ falls so low that the firm cannot pay its lease payment or the principal and interest on its debt); in this contingency, the asset is claimed by the creditors, who try to recover their payment by selling it in the secondary market. By assuming that different classes of creditors have different degrees of familiarity with this market (in particular, lessors are able to recover more of the asset's value than the less experienced bondholders),[1] there will be different bankruptcy costs associated with the different modes of asset acquisition. Furthermore, by making firm valuation a function of the expected bankruptcy costs, the mode of acquisition will affect the firm's market value, and for any particular asset the lease-or-buy decision will generally not be a matter of indifference.

It will be assumed, for convenience, that investors are risk neutral-- i.e.,they maximize the expected value of terminal wealth-- and have homogeneous expectations. Although this is a simplifying assumption, it does not affect the analysis in an essential way since, if the leasing decision is relevant under these circumstances, it will be relevant under more complex assumptions. Moreover, in this framework there is no ambiguity in evaluating firm decision making by the criterion of its impact on the market value of the firm's equity.[2] Finally, the marginal corporate tax rate t is constant, the personal tax rates on income from interest, dividends, and capital gains are equal,[3] and the investment tax credit associated with buying the asset is already reflected in the purchase price P.

Valuation under the Purchase Alternative

The method of demonstrating the relevance of the lease-or-buy decision will be to derive an expression for the value of the firm's equity (i.e.,the net present value of stockholder wealth) when it buys the asset and employs at least some degree of bond financing, explicitly allowing for the possibility of bankruptcy. Then, a second valuation expression will be obtained for the alternative of acquiring the asset through leasing. In both cases, the financing charges, i.e.,the promised interest payment or the lease payment, will be priced competitively so that the resulting net present value of the suppliers of capital is zero. Moreover, it will be assumed that the firm makes optimal decisions: i.e., it chooses the alternative which results in the highest market value of its equity. Under these circumstances, the lease-or-buy decision will be irrelevant only when the two valuation expressions are equivalent; otherwise the decision will matter.

If the firm buys the asset, it must pay the purchase price of P dollars at the beginning of the period to acquire it. This amount is at least partially offset by the proceeds it receives by issuing the optimal amount of debt, B_B.[4] Then, at the end of the period it receives its net operating profit $\tilde{R}$, which is taxable, and it must pay the promised interest payment, C_B, which is tax deductible, to the bondholders. Furthermore, since it owns the asset, it receives the depreciation-generated tax saving of t, the marginal corporate tax rate, times the amount of the depreciation. If δ is the depreciation rate, where $0 \leq \delta \leq 1$, the tax saving is given by $t\delta P$.[5] In addition, at the end of the period, the firm can sell the asset, which in an efficient real asset market will sell for the original price minus the depreciation, or $(1-\delta)P$.[6] Finally, it must repay the borrowed principal B_B to the bondholders.

The value of stockholder wealth in this firm V_B will depend upon whether it remains solvent at the end of the period. This will occur so long as its cash flow at the end of the period is positive, i.e., if $(\tilde{R} - C_B)(1-t) + t\delta P + (1-\delta) P - B_B > 0$

$$\text{or} \quad \tilde{R} > b = \frac{B_B - P + \delta P}{1-t} + C_B \qquad (1)$$

where b is the level of net operating profit below which the firm goes bankrupt. If inequality (1) holds, the firm is able to meet its obligations of B_B and C_B to its bondholders, and the stockholders receive the excess. On the other hand if (1) is violated, the firm is bankrupt, and the stockholders receive nothing at the end of the period. Formally, if $\tilde{S}'_B$ is the random cash flow to stockholders at the end of the period,

$$\tilde{S}'_B = \begin{cases} (\tilde{R}-C_B)(1-t)+t\delta P+(1-\delta)P-B_B & \text{if } \tilde{R} > b \\ 0 & \text{if } \tilde{R} \leq b \end{cases}$$

If stockholders are assumed to be risk-neutral, the value of the equity is the beginning of the period cash flow $-P+B_B$ plus the expected value of the end of period cash flow discounted one period at the risk-free rate of interest r:

$$V_B = -P + B_B + \frac{E[\tilde{S}'_B]}{1+r}$$

$$V_B = \frac{\hat{R}(1-t)(1-F)-C_B(1-t)(1-F)+(r+F)B_B - P[r+F+\delta(1-t)(1-F)]}{1+r} \qquad (2)$$

where F = the probability of bankruptcy (i.e., the cumulative probability that $\tilde{R}<b$), $\hat{R}$ is the conditional mean of $\tilde{R}$ (i.e., the expected value of $\tilde{R}$ given that the firm remains solvent), and $E[\cdot]$ is the expectation operator.

Expression (2) represents the value of the firm's equity for any arbitrary value of C_B, the promised interest payment. In a competitive equilibrium, however, C_B will be set so that the net present value of the bondholders' position is zero. The equilibrium value is determined as follows: at the beginning of the period, the bondholders expend B_B in loans to the firm; what they receive at the end of the period depends upon whether the firm remains solvent. If it does remain solvent, they receive their principal B_B plus their interest payment C_B; if the firm goes bankrupt, however, they receive the lesser of $B_B + C_B$ or $\ell_B P$,[7] the liquidation value of the asset when sold in an imperfect secondary market (N.B.: ℓ_B, the liquidation fraction appropriate for bondholders lies in the range $0 \leq \ell_B \leq 1$). Assuming the firm issues enough debt so that $B_B + C_B > \ell_B P$, $\tilde{B}_B$, the cash flow to bondholders at the end of the period, is

$$\tilde{B}'_B = \begin{cases} B_B + C_B & \text{if } \tilde{R} > b \\ \ell_B P & \text{if } \tilde{R} \leq b \end{cases}$$

Assuming bondholders are risk-neutral, the net present value of the bondholders' position is:

$$NPV_B = - B_B + \frac{E[\tilde{B}'_B]}{1+r}$$

Setting this expression equal to zero and solving for C_B,

$$C_B = \frac{B_B(r+F) - \ell_B PF}{1-F} \qquad (3)$$

Equation (3) expresses the promised interest payment C_B as a function of the outstanding amount of debt B_B, the pure time value of money r, the liquidation value of the asset $\ell_B P$, and the probability of bankruptcy F. Substituting the equilibrium value of C_B as given in (3) into valuation expression (2) yields

$$V_B = \frac{\hat{R}(1-t)(1-F)+tC_B(1-F) - P[r+(1-\ell_B)F+\delta(1-t)(1-F)]}{1+r} \qquad (4)$$

Equation (4) states the value of stockholder equity if the firm buys the asset. It is composed of the discounted value of three terms: the expected after tax profit from operations (N.B.:since $\hat{R}=\frac{E_b[\tilde{R}]}{1-F}$ = the semi-mean of $\tilde{R}$ above the bankruptcy point divided by the probability of solvency, the term $\hat{R}(1-t)(1-F)$ represents the expected value of $\tilde{R}$ on an after tax basis for all states in which the firm remains solvent) plus the expected tax savings from using debt, $tC_B(1-F)$, minus the user cost of capital.[8]

Valuation under the Leasing Alternative

It is instructive to consider the value of stockholder wealth under the assumption that the firm leases the asset since, in the process of specifying the cash flows for this alternative, most of the traditional advantages and disadvantages of leasing as opposed to buying are made explicit.[9] First, at the beginning of the period the firm does not need to spend funds to acquire the asset nor does it need to borrow funds by issuing bonds. However, at the end of the period it does not receive the cash inflows from depreciation-generated tax savings and the sale of the used asset for its salvage value. The cash flow it does receive at the end of the period is the net operating profit from using the asset $\tilde{R}$ minus the lease payment L to the lessor after taxes. The value of stockholder wealth under this alternative V_L depends upon whether the firm remains solvent at the end of the period. If $\tilde{R}$ exceeds the lease payment obligation, stockholders receive the excess after taxes; if not, the firm is declared bankrupt, and stockholders receive nothing. Formally,

$$\tilde{S}'_L = \begin{cases} (\tilde{R}-L)(1-t) & \text{if } \tilde{R} > L \\ 0 & \text{if } \tilde{R} \leq L \end{cases}$$

where $\tilde{S}'_L$ is the end of period cash flow to stockholders. For risk-neutral investors, V_L is the expected value of the end of period cash flow discounted at the risk free interest rate:

$$V_L = \frac{E[\tilde{S}_L']}{1+r}$$

$$V_L = \frac{\hat{R}(1-t)(1-F) - L(1-t)(1-F)}{1+r} \quad (5)$$

Equation (5) gives the value of stockholder wealth for any arbitrary value of L, the lease payment. In a competitive equilibrium, however, L will be set so that the net present value of the lessor which leases the asset to the firm is zero. To determine this value of L, an expression must be derived for the net present value of the lessor.

There is a parallel between the cash flows to the lessor and those of the firm under the purchase mode of acquisition. At the begining of the period, the lessor spends P dollars to acquire the asset to lease to the lessee (the firm using the asset) and receives B_L dollars from issuing the optimal amount of debt. However, its end-of-period cash flow will depend upon whether the lessee remains solvent. If it does, the lessor will receive L, its lease payment, minus the interest payment C_L to its bondholders after taxes, plus the depreciation-generated tax saving $t\delta P$ and the inflow from the sale of the asset $(1-\delta)P$; it must also repay its bondholders their principal B_L. On the other hand, if the lessee goes bankrupt, the lessor is still responsible for its obligations C_L and B_L to its bondholders and still receives the depreciation tax saving; but the lessee defaults on its lease payment,[10] and the proceeds from the sale of the used asset will be its liquidation value $(1-\delta)\ell_L P$, with $0 \leq \ell_L \leq 1$, so that only if the lessor's liquidation fraction

ℓ_L equals one will there be no loss on the salvage value.[11]

Formally, the lessor's end of period cash flow, $\tilde{L}'_L$ is given by

$$\tilde{L}'_L = \begin{cases} (L-C_L)(1-t)+t\delta P+(1-\delta)P - B_L & \text{if } \tilde{R} > L \\ -C_L(1-t)+t\delta P+(1-\delta)\ell_L P-B_L & \text{if } \tilde{R} \leq L \end{cases}$$

Assuming risk neutrality on the part of the lessor, the net present value of the position is

$$NPV_L = -P + B_L + \frac{E[\tilde{L}'_L]}{1+r}$$

$$NPV_L = \frac{L(1-t)(1-F) - C_L(1-t) + rB_L - P[r+(1-\delta)(1-\ell_L)F + \delta(1-t)]}{1+r} \quad (6)$$

where C_L, the lessor's promised interest payment to its bondholders, is determined in the same fashion as C_B, in accordance with equation (3), and depends upon the lessor's probability of bankruptcy. Setting (6) equal to zero and solving for L yields the equilibrium value of the lease payment:

$$L = \frac{P[r+(1-\delta)(1-\ell_L)F + \delta(1-t)] + C_L(1-t) - rB_L}{(1-t)(1-F)} \quad (7)$$

Finally, substituting (7) into (5) yields the equilibrium value of the stockholder wealth of the firm if it leases the asset:

$$V_L = \frac{\hat{R}(1-t)(1-F) - C_L(1-t) + rB_L - P[r + (1-\delta)(1-\ell_L) F + \delta(1-t)]}{1+r} \quad (8)$$

The issue of the relevance of the lease-or-buy decision can now be stated in terms of the two valuation expressions (4) and (8). Clearly, if both expressions are negative, the firm should not acquire the asset; if, on the other hand, at least one of these expressions is positive when both the firm under the purchase option and the lessor under the leasing option set their debt levels optimally, then the firm should acquire the asset through the mode that yields the highest net present value. Only if circumstances are such that (4) and (8) are equivalent will the lease-or-buy decision be irrelevant.

As a special case, it should be noted that the conclusions of Miller and Upton, Lewellen, Long and McConnell, and Myers, Dill and Bautista that (4) and (8) should be equivalent hold when the probability density function of $\tilde{R}$ is such that the firm acquiring the asset faces a zero probability of bankruptcy. In that case, it can be verified that both expressions reduce to

$$V = \frac{\bar{R}(1-t) + tC - P[r+\delta(1-t)]}{1+r} \qquad (9)$$

where $\bar{R}$ is the unconditional expected value of $\tilde{R}$, and $C = rB$, where B, the optimal debt level, depends further upon the nature of the probability density function of $\tilde{R}$. When F=0, it follows from the work of Modigliani and Miller [13] that firms (i.e., the firm buying the asset and the lessor) should issue as much debt as possible to take advantage of the tax saving on interest payments. While they should always finance their investments completely with debt under such conditions, if the probability density function of $\tilde{R}$ is such that the firm can earn monopoly profits on its investment, then it should issue more debt than is needed to finance the investment. This general case is allowed for in equation (9), which, given the single period framework, risk neutrality, and the specific consideration of capital costs, is equivalent to the Modigliani and Miller valuation model for a firm which can issue debt in a world in which interest payments are tax deductible and bankruptcy is impossible. (9) can be simplified further, however, for the case where the firm is a member of a competitive industry (i.e., the density function of $\tilde{R}$ is such that the firm earns a zero profit) since then it is optimal only to issue enough bonds to finance the asset completely with debt (i.e. B=P). In that case

$$V = \frac{[\bar{R} - P(r+\delta)]\ (1-t)}{1+r} = 0 \qquad (10)$$

and expected profit exactly covers the user cost of capital.[12]

Numerical Example

A numerical example will help to illustrate the nexus between the probability of bankruptcy, the value of the firm, and the relevance of the leasing decision. Let the purchase price of the asset P equal 100, the depreciation rate δ equal .1, the risk free rate of interest r = 10%, and the marginal corporate tax rate t equal .5. First, consider the decision of a competitive firm facing a zero probability of bankruptcy. The previous analysis suggests that the (net present) value of stockholder wealth should be zero under both the purchase alternative and the leasing alternative. This claim is verified by hypothesizing that at the end of the period there is only one state of the world, and in that state $\tilde{R} = 20$.

First, consider the value of the equity if the asset is purchased. The firm must decide how much debt to issue, which, in accord with the method described in Kraus and Litzenberger [6], is the maximum level allowable, considering the resulting probability of bankruptcy for each state. In this case, in the only state which can occur $\tilde{R} = 20$, so that for the maximum debt level corresponding to $\tilde{R} = 20$ the firm accepts a zero probability of bankruptcy. From equation (3), when F=0, $C_B = rB_B$, or $.1B_B$ in this example. Substituting this and the exogenously given data into criterion (1), it is found that B_B can equal 100 without causing insolvency when $\tilde{R} = 20$. Therefore, the optimal level of debt is 100: If $B_B > 100$, the firm will go bankrupt with certainty; if $B_B < 100$, the firm foregoes valuable tax savings from interest payments with no accompanying reduction in its probability of bankruptcy, since F=0 already. Once the optimal debt level is known, V_B can be determined; by substituting $B_B=100$, $C_B=10$, and F=0 into (4), it is found that $V_B=0$.

If the firm acquires the asset through leasing, the value of its equity depends upon the behavior of the lessor since the lessee firm's probability of bankruptcy depends upon the value of L, its lease payment: Since $\tilde{R} = 20$ with certainty, F=0 if $L \leq 20$, and F=1 if $L > 20$. In terms of the cash flows to the lessor, if $L > 20$, the lessor will receive

$$- C_L(1-t) + t\delta P + (1-\delta)\ \ell_L P - B_L$$

with certainty; assuming the best possible salvage value for the lessor's sale of the asset (i.e.,$\ell_L=1$), the lessor can set B_L = 90 without going bankrupt. If $L > 20$, it cannot issue more than 90 dollars of debt, since if it does it will go bankrupt with certainty. If it sets L no greater than 20, however, it will receive an additional L(1-t) with certainty, which raises the maximum level of debt which the lessor can issue while still accepting a zero probability of bankruptcy to 100. Since the higher debt level entails additional tax savings from the higher interest payment, it is optimal for both lessee and lessor to agree to set L=20 and accept a zero probability of bankruptcy, and for the lessor to set $B_L = 100$ and $C_L = 10$. Substituting these values into equation (8), it is found that $V_L = 0$.

The above example demonstrates that when there is no chance of bankruptcy, the value of stockholder wealth is unaffected by the mode of acquisition of an asset, as Miller and Upton, Lewellen, Long and McConnell, and Myers, Dill and Bautista suggest. Moreover, the (net present) value of stockholder wealth under both alternatives is zero if the firm only has access to investments which yield no monopoly profit. This latter condition is not essential for the lease-or-buy decision to be irrelevant: If the firm had some degree of monopoly power but faced a zero probability of bankruptcy, the value of stockholder wealth would be positive, but equal, under both alternatives.

In contrast to the previous example, if the firm, behaving optimally, faces a positive probability of bankruptcy regardless of its competitive posititon in its industry, the lease or buy decision will generally be relevant. While this is also true for competitive firms, it will be assumed here that the density function of $\tilde{R}$ is such that the firm's investment can yield monopoly profits.[13]

As an example of such a density function, consider the following:

State	1	2	3
$\tilde{R}$	−106	−11.67	236
Probability	.25	.5	.25

In addition to the other exogenously given data in the previous example, assume that ℓ_B=.2 and ℓ_L=1 (i.e.,bondholders receive 20 percent of the value of the asset if the firm goes bankrupt, while lessors incur no transaction costs).

The value of stockholder wealth if the asset is purchased depends upon the amount of debt which is issued and the resulting probability of bankruptcy. The optimizing firm considers the various alternatives, given the density function of $\tilde{R}$, and chooses the debt level which maximizes the value of stockholder wealth. The lowest level of $\tilde{R}$ will be realized if state 1 occurs; therefore, the firm accepts a zero probability of bankruptcy it if sets B_B so that it remains solvent in that state. Since $\tilde{R}$ = −106 in State 1, with F = 0 and C_B = $.1B_B$, the firm can issue up to $40.00 of debt. If more debt is issued, the probability of bankruptcy jumps to .25 and remains at that level until the point is reached where the firm would no longer be solvent if state 2 occurred. From equation (3), when F = .25, C_B = $.467B_B$−6.67, and if $\tilde{R}$ equalled −11.67, the firm would still be solvent if B_B were set at 75, at which level C_B would equal 28.33. If more debt is added, F jumps to .75, since if either state 1 or 2 occurred the firm would go bankrupt. With F = .75, C_B = $3.4B_B$−60, and the firm will remain solvent so long as $B_B \leq 90$, at which level the promised interest payment is 246. If more than \$90 of debt is issued, the firm will go bankrupt with certainty.

The optimal debt level is selected by evaluating V_B at the 3 different debt levels and their corresponding probabilities of bankruptcy. If F=0, the conditional mean of $\tilde{R}$ equals the unconditional mean, which is 26.7, and with a debt level of \$40, $V_B = 0.3$. If the firm sets the probability of bankruptcy at .25, the conditional mean of $\tilde{R}$ would be 70.9, and after issuing \$75 of debt, V_B would equal 3.14. Finally, when F=.75, $\hat{R} = 236$, and setting B_B equal to 90 yields $V_B = -10$. It is clear that V_B is maximized when $B_B = 75$ (and F=.25). If the firm buys the asset, it will issue \$75 of debt and its value will be 3.14.

If the firm leases the asset, it must accept a .75 probability of bankruptcy, since it will go bankrupt if $\tilde{R} < L$, and L must be positive. The lessor can ensure its solvency regardless of what happens to the lessee by setting B_L low enough so that

$$-C_L(1-t) + t\delta P + (1-\delta)P - B_L$$

(i.e., its cash flow in the event of the lessee's bankruptcy) is positive, which occurs if $B_L \leq 90$. If the lessor sets B_L higher, it will go bankrupt if the lessee goes bankrupt; in other words, the lessor must accept a .75 probability of bankruptcy (and C_L from equation (3) would equal $3.4\,B_L - 60$) if $B_L > 90$. Moreover, since F=.75 for the lessee as well, L would be set, from equation (7), at $120 + 4C_L - .8B_L$. Substituting these two expressions into the requirement that

$$(L - C_L)(1-t) + t\delta P + (1-\delta)P - B_L > 0$$

(i.e., the cash flow to the lessor when the lessee remains solvent must be positive if the lessor is to remain solvent), it is found that the largest allowable level of B_L is -17.6 (i.e. the firm must lend \$17.60). However, if it sets B_L this low, it will actually be accepting a zero probability of bankruptcy. In fact, as long as $B_L \leq 90$ this will be the case; unless the lessor can issue more than \$90 of debt when it accepts a .75 probability of bankruptcy, it is optimal to set B_L at 90 and accept a zero probability of bankruptcy. This conclusion follows from the observation that for a given level of bankruptcy probability a firm should issue the maximum amount of debt to take advantage of the tax savings, and follows as well from the competitive nature of the lease market. The latter consideration forces the lessor to charge the lowest possible lease payment which will still leave its net present value nonnegative; by substitution of the two values of B_L into equation (7) it can be seen that if $B_L = -17.6$, L would equal 127, while if $B_L = 90$, L = 84. Competition in the lease market would force the lessor to set $B_L = 90$, whereupon $C_L = 9$, since the lessor is accepting a zero probability of bankruptcy. Substituting these values, F = .75, and $\hat{R} = 236$ into equation (8), it can be verified that $V_L = 17.27$.

Given this density function of $\tilde{R}$ and the choice of buying the asset and issuin \$75 of debt or leasing the asset from a lessor which incurs no transaction costs in a distress sale of the asset, the firm does better by leasing the asset, since $V_L > V_B$. As Lewellen, Long, and McConnell [8] indicate, the origin of such an outcome is to be found in a comparison of the relative circumstances of the two buyers of the asset the firm of the purchase mode, and the lessor. In this case (and in general) each has an advantage over the other: the firm can deduct larger promised interest payments for tax purposes than the lessor, while the lessor incurs lower bankruptcy costs than the bondholders of the firm. In this case, it happens that the latter advantage outweighs the former, so that leasing is the preferred mode of acquisition.

Leasing may not always have such an advantage. If the lessor incurs costs as the result of the lessee's bankruptcy, the relative advantage of leasing may be lost. In this example, leasing is the favored alternative only so long as the lessor's liquidation fraction, ℓ_L, is greater than .784. If, for example, ℓ_L=.77, it can be verified that the lessor's optimal decision is to accept a zero probability of bankruptcy and issue \$70 of debt. With the lessee still accepting a .75 probability of bankruptcy, the equilibrium value of L will be 216, and V_L will equal 2.27. Here, the buying alternative is preferred by the firm, since the tax advantage of that mode exceeds the advantage of lower bankruptcy costs which could be gained from the leasing mode.

Three additional points can be noted here: first, the conclusion that $V_L > V_B$ does not follow simply because $\ell_L > \ell_B$. The existence of lower bankruptcy costs in the leasing mode is not sufficient to give that alternative an absolute advantage. Secondly, if ℓ_L exactly equalled .784, the lease-or-buy decision would again be irrelevant since the tax advantage of buying would exactly be offset by the lower bankrupty costs in leasing. The acceptance of a positive probability of incurring bankruptcy costs is not sufficient to ensure the relevance of the lease-or-buy decision, even though it will generally be relevant under those circumstances.

Finally, as Miller and Upton mention, the lease-or-buy decision varies from case to case; specifically, it depends on the nature of the asset (the density function of $\tilde{R}$) and the creditors (the values of ℓ_B and ℓ_L). So, while it may be preferable to lease one asset, it might be advantageous to buy a different one. To the extent that the firm employs heterogeneous assets, it should be expected not only that the leasing decision should be relevant but also that neither mode should dominate the other; i.e.,the firm may easily finance simultaneously with debt and leasing. Thus, when bankruptcy is possible and capital assets are heterogenous, not only is financing relevant but differentiated forms of financing are relevant.

Footnotes

1. The notion that lessors are able to receive a better liquidating price than bondholders for assets of a bankrupt firm to which these creditors have a claim originates in the legal restrictions of lease contracts and bond indentures with regard to bankruptcy. Bondholders will receive the full value of their claim -- principal plus interest -- only if the bonds are secured; otherwise, they will receive less. In the case of lessors, if the contract is ruled to be a "true lease," in that the lessee did not acquire an equity interest in the asset over the life of the contract, lessors are entitled to reclaim their assets and re-lease them to new lessees with no loss in income. If, however, the contract is ruled to be a "disguised secured purchase," lessors may receive less than the full value of the asset. In either case lessors fare better than unsecured bondholders since transacting in used assets is an integral part of the leasing business, whereas bondholders have no experience with this market. Furthermore, Scott [17] has noted that in actual practice the claims of lessors often supercede even those of secured bondholders. For a discussion of these legal issues, see Shapiro and Reisman [18].

2. With risk averse investors who have homogeneous expectations, firm value maximization is Pareto-optimal: investors are always made better off by a decision which results in the highest possible market value. If investors were risk averse or had heterogeneous expectations, value maximization would be appropriate only if the financial markets were complete (i.e., there existed a number of securities equal to the number of states of the world), or individual utility functions displayed the separation property, so that the composition of the risky portfolios of all individuals was identical. See Mossin [14] for a discussion of this issue.

3. Such a personal tax structure serves to neutralize the impact of personal taxes on valuation. Miller [9] derives the generalized expression for the firm's gain from the tax deductability of interest payments as

$$1 - \frac{(1-t_c)(1-t_{ps})}{1-t_{pb}}$$

where t_c is the corporate tax rate, t_{ps} is the personal tax rate applicable to income earned from common stock (dividends and capital gains), and t_{pb} is the personal tax rate applicable to income earned from bonds (interest payments). If $t_{ps} = t_{pb}$, as is assumed here, the expression reduces to t_c, the tax effect in the absence of personal taxes.

4. The determination of the optimal size of the debt issue when bankruptcy is costly is discussed in Kraus and Litzenberger [6] and will be explained here in the numerical example below.

5. This formulation assumes that accounting depreciation, which results in the tax saving, and economic depreciation, which relates to the gradual wearing out of the asset, are equal.

6. The distinction should be noted between the sale of the used asset for salvage by a solvent firm for price $(1-\delta)P$ and the distress sale of an asset by the creditors of a bankrupt firm for a lower price. Transactions costs are incurred only in the latter case, not in the former.

7. See Scott [17]. If the bondholders receive their full payment when the firm goes bankrupt (i.e. $B_B + C_B \leq \ell_B P$), the debt is said to be secured. If more debt is issued, so that the liquidation value of the asset is insufficient to pay the bondholders in full, at least some of the debt is unsecured. Since Scott [16] demonstrates that it is always optimal for the firm to exhaust its secured debt capacity, it will be assumed that $B_B + C_B > \ell_B P$. In either case, the model assumes either prepayment of or anticipation of the issuance of debt by the short term creditors (e.g.,wage earners and suppliers) who have a priority claim over bondholders in a bankruptcy proceeding. Otherwise, bondholders would receive $\ell_B P$ minus the negative cash flow from operations if the firm went bankrupt.

8. Jorgenson [4] gives the user cost of capital as $P(r+\delta)$, the foregone interest and depreciation from using a capital asset. Here, depreciation is on an after-tax basis, since accounting and economic depreciation are identical, and is multiplied by $(1-F)$ to reflect its expected value, since it is only realized if the firm remains solvent. The additional term $(1-\ell_B)F$ is the extra cost borne by the firm because of the possibility that it may go bankrupt while owning the asset; it should be noted that this extra term vanishes if either bankruptcy is impossible $(F=0)$ or the asset is perfectly liquid in the secondary market $(\ell_B=1)$ so that bankruptcy is costless.

9. See Johnson and Lewellen [3] for a description of these advantages and disadvantages. It should be noted that the work of Miller and Upton [11], Lewellen, Long and McConnell [8], and Myers, Dill and Bautista [15] demonstrate that, in general, these advantages and disadvantages are illusory when the required returns of suppliers of capital are considered.

10. This assumes the asset is uniquely suited to the needs of the lessee; otherwise, the lessor could petition to recover lost lease payments, the rationale being that the lessor's ability to re-lease a nonunique asset to a new lessee following the bankruptcy of the original lessee does not constitute recovery of lost profits, since another nonunique asset could always have been leased to the new lessee anyway. See Shapiro and Reisman [18] for a discussion of this issue.

11. In fact $\ell_L=1$ is the case of a "true lease." The value of ℓ_L when the contract is ruled a "disguised secured purchase" depends upon the lessor's familarity with the secondary market for used assets. In either case it is a safe assumption that $\ell_L > \ell_B$.

12. It follows that, in the case of competitive firms facing a zero probability of bankruptcy, if a firm leases the asset, its lease payment L will be set to equal $P(r+\delta)$, the user cost of capital -- point made by Miller and Upton [11]. The notion of irrelevance is clearest in this case, since the profit earned from the asset must pay for the cost of employing the asset whether that cost is borne by the producing firm or by the lessor.

13. If $F > 0$ for a competitive firm, it will be true that $V_L \neq V_B$, but the better alternative of the two will equal zero, while the other will be negative. Although such an example might demonstrate that the mode of asset acquisition is relevant, the investment decision will not be: it might turn out, for example, that if the firm decides to acquire the asset, it should lease it rather than buy it; but on the other hand, it will not matter whether it leases it or fails to acquire it at all.

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