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A NEW APPROACH TO NONLINEAR STRUCTURAL MODELING BY USE OF CONFIRMATORY MULTIDIMENSIONAL SCALING

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ABSTRACT

A new approach to structural modeling is presented. The confirmatory multidimensional scaling method upon which it is based handles nonlinear relationships in the sense that no functional form is specified. Further, no assumptions regarding independence of observations, variable distributions, sample size, or level of measurement are necessary. Without the restrictions of traditional statistical assumptions, a methodology for testing convergent, discriminant, and nomological validity, along with estimation procedures for structural parameters, epistemic relationships, and measurement errors, is developed.

INTRODUCTION

The decade following the introduction of nonmetric multidimensional scaling (NMDS) by Shepard (1962), saw numerous marketing applications utilizing a variety of algorithms (Green and Rao 1972). Major advantages of the nonmetric scaling methods over traditional metric models of analysis were that it was not necessary to specify a specific functional form such a linearity and that ordinal input data were sufficient. Another advantage of the nonmetric methods over previously fully nonmetric methods was that the nonmetric input produced metric output.

Yet nonmetric MDS has several shortcomings that have limited its use in recent years, the most serious of which is that the analysis cannot be effectively guided by substantive theory. There are also problems of local optima and the metric determinacy of the solution (which depends on the number of points to be scaled relative to the number of dimensions necessary to represent the data). Further, it has not been possible to use statistical theory in the evaluation of results. Hence, even in the case where a global optimum has been reached, one is still faced with a subjective evaluation with respect to the extent to which the obtained results are consistent with some a priori theory (Lingoes 1980).

As a result of these shortcomings, which have been discussed in the marketing literature by Green and Carmone (1970), Green and Rao (1972, 1977), and Green and Wind (1973), NMDS has been used essentially as a descriptive tool. For purposes of theory testing and statistical inference, other models, such as structural equations, are more useful. Because these models can analyze systems of relationships, estimate and remove errors in measurement, handle abstract or unobserved variables, and confront as well as combine empirical data with theoretical knowledge (Fornell 1982a), they are very powerful tools of analysis.

Before the development of structural equation models such as Jöreskog's LISREL and Wold's PLS, researchers had to settle for analysis techniques that incorporated some but not all of these capabilities. For example, multiple regression includes a theory for confronting data with a priori hypotheses via statistical testing, but assumes perfect measurement of predictor variables; traditional principal components can model unobservables but is essentially a descriptive tool without inferential powers.

This paper presents a new analysis methodology that not only overcomes the shortcomings of traditional NMDS but also offers an alternative or complement to existing structural equation models. Recent advances by Carroll, Purzansky, and Kruskal (1980), Bloxom (1978), Lee and Bentler (1980), Borg and Lingoes (1980), and Lingoes and Borg (1978) now make it possible to impute theoretical knowledge in the empirical analysis and to use statistical theory for hypothesis testing in a confirmatory MDS analysis. Fornell and Denison (1981) have shown that confirmatory NMDS can be used to assess convergent, discriminant, and nomological validity in a manner that is similar to the LISREL approach. In both cases constraints are imposed on a model according to certain validity criteria. The resulting and constrained model is then compared to a model without validity constraints. If the fit to the data of the constrained model is significantly worse than the fit of the unconstrained model, it is concluded that the difference in fit cannot be attributed to chance and the model is rejected.

The purpose of this paper is to develop the NMDS methodology further and demonstrate not only that it can handle systems of relationships, impute a priori (theoretical) knowledge, and subject the result to statistical testing, but that it can also deal with abstract variables and measurement error. In addition to incorporating a priori knowledge by imposing external constraints and making inferential statements about the results, a new approach to estimating measurement errors and structural parameters will be suggested.

Consequently, it will be argued that NMDS now offers a viable alternative to better known structural equation models. Its basic appeal is that it is virtually free of assumptions regarding level of measurement, specification of functional form, independence of observations, sample size, and variable distributions. It also offers a flexible method for determining the degree to which validity considerations are met by the data.

Following a brief and nontechnical introduction of confirmatory nonmetric multidimensional scaling, we present our approach to the analysis of unobservables with measurement error in the context of MDS. Subsequently, we illustrate the approach with empirical data.

CONFIRMATORY NONMETRIC MULTIDIMENSIONAL SCALING

The nonmetric MDS approach to the analysis of data structure rests on two basic assumptions: relationships between variables may be interpreted as distances between points in a multidimensional space; and proximity data can be taken as a basis for establishing a set of order relations on interpoint distances. The overall objective is to find a configuration whose rank order of ratio-scaled distances best reproduces the rank order of the input data in the fewest number of dimensions.

The interpretational difficulties posed by local optima, indeterminacy, and lack of statistical inference are overcome by confirmatory MDS, which makes it possible to impose external constraints on the solution. These external constraints may be derived from measurement requirements and/or substantive theory. In the case of multiple local minima that are not significantly different in terms of their ability to represent the data, the "preferred" configuration would be one that best satisfies <u>both</u> external constraints and data fit. Hotelling's t-test has been adapted (Lingoes and Borg 1980) as a means of comparing two configurations of points as representations of the same proximity matrix. Thus, one can express the difference between a configuration with and without external constraints in terms the the probability that both represent the same set of proximities presented in the original data.

There are several computer algorithms available for confirmatory MDS. In this paper we will use the Constrained/Confirmatory Montone Distance Analysis (CMDA) developed by Borg and Lingoes (1980). This algorithm constrains distances in an NMDS configuration to satisfy ordinal specifications between points by use of a penalty function. It differs from traditional MDS in that the minimization of the loss function is conditional upon target matrices (for a detailed discussion, see Borg and Lingoes 1980). However, the methodology we propose and the analyses we perform are not specific to this algorithm. MDSCAL-5 and KYST should produce equivalent results (see Carroll, Pruzansky, and Kruskal 1980). In addition, other algorithms are available for different types of constraints. For example, CANDELINC (Carroll, Pruzansky, and Kruskal 1980) and the technique of Bentler and Weeks (1978) impose linear constraints. An approach by Bloxom (1978) considers equality constraints. Finally, Lee and Bentler (1980) have developed a method that obtains solutions subject to nonlinear relations.

STRUCTURAL MODELING WITH CONFIRMATORY MULTIDIMENSIONAL SCALING

Much of the logic behind the analysis covariance structures derives from the multitrait-multimethod matrix of Campbell and Fiske (1959). Most analysis procedures can be seen as operationalizations of two fundamental principles: Multiple measures of the same construct must converge within as well as discriminate between the different constructs under consideration.

In addition, approaches to structural modeling include a system for imposing constraints implied not only by meausrement theory, but by substantive theory as well. That is, structural modeling specifies a methodology for determining which properties of the constraints constitute convergent, discriminant and nomological validity, along with estimation procedures for assigning a numerical value to the relationships between indicators and constructs, and to the relationships between constructs. These two steps (imposing constraints that operationalize the three types of validity and estimating the indicator-construct and construct-construct links) have yet to be explored in connection with multidimensional scaling. The remainder of this section of the paper develops an approach to accomplish this.

Convergent and discriminant Validity

One schema for imposing a set of order relations on an MDS solution can be derived within the framework of facet theory (Guttman 1959; Borg 1977). Consider first the requirements for convergent and discriminant validity, where the former is the degree to which two or more attempts to measure the same construct are in agreement and the latter is the degree to which a construct differs from other constructs. For the sake of simplicity, assume that there are two constructs, each represented by a number of indicators. In the terminology of MDS, a construct is a region and its indicators are points in that region. If R_1 and R_2 represent two separate regions, a definitional mapping schema for clustering points according to convergent-discriminant requirements can be expressed as:

[1] Each point of R_1 is closer to $\left\{\begin{matrix}\text{all}\\\text{some}\end{matrix}\right\}\begin{matrix}a_1\\a_2\end{matrix}$ points in R_1 than it is to $\left\{\begin{matrix}\text{all}\\\text{some}\end{matrix}\right\}\begin{matrix}b_1\\b_2\end{matrix}$ points of R_2.

Constraint conditions $a_1\ b_1$, for example, require each point within region 1 (or, stated differently, each indicator of construct 1) to be closer to all other points within that region than to any other point within region 2. Similarly, each point within region 2 should be closer to all other points within region 2 than to any point within region 1. $a_1\ b_1$ represents the most stringent form of convergent-discriminant validity, while the other possible constraint conditions ($a_1\ b_2$, $a_2\ b_1$, and $a_2\ b_2$) are operationalizations of weaker forms.[1]

Nomological Validity

Facet theory can also be used to derive a schema for nomological validity criteria. Nomological validity is concerned with the extent to which predictions from the theory embodied by the model are confirmed. In other words, the constraint conditions relative to convergent-discriminant validity are measurement criteria, while the constraint conditions for nomological validity are more concerned with the underlying substantive theory. For three regions, we operationalize the nomological criteria as:

[2] Each point of R_1 is closer to $\left\{\begin{matrix}\text{all}\\\text{some}\end{matrix}\right\}\begin{matrix}c_1\\c_2\end{matrix}$ points of R_2 than to $\left\{\begin{matrix}\text{all}\\\text{some}\end{matrix}\right\}\begin{matrix}d_1\\d_2\end{matrix}$ points of R_3, where R_2 is a region that is adjacent to R_1, and R_3 is a region that is distant from R_1.

For example, the constraint condition c_1 d_1 requires that each point of region 1 be closer to all points within region 2 than to any point within region 3. This operationalizes testing criteria for a theory postulating that consturct 1 is more closely related to construct 2 than it is to construct 3. As in the convergent-discriminant case, other possible conditions (c_1 d_2, c_2 d_1, and c_2 d_2) provide operationalizations of weaker forms of nomological validity.

Combining the mapping schemas for clustering (convergent-discriminant validity) and for proximity between regions (nomological validity) generates the matrix of constraints presented in Figure 1.

Parameter Estimation

Given the mapping of items in a geometrical space of the smallest dimensionality, there are many ways of assigning a numerical value to the relationships between items. Not only are there several possible distance functions (e.g., Euclidian, city block, Minkowski's p-metric), there are also different ways of conflating several distances into a single estimate. The approach we will describe addresses the problem of random measurement error and is able to handle abstract unobserved variables. It draws upon classical measurement theory, is simple to use, and is not statistically advanced.

If measurement errors are random, the expected value of the indicators of a construct will approximate the true value of the construct. This is calculated by using the Euclidian distance function to find the mean value on each dimension for the points in the relevant region, or

$$[3]\quad C_{ik} = \frac{\sum_{j=1}^{n} x_{ijk}}{n_i}$$

where:

C_{ik} is the projection of the i:th centroid in the k:th dimension

x_{ijk} is the projection of the j:th indicator of the i:th construct in the k:th dimension, and

n_i is the number of indicators in the i:th construct.

The relationships between constructs are estimated at the Euclidian distances between centroids:

$$[4] \quad d_{ij} = \left[\sum_{k=1}^{R} \left(C_{ik} - C_{jk} \right)^2 \right]^{1/2},$$

where:

d_{ij} is the distance between the i:th and the j:th constructs,

R is the number of dimensions,

C_{ik} is projection of the i:th centroid in the k:th dimension, and

C_{jk} is projection of the j:th centroid in the k:th dimension.

According to classical measurement theory, measurement error is equal to the difference between observed and true values. Having defined the true value as the centroid in equation 3, we may write

$$[5] \quad e_{iq} = \left[\sum_{k=1}^{R} \left(C_{ik} - X_{ikq} \right)^2 \right]^{1/2},$$

where:

e_{iq} is the distance between the i:th construct and its q:th indicator, and

X_{ikq} is the projection of the q:th indicator of the i:th construct in the k:th dimension.

Figure 2 illustrates indicator-construct and construct-construct relationships in a two-dimensional space. The distances between the centroids

($X_a - X_b$, $X_a - X_c$, $X_b - X_c$) are estimates of structural parameters. The distance between a centroid and its corresponding indicators (e.g., $X_a - A_1$, $X_a - X_2$, $X_a - A_3$) is an estimate of the degree to which an indicator represents the construct.

Since the interpoint distances in MDS are an inverse monotone function of the corresponding correlation coefficients, the interpretation of the distance between a regional centroid and a point in that region as relative measurement error is analogous to the conceptualization of measurement error in LISREL and PLS.

A PROPOSED ANALYSIS STRATEGY

Having defined the criteria for evaluating measurement quality and testing theory, and the procedure for parameter estimation, we now turn to the analysis strategy. At the outset, it may be helpful to establish two fundamental principles.

First, in simultaneous analysis procedures where parameter estimates may change throughout the model given a change in a part of the model (deletion or addition of a variable, a new constraint, etc.), it makes little sense to use a sequential assessment of validity. For example, it is entirely possible that convergent-discriminant validity may be satisfactory in a confirmatory factor analysis model but would fail in the context of nomological validity where the model is extended to structural relationships among the factors. A similar contention can be made for validity testing within CMDA. Thus, we propose that the validity testing should be conducted simultaneously rather than sequentially, i.e., both measurement and theory should be evaluated within the same model.

Second, there are no fixed or absolute criteria for determining whether or not a theory is false. The same is true for validity assessment of measurement (cf. Campbell and Fiske 1959). What is acceptable in one study may be unacceptable in another. Moreover, what is acceptable according to one method may be unacceptable according to another. In the final analysis, evaluation of theory and measurement quality is determined by subjective judgment. What we will propose is an approach to simultaneously assess theory and measurement and to provide objective statistics upon which the subjective judgments can be made. Our clustering schemas provide a means for assessing the <u>degree</u> of validity.

As with traditional theory testing, the analyst can have <u>predetermined</u> significance criteria. Referring to Figure 1, the validity requirement may, for example, be set to a_1 b_1 c_2 d_1 at a signifiance level of .10. Thus, if the difference betwen a solution with the external constraints a_1 b_1 c_2 d_1 and an unconstrained configuration is insignificant ($p > .10$), it is concluded that the validity is acceptable.

An alternative to use of predetermined criteria is use of the <u>strongest possible test</u>. Again referring to Figure 1, this analysis strategy begins by subjecting the data to the constraint condition specified by a_1 b_1 c_1 d_1 (the upper left-hand cell in Figure 1). If the probability of a solution with these constraints is unlikely, the analysis proceeds to less stringent measurement requirements (by moving horizontally in Figure 1) and/or to less stringent theoretical requirements (by moving vertically in Figure 1). The analysis continues until a probable solution has been found. The constraints associated with this solution measure the degree of convergent, discriminant, and nomological validity that is met by the data.

Evaluative Statistics

In determining whether or not a model with constraints fits the data, several statistics are available. The most important of these is a t-test based on Hotelling's (1940) model for determining regression predictor effectiveness. The null hypothesis is that there is no difference between the constrained and unconstrained solutions with respect to their representations of the raw order relations in the data. Thus, similar to LISREL solutions, CMDA solutions with different constraints can be evaluated statistically.

In applying Hotelling's model to CMDA, the estimated distances are the predictor variable and the original ordering of the proximity data is the criterion variable. The test considers whether the distances obtained under a particular constraint are as effective as predictors as are the distances obtained in an unconstrained solution. Although Hotelling's test assumes interval scaling, this problem is overcome by an isotonic transformation which linearizes the relationship. Further, simulation studies (Lingoes and Borg 1980) demonstrate that the test is insensitive to departures from normality in the dependent variable as well as to violations of the independence assumptions.

Other statistics which help distinguish the difference between CMDA solutions are the coefficient of alienation (Guttman 1964), the rank order correlation between the order of distances in a constrained solution and the order of distances in an unconstrained solution, and the rank order correlation between the order of distances in a constrained solution and the order of distances in the original data. These statistics are essentially descriptive, although differences in the size of the rank-order correlations might conceivably be evaluated through Fisher's Z transformation, which is robust against departures from normality and homoscedasticity.

ILLUSTRATION

We now present an illustration of structural modeling via multidimensional scaling. The CMDA method is used in tests of both convergent-discriminant and nomological validity. For comparison purposes, a parallel set of analyses using LISREL is also presented. Both models produce similar conclusions regarding overall structure, but lead to differences in individual parameter estimation. We discuss these differences in connection with the strengths and weaknesses of each method.

Nonmetric confirmatory multidimensional scaling can be used for many different types of studies. It is particularly useful in analyses where the researcher can impute a priori knowledge not only according to indicator-construct relationships (measurement), but also according to the relative strength of relationships between constructs (substantive theory). There are many theories/models in marketing that specify such relationships. One example is the model presented by Hauser and Urban (1977) in the area of consumer choice, which postulates a sequence of related constructs: perception —> preference —> choice. A test of this structure would require the distance between preference and perception to be less than the distance between perception and choice. The latter distance should also be greater than the distance between preference and choice.

In addition to hierarchy-of-effects models, hypotheses involving intervening variables are also well suited to confirmatory multidimensional scaling. It is also possible to evaluate theory when a priori knowledge about construct-construct relationships is lacking. This would, of course, be a weaker test since the constraint conditions derive solely from measurement considerations (i.e., convergent-discriminant validity).

The example used in this illustration is one in which relative distances between constructs are suggested by theory. The theory concerns organizational participation and is applied to consumer affairs influence in marketing decisions (Fornell 1976). Let us briefly describe the data and the hypotheses.[3]

One way for consumers to affect the behavior of a firm is to voice their concerns to an organizational boundary unit (Adams 1976) such as a corporate department for consumer affairs. This department may have access to different types of organizational communication channels which it can utilize to represent the consumer in marketing decision making. The theory postulates a link between a consumer affairs department's channel utilization and its active participation in marketing decisions. A second link concerns the veto power held by the consumer affairs department. Veto power is the strongest form of influence and is hypothesized to operate through active participation. Consequently, there are three constructs: channel utilization, active participation, and veto power. Nomological validity implies that the distance between channel utilization and veto power is greater than the distance between active participation and veto power.

Possession and utilization of communication channels were measured by three indicators: (1) the number of planning committee memberships held by consumer affairs, (2) the extent to which consumer affairs was involved in coordinating the activities of other departments, and (3) the extent to which consumer affairs was involved in educating company personnel in consumer matters (both of the last two items were measured on a five-point scale). Participation was measured by four composite indices representing active involvement in (1) new product development, (2) product management, (3) communication/advertising, and (4) pricing. Veto power was measured in a similar manner for the same decision areas. Thus, there are three indicators of channel utilization (x_1, x_2, x_3), four indicators of active participation (y_1, y_2, y_3, y_4), and four indicators of veto power (y_5, y_6, y_7, y_8). The correlation matrix and Cronbach's alpha for the composite variables are presented in Table 1.

The correlation matrix was transformed to a rank-order matrix of order relations and used as input into a multidimensional scaling algorithm whose solution provides a baseline model and a determination of the minimum dimensionality needed to represent the data. This part of the analysis used the MINISSA-1 algorithm (Lingoes 1973), which produced the three-dimensional configuration shown in Figure 3.

The three dimensional solution fits well. As shown in Figure 3, the coefficient of alienation (K) is .08, indicating that the order of the correlations can be represented by distances in three orthogonal dimensions with very few errors. In contrast to factor analysis, the dimensions are not interpreted; rather, regions or clusters of points represent constructs.[4] Thus, there may be more constructs than dimensions.

Inspection of the configuration in Figure 3 reveals no immediate pattern of points; certainly there is no evidence of the existence of three distinct constructs. The question now is: can we alter the solution to conform with validity criteria without significantly sacrificing goodness of fit (i.e., the order of interpoint distances)?

The most stringent test of convergent-discriminant validity implies that each indicator-point of channel utilization is closer to all other indicator-points of channel utilization than to any indicator-points of active participation and veto power. In addition, the indicator-points of active participation and veto power must conform to corresponding requirements. The most stringent test of nomological validity implies that each indicator-point of channel utilization must be closer to each indicator-point of active participation than to any indicator-point of veto power. Indicator-points of active participation must lie at a distance intermediate between channel utilization and veto power, and indicator-points of veto power must be closer to all indicator-points of active participation than to any indicator-points of channel utilization. The simultaneous test of all validity criteria according to the most stringent requirements is defined by a_1 b_1 c_1 d_1 in Figure 1. The results are presented in Figure 4.

The four statistics described earlier can be used to evaluate how well the data fit the model. p is the probability associated with Hotelling's t-test. In this case, $p < .000$ and the null hypothesis that there is no difference between the constrained solution and the unconstrained solution (Figure 3) is rejected. That is, the constrained model has a significantly inferior fit. K, the coefficient of alienation, is also larger. R_1 is the rank-order correlation between the order of the distances in the constrained solution and the order of the original data. R_2 is the rank-order correlation between distances in the constrained solution and distance in the unconstrained solution. These statistics indicate that the constraints have significantly altered the interpoint distances of the original input matrix as well as those of the unconstrained solution.

In sum, the four statistics suggest that the stringent validity constraints alter the original configuration substantially. Not only is the constrained solution highly improbable; it also introduces a good deal of alienation for a three-dimensional solution. A similar conclusion follows from the validity assessment via LISREL. The results of this analysis are presented in Figure 5. The chi-square is high, indicating a poor fit and an improbable model.

Since it is apparent that the stringent validity constraints cannot be satisfied, let us examine weaker criteria. Table 2 presents the statistics for a subset of the cells in Figure 1. As can be seen, all configurations resulting from constraints stronger than $a_1\ b_2\ c_2\ d_2$ are highly improbable, whereas other constraints are easily satisfied. Given the criterion that one should select the model that best satisfies measurement and theory considerations without significantly worsening data fit, it is clear that the $a_1\ b_2\ c_2\ d_2$ configuration should be selected for further analysis. The results, including parameter estimates, are given in Figure 6.

From the clustering schemas presented earlier we can determine the degree of validity met by the data. Specifically, we have met the constraints that each indicator-point of channel utilization is closer to all other indicator-points of channel utilization than it is to some indicator-points of active participation and veto power, and that each indicator-point of channel utilization is closer to some indicator points of active participation than to some indicator-points of veto power.

Thus, the CMDA analysis suggests that there is some degree of validity for our model. A nested LISREL model supports this conclusion. Since the two-stage model with three constructs (Figure 5) exhibited a very large chi-square, it was compared to a model postulating that all indicators belong to a single factor. If this model is a probable representation of the data, it means that discriminant validity (and, by implication, nomological validity) must be rejected. The results are shown in Figure 7, which depicts perfect correlations among the constructs (which is, of course, the same as constructing a single-factor model). The chi-square is higher than for the two-stage, three-construct model. According to the likelihood ratio test, it is clear that the two-stage, three-construct model represents a significant improvement over the one-factor model.

CONCLUSION

Both LISREL and CMDA suggests a similar overall interpretation. CMDA specifies exactly what constraints are met by the data, while LISREL indicates that the model in Figure 5 has a significantly better fit than the model in Figure 7. When all elements are specified according to a priori theory, both models suggest that validity has not been met. Only when we test by weaker operationalizations of validity measurement is the theory supported by the data.

However, if we attempt to isolate the location of errors in the model, LISREL and CMDA do, not surprisingly, provide different conclusions. Aside from pinpointing y_7 as having the largest measurement error (.77 in CMDA, see Figure 6 and $1-.42^2 = .82$ in LISREL, see Figure 5) the distribution of relative error size does not correspond across the two models. There are several reasons for this divergence. First, the models are not really compatible: the LISREL model is highly improbable (p = .000), whereas the CMDA model is able to recover the ordering of the original data quite well (p = .58). Second, the models have different objectives and different assumptions. LISREL attempts to account for input data in the form of a variance-covariance matrix and assumes multinormality, large samples, independent observations, and linear relationships. CMDA attempts to account for input data in the form of a proximity matrix and makes no assumptions except the requirement of monotonicity.

Further, the indicator-construct links are estimated via very different procedures. An unobserved variable in LISREL is assigned empirical meaning by all indicators in the model--not merely by the indicators to which it bears epistemic relationships. As a result, it may be difficult to separate the various sources of validity criteria that determine the construct-indicator links. In effect, estimated epistemic relationships will be stronger and measurement error smaller for indicators that covary with indicators of other, structurally related, constructs. This may lead to confounding problems (see Burt (1976)). For example, lack of discriminant validity may be interpreted as a significant structural relationship. In the language of MDS, it would be as if the centroid were not the expected value of the position of each of the indicator-points on all dimensions, but instead was estimated in a manner that optimized the relationships between constructs.

IMPLICATIONS

When the sample size is small and/or when the analyst is not able to specify the functional form, and when there is a system of relationships and many parameters to be estimated, confirmatory NMDS is an attractive alternative to structural equation models. For marketing, it should have a great deal of appeal--especially in studies of industrial markets where the number of observations across subjects often is small and the number of observations within subjects is large.

The price of liberating analysis from most of the assumptions of structural equation models is that confirmatory NMDS cannot provide all the evaluative (inferential and descriptive) statistics associated with, say, LISREL or PLS. Since the Euclidian distance is symmetric, the structural estimate does not in itself indicate directionality, let alone causality. This does not mean, however, that the testing of causal theory is irrelevant to confirmatory NMDS. Conceptualizing causal theories in terms of systems of relationships implies that there are certain structures among the components of the system. These structures can be specified via facet theory and subjected to rigorous tests of refutation within the methodology we described.

Compared to structural equation models, confirmatory NMDS is probably less able to evaluate individual parameter estimates. As with LISREL and PLS, an inferential statistic is available for the assessment of overall fit. Unlike these models, however, NMDS is not yet able to evaluate the significance of individual parameter estimates. Borg and Lingoes (1980) have found that confirmatory NMDS is not particularly powerful in discriminating among competing theories. Therefore, as is the case with LISREL (Fornell and Larcker 1981; Fornell 1982b), the usefulness of an overall goodness-of-fit measure may be limited. Therefore, it may be useful to apply a condition that constrains in a more continuous manner all but one or all but two of the points in a particular category, rather than a condition that changes from an all-point constraint to one that constrains some points. Notwithstanding, confirmatory NMDS is a promising method with much to offer, especially given the few assumptions that are required. It should find frequent application in market research. The methodology presented here is an attempt to demonstrate one way in which it can be used.

FOOTNOTES

1. Even more stringent forms of convergent-discriminant validity could be imposed by qualifying the term "closer." One might, for example, require that the distance between any two points within the same region be one-half (or any other value) the distance between points in different regions.

2. Again, there are theoretically many different constraint conditions that could be imposed. With three points per region, for example, separate conditions could constrain three, two, and then one of those points. The same could be true in the case where there are n points per region.

3. For the substantive theory behind this model, see Fornell (1976).

4. For an early illustration, see Guttman (1965).

Table 1

CORRELATION MATRIX (n = 138)
(CRONBACH'S ALPHA IN DIAGNONAL FOR COMPOSITES)

	VARIABLE NAME	1	2	3	4	5	6	7	8	9	10	11
1.	PLANNING COMMITTEES (x_1)											
2.	COORDINATE DEPARTMENTS[1] (x_2)	.44										
3.	EDUCATE PERSONNEL (x_3)	.32	.59									
4.	NEW PRODUCT DEVELOPMENT (y_1)	.54	.31	.27	.66							
5.	PRODUCT MANAGEMENT (y_2)	.53	.37	.20	.61	.57						
6.	COMM./ADVERTISING (y_3)	.45	.22	.11	.38	.39	.70					
7.	PRICING (y_4)	.41	.20	.12	.36	.31	.30	.59				
8.	VETO/NEW PRODUCT (y_5)	.25	.17	.11	.39	.23	.11	.18	.82			
9.	VETO/PRODUCT MANAGEMENT (y_6)	.48	.22	.11	.54	.50	.22	.33	.60	.66		
10.	VETO/COMM./ADVERTISING (y_7)	.24	.14	.11	.16	.14	.49	.42	.36	.38	.67	
11.	VETO/PRICING (y_8)	.20	.06	.10	.14	.12	.15	.57	.38	.40	.41	.75

TABLE 2

SUMMARY OF CMDA CONSTRAINT CONDITIONS

NOMOLOGICAL CONSTRAINTS	CONVERGENT-DISCRIMINANT CONSTRAINTS		
	$a_1\ b_1$	$a_1\ b_2$	$a_2\ b_2$
$c_1\ d_1$	P = .0000 K = .195 R_1 = .78 R_2 = .79	P = .0000 K = .195 R_1 = .78 R_2 = .79	P = .0000 K = .164 R_1 = .84 R_2 = .87
$c_2\ d_2$	P = .0000 K = .217 R_1 = .76 R_2 = .77	P = .5761 K = .140 R_1 = .93 R_2 = .93	P = .6723 K = .103 R_1 = .94 R_2 = .97
NONE	P = .0000 K = .181 R_1 = .81 R_2 = .82	P = .5549 K = .139 R_1 = .93 R_2 = .93	P = .7463 K = .105 R_1 = .94 R_2 = .97

P = Probability that the difference in the ability of the constrained and unconstrained solutions to predict the order in the original data would occur by chance.

K = Guttman-Lingoes coefficient of alienation.

R_1 = Correlation of the order of the distances in the constrained solution and the order of the original data.

R_2 = Correlation of the order of the distances in the constrained solution and the order of the distances in the unconstrained solution.

Figure 1

COMBINED VALIDITY CONSTRAINTS

Convergent-Discriminant Validity

Nomological Validity

	a_1 b_1	a_2 b_1	a_1 b_2	a_2 b_2
c_1 d_1	Strong Convergent-Discriminant Validity. Strong Nomological Validity.			Weak Convergent-Discriminant Validity. Strong Nomological Validity.
c_2 d_1				
c_1 d_2				
c_2 d_2	Strong Convergent-Discriminant Validity. Weak Nomological Validity.			Weak Convergent-Discriminant Validity. Weak Nomological Validity.

Figure 2

Euclidian Estimation Procedure

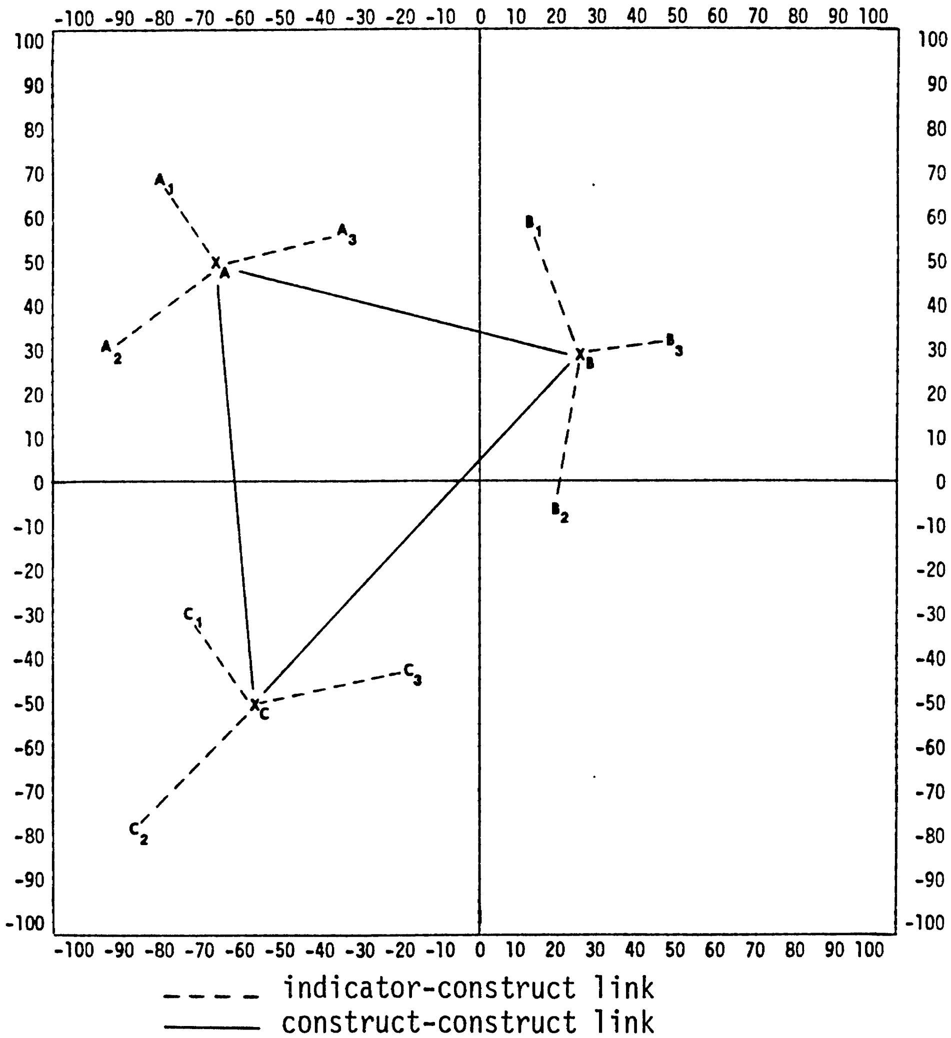
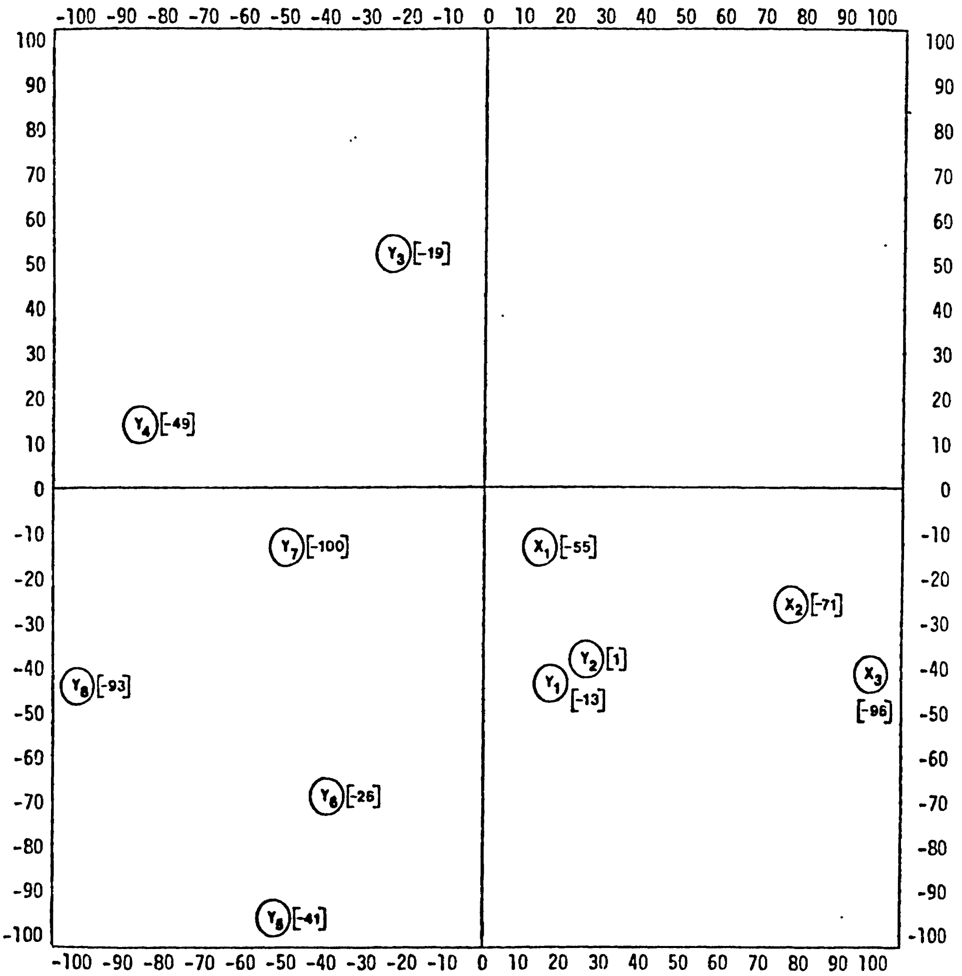


Figure 3

Unconstrained Solution

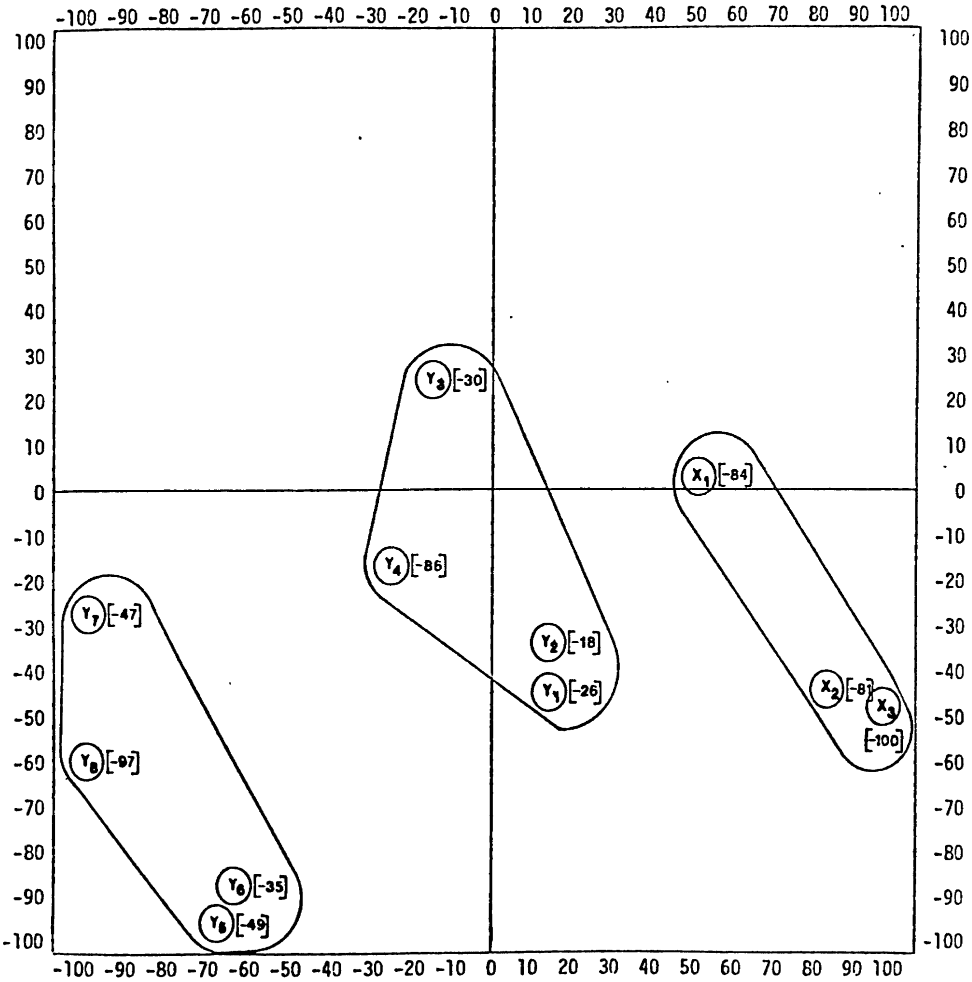


Coefficient of Alienation (K) = .08

Bracketed numbers are projections of each point on the third dimension

Figure 4

Combination of strongest form of both convergent-discriminant and nomological constraints



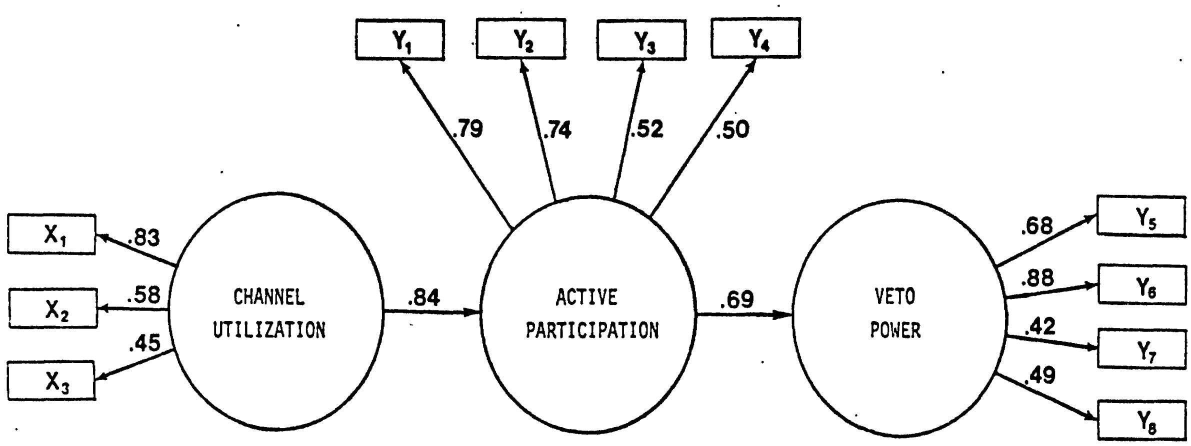
$p=.0000$ $R_1=.78$

$K=.195$ $R_2=.79$

Figure 5

Two-Stage Model

Combining convergent-discriminant and nomological validity.



$$\frac{\chi^2}{df} = \frac{184.2328}{42} = 4.39$$

p=.0000

Figure 6

$a_1b_2c_2d_2$ configuration

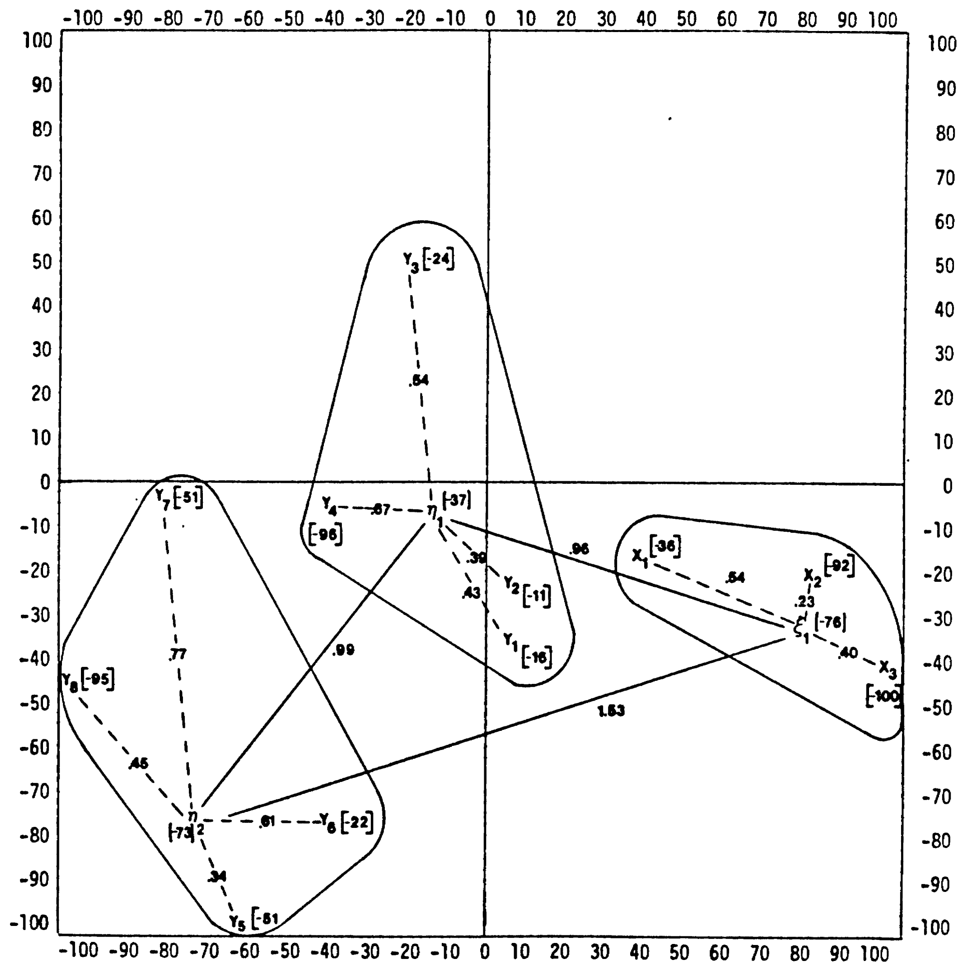
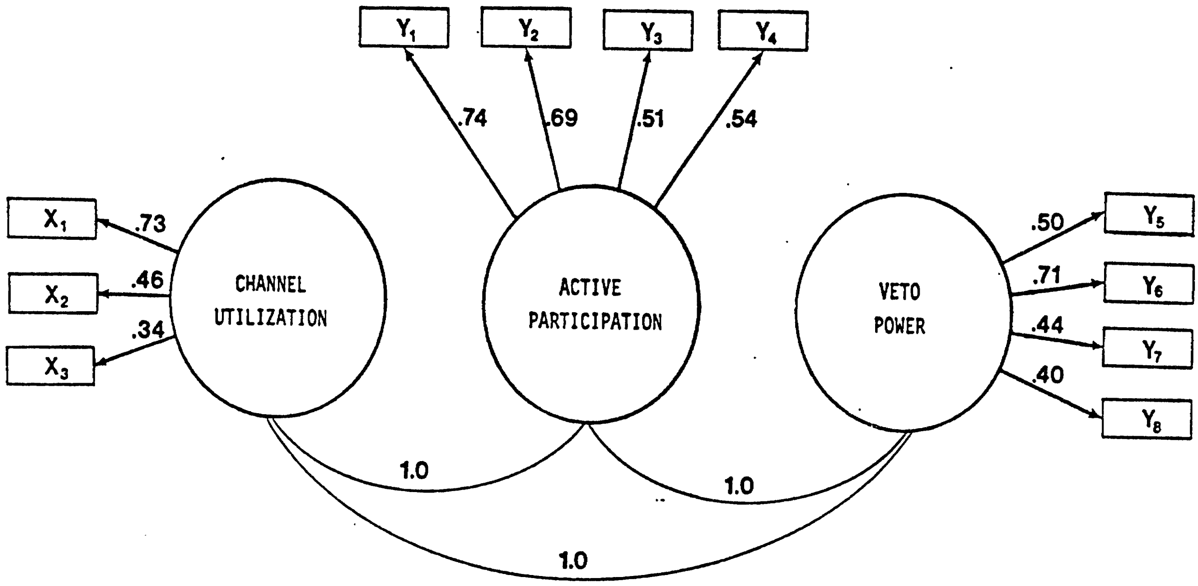


Figure 7

One-Factor Model



$$\frac{\chi^2}{df} = \frac{242.9643}{44} = 5.52$$

p=.0000

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