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AN INTEGRATED THEORY OF EXCHANGE RATE EQUILIBRIUM

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An Integrated Theory of Exchange Rate Equilibrium

This brief paper will show that a theoretical equilibrium state of the world exists in the absence of capital controls and trade barriers when (a) prices for the same goods in different markets are equal, after translation at the spot exchange rate; (b) differences in rates of aggregate price change in different markets eventually cause offsetting exchange rate changes which restore condition (a); (c) returns on equivalent securities denominated in different currencies but covered in the forward market are almost instantaneously equalized; (d) the market's expected rate of change of the exchange rate equals, to a close approximation, the control-free interest rate differential between the two currencies; (e) in the absence of predictable exchange market intervention by central banks, the interest rate differential is the best possible forecaster of the future spot rate; and (f) the forward rate also provides the best forecast of the future spot rate.[1] A final corollary also identifies a relationship between inflation rates and international interest rate differentials.

These results are based on four related theories:

(i) The purchasing power parity theorem;

(ii) The interest rate parity theorem;

(iii) The interest rate theory of exchange rate expectations;

(iv) The forward rate theory of exchange rate expectations.

Purchasing Power Parity

Monetarists argue that the pursuit of independent monetary policies by different governments is the cause of differing rates of price inflation. They also say that apart from occasional structural changes in demand and comparative advantage, different inflation rates are the causes of exchange rate changes. Because free international trade must, apart from transportation costs, equalize prices for equivalent goods in different countries, different rates of change in prices must eventually induce offsetting exchange rate changes in order to restore approximate price equality. Clearly this purchasing power parity equilibrium may exist irrespective of the sources of price changes--monetary, cost-induced, or the result of changes in demand. This results in the relative form of the purchasing power parity theorem which may be stated as follows:[2]

Rate of change of the exchange rate = Rate of change in relative price levels

$$\frac{X_{t+n} - X_t}{X_t} = \frac{P_{\$,t+n}/P_{f,t+n} - P_{\$,t}/P_{f,t}}{P_{\$,t}/P_{f,t}}$$

Where: X_t is the exchange rate at time t, in dollars ($) per foreign currency unit (f).

$P_{\$,t}, P_{f,t}$ are the price indices at time t of U.S. and foreign goods, respectively.

For the purposes of this paper a slightly modified version of the purchasing power parity theorem will be presented. It rests directly on the notion that for the same goods the rate of change of prices in dollars must, in equilibrium, equal the rate of change of prices in any other currency, translated into dollars. If $I_{\$,t}$ is the rate of change of the price level in dollars and $I_{f,t}$ the rate of change of the price level in the foreign currency, then

$$P_{\$,t+n} = P_{\$,t}(1+I_{\$,t})$$

and

$$P_{f,t+n} = P_{f,t}(1+I_{\$,t})$$

where, as before, P_t and P_{t+n} represent the present and future price level indices, respectively.

Now, if foreign and domestic prices are equal at the beginning, then

$$P_{\$,t} = X_t P_{f,t}.$$

And, if foreign and domestic prices are again equal at time t+n, then

$$P_{\$,t+n} = X_{t+n} P_{f,t+n}.$$

Hence

$$\begin{aligned} P_{\$,t}(1+I_{\$,t}) &= P_{f,t}(1+I_{f,t})X_{t+n} \\ &= (P_{\$,t}/X_t)(1+I_{f,t})X_{t+n} \end{aligned}$$

$$\therefore \quad \frac{1 + I_{\$,t}}{1 + I_{f,t}} = \frac{X_{t+n}}{X_t} .$$

Subtracting 1 from both sides gives

$$\frac{I_{\$,t} - I_{f,t}}{I + I_{f,t}} = \frac{X_{t+n} - X_t}{X_t}.$$

And if the foreign and domestic inflation rates are small fractions this may be reasonably approximated by

$$\frac{X_{t+n} - X_t}{X_t} = I_{\$,t} - I_{f,t}.$$

rate of change of the exchange rate = inflation rate differential.

Several recent empirical studies have provided support for the relative form of the purchasing power parity theorem in its <u>ex post</u> form, at least over the long run.[3] For obvious reasons, substantial (but perhaps temporary) deviations occur in the short run, and in all probability parity of foreign and domestic price levels is a shifting equilibrium and seldom attained in reality.

If the above model is to be used for forecasting exchange rates, the future exchange rate and price levels must be stated as random variables:

Expected future exchange rate $= E(\tilde{X}_{t+n}$

Expected inflation rate $= E(\tilde{I}_f) = (\tilde{P}_{t+n} - P_t)/P_t$

where a tilde (˜) designates a random variable,

and

$$\frac{E(\tilde{X}_{t+n}) - X_t}{X_t} = E(\tilde{I}_{\$,t}) - E(\tilde{I}_{f,t})$$

or

$$E[\frac{\tilde{X}_{t+n} - X_t}{X_t}] = E[\tilde{I}_{\$,t} - \tilde{I}_{f,t}].$$

Expected rate of change of the exchange rate = expected inflation rate differential.

Clearly the usefulness of this model depends on price forecasting. If commodity markets were efficient, price forecasting would not be possible. Moreover, as we shall see below, anticipated price changes will normally result in an exchange rate change in advance of, rather than in reaction to, the price change itself.

Interest Rate Parity

The interest rate parity theorem states quite simply that covered interest arbitrage will eliminate any covered interest rate differentials between equivalent interest-bearing securities denominated in different currencies. In the form presented here, this theory does not imply any undirectional causality. Observation of the Eurocurrency and foreign exchange markets makes it abundantly clear that interest rates and forward and spot exchange rates are determined simultaneously: it is inaccurate to suppose that changes in interest rate differentials cause adjustments in the forward rate to restore interest rate parity. Instead, the money and foreign exchange markets respond continously to one another so that in equilibrium:

Value at t+n of \$A earning interest rate $i_{\$,t}$ = Value at t+n of \$A converted into foreign currency and earning interest rate $i_{f,t}$ until t+n when it is converted back into dollars at the forward rate, F_t^n,

$$A(1 + i_{\$,t}) = (A/X_t)(1 + i_{f,t})\,F_t^n$$

$$\frac{1 + i_{\$,t}}{1 + i_{f,t}} = \frac{F_t^n}{X_t}\,.$$

Subtracting 1 from each side gives

$$\frac{i_{\$,t} - i_{f,t}}{1 + i_{f,t}} = \frac{F_t^n - X_t}{X_t} .$$

If $i_{f,t}$ is small, then, to a close approximation,

$$\frac{F_t^n - X_t}{X_t} = i_{\$,t} - i_{f,t}.$$

Forward premium or discount = interest rate differential.

Recent empirical evidence as well as the adamant assertions of bankers and foreign exchange traders provide substantiation for the validity of this theory.[4]

Interest Rate Theory of Exchange Rate Expectations

The notion that the foreign exchange market is highly efficient in the sense that information is disseminated quickly and acted on immediately suggests that the present spot exchange rate properly reflects all available information. Traders' and speculators' profit-seeking actions ensure that exchange rates fluctuate in anticipation of future demand and supply, rather than in reaction to present demand and supply. Put differently, exchange rates react to new information in an immediate and unbiased fashion, and since new information arrives randomly exchange rates fluctuate randomly. An analysis of future demand and supply is of no use in predicting exchange rate changes unless one is convinced of his ability to forecast changes in future demand and supply faster or more accurately than the market can.

Although exchange rates fluctuate randomly or unpredictably, they do so around a trend or rate of return in the same way as stock market prices, while effectively unpredictable, fluctuate about a long-run average upward trend. The trend or expected rate of return is easily derived from interest rates in the two currencies as follows:

Since exchange rate changes are unpredictable, investors (banks, foreign exchange traders, corporate treasurers, etc.) should be indifferent to the choices between holding or borrowing one currency or another as long as interest rates available or payable are identical. Spot exchange rates will be bid up or down to the point where the present rate is neither above nor below the expected future spot rate; otherwise someone could profit by selling or buying the currency. If, however, interest rates on equivalent securities in the two currencies are not equal, investors will have a preference for the currency having higher-yielding assets. Today's spot rate will not be bid up or down to the point of equality with the expected future spot rate. For example, one may be more willing to invest in sterling-denominated Eurodeposits yielding 14 percent than in dollar-denominated Eurodeposits with a yield of 8 percent, even though one expects sterling to depreciate somewhat. Similarly a corporate borrower may prefer to obtain funds in dollars at 9 percent rather than in dollars at Swiss francs at 4 percent, because some appreciation of the Swiss franc is expected.

Since many depositors and borrowers on both sides of the Atlantic (or English Channel) are making similar calculations continously, the spot exchange rate will generally be bid up or down to the point where no such incentive exists. In other words, spot exchange rates (and/or interest rates) will be bid to the point where the market's expected rate of depreciation (or appreciation) of the exchange rate is exactly offset by the interest rate differential between the two currencies.

If this is true, one may derive an equilibrium condition very similar to that for interest rate parity. As a result of unhedged borrowing and investing decisions made by individuals both at home and abroad, equilibrium will exist when:

Value at t+n of \$A earning interest rate $i_{\$,t}$ = Value at t+n of \$A converted into foreign currency and earning interest rate $i_{f,t}$ until t+n when it is converted back into dollars at the expected future spot rate, $E(X_{t+n})$.

$$A(1 + i_{\$,t}) = (A/X_t)(1 + i_{f,t})\ E(\tilde{X}_{t+n})$$

$$\frac{1 + i_{\$,t}}{1 + i_{f,t}} = \frac{E(\tilde{X}_{t+n})}{X_t}.$$

Subtracting 1 from each side:

$$\frac{i_{\$,t} - i_{f,t}}{1 - i_{f,t}} = \frac{E(\tilde{X}_{t+n}) - X_t}{X_t}.$$

And if $i_{f,t}$ is small, then to a close approximation,

$$\frac{E(\tilde{X}_{t+n}) - X_t}{X_t} = i_{\$,t} - i_{f,t}$$

or

$$E[\frac{\tilde{X}_{t+n} - X_t}{X_t}] = i_{\$,t} - i_{f,t}.$$

The expected rate of change of the exchange rate = the interest rate differential.

The interest rate differential is also the best forecaster of the rate of change of the exchange rate. This is the case because foreign exchange markets for the major currencies appear to be highly efficient and competitive markets in which present rates properly reflect all available information. No one can reasonably expect to earn a higher rate of return than the market, and so the market's estimate of the rate of change of the exchange rate is the best available one. Only if governments intervene in a systematic and predictable fashion would this statement be erroneous since governments, unlike traders and speculators, do not necessarily act in accordance with profit-seeking motives.

Since this theory, unlike the previous two, is an ex ante expectations hypothesis involving an unknown, $E(\tilde{X}_{t+n})$, it is not subject to direct empirical testing. The observed or ex post outcome of X_{t+n} will almost always differ from $E(\tilde{X}_{t+n})$. Nevertheless, tests of this formula as a forecasting method have shown it to produce results at least as accurate as other, more sophisticated methods.[5]

Forward Rate Theory of Exchange Rate Expectations

Speculators who think that the forward rate is above their prediction of the future spot rate will sell the foreign currency forward, thus bidding down the forward rate until it equals the expected future spot rate. Similarly those who believe the foreign currency is undervalued in the forward market will buy foreign currency forward, bidding the forward rate up until it reaches the expected future spot rate. Assuming sufficient funds are available to speculators, the forward rate (F_t^n) will be in equilibrium if, and only if:

foward rate = expected future spot rate.

$$F_t^n = E(\tilde{X}_{t+n}).$$

Also,

$$\frac{F_t^n - X_t}{X_t} = \frac{E(\tilde{X}_{t+n}) - X_t}{X_t}$$

$$= E[\frac{\tilde{X}_{t+n} - X_t}{X_t}].$$

Forward premium or discount = expected rate of change of the exchange rate.

There is little or no empirical evidence that provides either direct or indirect support of this particular theorem. On the other hand, studies purporting to disprove the hypothesis are suspect unless they have used actual spot and forward rates obtained from the trading desks of major foreign exchange dealers.[6]

Integration of the Four Postulates

The preceding discussion may be brought together in a single interest rate-exchange rate-inflation rate framework as follows:

$$\frac{E(\tilde{X}_{t+n}) - X_t}{X_t} = E(\tilde{I}_{\$,t}) - E(\tilde{I}_{f,t})$$

(from the purchasing power parity theorem)

$$\frac{F_t^n - X_t}{X_t} = i_{\$,t} - i_{f,t}$$

(from the interest rate parity theorem)

$$\frac{E(\tilde{X}_{t+n} - X_t}{X_t} = i_{\$,t} - i_{f,t}$$

(from the interest rate theory of exchange rate expectations)

$$\frac{F_t^n - X_t}{X_t} = \frac{E(\tilde{X}_{t+n}) - X_t}{X_t}$$

(from the forward rate theory of exchange rate expectations).

Hence the expected value and best forecast of the future spot rate, $\tilde{X}_{t+n}$, is either the interest rate differential or the forward rate, since these are identical. If the foreign exchange market is efficient, then any empirical result that is inconsistent with the previous statement is probably based on incorrect combinations of interest rates or exchange rates, or the result of capital controls inhibiting arbitrage.

A Residual Result

An interesting side result of the integrated theory concerns the relationship between interest rates and expected inflation rates. If, in equilibrium, it is true that the difference in expected inflation rates is equal to the expected rate of change of the exchange rate and that the interest rate differential also equals the expected rate of change of the exchange rate, then we must conclude that the interest rate differential equals the difference in expected inflation rates:

$$i_{\$,t} - i_{f,t} = E(\tilde{I}_{\$,t}) - E(\tilde{I}_{f,t})$$

hence
$$i_{\$,t} - E(\tilde{I}_{\$,t}) = i_{f,t} - E(\tilde{I}_{f,t}).$$

This result implies that after expected rates of inflation have been subtracted interest rates in different currencies are equal!

The interest rate-inflation rate relationship is, of course, consistent with Irving Fisher's theory that the nominal interest rate is equal to the sum of the real rate of interest (the rate reckoned in terms of goods) and the expected percentage change in prices.[7] If there is a single world capital market, real rates of interest will tend to be equalized and observed rates will differ from one another under a floating rate system only by their respective premiums for expected inflation.

Conclusion

Stated in this fashion, the above information seems fairly obvious. Indeed, while the integration of exchange rate and interest rate theories has received increasing attention during the recent years of international monetary turmoil, almost all that has been discussed about it in this paper was implied by the following sentence in Fisher's The Theory of Interest: [when rates of interest are measured in two diverging standards of value then] "the two rates of interest will, in a perfect adjustment, differ from each other by an amount equal to the rate of divergence between the two standards."[8]

Second, the empirical verification or otherwise of these notions depends importantly on the time horizon selected. While interest rate parity holds in the very short run, purchasing power parity appears to hold well only when periods of several years' length are employed. The interest rate-exchange rate relationship, on the other hand, may well hold both for a few days' or weeks' horizon and for intervals of several years.

Finally, it must be emphasized that all of the relationships stated here are <u>mutual</u> ones in which each variable acts on the others, although one direction of causality may predominate. Exchange rates affect prices as well as vice versa; interest rate changes cause price changes as well as the converse, and, as has been noted above, forward rates, spot rates, and interest rates interact simultaneously to bring about interest rate parity. To conclude with another quotation from Fisher, "All problems of local prices, exchange, and interest, act and react on each other in many ways."[9]

Footnotes

1. A similar approach is taken by Robert Z. Aliber in an unpublished manuscript entitled The Short Guide to Corporate International Finances (1974). See also Robert Z. Aliber and Clyde P. Stickney, "Accounting Measures of Foreign Exchange Exposure: The Long and Short of It," The Accounting Review, Vol. 50 (January 1975): 44-57.

2. The purchasing power parity theorem was first proposed and tested by Gustav Cassel in "The Present Situation of the Foreign Exchanges," Economic Journal, Vol. 26 (1916): 62-65, 319-23, and in subsequent articles and books. More recent expositions may be found in Bela Balassa, "The Purchasing Power Parity Doctrine: A Reappraisal," Journal of Political Economy, Vol. 72 (December 1964): 584-96, and in James M. Holmes, "The Purchasing Power Parity Theory: In Defense of Gustav Cassel as a Modern Theorist," Journal of Political Economy, Vol. 75 (October 1967): 686-95.

3. For recent tests of the purchasing power parity theorem see Aliber and Stickney, op. cit., Balassa, op. cit., Leland B. Yeager, "A Rehabilitation of Purchasing Power Parity," Journal of Political Economy, Vol. 66 (December 1958): 516-30, and Henry J. Galliot, "Purchasing Power Parity As An Explanation of Long-Term Changes in Exchange Rates," Journal of Money, Credit and Banking, Vol. 2 (August 1971), 348-57.

4. On the interest rate parity theorem see any recent international finance textbook. See also: Lawrence H. Officer and Thomas D. Willett, "The Covered-Arbitrage Schedule: A Critical Survey of Recent Developments," Journal of Money, Credit, and Banking, Vol. 2 (May 1970): 247-57; Robert Z. Aliber, "The Interest Rate Parity Theorem: A Reinterpretation," Journal of Political Economy, Vol. 81 (December 1973): 1451-59; Richard C. Marston, "Interest Arbitrage in the Eurocurrency Markets," Discussion Paper No. 259, The Wharton School, University of Pennsylvania, 1974; Richard J. Herring and Richard C. Marston, "The Forward Market and Interest Rate Determination in the Eurocurrency and National Money Markets," unpublished working paper, University of Pennsylvania, 1974; and Tamir Agmon and Saul Bronfeld, "International Mobility of Short Term Covered Arbitrage Capital," Journal of Business Finance and Accounting, Vol. 2 (Spring 1975).

5. See Ian H. Giddy and Gunter Dufey, "The Random Behavior of Flexible Exchange Rates: Implications for Forecasting," Journal of International Business Studies, Vol. 6 (Spring 1975); also: Aliber, The Short Guide, Chpt. 6, for long-run tests of this theory.

6. For example, David L. Kaserman, "The Forward Exchange Rate: Its Determination and Behavior as a Predictor of the Future Spot Rate," 1973 Proceedings of the Business and Economic Statistics Section (Washington, D.C.: American Statistical Association, 1973): 417-22.

7. A concise exposition of this notion may be found in George G. Kaufman, Money, the Financial System, and the Economy (Chicago: Rand McNally, 1973), pp. 136-43.

8. Irving Fisher, The Theory of Interest (New York: Macmillan, 1930), p. 39.

9. Ibid, p. 69.