Division of Research
Graduate School of Business Administration
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June 1985

# A COMPARISON OF BRIBERY AND BIDDING IN THIN MARKETS

Working Paper No. 431

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We are grateful for comments from Vic Bernard, Gary Biddle, Michael Bradley, Joel Demski, Ronald Dye, John Eichenseher, Aneel Karnani, William Kinney, Glenn Loury, Sam Peltzman, David Sappington, Thomas Stober, Martin Zimmerman, and participants in workshops at the Universities of Illinois and Michigan. This research was supported in part by the UCLA Graduate School of Management and the The University of Michigan Graduate School of Business Administration.

## ABSTRACT

This paper compares bribery to competitive bidding in a government purchasing context. While competitive bidding is one method of procurement, bribery is a common alternative in many Third World countries. Although bribery is often considered to be the ethical antithesis of competitive bidding, the analysis shows there is a fundamental isomorphism between bribery and competitive bidding on the supply side of the transaction.

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## 1. INTRODUCTION

Awarding contracts for goods and services on the basis of competitive bidding is a common procurement mechanism when there are thin markets. Government agencies are often required by law to obtain competitive bids, in part to reduce opportunities for bribe taking in such markets.[1] In many Third World countries, however, bribery is a common institutional arrangement for government procurement. In 1977, U.S. Congress enacted regulation that prohibited U.S. firms and individuals from paying bribes to foreign government officials. The Carter Administration, which proposed the regulation, claimed this type of bribery was morally repugnant and inefficient (U.S. House of Representatives, 1977, p. 175).

The purpose of this note is to model bribery as an arrangement for procurement in thin markets, and to compare bribery with competitive bidding as alternative institutional arrangements. Although bribery and bidding are considered to be ethical antitheses, our results show conditions under which there is a fundamental isomorphism between these two exchange mechanisms. Specifically, we show that the same supplier will win the contract under both exchange mechanisms and that the expected (net of bribes) price paid by the purchasing country will be equal to the expected value of the winning bid. Under these conditions, controversies about the relative efficiency of bribery versus bidding may be moot.[2]

## 2. THE BRIBERY MODEL

In the 1970s, several hundred U.S. firms disclosed that they were paying bribes to high level government officials to obtain contracts for selling aircraft, military hardware, health care products, and other goods to governments and government-controlled firms.[3] The most common bribery method in this type of transaction has been the commission bribe model. In the commission bribe model, the bribe paid is a function of sales. Typically, the government purchasing official sets an invoice price that includes an allowance for a bribe or kickback. The government pays the full invoice amount, but the supplier kicks back a portion of the invoice price to the official, a third party intermediary, or a designated bank account in a country specified by the customer (e.g., Switzerland, Lichtenstein) after the contract is awarded.[4]

We model the commission bribery transaction in a governmental procurement context in which the purchase contract could be awarded based upon either a bribery or competitive bidding process. Under the former, the contract is awarded at a predetermined price (P) to the firm paying the largest bribe; while under the latter, the contract is awarded competitively to the supplier submitting the lowest bid price. In the bribery model, each potential supplier is assumed to negotiate privately with a governmental official. Hence, the information available to bribery participants is similar to the information available to bidders in that firms do not know the bribe offers of other firms. Suppliers are assumed to know the government's policy of awarding the contract to the firm offering to pay the largest commission bribe.

We use the following notation in the models:

| | |
|---|---|
| P: | The contract price paid by the government. |
| c: | The cost of supplying the product (excluding bribes). |
| Z = P - c: | The gross profit from the contract. |
| B: | The bribe to be paid to the government official or third party intermediary. |
| F(z): | The cumulative probability distribution for z, defined over the interval $[\underline{z}, \bar{z}]$, where $\underline{z} \geq 0$. |
| n: | The number of firms competing for the contract. |

We assume the government official sets the contract price (P) and suppliers compete by offering bribes. Suppliers are assumed to know their own costs, but have incomplete information about competitors' costs and potential gross profits. Accordingly, our analysis is restricted to symmetric games in which all firms have the same information about competitors' gross profits, which is modelled by the distribution F(z). F(z) is twice differentiable and the bribe paid is an increasing and differentiable function of z; therefore, $B = B(z)$ where $dB(z)/dz = B'(z) > 0$. Given these assumptions, B(z) has an inverse which is denoted by $\pi(\cdot)$ and, by definition, $\pi(B(z)) = z$.

## Suppliers' Response Function

Given our information assumptions, we focus on symmetric Nash equilibria in which all firms employ a common (bribery) strategy. The probability that a representative (ith) firm would obtain the contract with a bribe of B is just the probability that the other n-1 firms offer smaller bribes. Assuming that all firms employ the common strategy, B(z), and submit their bribes independently, such an event occurs when all competitors have z's that are smaller than $z_i$ and the associted probability is $F(z_i)^{n-1}$. Given that $F(z_i)^{n-1} = F(\pi(B))^{n-1}$, the ith firm's expected payoff from submitting the bribe can be expressed as:

$$E[\phi(B)] = [z_i - B]F(\pi(B))^{n-1}, \qquad (1)$$

where E denotes the expectation operator and $\phi(B)$ is the payoff from bribe B.

## Proposition 1:

Given the above assumption, the (symmetric) equilibrium bribery strategy for the ith firm is

$$B(z_i) = z_i - \int_{\underline{z}}^{z_i} F(t)^{n-1}\, dt / F(z_i)^{n-1}, \qquad (i = 1,n) \qquad (2)$$

where t is a dummy variable of integration.

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Proof: (See Appendix A)

Proposition 1 indicates that, in equilibrium, each firm computes its own potential gross profit on the contract and then submits a bribe offer which represents a markdown from z. The markdown term can be analyzed further by manipulating equation (2) algebraically (see Appendix A) to obtain the following equivalent expression:

$$B(z_i)F(z_i)^{n-1} = (n-1)\int_{\underline{z}}^{z_i} t\cdot F(t)^{n-2}f(t)dt. \tag{3}$$

Since $B(z_i)$ is the bribe paid contingent upon winning the contract and $F(z_i)^{n-1}$ is the probability of winning, the left side of (3) is the expected value of the bribe paid by the ith firm. The right side of (3) is the expected value of the gross profit accruing to the firm submitting the second largest bribe (given that the ith firm submits the largest bribe). Thus, (3) indicates that the ith firm's expected bribe is effectively bounded by the expected gross profit of the second lowest cost firm (given that the ith firm is the lowest cost supplier among the set of n firms).

An important comparative statics property of the bribery model, which is evident in (2), is that the expected bribe paid to the governmental official is a non-decreasing function of the number of competing suppliers. Increased competition among firms affects the equilibrium bribe in two respects. First, as the number of suppliers increases, the probability of including the most profitable (lowest cost) producer increases. A statistical explanation for this result is that the largest (winning) bribe can be regarded as the nth order statistic for the "sample" of firms actually submitting bribes and, therefore, can be shown to converge in mean square to the (finite) upper support of the industry gross profit distribution as n becomes very large.

Second, the difference between the expected values of the nth and n-1 order statistics also can be shown to be a decreasing function of n. Therefore, since the potential profit captured by the winning firm is bounded by the gross profit of the next most profitable firm, increasing the number of suppliers forces firms to bid more aggressively. By pitting suppliers against each other, the governmental official, like a monopsonist exploits the thinness of the market by extracting producers' surplus in the form of a bribe.

## 3. BRIBERY AND BIDDING INSTITUTIONS

An alternative method of exchange in thin markets is competitive bidding. While competitive bidding has been portrayed as the legal and ethical antithesis of bribery, there is a basic isomorphism between bribery and bidding as institutions for procurement. The isomorphism between the bribery and bidding institutions is established in two propositions. Proposition 2 presents the equilibrium bidding strategy for a game in which competitors have the same amount of information and costs as in the bribery model presented above. Proposition 3 then shows that, for a given contract price, the expected return to each firm from the equilibrium bribery strategy is the same as the expected return from the equilibrium bidding strategy derived above.

Proposition 2:

Assuming that each firm knows its own cost as in the bribery model and assesses a common distribution of costs for competitors G(c), the equilibrium bidding strategy is

$$\hat{B}(c) = c + \int_{c}^{\bar{c}} [1 - G(s)]^{n-1} \, ds/[1 - G(c)]^{n-1}, \qquad (4)$$

where $\bar{c}$ denotes the upper support of G(•) and s is a dummy variable of integration.

Proof: (See Appendix A)

Having derived the equilibrium bidding strategy, we now compare the expected payoffs with those in the bribery game.

Proposition 3:

For a given contract price, P, firms' expected payoffs from the equilibrium bribery strategy are equal to the payoffs from the equilibrium bidding strategy.

Proof: (See Appendix A)

The isomorphism between the equilibrium outcomes of the bidding and bribery games can be explained by the fact that the bribe actually is a covert discount paid by the supplier to the government official, rather than to the state. In effect, the government official implicitly conducts a covert bidding game for his(her) own benefit by purchasing at the lowest bid price and then reselling to the state at the higher price, P. Since the contract is awarded to the same supplier and the same net of bribes price is paid by the government, both institutions are equally effective in extracting suppliers' surplus. Hence, in the absence of penalties for bribery, suppliers will be indifferent between paying bribes or discounting the selling price to the purchasing country.

## 4. CONCLUSIONS AND IMPLICATIONS

This paper has presented an equilibrium model of bribery in thin markets in which a government purchases from a group of suppliers. We compared the bribery model to a competitive bidding model and showed that, for a predetermined contract price, the bribery model was isomorphic to the bidding model in that the same firm won the contract and the government paid the same net-of-bribes purchase price. These results imply that, in the absence of penalties for bribery, supplier firms would be indifferent between bribery and bidding institutions. If all suppliers face the same penalty for paying bribes, then the equilibrium bribe would be reduced by the amount of the penalty, and the isomorphism between bribery and bidding would be retained. This isomorphism on the supply side may explain why exporting countries (except for the United States, after 1977) generally do not penalize their firms for paying bribes in importing countries where bribery is legal.

FOOTNOTES

[1]See Alchian (1977), Rose-Ackerman (1978), and Holt (1980).

[2]See Rose-Ackerman (1978) for a thorough discussion of the market versus "corrupt" methods of dealing with government officials.

[3]Detailed descriptions of these bribery activities can be found in U.S. Securities and Exchange Commission (1976), filings on 8-K and 10-K forms with the Securities and Exchange Commission, and in Greanias and Windsor (1982).

[4]See descriptions of bribery transactions in Lockheed Corporation (1979), In Re Sealed Case (1982), and Greanias and Windsor (1982).

## Appendix A

### Proof of Proposition 1

The equilibrium bribery strategy can be determined by differentiating equation (1) with respect to B,

$$\frac{d}{dB} E[\phi(B)] = -F(\pi(B))^{n-1} + [z_i - B](n-1)F(\pi(B))^{n-2}f(\pi(B))\pi'(B). \quad (1A)$$

Since $B = B(z)$ and $\pi(B(z)) = z$, one can verify that $\pi'(B(z)) = 1/B'(z)$.

After making the appropriate substitutions, (1A) is equivalent to the following expression:

$$E[\phi'(B)] = -B'(z_i)F(z_i)^{n-1} + [z_i - B(z_i)]F(z_i)^{n-2}f(z_i). \quad (2A)$$

The necessary condition for an interior optimum is that:

$$-B'(z_i)F(z_i)^{n-1} + [z_i - B(z_i)]F(z_i)^{n-2}f(z_i) = 0. \quad (3A)$$

Note that (3A) is a linear differential equation whose solution can be obtained readily by making use of the fact that:

$$B'(z_i)F(z_i)^{n-1} + B(z_i)(n-1)F(z_i)^{n-2}f(z_i) = \frac{d}{dz_i} [F(z_i)^{n-1}B(z_i)]. \quad (4A)$$

Thus, (3A) is equivalent to:

$$\frac{d}{dz_i} [F(z_i)^{n-1}B(z_i)] = (n-1)F(z_i)^{n-2}f(z_i). \quad (5A)$$

Integrating both sides of (5A) over the interval $[\underline{z}, z_i]$ where firm i has a positive probability of winning,

$$F(z_i)^{n-1}B(z_i) = \int_{\underline{z}}^{z_i} (n-1)F(t)^{n-2}f(t)t \, dt + k, \quad (6A)$$

where k is a constant of integration.

The integral of the right hand side of (6A) can be integrated by parts. Letting u = t and dv = $(n-1)F(t)^{n-2}f(t)\,dt$,

$$\int_{\underline{z}}^{z_i} (n-1)F(t)^{n-2}f(t)\,t\,dt = [tF(t)]_{\underline{z}}^{z_i} - \int_{\underline{z}}^{z_i} F(t)^{n-1}\,dt. \qquad (7A)$$

$$= z_iF(z_i) - \int_{\underline{z}}^{z_i} F(t)^{n-1}\,dt. \qquad (8A)$$

Substituting (8A) into (6A) and dividing by $F(z_i)^{n-1}$,

$$B(z_i) = z_i - \int_{\underline{z}}^{z_i} F(t)^{n-1}\,dt/F(z_i)^{n-1} + k/F(z_i)^{n-1}. \qquad (9A)$$

One can verify that the constant of integration in (9A) must be zero by taking the limit as $z_i \to \underline{z}$. The first term on the right side has a limit of $\underline{z}$ and the second term can be shown to have a finite limit using L' Hospital's rule. Therefore, a nonzero k would result in an infinitely negative bribe for $k < 0$ or violate the monotonicity property for $k > 0$. The resulting bribery strategy is consistent with a Nash equilibrium, which is verified by showing that $B(z_i)$ is a best-response when competitors are assumed to employ, $B(\cdot)$.

Proof of Proposition 2

Assuming that the contract is awarded to the firm submitting the lowest bid, the probability of winning with a bid of $\hat{B}$ is

$$[1 - G(\hat{\pi}(\hat{B}))]^{n-1}, \qquad (10A)$$

where $\hat{\pi}(\cdot)$ denotes the inverse of $\hat{B}(\cdot)$ and $G(\cdot)$ denotes the "industry cost distribution." The expected payoff is:

$$E[\phi(\hat{B})] = (\hat{B} - c)\ [1 - G(\hat{\pi}(\hat{B}))]^{n-1} \tag{11A}$$

Differentiating (11A) with respect to $\hat{B}$ and evaluating the resulting expression at $\hat{B} = \hat{B}(c)$, the first-order (necessary) condition is:

$$\left.\frac{dE[\phi(\hat{B})]}{d\hat{B}}\right|_{\hat{B} = \hat{B}(c)} = [1 - G(\hat{\pi}(\hat{B}))]^{n-1} \tag{12A}$$

$$+ [\hat{B}(c) - c](n-1)[1 - G(\hat{\pi}(\hat{B}(c)))]^{n-2}$$

$$g(\hat{\pi}(\hat{B}(c))) \cdot \hat{\pi}'(\hat{B}(c)) = 0.$$

Making use of the facts that

$$\hat{\pi}(\hat{B}(c)) = c \text{ and } \frac{d\hat{\pi}}{dc}(\hat{B}(c)) = \frac{d\hat{\pi}}{d\hat{B}} \cdot \frac{d\hat{B}}{dc} = 1,$$

(12A) can be written as:

$$[1 - G(c)]\hat{B}'(c) = [\hat{B}(c) - c]g(c)(n-1). \tag{13A}$$

One can verify that the following solution satisfies the differential equation in (13A) on the interval $(c_i, \bar{c})$:

$$\hat{B}(c_i) = c + \int_{c_i}^{\bar{c}} [1 - G(s)]^{n-1} ds/(1 - G(c))^{n-1} + k/(1 - G(c))^{n-1}. \tag{14A}$$

Notice that the integration is over the interval $[c_i, \bar{c}]$ where firm i has a positive probability of winning. The constant of integration in (14A) must be non-negative, since the monotonicity property does not permit $\hat{B}(c)$ to decrease as $c_i \to \bar{c}$. Alternatively, if $k > 0$, $\hat{B}(\bar{c}_i)$ becomes infinite, so the constant of integration must be identically zero. Q.E.D.

Proof of Proposition 3

The expected net payoff from the equilibrium bidding strategy in Proposition 2 is equal to the expected net payoff from the equilibrium bribery strategy in Proposition 1. The expected net payoff to a firm employing the equilibrium bribery strategy is obtained by substituting (2) into (1):

$$E[\phi(B(z_i))] = [z_i - (z_i - \int_{\underline{z}}^{z_i} F(t)^{n-1}\, dt/F(z_i)^{n-1})]\ F(\pi(B(z_i)))^{n-1} \qquad (15A)$$

$$= \int_{\underline{z}}^{z_i} F(t)^{n-1}\, dt. \qquad (16A)$$

The expected net payoff from the equilibrium bidding strategy is:

$$E[\phi(\hat{B}(c_i))] = \int_{c_i}^{\bar{c}} (1 - G(s))^{n-1}\, ds. \qquad (17A)$$

We must show that (16A) and (17A) are equal. By definition of the cumulative distribution function,

$$F(z_i) = Pr(z < z_i). \qquad (18A)$$

Recalling that $z = P - c$ and $z_i = P - c_i$,

$$Pr(z < z_i) = Pr[(P - c) < (P - c_i)] \qquad (19A)$$

$$= Pr(c_i > c) \qquad (20A)$$

$$= 1 - G(c_i). \qquad (21A)$$

Notice that $\underline{z} = P - \bar{c}$ and $\bar{z} = P - \underline{c}$, so the range of integration in (16A) is consistent with (17A). To verify this, consider that for firm i to win the symmetric bidding game, $c_j > c_i$ for $i \neq j$, $j = 1, n$. But $c_j > c_i$ implies that $P - c_j < P - c_i$, so $z_j < z_i$, for $i \neq j$, $j = 1, n$. Thus, $F(z) = 1 - G(c)$. Finally, the reversal in the direction of integration in (17A) relative to (16A) takes into account that $F(z_i)^{n-1}$ is an increasing function of $z_i$ over $[\underline{z}, \bar{z}]$ while $[1 - G(c_i)]^{n-1}$ is a decreasing function of $c_i$ over $[\underline{c}, \bar{c}]$. Thus, it follows that:

$$\int_{c_i}^{\bar{c}} (1 - G(s))^{n-1} ds = \int_{\underline{z}}^{z_i} F(t)^{n-1} dt. \quad \text{Q.E.D.} \qquad (22A)$$

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