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THE USE OF MARKET DATA AND ACCOUNTING DATA IN HEDGING AGAINST CONSUMER PRICE INFLATION

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1.0 INTRODUCTION

This paper examines the use of alternative information sets in the construction of inflation hedge portfolios. The study is motivated by consideration of the investor's problem in a multi-period world. Several authors (e.g., Merton [1973], Breeden [1979]) have shown that in a multi-period setting, optimal investment behavior will, in general, involve holding portfolios that can be used to hedge against changes in certain relevant states of nature. One potentially relevant state of nature is the rate of inflation in general prices (Jones [1982]).

In contrast to prior related research, the empirical results indicate that it is possible to construct inflation hedge portfolios successfully, if certain accounting information is used. However, portfolios constructed on the basis of historical security price information do not serve as effective hedges.

One contribution of this paper is to demonstrate the potential usefulness of accounting information to a price-taking investor. Although financial statements play an important role in the setting of equilibrium security prices, it is not clear what additional value, if any, the financial statements offer to the investor, once information contained therein has been impounded in prices. For example, within the world of the Sharpe-Lintner-Mossin singleperiod capital asset pricing model (CAPM), the investor buys a share of the market portfolio and then levers that portfolio to achieve a desired level of risk. It has long been recognized by accountants that this simple investment strategy could be adopted without using any firm-specific accounting information.[1] In contrast, more general (multi-period) forms of the CAPM assume the existence of hedge portfolios that could be constructed only on the basis of certain firm-specific information, possibly including accounting information.

Within this context, accounting information may be valuable even to a price-taker. While such an investor does not seek abnormal returns, (s)he might increase expected utility by using accounting information to construct hedge portfolios, thus achieving an optimal balance between expected return, aggregate market risk, and risk associated with changes in relevant state variables.

A second contribution of this paper concerns the development of asset pricing theory. In providing evidence that it is possible to construct inflation hedge portfolios, the paper demonstrates the existence of a necessary (though not sufficient) condition for the descriptive superiority of multi-period CAPM's that explicitly consider uncertainty in consumer prices. This evidence also indicates which sources of information are useful to investors actually attempting to construct inflation hedge portfolios.

The paper is organized into six sections. In Section 2, I discuss the rationale for hedging against inflation, and review previous attempts to establish the existence of inflation hedge portfolios. Section 3 describes three hedge portfolio construction techniques, based on three alternative information sets. Data and measurement issues are discussed in Section 4, and the empirical results appear in Section 5. Section 6 contains the summary and conclusions.

2.0 ASSET PRICING AND THE EXISTENCE OF HEDGE PORTFOLIOS

The Sharpe-Lintner-Mossin CAPM is developed within a single-period framework. More general models are developed within a setting where investment and consumption take place over several periods (Merton [1973], Long [1974], Breeden [1979], Jones [1982]). Unless certain restrictions hold, the utility-maximizing investment strategy in the multi-period setting differs from that in the single-period models. In the single-period model, the investor holds only combinations of a riskless asset and a share of the market portfolio. In a multi-period setting, however, the investor may also hold long or short positions in certain hedge portfolios.

Hedge portfolios are used in the multi-period setting to reduce risk associated with unexpected changes in states of the world that affect the marginal utility of wealth. It is not possible to specify which states of the world are relevant without knowledge of the forms of investors' utility functions. However, consumer prices represent states of nature that very likely affect the utility of wealth. A multi-period asset pricing model developed by Long [1974] explicitly introduces uncertainty in the prices of specific consumer goods. In that model, investors' utility of wealth depends on prices, so they hold long (short) positions in portfolios to hedge against (speculate on) changes in specific prices. Within the context of an asset pricing model developed in a monetary economy, Jones [1982] shows how the utility of wealth may also depend upon inflation in the general price level.[2] In that economy, investors would desire to hold long or short positions in portfolios designed to hedge against the stochastic element of inflation in general prices.

In a world of incomplete markets, the existence of hedge portfolios is open to question. As a result, prior research has focused on whether investors can actually form inflation hedge portfolios (Gouldey [1980], Schipper and Thompson [1981], Gay and Manaster [1982], Bernard and Frecka [1983]). Some support for the existence of inflation hedge portfolios is provided by Gouldey, who states that "it is highly likely that consumer-investors can economically form portfolios in order to hedge against consumer price level inflation" (Gouldey [1980], p. 257). However, Gouldey's tests were designed to examine the validity of the Long CAPM, and did not involve the actual identification of the components of the inflation hedge portfolios. Nor did they require the construction of hedge portfolios using only information from other periods.

Those who have actually attempted to construct common stock inflation hedge portfolios have generally met with little success.[3] All of these previous attempts have sought to derive hedge portfolio weights on the basis of the covariability of security returns with inflation in prior, or subsequent periods. Due to the instability of this covariability, portfolios constructed on the basis of security price data offer either little or no hedging potential (Schipper and Thompson [1981], Gay and Manaster [1982]) or hedging potential over a brief period (Bernard and Frecka [1983]). Schipper and Thompson, pointing to the lack of stability in the covariances of stock returns with unexpected inflation, conclude that "additional information besides the past history of rate of return covariability should be sought by investors attempting actually to construct a hedge portfolio" (Schipper and Thompson [1981], pp. 325-326).

In the following section, additional information besides the past history of rate of return covariability is incorporated into hedge portfolio construction techniques. The additional information consists of accounting variables which, according to a model of the firm, should be useful in deriving hedge portfolio weights. Hedging strategies are developed which depend 1) solely on security price data 2) solely on the predictions of the model of the firm and 3) on a combination of security price data and accounting data.

3.0 CONSTRUCTION OF HEDGE PORTFOLIOS

3.1 Definition of a hedge portfolio

If investors' marginal utility of wealth is conditional only upon the state variable S, then the multi-period CAPM can be written:

$$\tilde{R}_{jt} = \alpha_j + \beta_j \tilde{R}_{Mt} + \gamma_j \Delta\tilde{S}_t + \tilde{e}_{jt} \tag{1}$$

where

$\tilde{R}_{jt}$ = return on security j in period t;

$\tilde{R}_{Mt}$ = return on the market portfolio in period t;

$\tilde{\Delta S}_t$ = the stochastic (unanticipated) portion of the change in the state variable in period t;

$\tilde{e}_{jt}$ = error term.

In this context, optimal investment strategy involves combinations of a systematic-risk-free portfolio, a share of the market portfolio, and a single hedge portfolio.[4] The content of the hedge portfolio can be defined by appealing to the work of Breeden (1979) or Long (1974), among others. The essence of all definitions of hedge portfolios is to maximize the correlation between the hedge portfolio return and changes in the state variable.[5] For example, when the Long model is simplified to include only one relevant state variable, the hedge portfolio is equivalent (up to a factor of proportionality) to the zero-β portfolio with maximum correlation of its return with changes in the state variable. That hedge portfolio can be found by minimizing the variance of return while preserving a fixed positive covariance between the portfolio return and changes in the state variable, and constraining the portfolio β to be equal to zero. Specifically, the hedge portfolio weights $[w_1, w_2, \ldots w_N]$ solve the following problem:[6]

$$\text{Minimize Var } [\sum_{j=1}^{N} w_j R_{jt}] \tag{2}$$

subject to:

$$[w_1, w_2, \ldots w_N] \begin{bmatrix} \beta_1 & \gamma_1 \\ \beta_2 & \gamma_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \beta_N & \gamma_N \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where

w_i = hedge portfolio weight assigned to security i;

N = number of securities;

and R_{jt}, β_j, and γ_j are as defined by equation (1).

The solution to this problem is the minimum-variance, zero-β, unit-γ portfolio.

In order to calculate hedge portfolio weights that satisfy problem (2), consider a cross-sectional regression equation of the following form:

$$R_{jt} = a_{0t} + a_{1t}\,\beta_j + a_{2t}\,\gamma_j + e_{jt} \tag{3}$$

where

R_{jt}, β_j, and γ_j, are as defined in equation (1);

a_{0t}, a_{1t}, and a_{2t} are regression coefficients;

e_{jt} is a disturbance term.

The estimated coefficients a_{0t}, a_{1t}, and a_{2t} can be interpreted as portfolio returns. (This interpretation of regression coefficients is analogous to that used by Fama and Macbeth [1973].) The coefficient a_{2t} can be viewed as the realized return on the zero-investment hedge portfolio that solves problem (2). The portfolio with return a_{2t} has a β equal to zero, and a γ equal to one. In addition, given the assumptions of the basic linear regression model, the

Gauss-Markov theorem states that the least-squares estimate of a_{2t} has minimum variance of all zero-investment, zero-β, unit-γ portfolios.

The individual security weights associated with the portfolio with return a_{2t} are implicit in the calculations used to estimate regression equation (3).[7] Those weights can be determined at the beginning of period if one knows the values of β_j and γ_j for j = 1,2,...N. In the remainder of this section, it is assumed that the relevant state variable, against which the investor desires to hedge, is unanticipated inflation. Three alternative methods of estimating β_i and γ_i are developed under this assumption, leading to three alternative inflation hedge portfolio construction techniques.

3.2 STRATEGY 1: Hedging strategy based on securities market data

The first hedging strategy is based upon estimates of β_j and γ_j derived from a set of firm-specific time series regressions of the following form:

$$R_{jt} = \alpha_j + \beta_j (R_{mt}) + \gamma_j (u_t) + e_{jt} \qquad j=1, 2, \ldots N \qquad (4)$$

where

R_{jt} = nominal return on security j for period t;
R_{mt} = nominal return on a value-weighted NYSE index for period t;
u_t = unexpected inflation for period t;
e_{jt} = disturbance term;
α_j, β_j, γ_j = regression coefficients.

In certain tests of hedging potential, the estimates of β_j and γ_j used to construct a hedge portfolio for a given period will be based on time series regressions which include all observations except those from that period. The use of information from both prior and subsequent periods improves the efficiency of the estimations and provides information about the intertemporal stability of γ.[8] If γ_j is not sufficiently stable over time, a hedging strategy that depends upon values of γ_j estimated in prior periods will not succeed. To assess the feasibility of such a hedging strategy, I will also derive hedge portfolio weights by using only observations of prior periods to estimate equation (4).

Once estimates of β_j and γ_j are derived for each period, the actual hedge portfolio return for the period can be calculated by applying OLS to equation (3). The estimate of a_{2t} is equal to the hedge portfolio return.

3.3 STRATEGY 2: Hedging strategy based on a model of the firm

In STRATEGY 2, estimates of β_j are again derived using equation (4). The estimates of γ_j, which reflect the reaction of security prices to unexpected inflation, are now derived, however, by appealing to a model of the firm.

The model predicts that the impact of unexpected inflation upon the value of a firm's common stock depends upon three factors: the firm's net monetary position (NMP); the magnitude of the firm's depreciation tax shields (NPE); and a set of "inflation response parameters" (θ_k) that predict the impact of unexpected inflation upon operating profitability. More precisely, the model predicts that when unexpected inflation for period t (u_t) occurs, the price of security j should change, as a fraction of the beginning-of-period price, by the amount $\gamma_{jt}\, u_t$, where

$$\gamma_{jt} = 1 + [(-NMP_{j,t-1} - F(NPE_{j,t-1}) + \sum_{k=1}^{n} \theta_{jk}]/V_{j,t-1} \qquad (5)$$

and where

$NMP_{j,t-1}$ = beginning-of-period-t net monetary position for firm j;

$NPE_{j,t-1}$ = beginning-of-period-t tax basis of depreciable assets for firm j;

F = tax rate (assumed equal to .5);

θ_{jk} = inflation response parameters for firm j, estimated either while excluding data from period t, or while using only data from periods prior to period t;

k = lag parameter to be defined below in equation (6);

n = 2 in case of annual data; 4 in case of quarterly data;

$V_{j,t-1}$ = beginning-of-period t value of firm j's common shares.

Details of the model development appear in the Appendix and in Bernard [1983]. The essence of the model, the logic for inclusion of the given independent variables, and the meaning of the inflation response parameters (θ_{jk}) are described below.

Consider a model in which the real (inflation-adjusted) value of the firm's common shares is represented as a function of the firm's expected future cash flows. These expected cash flows are affected by the impact of unexpected inflation upon the real value of their three major components: cash flows from operations, cash flows associated with the issue and retirement of debt (and/or loans), and cash flows used to pay taxes.

The impact of unexpected inflation upon the real value of cash flows required to retire debt is described by the debtor-creditor hypothesis. (See Kessel and Alchian [1962], French, Ruback, and Schwert [1983].) That is, unanticipated positive inflation should transfer wealth from net creditors to net debtors, so the impact of unanticipated inflation upon the value of a firm's equity shares should depend upon the firm's net monetary position (NMP).

The effect of unanticipated inflation upon the real value of cash flows used to pay taxes is described by the "tax effects" hypothesis. (See Feldstein and Summers [1979], Gonedes [1981], French, Ruback, and Schwert [1983].) Under this hypothesis, the real value of historical-cost-based tax shields falls with increases in the price level. If the inflation is unanticipated, then the value of the firm's shares should also fall. The magnitude of this impact of unanticipated inflation should thus depend upon the magnitude of the firm's historical-cost-based tax shields (F•NPE).

Unanticipated inflation can affect the real value of the firm's cash flows from operations in two ways. First, unanticipated inflation can alter the real value of nominal contracts (other than those mentioned above), such as sales contracts, labor contracts, and purchase commitments. Second, unanticipated inflation can transfer wealth to or from a firm's customers and/or suppliers, thus shifting the demand for a firm's output and/or the supply of a firm's input. The impact of unexpected inflation upon real cash flows from operations is measured by firm specific "inflation response parameters." These response parameters are denoted by vectors θ, estimated using the following time series regressions:

$$\left[\frac{\bar{C}_t}{S_t} - \frac{C_{t-n}}{S_{t-n}}\right] S_{t-n} = d_0 + \theta_1 u_t + \ldots + \theta_n u_{t-n+1} + e_t \qquad (6)$$

where

C = real cash flows from operations, as proxied by current cost operating income before depreciation, interest, and taxes;

S = number of shares outstanding;

u = unexpected inflation;

n = 2 when annual data are employed, and 4 when quarterly data are employed.

Equation (6) can be derived from an empirical analogue of the equation supplied in the Appendix to define the inflation response parameters. In equation (6), the term C_{t-n}/S_{t-n} serves as a proxy for the expected value of C_t/S_t, where the expectation is formed in period t-n. Thus, the dependent variable represents the change in expected cash flows from period t-n through period t. This change is expressed as a function of unexpected inflation over periods t-n+1 through period t. Note that unexpected inflation is allowed to affect expected contemporaneous cash flows (as reflected by θ_1) and expected future cash flows (as reflected by θ_2 through θ_n).

In some tests, the estimated values of θ used in the construction of hedge portfolios for any given period were derived using only data from prior periods for which accounting reports were available. In other tests, data from both prior and subsequent periods were used.[9]

As in STRATEGY 1, actual hedge portfolio returns were calculated by using the estimates of β_j and γ_j in equation (3). The estimate of a_{2t} is equal to the hedge portfolio return.

3.4 STRATEGY 3: Hedging strategy based upon joint information sets

The third and final hedging strategy is based both on securities market data and on accounting data. Once again, hedge portfolio returns are calculated in cross-sectional regressions of returns against the parameters β_j and γ_j. However, neither the estimates of γ_j based upon securities market data nor the estimates of γ_j based upon the model of the firm are used directly to calculate hedge portfolio weights. Rather, an error-in-variables approach is adopted which is designed to yield estimates of γ_j that should be more accurate than those used in either STRATEGY 1 or STRATEGY 2. The approach assumes that estimates of γ_j (say $\hat{\gamma}_j$) that are derived using security price data as in equation (4) measure the true value of γ_j (say γ_j^*) only with error (z_j). However, the true value γ_j^* can be expressed as a linear function of the variables NMP_j, NPE_j, and $\sum_k \theta_{jk}$. The following system of equations represents the estimation procedures used for Strategy 3:

$$R_{jT} = a_{OT} + a_{1T}\,\beta^*_{jT} + a_{2T}\,\gamma^*_{jT} + e_T \tag{7.1}$$

$$R_{jt} = \hat{\alpha}_0 + \hat{\beta}_{jT}\,R_{mt} + \hat{\gamma}_{jT}\,u_t + \hat{\varepsilon}_t \qquad t \neq T \tag{7.2}$$

$$\hat{\beta}_{jT} = \beta^*_{jT} \tag{7.3}$$

$$\hat{\gamma}_{jT} = \gamma^*_{jT} + z_{jT} \tag{7.4}$$

$$\gamma^*_{jT} = d_{OT} + d_{1T}\left[\frac{NMP_{j,T-1}}{V_{j,T-1}}\right] + d_{2T}\left[\frac{NPE_{j,T-1}}{V_{j,T-1}}\right] + d_{3T}\left[\frac{\sum_{k=1}^{n}\theta_{jk}}{V_{j,T-1}}\right]. \tag{7.5}$$

Equation (7.1) is the cross-sectional regression used to estimate the hedge portfolio return, a_{2T}. It is equivalent to equation (3). Equation (7.2) represents a set of time series regressions used to generate estimates $\hat{\beta}_{jT}$ and $\hat{\gamma}_{jT}$ for each firm j and each period T, and is equivalent to equation (4). Equation (7.3) asserts that the estimate $\hat{\beta}_{jT}$ is equal to the true value β^*_{jT}. In contrast, equation (7.4) indicates that the estimate $\hat{\gamma}_{jT}$ in equation (7.2) measures the true value γ^*_{jT} with error. In equation (7.5), the true value γ^*_{jT} is written as a linear function of scaled values of $NMP_{j,T-1}$, $NPE_{j,T-1}$, and $\sum\theta_{jk}$.

Given this system of equations, a two-stage least-squares approach can be used to estimate (7.1). (See Judge, Griffiths, Hill, and Lee [1980], section 13.7.) The first-stage regression is:

$$\hat{\gamma}_{jT} = d_{0T} + d_{1T}\left[\frac{NMP_{j,T-1}}{V_{j,T-1}}\right] + d_{2T}\left[\frac{NPE_{j,T-1}}{V_{j,T-1}}\right] + d_{3T}\left[\frac{\sum_{k=1}^{n}\theta_{jk}}{V_{j,T-1}}\right] + z_{jt} \tag{8}$$

The dependent variable in (8) is the estimate $\hat{\gamma}_{jT}$ from the time series regression equation (7.2). Equation (8) is estimated using cross-sectional data. Once the estimates $\hat{d}_{0T}$, $\hat{d}_{1T}$, $\hat{d}_{2T}$, and $\hat{d}_{3T}$ are derived, the values of γ^*_{jT} actually used to construct a hedge portfolio for period T are calculated as follows:

$$\gamma^*_{jT} = \hat{d}_{0T} + \hat{d}_{1T}\left[\frac{NMP_{j,T-1}}{V_{j,T-1}}\right] + \hat{d}_{2T}\left[\frac{NPE_{j,T-1}}{V_{j,T-1}}\right] + \hat{d}_{3T}\left[\frac{\sum_{k=1}^{n}\theta_{jk}}{V_{j,T-1}}\right] \tag{9}$$

Thus, the hedge portfolio return a_{2T} can be derived using a second-stage cross-sectional regression like equation (7.1). Given equation (7.3), the values of β^*_{jT} used in the second stage regression are derived by estimating equation (7.2).

Note that the essence of the approach is to use appropriate linear combinations of NMP_j, NPE_j, and θ_j to estimate γ^*_j. Which linear combination is appropriate is determined by using security-price-based estimates $\hat{\gamma}_j$ in equation (8). Of course, since equations (7.3) and (7.5) will not hold precisely, the approach does not eliminate all measurement error. Nevertheless, some reduction in measurement error should occur.

4.0 DATA AND MEASUREMENT ISSUES

4.1 Test Period and Test Sample

The empirical tests employ annual data from the 1961-1980 period and quarterly data from the 1966-1980 period. The test sample includes 136 firms from 27 industries, including mining, manufacturing, transportation, utilities, financial, and service industries. The number of firms included from each of the 27 industries is listed in Table 1. The firms were chosen primarily on the basis of data availability[10] and in order to obtain 2 to 10 representatives of each of the 27 industries. The final sample size reflects the elimination of 14 firms that experienced major changes in lines of business. Such changes were expected to lead to unstable inflation response parameters.

4.2 Measurement of Firm-specific Variables

4.2.1 Net Monetary Position

The net monetary position was measured as cash, plus short-term investments, plus long-term monetary assets, minus the sum of current liabilities, long-term debt, and preferred stock.[11] With the exception of data on long-term monetary assets, all data were available on COMPUSTAT. For companies with long-term investments in excess of 10 percent of total assets, data on the monetary portion of those investments were gathered from annual reports. Also included in long-term monetary assets were investments in wholly owned but unconsolidated financial subsidiaries.

4.2.2 Tax Basis of Depreciable Assets

The tax basis of depreciable assets was estimated by adjusting book values of plant, property, and equipment as disclosed in financial reports. Book values were first reduced by the investment in land and assets subject to percentage-depletion.[12] The remaining book value was converted to a tax basis based upon information about the firm's depreciation accounting methods, estimated average age of assets, and estimated useful life of assets.[13]

4.2.3 Measurement of Cash Flows from Operations

Estimation of inflation response parameters required a measure of real (inflation-adjusted) cash flows from operations. Real cash flows from operations were approximated by current-cost constant-dollar operating income before depreciation, interest, and taxes. The latter had to be estimated since only historical-cost-based income was available over the test period chosen.[14] This entailed restating cost of goods sold (excluding depreciation) on a replacement cost basis and then adjusting sales, cost of sales, and other operating expenses for changes in the general price level.

Restatement of cost of goods sold was carried out using methods similar to those of Falkenstein and Weil (1977). Inventory for each firm was matched with one or more of a list of over 1,000 specific price indexes, based upon descriptions of business found in annual reports, 10-K's, and Moody's Manuals. The inventory was then aged and restated on the basis of the change in the specific price index since date of purchase.

To assess the accuracy of the restatement procedure, these estimated amounts were compared to actual replacement cost disclosures required by the SEC in 1976-1979 and by the FASB in 1979-1980. The mean relative difference, as a fraction of the reported replacement cost of sales (excluding depreciation), was -.0003; the mean absolute relative difference was .0070. These small differences reflect the accuracy of the restatement procedure and the small magnitude of the difference between cost of sales on a historical cost basis and a current cost basis.

4.3 Measurement of Unexpected Inflation

The unexpected component of inflation was measured using inflation forecasts based upon Treasury bill rates.[15] Unexpected inflation was assumed equal to the expected real rate of return on Treasury bills outstanding during the given quarterly or annual period, minus the actual real rate of return on those bills. Real interest rates were calculated by subtracting the percentage change in the Consumer Price Index from nominal interest rates. The model is similar to one employed by Fama and Gibbons (1983), in that it assumes interest rates include an inflation premium and a fluctuating real interest rate.[16]

5.0 EMPIRICAL RESULTS

Hedge portfolios were constructed for each year in the 1961-1980 period and each quarter in the 1966-1980 period according to the strategies described in section 3:

STRATEGY 1: based on securities market data;
STRATEGY 2: based on a model of the firm;
STRATEGY 3: based on joint information sets.

Table 1 lists portfolio weights derived using quarterly data for the fourth quarter of 1980. Although portfolio weights were actually calculated for each of the 136 firms in the sample, those weights were averaged within industries to facilitate the presentation in Table 1. Those stocks assigned positive weights were predicted to have higher-than-average covariability of returns with unexpected inflation, and vice-versa. For example, consider the aircraft manufacturers in the sample. Since those firms have large net debtor positions and (in some cases) hold cost-plus contracts that would tend to buffer the impact of unexpected inflation upon operating profitability, we would expect those firms' returns to have a higher-than-average covariance with unexpected inflation. Indeed, the average predicted value of that covariance (as measured by γ) for aircraft manufacturers was positive, whether based on security price information (equation (4)) or accounting information (equation (5)). Thus, the inflation hedge portfolios include (on average) a long position in the common stock of the aircraft manufacturers.

[Table 1 about here.]

Tests of the ability of the portfolios to hedge against unexpected inflation were conducted as follows. For each annual or quarterly period, realized hedge portfolio returns were calculated using a cross-sectional regression like equation (3). Once these hedge portfolio returns (denoted a_{2t}) were known, the success of the hedging strategies was assessed by observing the association over time between the portfolio returns and unexpected inflation, as estimated in the following regression equation:[17]

$$a_{2t} = g_0 + g_1 u_t + z_t \tag{10}$$

where

a_{2t} = hedge portfolio return from equation (3);

u_t = unexpected inflation.

Table 2 presents statistics that summarize the estimation of equation (10). The results that appear in Panel 1 and Panel 2 are based on tests where hedge portfolio weights for period t were derived using information of test periods prior to and subsequent to period t. For all three of the strategies assessed in Panels 1 and 2, the hedge portfolio returns covary positively and significantly (at the .05 level) with unexpected inflation. Thus, Panels 1 and 2 provide evidence that it is possible to construct common stock portfolios, using out-of-sample data, that have returns that are significantly correlated with unexpected inflation. These results are somewhat in contrast to results obtained by Schipper and Thompson [1981]. Further analysis of tests based on quarterly data indicated that all of the hedging strategies perform better during the 1973-1980 period than during previous years. Thus the difference between my results and those of Schipper and Thompson could be due primarily to the use of a test period which is more recent than (but overlaps with) the 1954-1975 period which they used. Recent years have been characterized by unexpected inflation of higher absolute magnitude, which gives rise to more powerful tests of hedging potential.

[Table 2 about here.]

The tests summarized above suggest sufficient stability in the association between security returns and unexpected inflation to allow investors to successfully construct common stock inflation hedge portfolios. However, the tests were based upon information from prior and <u>subsequent</u> periods. The latter information would not be available to investors until after the fact.

Panel 3 presents the results of tests using only historical information available at the time of forming the hedge portfolios. Tests were limited to those employing quarterly data, since an insufficient number of annual observations were available both to estimate hedge portfolio weights with historical data <u>and</u> to test hedge portfolio performance. The tests summarized in Panel 3 estimated hedge portfolio weights using quarterly data from periods subsequent to 1965, but which were publicly available as of the portfolio formation date. For example, when estimating hedge portfolio weights for the first quarter of 1978, I used securities market data from the 1966-1977 period and/or accounting data for periods subsequent to 1965 which had been publicly released as of the end of 1977. The hedging strategies were tested using the last half of the 1966-1980 period of data availability. Thus, 30 quarterly observations of hedge portfolio returns were available.

The results of Panel 3 show that the hedge portfolio returns were positively correlated with unexpected inflation, regardless of the source of historical information used to construct the hedge portfolios. However, only when accounting data and the model of the firm were used to construct the portfolios were the correlations significantly different from zero at the .05 level.[18,19]

The better performance of the accounting-information-based hedging strategy appears to be due to two reasons. First, the response parameters that measure the impact of inflation upon accounting profitability must have been less noisy and/or more stable predictors of security price reaction to unexpected inflation than predictors based on prior period security price behavior. This was supported by supplemental tests in which the accounting-information-based strategy outperformed the security-price-based strategy, even when the net monetary position and depreciation tax shields were ignored and hedge portfolio weights were constructed solely on the basis of inflation response parameters. Second, since the net monetary position and depreciation tax shields do vary over time, the accounting-information-based strategy allows for some instability in the association between security returns and unexpected inflation.

When the combined information sets were used to construct hedge portfolios, the coefficient obtained by regressing the hedge portfolio return against unexpected inflation is highest. However, this does not imply that the strategy based on the combined information sets offers the best hedge against unexpected inflation. Recall from Section 3 that the desired hedge portfolio is the minimum variance, zero-β, unit-γ portfolio. Since the regression coefficients in Table 2 ($\hat{g}_1$) are always below unity, none of the hedging strategies yield a unit-γ portfolio; this is indicative of measurement error in unexpected inflation and/or prediction errors in the estimates of γ_{jt}. However, there always exists some scalar by which all hedge portfolio weights can be multiplied to achieve a g_1 equal to unity.[20] Once this operation is carried out, the hedging strategy with the highest correlation of return with unexpected inflation will have the minimum variance of return. To emphasize this point, consider multiplying hedge portfolio weights derived under each strategy in Panel 3 by a scalar so as to obtain an estimated value for g_1 equal to one. Then compare the variance of the resulting hedge portfolio return across strategies. Table 3 shows that when the hedge portfolio weights are increased so as to obtain a value of g_1 equal to one, the strategy based upon the model of the firm offers the hedge portfolio return with the lowest variance.

[Table 3 about here.]

6.0 SUMMARY AND CONCLUSIONS

The results of this study indicate that it is possible to construct common stock portfolios that are capable of hedging against inflation. The results also provide evidence on the relative usefulness of alternative information sets for purposes of constructing inflation hedge portfolios.

Portfolios constructed on the basis of the covariability of security returns with unexpected inflation of prior and future periods did provide returns that were positively and significantly correlated with unexpected inflation. However, those based on covariability estimated solely from securities data of prior periods did not. These results indicate that while some stability exists in the association between inflation and security returns, the degree of stability may not be sufficient to support a feasible hedging strategy.

The use of only prior period data did lead to a successful hedging strategy when that strategy was based on certain accounting information and a model of the firm. Thus, investors seeking to construct inflation hedge portfolios should assess information concerning the fundamental relationships between inflation and the value of the firm rather than simply relying on prior period security price behavior.

An implication of these results is that accounting information can be useful to a price-taker who adopts literally the expected-utility-maximizing strategy that exists under the assumptions of an asset pricing theory. That is, even after accounting information has been impounded in security prices, it can still be used to increase investors' expected utility by denoting which assets should be included in inflation hedge portfolios. In general, investors may desire to hold portfolios that can be used to hedge against or speculate on a variety of potentially relevant state variables. Hence, investors may be concerned not just with simply predicting future cash flows, but also with predicting how the probability distribution of cash flows varies across possible states of nature.

The results reported here are also potentially important for the development of capital asset pricing theory, since the existence of hedge portfolios is a necessary condition for the descriptive superiority of the multi-period CAPM over the single-period model. A sufficient condition for the superiority of the multi-period CAPM would be the existence of risk premia or discounts associated with zero-β hedge portfolios. The methods outlined here were not designed to be sufficiently powerful to support a complete test of the descriptive validity of alternative asset pricing theories. However, such tests represent a natural step for future research.

TABLE 1

Content of Sample And Average Hedge Portfolio
Weights By Industry For Fourth Quarter 1980

SIC Code	Industry	No. Firms	Portfolio Weights Derived From: Combined Information Sets	Model of Firm	Security Price Information
3330-3399	Nonferrous Metals	4	.0065	.0010	.0021
3720-3729	Aircraft Manufacturing	4	.0063	.0004	.0026
5400-5499	Grocery Stores	5	.0059	-.0001	.0005
1000-1299	Mining	3	.0037	.0004	.0011
2830-2899	Drugs, Toiletries & Related	7	.0031	-.0001	-.0004
2911	Petroleum Refining	7	.0030	.0001	.0016
6100-6299	Financial, Except Banks	2	.0026	-.0000	-.0004
2000-2049	Food Processing	6	.0026	-.0002	-.0001
3510-3549	Machinery	5	.0019	.0002	-.0001
2100-2199	Tobacco	2	.0015	-.0003	-.0003
3310	Steel	5	.0015	.0016	-.0002
2800-2829	Chemicals & Related Products	5	.0011	.0003	.0001
3800-3899	Instruments & Related	5	.0011	-.0001	-.0009
3570-3579	Business Machines & Equipment	5	.0011	-.0001	.0001
3200-3299	Stone, Clay, Glass Products	8	.0005	.0007	.0003
3600-3629	Electrical Equipment	2	.0004	-.0001	.0007
7000-7999	Consumer Services	4	-.0000	.0002	-.0013
2600-2699	Paper & Related Products	5	-.0000	.0004	.0006
3630-3669	Home Appliances	5	-.0001	-.0002	.0001
5300-5399	Retail Stores	7	-.0004	-.0002	-.0010
3000-3099	Tires & Rubber Products	4	-.0012	-.0002	.0008
2065-2086	Soft Drinks & Candy	5	-.0012	-.0004	-.0025
4511	Airlines	7	-.0046	.0003	-.0020
4800-4999	Utilities	10	-.0046	-.0002	.0001
2200-2399	Textiles-Apparel	5	-.0047	-.0015	-.0000
6020-6029	Banks	5	-.0074	-.0003	.0002
3711	Auto Manufacturing	4	-.0110	-.0021	-.0001

TABLE 2

Association of Hedge Portfolio Returns
And Unexpected Inflation

Strategy based upon:	$\hat{g}_1$	$t(\hat{g}_1)$	Significance Level	Correlation (a_{2t}, u_t)	D-W
PANEL 1: Annual Data (1961-1980)					
Securities market data:	.45	2.97	.00	.57	1.84
Model of firm:	.41	2.68	.01	.53	2.07
Combined information sets:	.62	3.18	.00	.60	1.56
PANEL 2: Quarterly Data (1966-1980)					
Securities market data:	.35	3.17	.00	.39	1.47
Model of firm:	.19	1.77	.04	.22	1.95
Combined information sets:	.63	2.36	.01	.30	1.87
PANEL 3: Quarterly Data (1973:III - 1980:IV)					
Hedge Portfolios Constructed Using Only Historical Data					
Securities market data:	.16	1.23	.11	.22	1.98
Model of firm:	.30	3.37	.00	.54	2.15
Combined information sets:	.70	2.15	.02	.37	1.69

Note: Statistics summarize the following regressions:

$$a_{2t} = g_0 + g_1 u_t + e_t$$

where a_{2t} = hedge portfolio return
u_t = unexpected inflation
e_t = error term

TABLE 3

Comparison of Standard Deviation of Hedge Portfolio Returns

Strategy based upon:	Standard deviation of hedge portfolio return		Scalar required to achieve $\hat{g}_1$ equal to one		Standard deviation of scaled hedge portfolio return
securities market data:	.00442	x	(1.0 ÷ .16)	=	.0276
model of firm:	.00349	x	(1.0 ÷ .30)	=	.0116
combined information sets:	.01156	x	(1.0 ÷ .70)	=	.0165

Note: Scalars required to achieve $\hat{g}_1$ equal to one are reciprocals of $\hat{g}_1$ from Panel 3 of Table 2.

NOTES

[1] Beaver (1972), page 425. Note that since no asset pricing theory to date has explicitly considered the economics of information, asset pricing theories are silent on the issue of the value of information. Statements made here concerning the value of information are based on a consideration of the information which would be required by an investor who attempts to adopt literally the investment strategy that is optimal in a given asset pricing model.

[2] Jones shows that the utility of wealth will depend upon the rate of inflation in general prices unless the economy is dichotomized (i.e., the monetary sector has no effect on real variables). Thus, so long as inflation rates affect opportunities for real investment and growth, a rationale for hedging against or speculating on inflation exists. For a discussion of the link between unexpected inflation and real activity, see Geske and Roll [1983].

[3] To date, no research has investigated the use of assets other than common stocks to construct inflation hedge portfolios as defined within the context of multi-period asset pricing theory. However, prior research has investigated the correlation between unexpected inflation and the returns on a variety of assets (Fama and Schwert [1977]).

[4] In general, the utility of wealth may depend upon more than one state variable and thus the CAPM could include a vector of state variables. In that case, the investor may desire to hold several hedge portfolios. (For example, see Long [1974].)

[5] The most general multi-period CAPM's were developed by Merton [1973] and Breeden [1979]. Merton (page 879) states that perfect correlation with changes in the state variable is a sufficient but not necessary characteristic of the hedge portfolio. Breeden (pages 292-293) states that his hedge portfolios effectively maximize the correlation of return with changes in the state variable. A similar argument can be applied to Long's definition, under the constraint that the hedge portfolio be a zero-β portfolio.

[6] The constraint in the problem is developed in Long [1974], Appendix B and is equivalent to equation (9) of Gay and Manaster [1982], page 327. When the constraint can be satisfied by more than one portfolio, investors choose the minimum-variance solution (Long, page 157; Gay and Manaster, page 328).

[7] When ordinary least squares is used to estimate equation (3), a_{2t} is equal to the third element of the (3x1) vector $(X'X)^{-1}X'R$, where R is the vector of security returns, and X is the matrix of independent variables. (X includes a column of ones, a column of β_j's, and a column of γ_j's.) The portfolio weights associated with a_{2t} are equal to the third row of the (3xN) matrix $(X'X)^{-1}X'$.

[8] The use of prior and future period data also allows comparison of the empirical results with those of Schipper and Thompson [1981], who adopted the same approach.

[9] Since security returns are correlated with the cash flow variable used to estimate the response parameters, inclusion of data from certain periods when estimating the response parameters could cause a spurious correlation between hedge portfolio returns and unexpected inflation. This was not a problem when only historical data were used to calculate the response parameters, since market returns should not be correlated with information made public in previous periods. However, when cash flow data from subsequent periods were used in estimating the response parameters, certain observations were excluded, since market returns can be correlated with cash flows of subsequent periods. When estimating response parameters used to construct a hedge portfolio for period t, observations must be excluded in estimating equation (6) if any of the current or lagged independent variables associated with those observations represent unexpected inflation of periods that intersect with period t. When annual data were used, this required exclusion of observations of period t and t+1 for all firms, as well as period t+2 for firms which did not have fiscal years ended in December. When quarterly data were used, observations of periods t through t+4 were excluded for all firms, as well as observations from period t+5 for firms which did not have fiscal quarter-ends corresponding to calender quarter-ends.

[10] Security returns were available on CRSP on a quarterly basis from 1961 through 1980 for all sample firms with the exception of three of the five banks in the sample. Security price data for those banks were gathered from Barron's and Moody's Dividend Record. Annual income for years 1960-1980 and quarterly income for years 1966-1980 was available on COMPUSTAT for all firms. A large volume of accounting data was also gathered from annual reports and 10-K's.

[11] When unexpected inflation causes revisions in expected inflation, the change in the real value of monetary claims will vary with the time-to-maturity of those claims. Thus, it would be preferable to apply differing weights to monetary accounts with differing maturities for purposes of predicting the response of the value of the firm to unexpected inflation. Unfortunately, construction of an appropriate weighting scheme requires a priori knowledge of the relationship between unexpected inflation and changes in expected inflation. This relationship appears to change significantly over time (based upon time series analysis of inflation). The relationship also varies with the perceived cause of the unexpected inflation, which, of course, is not known in advance. This difficulty and the lack of reliable data on the maturity of many monetary claims over most of the test period led to the simplified approach used here.

If anything, the failure to consider differing duration of monetary claims should tend to weaken the power of the hedging strategy based upon accounting information. In spite of this, the strategy was successful.

[12] A depletion tax shield is not a fixed-dollar claim, so its real value is not necessarily affected by inflation.

[13] The adjustment procedure for some firm-years is complex because of changes in depreciation policy accounted for prospectively. However, in a simple case of a firm using straight-line depreciation for financial reporting purposes and accelerated depreciation for tax purposes, the adjustment is:

$$\text{Estimated Tax Basis} = [\text{Gross Book Value}]\ [(1 - (\text{R/LIFE}))^{\text{Age}}]$$

where

Age = Accumulated depreciation/Depreciation expense;

Life = Gross book value/Depreciation expense;

R = Depreciation rate assumed for tax purposes (for years prior to 1970, R = 2; for years after 1970, R declines gradually to 1.8).

In the absence of information to the contrary, firms were assumed to use accelerated depreciation for tax purposes.

[14] Historical cost income before depreciation, interest, and taxes was available on COMPUSTAT on an annual basis for all nonfinancial institutions and on a quarterly basis for about half of all firm-quarters. Income before depreciation, interest, and taxes for financial institutions was calculated using data from annual reports. For those firm-years when quarterly income before depreciation, interest, and taxes was not disclosed on COMPUSTAT, the amount was estimated by adding to quarterly income before tax an amount equal to one fourth of annual depreciation and interest.

[15] The tests were repeated using two other inflation forecasting techniques: one based on Livingstone's surveys (see Carlson [1977]), and another based upon distributed lags of past inflation rates and money supply growth. All results based upon annual data were similar to those reported here. Evidence of hedging ability was weaker than that reported here when the quarterly distributed lag forecasts were used. However, since those forecasts are less accurate than the Treasury bill forecasts, the weaker results may be due to measurement error in unexpected inflation.

[16] Nominal returns on 90-day Treasury bills and one-year Treasury notes were available on the CITIBANK tapes. Prior to the issue of one-year notes in 1963, estimated rates on one-year notes were developed by annualizing rates on 9-, 10-, or 11-month Treasury bills.

For tests based on quarterly data, expected real rates were generated using a univariate model with moving average terms at lags 1, 2, and 4. The models were reestimated each month, generally using the 60 most recent quarterly observations. Fewer than 60 quarterly observations were used to generate forecasts for annual periods 1960 through 1964. The forecast base period was reduced to eliminate observations of interest rates that were essentially pegged by the government until the Treasury-Federal Reserve Accord of 1951.

Expected annual real rates were developed by combining one-step-ahead through four-step ahead forecasts of quarterly real rates, and then adding a constant liquidity risk premium equal to the average difference over the test period between returns on one-year notes and 90-day bills.

[17] An alternative method of evaluating hedging ability would be to regress the hedge portfolio return simultaneously against both the return on the market portfolio and unexpected inflation. If the hedge portfolios are zero-β portfolios (as required by equation (2)), then the coefficient of the return on the market would be zero and the results would be equivalent to those obtained with equation (10). When this alternative approach was used, hedge portfolio β's ranged from -.03 to +.03 and the results were in fact similar to those obtained with equation (10).

[18] In order to assess the importance of the difference between hedging strategies, returns on the security-price-based strategy were subtracted from those of the accounting-information-based strategy. The resulting differences were positively correlated with unexpected inflation at the 20 percent level, using a one-tailed test. When the differences were regressed against both the return on the market and unexpected inflation, the positive association between the return differences and unexpected inflation was significant at the 6 percent level, using a one-tailed test.

[19] The strategy based on the model of the firm performs much better when only prior period data are used (Panel 3) than when both historical and future period data are used (Panel 2). However, this apparent improvement is due primarily to the use of different time periods in the two panels.

[20] In order to conform strictly to the theory underlying the hedge portfolio construction techniques, the investor would have to be able to predict the value of the scalar required to achieve a g_1 equal to one. However, it is not required that the investor predict the scalar perfectly in order to increase expected utility by adopting the hedging strategy.

If it is difficult to predict this scalar, then the value of a hedging strategy would depend not only upon the correlation between the hedge return and unexpected inflation, but also upon how closely g_1 approximates unity. In that case, it would not be clear that the strategy based solely on accounting information is superior to the strategy based on combined information sets.

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APPENDIX

A Model Of The Impact Of Unanticipated Inflation Upon The Value Of The Firm

Below, the value of the firm at time 0 (V_0) is expressed as the present value of four basic components of future cash flows: pretax cash flows from operations (C_t), less associated income tax (TC_t); tax savings due to depreciation deductions ($T(D_t^0+D_t^f)$); net cash inflows (outflows) associated with the issue and service of debt ($M_t^0+M_t^f$); and capital expenditures (K_t). Depreciation deductions associated with assets purchased prior to time 0 (D_t^0) are segregated from deductions associated with assets not yet purchased (D_t^f). Similarly, cash flows associated with debt already outstanding at time 0 (M_t^0) are segregated from cash flows associated with debt to be issued after time 0 (M_t^f).

All future cash flows are denominated in time 0 dollars. Since the cash flows are measured in real terms, the discount rate (p_t) is a real discount rate, with no adjustment for inflation.

$$V_0 = \sum_{t=1}^{\infty} p_t \{C_t (1-T) + T (D_t^0 + D_t^f) + M_t^0 + M_t^f - K_t\}. \qquad \text{(A1)}$$

where V_0 = value of the firm's shares at time 0;

p_t = present (time 0) value of one unit of purchasing power to be received at time t. The discount rate p_t is equal to $(1/(1+r))^t$, where r is the real rate of interest;

T = tax rate;

C_t = expected pretax cash flows from operations;

D_t^0 = expected amount of depreciation deduction at time t associated with assets owned at time 0;

D_t^f = expected amount of depreciation deduction at time t associated with assets to be purchased after time 0;

M_t^0 = expected net after-tax cash inflows (outflows) at time t associated with interest and principal receipts (payments) on monetary claims outstanding at time 0;

M_t^f = expected net after-tax cash flows at time t associated with monetary claims to be issued after time 0;

K_t = expected capital expenditures at time t.

Using this simple valuation model as a basis, the impact of unanticipated inflation upon the value of the firm is examined.

Assume that immediately after time 0, unexpected inflation in the general price level in the amount u occurs. (Of course, u could be positive or negative.) Then, using an asterisk to denote that the value of a variable has changed, the value of the firm is written as:

$$V_0* = \sum_{t=1}^{\infty} p_t \; [C_t* \; (1-T) + T \; (D_t^o* + D_t^f) + M_t^o* + M_t^f - K_t]. \qquad \text{(A2)}$$

Certain variables are assumed not to be affected by the dose of unexpected inflation. Although impacts upon p_t and T could exist, they are not considered in the empirical work which follows. The expected nominal cash flows associated with monetary claims not yet issued and assets not yet purchased are assumed to change in proportion to the unexpected change in the price level, thus leaving the real cash flows M_t^f, K_t, and TD_t^f unchanged.

All remaining variables (C_t*, D_t^o*, and M_t^o*) are allowed to change as a result of unexpected inflation. When the price level increases unexpectedly by the amount u, the expected real values of outstanding monetary claims and depreciation tax shields should decline to 1/(1+u) of their previous real value:

$$M_t^o* = (\frac{1}{1+u}) \; M_t^o \qquad \text{(A3)}$$

$$D_t^o* = (\frac{1}{1+u}) \; D_t^o \qquad \text{(A4)}$$

Now consider the impact of unexpected inflation upon expected real cash flows from operations (C_t). Unexpected inflation can influence real cash flows directly through impacts on real prices of the firm's inputs and outputs, and indirectly through impacts upon the real wealth of the firm's customers. The direct impact upon real prices of inputs and outputs depends on the terms of the firm's implicit or explicit nominal sales contracts, labor agreements, purchase agreements, or even informal pricing policies. If C_t is also influenced because unexpected inflation affects the real wealth of the firm's customers, the relation between unexpected inflation and real cash flows then depends upon the income elasticity of demand for the firm's products.

The above discussion indicates that the impact of unexpected inflation upon real cash flows from operations will vary from firm to firm. The impact of unexpected inflation upon C_t is denoted here by firm-specific inflation response parameters (θ), the values of which will be estimated empirically:

$$C_t* = C_t \; (1+\theta_t u) \text{ for } t = 1, 2, 3... \qquad \text{(A5)}$$

If θ_t is zero, then unexpected inflation (u) at time zero does not alter expected real cash flows for time t. If unexpected inflation causes an increase (decrease) in expected real cash flows from operations for time t, θ will be positive (negative).

Note that equation (A5) allows for both current and lagged impacts of u upon real cash flows by using a vector of inflation response parameters. However, one would expect θ_t to approach zero as t grows larger. This is because many of the effects of u upon real cash flows (such as price-wage lags) are temporary. As these effects dissipate, the impact of unexpected inflation upon real cash flows is diminished.

Substitution of equations (A3), (A4), and (A5) into equation (A2) and subtraction of equation (A1) yields:

$$V_0^*-V_0 = \sum_{t=1}^{\infty} P_t C_t \theta_t u - \left(\frac{u}{1+u}\right) \sum_{t=1}^{\infty} P_t M_t^0 - \left(\frac{u}{1+u}\right) \sum_{t=1}^{\infty} P_t D_t^0 \qquad \text{(A6)}$$

or

$$R_0 = \frac{V_0^*-V_0}{V_0} = \sum_{t=1}^{\infty} \frac{P_t C_t \theta_t u}{V_0} - \left(\frac{u}{1+u}\right) \frac{PVMP}{V_0} - \left(\frac{u}{1+u}\right) \frac{PVTS}{V_0}$$

where PVMP = present value of monetary position and

PVTS = present value of depreciation tax shields.

Equation (A6) states that the change in the value of the firm which results from unexpected inflation is a linear function of the present value of the firm's monetary position, the present value of the firm's depreciation tax shields, and the firm's inflation response parameters.

The empirical analogue of equation (A6) used in the construction of hedge portfolios uses proxies for PVMP, PVTS, and θ. The firm's net monetary position (NMP) is used as a proxy for PVMP. The product of the tax rate (F) and the firm's estimated tax basis of depreciable assets (NPE) is used as a proxy for PVTS. Estimation of θ is discussed in the main body of the paper. The empirical analogue of equation (9) is written:

$$R_0 = \left[\frac{\Sigma\, C\theta}{V_0} - \frac{NMP}{V_0} - \frac{F \cdot NPE}{V_0}\right] u \qquad \text{(A7)}$$

If the real return R_0 is converted to a nominal return R_0^N, one obtains:

$$R_0^N = 1 + \left[(\Sigma C\theta - NMP - F \cdot NPE)/V_0\right] u \qquad \text{(A8)}$$

This expression serves as the basis for equation (5) in the main body of the paper. In the main body of the paper, the inflation response parameter θ already reflects the scaling effect of the variable C.

The explanatory power of a model similar to the one described above is examined in Bernard (1983). In that study, it is shown that inflation response parameters estimated in one time period can be used to explain cross-sectional differences in the reaction of security prices to unexpected inflation of a different time period. Furthermore, when the inflation response parameters are included in the model, cross-sectional differences in the association of security prices and unexpected inflation are consistent with cross-sectional differences in firms' net monetary positions and the magnitudes of firms' depreciation tax shields.