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WHY DO FIRMS SMOOTH EARNINGS?

By

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ABSTRACT

We develop a model that explains why the manager of a firm may smooth reported earnings by reducing its variability through time. Greater earnings volatility leads to a bigger informational advantage for informed investors over uninformed investors. If a sufficient number of current shareholders are uninformed and face some likelihood of trading in the future for liquidity reasons, then an increase in the volatility of reported earnings will magnify the trading losses these uninformed shareholders perceive. They will, therefore, want their firm's manager to produce as smooth a reported earnings stream as possible. Interestingly, it is a concern with long-term stock price performance rather than a preoccupation with the short term performance that causes smoothing. Empirical implications are drawn out that link earnings smoothing to managerial compensation contracts, uncertainty about the volatility of earnings and the ownership structure.

I. Introduction

Corporate earnings management has been much in the news lately. For example, Business Week (October 5, 1998) ran a cover story titled "Who Can You Trust?" that suggested that the credibility of earnings reports was being eroded by earnings management. Arthur Levitt Jr., Chairman of the Securities and Exchange Commission (SEC) commented in 1998[1]:

"Too many corporate managers, auditors, and analysts are participants in a game of nods and winks. In the zeal to satisfy consensus earnings estimates and project a smooth earnings path, wishful thinking may be winning the day over faithful representation."

Earnings management means manipulating reported earnings so that they don't accurately represent economic earnings at every point in time. Earnings smoothing is a special case of earnings management involving intertemporal smoothing of reported earnings relative to economic earnings; it attempts to make earnings look less variable through time. Earnings smoothing is extensively documented; see Bannister and Newman (1996), Beidlerman (1973) and Subramanyam (1996). Moses (1987) studies how various firm-specific factors affect the extent of earnings smoothing. This raises the question we address: why is earnings smoothing so prevalent?

If earnings are being smoothed, reported earnings must be sometimes higher than economic earnings and sometimes lower. It is not difficult to see why managers may want to report inflated earnings. But it is a lot harder to explain why a manager reports lower earnings than what he observes. Yet, numerous such instances have recently been discussed. For example, in 1998, the SEC delayed approval of SunTrust Banks Inc.'s acquisition of Crestar Financial Corp., until the company agreed to reduce loan loss reserves by $100 million and restate higher earnings for last three years[2]. The SEC also criticized W. R. Grace & Co. for underreporting its 1998 profits by $20 million. The SEC alleged that the company was attempting to exploit apparently diminishing marginal returns to reported earnings. When reported earnings are high, reporting even higher earnings tends to elicit a relatively small positive market reaction. The company may therefore want to "hide" some of its current earnings for reporting it in a future period when earnings are lower and the marginal impact of a higher report is greater[3].

Earnings smoothing can be either "artificial" or "real". Real smoothing involves decisions that affect the cash flow distribution and dissipate firm value. Examples include changing the timing of investments, providing promotional discounts to pump up sales towards the end of the quarter, etc. When earnings are artificially smoothed, cash flows are unaffected. This kind of smoothing is achieved primarily by using the reporting flexibility provided by Generally Accepted Accounting Principles (GAAP).[4] Real smoothing has costs that are obvious whereas artificial smoothing has costs that are subtler, related to loss of credibility or consumption of the manager's time in such activities.

The extent to which earnings can be artificially smoothed is limited by realized cash flows, which cannot be misreported; there are only so many earnings values compatible with realized cash flows. But because earnings cannot be inferred unambiguously from cash flows, the manager has some discretion in reporting earnings. Though economic earnings and cash flows converge in the long run, they differ due to revenue recognition discretion, accruals, extraordinary items etc.; unlike cash flows, economic earnings attempt to match costs and benefits through time. Thus, earnings may carry information that cannot be inferred directly from cash flows. To see this, consider a firm that makes a sale of $1 million and delivers goods on December 24, 1999. The sale revenue will be received in year 2000. The firm incurs period costs (in cash) of $900,000. There are no other transactions. The firm uses calendar years as financial years. If we ignore taxes, these transactions affect 1999 cash flow in the form of a cash outflow of $900,000. If the sale is recognized in the 1999 report, these transactions cause earnings go up by $100,000.

An investor who observes only the cash flow of the firm sees a different picture from that seen by an investor who sees only the earnings. In this case, earnings and cash flow together tell us much more than does any single statement. In particular, if future sales are correlated with current sales, earnings may provide information about future *cash flows* that the current cash flow does not.

This illustrates that earnings may have valuation-relevant information not contained in cash flow. It is this information that the manager in our model exercises his reporting discretion on. Thus, the manager artificially smoothes earnings even when he cannot smooth cash flows.

We explain artificial smoothing. The basic intuition is as follows. In a perfect world with symmetric information, the volatility of the firm's earnings will be irrelevant if the shareholders are risk neutral or if they are risk averse but can costlessly diversify away their exposure to the firm. So we start with the assumption that there is some valuation-relevant information about the firm that cannot be costlessly and credibly communicated to shareholders. Investors can acquire this information at a personal cost or can choose to remain uninformed.

Some investors may have to sell their shares in the future due to (exogenous) liquidity reasons, such as an unexpected contingency. We assume that these liquidity investors are mostly shareholders of the firm. As they trade with informed investors, they lose money on average. In fact, it is their trading loss that compensates the informed investors for their information acquisition cost. Competition among the informed investors causes their expected trading profit to equal their information acquisition cost, producing zero ex ante expected profits. Thus, the resources devoted to information acquisition are a welfare loss absorbed mostly by the firm's shareholders.

The key to the analysis is that when the volatility of the firm's earnings[5] is high, private information about the firm is more valuable, and more investors become informed. This means higher expected losses for shareholders who trade for liquidity. Shareholders, therefore, abhor earnings volatility and pay less for firms with higher earnings volatility.

The manager responds by smoothing earnings to affect market perceptions of earnings volatility and hence the firm's stock price. However, the market understands this in equilibrium and is not fooled. This means there is no overall benefit from smoothing in equilibrium. The phenomenon persists nonetheless because not smoothing when the market expects smoothing can result in the firm's stock price being lower than its true value. Interestingly, what causes smoothing in our analysis is the manager's concern about long-term stock price performance rather than just the current stock price. A "myopic" manager would simply inflate earnings.

Smoothing reduces the *expected* value of the time-series volatility of reported earnings. The effect of smoothing on volatility is state contingent in that it depends on the shocks to earnings realized in the future. Smoothing reduces measured times-series volatility when shocks in successive periods tend to offset each other. However, if the shocks are positively serially correlated, smoothing in early periods may increase the variability of reported earnings in later periods. In such cases, smoothing may raise the time-series volatility of reported earnings. Since earnings shocks are not expected to be positively serially correlated, smoothing reduces the expected time-series volatility of reported earnings.[6]

The existing literature has provided alternative explanations for earnings smoothing. Barnea, Ronen and Sadan (1975) argue that earnings smoothing is a signaling device. In an overlapping generations model, Dye (1988) shows that current shareholders may demand earnings smoothing to influence perceptions of potential shareholders about firm value when the manager's contract with current shareholders is unobservable. Fudenberg and Tirole (1995) assume that management derives incumbency rents from continuing in the firm. Management can minimize the probability of being fired by developing a smooth performance record because recent performance is a stronger determinant of performance than current performance. Dividend and earnings smoothing arise since these are criteria for performance judgment.

Trueman and Titman (1988) point out that high perceived earnings volatility increases the perceived bankruptcy probability of the firm and hence its borrowing cost, so earnings smoothing is cost-minimizing. Lambert (1984) explains real smoothing in a two-period moral hazard setting where the optimal contract has second-period managerial compensation increasing in first-period output. Thus, when first-period performance is good, the marginal utility of consumption in the second period is low for the manager, and he reduces effort. Rozycki (1997) uses the convexity of the tax code to explain smoothing.

What distinguishes our work from the existing literature is that earnings smoothing is not driven by issues related to managerial self-interest, tax incentives or leverage concerns. Rather, it is solely the consequence of the manager trying to increase his firm's stock price by reducing the potential loss shareholders may suffer when they trade for liquidity reasons[7]. This is not to say that the factors examined by others are unimportant. Taken together with the existing literature, our analysis suggests that there may be many reasons why firms smooth earnings.

Section II lays out the model. Section III discusses the information acquisition and liquidity trading process and relates it to the perceptions about firm's earnings stream. Section IV analyzes the earnings smoothing. Section V provides a numerical example. Section VI examines robustness issues. Section VII concludes. All proofs are in the Appendix.

II. Model

Timeline

There are four dates in the model: $t = 0$, $t = 1$, $t = 2$ and $t = 3$ corresponding to three time periods: the first ($t = 0$ to $t = 1$), the second ($t = 1$ to $t = 2$), and the third ($t = 2$ to $t = 3$). A firm was started in the past (say $t = 0$). The earnings of the firm for the three periods of its existence are realized at $t = 1$, 2 and 3, respectively. After the earnings realization at $t = 3$, the firm is liquidated. Sometime during the third period, (say at $t = 2.5$), there is liquidity trading in the firm's shares. We will explain this later.

Figure 1: Timeline of Events

$t=0$	$t=1$	$t=2$	$t=2.5$	$t=3$
Firm is started.	Manager observes first-period earnings. He then reports and distributes earnings to investors.	Manager observes second-period earnings. He then reports and distributes earnings to investors.	Some investors choose to become informed and receive a private signal about the shock earnings will experience at t=3. Some shareholders face liquidity shocks. Trading occurs and a market-clearing price is determined.	Firm is liquidated and third-period earnings are distributed to shareholders.

The Players

The firm is all-equity financed, with the number of shares outstanding normalized to one. The manager of the firm privately observes the economic earnings e_{at} in period t and then reports e_{rt} to the market. Besides the manager and the shareholders, there are many competing investors in the market who can acquire costly private information about the firm. Since we use a market microstructure model of trading similar to that in Boot and Thakor (1993), we adopt their assumption that all investors are capital-constrained and cannot short-sell or trade on margin. Any investor choosing not to get informed can still trade competitively based on publicly available information. We assume universal risk neutrality.

Earnings Distribution

The earnings stream of the firm is stationary. The form of the earnings distribution is common knowledge but the parameters of this distribution are unknown. The earnings distribution at time t is

$$e_{at} = \begin{cases} \mu + \delta & \text{if } \lambda_t = \text{high} \quad \text{(positive shock)} \\ \mu - \delta & \text{if } \lambda_t = \text{low} \quad \text{(negative shock)} \end{cases}, \tag{1}$$

$$\lambda_t | \lambda_1 \cdots \lambda_{t-1} = \begin{cases} \text{high with probability } 0.5 \\ \text{low with probability } 0.5, \end{cases} \tag{2}$$

for $t = 1, 2$, and 3 so the distribution is time-invariant.

The parameters μ and δ are the mean and the volatility of the distribution and are unknown to everyone including the manager. The common prior beliefs about the distribution of μ and δ are:

$$\begin{aligned} \delta &\sim g(\delta) \quad \text{with support } (\underline{\delta}, \overline{\delta}) \\ \mu &\sim h(\mu) \quad \text{with support } (\underline{\mu}, \overline{\mu}), \end{aligned} \tag{3}$$

and μ and δ are independently distributed.

Managerial Discretion

The manager reports only a single number, the firm's earnings. He has some discretion in choosing what to report at $t = 1$. The manager's report can differ from the economic earnings by at most d, where $d > 0$ is common knowledge. Thus, for simplicity, we are assuming that earnings can be costlessly manipulated by an amount up to d, but it is prohibitively costly to manipulate more. This captures in a stylized way the notion that the manager has *limited* reporting discretion.

In exercising his reporting discretion, the manager is "rolling over" an earnings "surplus" or "deficit" to another time period. We assume that the manager cannot "roll over" earnings forever and must settle accounts at $t = 2$ [8]. Thus, misreporting at $t = 1$ must be followed by an offsetting misreporting at $t = 2$.

The discount rate is zero. Therefore, any inflation of earnings by one dollar at $t = 1$ must cause a deflation of earnings by one dollar at $t = 2$ and vice-versa. There is no investment or share repurchase, and *reported* earnings are distributed as dividends to the shareholders. Since total earnings equal total cash flows over the life of the firm, total dividends paid equal total cash flows. We assume that the firm is not cash-constrained so it can pay dividends in the first period that exceed first-period cash flow. Everyone sees the earnings for the third period at $t = 3$ as the firm is liquidated and the proceeds distributed to the shareholders.

The assumption that the manager reports earnings as the only measure of firm performance is a simplification. In practice, investors receive multiple reports, including reported earnings and cash flows. These reports provide cross-checks that limit the manager's ability to reduce the information content of any report. However, all that we need is that an earnings report contain information that is valuable to investors but cannot be recovered from cash flows.

As mentioned in the introduction, when firm performance is reported at discrete points in time, current earnings may contain information to predict future cash flows that current cash flows do not contain. There is evidence that earnings and cash flow both have incremental information over each other in predicting future cash flows (see Bowen, Burgstahler, and Daley (1987) and

Dechow (1994)). Thus, introduction of other reports like cash flow would not qualitatively change the results. The earnings report in this model should be interpreted as the information provided by the manager that is not already contained in cash flow.

Liquidity Selling

During period 3, some shareholders face liquidity shocks, and are forced to sell their shares; these shares are sold at the market-clearing price at $t = 2.5$, and their supply is stochastic. The supply l in dollar terms has the distribution:

$$l \sim f(l) \quad \text{with support } \left(\underline{l}, \bar{l}\right), \tag{4}$$

and is independent of the earnings history (economic or reported) of the firm.

Information Structure

Let ϕ^j and $\hat{\phi}^j$ denote the information sets of the manager and the investors respectively just after $t = j$ where $j = 1, 2$ or 3. Everyone remembers past information, so the information sets do not decay over time. Everyone starts with common prior knowledge about the distributions of μ and δ. The manager gets to see the economic earnings e_{aj} at $t = j$, $j = 1, 2$ and 3. Investors do not observe e_{a1} and e_{a2}. They see only the earnings reports e_{r1} and e_{r2} at $t = 1$ and $t = 2$, respectively. The economic earnings at $t = 3$, e_{a3}, are seen by everyone during liquidation. The market fixes the price P_j of the shares of the firm at $t = j$ using the information set $\hat{\phi}^j$, and this price is observed by everyone. The price P_j depends deterministically on $\hat{\phi}^j$, so it is also a part of $\hat{\phi}^j$.

Some investors choose to observe a private signal about the firm's prospects in period 3 before liquidity trading. Each of them incurs a fixed cost $M > 0$ (in dollars) and gets a signal γ. Each investor starts with dollars $1 + M$. The signal γ tells with certainty whether the firm will experience a positive or a negative shock to earnings at $t = 3$. Note that the signal does not convey any information about the value of μ or δ or the exact level of earnings. The information set of informed investors is $\hat{\phi}^2 \cup \{\gamma\}$ before trading occurs. These investors cannot observe the liquidity supply while making their trading decisions. The uninformed investors, however, observe demand or supply that is net of liquidity investors' supply and any demand from informed investors. Let S denote the net supply seen by the uninformed investors ($S < 0$ if there is net demand). The uninformed investors use information $\hat{\phi}^2 \cup \{S\}$ to set a market-clearing price[9]. We shall use E^j as the expectation conditional on the information set ϕ^j of the manager and $\hat{E}^j$ as the expectation conditional on the information set $\hat{\phi}^j$ of investors. We denote by θ the measure of informed investors; θ is common knowledge at the time of liquidity trading because everyone can solve the optimization problem of an investor who has to choose whether to become informed or not, and thus determine how many investors become informed.

Investors must decide whether to become informed or not before liquidity shocks occur. We assume for simplicity that no shareholder becomes informed.[10] The implications of relaxing this assumption are discussed in Section VI.

We want to analyze the behavior of investors and the manager. The first step is to study the behavior of informed and uninformed investors during liquidity trading. The common information pieces that affect investor behavior are their beliefs about the mean and the volatility of the firm's earnings. Investors anticipate the outcome of the liquidity trading at $t = 2.5$ and set share prices at $t = 1$ and $t = 2$, which depend on the perceptions of the firm's mean and volatility. The manager knows this and his objective is to maximize the prices of the shares. He does this by using his earnings reporting discretion to influence investors' perceptions.

III. Equilibrium Analysis of Liquidity Selling

The manager reports earnings at $t = 1$ and $t = 2$. Investors try to infer the mean μ and the volatility δ of the firm's earnings from these reports. Let the expectations of the mean and the volatility based on investor perceptions be $\hat{\mu}$ and $\hat{\delta}$, respectively. Let us take these beliefs as given here. We shall see in the next section how these are determined.

We shall show below that there is a Nash equilibrium in which the measure of investors who become informed depends on the characteristics of the firm's earnings. The reason is that these characteristics determine the ex-post trading profits of the informed investors at t = 2.5 and hence the marginal return to getting informed. The trading profits of the informed investors come at the expense of liquidity sellers who systematically suffer losses in equilibrium. Thus, the expected losses suffered by liquidity investors are a function of the characteristics of earnings.

The net supply of shares S observed by the uninformed investors depends on the liquidity sellers' supply l and any demand by informed investors. We have

$$S = S(l,\gamma,\theta) = \begin{cases} l & \text{if } \gamma \text{ signals negative shock} \\ l-\theta & \text{if } \gamma \text{ signals positive shock}, \end{cases} \tag{5}$$

As we explain later, informed investors demand shares only when the private signal is favorable. If uninformed investors knew the liquidity supply l or the signal γ, they could figure out whether earnings will experience a positive or a negative shock at t = 3, and set price equal to the expected value of earnings, $\hat{\mu}+\hat{\delta}$ or $\hat{\mu}-\hat{\delta}$. However, uninformed investors observe only the net supply S and not l or γ. Therefore, they use their prior beliefs about the distribution of liquidity supply to arrive at an unbiased Bayesian posterior estimate of share value. The price function is:

$$P(S,\theta) = \frac{\frac{1}{2} f(S+\theta)(\hat{\mu}+\hat{\delta}) + \frac{1}{2} f(S)(\hat{\mu}-\hat{\delta})}{\frac{1}{2} f(S+\theta) + \frac{1}{2} f(S)}. \tag{6}$$

The net supply S equals the liquidity supply l if informed investors observe a low signal and submit no demand. The likelihood of l being S and the signal being low is $f(S)/2$, and the firm value in this case is $\hat{\mu}-\hat{\delta}$. If informed investors observe a high signal, they demand θ, so the liquidity supply must be $S + \theta$. The likelihood of this is $f(S + \theta)/2$, and the firm value in this case is $\hat{\mu}+\hat{\delta}$. Using Bayes rule and setting price equal to the firm's expected value yields (6).

Lemma 1: *If liquidity supply has a log-concave density function, then price is a decreasing function of net supply.*[11]

Assumption 1: *The sufficient condition in Lemma 1 is true, i.e., f is log-concave.*

From (6), we see that the share price is always between $\hat{\mu} - \hat{\delta}$ and $\hat{\mu} + \hat{\delta}$. Therefore, an informed investor will never buy when he sees an unfavorable signal and will always buy when he sees a favorable signal. As the two cases are equally likely, the expected trading profit of an informed investor is given by

$$m(\hat{\mu}, \hat{\delta}, \theta) = \frac{1}{2} \int_{\underline{l}}^{\bar{l}} \frac{(\hat{\mu} + \hat{\delta}) - P(l - \theta, \theta)}{P(l - \theta, \theta)} f(l) dl. \tag{7}$$

Each informed investor submits a \$1 demand for shares when he observes a high signal. If liquidity investors supply l, the net supply S observed by the uninformed investors is $S = l - \theta$, and the price is set at $P(l - \theta, \theta)$. Each informed investor ends up buying $1/P(l - \theta, \theta)$ shares and makes a profit of $(\hat{\mu} + \hat{\delta}) - P(l - \theta, \theta)$ per share. This trading profit is conditional on liquidity supply l and a high signal. The expected trading profit is obtained by taking expectation over l and then multiplying by ½, the probability of a high signal.

The measure of informed investors, θ, taken as exogenous thus far, is uniquely determined in equilibrium by a condition of competition among investors that says that the *marginal* informed investor must earn zero ex-ante expected profit, net of his information acquisition cost. Otherwise, we would not be at equilibrium; if the marginal informed investor is earning a positive expected profit, there would be an incentive for others to become informed, while if the marginal informed investor is earning negative expected profit, there are too many informed investors, and some will opt out of becoming informed. With information-acquisition costs that vary cross-sectionally across investors, *inframarginal* informed investors make positive ex-ante expected profits in equilibrium. However, we assume the same information-acquisition cost for all investors. In this case, all informed investors make zero expected profit in equilibrium and investors are indifferent between becoming informed and staying uninformed. A key result is the following.

Lemma 2: *The expected trading profit of each informed investor is decreasing in the measure of informed investors.*

The intuition is that as more investors get informed, more of their information enters the price. This reduces mispricing as well as the trading profit of each informed investor that is based on this mispricing. We can now define equilibrium.

Definition 1: *A Nash Equilibrium is:*

- *a measure θ^* of informed investors such that $m(\hat{\mu}, \hat{\delta}, \theta^*) = M$*
- *a net supply of shares $S(l, \gamma, \theta^*)$ as in (5)*
- *a market clearing price $P(S, \theta^*)$ as in (6)*

We shall now show that there is a unique equilibrium as defined above. To avoid the uninteresting case in which cost of getting informed is too high for anyone to get informed, we make the following assumption.

Assumption 2: $M < \hat{\delta}/2\hat{\mu}$.

Proposition 1: *There is a unique Nash Equilibrium with the measure of informed investors,* $\theta^* = \theta^*(\hat{\mu}, \hat{\delta})$.

Investors know the distribution of liquidity supply, which enables them to compute their expected trading profit as a function of the measure of informed investors θ, using (7). The expected trading profits of the informed are decreasing in θ, so the number of investors getting informed adjusts to a level so that the expected trading profit for the marginal informed investor equals M. Those who become informed in equilibrium receive a signal about the earnings shock the firm will receive and demand shares only if the signal is favorable. The uninformed investors see aggregate demand and set a market-clearing price such that they expect zero profit.

The liquidity investors suffer losses in equilibrium. The reason is that their trades are induced by exogenous liquidity shocks, whereas those they trade with are trading strategically based on information. If there are no informed investors, the price does not depend on the order flow because there is nothing to be learned from the order flow. Good and bad firms are priced alike. Bad firms are overpriced and good firms are underpriced, but since all investors are trading on a "level playing field", nobody makes expected losses or gains. The presence of informed investors reduces mispricing because the trades of these investors cause their information to be noisily reflected in prices. The mispricing does not vanish altogether, however, because the number of informed investors is finite and uninformed investors cannot distinguish between liquidity trades and informationally-motivated trades. In fact, this mispricing is needed to attract some investors to become informed, since these investors recover their information-acquisition costs from the expected trading profits generated by mispricing. If there were no cost of information, the number of informed investors would be so large that order-flow would reveal all private information and there would be no mispricing. Alternatively, the order flow would be perfectly revealing if there were no liquidity investors. We now characterize the dependence of the measure of informed investors on the characteristics of the firm's earnings stream.

Lemma 3: *The expected trading profit of an informed investor, m, is increasing in the perceived earnings volatility $\hat{\delta}$ and decreasing in the perceived earnings mean $\hat{\mu}$.*

Lemma 4: *The equilibrium measure of informed investors, θ^*, is increasing in the perceived volatility $\hat{\delta}$ and decreasing in the perceived earnings mean $\hat{\mu}$.*

The intuition behind these lemmas is as follows. The higher the volatility of the earnings, the higher is the informational advantage of the informed investors over the uninformed. The reason is that the signal the informed privately observe reveals whether earnings will experience a positive or negative shock, and the *size* of this shock is increasing in the volatility. Thus, if the measure of informed investors is fixed, their expected trading profit increases with earnings volatility. Competition among informed investors however, results in more investors getting informed when perceived earnings volatility is high, so that the marginal informed investor's expected trading profit equals the cost of acquiring the signal.

An increase in the earnings mean has a different effect, however. Since the signal the informed investors receive contains no information about mean earnings, these investors have no informational advantage over other investors when it comes to the mean. But as the mean μ increases, keeping the size of the earnings shock δ fixed, the impact of δ relative to μ on the firm's stock price decreases. This diminishes the informational advantage of the informed investors and hence their ex-post trading profits. This causes the measure of informed investors to shrink as μ, and hence, the volatility-to-mean ratio, increases. This suggests the following result.

Proposition 2: *The measure of informed investors, θ^*, depends on the ratio of the earnings volatility to the earnings mean, i.e.,*

$$\theta^*(\hat{\mu},\hat{\delta})=\theta^*(\hat{\delta}/\hat{\mu}). \tag{8}$$

Further, θ^* is an increasing function of this ratio, $\hat{\delta}/\hat{\mu}$.

The following result shows that the equilibrium measure of informed investors determines the expected trading losses of liquidity investors at $t = 2.5$.

Lemma 5: *The expected trading losses of liquidity investors equal the information-acquisition cost of informed investors.*

The expected trading profit of any informed investor equals the marginal informed investor's cost of acquiring the signal in equilibrium. All investors face same information-acquisition cost, so the expected trading profits of all informed investors equal their information-acquisition cost. The profits come entirely at the expense of liquidity investors because uninformed investors make zero expected profit when they clear market. The expected losses of liquidity investors thus equal the information-acquisition cost of informed investors. This means that these expected losses are proportional to the measure of informed investors. In light of Proposition 2, the expected losses of liquidity investors are thus increasing in the earnings volatility-to-mean ratio.

IV. Earnings Smoothing

To understand the manager's incentives to smooth earnings, we need to discuss the form of his compensation contract, the choices available to him in reporting earnings and the impact of earnings reports on investors' perceptions of the firm's earnings stream. The manager will choose a reporting strategy that maximizes his expected compensation, taking into account the effect of earnings reports on the investors' perceptions and consequently on his compensation.

Manager's Compensation

Since moral hazard due to unobservable effort is precluded by assumption, current shareholders design the compensation contract to induce the manager to report earnings to maximize the value of their shares. The compensation contract may, in principle, be a function of all the observable variables. The shareholders may make the compensation contingent on the reported earnings in the three periods and on the prices after earnings reports at $t = 1$ and $t = 2$.

The shareholders' claims are residual to the manager's compensation. To avoid the complication of considering the effect of the manager's compensation on the firm's cash flows, we assume that the manager's compensation is very small compared to the earnings level. This enables us to ignore the effect of a change in the manager's compensation on the earnings.

We assume that the manager's compensation is proportional to

$$\alpha(e_{r1} + P_1) + (1-\alpha)P_2, \tag{9}$$

where $0 \leq \alpha \leq 1$. The compensation is linked to prices at $t = 1$ and $t = 2$. The earnings and price at $t = 1$ are added to remove the obvious distortions in manager's reporting that would arise if a dollar paid as dividend were weighed differently from a dollar saved for the future.

Managerial Discretion

The earnings reported by the manager may vary from the economic earnings by an amount $d > 0$, i.e., $| e_{a1} - e_{r1} | \leq d$. The manager, however, has to "clear the position" created by this manipulation in earnings. If reported earnings at $t = 1$ were higher than economic earnings by an amount x, then reported earnings at $t = 2$ must be less than the economic earnings by an amount x. Similarly, lower-than-economic first-period reported earnings must be followed by higher-than-economic second-period-earnings. The total economic earnings in the first two periods thus equal the total reported earnings in the first two periods.

$$e_{a1} + e_{a2} = e_{r1} + e_{r2} . \tag{10}$$

The discretion available to the manager is assumed low enough to permit investors to infer from the reported earnings whether the earnings shock was positive or negative in any period.

Assumption 3:

(a) $(\bar{\mu} - \underline{\mu}) < 2(\underline{\delta} - d)$

(b) $(\bar{\mu} - \underline{\mu}) < \underline{\delta}$

Condition (a) ensures that the market can always use reported earnings at $t = 1$ to infer if there is a positive or a negative shock. This is because the highest earnings report possible with a negative shock is less than the lowest earnings report possible with a positive shock, i.e.,

$$\bar{\mu} - \underline{\delta} + d < \underline{\mu} + \underline{\delta} - d .$$

Thus, investors use the following rule to infer the kind of shock experienced by earnings at $t = 1$:

$$\lambda_1 = \begin{cases} \text{high} & \text{if } e_{r1} \geq \underline{\mu} + \underline{\delta} - d \\ \text{low} & \text{if } e_{r1} \leq \bar{\mu} - \underline{\delta} + d . \end{cases} \tag{11}$$

The manager, however, observes the economic earnings and thus updates as follows:

$$\lambda_t = \begin{cases} \text{high} & \text{if } e_{at} \geq \underline{\mu} + \underline{\delta} \\ \text{low} & \text{if } e_{at} \leq \overline{\mu} - \underline{\delta}. \end{cases} \tag{12}$$

Condition (b) ensures that the market can infer whether the earnings at $t = 2$ experienced a positive or a negative shock after observing the earnings reports for the first two periods. Investors use the following rule to determine the kind of earnings shock experienced at $t = 2$:

$$\begin{aligned} &\lambda_1 = \lambda_2 = high \quad \text{if } \frac{e_{r1} + e_{r2}}{2} \geq \underline{\mu} + \underline{\delta} \\ &\lambda_1 = \lambda_2 = low \quad \text{if } \frac{e_{r1} + e_{r2}}{2} \leq \overline{\mu} - \underline{\delta} \\ &\lambda_1 \text{ and } \lambda_2 \text{ are different if } \underline{\mu} \leq \frac{e_{r1} + e_{r2}}{2} \leq \overline{\mu}. \end{aligned} \tag{13}$$

Perceptions about Earnings Stream

Everyone has common prior beliefs about the parameters of the earnings stream, given by (1) and (3). The economic earnings value conveys some information about the mean μ and the volatility δ. When earnings experience a positive shock, the earnings value equals $\mu + \delta$. Therefore, a high economic earnings value is likely to be associated with a high mean and a high volatility. When earnings experience a negative shock, the earnings value equals $\mu - \delta$. Therefore, a high economic earnings value is likely to be associated with a high mean and a low volatility. The exact effect depends on the distributions of μ and δ. We assume that the uncertainty about the mean is low compared to the uncertainty about the volatility in a sense made precise below.

Assumption 4:

(a) The probability distributions g and h are such that when earnings experience a positive shock, a marginal increase in economic earnings increases the ratio of the conditional expected value of volatility to the conditional expected value of the mean of earnings. That is,

$E[(\delta/\mu) \mid \mu + \delta = S]$ increases with S.

(b) The density functions of μ and δ are sufficiently smooth, so that

$$E\left[\delta \middle| \delta \geq \bar{\delta} - r, \mu + \delta = \bar{\mu} + \bar{\delta} - s\right] \leq \bar{\delta} - kr \quad \forall s \geq r, r \leq 2d, \text{ where } k \in (0,1).$$

(c) $\frac{d}{dS}E[\delta \mid \mu - \delta = S] \leq 0, \ \frac{d}{dS}E[\mu \mid \mu - \delta = S] \geq 0.$

Conditions (a), (b) and (c) refer to rational beliefs formed after observing economic earnings. Condition (a) ensures that during positive earnings shocks, any increase in earnings increases the perceived coefficient of variation (volatility/mean) of the earnings stream. Condition (b) ensures that the density functions of μ and δ are smooth so that, conditional on the sum $\mu + \delta$ (the economic earnings in a high shock period) and a lower bound on δ, the expected value of δ is sufficiently close to the lower bound. Condition (c) means that with a negative earnings shock, a higher value of economic earnings leads to a perception of lower volatility and higher mean.

Assumption 4 is needed to ensure that the manager's incentive to smooth earnings is strong compared to his incentive to inflate earnings. Earnings smoothing reduces the perceived volatility of earnings, and earnings inflation increases the perceived mean of earnings. The former strategy is effective when there is significant uncertainty about the volatility of earnings and the latter is effective when there is significant uncertainty about the mean of earnings. The conditions in Assumption 4 assert that the uncertainty about the volatility is high compared to the uncertainty about the mean, so the manager's incentive to smooth earnings is strong.

Assumption 4 formalizes how the value of economic earnings affects investors' beliefs about μ and δ. Recall that investors observe only reported earnings, not economic earnings. The effect, however, is qualitatively the same if the manager's report is a monotone function of the economic earnings. Suppose that the manager's reporting strategy is to report higher earnings, e_{r1}, for higher values of economic earnings e_{a1}. Then the beliefs formed by investors based on observing the earnings report have the following characteristics:

1. With a positive shock to earnings, higher reported earnings e_{r1} leads to higher $\hat{E}^1[\delta / \mu]$.

2. With a negative shock to earnings, higher reported earnings e_{r1} leads to higher $\hat{E}^1[\mu]$ and lower $\hat{E}^1[\delta]$.

These characteristics determine how reported earnings affect prices P_1 and P_2. Suppose investors have some beliefs about the manager's reporting strategy. If the manager does not follow this strategy and reports higher earnings at $t = 1$, then he fools investors into believing that the mean level of earnings is higher. At $t = 2$, the manager will have to offset this effect and then investors will form correct beliefs about the mean μ. Thus, the higher perception about μ has a positive effect on P_1 but no effect on P_2.

The effect of the manager's report on investors' perceptions about earnings volatility is not so straightforward. When there is a negative shock to earnings, a high report lowers perceived volatility δ. When there is a positive shock to earnings, a high report raises perceived volatility. A high volatility is undesirable from the perspective of shareholders because then they expect to lose more at $t = 2.5$, as discussed in the previous section. These expected losses affect prices P_1 and P_2. Thus, if the manager increases reported earnings, its effect on perceived volatility increases expected prices during negative shocks but decreases expected prices during positive shocks.

The manager's report affects P_1 and P_2 through its combined effect on the perceived mean and volatility of earnings. During a negative shock, a high earnings report has an unambiguously favorable effect on prices. During a positive shock, a high earnings report has a favorable effect on P_1 because of a higher perceived μ, but an unfavorable effect on both P_1 and P_2 because of a higher perceived δ. Since the manager's compensation is an increasing function of P_1 and P_2, he has an incentive to inflate earnings if earnings experience a negative shock, but the incentives are less clear if earnings experience a positive shock. In Proposition 3, we specify conditions under which the manager wants to report low earnings during a positive shock to earnings. We show that the manager smoothes the earnings to the maximum extent possible. The earnings report at $t = 1$ as a function of the economic earnings under smoothing strategy is characterized by:

$$e_{r1} = \begin{cases} e_{a1} - d & \text{if} \quad \lambda_1 = high \\ e_{a1} + d & \text{if} \quad \lambda_1 = low. \end{cases} \tag{14}$$

The market is assumed to set the prices competitively equal to the expected value of future cash flows from a share. Thus, the pricing function is:

$$\begin{aligned} P_1 &= \hat{E}^1\left[e_{a2} + e_{a3} + (e_{a1} - e_{r1}) - M\,\theta^*(\mu,\delta)\right], \\ P_2 &= \hat{E}^2\left[e_{a3} - M\,\theta^*(\mu,\delta)\right]. \end{aligned} \tag{15}$$

The prices in (15) are set as expected values of future cash flows to the shareholders conditional on the information available to them; what an investor is willing to pay for a share depends on his expected payoff from owning that share. Given the possibility of a liquidity shock, the investor cannot be sure of holding the share until the terminal date. This means that the investor's expected payoff differs from the firm's expected terminal cash flow; it also depends on the possible trading losses the investor could incur, given a liquidity shock *prior* to the terminal date. The price at $t = 1$ equals the expected value of the economic earnings in the second and third periods plus any balance from the first period's earnings (positive if the manager reported lower than economic earnings and negative if the manager reported higher than economic earnings) minus shareholders' expected liquidity-trading losses. The price at $t = 2$ equals the expected value of third-period earnings minus expected liquidity-trading losses.

Proposition 3: *For sufficiently low values of α, the weight placed in the manager's compensation contract on the cum-dividend share price at $t = 1$, the following constitutes a sequential equilibrium for the game in the first two periods:*

- *The manager's strategy is to smooth and report earnings according to (14) and (10).*
- *The market's strategy is to price competitively according to (15), using (1) and (2) to form expectations about past and future earnings, with its beliefs about μ, δ and λ_1 as specified below.*

- *The manager's beliefs: The manager observes economic earnings (only e_{a1} at $t = 1$ while both e_{a1} and e_{a2} at $t = 2$), uses (12) to infer λ_1 and uses (1), (3) to form beliefs about the distributions of μ and δ.*
- *The market's beliefs:*

At $t = 1$:

The market uses (11) to infer λ_1. Its beliefs about economic earnings are:

$$e_{a1} = \begin{cases} \min\left(e_{r1} + d, \overline{\mu} + \overline{\delta}\right) & \text{if } \lambda_1 = high \\ \max\left(e_{r1} - d, \underline{\mu} - \overline{\delta}\right) & \text{if } \lambda_1 = low \end{cases},$$

and its beliefs about μ and δ are based on (1) and (3).

At $t = 2$:

The market uses (11) and (13) to infer λ_1 and λ_2. Its beliefs about μ, δ and economic earnings are:

If $\lambda_1 = \lambda_2$, *then* $e_{a1} = e_{a2} = \dfrac{e_{r1} + e_{r2}}{2}$ *and use (1), (3) to form beliefs about μ and δ.*

If $\lambda_1 \neq \lambda_2$, *then* $\mu = \dfrac{e_{r1} + e_{r2}}{2}, \delta = \begin{cases} \min\left(e_{r1} + d - \mu, \overline{\delta}\right) & \text{if } \lambda_1 = high \\ \min\left(\mu - e_{r1} + d, \overline{\delta}\right) & \text{if } \lambda_1 = low, \end{cases}$

and use (1) to infer e_{a1} and e_{a2}.

The above proposition shows that when the manager's compensation relies sufficiently heavily on the price P_2 at $t = 2$, the manager smoothes earnings to the maximum extent possible. Recall that the manager always has an incentive to report higher-than-economic earnings when the economic earnings experience a negative shock at $t = 1$. However, when earnings experience a positive shock, a high report has two opposing effects. On the one hand, it decreases P_1 and P_2 due to a high perceived volatility. On the other hand, it increases P_1 due to a high perceived mean. When the impact of volatility is more significant (see Assumption 4(a)) and the manager's compensation is very sensitive to price P_2, the first effect dominates and the manager reports lower-than-economic earnings. Investors, however, rationally anticipate the smoothing behavior of the manager in equilibrium and infer the economic earnings from the reported earnings to form correct beliefs about the mean and the volatility.

It would appear that earnings smoothing reduces the perceived volatility of earnings and discourages information acquisition. If this were true, earnings smoothing would enhance welfare because the information-acquisition cost is a welfare loss in this model. Information production simply consumes resources and redistributes wealth across investors; the improved transparency of prices due to informed trading has no welfare effect in this model. However, in our analysis earnings smoothing does not improve or worsen social welfare. This is because investors anticipate smoothing in equilibrium and therefore account for it in forming their beliefs about volatility.

It may seem surprising that smoothing would occur in a model in which there is a one-to-one mapping from economic to reported earnings and investors are not fooled by the smoothing in equilibrium. The reason is that, given investors' equilibrium expectation that smoothing is taking place, any firm that does not smooth is worse off. Thus, what we have is a form of prisoner's dilemma. Smoothing is initially motivated by the firm's desire to reduce perceived earnings volatility, but since all firms smooth, investors rationally "undo" the effect of smoothing, thereby leaving volatility perceptions unchanged. Knowing that this is the equilibrium outcome, however, does not make it privately optimal for any firm to avoid smoothing, since not smoothing when everybody else is smoothing is not the same as not smoothing when no one is smoothing.

An interesting question this raises is whether there are multiple equilibria in this model, one of which is an equilibrium in which no firm smoothes. The following result says no.

Proposition 4: *For sufficiently low α, there is no Nash Equilibrium in which the manager does not smooth reported earnings.*

The intuition is as follows. If no firm smoothes in equilibrium, then investors expect all firms to accurately report economic earnings. It would now pay for a firm with high earnings volatility to smooth reported earnings to fool investors into giving it a higher price than its value to shareholders. Thus, it cannot be an equilibrium for all firms to eschew smoothing.

It is possible to visualize an alternative model in which there is partial pooling so that a particular report is associated with many possible values of economic earnings. Investors will now be unable to perfectly distinguish between firms with smooth and volatile earnings. The firms with the most volatile earnings (bad-type) will get their first-best outcome whereas firms with truly smooth earnings (good-type) will be worse-off than in a world with no smoothing.

The effect of earnings smoothing is to reduce the time-series volatility of reported earnings. This is true however, only in an *expected* sense. Sometimes smoothing will not lower volatility because the ability of the manager to smooth is limited. Since the manager can only shift earnings from one period to another, he cannot always report earnings that are higher (or lower) than economic earnings. If the earnings are high in first period, the manager reports lower earnings and tries to spread the positive shock over two periods. The manager must then report earnings higher than the economic earnings in the second period. If second-period economic earnings turn out to be even higher than first-period earnings, the manager's report may end up increasing the measured time-series volatility of reported earnings. Thus, there can be instances where smoothing fails in *hindsight*. But the ex ante probability of such positive serial correlation is low, so smoothing does reduce the expected value of the time-series volatility of reported earnings.

We now justify the manager's compensation scheme. We assume that the manager's compensation contract, written by representatives of the shareholders, is not visible to other investors. The shareholders cannot do better than writing a contract to align the manager's interests with their interests.[12]

We have assumed that no trading takes place before t = 2.5. The manager's reporting strategy affects e_{r1} and e_{r2}, which in turn determine the price P_2. Shareholders want to write a contract that maximizes their payoffs up to $t = 2$. The payoff of a person holding a share until $t = 2$ is $e_{r1} + e_{r2} + P_2 = e_{a1} + e_{a2} + P_2$. The manager cannot influence e_{a1} or e_{a2}, so shareholders want to maximize P_2 and write a compensation contract proportional to P_2, i.e., set $\alpha = 0$.

We can relax the assumption of no trading between $t = 1$ and $t = 2$. Suppose that when the contract is written at $t = 0$, shareholders expect to sell π shares between $t = 1$ and $t = 2$. The payoff from selling a share in this period is $e_{r1} + P_1$, whereas the expected payoff from holding a share beyond $t = 2$ is $e_{r1} + e_{r2} + P_2 = e_{a1} + e_{a2} + P_2$. Therefore, shareholders write a compensation contract proportional to $\pi(e_{r1} + P_1) + (1 - \pi)P_2$. This is exactly the form we have in (9) with $\alpha = \pi$. Thus, if the fraction of shares expected to be sold between $t = 1$ and $t = 2$ is sufficiently small, the compensation contract written by the shareholders induces the manager to smooth earnings.

V. A Numerical Example

In this section we illustrate the smoothing equilibrium with an example. The earnings in any period are $\mu + \delta$ with a positive shock (probability 0.5), and $\mu - \delta$ with a of negative shock (probability 0.5). The earnings mean, μ, is \$4 or \$5 with probability 0.5 each and the earnings volatility, δ, is \$2 or \$3 with probability 0.5 each. The manager can report earnings that differ from the economic earnings by at most \$1. The cost of acquiring a private signal is \$0.20 for each investor. Investors have \$1.20 each so that any investor getting informed is left \$1.00 to invest. The dollar measure of liquidity supply at $t = 2.5$ varies between 0 and 1. Liquidity supply l has a triangular distribution that peaks in the middle.

$$f(l) = \begin{cases} 4(0.5 - |l - 0.5|) & \text{if } 0 \leq l \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Investors who spend \$0.20 to become informed at $t = 2.5$ observe their private signal about the earnings shock at $t = 3$ and submit their demand for shares only if earnings are to experience a positive shock. Discretionary uninformed investors see the sum of the flow of liquidity and informed trades. They use the aggregate order flow to form Bayesian beliefs about the nature of the shock to earnings in the next period and then set a market-clearing price. The equilibrium measure of investors who choose to become informed, $\theta^*(\mu,\delta)$, is such that the expected profit from trading in the future equals \$0.20. The expected profit and hence the measure of informed investors depends on the characteristics of earnings. We numerically compute $\theta^*(\mu,\delta)$ for different μ and δ combinations.

$$\begin{aligned}
&\theta^*(5,2)=0,\\
&\theta^*(4,2)=0.21566,\\
&\theta^*(5,3)=0.31163,\\
&\theta^*(4,3)=0.40824.
\end{aligned} \tag{17}$$

This measure of informed investors is increasing in the ratio of the earnings volatility to the earnings mean. Liquidity investors suffer the cost of information-acquisition so their expected trading losses equal the measure of informed investors times the individual cost of information acquisition. The price at $t = 2$, which represents the expected future payoffs to the holder of a share, equals the mean of earnings minus expected trading losses. Denoting this function by $P^*(\mu,\delta)$, we have

$$\begin{aligned}
&P^*(5,2)=\$5,\\
&P^*(4,2)=\$3.957,\\
&P^*(5,3)=\$4.938,\\
&P^*(4,3)=\$3.918.
\end{aligned} \tag{18}$$

We assume that the manager's objective is linearly increasing in the price at $t = 2$. The manager will report earnings smoothed to the maximum extent possible. The equilibrium smoothing strategy is defined as follows:

If first-period economic earnings experience negative shock ($3 or less),

the manager reports earnings $1 higher than the economic earnings.

If first-period economic earnings experience positive shock ($6 or more), (19)

the manager reports earnings $1 lower than the economic earnings.

In the second period, the manager has no choice. He must report the aggregate of any outstanding "earnings balance" from the previous period and the second-period earnings. To show that the manager smoothes in a sequential equilibrium, we specify the beliefs held by investors.

<u>*Beliefs of Investors at $t = 1$:*</u>

If reported earnings in the first period are low ($4 or less), economic earnings are $1 lower than the reported earnings.

If reported earnings in the first period are high ($5 or more), economic earnings are $1 higher than the reported earnings.

<u>*Beliefs of Investors at $t = 2$:*</u> (20)

If reported earnings are high or low in both periods, the economic earnings in each period equal the average of the two reported earnings.

If reported earnings are high in one period and low in the other, then

(i) *the mean of economic earnings is the average of the two reported earnings,*

(ii) *the economic earnings are $1 less than the lower reported earnings and $1 more than the higher reported earnings, and*

(iii) *the volatility is the difference between the mean and the economic earnings in either of the two periods.*

With these beliefs, investors set the price as the expected value of future payoffs. At $t = 2$, this price (ex-dividend) is given by the function in (18) if the mean and the volatility are both known, and by a weighted average of these values if there is uncertainty about the mean and the volatility. Thus, if investors know that the sum of the mean and the volatility is $7, a mean of $5 and a volatility of $2 is as likely as a mean of $4 and a volatility of $3. In that case, the price at $t = 2$ is the average of $5 and $3.918, that is, $4.459.

The price at $t = 1$ is the sum of the expected reported earnings at $t = 2$ and the expected price at $t = 2$. Thus, if the manager reports earnings of $2 at $t = 1$, investors believe that economic earnings are $1 and that the manager has inflated his report by $1, which he will have to offset in his report of next period's earnings. With $1 earnings, the mean must be $4 and the volatility must be $3. Therefore, the expected value of next period's reported earnings is $4 - $1 = $3 and the expected price at $t = 2$ is $3.918. Therefore, the price at $t = 1$ is $6.918.

Given the beliefs of investors, the manager smoothes earnings. To conserve space, we show this only for one case of high first-period earnings and one case of low first-period earnings.

Consider economic earnings of $7 in the first period. This means that earnings experienced a positive shock. As seen from (20), investors will form correct beliefs about the mean of the earnings after observing the reported earnings in the two periods. The manager wants to minimize investors' estimate of volatility. If earnings are again high ($7) in the second period, investors will infer that the economic earnings were $7 in each period regardless of what the manager reports. The only case in which manager's report may influence investor perceptions is when earnings are low in the second period. If the manager smoothes as expected and reports $6, investors will deduce that economic earnings were $7 in first period and form correct beliefs. If, however, the manager doesn't smooth, investors may sometimes overestimate volatility.

For example, suppose the manager truthfully reports $7. Investors will think that the economic earnings were actually $8. Suppose economic earnings in the second period are $3. The manager reports $3 truthfully but this time the investors will think economic earnings were $2. In this case, the investors correctly infer the mean to be $5 but overestimate the volatility to be $3 instead of the actual $2. The price set in the second period will then be $4.938 although it should have been $5. The manager will smooth in order to maximize the second-period price.

Smoothing also occurs in the case of low earnings. Suppose earnings in the first period are $2. If the manager smoothes and reports $3, investors infer economic earnings correctly. If, however, the manager doesn't smooth, investors may overestimate volatility. Suppose the manager chooses to truthfully report $2. Investors then think that economic earnings were $1. If economic earnings in the second period turn out to be $6, the manager again truthfully reports $6. This time investors think that the economic earnings were \$7. They then correctly infer the mean to be \$4 but overestimate the volatility to be \$3 instead of the true value of \$2. The price set at $t = 2$ is then \$3.918, whereas it should have been \$3.957. The manager is once again better off smoothing.

VI. Robustness and Qualifications

Capital Structure Assumptions

We have assumed the firm has no debt. One implication of introducing debt is that it could act as a signal of future cash flows (e.g., Ross (1977)). However, in our model there is no informational asymmetry about future cash flows at $t = 0$ when the debt issue decision would presumably be made. The manager becomes privately informed only when he observes the economic earnings at $t = 1$. Thus, there is no apparent signaling role for debt in the usual sense.

There is another aspect of debt that could, however, affect the analysis. Boot and Thakor (1993) show that debt magnifies the volatility of shareholders' claims, encourages information acquisition and thereby increases the expected trading losses of liquidity investors. Thus, keeping the volatility of earnings fixed, a higher debt-to-equity ratio increases the price discount of equity due to expected trading losses[13], strengthening the manager's smoothing propensity.

Acquisition of Private Information by Shareholders

An apparently strong assumption in our analysis is that shareholders are selling for liquidity reasons and none acquires private information. This simplifying assumption can be relaxed, however. The analysis uses the fact that the expected losses of shareholders due to liquidity trading are proportional to the information-acquisition costs of the informed investors. An alternative would be to allow some of the shareholders to get informed and endogenously determine how many shareholders become informed. In such a model, the marginal benefit of getting informed equals the cost of information acquisition in equilibrium, taking into account the possibility of liquidity-motivated trading losses in the future. It is difficult to specify choice of information acquisition, trading and pricing in this setting without imposing further structure. However, the analysis goes through even if some informed investors are shareholders as long as all the liquidity investors are also shareholders of the firm, so that the information-acquisition costs of all informed investors are still paid by the shareholders. The key is that the measure of informed investors should be increasing in earnings volatility; this seems likely because high earnings volatility increases the ex ante value of private information.

Buying and Selling by Liquidity Investors

We can also relax the restriction that all liquidity-motivated investors are sellers; we could allow some to be buyers, and some of these buyers to be existing shareholders. In this case, the losses of liquidity trading will be borne by all these noise (liquidity-motivated) investors. What we need is the assumption that a person owning the shares of the firm is expected to lose more due to liquidity trading than a person not owning shares. In this case, what matters to current shareholders is the expected loss due to liquidity trading that is in excess of the expected loss due to liquidity trading if they didn't own shares.

Cost of Smoothing

We have assumed that the discretion exercised by the manager does not have any real effects on firm performance. In practice, there will be real costs of smoothing, like the cost of manipulating accounts and the cost of suboptimal decisions. We have assumed that these costs are zero when the manager manipulates earnings by less than d and infinite when the manager manipulates earnings by more than d. We suspect that a continuous cost function will not alter the flavor of the results, although the extent of smoothing may be reduced.

Observability of Manager's Compensation Contract

We have also assumed that the compensation contract is not observable to outsiders. However, this may be relaxed. If the contract becomes observable to outsiders after the trading at $t = 2.5$, it does not change the analysis. Usually the details of the compensation received by senior corporate executives in a given year are released to investors in subsequent years. Further, executive compensation that is reported to investors concentrates on the easily quantifiable monetary benefits only. In practice, managers enjoy many non-monetary benefits, which can be influenced by the shareholders but cannot be seen by all investors.

Benefits of Information Acquisition

We did not assume any public benefits to information acquisition. When information-acquisition results in spillover externalities that lead to better decisions within the firm, some of the benefits of information acquisition will also accrue to shareholders (see Allen (1993), Allen and Gale (1999), and Boot and Thakor (1997)). Earnings smoothing should be less for such firms.

Extension to Multiple Periods

It is natural to wonder how robust our results are when the model is extended from three periods to perpetuity. So we now provide a discussion of this issue. This requires that we make some simplifying assumptions. First, we assume that the mean is known with certainty and is common knowledge. As discussed earlier, uncertainty about the mean relative to the uncertainty about the volatility determines whether the manager smoothes or inflates earnings. A known mean eliminates the manager's incentive to inflate earnings. We also modify the earnings distribution so that a finite number of observations are not sufficient to completely infer the distribution. Assume that earnings in each period are independent and identically distributed. The distribution is normal with mean taken to be zero without losing generality and an unknown variance. Everyone has common prior beliefs about the volatility.

Only the manager sees the economic earnings, based on which he reports, with some discretion, to investors. He can report higher or lower earnings and carry a balance to the next period. This balance, either a surplus or a deficit, cannot exceed a fraction α of the current-period earnings. In the next period, the balance carried forward and the new earnings are available to the manager. The manager must now report total earnings that include the balance and at least a fraction $1 - \alpha$ of the new earnings and at most a fraction $1 + \alpha$ of the new earnings.

Investors rationally form beliefs about economic earnings by observing reported earnings. Some shareholders experience liquidity shocks in a future period, and this is followed by trading. Some investors can acquire a private signal about the firm's prospects before trading in that period; they don't buy such signals in prior periods as nobody would trade with them in the absence of liquidity shocks. The trading mechanism is exactly the same as in our three-period model[14]. It can be shown that the incentives for getting informed are increasing in the perceived volatility of the earnings stream. Thus, the higher the perceived volatility of the earnings distribution, the higher the measure of informed investors and the higher the expected losses of liquidity sellers, leading to a lower price of the shares. If the manager's objective function is some weighted average of prices in different periods, he wants to reduce perceived volatility in those periods. One should intuitively expect the manager to report smoothed earnings. We show that this is the case.

Define the equilibrium smoothing strategy as one in which manager reports earnings equal to the sum of the balance from the previous period and a fraction 1- α of the current period's earnings. Suppose investors expect the manager to follow this strategy in every period.

The smoothing strategy is a one-to-one mapping from economic earnings to reported earnings. Further, investors and the manager agree that the previous balance in the first period is zero. If the manager smoothes in the first period, investors infer the economic earnings as well as the true balance. Then, if the manager again smoothes in the second period, investors again infer the economic earnings and the new balance in the second period. Thus, if the manager smoothes consistently, investors infer the economic earnings in each period. The volatility they infer from reported earnings is an unbiased estimate of the true earnings volatility. With a normal distribution, it can be shown that the sum of the squared economic earnings is a sufficient statistic for estimating volatility. The higher this sum, the higher is the estimate of volatility.

Consider a deviation by the manager from the equilibrium smoothing strategy in any period. Suppose that when the economic earnings in that period are positive, the manager reports higher earnings than that dictated by the smoothing strategy. Investors now infer earnings to be higher than economic earnings. If the economic earnings are negative, suppose the manager reports lower earnings than dictated by the smoothing strategy. In this case, investors would infer earnings to be lower than economic earnings. Thus, investors perceive earnings to have a wider dispersion than that of economic earnings, which means that their estimate of volatility is biased upward. Further, we show that after such a deviation, this bias tends to increase in future periods.

Consider a period in which the balance from the previous period is B but investors believe it is C. First assume that $B > C$. Suppose economic earnings are x and the manager reports y. Investors expect the manager to smooth and report $C + (1 - \alpha)z$ when economic earnings are z. Therefore, from the report y, they infer that economic earnings are $(y - C)/(1 - \alpha)$. If the manager wants to minimize the magnitude of inferred earnings z, the best he can do is to report a value as close as possible to C. With this strategy, the manager's report and investors' beliefs are:

$$
\begin{array}{ll}
y = B + x(1-\alpha),\ z = \dfrac{B-C}{1-\alpha} + x,\ B' = \alpha x,\ C' = \alpha z & \text{if } x > 0 \\
y = B + x(1+\alpha),\ z = \dfrac{B-C}{1-\alpha} + \dfrac{x(1+\alpha)}{(1-\alpha)},\ B' = -\alpha x,\ C' = \alpha z & \text{if } \dfrac{C-B}{1+\alpha} < x < 0 \\
y = C,\ z = 0,\ B' = B - C + x,\ C' = 0 & \text{if } \dfrac{C-B}{1-\alpha} < x < \dfrac{C-B}{1+\alpha} \\
y = B + x(1-\alpha),\ z = \dfrac{B-C}{1-\alpha} + x,\ B' = \alpha x,\ C' = \alpha z & \text{if } x < \dfrac{C-B}{1-\alpha}
\end{array}
$$

where B' is the balance carried to next period and C' is what investors think it is. We can show that for any value of B and C, the expected value of z^2 is more than the expected value of x^2 when x is normally distributed. We can also see that the investors' beliefs about next period's balance are also incorrect. Thus, even when the manager tries his best to reduce the magnitude of z, the signal about volatility from this period is biased upward.

We can similarly consider the case where investors' perception of the balance C is higher than the actual balance B. In this case, the manager can reduce investors' perception of economic earnings z using the following strategy:

$$y = B + x(1-\alpha),\ z = \frac{B-C}{1-\alpha} + x,\ B' = \alpha x,\ C' = \alpha z \qquad \text{if } x > \frac{C-B}{1-\alpha}$$

$$y = C,\ z = 0,\ B' = B - C + x,\ C' = 0 \qquad \text{if } \frac{C-B}{1+\alpha} < x < \frac{C-B}{1-\alpha}$$

$$y = B + x(1+\alpha),\ z = \frac{B-C}{1-\alpha} + \frac{x(1+\alpha)}{(1-\alpha)},\ B' = -\alpha x, C' = \alpha z \qquad \text{if } 0 < x < \frac{C-B}{1+\alpha}$$

$$y = B + x(1-\alpha),\ z = \frac{B-C}{1-\alpha} + x,\ B' = \alpha x,\ C' = \alpha z \qquad \text{if } x < 0$$

We can show in this case also that the expected value of z^2 is more than the expected value of x^2. The balance for next period is also different from what investors think. Thus, we see that whenever investors' beliefs about the balance in a given period are incorrect, they overestimate the volatility in that period. The manager can thus minimize perceived volatility by smoothing.

VII. Conclusion

We have developed a model in which earnings smoothing is motivated by the desire to reduce the perceived volatility of the firm's earnings stream and discourage investors from spending resources to acquire private information that could then be used to trade against shareholders selling for liquidity reasons.

We can extract a few empirical implications from our analysis. Proposition 3 shows that the manager smoothes earnings when his compensation is tied to P_2.[15] Thus, an empirical implication is that a firm whose manager's compensation contract is tied to long-run performance is more likely to smooth earnings than a firm whose manager's compensation contract is tied to short-term performance. That is, somewhat surprisingly, earnings smoothing is just not something that arises from a preoccupation with short-term performance.

We have also argued that the manager's compensation contract should be tied to long-term performance.[16] This appears to be the case as a major component of executive compensation these days is in the form of stock options that are given when the firm reports good results in a period, but are valuable only if the firm continues to perform well in future periods.

A second prediction of the analysis is that the degree of earnings smoothing will be higher for firms with higher uncertainty about the volatility of their earnings stream. In contrast, the managers of firms with high uncertainty about their earnings mean and low uncertainty about their earnings volatility are more likely to report inflated earnings.

A third prediction is that firms with large institutional ownership will smooth less because institutions are less likely to sell for liquidity reasons. When there are few liquidity investors, trading is driven largely by informed investors and the order flow reflects a lot of private information of informed investors. The advantage of getting informed is low in such a setting and few non-institutional investors acquire costly information. Thus, if institutional investors have owned the firm's shares for a while and enjoy "incumbency informational advantages," the manager will not smooth earnings significantly. If the volatility of economic earnings is not correlated with the degree of institutional ownership, we should expect reported earnings to be more volatile for firms with large institutional holdings[17]. This however, seems to be at odds with the existing evidence[18]. We suspect this is because the volatility of earnings is correlated with some other attribute (such as firm size) that is considered by institutional investors in choosing stocks.

It might, however, be better to test our prediction more directly. The prediction is that greater institutional ownership is associated with less earnings smoothing. Rajgopal, Venkatachalam, and Jiambalvo (1999) test this prediction and find supporting results.

A fourth prediction is that firms with more diffuse ownership structures – numerous shareholders with each owning a relatively small fraction of the firm – will smooth earnings more. This is because we would expect small shareholders to be more likely to sell their shares for liquidity reasons. This prediction can be tested with shareholder concentration data.

Future research could be directed at examining the potential asset pricing implications of earnings smoothing. For instance, smoothing may affect the liquidity of the firm's stock at different points in time, and this could affect stock price dynamics. It would also be interesting to examine why firms that smooth reported earnings also smooth dividends relative to reported earnings.

Appendix

Proof of Lemma 1: From (6),

$$P(S,\theta)=\frac{\frac{f(S+\theta)}{f(S)}(\hat{\mu}+\hat{\delta})+(\hat{\mu}-\hat{\delta})}{\frac{f(S+\theta)}{f(S)}+1}$$

is increasing in $f(S+\theta)/f(S)$ so it suffices to show that $f(S+\theta)/f(S)$ is decreasing in S. Its first derivative with respect to S is

$$\frac{f(S+\theta)}{f(S)}\left\{\frac{f'(S+\theta)}{f(S+\theta)}-\frac{f'(S)}{f(S)}\right\}$$

which is negative as $f'(x)/f(x)$, the slope of $\ln(f(x))$, is decreasing in x if f is log-concave.

Proof of Lemma 2: We shall show that

$$\frac{-\hat{\delta} f^*}{(\hat{\mu}+\hat{\delta})}<\frac{\partial}{\partial\theta}m(\hat{\mu},\hat{\delta},\theta)<0, \tag{A1}$$

where $f^*=f(l^*)$.

From (6) and (7),

$$m(\mu,\delta,\theta)=\frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}}\frac{f(l)}{P(l-\theta,\theta)}dl\quad-\frac{1}{2}$$

$$=\frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}}\frac{f(l)+f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl\quad-\frac{1}{2}.$$

Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta)=\frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}}\frac{-f'(l-\theta)[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]+f'(l-\theta)(\mu-\delta)[f(l)+f(l-\theta)]}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}f(l)dl$$

$$=-(\mu+\delta)\delta\int_{\underline{l}}^{\bar{l}}\frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl \tag{A2}$$

Let f achieve its maximum at l^*. Breaking the integral in (A2) into two parts, we get

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta)=-(\mu+\delta)\delta(I_1+I_2),$$

where

$$I_1 = \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$

$$= \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)}{\left[(\mu+\delta)+\frac{f(l-\theta)}{f(l)}(\mu-\delta)\right]^2}dl$$

$$> \frac{1}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} \int_{\underline{l}}^{l^*+\theta} f'(l-\theta)dl$$

since log-concavity of f implies that $f'(l) > 0$ for $l < l^*$ and $f(l-\theta)/f(l)$ is increasing in l (see Proof of Lemma 1). Moreover,

$$I_2 = \int_{l^*+\theta}^{\bar{l}} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$

$$> \frac{1}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} \int_{l^*+\theta}^{\bar{l}} f'(l-\theta)dl$$

since log-concavity of f implies that $f'(l) < 0$ for $l < l^*$ and $f(l-\theta)/f(l)$ is increasing in l. Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) < \frac{-(\mu+\delta)\delta}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} \int_{\underline{l}}^{\bar{l}} f'(l-\theta)dl$$

$$= \frac{-(\mu+\delta)\,\delta\, f(\underline{l}-\theta)}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} < 0.$$

To prove the other inequality, we have

$$I_1 = \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$

$$< \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)]^2}dl$$

$$=\frac{1}{(\mu+\delta)^2}\int_{\underline{l}}^{l^*+\theta} f'(l-\theta)dl \quad =\frac{f(l^*)}{(\mu+\delta)^2},$$

and $I_2 < 0$. Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) > -(\mu+\delta)\delta\frac{f(l^*)}{(\mu+\delta)^2} = \frac{-\delta f^*}{\mu+\delta}.$$

Proof of Proposition 1: We want to show that there is a unique value of θ^* satisfying the conditions of equilibrium in Definition 1. Taking the supply and price functions as asserted, consider the expected trading profit m of each informed investor as a function of θ. This is:

$$m(\hat{\mu},\hat{\delta},0)=\frac{1}{2}\int_{\underline{l}}^{\bar{l}}\frac{\hat{\mu}+\hat{\delta}-P(l,0)}{P(l,0)}f(l)dl$$

$$=\frac{1}{2}\int_{\underline{l}}^{\bar{l}}\frac{\hat{\mu}+\hat{\delta}-\hat{\mu}}{\hat{\mu}}f(l)dl \quad =\frac{\hat{\delta}}{2\hat{\mu}} \quad as\ P(S,0)=\hat{\mu}$$

$> M$, by Assumption 2.

Further, $m(.,.,\theta)$ is a strictly decreasing function of θ by Lemma 2. Therefore, there is a unique $\theta^* > 0$ that satisfies the following condition for competition among informed investors:

$$m(\hat{\mu},\hat{\delta},\theta^*) = M$$

Proof of Lemma 3: (a) For the effect of perceived volatility on expected trading profit, we shall show that

$$\frac{\partial}{\partial\delta}m(\mu,\delta,\theta) > \frac{\sigma_f^2}{4(\mu+\delta)(\bar{l}-\underline{l})^2} > 0, \tag{A3}$$

where σ_f^2 is the variance of liquidity supply l.

From (6) and (7),

$$m(\mu,\delta,\theta)=\frac{\mu+\delta}{2}\int^{\bar{l}}\frac{f(l)+f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl - \frac{1}{2}$$

$$=\frac{1}{2}\int^{\underline{l}+\theta} f(l)dl + \frac{1}{2}\int_{\underline{l}+\theta}^{\bar{l}}\frac{f(l)\ \ldots\ f(l-\theta)+\ldots}{f(l)\ \ldots\ +\mu-\delta}f(l)dl - \frac{1}{2} \tag{A4}$$

Thus,

$$\frac{\partial}{\partial\delta}m(\mu,\delta,\theta)=\frac{1}{2}\int_{\underline{l}+\theta}^{\bar{l}}\frac{\partial}{\partial\delta}\left[\frac{f(l)/f(l-\theta)+1}{f(l)/f(l-\theta)+\mu-\delta/\mu+\delta}\right]f(l)dl$$

$$=\frac{\mu}{2(\mu+\delta)^2}\int_{\underline{l}+\theta}^{\bar{l}}\frac{f(l)/f(l-\theta)+1}{\left(f(l)/f(l-\theta)+\mu-\delta/\mu+\delta\right)^2}f(l)dl$$

$$>\frac{\mu}{2(\mu+\delta)^2}\int_{\underline{l}+\theta}^{\bar{l}}\frac{1}{f(l)/f(l-\theta)+\mu-\delta/\mu+\delta}f(l)dl$$

$$=\frac{\mu}{2(\mu+\delta)}\int_{\underline{l}+\theta}^{\bar{l}}\frac{f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl$$

$$=\frac{\mu}{2(\mu+\delta)}\left\{\int_{\underline{l}+\theta}^{l^*}\frac{f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl+\int_{l^*}^{\bar{l}}\frac{f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl\right\}$$

$$\geq\frac{\mu}{2(\mu+\delta)}\left\{\int_{\underline{l}+\theta}^{l^*}\frac{f(l-\theta)f(l)}{f(l)(\mu+\delta)+f(l)(\mu-\delta)}dl+\int_{l^*}^{\bar{l}}\frac{f(l-\theta)f(l)}{f(l-\theta)(\mu+\delta)+f(l-\theta)(\mu-\delta)}dl\right\}$$

$$=\frac{1}{4(\mu+\delta)}\left\{\int_{\underline{l}}^{l^*-\theta}f(l)dl+\int_{l^*}^{\bar{l}}f(l)dl\right\}$$

$$=\frac{\text{Probability}\left(\left|l-\left(l^*-\theta/2\right)\right|>\theta/2\right)}{4(\mu+\delta)}$$

$$=\frac{Probability\left(|y|>\theta/2\right)}{4(\mu+\delta)},\ where\ y=l-\left(l^*-\theta/2\right)$$

$$\geq\frac{E\left[y^2\,\middle|\,|y|>\theta/2\right]}{y_{\max}^2 4(\mu+\delta)},\ \text{where } y_{\max}=\max|y|\leq\bar{l}-\underline{l}.$$

$$\geq\frac{E\left[y^2\right]}{4(\mu+\delta)\left(\bar{l}-\underline{l}\right)^2}=\frac{Var(y)+E[y]^2}{4(\mu+\delta)\left(\bar{l}-\underline{l}\right)^2}$$

$$\geq\frac{\sigma_f^2}{4(\mu+\delta)\left(\bar{l}-\underline{l}\right)^2},\ \text{where}\ \ \text{Var}(y)=Var(l)=\sigma_f^2.$$

(b) From equation (A4), we see that $m(\mu,\delta,\theta)$ is decreasing in $(\mu-\delta)/(\mu+\delta)$ which is increasing in μ.

Proof of Lemma 4: (a) We shall show that

$$\frac{\partial}{\partial\delta}\theta^*(\mu,\delta)>\frac{\sigma_f^2}{4f^*\bar{\delta}\left(\bar{l}-\underline{l}\right)^2}>0. \tag{A5}$$

In equilibrium, the measure of informed investors is such that the profit of marginal informed investor is M, a constant. This implies

$$\frac{\partial}{\partial\delta}\theta^*(\mu,\delta)=\frac{-\frac{\partial}{\partial\delta}m\left(\mu,\delta,\theta^*\right)}{\frac{\partial}{\partial\theta^*}m\left(\mu,\delta,\theta^*\right)}>\frac{-\frac{\sigma_f^2}{4(\mu+\delta)\left(\bar{l}-\underline{l}\right)^2}}{\frac{-\delta f^*}{\mu+\delta}}=\frac{\sigma_f^2}{4\delta f^*\left(\bar{l}-\underline{l}\right)^2},$$

using equations (A1) and (A3).

(b) Using Lemma 2 and Lemma 3 and noting that

$$\frac{\partial}{\partial\mu}\theta^*(\mu,\delta)=\frac{-\frac{\partial}{\partial\mu}m\left(\mu,\delta,\theta^*\right)}{\frac{\partial}{\partial\theta^*}m\left(\mu,\delta,\theta^*\right)}<0.$$

Proof of Proposition 2: From (A4), we see that $m(\mu,\delta,\theta)$ depends on μ and δ only through $(\mu-\delta)/(\mu+\delta)$ which equals $(1-\mu/\delta)/(1+\mu/\delta)$. Further, $m(\mu,\delta,\theta)$ is decreasing in $(\mu-\delta)/(\mu+\delta)$ which is again decreasing in δ/μ. Thus, m is increasing in δ/μ. We know from Lemma 2 that m is decreasing in θ^*. So when δ/μ increases, θ^* must also increase to keep m unchanged at M, a condition for equilibrium.

Proof of Lemma 5: The unconditional value of the firm is μ. The expected value of future payoffs to the shareholders is however lower and the difference constitutes the expected value of losses suffered by liquidity investors. When liquidity induced supply is l, shareholders get \$$l$ at $t = 2.5$ by selling some shares at market price and get liquidating dividend for the remaining shares at $t = 3$.

The market price at t = 2.5 depends on whether the informed demand shares or not and is $P(l-\theta^*, \theta^*)$ when third period earnings experience positive shock and $P(l,\theta^*)$ when third period earnings experience negative shock. The liquidating dividend per share is $\mu + \delta$ in the former case and $\mu - \delta$ in the latter. The expected losses L are thus given by

$$L=\mu-\frac{1}{2}\int\left\{l+\left(1-\frac{l}{P(l)}\right)(\mu-\delta)\right\}f(l)dl-\frac{1}{2}\int\left\{l+\left(1-\frac{l}{P(l-\theta^*)}\right)(\mu+\delta)\right\}f(l)dl$$
$$=-\frac{1}{2}\int\left\{l-\frac{l(\mu-\delta)}{P(l)}\right\}f(l)dl-\frac{1}{2}\int\left\{l-\frac{l(\mu+\delta)}{P(l-\theta^*)}\right\}f(l)dl.$$

Thus,

$$-2L=\int\left\{(l-\theta^*)-\frac{(l-\theta^*)(\mu-\delta)}{P(l-\theta^*)}\right\}f(l-\theta^*)dl+\int\left\{l-\frac{(l-\theta^*)(\mu+\delta)}{P(l-\theta^*)}\right\}f(l)dl-\theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl$$
$$=\int(l-\theta^*)f(l-\theta^*)dl+\int lf(l)dl-\int(l-\theta^*)\frac{(\mu-\delta)f(l-\theta^*)+(\mu+\delta)f(l)}{P(l-\theta^*)}dl-\theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl$$
$$=\int(l-\theta^*)f(l-\theta^*)dl+\int lf(l)dl-\int(l-\theta^*)\{f(l-\theta^*)+f(l)\}dl-\theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl$$
$$=\theta^*\int\left\{1-\frac{\mu+\delta}{P(l-\theta^*)}\right\}f(l)dl$$
$$=-2\theta^* m(\mu,\delta,\theta^*)$$

The third equality is obtained by substituting for price function from equation (6) and the last equality uses equation (7). Since, $m(\mu,\delta,\theta^*)=M$ in equilibrium, we get $L = M\theta^*$, the cost of information-acquisition by informed investors.

Proof of Proposition 3: We shall first show that the beliefs formed by the manager and the market are rational given their strategies, and then show that their strategies are optimal given their beliefs.

Manager's Beliefs

The manager observes e_{a1} at t = 1 and e_{a2} at t = 2 and can correctly infer λ_1 at t = 1 and λ_2 at t = 2 using (12). With his knowledge of e_{at} and λ_t, he forms Bayesian beliefs about μ and δ using (1) and (3).

Market's Beliefs

The market observes only the reported earnings e_{rt}, yet it can also correctly infer the nature of the shock to earnings, i.e., λ_1 and λ_2 at $t = 1$ and $t = 2$, respectively using (11) and (13). The manager's strategy at $t = 1$ (in accordance with (14)) is shown in Figure 2. When investors observe an equilibrium earnings report, they form beliefs in accordance with the manager's strategy and infer the following:

If $\lambda_1 = \text{high}$, then $e_{a1} = e_{r1} + d$. (Note $e_{r1} + d \le \overline{\mu} + \overline{\delta}$ for equilibrium e_{r1})

If $\lambda_1 = \text{low}$, then $e_{a1} = e_{r1} - d$. (Note $e_{r1} - d \ge \underline{\mu} - \overline{\delta}$ for equilibrium e_{r1})

When investors observe an off-equilibrium earnings report, the sequential equilibrium concept allows them to hold any plausible beliefs about e_{a1} (see Kreps and Wilson (1982)), which can lead to observed reported earnings. One possible off-equilibrium earnings report is:

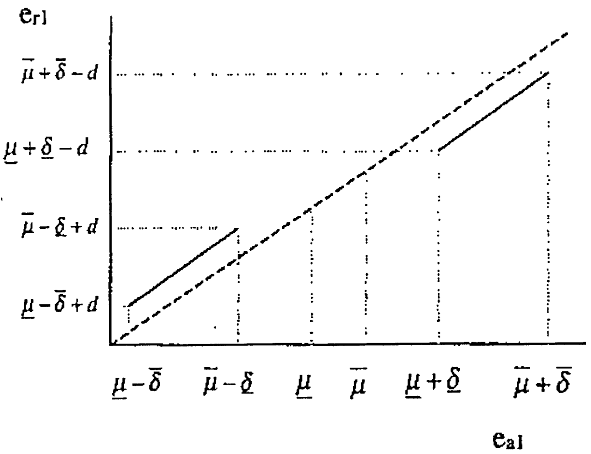
- $e_{r1} \ge \overline{\mu} + \overline{\delta} - d$

Here reported earnings are greater than the maximum possible in the equilibrium set. The market holds the plausible belief that the economic earnings are at the highest level possible i.e., $e_{a1} = \overline{\mu} + \overline{\delta}$. Another possible off-equilibrium earnings report is:

- $e_{r1} \le \underline{\mu} - \overline{\delta} + d$

Here reported earnings are lower than the minimum possible in the equilibrium set in. The market holds the plausible belief that the economic earnings are at the lowest level possible i.e., $e_{a1} = \underline{\mu} - \overline{\delta}$.

Figure 2: Smoothing Strategy



Having formed beliefs about λ_1 and e_{a1}, the market must use (1) and (3) to form Bayesian beliefs about μ and δ. At $t = 2$, the market can correctly infer λ_1 and λ_2 using (11) and (13).

If $\lambda_1 = \lambda_2$, then $e_{a1} = e_{a2}$. Now, from (10) we get

$$e_{a1} = e_{a2} = \frac{e_{r1} + e_{r2}}{2}.$$

Once the market knows e_{a1} and e_{a2}, it has all the information the manager has and can form Bayesian beliefs about μ and δ using (1) and (3).

If λ_1 and λ_2 are different, economic earnings are $\mu + \delta$ in one period and $\mu - \delta$ in the other. Therefore, the sum of earnings is 2μ. The market can use (10) to correctly infer μ as follows:

$$\mu = \frac{e_{r1} + e_{r2}}{2}.$$

Thus, at $t = 2$ the market always has same beliefs about μ as the manager. The market forms rational beliefs about δ. If λ_1 is high, the manager's equilibrium strategy is $e_{r1} = e_{a1} - d = \mu + \delta - d$, so the market uses the rule $\delta = e_{r1} + d - \mu$. If this value of δ is too high to be feasible, the market infers that the manager didn't smooth as expected and reported higher than what he should have in equilibrium. In this case, the market assumes that the economic earnings are the highest feasible value conditional on μ and that δ is the highest possible value. Thus, the market believes

$\delta = \min(e_{r1} + d - \mu, \bar{\delta})$.

If λ_1 is low, the manager's equilibrium strategy is $e_{r1} = e_{a1} + d = \mu - \delta + d$ so the market uses the rule $\delta = \mu - e_{r1} + d$. If this value of δ is too high to be feasible, the market infers that the manager didn't smooth as expected and reported lower than what he should have in equilibrium. In this case, the market assumes that the economic earnings are the lowest feasible value conditional on μ and that δ is the highest possible value. Thus, the market believes

$\delta = \min(\mu - e_{r1} + d, \bar{\delta})$.

Market's Strategy

The market forms rational expectations about future earnings using (1) and (2) with its beliefs about μ and δ. The price set by market in (15) is the competitive price set by the marginal shareholder. Using the fact that expected value of future earnings is μ, we can simplify (15) to

$$P_1 = \begin{cases} \hat{E}^1\left[3\mu + \delta - M\theta^*(\mu,\delta)\right] - e_{r1} & \text{if } \lambda_1 = high \\ \hat{E}^1\left[3\mu - \delta - M\theta^*(\mu,\delta)\right] - e_{r1} & \text{if } \lambda_1 = low \end{cases} \qquad \text{(A6)}$$
$$P_2 = \hat{E}^2\left[\mu - M\theta^*(\mu,\delta)\right]$$

Manager's Strategy

The manager has no choice in his second-period report. For the first period, let $Z(x,y)$ denote the manager's objective function, dependent on reported earnings x and economic earnings y. The manager's problem is to choose

$$e_{r1} = \underset{x \in [e_{a1} - d, e_{a1} + d]}{\arg\max} Z(x, e_{a1}).$$

The function Z is

$$Z(x, y) = E\left[\alpha(e_{r1} + P_1) + (1-\alpha)P_2 \middle| e_{r1} = x, e_{a1} = y\right]$$

We first consider the case of a positive earnings shock in the first period, that is, $\lambda_1 = high$. We shall analyze the two parts in $Z(x,y)$ separately. First, we consider the value of shares at $t = 1$.

$$
\begin{aligned}
&E\left[e_{r1}+P_1 \middle| e_{r1}=x, e_{a1}=y\right] \\
&= E\left[e_{r1}+\hat{E}^1\left[3\mu+\delta-M\theta^*(\mu,\delta)\right]-e_{r1} \middle| e_{r1}=x, e_{a1}=y\right] \\
&= E\left[\hat{E}^1[\mu+\delta]+\hat{E}^1\left[2\mu-M\theta^*(\mu,\delta)\right] \middle| e_{r1}=x, e_{a1}=y\right] \\
&= \hat{E}^1\left[e_{a1} \middle| e_{r1}=x\right]+\hat{E}^1\left[2\mu \middle| e_{r1}=x\right]-M\theta^*\left(\hat{E}^1\left[\mu \middle| e_{r1}=x\right], \hat{E}^1\left[\delta \middle| e_{r1}=x\right]\right).
\end{aligned}
$$

In the last equality, we have used the fact that θ^* is linear in μ and δ, so we can use the expected values of μ and δ to get the expected value of θ^*. At $t = 1$, the market believes that

$$e_{a1}=\min\left(e_{r1}+d, \overline{\mu}+\overline{\delta}\right)$$

Therefore,

$$
\begin{aligned}
&E\left[e_{r1}+P_1 \middle| e_{r1}=x, e_{a1}=y\right] \\
&= \min\left(x+d, \overline{\mu}+\overline{\delta}\right)+2E\left[\mu \middle| \mu+\delta=\min\left(x+d, \overline{\mu}+\overline{\delta}\right)\right] \\
&\quad -M\theta^*\left(E\left[\mu \middle| \mu+\delta=\min\left(x+d, \overline{\mu}+\overline{\delta}\right)\right], E\left[\delta \middle| \mu+\delta=\min\left(x+d, \overline{\mu}+\overline{\delta}\right)\right]\right)
\end{aligned}
\tag{A7}
$$

The above expression becomes independent of x for $x > \overline{\mu}+\overline{\delta}-d$. For lower values of x,

$$
\begin{aligned}
&\frac{\partial}{\partial x}E\left[e_{r1}+P_1 \middle| e_{r1}=x, e_{a1}=y\right] \\
&= 1+2\frac{d}{dx}E\left[\mu \middle| \mu+\delta=x+d\right]-M\frac{d}{dx}\theta^*\left(E\left[\mu \middle| \mu+\delta=x+d\right], E\left[\delta \middle| \mu+\delta=x+d\right]\right) \\
&\le 3-M\frac{d}{dx}\theta^*\left(E\left[\mu \middle| \mu+\delta=x+d\right], E\left[\delta \middle| \mu+\delta=x+d\right]\right) \\
&\le 3.
\end{aligned}
$$

The first inequality follows from Assumption 4(a). The second inequality follows from Assumption 4(a) and Proposition 2. Thus,

$$
\frac{\partial}{\partial x}E\left[e_{r1}+P_1 \middle| e_{r1}=x, e_{a1}=y\right] \begin{cases} =0 & \text{if } x>\overline{\mu}+\overline{\delta}-d \\ <3 & \text{if } x<\overline{\mu}+\overline{\delta}-d. \end{cases}
\tag{A8}
$$

Now, we consider the expected value of price at $t = 2$.

$$
\begin{aligned}
&E\left[P_2 \middle| e_{r1}=x, e_{a1}=y\right] \\
&= E\left[\hat{E}^2\left[\mu-M\theta^*(\mu,\delta)\right] \middle| e_{r1}=x, e_{a1}=y\right] \\
&= E\left[\hat{E}^2[\mu] \middle| e_{r1}=x, e_{a1}=y\right]-M\theta^*\left(E\left[\hat{E}^2[\mu] \middle| e_{r1}=x, e_{a1}=y\right], E\left[\hat{E}^2[\delta] \middle| e_{r1}=x, e_{a1}=y\right]\right).
\end{aligned}
\tag{A9}
$$

We saw earlier that at $t = 2$ the market has the same beliefs about μ as the manager. Therefore,

$$E\left[\hat{E}^2[\mu]\middle| e_{r1} = x, e_{a1} = y\right] = E\left[\mu \middle| e_{r1} = x, e_{a1} = y\right] = E\left[\mu \middle| e_{a1} = y\right].$$

The market's beliefs about δ at $t = 2$ will match the manager's beliefs if earnings experience another positive shock in the second period. Otherwise, the market believes that

$$\delta = \min\left(e_{r1} + d - \mu, \bar{\delta}\right).$$

Thus,

$$E\left[\hat{E}^2[\delta]\middle| e_{r1} = x, e_{a1} = y\right] = \frac{1}{2}E\left[\delta \middle| e_{a1} = y\right] + \frac{1}{2}E\left[\min\left(x + d - \mu, \bar{\delta}\right)\middle| e_{a1} = y\right].$$

Making the above substitutions in equation (A9) we get

$$E\left[P_2 \middle| e_{r1} = x, e_{a1} = y\right] \tag{A10}$$
$$= E\left[\mu \middle| e_{a1} = y\right] - M\theta^*\left(E\left[\mu \middle| e_{a1} = y\right], \frac{1}{2}E\left[\delta \middle| e_{a1} = y\right] + \frac{1}{2}E\left[\min\left(x + d - \mu, \bar{\delta}\right)\middle| e_{a1} = y\right]\right).$$

We see from equations (A7) and (A10) that the manager's objective function is independent of x for $x > \bar{\mu} + \bar{\delta} - d$. Therefore, we only need to compare reporting strategies with $x \leq \bar{\mu} + \bar{\delta} - d$. We shall now show that when the economic earnings are y, the manager prefers to report $y - d$ rather than any other report x with $|y - x| \leq d$ and $x \leq \bar{\mu} + \bar{\delta} - d$. Consider the incremental benefit to the manager of reporting x rather than following the equilibrium strategy.

$$\begin{aligned} &Z(x, y) - Z(y - d, y) \\ &= \alpha\left\{E\left[e_{r1} + P_1 \middle| e_{r1} = x, e_{a1} = y\right] - E\left[e_{r1} + P_1 \middle| e_{r1} = y - d, e_{a1} = y\right]\right\} \\ &\quad + (1 - \alpha)\left\{E\left[P_2 \middle| e_{r1} = x, e_{a1} = y\right] - E\left[P_2 \middle| e_{r1} = y - d, e_{a1} = y\right]\right\}. \end{aligned}$$

Using equations (A8) and (A10) we get

$$Z(x, y) - Z(y - d, y)$$
$$\leq 3\alpha\{x - (y - d)\} + (1 - \alpha)M\left\{\begin{array}{l} \theta^*\left(E\left[\mu \middle| e_{a1} = y\right], \frac{1}{2}E\left[\delta \middle| e_{a1} = y\right] + \frac{1}{2}E\left[\min\left(x + d - \mu, \bar{\delta}\right)\middle| e_{a1} = y\right]\right) \\ -\theta^*\left(E\left[\mu \middle| e_{a1} = y\right], \frac{1}{2}E\left[\delta \middle| e_{a1} = y\right] + \frac{1}{2}E\left[\min\left(y - \mu, \bar{\delta}\right)\middle| e_{a1} = y\right]\right) \end{array}\right\}$$

In the above expression

$$y - \mu = e_{a1} - \mu = \delta \leq \bar{\delta}.$$

Thus,

$$
\begin{aligned}
&Z(x,y)-Z(y-d,y)\\
&\le 3\alpha(x-y+d)-(1-\alpha)M\begin{Bmatrix}\theta^*\left(E[\mu|e_{a1}=y]\ ,\frac{1}{2}E[\delta|e_{a1}=y]+\frac{1}{2}E[\min(x+d-\mu,\bar{\delta})\ |\ e_{a1}=y]\right)\\ -\theta^*\left(E[\mu|e_{a1}=y]\ ,E[\delta|e_{a1}=y]\right)\end{Bmatrix} \qquad \text{(A11)}\\
&\le 3\alpha(x-y+d)-\frac{(1-\alpha)M\sigma_f^2}{4f^*\bar{\delta}(\bar{l}-\underline{l})^2}\left\{\frac{1}{2}E[\min(x+d-\mu,\bar{\delta})\ |\ e_{a1}=y]-\frac{1}{2}E[\delta|e_{a1}=y]\right\}.
\end{aligned}
$$

The last inequality is obtained using equation (A5). Now we analyze the last term in the above expression.

$$
\begin{aligned}
&E[\min(x+d-\mu,\bar{\delta})\ |e_{a1}=y]-E[\delta|e_{a1}=y]\\
&=E[x+d-\mu|x+d-\mu<\bar{\delta},e_{a1}=y]P(x+d-\mu<\bar{\delta}|e_{a1}=y)\\
&\quad+\bar{\delta}P(x+d-\mu\ge\bar{\delta}|e_{a1}=y)\\
&\quad-E[\delta|x+d-\mu<\bar{\delta},e_{a1}=y]P(x+d-\mu<\bar{\delta}|e_{a1}=y)\\
&\quad-E[\delta|x+d-\mu\ge\bar{\delta},e_{a1}=y]P(x+d-\mu\ge\bar{\delta}|e_{a1}=y).
\end{aligned}
$$

Let us denote by r the excess of reported earnings x over the equilibrium report $y-d$. That is, $r = x - y + d$. We know $r \le 2d$ because $x \le y + d$. Also denote by ϕ the probability $P(x+d-\mu\ge\bar{\delta}|e_{a1}=y)$. Then

$$
\begin{aligned}
&E[\min(x+d-\mu,\bar{\delta})e_{a1}=y]-E[\delta|e_{a1}=y]\\
&=E[x+d-\mu-\delta|x+d-\mu<\bar{\delta},e_{a1}=y](1-\phi)+\left(\bar{\delta}-E[\delta|x+d-\mu\ge\bar{\delta},e_{a1}=y]\right)\phi \qquad \text{(A12)}\\
&=(x+d-y)(1-\phi)+\left(\bar{\delta}-E[\delta|x+d-\mu\ge\bar{\delta},e_{a1}=y]\right)\phi\\
&=r(1-\phi)+\left(\bar{\delta}-E[\delta|r+y-\mu\ge\bar{\delta},e_{a1}=y]\right)\phi\\
&=r(1-\phi)+\left(\bar{\delta}-E[\delta|\delta\ge\bar{\delta}-r,e_{a1}=y]\right)\phi.
\end{aligned}
$$

But,

$$
E[\delta|\delta\ge\bar{\delta}-r,e_{a1}=y]=E[\delta|\delta\ge\bar{\delta}-r,\mu+\delta=y]=E[\delta|\delta\ge\bar{\delta}-r,\mu+\delta=x+d-r],
$$

and

$$
x+d-r\le(\bar{\mu}+\bar{\delta}-d)+d-r=\bar{\mu}+\bar{\delta}-r,
$$

so we can use Assumption 4(b) to get

$$E\left[\delta \middle| \delta \geq \bar{\delta} - r, e_{a1} = y\right] \leq \bar{\delta} - kr.$$

Using the above inequality and (A12) yields:

$$E\left[\min\left(x + d - \mu, \bar{\delta}\right) \middle| e_{a1} = y\right] - E\left[\delta \middle| e_{a1} = y\right] \geq r(1 - \phi) + kr\phi = r\{1 - (1 - k)\phi\} \geq kr.$$

Using the above inequality and (A11) we get:

$$\begin{aligned} Z(x, y) - Z(y - d, y) &\leq 3\alpha r - \frac{(1 - \alpha) M \sigma_f^2 kr}{8 f^* \bar{\delta} (\bar{l} - \underline{l})^2} \\ &\leq 0 \quad \text{if } \alpha \leq \alpha^* \equiv \frac{M \sigma_f^2 k}{M \sigma_f^2 k + 24 f^* \bar{\delta} (\bar{l} - \underline{l})^2}. \end{aligned}$$

Therefore, if $\alpha < \alpha^*$, the manager prefers to report $e_{a1} - d$ rather than x. This means that the manager's strategy is incentive compatible when first-period earnings experience a positive shock.

We now consider the case of a negative shock to first-period earnings. We again analyze the values of shares at $t = 1$ and $t = 2$ separately.

$$\begin{aligned} & E\left[e_{r1} + P_1 \middle| e_{r1} = x, e_{a1} = y\right] \\ &= E\left[e_{r1} + \hat{E}^1\left[3\mu - \delta - M\theta^*(\mu, \delta)\right] - e_{r1} \;\middle|\; e_{r1} = x, e_{a1} = y\right] \\ &= E\left[\hat{E}^1[\mu - \delta] + \hat{E}^1\left[2\mu - M\theta^*(\mu, \delta)\right] \middle| \; e_{r1} = x, e_{a1} = y\right] \\ &= \hat{E}^1\left[e_{a1} \middle| e_{r1} = x\right] + \hat{E}^1\left[2\mu \middle| e_{r1} = x\right] - M\theta^*\left(\hat{E}^1\left[\mu \middle| e_{r1} = x\right], \hat{E}^1\left[\delta \middle| e_{r1} = x\right]\right) \end{aligned}$$

At $t = 1$ the market believes that

$$e_{a1} = \max\left(e_{r1} - d, \underline{\mu} - \bar{\delta}\right).$$

Therefore,

$$\begin{aligned} & E\left[e_{r1} + P_1 \middle| e_{r1} = x, e_{a1} = y\right] \\ &= \max\left(x - d, \underline{\mu} - \bar{\delta}\right) + 2E\left[\mu \middle| \mu - \delta = \max\left(x - d, \underline{\mu} - \bar{\delta}\right)\right] \\ &\quad - M\theta^*\left(E\left[\mu \middle| \mu - \delta = \max\left(x - d, \underline{\mu} - \bar{\delta}\right)\right], E\left[\delta \middle| \mu - \delta = \max\left(x - d, \underline{\mu} - \bar{\delta}\right)\right]\right) \end{aligned}$$

In the above expression the first term is clearly increasing in x. The second term is also increasing in x because of Assumption 4(c). The third term is again increasing in x because of Assumption 4(c) and Proposition 2. Thus, $E[e_{r1} + P_1 \mid e_{r1} = x, e_{a1} = y]$ is clearly increasing in x.

Now we consider the expected value at $t = 1$ of the price at $t = 2$.

$$\begin{aligned}
&E[P_2 | e_{r1} = x, e_{a1} = y] \\
&= E\left[\hat{E}^2[\mu - M\theta^*(\mu,\delta)] \;|e_{r1} = x, e_{a1} = y\right] \\
&= E\left[\hat{E}^2[\mu] \;|e_{r1} = x, e_{a1} = y\right] - M\theta^*\left(E\left[\hat{E}^2[\mu] \;|e_{r1} = x, e_{a1} = y\right], E\left[\hat{E}^2[\delta] \;|e_{r1} = x, e_{a1} = y\right]\right).
\end{aligned} \tag{A13}$$

But as we saw earlier,

$$E\left[\hat{E}^2[\mu] \mid e_{r1} = x, e_{a1} = y\right] = E[\mu | e_{a1} = y].$$

The market's beliefs about δ at $t = 2$ match the manager's beliefs if earnings experience another negative shock in the second period. If earnings experience a positive shock in the second period, the market believes

$$\delta = \min(\mu - e_{r1} + d, \bar{\delta})$$

Thus,

$$E\left[\hat{E}^2[\delta] \mid e_{r1} = x, e_{a1} = y\right] = \frac{1}{2}E[\delta | e_{a1} = y] + \frac{1}{2}E\left[\min(\mu - x + d, \bar{\delta}) \;|e_{a1} = y\right].$$

Making the above substitutions in equation (A13) we get

$$\begin{aligned}
&E[P_2 | e_{r1} = x, e_{a1} = y] \\
&= E[\mu | e_{a1} = y] - M\theta^*\left(E[\mu | e_{a1} = y], \frac{1}{2}E[\delta | e_{a1} = y] + \frac{1}{2}E\left[\min(\mu - x + d, \bar{\delta}) \;|e_{a1} = y\right]\right).
\end{aligned}$$

In the above expression, the first term is independent of x and the second term is increasing in x because of Lemma 4. Thus, $E[P_2 \mid e_{r1} = x, e_{a1} = y]$ is increasing in x. The manager's compensation increases in $E[e_{r1} + P_1 \mid e_{r1} = x, e_{a1} = y]$ and $E[P_2 \mid e_{r1} = x, e_{a1} = y]$, both increasing in x. Therefore, the manager reports the highest possible earnings i.e., $x = y + d$. This proves that the manager's strategy is incentive compatible in the case of a negative shock to earnings in the first period.

Proof of Proposition 4: Suppose there is an equilibrium in which the manager reports truthfully (no smoothing). Then investors believe that the economic earnings are the same as reported earnings. Consider a negative shock to earnings in the first period. If the manager reports earnings slightly higher than economic earnings, it increases the price at $t = 1$ as well as the expected price at $t = 2$, thereby increasing the manager's expected compensation. At $t = 2$, investors will be able to determine the mean μ correctly, whereas they may underestimate volatility due to smoothing by the manager. Since perceived volatility adversely affects price as discussed in Section IV, smoothing increases the expected price at $t = 2$. At $t = 1$, smoothing increases investors' perception about the mean and reduces their perception about the volatility. Both effects make the shares more attractive to investors, and thus smoothing unambiguously increases the price at $t = 1$.

Now consider a positive shock to earnings. Suppose economic earnings are $y > \overline{\mu} + \underline{\delta}$, so that the volatility is clearly above its lowest bound. Consider the effect of reporting x, slightly smaller than y, on the manager's expected compensation. If the firm experiences a negative shock next period, the manager will report higher than economic earnings and thus would be able to fool investors by reducing the perceived volatility of the firm. This increases the expected price of shares at $t = 2$. Smoothing reduces the perceived mean μ as well as the perceived volatility δ at $t = 1$. These two effects influence the price at $t = 1$ in opposite directions. The expressions for the prices at $t = 1$ and $t = 2$ are given in (A7) and (A10) respectively when investors expect the manager to smooth by d. We cancel terms with d because investors expect the manager to report truthfully, and ignore boundary conditions to get the manager's objective as

$$\alpha\{x + 2E[\mu|\mu+\delta = x] - M\theta^*(E[\mu|\mu+\delta = x], E[\delta|\mu+\delta = x])\} +$$
$$(1-\alpha)\left\{E[\mu|\mu+\delta = y] - M\theta^*\left(E[\mu|\mu+\delta = y], \frac{1}{2}E[\delta|\mu+\delta = y] + \frac{1}{2}(x - E[\mu|\mu+\delta = y])\right)\right\}$$

Differentiating the above expression with respect to x and using Assumption 4(a) and Proposition 2, the slope of the manager's objective with respect to the reported earnings x is at most

$$3\alpha - \frac{(1-\alpha)M}{2}\frac{\sigma_f^2}{4f^*\bar{\delta}(\bar{l}-\underline{l})^2},$$

which is negative if

$$\alpha < \frac{M\sigma_f^2}{M\sigma_f^2 + 24f^*\bar{\delta}(\bar{l}-\underline{l})^2}.$$

Thus, the manager strictly prefers to smooth if α is sufficiently small. When the market expects truthful reporting, the shareholders of the firm have an incentive to structure the manager's compensation with a small enough α to induce smoothing, discourage informed acquisition and thus reduce future trading losses when they experience liquidity shocks.

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Endnotes

[1] See *CPA Journal*, December 1998, pp. 14-19.

[2] See *American Banker*, March 11, 1999, page 1.

[3] See *The Boston Globe*, December 23, 1998, page D1.

[4] Bannister and Newman (1996) and Defond and Park (1997) use discretionary accruals as estimators of smoothing. Moses (1987) considers accounting changes as smoothing device. Michelson, Jordan-Wagner and Wooton (1995) consider the ratio of coefficient of variation in income variable to the coefficient of variation in sales as a measure of smoothing.

[5] Hereafter, earnings shall mean economic earnings and not reported earnings.

[6] As an example, let us consider a firm with stationary earnings with a mean of 100 and an unknown volatility. Suppose the manager wants to smooth earnings. If the earnings realization is x_1 in first period, what can the manager do to make earnings look as smooth as possible? If the firm lives for two periods, an econometrician will estimate volatility by the standard deviation of the reported earnings. If y_1 and y_2 are the reported earnings in the two periods, their estimate of volatility will be $(y_1 - 100)^2 + (y_2 - 100)^2$.

If the manager reports y_1 instead of x_1 in the first period, he must report $y_2 = x_1 + x_2 - y_1$ in the second period. The estimate of volatility will be $(y_1 - 100)^2 + (x_1 + x_2 - y_1 - 100)^2$. Its expected value, $(y_1 - 100)^2 + (x_1 - y_1)^2 + \mathrm{Var}(x_2)$ is minimized when the manager chooses $y_1 = (x_1 + 100)/2$. Thus, the manager divides total expected earnings equally across two periods. For instance, if the realized earnings are 110 in the first period, the manager "spreads" the positive shock of 10 over two periods and reports 105 in first period. This minimizes manager's expectation of the volatility estimate that an econometrician will calculate.

Smoothing however does not always reduce volatility estimate calculated by investors. If the earnings are realized to be 110 in first period, the manager optimally reports earnings of 105. If second-period earnings are x_2, the manager will report $x_2 + 5$. The volatility estimate with smoothing is $(105 - 100)^2 + (x_2 + 5 - 100)^2$. The estimate without smoothing would be $(110 - 100)^2 + (x_2 - 100)^2$. If the second period earnings turn out to be 105 or lower, smoothing reduces the volatility estimate. However, if the second period earnings turn out to be higher than 105, smoothing increases volatility estimate. Since realized earnings are more likely to be below 105 than above (the mean is 100), smoothing is more likely to reduce volatility estimate than to raise it. Further, the reduction in volatility estimate is likely to be larger than the increase in volatility estimate. Thus, the manager can minimize the expected value of estimate of volatility by smoothing.

[7] The intuition is reminiscent of the idea in Brennan and Thakor (1990) that, in choosing between a stock repurchase and a dividend as a cash disbursement mechanism, the manager takes into account the fact that a repurchase forces uninformed shareholders to possibly trade against informed shareholders whereas a dividend does not cause one group of shareholders to be disadvantaged relative to another.

[8] We do not consider rollover of earnings to t = 3 to simplify analysis. This assumption narrows the set of reporting strategies available to the manager. We believe this assumption is not critical for our results.

[9] Some of the existing shareholders may also trade as uninformed investors. In fact, they will be required to clear the market when the demand by informed investors exceeds the supply by liquidity sellers.

[10] This assumption is not critical to the analysis, as we discuss later. We can justify it, however by assuming that current shareholders have exorbitant information acquisition costs or are otherwise wealth-constrained.

[11] The density function f is log-concave if $\log(f(x))$ is concave in x. Some of the standard distributions satisfying this property are uniform, (truncated) normal, and (truncated) exponential.

[12] This holds because we have assumed that the manager's compensation is negligible compared to the earnings level so the only way that compensation affects shareholders' interests is through its impact on manager's reporting behavior. We may equivalently assume that the manager acts in shareholders' interests.

[13] This result *does* depend on the assumption that current shareholders are the liquidity investors.

[14] We shall need a positive mean so that the price is never negative for the trading mechanism to work. We are assuming zero mean just to simplify the expressions for smoothing strategy and a positive mean will not change the results in any way.

[15] Prices formed beyond $t = 2$ don't enter the contract because manager's actions do not directly affect them.

[16] The compensation will be tied to short term performance for firms with high uncertainty about mean and low uncertainty about the volatility.

[17] We thank an anonymous referee for pointing this out.

[18] See, for example, Potter (1992).