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June 1978

THE U.S. TREASURY BILL FUTURES MARKET AND HYPOTHESES REGARDING THE TERM STRUCTURE OF INTEREST RATES

Working Paper 160

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ACKNOWLEDGEMENTS

The authors wish to thank David Aker, M.B.A., for his help in data collection and computer programming and Mr. Charles Curtiss for a discussion of the operations of the T-bill futures market.

Abstract

Comparison of the term structure of interest rates derived from the U.S. Treasury Bill spot and futures markets provides a vehicle for testing term structure hypotheses. Market-generated interest rate expectations observed in the T-bill futures market may be used in tests of hypotheses, thus removing the need for model-generated expectations. In this paper we demonstrate the use of this approach in the testing of certain hypotheses about term structure. Our results show that, under the assumption of efficiency in the U.S. Treasury Bill spot and futures markets, these tests do not substantiate the pure expectations hypothesis and the stationary variant of the market segmentation hypothesis.

Introduction

Modern theory of the term structure of interest rates dates at least from the work of Irving Fisher [1]. His proposition that expectations of future interest rates influence the term structure of rates became the foundation of the "expectations" hypothesis. In the 1930s, building on the Keynesian notion of "normal backwardation" in the futures market [2], J.R. Hicks suggested the "liquidity premium" term structure hypothesis [3]. He argued that the forward rate would normally exceed the expected interest rate by a risk premium to compensate the investor for assuming the uncertainty of price fluctuations.

During the past two decades extensive studies of the term structure have appeared. While these have resulted in the development and refinement of many variants to interpret and many techniques to analyze the term structure, they have not produced a consensus. Among the significant contributors, John Culbertson [4] has formalized the "market segmentation" hypothesis while David Meiselman [5] has cast the expectations hypothesis in its best light. Between these two positions, Reuben Kessel [6] and Burton Malkiel [7] have incorporated both concepts in interpretations of the term structure.

We believe that a factor contributing to the lack of consensus in term structure interpretation has been the lack of independent measures of interest rate expectations. Tests of the hypotheses cited above require the inclusion of such expectations, and the usual procedure has been to generate expectations with a model of some sort. Consequently, tests of term structure hypotheses turn out to be joint tests of the hypothesis in question and of the interest rate expectations model which the researcher is employing. Since the expectations model used is always open to challenge, the results of any test of term structure hypotheses done in this way are, at best, conditional.

In this paper we examine the proposition that observed interest rate expectations, generated by participants in an organized futures market, can be used in tests of term structure hypotheses to remove effectively the conditionality which is inherent in the model-generated expectations described above. We direct our tests specifically to the pure expectations hypothesis and the stationary--or liquidity preference--variant of the market segmentation hypothesis.

Our source of market-generated interest rate expectations is the U.S. Treasury Bill futures market. This market has been in operation since 1976 and is maintained by the International Monetary Market of the Chicago Mercentile Exchange. Participants in it deal in contracts involving U.S. Treasury Bills with maturity of three months (13 weeks, 91 days) for delivery on specified dates in March, June, September, and December. The longest-term contract is for delivery in 18 months. The data used in this study are described in detail in Section IV.

The body of this paper proceeds with sections devoted to each of the following: a specification and discussion of the pure expectations hypothesis; the formulation of tests of the pure expectations hypothesis; description of the data and the results of the tests of the pure expectations hypothesis; effects due to transactions costs; a test of the stationary variant of the market segmentation hypothesis; and conclusions.

II. Pure Expectations Hypothesis

The fundamental dynamic equation for interest rates in an efficient market can be stated [8] as:

$$E_{t-1}(\tilde{r}_{j,t}-\tilde{L}_{j,t} \mid B_{t-1}) = r_{j+1,t-1} - L_{j+1,t-1} \qquad \text{(II-1)}$$

where $r_{j,t}$ is the one-period forward rate for j-periods hence, observed in period t, and $L_{j,t}$ is the j-period liquidity premium observed in period t. The E_{t-1} is the mathematical expectations operator as of period t-1. B_{t-1} represents all available information about interest rates at time t-1. All random variables have a tilde superscribed above them. We follow Roll's definition of an efficient market, namely, one which has: (1) zero transactions costs (an assumption we will relax in section V below); (2) symmetric market rationality, which means that every trader acts rationally (i.e., desires more, rather than less, wealth, and uses all available information) and believes all others do likewise; (3) information is free and becomes available to everyone at the same instant. These conditions are sufficient grounds for assuming that no trading rule which earns excess profits can be developed. Equation (II-1) states that the forward rate applicable to a fixed future date, less a liquidity premium, follows a martingale sequence, i.e.,

$$E_{t-1}(\tilde{X}_{j,t}) = X_{j+1,t-1}$$

$$E_{t-2}(\tilde{X}_{j+1,t-1}) = X_{j+2,\ t-2}; \text{ etc.} \qquad \text{(II-2)}$$

In other words, $r_{j+1,t-1}-L_{j+1,t-1}$ is an unbiased estimator of the random variable $\tilde{r}_{j,t}-\tilde{L}_{j,t}$.

The pure expectations hypothesis can be stated mathematically as:

$$L_{j,t} = 0 \text{ for all } j \text{ and } t. \qquad \text{(II-3)}$$

Thus, equation (II-1) can be written as:

$$E_{t-1}(\tilde{r}_{j,t} | B_{t-1}) = r_{j+1,t-1}. \qquad \text{(II-4)}$$

In words, Meiselman [5, p. 10] has stated the expectations hypothesis succinctly:

> . . . [it] follows from the assumptions that short- and long-term securities can be treated as if they were perfect substitutes and that transactors, indifferent to uncertainty and holding similar expectations, equate the forward rates in the market to the expected rates. As a matter of descriptive reality, individual transactors may still speculate or hedge on the basis of risk aversion, but the speculators who are indifferent to uncertainty will bulk sufficiently large to determine market rates on the basis of the mathematical expectations alone.

III. Formulation of a Test of the Pure Expectations Hypothesis

Fundamental to our test approach is the assumption--which we feel to be reasonable--that both the T-bill spot and futures markets are efficient in the sense attributed to Roll above. If both markets are efficient and open to the same body of investors, it follows that the same term structure theory should apply in both markets. Different basic theories in the two markets would permit the development of trading rules capable of producing excess profits. This, of course, is inconsistent with the efficient market assumption. On this basis, then, we expect the term structure derived from the T-bill spot market on any given data should be identical to the term structure derived from the T-bill futures market at the same time.

For test purposes we derive the T-bill spot market term structure through the use of a pure expectations hypothesis formulation. We then derive the contemporaneous term structure from the T-bill futures market and test for differences between the two term structures. We argue that the existence of any such differences indicates that investors must have used factors additional to expectations in valuing the futures contracts. If investors are using different theories in the spot and futures markets, the pure expectations theory cannot hold: that is, expectations are not the <u>sole</u> factor determining the term structure.

The major difference between this approach and the model-generated expectations approach is, of course, that the futures interest rates are market-based and immediately observable. Unless and until the T-bill futures market is proved to be inefficient, such observations may be considered reliable and accurate reflections of investors' actual expectations on a certain date. The question of whether the expectation coincides with actual spot rates observed later is, in a significant sense, not relevant to the question at hand. The T-bill futures market is important, therefore, in capital market analyses because it is both a source of reliable expectations information and a vehicle for tests of consistency when compared to the T-bill spot market.

The prices in both the Treasury-bill spot and futures markets are quoted on a discount basis. The interest earned on a Treasury bill is the difference between the par price (face value) and the purchase price, if it is held to maturity. Prices in the T-bill futures market are quoted in terms of the International Monetary Market Index, which is 100 minus the T-bill yield (discount) on an annual basis.

Let $D_{n_i,\Delta_i}(t)$ be the discount yield at time t for a bill to be delivered according to the terms of a futures contract n_i days from t and which will mature $n_i+\Delta_i$ days from t. The price of a $100 (at maturity) bill at time t, $P_{n_i,\Delta_i}(t)$ is:

$$P_{n_i,\Delta_i}(t) = 100-\frac{\Delta i}{360} D_{n_i,\Delta_i}(t). \qquad \text{(III-1)}$$

Furthermore, let us define the true rate of return $R_{n_i,\Delta_i}(t)$ as the internal rate of return at which the maturity value of the T-bill must be discounted to equal its price at time t. The subscripts of $P_{n_i,\Delta_i}(t)$ and $R_{n_i,\Delta_i}(t)$ have the same meaning as those of $D_{n_i,\Delta_i}(t)$. For mathematical simplification we will use continuous compounding. Other methods of compounding do not affect our conclusions. The true rate of return can be expressed in terms of the price of a T-bill as follows:

$$P_{n_i,\Delta_i}(t)e^{\frac{R_{r_i,\Delta_i}(t)\Delta_i}{36,500}} = 100$$

or,

$$R_{n_i,\Delta_i}(t) = -\frac{36,500}{\Delta_i}\ln\frac{P_{n_i,\Delta_i}(t)}{100}, \qquad \text{(III-2)}$$

where $R_{n_i,\Delta_i}(t)$ is in percentage per annum and ln is the natural logarithm. The subscripts of R also have the same meanings as those of D.

According to the pure expectations hypothesis, short- and long-term securities can be treated as perfect substitutes. Therefore:

$$e^{\frac{R_{o,\Delta_1+\Delta_2+\ldots\ldots\Delta_k}(t)}{36,500}(\Delta_1+\Delta_2+\ldots\ldots\Delta_k)}$$

$$= e^{\frac{R_{o,\Delta_1}(t)\Delta_1}{36,500}}\; e^{\frac{R_{\Delta_1,\Delta_2}(t)\Delta_2}{36,500}}\; e^{\frac{R_{\Delta_1+\Delta_2,\Delta_3}(t)\Delta_3}{36,500}}$$

$$\ldots\ldots e^{\frac{R_{\Delta_1+\Delta_2+\ldots\ldots\Delta_{k-1},\Delta_k}(t)\Delta_k}{36,500}}; \qquad \text{(III-3)}$$

or,

$$R_{o,\Delta_1+\Delta_2+\ldots\ldots\Delta_k}(t) = \Big[R_{o,\Delta_1}(t)\Delta_1+R_{\Delta_1,\Delta_2}(t)\Delta_2$$

$$+R_{\Delta_1+\Delta_2,\Delta_3}(t)\Delta_3+\ldots\ldots+R_{\Delta_1,+\Delta_2+\ldots\ldots\Delta_{k-1},\Delta_k}(t)\Delta_k\Big]$$

$$\div\ \big[\Delta_1+\Delta_2+\ldots\ldots+\Delta_k\big]. \qquad \text{(III-4)}$$

In words, equation (III-4) states that the true rate of return of a bill which will mature $\Delta_1+\Delta_2+ \ldots\ldots \Delta_k$ days is the weighted average of the true rates of return of the T-bill future contracts. Since the T-bills are quoted in discount yields, $D_{n_1,\Delta_i}(t)$ instead of true rates of returns, $R_{n_i,\Delta_i}(t)$, we proceed to derive a relation among discount yields. Substituting equations (III-1) and (III-2) into (III-4), we obtain:

$$D_{o,\Delta_1+\Delta_2+\ldots\ldots+\Delta_k}(t) = \frac{36,000}{\Delta_1+\Delta_2+\ldots\ldots\Delta_k}\left[1-(1-\frac{\Delta_1 D_{o,\Delta_1}(t)}{36,000})\right.$$
$$(1-\frac{\Delta_2 D_{\Delta_1,\Delta_2}(t)}{36,000})\ (1-\frac{\Delta_3 D_{\Delta_1+\Delta_2,\Delta_3}(t)}{36,000})$$
$$\left.\ldots\ldots(1-\frac{\Delta_k\ D_{\Delta_1+\Delta_2+\ \ldots\ldots\Delta_{k-1},\Delta_k}(t)}{36,000})\right], \quad \text{(III-5)}$$

where $D_{\Delta_1+\Delta_2+\ \ldots\ldots\Delta_{k-1},\Delta_k}(t)$ is the discount yield of a bill at time t to be delivered at $\Delta_1+\Delta_2+\ \ldots\ldots\Delta_{k-1}$ days from t and which will mature at $\Delta_1+\Delta_2+\ \ldots\ldots\Delta_k$ days from t.

Let us introduce a superscript s and f to the discount yield D to signify its origin. Thus, $D^s_{o,\Delta_i}(t)$ is the spot (observed) discount yield at time t while $D^f_{o,\Delta_i}(t)$ is the expected discount yield derived from Treasury-bill futures markets at time t by means of equation (III-5). If the pure expectations hypothesis is the sole determinant of interest rates and discount yields in the spot and futures markets, it requires:

$$D^s_{o,\Delta_i}(t) = D^f_{o,\Delta_i}(t)$$
$$\text{for all i and t.} \qquad \text{(III-6)}$$

Since $D^s_{o,\Delta_i}(t)$'s are directly observable and $D^f_{o,\Delta_i}(t)$'s are completely determined by the observable discount yields in the T-bill futures market via equation (III-5) plus one spot yield, equation (III-6) is a testable hypothesis. The test of this hypothesis is referred to below as Test 1.

Another way to test term structure hypotheses is by comparing a "piggy back" combination of a short-term spot transaction and a futures contract, with a spot transaction in a T-bill of maturity equal to that of the combination.

Consequently, one can test:

$$\bar{D}_{\Delta_i-91,\Delta_i}(t) \equiv D^f_{\Delta_i-91,\Delta_i}(t) - \frac{36,000}{91}\left[1-\frac{1-\dfrac{\Delta_i D^s_{o,\Delta_i}(t)}{36,000}}{1-\dfrac{(\Delta_i-91)D^s_{o,\Delta_i-91}(t)}{36,000}}\right]$$

$$= 0 \qquad \text{(III-7)}$$

for all Δ_i and t, where $\bar{D}_{\Delta_i-91,\Delta_i}(t)$ is the difference between the observed discount yield of a 13-week T-bill futures contract to be delivered Δ_i days from now and the discount yield deduced from the spot T-bill market yields. The test of the hypothesis represented by equation (III-7) is referred to below as Test 2.

Parenthetically we note that with Test 1 the researcher can use all the data available in the T-bill spot market while Test 2 uses only a small subset of it. On the other hand, Test 2 is easier to correct for transactions costs and is a more convenient vehicle for testing whether the liquidity premium differs in the spot and futures markets. The following section describes the data and presents the results of hypothesis testing.

IV. Description of Data and Results of Tests on the Pure Expectations Hypothesis

The data used here are the discount yields on U.S. Treasury bills available for direct transactions in the spot market and through contracts for future execution in the T-bill futures market. U.S. Treasury bills can be considered as a homogeneous group of riskless securities with the yield differential among bills fully attributable to difference in maturity. For the spot market data, the average of bid and ask yields is used here. For the future contracts market data, the yields at market close are used. Transaction costs are ignored in these tests but are discussed in Section V. It should be noted that the bid-ask spread for T-bills which will mature in less than 30 days is very large because of transaction costs: consequently these securities are not included in the test data.[1] Our sample consists of bi-weekly yield curves from the spot market and the corresponding yield curves from the futures market covering the period from January 8, 1976, through January 26, 1978.

We proceed with Test 1 by using the following regression equation:

$$D_{\Delta_i}(t) \equiv D^s_{o,\Delta_i}(t) - D^f_{o,\Delta_i}(t) = \alpha + \Delta_i \beta \,, \qquad \text{(IV-1)}$$

where $D_{\Delta_i}(t)$ is the difference between the discount yields derived from the T-bill spot market and futures markets respectively. The discount yields from the T-bill futures market are calculated by means of equation (III-5).

If the pure expectations hypothesis is valid, according to equation (III-6), α and β in equation (IV-1) will not be statistically different from zero. Our results are as follows:

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[1] We have also run a test which included T-bills with maturity of less than 30 days. The results show nothing which would alter the conclusions presented in this study.

$$D_{\Delta_i}(t) = .1038 - .00027\Delta_i \qquad \text{(IV-2)}$$

Std. Errors	(.0052)	(.00003)
t-Statistics	(19.8)	(-9.2)
Number of observations	1,308	

where $D_{\Delta_i}(t)$ is in percentage per annum and Δ_i in number of days to maturity. The standard errors and t-statistics are shown in brackets beneath the coefficients. The t-statistics are 19.8 and -9.2 for the intercept and slope respectively. For both coefficients, the significance level is zero to four decimal places, which means that the chance for either α or β to be zero is practically nil. Thus, the pure expectations hypothesis predicting zero intercept and zero slope is not substantiated by this test.

Strictly speaking, a statistical test is not necessary. So long as $D_{\Delta_i}(t)$ is non-zero for any particular Δ_i and t, the pure expectations hypothesis is invalidated at that point. It is, of course, this ability to observe the expectations of market participants directly which makes the T-bill futures market useful in testing term structure hypotheses.

We proceed with Test 2 by using the following regression equation:

$$\bar{D}_{\Delta_i-91,\Delta_i}(t) = \bar{\alpha} + \bar{\beta}\,(\Delta_i-91), \qquad \text{(IV-3)}$$

Our results are as follows and lead to the same conclusion as Test 1.

$$\bar{D}_{\Delta_i-91,\Delta_i}(t) = .199 - .00180\,(\Delta_i-91) \qquad \text{(IV-4)}$$

Std. Errors	(.025)	(.00016)
t-Statistics	(7.8)	(-11.4)
Number of observations	351	

V. Effects Due to Transaction Costs

In the T-bill spot market, the dealer makes a profit through the bid-ask spread. Thus, it is reasonable to assume that the spot interest rates, in the absence of transaction costs, are the average of bid and ask rates if the spread is small. A typical spread for a T-bill with a week to maturity is around .4 percent while that with three months or more to maturity is around .1 percent or less. In Test 1, bills with maturity of less than one month are not included while in Test 2 bills with maturity of less than one week are not included.

In the T-bill futures market, although the size of a single contract is $1 million, one can buy or sell a contract on margin, and the round-turn commission per contract is $60. However, the commission rates vary from one brokerage house to another. Furthermore, marketable securities, treasury stock, or a letter of credit may be substituted for cash in fulfilling the margin requirement. Thus, the opportunity cost varies. For our purpose, however, it is not necessary to determine precisely the weighted-average opportunity cost for the T-bill futures transactions, because our conclusions are not sensitive to the opportunity cost. We thus assume that the average margin was $2,000 and commission was $60 per contract during the period of our investigation.

Let $\bar{D}_{\Delta_i+\Delta_2+\ldots\Delta_{k-1},\Delta_k}(t)$ be the discount yield, after transaction cost adjustment, of a T-bill at time t to be delivered at $\Delta_i+\Delta_2+\ldots+\Delta_{k-1}$ days from t and which will mature at $\Delta_i+\Delta_2+\ldots+\Delta_k$ days from t. If the pure expectation hypothesis is valid, a piggyback combination of a spot transaction and a future contract will have the same yield as a spot contract of maturity equal to that of the combination. If the margin for a future contract is assumed to have been invested in the spot T-bill market to earn an interest rate of 6 percent per annum, the quoted discount yield for a T-bill futures contract with a maturity value of $1 million and a delivery date Δ_i-91 days from now, $D^f_{\Delta_i-91,\Delta_i}(t)$, should be adjusted downward by a percentage factor $A^f_{\Delta_i}$ according to the following formula to represent the discount yield net of transactions costs to the investor.

$$A^f_{\Delta_i} = \left[\frac{60 + (2{,}000)(.06)(\frac{\Delta_i-91}{365})}{1{,}000{,}000} \right] \cdot \left[\frac{12}{3} \right] \cdot [100\%]$$

$$= 0.024 + .0001315\ (\Delta_i - 91) \qquad \text{(V-1)}$$

For example, the adjustment for T-bill contracts to be delivered 13 weeks, 26 weeks, and 39 weeks from now are 0.036 , 0.048, and 0.06 percent respectively. In the present study we will apply this modification to Test 2 to account for transactions costs in the T-bill futures market.

In equation (III-7) $D^f_{\Delta_i-91,\Delta_i}(t)$ should be adjusted downward by the value of $A^f_{\Delta_i}$. Alternatively, if we use the same $D^f_{\Delta_i-91,\Delta_i}$ values as data for testing, we are really testing that:

$$\bar{\alpha} + \bar{\beta}(\Delta_i - 91) = A^f_{\Delta_i} = .024 + .0001315\ (\Delta_i - 91)\ .$$

But we have already found, in equation (IV-4) that $\bar{\alpha}$ = .199 and $\bar{\beta}$ = -.0018. The t statistics of the hypothesis test that $\bar{\alpha}$ and $\bar{\beta}$ are different from .024 and .0001315 are 6.87 and -12.18 respectively. For both coefficients the significance level remains zero to four decimal places. Thus, even with an adjustment for transactions costs, the chance of the pure expectations hypothesis being valid is practically nil--provided, of course, that the spot and futures markets are otherwise efficient.

VI. Test of the Stationary Variant of the Market Segmentation Hypothesis

The basic premise of the market segmentation hypothesis is that securities which differ only with respect to maturity are imperfect substitutes. Stated in terms of equation (II-1),

$$E_{t-1}(\tilde{r}_{j,t}-\tilde{L}_{j,t}|B_{t-1}) = r_{j+1,t-1}-L_{j+1,t-1}\,, \qquad \text{(II-1)}$$

the segmentation hypothesis implies that at least some of the L's are non-zero. There are two major variants of the market-segmentation hypothesis. The time-dependent variant [8, p. 38] states that the maturities of the payment streams of many assets and liabilities depend on calendar time. If investors with calendar time-dependent maturity habitats dominate the market, the j-period liquidity premium observed in period t, $L_{j,t}$, should be about equal to the j + 1-period liquidity premium observed in period t - 1, aside from random unexpected fluctuations.

In this paper we have chosen to test the stationary variant of the market segmentation hypothesis--known also as the liquidity preference hypothesis. Roll has shown this to be well supported in the T-bill spot market and to be considered the "best" term structure hypothesis [8, p. 109].

Roll has stated that:

> The stationary variant of the market segmentation hypothesis is based on the observation that some asset and liability payment streams are independent of calendar time, retaining approximately the same maturity by being periodically renewed. [8, p.41]

For example, many companies maintain "lines of credit" or periodically renewable loans with banks, which are actually bonds with fixed maturities. If such behavior dominates the market, the liquidity premium for a given maturity should remain unchanged over time except for random shifts in the types of investors participating in the market and shifts in their maturity preferences.

Mathematically,

$$E_{t-1}(\tilde{L}_{j,t}) = L_{j,t-1} . \qquad \text{(VI-1)}$$

Substituting equation (VI-1) into (II-1) we obtain:

$$E_{t-1}(\tilde{r}_{j,t}) = r_{j+1,t-1} + L_{j,t-1} - L_{j+1,t-1} , \quad \text{(VI-2)}$$

or

$$E_t(\tilde{r}_{j,t+1} - r_{j+1,t}) = L_{j,t} - L_{j+1,t} .$$

Since $\qquad L_{1,t} = 0$

then $\qquad -L_{2,t} = E_t(\tilde{r}_{1,t+1} - r_{2,t}) .$

Let the estimate of $L_{2,t}$ be $\hat{L}_{2,t}$;

then, $\qquad \hat{L}_{2,t} = \overline{r_{2,t} - r_{1,t+1}}$,

where the bar denotes the sample average.

Similarly,

$$\hat{L}_{2,t} - \hat{L}_{3,t} = \overline{r_{2,t+1} - r_{3,t}}$$

$$\begin{aligned} \hat{L}_{3,t} &= L_{2,t} + \overline{r_{3,t} - r_{2,t+1}} \\ &= \overline{L_{2,t} - r_{1,t+1}} + \overline{r_{3,t} - r_{2,t+1}} \\ &= \sum_{j=2}^{3} (\overline{r_{j,t} - r_{j-1,t+1}}) . \end{aligned}$$

In general,

$$\hat{L}_{j,t} = \sum_{i=2}^{j} (\overline{r_{i,t} - r_{i-1,t+1}}) . \qquad \text{(VI-3)}$$

If the stationary variant of the market segmentation hypothesis is the determinant of the term structure, investors would use the same hypothesis to determine the term structures of interest rates in the spot and futures markets. That is to say, the expected interest rates after adjustment for liquidity premiums, derived from the spot and futures markets should be the same. Mathematically, we have,

$$E^{s}_{t-1}(\tilde{r}_{j,t}) = E^{f}_{t-1}(\tilde{r}_{j,t}) \quad \text{for all j and t,}$$

or

$$r^{s}_{j+1,t-1} + L^{s}_{j,t-1} - L^{s}_{j+1,t-1} = r^{f}_{j+1,t-1} + L^{f}_{j,t-1} - L^{f}_{j+1,t-1} \quad \text{for all j and t} \quad \text{(VI-4)}$$

where the superscripts s and f denote quantities in the spot and futures market respectively.

The liquidity premiums in the spot and futures markets have to be identical. In fact, it was by utilizing the forward rates that J.R. Hicks [3] derived the liquidity premium. Therefore, equation (VI-4) becomes:

$$r^{s}_{j+1,t-1} = r^{f}_{j+1,t-1} \,,$$

or

$$r^{s}_{j,t} = r^{f}_{j,t} \,, \qquad \text{for all j and t .} \qquad \text{(VI-5)}$$

We have thus shown the testing of the stationary variant of the market segmentation hypothesis is identical to that of the pure expectation hypothesis. Since the latter is not substantiated in Sections III to V, the former is not substantiated in the present study. Essentially, these results show that the chances of the stationary variant of the market segmentation hypothesis to be valid is practically nil.

It may be argued that the results of our tests are suspect because they depend upon the assumption of efficiency in both the T-bill spot and futures markets. Such inefficiency, if it exists, may be more likely to be found in the T-bill futures market, simply because the market is only two years old at this writing.

There are two approaches which might be used to test for inefficiency in the T-bill futures market. As a first approach we would have to show that price changes of futures contracts are serially correlated or that prices do not fully and quickly adjust to new information. However, the prerequisite for such an approach is to have a term structure hypothesis which can be validly used to determine the expected discount yield $D^f_o, \Delta_i(t)$.

The additional assumption we propose in this study is that the same term structure hypothesis must be used in both the spot and futures markets. In other words, if an hypothesis does not hold in both markets--as we have shown to be the case with the pure expectations hypothesis and the stationary variant of the market segmentation hypothesis--that hypothesis cannot be used in the testing of market efficiency.

In summary, we argue that if the T-bill futures market is efficient then the two term structure hypotheses tested are invalid. On the other hand, on the basis of our results, if we wish to test for T-bill futures market efficiency neither of these hypotheses may be used [9].

The second approach to demonstrating market inefficiency in the T-bill futures market is to develop a trading rule by which excess profits may be generated. To date, despite efforts by researchers and practitioners, published proof of the existence of such a trading rule has not appeared. In one recent paper [10] the author suggests the existence of such a trading rule based on market inefficiency which he believes to be temporary and a reflection of the immaturity of the market. The results shown in that paper, however, provide no evidence that the trading rule developed produced excess--or even very consistent and satisfactory--profits.

Even if such a trading rule were found--and if it reflected "temporary inefficiency"--we may still conclude that the mature T-bill futures market provides an important medium for testing term structure hypotheses. However, if we <u>assume</u> (rather than prove) that the market is inefficient, and thus ascribe results to that factor, we raise an issue the implications of which go far beyond this paper. Much of modern finance theory is based on the assumption of efficient markets--for example, Merton's dominance principle, and most of option-pricing theory. Until and unless excess profits-generating trading rules are proven to exist, we should continue to use the efficient markets assumption in the testing of term structure hypotheses through the use of T-bill and futures market information.

VII. Conclusions

We have sought to demonstrate that the comparison of the term structure of interest rates derived from the T-bill spot and futures markets offers a critical empirical test of term structure hypotheses. The value of this approach stems from the fact that observations of market-generated interest rate expectations are used rather than model-generated expectations. Thus, the results of hypotheses testing are not conditional upon the validity of the model underlying generation of interest rate expectations. This dependence has flawed previous tests of term structure hypotheses.

Our results show that the pure expectations hypothesis and the stationary variant of the market segmentation hypothesis are not substantiated under this approach. These results rest on the assumption that both the T-bill spot and futures markets are efficient--an assumption which we believe has not been discredited and is therefore reasonable.

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