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INVESTOR PSYCHOLOGY AND CAPITAL ASSET PRICING

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Investor Psychology and Capital Asset Pricing

- *Abstract* -

This paper offers a model in which securities prices reflect both risk and misperceptions of firms' prospects. Based on psychological evidence, our premise is that individuals are overconfident about their ability to evaluate securities, and hence overestimate the precision of their private information signals. The rational expectation of price changes increases linearly with factor sensitivities, with a term reflecting mispricing of security-specific risk, and with a set of terms that are proportional to the product of the security's sensitivity to a factor and the mispricing of that factor. The model thereby implies a multivariate relation between expected return and beta and with variables that proxy for market mispricing. The model is consistent with several anomalous findings regarding the cross-section of equity returns, including: the empirically observed ability of fundamental/price ratios to forecast aggregate and cross-sectional returns, and of market value to forecast cross-sectional patterns; the ability in some studies of fundamental/price ratios and market value to dominate traditional measures of security risk; and the behavior of closed-end fund discounts.

1 Introduction

In the last decade a body of evidence about the behavior of securities prices has sharply challenged both the leading theory of equilibrium pricing (the Capital Asset Pricing Model), and the premise that markets process information very rapidly and accurately (the Efficient Markets Hypothesis). To address this evidence, Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1997) have proposed theories of securities market under- and overreactions based on imperfect rationality. These papers focus on puzzling time series patterns in stock returns that have been interpreted in isolation as being either underreactions or overreactions to new information. Such patterns include short-term momentum, long-term reversals, and apparent underreaction to public news announcements in event studies.

The above papers, however, are cast in risk neutral, single-security settings, so they do not address recent evidence on the importance of risk measures vis-a-vis other security characteristics in predicting returns. This evidence includes (Appendix C discusses the relevant literature): (1) the ability of fundamental/price ratios (dividend yield, earnings/price, and book/market) to predict cross-sectional differences in future returns; (2) univariate evidence suggesting a positive relation between beta and returns; (3) domination, in some studies, of CAPM β as a cross-sectional predictor of stocks' future returns by variables based on price, such as market value and price-scaled variables (book/market, earnings/price ratios), and (4) the ability of firm size to predict future returns when size is measured by market value, but not when measured by non-market measures such as book value. Also, closely related to the cross-sectional phenomena is the aggregate pattern of a positive association between fundamental-scaled price measures and future aggregate stock market returns.

The Efficient Markets Hypothesis contends that prices rationally reflect public information. There have been previous efforts to explore the alternative hypothesis that investor behavior is not fully rational. Specifically, the noise trader approach to securities markets models market misvaluation as a consequence of random exogenous trades (see, e.g., Black (1986), De Long, Shleifer, Summers, and Waldmann (1990)). The single security setting of much of the noise trading literature precludes the underpinning of much of asset pricing theory, the pricing of covariance risk. Furthermore, since there always exists a specification of exogenous trades that will match any given empirical pattern in returns, to have content, the noise trading approach needs to be constrained in some way. Most such models constrain noise trades to be independent of information and prices. However, the psychological basis of this assumption is unclear, and it is doubtful that anyone actually behaves this way. We believe that it is valuable, rather than specifying exogenous trades, to base assumptions about individual behavior on psychological evidence. The consistency of the model's implications with existing and future evidence will then help us evaluate specific hypotheses about individual decision making in markets.

Since people deviate from pure rationality in many ways, there is a question of where to start in incorporating psychological patterns in a model of asset pricing. According to DeBondt and Thaler (1995), "perhaps the most robust finding in the psychology of judgment is that people are overconfident." There is pervasive psychological evidence that individuals are overconfident about their own abilities, including their ability to make judgements about and forecasts of uncertain quantities.[1] Specifically, our premise is that overconfident investors overestimate the quality of information signals they have generated about security values.[2] This suggests that individuals put greater weight on information which they themselves have generated or which reflects their personal analytical skills. Thus, we assume that individuals overestimate the precision of private information signals.[3] Individuals are 'quasi-rational' in the sense that they rationally maximize expected utility under this mistaken perception.

Groups of traders often use related methods of data gathering and analysis (*e.g.*, analysts talking to management). Thus, consistent with the assumption of several securities markets models, we assume that private signals are correlated across investors. This commonality implies that if investors misinterpret their signals, there can be nontrivial consequences for market prices.

The model allows for the possibility that investors receive private information about systematic factors as well as firm-specific risk.[4] Furthermore, investors may believe they observe meaningful factor signals when in fact they observe pure noise.

In equilibrium, securities are linearly increasing in the 'overconfident' beta of the security

[1]The psychological literature on individual overconfidence is summarized in Appendix B. See also Odean (1998) for a good summary of this literature.

[2]Interpreted broadly, private information in our context may refer to conclusions that a trader obtains by applying his processing abilities to purely public information, as with a trader who has special abilities to recognize the business implications of a firm's accounting performance. Alternatively, a trader who visits a firm's plants and talks with its managers has to interpret the information he receives and generate an estimate of the firm value. Again, since the trader is using his skill in generating this estimate, it may be regarded as private information about which he may be overconfident.

[3]For alternative specifications of overconfidence see Hirshleifer, Subrahmanyam, and Titman (1994), Kyle and Wang (1997), Odean (1998), Wang (1998), and Benos (1998).

[4]Chen, Roll, and Ross (1986) identify macroeconomic variables that have systematic influence on security returns. The existence of an active industry selling macroeconomic forecasts to investors suggests that, rightly or wrongly, investors believe that something akin to private information about aggregate factors exists. Consistent with genuine private information about aggregate factors, several studies have provided evidence that aggregate insider trading forecasts future industry and aggregate stock market returns (see, e.g., Lakonishok and Lee (1998)).

with the market portfolio. If investors are equally overconfident about all signals (in a sense made precise in the model), then the overconfident beta is equal to the conventional CAPM beta. Thus, despite the fact that investors misperceive the covariance matrix of returns, the conventional beta still prices securities.

Better predictors of return can be obtained by conditioning on the investor information signals that cause mispricing. This leads to an additive separation of risk and mispricing components of expected returns. The model is quite simple to implement, the main challenge being to identify measures of market misvaluation of securities. Indeed, empirical studies involving multiple regression on CAPM β and on firm size or fundamental-scaled price variables (such as Fama and French (1992)) can be interpreted as implementing a special case of this framework.[5]

To understand why price-related variables proxy for mispricing, suppose that traders start with a common prior, and at date 1 informed traders revise their belief about the value of a security or securities based upon a common private signal. Since they overestimate the precision of this signal, they overweight it relative to the prior, causing the stock price to overreact.

Following favorable information, the price of the security increases, and owing to overreaction, the security becomes overvalued. For example, measures of misvaluation that contain price in the denominator decrease. (The numerator may normalize by a non-market fundamental such as book value, earnings or dividends.) In this setting, firms with low (high) fundamental/price ratios will generally have low (high) future common stock returns, and vice-versa.

A prevailing intuition is that whenever the market makes systematic valuation errors, high fundamental/price ratios will predict high returns.[6] This argument implicitly assumes that the pricing errors are unrelated to a firm's fundamentals. This may not be true if errors arise from misinterpretation of new information. In our setting, informed investors generally have favorable private information about low book/market firms. If the informed were to

[5]An alternative approach to securities pricing is offered by Shefrin and Statman (1994), who analyze the effect of mistaken beliefs on equilibrium in stock, option and bond markets. Their model allows for arbitrary beliefs, and therefore for a wide range of possible patterns. However, the focus of the paper is not on empirically predicting the direction of pricing errors or addressing evidence on the cross-section of security returns. In contrast, our approach provides guidance regarding empirical implementation based on linear regression analysis (or, as in Fama and French (1992), cross-classification) which can be used to evaluate the performance of the model. In a recent paper, Shumway (1998) examines the effects of loss-aversion on securities prices. He does not, however, examine whether this approach can explain the known anomalous patterns in the cross-section of securities prices.

[6]This argument is related to Berk (1995a), who shows that if a misspecified test is applied to a market that obeys a rational asset pricing model, under certain conditions a firm's fundamental/price ratio will predict future abnormal returns.

systematically underreact to this private information (because of *under*confidence) then a low book/market ratio would forecast high future returns. Thus, we view the empirical direction of these effects as supporting theories based on overreaction and correction (such as the psychological theory proposed here) as opposed to the hypothesis of pure underreaction.

The model has several other implications consistent with existing evidence. If a fundamental such as book value measures expected future cash flows with error, the theory implies that size and the fundamental/price ratio jointly predict future expected returns, and provides plausible conditions under which the fundamental/price ratio will be a better predictor. Furthermore, the theory explains why fundamental-scaled price measures can be better forecasters of future returns than systematic risk measures such as β (Appendix C describes existing evidence). Intuitively, a high fundamental/price ratio (e.g., high book/market) can arise from high risk and/or overreaction to an adverse signal. If price is low due to a risk premium, on average it will rise back to unconditional expected value. If there is an overreaction to genuine adverse information, then the price will on average recover only part way toward the unconditional expected terminal cash flow. Knowing the level of risk (beta) generally helps disentangle these two effects. However, this is no longer the case if overconfidence becomes extreme in the sense that investors treat a pure noise signal as if it were informative. In this case, after an adverse signal price will on average rise back to the unconditional expected value, just as under the risk effect. Thus, even though covariance risk is priced by the market, the fundamental/price ratio, which reflects both risk and overreaction effects, will dominate beta, which reflects only risk. Subsection 2.3.1 provides a numerical illustration of the basic intuition for these implications.

Investor overconfidence is also potentially consistent with anomalous evidence concerning closed-end fund discounts. A fund discount is somewhat like a market/book ratio, except that the 'book' value (net asset value) is also set in the market. Under some auxiliary assumptions regarding market segmentation, the overconfidence approach shares some of the implications of the noise trader approach (see, for example, De Long, Shleifer, Summers, and Waldmann (1990)) for closed-end funds. In a pure noise approach, discounts reflect mispricing and therefore forecast future returns. Since overconfident traders may trade based on genuine information rather than noise, a high discount on closed-end funds will forecast both *low* future fundamentals such as earnings and *high* future stock returns.

There is still considerable disagreement over whether the anomalous empirical patterns we explain here result from stochastically varying risk-premia, imperfect markets, or imperfect rationality. While in principle these patterns of return predictability may reflect risk, efforts to identify an economically relevant risk factor have met with limited success. For the size or book/market ratio of a firm to be a good proxy for risk, the returns of small and high book/market firms' stocks would have to be negatively correlated with marginal utility, meaning the returns should be particularly high in good times (relative to other stocks) and low in bad times. No such correlation is obvious in the data. [See, for example, Lakonishok, Shleifer, and Vishny (1994).] Also, fundamental-scaled price variables may be related to the liquidity of a firm's stock. However, Daniel and Titman (1997) find that, if anything, the common stocks of firms with higher book/market ratios are more liquid. Thus, while we cannot rule out explanations for these phenomena based on risk or market imperfections, it seems reasonable to consider explanations based on imperfect rationality.

A common objection to models that explain price anomalies as a result of imperfect rationality of market participants is that rational individuals should be able to trade profitably against the mispricings, and thereby eliminate the mispricing. Furthermore, if such trading causes wealth to flow from irrational to smart traders, eventually the smart traders may dominate price-setting. However, risk aversion limits the extent to which trading by fully rational would eliminate the patterns created by the imperfectly rational traders. Furthermore, in some settings overconfident traders earn higher expected profits than do fully rational traders.[7]

The remainder of the paper is structured as follows. Section 2 presents a pricing model based on investor psychology. Section 3 examines forecasting future returns using both mispricing measures and traditional risk-based return measures (such as the CAPM beta). Section 4 discusses closed-end funds, and Section 5 concludes.

2 The Model

2.1 The Economic Setting

We have argued that the psychological basis for overconfidence is that people overvalue their own expertise. This suggests that people will tend to be overconfident about private signals. A signal that only a subset of individuals receive presumably reflects special expertise on the part of the recipients. Thus, the *motivating scenario* involves a set of risk averse traders, some of whom are endowed with private information and some of whom are not. The traders who possess private information are *overconfident*: they overestimate the precision of their

[7]Several papers argue that there are limits to the degree that arbitrage reduces market inefficiencies, and that imperfectly rational or overconfident traders can earn higher expected profits than fully rational traders and therefore can be influential in the long run; see, e.g., De Long, Shleifer, Summers, and Waldmann (1990), Kyle and Wang (1997), Shleifer and Vishny (1997). Daniel, Hirshleifer, and Subrahmanyam (1997) analyze the profitability of overconfident trading in a competitive equilibrium setting with endogenous prices. They show that overconfident informed traders may make greater expected profits (though lower utility) than rational informed traders because overconfident traders exploit their information more aggressively.

signals. The uninformed traders have no signals to be overconfident about.

For tractability, our formal analysis consists of a limiting case of this motivating scenario in which the fraction of the population that is uninformed approaches zero. Thus, equilibrium is determined by a set of identical informed individuals with exponential utility. In the spirit of the motivating scenario, we will refer to the signals the informed individuals receive as 'private'.[8,9] The common risk aversion coefficient of the investors is denoted by A.[10] Each individual is endowed with a basket containing shares of the risky securities and riskfree consumption claims. At each date individuals can trade the consumption claims for shares. For simplicity we assume that the riskfree rate is zero.

Timing

There are two dates. At date 1, individuals with identical prior beliefs receive common noisy private signals about the risky security payoffs.[11] At date 2, conclusive public information arrives, each security i pays a liquidating dividend of θ_i, and consumption occurs.

Security Payoffs

There are N securities. Let $\mu_i^C \equiv E^C[\theta_i|\phi]$ denote the overconfident expected payoff on security i, where ϕ is all information available to investors at date 1 (the signals about the factors and securities). The signals s_i and the security payoffs θ_i are multivariate normally distributed. Let $\bar{\theta}_i$ denote security i's unconditional expected value (the expected liquidating dividend). The distribution of $\bar{\theta}_i$ across securities is defined by

$$\bar{\theta}_i \sim \mathcal{N}(\bar{\bar{\theta}}, V_{\bar{\theta}}), \quad i = 1, \ldots N, \tag{1}$$

where $\bar{\bar{\theta}}$ is the cross-sectional expectation of the unconditional expected values, and $V_{\bar{\theta}}$ is the variance of $\bar{\theta}$. All individuals know $\bar{\theta}_i$ $(i = 1, \ldots, N)$, so $\bar{\theta}_i$ (for all i) is non-stochastic from the perspective of investors at the time of their decisions.

The Portfolio Problem and Market Equilibrium

Let P_i denote the date 1 price of security i, and x_i and $\bar{x}_i$, denote the identical investors' demands for and endowments of security i, respectively. The overconfident covariance between securities i and j is $\sigma_{ij}^C \equiv E^C[(\theta_i - \mu_i^C)(\theta_j - \mu_j^C)]$, where a C superscript denotes an expectation calculated with respect to the over$C$onfident beliefs of the informed. An

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[8]We believe that similar results would obtain if there were a positive mass of uninformed investors. Intuitively, prices would reflect a weighted average of the beliefs of the overconfident and the uninformed, so prices would deviate in the same direction from efficient prices but the magnitudes of the deviations would be attenuated. Unfortunately, such an analysis is not very tractable.

[9]Overconfident investors recognize that other overconfident investors perceive a similarly high precision for the signal. Since they share this perception, they do not regard the others as overconfident.

[10]Appendix A provides a guide to the model notation.

[11]Some previous models with common private signals include Grossman and Stiglitz (1980), Admati and Pfleiderer (1988), and Hirshleifer, Subrahmanyam, and Titman (1994).

investor solves

$$\max_{x_1,\dots,x_N} \sum_{i=1}^{N} x_i \mu_i^C - \left(\frac{A}{2}\right) \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}^C \quad \text{subject to} \sum_{i=1}^{N} (x_i - \bar{x}_i) P_i = 0.$$

Since there are N securities and prices, one price is redundant. Taking the first order condition, rescaling prices to eliminate the Lagrangian multiplier, and imposing the market clearing condition $x_i = \bar{x}_i$ for all securities gives

$$P_i = \mu_i^C - A \sum_{j=1}^{N} \sigma_{ij}^C \bar{x}_j. \tag{2}$$

Further Structure on Security Payoffs

Now suppose that the security payoffs follow a K-factor structure

$$\theta_i = \bar{\theta}_i + \sum_{k=1}^{K} \beta_{ik} f_k + \epsilon_i, \tag{3}$$

where β_{ik} is the loading of the i'th security on the k'th factor, and f is the realization of the k'th factor. Without loss of generality, we set:

$$\begin{aligned} E[f_k] &= 0 \text{ for all } k & \qquad E[\epsilon_i] &= 0 \text{ for all } i \\ E[f_k f_l] &= 0 \text{ for all } k \neq l & E[f_k^2] &= 1 \\ E[f_k \epsilon_i] &= 0 \text{ for all } i, k & E[\epsilon_i \epsilon_j] &= 0 \text{ for all } i \neq j. \end{aligned}$$

Also, let $V_i^\epsilon \equiv var(\epsilon_i)$.

Information Signals

The individuals receive signals both about the idiosyncratic components ϵ_i as well as the factor realizations f_k. The signals about the idiosyncratic components are of the form

$$s_i^\epsilon = \epsilon_i + e_i^\epsilon.$$

The true variance of e_i^ϵ is $V_i^{R\epsilon}$ (R denotes "Rational"), but because the informed investor is overconfident, he mistakenly believes it to be $V_i^{C\epsilon} < V_i^{R\epsilon}$. The noisy signals about each of K systematic factors $f_1, \dots, f_K$ take the form

$$s_k^f = f_k + e_k^f,$$

and again the signal noise variance is underestimated by the overconfident individuals, so that the estimated noise variance is $V_k^{Cf} < V_k^{Rf}$, where the latter expression is the true (rational) noise variance. Finally we assume that $cov(e_i^\epsilon, e_j^\epsilon) = 0$ for $i \neq j$, $cov(e_k^f, e_l^f) = 0$ for $k \neq l$, and $cov(e_i^\epsilon, e_k^f) = 0$ for all i, k

2.2 Pricing Relationships

To solve for price in terms of exogenous parameters, we calculate the overconfident expectation of security i's terminal value,.

$$\mu_i^C = E^C[\theta_i | s_i^\epsilon, s_1^f, \ldots, s_K^f] = \bar{\theta}_i + \lambda_i^{C\epsilon} s_i^\epsilon + \sum_{k=1}^{K} \beta_{ik} \lambda_k^{Cf} s_k^f,$$

where

$$\lambda_i^{C\epsilon} \equiv \frac{V_i^\epsilon}{V_i^\epsilon + V_i^{C\epsilon}}, \qquad \lambda_k^{Cf} \equiv \frac{1}{1 + V_k^{Cf}}. \tag{4}$$

Applying (2) yields

$$P_i = \bar{\theta}_i + \lambda_i^{C\epsilon} s_i^\epsilon + \sum_{k=1}^{K} \beta_{ik} \lambda_k^{Cf} s_k^f - A\mathrm{cov}^C(\theta_i, R_M), \tag{5}$$

where $R_M \equiv \sum_{j=1}^{N} \bar{x}_j(\theta_j - P_j)$ is defined to be the price change of the market portfolio. This relationship says that the price of the security is the unconditional expected payoff plus $K+1$ adjustments reflecting the various signals relevant for the expected payoff of security i, interpreted according to overconfident beliefs, and with a risk premium adjustment based on (overconfident) covariance with the market price change.

Now, let

$$\lambda_i^{R\epsilon} \equiv \frac{V_i^\epsilon}{V_i^\epsilon + V_i^{R\epsilon}} \quad \text{and} \quad \lambda_k^{Rf} \equiv \frac{1}{1 + V_k^{Rf}}. \tag{6}$$

The λ^R's are weights that a rational individual would place on the signals such that equation (5) would hold with all λ^C's replaced by λ^R's.

Let $R_i \equiv \theta_i - P_i$, and let the information set available to investors be denoted by ϕ. Then the true conditional expected price change is

$$E[R_i|\phi] = (\lambda_i^{R\epsilon} - \lambda_i^{C\epsilon}) s_i^\epsilon + \sum_{k=1}^{K} \beta_{ik} (\lambda_k^{Rf} - \lambda_k^{Cf}) s_k^f + A\mathrm{cov}^C(\theta_i, R_M) \tag{7}$$

for all $i = 1, \ldots, N$. This equation states that the true expected price change on a security includes $K+1$ terms involving correction of the mispricing that arises from the $K+1$ signals, and a risk premium term associated with an overconfident covariance with the stock market as a whole.

These relations can be expressed more compactly using the following definitions:

$$S_i^\epsilon \equiv \lambda_i^{R\epsilon} s_i^\epsilon, \qquad S_k^f \equiv \lambda_k^{Rf} s_k^f$$

and

$$\omega_i^\epsilon \equiv \frac{\lambda_i^{C\epsilon} - \lambda_i^{R\epsilon}}{\lambda_i^{R\epsilon}}, \qquad \omega_k^f \equiv \frac{\lambda_i^{Cf} - \lambda_k^{Rf}}{\lambda_k^{Rf}}$$

for all $i = 1, \ldots, N$, $k = 1, \ldots, K$. Here S_i^ϵ and S_k^f are rescaled so that a unit increase in any signal has the same effect on the rational price, and ω's are the fractional excess sensitivity of the overconfident price relative to the rational sensitivity of price to a unit increment of the signal. We also define $\alpha \equiv A\mathrm{var}^C(R_M)$. The exogenous share composition of the market (shares per person of each security, $\bar{x}_i$) is independent of signal realizations. Since all second moments of a normal distribution are constants independent of conditioning variables, α is also nonstochastic and independent of signal realizations. Let $\beta_{iM}^C \equiv \mathrm{cov}^C(\theta_i, R_M)/\mathrm{var}^C(R_M)$ and $\beta_{iM} \equiv \mathrm{cov}(\theta_i, R_M)/\mathrm{var}(R_M)$ be the overconfident and rational market β's respectively.

We define investors as being *equally overconfident* about all signals if the ratios of overconfident to rational conditional variances of factors and residuals are equal. Using standard expressions for the conditional variances of normal distributions, we have

$$\frac{\lambda_k^{Cf} V_k^{Cf}}{\lambda_k^{Rf} V_k^{Rf}} = \frac{\lambda_i^{C\epsilon} V_i^{C\epsilon}}{\lambda_i^{R\epsilon} V_i^{R\epsilon}} \equiv \rho \tag{8}$$

for all $i = 1, \ldots, N$, $k = 1, \ldots, K$. Intuitively, these quantities measure overconfidence inversely. *Ceteris paribus*, if investors are more confident about a factor or idiosyncratic signal they assess a lower noise variance. Lower noise variance causes this measure to decrease (as can be seen by substituting for the λ's using (4) and (6)).

Lemma 1 *If investors are equally overconfident about all signals, all securities' overconfident market betas are equal to the corresponding rational market betas.*

It follows from equations (5) and (7) that

Proposition 1 *If risk averse investors with exponential utility are overconfident about the signals they receive regarding K factors and about the idiosyncratic payoff components of N securities, then securities obey the following relationships:*

$$\begin{aligned} P_i &= \bar{\theta}_i - \alpha\beta_{iM}^C + (1+\omega_i^\epsilon)S_i^\epsilon + \sum_{k=1}^{K} \beta_{ik}(1+\omega_k^f)S_k^f \qquad (9) \\ E[R_i|\phi] &= \alpha\beta_{iM}^C - \omega_i^\epsilon S_i^\epsilon - \sum_{k=1}^{K} \beta_{ik}\omega_k^f S_k^f, \qquad (10) \end{aligned}$$

for all $i = 1, \ldots, N$. If investors are equally overconfident about all signals, then the overconfident beta can be replaced with the rational CAPM beta.

In the price equation, all signals are multiplied by overreaction coefficients of the form $1+\omega$. The true expected return therefore contains the negative of the sum of the overreactions.

This security valuation model implies that true expected return decomposes additively into a risk premium (the first term) and components arising from mispricing (the next two terms). The mispricing due to overconfidence about factor information is proportional to the security's sensitivity to that factor. A security's expected return is also influenced by overconfidence about information about its idiosyncratic risk, and includes a premium for market risk. Mispricing arises from the informed's overreaction to signals about the factors and the idiosyncratic components.[12] If there were no overconfidence (ω terms all zero), this equation would be identical to the Sharpe/Lintner/Black CAPM with zero riskfree rate.

Since the expected value of the signals is zero, the following Corollary follows by taking the rational expectation of (10) when investors are equally overconfident about all signals; and then taking a weighted average of security expected returns to show that $\alpha = E[R_M]$. In this case, even though investors misperceive the covariance matrix of security returns, the true CAPM beta is still a valid predictor of future returns.

Corollary 1 *If investors are equally overconfident about all signals, then unconditional security expected returns obey the CAPM pricing relationship*

$$E[R_i] = E[R_M]\beta_{iM}, \quad i = 1, \ldots, N,$$

where $E[R_M]$ is the true expected return on the market portfolio, and β_{iM} is the security's true beta with the market portfolio.

Thus, as in the CAPM, beta is predicted to be positively related to future returns, and the predicted slope of the security market line is the risk premium (unconditional expected return) of the market. Thus, unconditionally the model is consistent with the cross-sectional return predictions of the CAPM.

Although the unconditional pricing equation of Corollary 1 is identical in form to that of the CAPM (with a zero riskfree rate), securities and the market are still mispriced. Since overconfident investors place too much weight on signals, they underestimate the conditional variability of all securities, and demand too low a risk premium on the market (lower $E[R_M]$).[13] Thus, the security market line is less steeply sloped than in a fully rational capital market.

[12]The coefficient β_{iM} is a price-change beta, not the CAPM return beta. That is, β_{iM} is the regression coefficient in $\theta_i - P_i = \alpha_i + \beta_i(\theta_M - P_M)$ where P_i and P_M are known. The CAPM return beta is the coefficient in the regression $(\theta_i - P_i)/P_i = \alpha_i^R + \beta_{iM}^{\mathrm{Ret}}(\theta_M - P_M)/P_M$, and is equal to $(P_M/P_i)\beta_{iM}$.

[13]Mehra and Prescott (1985) find that the market risk premium is high relative to market risk and plausible levels of risk aversion. Some possible explanations are offered by Constantinides (1990), Benartzi and Thaler (1995), Brown, Goetzmann, and Ross (1995), Goetzmann and Jorion (1997), and Constantinides, Donaldson, and Mehra (1997). In our model, suppose date 1 as currently defined is divided into a date 0 prior to any informational arrival, and a date 1 after the private signal has been received. Then the

The model is consistent with the univariate evidence that mean returns are increasing with beta (see Appendix). However, the model implies that there are better ways to predict future returns than the CAPM security market line. Proposition 1 suggests that better predictors can be obtained by regressing not just on beta, but on proxies for market misvaluation. This is the topic of the next subsection.

2.3 Proxies for Mispricing

The model so far is described in terms of unconditional mispricing induced by signals that are not directly observable by the econometrician. We now examine how expected returns are related to observable proxies for mispricing that correlate with the $K+N$ signals. In the following subsections, we consider the question of how the returns will appear to be related to various risk and mispricing measures. We begin with a simple numerical illustration of the basic reasoning.

2.3.1 A Simple Example

We illustrate here (and formalize later) three points:

1. High fundamental/price ratios predict high future returns if investors are overconfident, and low future returns if investors are underconfident.

2. Regressing on β as well as a fundamental-scaled price ratio in general helps an econometrician disentangle risk premium versus mispricing effects, with positive coefficients on both β and fundamental/price ratios.

3. However, if overconfidence about signals is extreme and the fundamental is measured perfectly, even though β is priced, it has no incremental power to predict future returns.

The reasoning for part 3 is *not* that with infinite overconfidence, investors would perceive risk to be zero, which would cause β to be unpriced. Even if investors are only overconfident about idiosyncratic risk, so that covariance risk is rationally priced, we will see that β has no incremental power to predict returns. More generally, overconfidence about factors that

analysis of this paper focuses on price changes from dates 1 to 2. This is where the 'action' of the analysis occurs. If instead we were to examine the empirical problem of an econometrician who is equally likely to observe a date 0-1 return versus a date 1-2 return, the qualitative results of the analysis would be essentially unchanged. However, if the econometrician were to randomly pick market returns from 0-1 or 1-2, the mean return on the market would be equal to the rational expected return on the market.

weakens but does not eliminate the pricing of β risk can completely eliminate the incremental ability of β to forecast returns.

To understand parts (1)-(3), suppose that there is only information about idiosyncratic risk, and consider a stock that is currently priced at 80. Suppose that its unconditional expected cash flow is known to be 100, and that the econometrician can observe this unconditional fundamental without noise. The fact that the price is below the unconditional fundamental could reflect a rational premium for factor risk, an adverse signal, or a combination of effects.

Suppose now that investors are overconfident, and let us begin by assuming that the econometrician knows that $\beta = 0$. Then the price of 80 resulted from an adverse signal. Since investors overreacted to this signal, the true conditional expected value is greater than 80. Thus, an above-average fundamental/price ratio ($100/80 > 1$) indicates a positive abnormal expected return. Of course, the reverse reasoning indicates that a below-average value (e.g, $100/120 < 1$) predicts a negative expected return. Thus, under overconfidence, a high fundamental/price ratio predicts high future returns, consistent with a great deal of existing evidence.

If instead investors were underconfident, then the price of 80 would be an *under*reaction to the signal, indicating that in the future it can be expected to fall. Thus, the high fundamental/price ratio would predict *low* returns, inconsistent with the evidence.

If we allow for differing β's, there is a familiar interfering effect (see Ball (1978), Berk (1995a)): a high beta also reduces price relative to fundamentals. Thus, even if there is no information signal, a high fundamental/price ratio will predict high returns. This illustrates point (1) above.

These confounding effects suggest regressing future returns on both the fundamental/price ratio and beta, which leads to point (2) above. Henceforth we assume overconfidence. If the econometrician knew that the price of 80 was a pure risk premium effect (i.e., the signal happens to equal zero), then the expected terminal cash flow would still be 100. In contrast, if $\beta = 0$, the price of 80 would be purely the result of an adverse signal. The conditional expected value (90, say) lies between 80 and 100 because the signal is adverse, and investors overreact to it. Thus, the expected return is positive, but not as large as in the case of a pure risk premium. By controlling for β as well as the fundamental/price ratio, the econometrician can disentangle whether the price will rise to 100 or only to 90.

The multiple regression coefficient on β will be positive because a high β indicates, for a given fundamental/price ratio, a higher conditional fundamental (e.g., 100 instead of 90). The multiple regression coefficient on the fundamental/price ratio will be positive because a high ratio indicates, for a given β, a more adverse signal. A more adverse signal implies more overreaction to be corrected.

To understand point (3), consider now the extreme case where overconfidence is infinite, in the sense that the 'signal' is pure noise ($V_i^{R\epsilon} \to \infty$) but investors believe that the noise variance is finite. In this case, even when the price decreases to 80 purely because of an adverse 'signal,' it will still on average recover to 100. This leads to exactly the same expected return as when the price of 80 is a result of a high beta. Both effects are captured fully and equally by the fundamental/price ratio, whereas beta captures only the risk effect. Thus, the fundamental/price ratio will have incremental explanatory power over beta alone, but beta will have no incremental explanatory power over the fundamental/price ratio alone.

This scenario is too extreme; we would expect at least some beta effect either because overconfidence is not so extreme or because of noise in fundamental measures such as book value. However, it does offer an explanation for why the incremental beta effect can be small and therefore hard to identify statistically. The following subsections derive these results rigorously.

2.3.2 Fundamental Value Observable

Consider an econometrician who seeks the best linear forecast of a firms' return using one or several variables. Since market price reflects mispricing, it is natural to use a measure that contains price to predict future returns. However, a firm may have a low price not only because of a low signal and undervaluation, but also because the unconditional expected payoff $\bar{\theta}_i$ is low. When this unconditional fundamental is observable, this possibility can be filtered away perfectly so that a purer measure of mispricing is obtained. We assume for now that the econometrician observes a perfect *fundamental measure* which is equal to the unconditional expected value (an assumption we relax in Subsection 2.3.3).

Consider the case in which econometrician wishes to predict returns given only the value of the fundamental-scaled price variable $P_i - \bar{\theta}_i$. We assume here that the econometrician does not directly observe or, more importantly, make use of the overconfidence levels for different securities and factors (ω's), or other security specific information such as the security's β, signals, and $\bar{\theta}$. This can be viewed as a setting where the econometrician uses the fundamental-scaled price measure for a randomly selected security to predict its return. We now let variables with i subscripts *omitted* denote random variables whose realizations are determined in two stages: (1) a security is randomly selected from the pool with equal probability (i.e., there is a random selection of security characteristics), and (2) the payoff outcome is realized.

We denote the moments of the factor loading distribution β_k (the loading of a randomly selected security on factor i) by $E[\beta_k], V_{\beta_k}$; of β_M^C (the overconfident beta of a randomly selected security with the market price change) by $E[\beta_M^C], V_{\beta_M^C}$; of ω^ϵ (the excess sensitivity of price to a unit increase in the signal about idiosyncratic risk for a randomly selected security) by $E[\omega^\epsilon], V^{\epsilon\omega}$; of ω_k^f (the excess sensitivity of prices to a unit increase in the signal about factor k) by $E[\omega^f], V^{f\omega}$ (moments assumed to be independent of k); of S_k^f (the normalized signal about factor k) by V_k^{Sf}; and of S^ϵ (the normalized signal about idiosyncratic risk for a randomly selected security i) by $V^{S\epsilon}$ (as per our earlier assumptions, the last two random variables have means of zero). Further, we assume that the choice of firm is independent of the signal realization, so that the covariance between all signal realizations and variances of signals and noises, and, in turn, the ω's and β's are zero.

Since the unconditional expected value of a security $\bar{\theta}_i$ is measured without noise, the econometrician observes the deviation of price from true fundamental $P_i - \bar{\theta}_i$, which reflects both mispricing and risk (see equation (9)). To the extent that investors are generally overconfident, this will also proxy for how overpriced a security is now, and will therefore predict whether the security will experience low risk-adjusted returns in the future.

Consider the linear projection of the security return R onto $P - \bar{\theta}$,

$$R = a + b_{P-\bar{\theta}}(P - \bar{\theta}) + e.$$

The value of the slope coefficient that minimizes the variance of the projection error is:

$$b_{P-\bar{\theta}} = \frac{cov(R, P - \bar{\theta})}{var(P - \bar{\theta})}. \tag{11}$$

We then have the following proposition (The proof details are presented in Appendix D).

Proposition 2 *The regression of the return R on the fundamental-scaled price $P - \bar{\theta}$ yields the following coefficient:*

$$b_{P-\bar{\theta}} = \frac{-\alpha^2 V_{\beta_M^C} + cov_{OC}}{\alpha^2 V_{\beta_M^C} + var_{OC}}, \tag{12}$$

where

$$cov_{OC} \equiv -\left(E[\omega^\epsilon] + E[(\omega^\epsilon)^2]\right) E[(S^\epsilon)^2] - \left(E[\omega^f] + E[(\omega^f)^2]\right) \sum_{k=1}^{K} E[\beta_k^2] E[(S_k^f)^2] \tag{13}$$

$$var_{OC} \equiv \left[(1 + E[\omega^\epsilon])^2 + V^{\epsilon\omega}\right] E[(S^\epsilon)^2] + \left[(1 + E[\omega^f])^2 + V^{f\omega}\right] \sum_{k=1}^{K} E[\beta_k^2] E[(S_k^f)^2]. \tag{14}$$

The coefficient is negative if investors are on average overconfident, $E[\omega^\epsilon] > 0$ and $E[\omega^f] > 0$. If investors are not overconfident about any signal and are underconfident about at least one signal ($\omega_k^f \leq 0$, $\omega_i^\epsilon \leq 0$, with at least one inequality strict), and if $V_{\beta_M^C}$ is sufficiently small, then the coefficient is positive.

The first term of both numerator and denominator is the contribution of the rational cross-sectional variation in risk premia: high beta firms will have a low $P-\bar{\theta}$ and a high expected return. The second terms, cov_{OC} and var_{OC} respectively, are the contributions of the individuals' private information to the covariance and variance. If there is no overconfidence, then $E[\omega]$ and V^{ω} are zero for all signals. In this case, cov_{OC} will be zero. If the $E[\omega]$'s are positive for all signals, then cov_{OC} will be negative; overreaction to positive private information will mean that firms with a positive private signal (and high price) will have a negative future return.

Equation (12) shows that, even if there is no overconfidence, there will still be a risk-based relationship between $P-\bar{\theta}$ and R. As Ball (1978) and Berk (1995a) have pointed out, in asset pricing models such as the CAPM, low price firms are those that are discounted heavily, i.e., high beta/high return firms.

To interpret (12) further, suppose that the individuals' signals concern only the idiosyncratic terms of the factor structure, meaning that $E[(S_k^f)^2] = 0$ for all K factors; and eliminate risk effects by making all market betas be equal ($V_{\beta_M} = 0$).

Corollary 2 *If all security market beta's are equal ($V_{\beta_M} = 0$), and if individuals only observe signals about firm-specific components of asset values, the regression coefficient in Proposition 2 becomes*

$$b_{P-\bar{\theta}} = -\frac{E[\omega^{\epsilon}] + E[(\omega^{\epsilon})^2]}{1 + 2E[\omega^{\epsilon}] + E[(\omega^{\epsilon})^2]}, \text{ or, } b_{P-\bar{\theta}} = -\frac{E[\omega^{\epsilon}](1 + E[\omega^{\epsilon}]) + V^{\epsilon\omega}}{(1 + E[\omega^{\epsilon}])^2 + V^{\epsilon\omega}}.$$

Omitting $P-\bar{\theta}$ subscripts and ϵ superscripts for convenience, it follows that:

1. *If $E[\omega] = 0$ and $V^{\omega} = 0$, $b = 0$.*
2. *If $E[\omega] > 0$, $b < 0$.*
3. $\lim_{E[\omega]\to\infty} b = -1$
4. *For $-1 < E[\omega] < 0$, and $V^{\omega} = 0$, $b > 0$.*

If investors are on-average overconfident ($E[\omega] > 0$), then by 2, b will be negative. Intuitively, overconfident investors overreact to their private information, so a high price probably means too high a price, and consequently that expected risk-adjusted returns will be negative.

Under sufficiently intense overconfidence, by Part 3 of the Corollary the coefficient becomes more negative (close to -1). Intuitively, the overreaction becomes so strong that price moves are almost completely reversed in the long run.

On the other hand, if investors were *under*-confident then Part 4 shows that a high price to fundamental ratio would be associated with a high future return (if V^{ω} is not too large). Intuitively, if investors are underconfident and hence underreact to their private information, then a high-price will mean that the price is likely to increase still more.

Proposition 2 and Corollary 2 provide a theoretical motivation for the well-known regressions (and cross-classifications) of Fama and French (1992). The analysis suggests that the relationship between book/market and future returns is a valid one rather than an ex post relationship arising from data-snooping. However, there is no implication that there will exist any meaningful book/market factor, nor that sensitivities with respect to a factor constructed from book/market portfolios can be used to price assets. In this regard, our analysis is consistent with the evidence of Daniel and Titman (1997) that the book-market effect is a result of the book/market characteristic, not an underlying factor (distinct from the market return). Specifically, our analysis suggests that book/market may instead capture a combination of market risk and mispricing.

To see this, consider the special case in which there is just a single factor (essentially the market) and in which there is no information about the factor, only about idiosyncratic security return components. Clearly there is no book/market factor as distinct from the market factor. Nevertheless, a price-scaled fundamental (such as book/market) will predict future returns, and may dominate beta.

Winners versus Losers

We have interpreted $P_i - \bar{\theta}_i$ as a fundamental-scaled price, where the unconditional fundamental is perfectly observed by the econometrician. A possible proxy for the unconditional fundamental is an earlier stock price that obtained prior to the arrival of recent signals. In this interpretation the fundamental-scaled price variable can be interpreted as past price runup. Thus, misvaluation effects imply that past 'winners' should tend to be losers in the future. However, an interfering risk effect goes in the opposite direction. Past price is somewhat different from non-price fundamental measures, because past price discounts for risk. Thus, runup will tend to be greater for riskier stocks, which should earn high returns in the future as well. Although there is evidence supporting the misvaluation implication of reversals at long lags, at shorter lags there is a momentum effect in the opposite direction (Jegadeesh and Titman (1993)).[14]

[14]These results have been developed in a setting where confidence is constant. There is psychological evidence that confidence varies over time depending on whether outcomes confirm actions, and that individuals interpret evidence about their abilities in a favorably biased way. Daniel, Hirshleifer, and Subrahmanyam (1998) provide a model in which variations in confidence as a result of news arrival tends to cause temporary continuation of overreaction before reversal occurs. Thus, past winners may on average temporarily become even bigger winners, but in the long run mispricing is corrected so they suffer reversal. In contrast, the static confidence model provided here does not allow for shifts in confidence and temporary continuation of overreaction. This suggests that the winner/loser implications derived here will work best using long-term

2.3.3 Econometrician Observes a Noisy Fundamental Measure

Fundamental Proxy is Fundamental Plus Noise

Subsection 2.3.2 assumed that the unconditional fundamental value could be observed by the econometrician. This will generally not be the case. This makes it harder to disentangle whether a low price arises because $\bar{\theta}_i$, its unconditional expected payoff, is low, or because a low signal has been received. The econometrician can use a fundamental measure as a noisy proxy for the unconditional expected value. (As indicated at the end of the preceding subsection, a past stock price is one possible proxy for unconditional fundamentals.) We assume that the noisy fundamental measure is the true expected cash flow plus noise,

$$F_i = \bar{\theta}_i + e_i^F,$$

where e_i^F is *i.i.d.* normal noise with zero mean, and $V^F \equiv E[(e^F)^2]$ is the variance of the error in the fundamental measure of a randomly selected security.

In the OLS regression

$$R = a + b_{P-F}(P - F) + \gamma,$$

where γ is independent noise with zero mean, the slope coefficient is

$$b_{P-F} = \frac{cov(R, P - F)}{var(P - F)}. \tag{15}$$

Since $P_i - F_i = P_i - \bar{\theta}_i - e_i^F$, and since e_i^F is uncorrelated with the other variables, the covariance in the numerator of b_{P-F} will be identical to that in equations (11) and (12), and the variance in the denominator is the same except for the additional V^F term.

Proposition 3 *When unconditional expected fundamental values are measured with noise, the regression of the return R on the price scaled fundamental $P - F$ yields the coefficient*

$$b_{P-F} = \frac{-\alpha^2 V_{\beta_M^C} + cov_{OC}}{\alpha^2 V_{\beta_M^C} + var_{OC} + V^F} = \left(\frac{var(P - \bar{\theta})}{var(P - F)}\right) b_{P-\bar{\theta}}. \tag{16}$$

The R^2 of the regression is

$$R^2_{P-F} = \frac{[cov(R, P - F)]^2}{var(R)var(P - F)} = \frac{[cov(R, P - \bar{\theta})]^2}{var(R)var(P - F)} = \left(\frac{var(P - \bar{\theta})}{var(P - F)}\right) R^2_{P-\bar{\theta}}.$$

The slope coefficient is smaller in magnitude than that in (12) because F_i is a noisy proxy for $\bar{\theta}_i$, so that $var(P - F) > var(P - \bar{\theta})$; similarly, the R^2 of the regression is also lower.

measures of past performance which allow enough time for temporary mispricings to correct out fully.

Any adjustment of the fundamental proxy F that decreases the measurement error variance will improve R^2_{P-F}. One method of doing so is to adjust fundamental ratios relative to industry values. Accounting measures of value differ across industry for accounting reasons that do not reflect differences in fundamental value. For example, different businesses have differing importance of intangible assets, which are imperfectly reflected in accounting measures of value. This suggests that industry adjustment can improve the ability of fundamental measures to predict return, by filtering out measurement noise that is correlated for firms within an industry. This is consistent with the evidence of Cohen and Polk (1995) and of Dreman and Lufkin (1996).

Firm Size

Suppose instead that the proxy used by the econometrician is simply firm size (P), which is a special case of a fundamental scaled measure in which the fundamental proxy is a constant.[15] In this case, we have:

Proposition 4 *Suppose that investors are on average overconfident, $E[\omega^f] > 0, E[\omega^\epsilon] > 0$. Then, if the variance the noise in the fundamental proxy is smaller than the variance of the security expected cash flow, $V^F < V_{\bar{\theta}}$, then the R^2 from the regression of the return R on the fundamental-scaled price measure $F - P$ is greater than the R^2 from the regression of R on market value P.*

The above proposition thus suggests that the fundamental scaled variable will have greater explanatory power to forecast future returns than firm size so long as the fundamental proxy is not too noisy. This is consistent with the findings of some recent studies that the relation between size and expected returns is weaker and less reliable than the relation of book/market with expected returns (Fama and French (1992), Davis, Famma, and French (1998)).

2.4 Aggregate Fundamental-Scaled Price Variable Effects

This subsection examines the special case of a single security ($N = 1$), which we interpret as the aggregate stock market portfolio. The implications for the aggregate market are obtained from the general model by deleting all i subscripts.

[15] Whether we interpret P as per-share price or market value depends on whether we are modeling this on a basis of per-share or total firm value. Here we assume that θ_i is the total value of the firm. However, this analysis is equally valid on a per-share basis, and is therefore consistent with the empirical evidence that, cross-sectionally, share price is negatively correlated with future returns. A fuller analysis of this topic would include the number of shares relative to total firm value as a source of cross-sectional noise, so that firm value versus share price could have different degrees of predictive power for future returns.

The model then predicts that future aggregate market returns will be predicted by variables of the form F/M, where F is a publicly observable *non*-market measure of expected fundamental value and M is aggregate market value. Four examples are aggregate dividend yield, earnings/price ratio, the aggregate book/market ratio, and the reciprocal of market value (where the numerator $F \equiv 1$ is a constant). Noisy public information may moderate, but does not eliminate this effect. These deviations will gradually be corrected, though this correction may be slow. Thus, the model explains the empirical finding of a dividend yield effect, and predicts aggregate earnings yield and book/market effects as well. The evidence regarding past winners and losers depends on the length of lag; see also the discussion at footnote 14.

3 Risk Measures versus Fundamental-Scaled Price Ratios

Often tests of return predictability look simultaneously at standard risk measures and measures of mispricing. These regressions usually involve the CAPM β, a measure of market capitalization, and a fundamental-scaled variable such as the book-to-market ratio (see, *e.g.*, Fama and French (1992)). In our setting, if the expected fundamental value is measured with noise, as in Section 2.3.3, the fundamental-scaled variable will be an imperfect proxy for the private signal, and both size (P) and a fundamental-scaled variable ($F_i - P_i$), in addition to the risk measure β, will predict future returns. To see this, consider the linear projection of R onto β_M, $P-F$ and P:

$$R = a + b^{\dagger}_{\beta}\beta_M + b^{\dagger}_{P-F}(P-F) + b^{\dagger}_{P}P + e. \tag{17}$$

The set of optimal coefficients come from the standard matrix equation (details are in the Appendix D).

Proposition 5 *The regression of the return R on β_M, the price-scaled fundamental $P-F$, and the price P yields the following set of coefficients:*

$$b^{\dagger}_{\beta} = \alpha\left(\frac{var_{OC} + cov_{OC} + K_1}{var_{OC} + K_1}\right) \tag{18}$$

$$b^{\dagger}_{P-F} = \frac{V_{\bar{\theta}}}{V_{\bar{\theta}} + V^F}\left(\frac{cov_{OC}}{var_{OC} + K_1}\right) \tag{19}$$

$$b^{\dagger}_{P} = \frac{V^F}{V_{\bar{\theta}} + V^F}\left(\frac{cov_{OC}}{var_{OC} + K_1}\right) \tag{20}$$

where

$$K_1 = \frac{V_{\bar{\theta}}V^F}{V_{\bar{\theta}} + V^F}$$

is half the harmonic mean of $V_{\bar{\theta}}$ and V^F, and $K_1 = 0$ if either $V^F = 0$ or $V_{\bar{\theta}} = 0$. Under overconfidence, the coefficients on the fundamental-scaled price and the unadjusted price level are both negative.

If individuals are on average overconfident $E[\omega^f] > 0$, equation (13) shows that cov_{OC} will be negative. Also, provided the fundamental measure that we use is not perfect (*i.e.*, $V^F > 0$), these equations show that: (1)the regression coefficient on β will be less than α (the CAPM 'market price of risk'); (2) the coefficient on size (P) will be negative; and (3) the coefficient on a $P - F$ variable (such as market-to-book) will be negative. These results are thus consistent with the evidence of Fama and French (1992).

If $V^F = 0$, then the coefficient on P_i is zero, as $F_i - P_i$ captures all variability in the expected future return (conditional on the private signal). Alternatively, if there is no variability in fundamental values across firms (if $V_{\bar{\theta}} = 0$), then P_i will be a perfect proxy for s_i, and the fundamental-scaled price variable will have no additional explanatory power for future returns. Further, if a multiple regression is run with any number of fundamental-scaled price variables, such as book-to-market and price-to-earnings ratio, and if the errors of the different fundamental proxies are imperfectly correlated, so that each proxy adds some extra information about $\bar{\theta}_i$, the coefficient on each variable should be non-zero. If the errors are independent, then the coefficients on each price regressor will be negative.

As $V_{\bar{\theta}} \to \infty$, $b_P^{\dagger} \to 0$ and $b_{P-F}^{\dagger}$ does not. Thus, if the variance of expected cash flows across securities is large relative to the noise in the fundamental proxy, the size variable becomes dominated in the multiple regression. We therefore expect that for large $V_{\bar{\theta}}$, the coefficient on size will be less significant than the coefficient on a fundamental/price variable such as book market. Intuitively, if securities have very different expected cash flows, it becomes very important to find a proxy to eliminate this effect in order to locate mispricing effects. Two recent studies find that in a multivariate regression or cross-classification that book/market is more significant than size (Fama and French (1992), Brennan, Chordia, and Subrahmanyam (1998))

Overconfidence implies that future returns should be related to market-value, but not to non-price measures of size. Non-price measures of size such as number of employees or book-value are unrelated to the error in the informed's signal e_i, and are therefore also unrelated to the future return on the security. This is consistent with the empirical findings of Berk (1995b).

Suppose that the tendency to overreact is equal for all factor and idiosyncratic signals (*i.e.*, $\omega_i^{\epsilon} = \omega_k^f = \omega$ for all i and k),[16] and that the fundamental measure is noiseless ($F_i = \bar{\theta}_i$).

[16]A sufficient condition for the market to have an equal tendency to overreact to all signals is that (1) investors are equally overconfident about all signals as defined in equation (8), and (2) the ratio of true

Equations (25) and (26) then reveal that

$$cov_{OC} = -\left(\frac{\omega}{1+\omega}\right) var_{OC}.$$

Substituting this equality into equations (18) and (19) yields the following corollary.

Corollary 3 *If the tendency to overreact is equal for all factor and idiosyncratic signals ($\omega_i^\epsilon = \omega_k^f = \omega$ for all i and k), and if the fundamental measure is noiseless, the regression coefficients in Proposition 5 are*

$$b_\beta^\dagger = \frac{\alpha}{1+\omega}, \quad b_{F-P}^\dagger = \frac{\omega}{1+\omega}, \quad \text{and } b_P^\dagger = 0.$$

As the degree of overconfidence becomes very large ($\omega \to \infty$), the coefficient on beta approaches zero, whereas the coefficient on the fundamental-scaled price approaches unity.

The limiting case of infinite ω occurs with λ^C constant and $\lambda^R \to 0$. In other words, investors perceive the signals as informative when actually they are virtually pure noise.[17] In the limit as investors become highly overconfident in this sense, their behavior becomes close to noise trading, because nontrivial price revisions are triggered by very little information. As this occurs, the coefficient on β approaches zero, and the coefficient on $F - P$ approaches 1. Intuitively, if investors push prices away from fundamentals based on pure noise alone, then $F - P$ will be equal to the sum of (1) the future price change due to the correction of the mispricing, and (2) the future price change due to risk. Since $F - P$ is a proxy for both terms, beta adds no information, and its coefficient is therefore zero.

With regard to the last point, if investors' private signals are very noisy, the statistical relationship between fundamental/price ratios and expected return will be strong, and the relation between β and expected return will be weak. Thus, the theory is consistent with the differing findings of several studies regarding the existence of a cross-sectional relation between return and β after controlling for book-to-market or for market-value (see Appendix C).

Intuitively, a high value of fundamental/price (low value of market price relative to unconditional expected terminal cash flow) could arise from adverse private information, or from high risk. In either case, price should rise (because overreaction is reversed, or

value variability to signal noise variance is constant across signals,

$$\frac{1}{V_k^{Rf}} = \frac{V_j^\epsilon}{V_j^{R\epsilon}} \text{ for all } j = 1, \ldots, N \text{ and all } k = 1, \ldots, K, \tag{21}$$

i.e., for all signals. (This is proved after the proof of Proposition 5 in the appendix.)

[17] Alternatively, λ^C could approach infinity, but this would lead to infinitely volatile prices.

because of a risk premium), but in general by different amounts. If price is low because of a risk premium, on average it should rise back to the unconditional expected terminal value. But if price is low because of adverse information, then the expected value is below the unconditional expected value. A risk measure such as β will, in general, help disentangle these two cases, so it should have incremental explanatory power.

However, when overconfident individuals trade based on pure noise ($V^C \to \infty$), the conditional expected value of θ is equal to the unconditional value. Thus, risk measures such as β provide no incremental explanatory power for future returns. In contrast, fundamental/price does have explanatory power when β is held constant, because the fundamental/price ratio reflects both the risk premium and the mispricing.

Analogous to the discussion in Subsection 2.4, it is sometimes asserted that almost any model with valuation errors will cause high fundamental/price to predict high returns. However, our analysis shows that this is not the case. The conclusion would reverse in settings where investors *under*reacted to private information (such as an *under*confidence setting). In such a setting, a low fundamental/price ratio could reflect favorable private information, so that in the long run market price needs to rise still further. Thus, the direction of the fundamental/price effect is *not* a general implication of irrationality, but an implication of a specific type of bias (overconfidence).

We have assumed that the public information signal is conclusive. However, similar results would apply with noisy public information arrival. Basically, a noisy public signal only partly corrects the initial overreaction to the private signal. So high fundamental/price still indicates undervaluation (even though the public signal on average has partly corrected the market price upwards). However, as discussed in footnote 14, in a setting with dynamic overconfidence it is possible that overreactions can temporarily continue before eventually being corrected. These temporary continuations can be triggered by noisy public signals. Thus, in a more general model it is possible that high fundamental/price ratios could predict *low* returns at short lags. However, since misvaluation must eventually be corrected, high fundamental/price ratios should still predict high returns at sufficiently long horizons.

4 Closed-End Fund Discounts and Small Firm Returns

The behavior of discounts and premiums in closed-end mutual funds has been adduced as evidence that noise trading or 'market sentiment' are important in equity markets (see De Long, Shleifer, Summers, and Waldmann (1990), and Lee, Shleifer, and Thaler (1991)). The latter paper argues that unpredictable noise trading is more important for the pricing of fund shares than for the underlying assets of the fund. Assuming that individual investors, who are more active in the markets for fund shares and small stocks than larger stocks, are prone to trading on noise (or faulty 'popular models'), this imposes non-fundamental risk on rational holders of fund shares. Closed-end fund discounts are premia for this noise trader risk.

A fund discount is similar to a market/book ratio, except that the 'book' value (net asset value) may also be influenced by overconfidence. Their finding that changes in fund discounts are correlated with contemporaneous returns on small stocks (for which individual investors are especially important) suggests that an underlying sentiment/noise factor causes individual investors to buy/sell funds and small shares together. Several studies have provided further evidence consistent with sentiment/noise effects.[18]

Since noise trading can be viewed as a special type of overconfidence, it is interesting to consider whether the intuition and predictions of the market sentiment theory will carry through when investors are overconfident about *genuine* information. Intuitively, if different overconfident traders with different information are involved in segmented markets for fund shares and for underlying assets, then a layer of mispricing risk is present at the fund level that is not present for individual stocks. Fund discounts could partly be premia for such risk, and partly reflect mispricing due to overreaction. However, since the mispricing arises from overreaction to *genuine* information, changes in fund discounts should be able to predict not just future stock performance, but also measures of fundamentals such as future accounting performance. Swaminathan (1996) finds such predictive power: "Thus, discounts on closed-end funds may provide important fundamental information that is not contained in any of the commonly used expected return variables." He interprets this evidence as tending to support a rational risk premium hypothesis more than a noise/sentiment approach. His evidence at lags of greater than one year is surprising, because high discounts predict both low future accounting profits and high future stock returns. This evidence is consistent with an overconfidence approach where genuine adverse information is associated with large discounts and low future profitability, yet high future stock returns as the market corrects its overreaction.

[18]Thompson (1978) finds that abnormal profits can be made by trading based on fund discounts. Pontiff (1997) finds that deviations of fund market value from asset value is greater for funds with portfolios that are harder to replicate and for funds with lower market values. Swaminathan (1996) finds that fund discounts forecast future small firm returns. Bodurtha, Kim, and Lee (1995) find that the stock prices of foreign country funds covary with the U.S. stock market but their net asset values do not; that changes in foreign country fund premia covary positively with the returns on small U.S. firms, and that premia are negatively correlated with future country fund stock returns.

5 Conclusion

Empirical securities markets research in the last three decades has presented a body of evidence that is not easy to explain on the basis of purely rational asset pricing theory. We have lacked a consistent alternative theory to integrate this evidence. This paper offers a theory based on well established evidence about the psychology of judgement for several anomalous findings concerning cross-sectional predictors of expected security returns. Individuals in the model are imperfectly rational, but make optimal investment decisions subject to their mistaken expectations. In contrast with several other recent papers on investors with mistaken expectations, the analysis allows for multiple securities and risk aversion, which are important for analyzing the relative ability of risk versus mispricing measures to predict future returns.

Despite the fact that the investors misperceive the covariance matrix of returns, in some cases the traditional CAPM β will predict future returns. In addition, and in contrast with the CAPM, the model implies that measures of market misvaluation will also predict returns. In the model, variables which contain market price can proxy for misvaluation. The model therefore implies that high values of price relative to fundamental measures predicts low future returns, and that this effect is incremental to the effect of risk measures such as beta. Furthermore, if the proxy for unconditional fundamental value is perfect and investors are highly overconfident (in the sense that they trade on pure noise signals that they believe are informative) then beta will have no incremental explanatory value relative to price variables such as book/market.

The noise trader approach to securities pricing is based on the recognition that there is variability in prices that seems unrelated to the arrival of valid information. We think that an important class of mistakes by investors involves the misinterpretation of *genuine* new private information. Thus, irrational errors may be correlated with fundamentals, and informed traders may not be free of bias. Further, as mentioned in the introduction, so long as rational traders are risk averse, and imperfectly rational informed traders are a substantial part of the market, prices will be influenced substantially by investors' cognitive errors.

Our approach is broadly consistent with the more traditional noise-trader approach in the limiting case where, holding constant the investors' perceived precision, the noise in the signal observed by the informed approaches infinity. This implies that a negligible informational stimulus can lead to significant trades.[19] Thus, the pure noise trader approach can

[19]However, in an explicit overconfidence model the trades of the "noise trades" are endogenously determined through optimization.

be viewed as a special kind of overconfidence model. This paper derives some implications of both overconfident noise trading, and of overconfident trading based upon non-negligible private information.

Some of the implications of our psychological approach to security pricing are consistent with casual intuition about securities market inefficiency: for example, the book/market variable in Fama and French (1992) has been widely interpreted as a possible measure of mispricing. We believe the benefits of formalizing casual ideas about mispricing in an explicit multi-security setting are as follows:

- Given the lack of any clear evidence for a consistently high level of investor rationality, it is reasonable to consider models which incorporate both risk and mispricing. We believe it is important in such a task to base model assumptions on psychological evidence. Integrating psychological misvaluation effects in an explicit asset pricing model extends to these effects the same sort of careful analytical attention that has, in numerous papers, been bestowed upon cross-sectional differences in risk. The implications of such an approach also allow the testing of alternative hypotheses about investor behavior in financial markets.

- Such an analysis shows that the effects of risk and mispricing separate additively into a 'beta' term and a set of 'mispricing' terms, where factor mispricing is inherited by securities according to their factor sensitivities.

- An explicit model allows careful analysis of the interaction of risk and mispricing in the cross-section of security returns, which leads to the result that when traders are overconfident about pure noise, fundamental/price ratios completely dominate beta. Despite the fame of the Fama and French result and the familiarity of the 'noise trader' approach, we think this story has not clearly been suggested as an explanation for the weakness of the beta effect.

- The analysis provides a conceptual basis for choosing between alternative measures of mispricing as predictors of future returns. This consists of trading off the benefit of filtering out variation in market price that arises from differences in scale rather than mispricing, versus the cost arising from errors in fundamental proxies as measures of unconditional expected value. The theory therefore offers predictions about the relative predictive power of firm 'size' (market value), variables that normalize market price by a noisy measure of fundamentals such as book value, relative measures that compare fundamental/price ratios to industry averages, and variables that use constructed accounting measures of fundamentals.

- An approach that quantifies the pricing of multiple securities when investors are imperfectly rational may be useful for the great preponderance of practitioners who do not believe markets are perfectly efficient.[20] We believe our theory offers a reassurance that it is possible to provide a logically consistent model of asset pricing in which investors make systematic errors, and in which measures of mispricing can be used to predict future returns, and thereby achieve superior investment performance.

The challenge for implementing the model empirically is to identify good proxies for security and/or factor mispricing. Our main focus in this paper has been on fundamental/price ratios as possible proxies for market mispricing. However, fundamentals such as dividends, earnings and book value are very crude proxies for unconditional expected value. An accounting index in which some of the noise is removed should be a better forecaster of future returns. Abarbanell and Bushee (1997) construct an accounting-based measure of fundamental value, and find that this predicts future returns. Frankel and Lee (1996) construct an index which includes accounting measures and analyst forecasts of accounting variables, and find that their measure dominates book/market as a predictor of future stock market returns.

Closely related to fundamental/price ratios is past stock price performance (winners versus losers). Past winners/losers should also provide a proxy for misvaluation (see, however, footnote 14). Further, insider trading may also proxy for misvaluation; Lakonishok and Lee (1998) provide evidence that imitation of insider trades for up to about two years after these are publicly disclosed is a profitable strategy. They also find that controlling for size and book/market ratios does not eliminate the ability of insider trading to predict future returns.

Another set of possible proxies for market misvaluation involve corporate actions such as aggregate new-issue versus repurchase activity. Indeed, Loughran and Ritter (1995) explicitly propose that managers time new issues in order to exploit market misvaluation.

The variables mentioned above would be expected to predict returns in almost any imperfectly rational model where prices deviate from true underlying expected value. The theory provides additional restriction because it is based on an overconfidence-induced overreaction. Thus, the predicted sign of the coefficients on fundamental-scaled price variables is negative. In contrast, a model involving only underreaction would imply the reverse, inconsistent with existing evidence. Focusing specifically on overconfidence as a source of initial overreaction provides at least one way to go further, by deriving a linear decomposition of expected returns into a risk premium and mispricing terms.

[20]Jones (1997) offers a recent 'quant' practitioner's perspective on overconfidence and market inefficiency.

If overconfidence levels can be measured, our approach offers further implications about how to predict returns in different markets. Holding constant signal accuracy, greater overconfidence leads to greater variability in price relative to unconditional fundamental measures. Thus, the variance of fundamental/price ratios across securities within an industry could proxy for overconfidence. This proxy will be more accurate if this variance is calculated as an excess over the variance in fundamental/price ratios implied by variation in betas across firms in the industry. Fundamental/price ratios should predict future returns more strongly in industries in which investors are highly overconfident about their signals. Similarly, high time series volatility of a single stock's fundamental/price ratios relative to what would be predicted based on time variation in its beta may be a measure of overconfidence about information related to that stock. We believe that an interesting direction for further research will be to identify possible measures of misvaluation and of market overconfidence.

The analysis offered here was based on a representative overconfident investor. It is likely that the relative numbers of overconfident individuals and fully rational 'arbitrageurs' varies with stock characteristics such as liquidity and firm size. Thus, explicit modeling of investor heterogeneity may offer further empirical implications. A second useful extension of the theory relates to evidence that individuals tend to be more overconfident in settings where feedback on their information or decisions is slow or inconclusive than where the feedback is clear and rapid (Einhorn (1980)). Thus, mispricing may be stronger in stocks which require more judgment to evaluate, and where the feedback on this judgment is ambiguous in the short run. This line of reasoning suggests that fundamental/price effects should be stronger for stocks that are difficult to value, such as those with high R&D expenditures or intangible assets. A third possible extension would be to allow for changes in confidence arising from biased self-attribution (as in Daniel, Hirshleifer, and Subrahmanyam (1998)). This would allow incorporation of long-term versus short-term stock price runup as separate cross-sectional predictors distinct from fundamental/price ratios. In other words, it would allow for short-term momentum as well as long-term reversal effects.

Appendices

A A Guide to the Model Notation

Symbol	Definition	Description
θ_i	$= \bar{\theta}_i + \sum_k \beta_{ik} f_k + \epsilon_i$	Security i payoff at date 2; price at date 2
μ_i^C	$\equiv E^C[\theta_i \vert \phi]$	Overconfident expected payoff based on all (public and private) information at date 1
P_i		Security Price at date 1
R_i	$\equiv \theta_i - P_i$	date 1-2 price change
R_M	$\equiv \sum_i x_j(\theta_j - P_j)$	Market Price Change
$\bar{\theta}_i$		Fundamental value, or expected security i payoff based on all public (but not private) information
$\bar{\bar{\theta}}$		Mean of $\bar{\theta}_i$ distribution
$V_{\bar{\theta}}$		Cross-sectional variance of the fundamental values
V_i^ϵ	$= E[(\epsilon_i)^2]$	Variance of Firm Specific (FS) Component for firm i
F_i	$= \bar{\theta}_i + e_i^F$	"Fundamental" measure – a noisy measure of the $\bar{\theta}_i$
V^F	$= E\left[(e^F)^2\right]$	Variance of the fundamental measure of a randomly selected security
s_i^ϵ	$= \epsilon_i + e_i^\epsilon$	Informed's Signal os FS component for firm i
s_k^f	$= f_k + e_k^f$	Informed's Signal on factor k
e_i^ϵ		Error in Signal of (FS) Component for firm i
e_k^f		k'th factor-signal error
$V_i^{C\epsilon}$	$= E_C[(e_i^\epsilon)^2]$	I's over$C$onfident assessment of i'th FS component error variance
$V_i^{R\epsilon}$	$= E_R[(e_i^\epsilon)^2]$	True (Rational) assessment of i'th FS component error variance
V_k^{Cf}	$= E_C[(e_k^f)^2]$	I's over$C$onfident assessment of the k'th factor signal error variance
V_k^{Rf}	$= E_R[(e_k^f)^2]$	True (Rational) assessment of k'th factor-signal error variance
$\lambda_i^{C\epsilon}$	$= V_i^\epsilon/(V_i^\epsilon + V_i^{C\epsilon})$	Weight placed by Over$C$onfident I on firm i specific signal
$\lambda_i^{R\epsilon}$	$= V_i^\epsilon/(V_i^\epsilon + V_i^{R\epsilon})$	Weight that would be placed by Rational's on firm i specific signal
λ_k^{Cf}	$= 1/(1 + V_k^{Cf})$	Weight placed by Over$C$onfident I on signal for factor k
λ_k^{Rf}	$= 1/(1 + V_k^{Rf})$	Weight that would be placed by Rational's on signal for factor k
S_i^ϵ	$\equiv \lambda_i^{R\epsilon} s_i^\epsilon$	Normalized signal of FS component for firm i
S_k^f	$\equiv \lambda_k^{Rf} s_k^f$	Normalized signal of factor k realization
ω_i^ϵ	$\equiv (\lambda_i^{C\epsilon} - \lambda_i^{R\epsilon})/\lambda_i^{R\epsilon}$	Informed's relative overconfidence for FS component of firm i
ω_k^f	$\equiv (\lambda_k^{Cf} - \lambda_k^{Rf})/\lambda_k^{Rf}$	Informed's relative overconfidence for factor k
α	$\equiv A\mathrm{var}^C(R_M)$	Coefficient of absolute risk aversion times the market variance
β_{iM}	$= cov(\theta_i, R_M)/var(R_M)$	price-change beta of firm i with respect to the market
$E[\beta_k], V_{\beta_k}$		the first and second moments of the loading of a randomly selected security on factor k
$E[\beta_M^C], V_{\beta_M^C}$		moments of the overconfident estimates of the market beta of a randomly selected security
$E[\omega^\epsilon], V^{\epsilon\omega}$		moments of the excess sensitivity to residual variance.
$E[\omega^f], V^{f\omega}$		moments of the excess sensitivity of prices to factor k
V_k^{Sf}		the second moment of the the normalized factor k signal
$V^{S\epsilon}$		the second moment of the the normalized residual i signal
cov_{OC}	eqn (13), page 14	Covariance between R and $P - \theta$ attributable to Is overconfidence in their private signals
var_{OC}	eqn (14), page 14	Variance of $P - \bar{\theta}$ that can be attributed to signals received by Is
$b_{P-\bar{\theta}}$	eqn (11), page 14	Coefficient in linear projection of R on $P - \bar{\theta}$
b_{P-F}	eqn (15), p. 17	Coefficient in linear projection of R on $P - F$
$b_\beta^\dagger, b_{P-F}^\dagger, b_P^\dagger$	eqns (18)-(20), page 19	Coefficients on β_M, $P - F$, and P in linear projection of R on β_M, $P - F$ and P

B Psychological Evidence of Overconfidence

This appendix cites relevant literature on the psychological patterns mentioned in the first paragraph of the introduction. Evidence that people perceive themselves as more able than they actually are, as more able than average and more favorably than they are viewed by others is found in Greenwald (1980), Svenson (1981), Cooper, Woo, and Dunkelberg (1988), and Taylor and Brown (1988). Evidence from experimental studies indicates that individuals underestimate their error variance in making predictions.[21]

Evidence of overconfidence has been found in psychologists (Oskamp 1965), physicians and nurses (Christensen-Szalanski and Bushyhead 1981, Baumann, Deber, and Thompson 1991), engineers (Kidd 1970), attorneys (Wagenaar and Keren 1986), negotiators (Neale and Bazerman 1990), entrepreneurs (Cooper, Woo, and Dunkelberg 1988), managers (Russo and Schoemaker 1992), investment bankers (Stael von Holstein 1972), market professionals such as security analysts and economic forecasters (Ahlers and Lakonishok 1983, Elton, Gruber, and Gultekin 1984, Froot and Frankel 1989, DeBondt and Thaler 1990, DeBondt 1991), and investors in experimental asset markets (Bloomfield, Libby, and Nelson 1998).

C Some Empirical Literature on Securities Price Patterns

A positive univariate relation of beta with expected returns is found in some studies and not others, depending on the country and the time period. One would expect on theoretical grounds that the incremental relation of beta to future returns after controlling for market value or fundamental/price ratios would be weaker. Empirically, an incremental effect of beta is found in some studies but not others.

Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) provide evidence of a significant positive univariate relation between security betas and expected returns. Both studies find significant time variation in this effect. Fama and French (1992) also find a positive but insignificant unconditional relation between return and market beta in a more recent sample. Internationally, Rouwenhorst (1998) finds no significant unconditional relation between average return and beta, relative to a local market index, on common stocks from 20 emerging markets. Heston, Rouwenhorst, and Wessels (1998) find some evidence of an unconditional univariate relation between market beta and future returns across stocks in 12 European countries.

[21]See Alpert and Raiffa (1982); Fischhoff, Slovic, and Lichtenstein (1977), and the discussions of Lichtenstein, Fischoff, and Phillips (1982) and Yates (1990)).

On the incremental importance of conventional risk measures versus fundamental-scaled price variables: Fama and French (1992) find that size and book/market predict future returns, and that when firms are sorted simultaneously by β and size, or by β and book/market, the CAPM β has no power to explain cross-sectional return differences. However, in contrast to the Fama and French (1992) results, Jagannathan and Wang (1996) find that the incremental effect of beta on future returns is significant when human capital is included in the definition of the market, and conditional rather than unconditional beta's are calculated. Knez and Ready (1997) present evidence that market β is priced after controlling for size if robust test statistics are used. Heston, Rouwenhorst, and Wessels (1998) find that size and international market beta are both positively associated with future returns in 12 European countries.

There is strong evidence, beginning with Banz (1981), that firm size as measured by market value predict future returns. This predictive power vanishes when size is measured by book value or other non-market measures (see Berk (1995b)).

Fama and French (1993) provide evidence that a three-factor model explains the average returns of stocks sorted on market equity and book/market ratio, which they interpret as a model of equilibrium risk premia. However, Daniel and Titman (1997) argue that the Fama and French (1993) results are also consistent with a "characteristics" model, and present evidence that, after controlling for the size and book/market ratios, returns are not related to loadings on the Fama and French (1993) factors.

Furthermore, MacKinlay (1995) finds that high Sharpe-ratios (relative to the market) can apparently be achieved with strategies based on fundamental-scaled price variables. As Hansen and Jagannathan (1991) point out, high Sharpe-ratios are only possible in a rational asset pricing model when there is highly variable marginal utility across states. Brennan, Chordia, and Subrahmanyam (1998) show that these strategies produce Sharpe ratios about three times as high as what is achievable with the market, and argue, like MacKinlay, that these are too high relative to the market Sharpe ratio to be plausible within a rational, frictionless asset pricing model.

The ability of fundamental-scaled price variables to predict cross-sectional differences in future returns is confirmed by numerous studies. Jaffe, Keim, and Westerfield (1989) find that the ratio of earnings to price has predictive power for the future cross-section of returns. Rouwenhorst (1998) finds evidence that firm size and fundamental-scaled price measures predict returns on common stocks from 20 emerging markets. He finds little correlation between book/market- and size-sorted portfolios across 20 countries.

Earlier studies which found evidence of size and market-to-book effects were Banz (1981), Basu (1983) (size); Stattman (1980), Rosenberg, Reid, and Lanstein (1985), and DeBondt and Thaler (1987); (book/market). Also, Davis (1994) finds that the book/market effect is present in pre-COMPUSTAT US common-stock returns, and Fama and French (1997) find evidence of an international book/market effect in the 1975-95 period. Fama and French found that other fundamental-scaled price variables also had power to forecast the future cross-section of returns, but that these other variables had no predictive power over and above book/market and size.

On the closed-end-fund puzzle see Lee, Shleifer, and Thaler (1991), Thompson (1978), Swaminathan (1996) and Bodurtha, Kim, and Lee (1995). Also, Pontiff (1997) finds that discounts and premia are larger when it is more costly to arbitrage between the fund and its portfolio.

D Proofs

Proof of Lemma 1: The expected value of the factors and idiosyncratic terms are

$$E^C[f_k|s_k^f] = \lambda_k^{Cf} s_k^f \quad \text{and} \quad E^C[\epsilon_i|s_i^\epsilon] = \lambda_i^{C\epsilon} s_i^\epsilon.$$

Consider security price-changes between time 1 and 2. Combining (3), (5), and the above shows that price changes are also given by a factor structure:

$$R_i \equiv \theta_i - P_i = \sum_{k=1}^{K} \beta_{ik}(f_k - E^C[f_k|s_k^f]) + (\epsilon_i - E^C[\epsilon_i|s_i^\epsilon]) + A\mathrm{cov}^C(\theta_i, R_M). \tag{22}$$

Similarly, market price changes can be written as:

$$R_M \equiv \theta_M - P_M = \sum_{k=1}^{K} \beta_{Mk}(f_k - E^C[f_k|s_k^f]) + \sum_{i=1}^{N} \bar{x}_i(\epsilon_i - E^C[\epsilon_i|s_i^\epsilon]) + A\mathrm{var}^C(R_M), \tag{23}$$

where $\beta_{Mk} = \sum_{i=1}^{N} \bar{x}_i \beta_{ik}$ is the loading of the market on the k'th factor.

To calculate date 1 overconfident and rational security betas with the market portfolio, we will need to calculate the conditiaonl variances of the terms in equation 23. Overconfident investors believe that the conditional variances of the factor and idiosyncratic realizations between 1 and 2 are given by:[22]

$$var^C(f_k|s_k^f) = \frac{V_k^{Cf}}{1+V_k^{Cf}} = \lambda_k^{Cf} V_k^{Cf}; \qquad var^C(\epsilon_i|s_i^\epsilon) = \frac{V_i^\epsilon \, V_i^{C\epsilon}}{V_i^\epsilon + V_i^{C\epsilon}} = \lambda_i^{C\epsilon} V_i^{C\epsilon}.$$

The rationally assessed conditional variance of the factors and idiosyncratic terms are:

$$var^R(f_k|s_k^f) = \frac{V_k^{Rf}}{1+V_k^{Rf}} = \lambda_k^{Rf} V_k^{Rf}; \qquad var^R(\epsilon_i|s_i^\epsilon) = \frac{V_i^\epsilon \, V_i^{R\epsilon}}{V_i^\epsilon + V_i^{R\epsilon}} = \lambda_i^{R\epsilon} V_i^{R\epsilon}.$$

[22] The precision of the posterior distribution of f is equal to the sum of the precision of the prior (which is 1 here), and the signal precision.

The rationally assessed market β of the i'th security is defined as β^R_{iM}.

To determine when the rationally assessed and overconfidence assessed beta (β^C_{iM}) will be equal, we first, consider the market betas of the k'th factor, which we denote β^R_{kM} and β^C_{kM}:

$$\beta^R_{kM} = \frac{cov^R(f_k, R_m)}{var^R(R_M)} = \frac{\beta_{Mk}\lambda^{Rf}_k V^{Rf}_k}{\sum_{l=1}^K \beta^2_{Ml}\lambda^{Rf}_l V^{Rf}_l + \sum_{i=1}^N \bar{x}^2_i \lambda^{R\epsilon}_i V^{R\epsilon}_i}$$

and

$$\beta^C_{kM} = \frac{cov^C(f_k, R_m)}{var^C(R_M)} = \frac{\beta_{Mk}\lambda^{Cf}_k V^{Cf}_k}{\sum_{l=1}^K \beta^2_{Ml}\lambda^{Cf}_l V^{Cf}_l + \sum_{i=1}^N \bar{x}^2_i \lambda^{C\epsilon}_i V^{C\epsilon}_i}$$

Now, we can write the overconfident and rational market betas of the i'th security as

$$\beta^C_{iM} = \frac{\sum_{l=1}^K \beta_{il}\beta^C_{lM} + \bar{x}_i \lambda^{C\epsilon}_i V^{C\epsilon}_i}{var^C(R_M)}$$

and

$$\beta^R_{iM} = \frac{\sum_{l=1}^K \beta_{il}\beta^R_{lM} + \bar{x}_i \lambda^{R\epsilon}_i V^{R\epsilon}_i}{var^R(R_M)}.$$

The ratio of these two quantities is

$$\frac{\beta^C_{iM}}{\beta^R_{iM}} = \frac{\sum_{l=1}^K \beta^2_{Ml}\lambda^{Rf}_l V^{Rf}_l + \sum_{i=1}^N \bar{x}^2_i \lambda^{R\epsilon}_i V^{R\epsilon}_i}{\sum_{l=1}^K \beta^2_{Ml}\lambda^{Cf}_l V^{Cf}_l + \sum_{i=1}^N \bar{x}^2_i \lambda^{C\epsilon}_i V^{C\epsilon}_i} \times \frac{\sum_{l=1}^K \beta_{il}\beta_{Ml}\lambda^{Cf}_l V^{Cf}_l + \bar{x}_i \lambda^{C\epsilon}_i V^{C\epsilon}_i}{\sum_{l=1}^K \beta_{il}\beta_{Ml}\lambda^{Rf}_l V^{Rf}_l + \bar{x}_i \lambda^{R\epsilon}_i V^{R\epsilon}_i}.$$

A sufficient condition for this ratio to be equal to one is that the degree of overconfidence, defined as the ratio of posterior variances (i.e., the ratio of the conditional variances of the f's or the ϵ's under overconfidence and rationality)

$$\rho_k = \frac{\lambda^{Cf}_k V^{Cf}_k}{\lambda^{Rf}_k V^{Rf}_k} \text{ for factors, or } \rho_i = \frac{\lambda^{C\epsilon}_i V^{C\epsilon}_i}{\lambda^{R\epsilon}_i V^{R\epsilon}_i} \text{ for idiosyncratic terms} \tag{24}$$

be equal for all factors and all idiosyncratic terms.

We can substitute for the λ's to rephrase the numerator of ρ_k as

$$\frac{1}{\frac{1}{V^{Cf}_k} + 1},$$

which is monotonically increasing in the overconfidently-assessed noise variance. Since ρ_k is monotonically increasing in the overconfidently noise variance, in this sense equal ρ's imply equal overconfidence. A similar point applies to ρ_i.

Proof of Proposition 2 Since the signals in the model are mean zero, by equation (10), $E[R] = -E[P-\bar{\theta}] = \alpha E[\beta_M]$. From equations (9) and (10) (and applying the law of iterated expectations),

$$cov(R, P-\bar{\theta}) = -E\left[\omega^{\epsilon}(1+\omega^{\epsilon})(S^{\epsilon})^2 + \sum_{k=1}^{K}\beta_k^2\omega_k^f(1+\omega_k^f)(S_k^f)^2 + \alpha^2\beta_M^2\right] + \alpha^2 E[\beta_M]^2..$$

Now, applying $V^{\epsilon\omega} \equiv var(\omega^{\epsilon}) = E[(\omega^{\epsilon})^2] - (E[\omega^{\epsilon}])^2$ and $V_{\beta_M^C} \equiv E\left[\left(\beta_M^C\right)^2\right] - (E[\beta_M^C])^2$ we have

$$\begin{aligned} cov(R, P-\bar{\theta}) &= -\alpha^2 V_{\beta_M^C} + cov_{OC} && (25)\\ var(P-\bar{\theta}) &= \alpha^2 V_{\beta_M^C} + var_{OC} && (26) \end{aligned}$$

where cov_{OC} and var_{OC} are as defined in the text.

Proof of Proposition 4 As in Proposition 3, denote the R^2 from the regression of the return R on the fundamental scaled measure $F-P$ as R^2_{F-P}. Further, denote the R^2 from the regrssion of the return R on P to be R^2_P. Note from (25) and (26) that

$$\begin{aligned} cov(R,P) &= -\alpha^2 V_{\beta_M^C}\\ cov(R,P-F) &= cov_{OC} - \alpha^2 V_{\beta_M^C}\\ var(P) &= var_{OC} + \alpha^2 V_{\beta_M^C} + V_{\bar{\theta}}\\ var(P-F) &= var_{OC} + \alpha^2 V_{\beta_M^C} + V^F \end{aligned}$$

From the above we have that

$$\frac{R^2_{P-F}}{R^2_P} = \frac{var_{OC} + \alpha^2 V_{\beta_M^C} + V_{\bar{\theta}}}{var_{OC} + \alpha^2 V_{\beta_M^C} + V^F} \times \frac{(cov_{OC} - \alpha^2 V_{\beta_M^C})^2}{\alpha^2 V^2_{\beta_M^C}}$$

Note that cov_{OC} is negative so long as $E(\omega^f)$ and $E(\omega^{\epsilon})$ are positive. Further, if cov_{OC} is negative, the above ratio is greater than unity $V_F < V_{\bar{\theta}}$.

Proof of Proposition 5 Let the vector $X \equiv [\beta_M, P-F, P]$ and define

$$\Sigma_{YX} \equiv [\text{cov}(R,\beta_M),\ \text{cov}(R,P-F),\ \text{cov}(R,P)].$$

Further, let Σ_{XX} denote the variance-covariance matrix of $\beta_M, P-F$ and P. Then the OLS predictor of R is given by $\Sigma_{YX}\Sigma_{XX}^{-1}X$. The vector of regression coefficients in (17) can therefore be written as

$$[b_{\beta}^{\dagger},\ b_{P-F}^{\dagger},\ b_P^{\dagger}] = \Sigma_{YX}\Sigma_{XX}^{-1}. \qquad (27)$$

Some of the covariances and variances required to calculate Σ_{YX} and Σ_{XX}, were given in the proof of Proposition 4. The additional variances and covariances, from (25) and (26), are given by:

$$\begin{aligned} var(\beta_M^C) &\equiv V_{\beta_M^C} \\ cov(R,\beta) &= \alpha V_{\beta_M^C} \\ cov(\beta, P-F) &= -\alpha V_{\beta_M^C} \end{aligned}$$

Explicitly calculating these coefficients and substituting into (27) yields the expressions in Proposition 5.

Proof of 'sufficient condition' for equal ω's given below Corollary 3: The quantity ρ can be rewritten as:

$$\rho_k = \frac{V_k^{Cf}}{V_k^{Rf}} \frac{\frac{1}{V_k^{Rf}}+1}{1+\frac{V_k^{Cf}}{V_k^{Rf}}} \text{ for factors, or } \rho_i = \frac{1+\frac{V_i^\epsilon}{V_i^{R\epsilon}}}{1+\frac{V_i^{C\epsilon}}{V_i^{R\epsilon}}} \text{ for idiosyncratic terms.}$$

Holding constant the ratios $1/V_k^{Rf}$ and $V_i^\epsilon/V_i^{R\epsilon}$ across all factor and idiosyncratic signals, ρ is monotonically strictly decreasing in V_k^{Cf}/V_k^{Rf} and $V_i^{C\epsilon}/V_i^{R\epsilon}$. So equality of the ρ's implies equality of the V^C/V^R ratios.

The ω's can be rewritten as

$$\omega_k = \frac{1-\frac{V_k^{Cf}}{V_k^{Rf}}}{\frac{1}{V_k^{Rf}}+\frac{V_k^{Cf}}{V_k^{Rf}}} \text{ for factors, or } \omega_i = \frac{1-\frac{V_i^{C\epsilon}}{V_i^{R\epsilon}}}{\frac{V_i^\epsilon}{V_i^{R\epsilon}}+\frac{V_i^{C\epsilon}}{V_i^{R\epsilon}}} \text{ for idiosyncratic terms,} \qquad (28)$$

so equality of the ratios $1/V_k^{Rf}$ and $V_i^\epsilon/V_i^{R\epsilon}$ across all signals and equality of the V^C/V^R ratios implies equal ωs.

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