Bureau of Business Research
Graduate School of Business Administration
University of Michigan

March, 1971

SOME NEW STATISTICAL METHODS FOR ANALYZING INCOMPLETE BRAND PREFERENCE DATA

WORKING PAPER NO. 28

by

M. J. Karson
Assistant Professor of Statistics
University of Michigan

W. J. Wrobleski
Associate Professor of Statistics
University of Michigan

FOR DISCUSSION PURPOSES ONLY

None of this material is to be
quoted or reproduced without
the express permission of the
Bureau of Business Research

BACKGROUND OF THIS PAPER

This paper presents several new research findings in statistics developed through nonsponsored faculty research activities in the University of Michigan Graduate School of Business Administration. The primary goal of the authors is to extend mathematical methods of statistics to the analysis of data-analytic problems which occur in other functional fields of business administration. Thus it is the intent of the authors to contribute both to the theory of statistics as well as to provide the active business manager with more powerful statistical aids for actually carrying out quantitative decision analyses related to practical problems in business administration involving incomplete information and uncertainty. The authors wish to acknowledge the assistance of Mr. W. E. Miklas in preparing Figures 1-6 and aiding with other helpful computations. Mr. Miklas' help was partially supported by the Faculty Assistance Fund of the University of Michigan

---

Since this paper is going to be submitted for publication elsewhere, its format differs from that usually prescribed by the Bureau.

ABSTRACT

In this paper a probability model is introduced for sampling data frequently encountered where some observations have not been classified into existing population categories and, therefore, are only partially informative. By using this sampling model, which involves a certain conditional probability appearing as an allocation parameter, the problem of estimating various underlying population parameters is discussed in terms of identifiability, consistency, and unbiased estimation. Several estimators related to constrained maximum likelihood estimation and minimum average (weighted) bias estimation are derived and their properties discussed.

1. Introduction

This paper is concerned with statistical inference in sampling situations where a response actually belongs in one of a number of mutually exclusive population categories, but, when the sampling is performed, other categories not included in the original set appear. Consider, for example, the following typical marketing problem. We want to estimate, by means of a two-brand preference study, the proportions of potential product users who prefer Brand A, Brand B, or who have no preference. A random sample of size n is chosen from the population and each sampled individual, after using the products, is asked to state whether he prefers A, B, or has no preference. The philosophical question raised by Odesky [5] of permitting a "no preference" category as an easy way out for the respondent is of no concern here. What is of concern is that new categories arise upon sampling. For example, some people are not at home at the time of the interview or refuse to answer. The former often results in costly and frequently wasteful call-back procedures, while the latter often occurs when income classifications are needed. A survey of the brand preferences of 200 individuals might yield the following data:

	Number of People
Brand A	79
Brand B	66
No preference	7
Not at home	28
Refusal	20

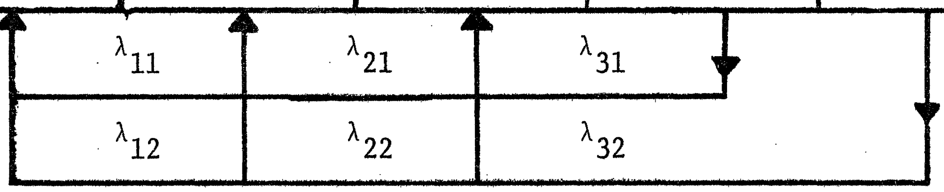
The problem of analyzing such data for the purpose of statistical inference--particularly for estimation of the population proportions for Brand A, Brand B, and "no preference"--is apparent.

A probability model describing, in general, brand preference market surveys which involve I population categories and J new sampling categories can be developed. Consider a random sample of size n drawn from a multinomial population with I categories $K_1, K_2, \ldots, K_I$ and corresponding probabilities $p_1, p_2, \ldots, p_I$, with $\sum_{i=1}^{I} p_i = 1$. In the sample m_i observations fall in K_i, $i=1,2,\ldots,I$, and, for various reasons, J new categories $K_1^*, K_2^*, \ldots, K_J^*$ arise with m_j^* observations in K_j^*. These new categories are called partially informative categories since an observation in K_j^* actually belongs in one of the K_i, but has not been classified in its proper category. Denote by λ_{ij} the probability that a response, observed in K_j^*, actually belongs in K_i; in other words, $\lambda_{ij} = P(K_i | K_j^*)$. Furthermore, let p_j^* be the probability that an observation falls in K_j^*, or $p_j^* = P(K_j^*)$.

In the brand preference example, there are I=3 population categories (Brand A, Brand B, and no preference). Upon sampling, J=2 new sampling categories arise ("not-at-home" and "refusal-to-answer"). Schematically this situation may be represented by the diagram on page 3. Thus the cell probabilities for the I+J=5 categories which actually occur in the market survey explicitly display the allocation parameters λ_{ij} related to the two partially informative sampling categories.

In general, if we let $p = (p_1, p_2, \ldots, p_I)$, $p^* = (p_1^*, p_2^*, \ldots, p_J^*)$, $\lambda = (\lambda_{11}, \lambda_{12}, \ldots, \lambda_{IJ})$ and $\theta = (p, p^*, \lambda)$, then the sampling model is given

Population Categories (I=3)			Sampling Categories (J=2)	
Brand A	Brand B	No Preference	Not at Home	Refusal to Answer
p_1	p_2	p_3	$(\lambda_{11},\lambda_{21},\lambda_{31})$	$(\lambda_{12},\lambda_{22},\lambda_{32})$
$p_1 - \sum_{j=1}^{2} \lambda_{1j} p_j^*$	$p_2 - \sum_{j=1}^{2} \lambda_{2j} p_j^*$	$p_3 - \sum_{j=1}^{2} \lambda_{3j} p_j^*$	p_1^*	p_2^*



by

$$f(m,m^*;\,\theta) = \frac{n!}{\prod_{i=1}^{I} m_i! \prod_{j=1}^{J} m_j^*!} \prod_{i=1}^{I} \left(p_i - \sum_{j=1}^{J} \lambda_{ij} p_j^*\right)^{m_i} \prod_{j=1}^{J} (p_j^*)^{m_j^*} , \qquad (1.1)$$

where $m = (m_1, m_2, \ldots, m_I)$ and $m^* = (m_1^*, m_2^*, \ldots, m_J^*)$ are the data vectors of classified and partially classified observations. Moreover, the parameters in (1.1) are subject to the following restrictions:

$$\sum_{i=1}^{I} \lambda_{ij} = 1, \quad \sum_{j=1}^{J} \lambda_{ij} p_j^* < p_i < 1, \quad 0 < p_j^*, \lambda_{ij} < 1. \qquad (1.2)$$

The model is simply a multinomial distribution for the observed sampling categories with the usual restriction that the sum of the category probabilities for the I+J cells be one. In other words, if K is the event "partially informative," or $K = \bigcup_{j=1}^{J} K_j^*$, then $p_i - \sum_{j=1}^{J} \lambda_{ij} p_j^*$ is the probability of the event "K_i and not K."

This formulation of the general model is distinct from other approaches which have been proposed for analyzing other kinds of data. In particular it differs from multinomial sampling with misclassification, as in Bryson [2] for example. It also differs from the assumption of Draper, Hunter, and Tierney [3] that population categories themselves include the new partially informative sample categories. This last approach does not explicitly introduce allocation parameters for the non-population categories which occur through the sampling process. Finally, it differs from a model of Blumenthal [1] for "nested" kinds of sampling data where the observed measurements are known to fall into certain primary categories but cannot be further subclassified into secondary categories.

The model formulation in the present paper, for example, enables the marketing analyst to explicitly recognize that different patterns of preference behavior might be associated with the different partially informative categories, and that these distinctions can and should be maintained whenever one desires to allocate these partially informative responses back to the original cells. The distinct feature of the model discussed in this paper is the explicit use of the conditional probability parameters, λ_{ij}, which serve as basic allocation parameters from partially informative categories back into the original population categories.

2. Binomial Model with One Partially Informative Category

In this paper we are concerned primarily with the estimation of p_1 in the case when (1.1) corresponds to a binomial model with two informative categories, K_1 and K_2, and a single partially informative category, $K_1^*=K$. For example, a brand preference study might involve only two brands, lack a "no preference" category, and restrict sampling to consumers who have a brand preference among the two products. In the course of sampling, however, a single partially informative category occurs such as "refusal to specify a preference," "can't be reached for interview," or some other kind of nonresponse.

Consider (1.1) when there are two informative categories with one partially informative category occurring in the sample, that is, when I=2, J=1, $\lambda_{11}=\lambda$, $\lambda_{21}=1-\lambda$, and $p_1^*=p_o$, $m_1^*=m_o$. Then the sampling model for a binomial population in which one partially informative sampling category occurs is simply

$$f(m_1,m_o;\ \theta) = \frac{n!}{m_o!m_1!m_2!}\, p_o^{\,m_o}\, (p_1-\lambda p_o)^{m_1}\, (p_2-(1-\lambda)p_o)^{m_2}, \qquad (2.1)$$

where the parameter $\theta = (p_1,p_o,\lambda)$ is restricted to the admissable parameter space given by

$$\Theta = \{(p_1,p_o,\lambda)\,|\,0<p_o,p_1,\lambda<1, \lambda p_o<p_1<\lambda p_o+(1-p_o)\}. \qquad (2.2)$$

3. Estimation of Constrained Maximum Likelihood under Identifiability Conditions

The sampling model (2.1) is unidentifiable with respect to $\theta = (p_1,p_o,\lambda)$ since different values of θ may correspond to the same sampling density. In fact, of the three parameters, only p_o is identifiable. One consequence of this is that there is no consistent estimator of p_1 or of λ. Furthermore, the maximum likelihood equations $\delta f/\delta p_1 = 0$, $\delta f/\delta p_o = 0$, and $\delta f/\delta\lambda = 0$ are not linearly independent, and there are infinitely many solutions, $\hat{\theta} = (\hat{p}_1, \hat{p}_o, \hat{\lambda})$.

In this section we assume certain a priori knowledge about the parameters of (2.1) which leads to the model's identifiability, and the constrained maximum likelihood estimators for p_1 are found. The four following cases are considered:

1. When p_o and λ are known
2. When λ alone is known
3. When λ is unknown and $\lambda = p_1$
4. When $\lambda/(1-\lambda) = \delta(p_1/p_2)$ with $\delta>0$ known

When p_o and λ are known, the single maximum likelihood equation for p_1 has a unique solution given by

$$\hat{p}_1 = \lambda p_o + (m_1/(m_1+m_2))(1-p_o), \tag{3.1}$$

provided not all observations are unclassified, namely, provided $m_o \neq n$. Noting that

$$P(K_1) = P(K_1|K)P(K) + P(K_1|\overline{K})P(\overline{K}), \tag{3.2}$$

where $\overline{K}$ is the complement of K, or the union of the informative categories, the maximum likelihood estimator of p_1 simply estimates $P(K_1|\overline{K})$ by $m_1/(m_1+m_2)$. In this case $\hat{p}_1$ is the uniformly minimum variance unbiased estimator of p_1.

In the case when λ is known the maximum likelihood equations yield the solutions

$$\hat{p}_o = m_o/n \quad \text{and} \quad \hat{p}_1 = (m_1+\lambda m_o)/n. \tag{3.3}$$

For these estimators, $E(\hat{p}_o) = p_o$ and $E(\hat{p}_1) = p_1$. In addition, $\hat{p}_o$ and $\hat{p}_1$ are complete and sufficient statistics for p_o and p_1, and, consequently, they are the uniformly minimum variance unbiased estimators of p_o and p_1.

This estimator of p_1 weights the observed proportion of partially informative observations by λ and allocates this fraction to the observed proportion of classified observations in K_1.

The condition $\lambda = p_1$ means that whether an observation belongs to one population category or the other is independent of whether or not it is a partially classified observation. In this case the maximum likelihood estimators of p_o and p_1 are

$$\hat{p}_o = m_o/n \quad \text{and} \quad \hat{p}_1 = m_1/(m_1+m_2). \tag{3.4}$$

Again, these estimators are the uniformly minimum variance unbiased estimators of p_o and p_1.

In the fourth case above, the ratio of the conditional category probabilities is assumed to be proportional to the ratio of the unconditional category probabilities, and it is assumed that the proportionality constant δ is known. The previous assumption, $\lambda = p_1$, is equivalent to $\delta = 1$. In general, when $\delta \neq 1$, the maximum likelihood equations are quadratics in p_1 and the solutions to these equations lead to the estimators

$$\hat{p}_o = m_o/n \quad \text{and} \quad \hat{p}_1 = \frac{\Delta-[\Delta^2-4nm_1(1+\delta)]^{1/2}}{2n(1-\delta)}, \tag{3.5}$$

where $\Delta=m_1+\delta m_2+(1-\delta)n$. For δ between 0 and 1, the numerical values of this estimator of p_1 are bounded by m_1/n, the maximum likelihood estimator of p_1 in a model with no partially informative categories, and $m_1/(m_1+m_2)$, the maximum likelihood estimator of p_1 when $\lambda = p_1$.

4. Unbiased Estimation of p_1 without Identifiability Conditions

As previously discussed in Section 2, no consistent estimator for p_1 exists in (2.1) without a priori identifiability conditions.

Besides consistency the question of unbiased estimation of p_1 also arises. Clearly, an unbiased estimator of p_1 or of any polynomial of finite degree in p_1 does not exist. This follows since the expected value of any function of the data contains multinomial terms of nonzero degree in λ and p_o with nonzero coefficients. Nor can the ratios p_1/p_2 or p_2/p_1 be estimated without bias. Actually, there does not exist any unbiased estimator for any function $h(p_1, p_o, \lambda)$ of the three parameters p_1, p_o, and λ which is partially differentiable in p_1 and for which $\delta h/\delta p_1 \neq 0$ when evaluated at $\lambda = 1$ and $p_o = 1$.

Alternately, since unbiased estimators for p_1 do not exist, one might consider families of estimators having given parametric forms for their bias and seek an estimator whose variance achieves the Cramer-Rao lower bound. Since the model (2.1) is unidentifiable, however, it can be shown that the information matrix which appears in the expression for the Cramer-Rao lower bound is singular, and that consequently no unique lower bound of the Cramer-Rao type exists.

5. Mean Square Error Comparisons of Two Estimators

In this section we consider the general problem of estimating p_1 when no a priori knowledge of any form about the parameters is available. We proceed in this case by developing certain canonical estimators for p_1 and comparing these estimators in terms of their mean square errors, with particular attention on the bias component of the mean square error. In particular, we compare a so-called "natural" estimator and an estimator selected as a canonical representative with respect to a statistical optimality criterion from a family of potential estimators of p_1.

In many brand preference studies leading to (2.1) for the probability density of the data, a common procedure for estimating p_1 is to ignore the m_o observations falling in K and to estimate p_1 by the "natural" estimator,

$$\hat{p}_1 = m_1/(m_1+m_2), \text{ when } m_o<n. \tag{5.1}$$

For example, suppose that 20 respondents in a brand preference study expressed preference for Brand A, 48 for Brand B, and 32 did not express their preference. For these data, the estimated proportion of buyers in the population preferring Brand A to Brand B would be $\hat{p}_1 = 0.294$ using (5.1).

The estimator (5.1) is used widely since it would be the uniformly minimum variance unbiased estimator if the m_o observations belonging to category K were ignored and the original fixed sample size n considered to be the reduced sample size $n-m_o=m_1+m_2$.

If the estimator (5.1) is expressed in the form,

$$\hat{p}_1 = (1/n)(m_1+(m_1/(m_1+m_2))m_o), \tag{5.2}$$

one finds this same estimator is also obtained when the m_o partially informative observations in category K are allocated to the categories K_1 and K_2 in proportion to the sample proportions m_1/m_1+m_2 and m_2/m_1+m_2 observed in these categories.

It is of interest to recall that the estimator (5.1) or, equivalently, (5.2), would be the constrained maximum likelihood estimator of p_1 if it were assumed that $p_1 = \lambda$. In other words, when the conditional probability that an observation belongs in K_1 given that it is observed in K is the same as the unconditional probability that it belongs in K_1, constrained maximum likelihood estimation would yield the estimator (5.1). This, of course, may be an unreasonable assumption in many applications. Indeed, the marketing analyst who uses (5.1) is assuming implicitly that whether a respondent prefers Brand A to Brand B or Brand B to Brand A is independent of whether or not the respondent is willing to reveal his preference. Of course, under this assumption, allocation of responses in the undecided category according to the proportion of the respondents expressing a preference is clearly a reasonable estimation procedure.

The expected value of $\hat{p}_1$ in (5.1) is

$$E(\hat{p}_1) = (P_1-\lambda p_o)/(1-p_o) \tag{5.3}$$

while its squared bias, denoted by $B(\hat{p}_1)$, is given by

$$B(\hat{p}_1) = [(p_o(\lambda-p_1)/(1-p_o)]^2. \tag{5.4}$$

The variance of $\hat{p}_1$ can be expressed as

$$\mathrm{Var}(\hat{p}_1) = \left(\frac{p_1-\lambda p_o}{1-p_o}\right)\left(1-\frac{p_1-\lambda p_o}{1-p_o}\right) E((m_1+m_2)^{-1}|m_1+m_2>0). \tag{5.5}$$

Since the conditional expectation in (5.5) can only be represented as a finite sum and can't be expressed explicitly in a closed form involving p_1, p_o, and λ, we use an approximation for inverse binomial moments suggested by Mendenhall and Lehman [4] to approximate Var $(\hat{p}_1)$ by

$$\mathrm{Var}(\hat{p}_1) \cong \left(\frac{n-2}{n}\right)\left(\frac{1}{(n-1)\ (1-p_o)-1}\right)\left(\frac{p_1-\lambda p_o}{1-p_o}\right)\left(1-\frac{p_1-\lambda p_o}{1-p_o}\right). \tag{5.6}$$

We observe for large n that $\mathrm{Var}(\hat{p}_1)$ approaches zero, but that $B(\hat{p}_1)$ does not. Hence for large n it becomes more important to consider the bias contribution to the mean square error of the "natural" estimator (5.1).

Recalling, when λ is known, that the minimum variance unbiased estimator of p_1 is simply $\hat{p}_1 = (m_1 + \lambda m_o)/n$ suggests investigating estimators of the form,

$$\hat{p}_1(a) = (m_1 + am_o)/n, \tag{5.7}$$

where a is chosen to satisfy a statistical criterion for optimality related to the variance and bias of $\hat{p}(a)$ in (5.7). Since $E(\hat{p}_1(a)) = p_1 + p_o(a-\lambda)$, the squared bias of $\hat{p}_1(a)$ is

$$B(\hat{p}_1(a)) = p_o^2\ (a-\lambda)^2. \tag{5.8}$$

The variance of $\hat{p}_1(a)$ is given by

$$Var(\hat{p}_1(a)) = \frac{1}{n}\ [a^2 p_o(1-p_o) - 2ap_o(p_1-\lambda p_o) + (p_1-\lambda p_o)\ (p_2+\lambda p_o)], \tag{5.9}$$

while its mean square error is $MSE(\hat{p}_1(a)) = B(\hat{p}_1(a)) + Var(\hat{p}_1(a))$.

One criterion for choosing a, which leads to a canonical representative from this family of estimators, is to minimize an average weighted mean square error criterion over the admissable parameter space Θ given by (2.2). In other words, if $\omega(\theta)$ is some weighting function defined over the admissable parameter space, then a is to be chosen to satisfy,

$$\min_a \int_0^1 \int_0^1 \int_{\lambda p_o}^{1-(1-\lambda)p_o} MSE(\hat{p}_1(a))\omega(p_1,p_o,\lambda)dp_1 dp_o d\lambda. \tag{5.10}$$

Assuming $\omega(\theta)$ is continuous with continuous partial derivatives over Θ, one may differentiate with respect to a under the integral appearing in (5.10). Now MSE $(\hat{p}_1(a))$ is a quadratic in a of the form $\alpha\ a^2 + \beta a + \gamma$, whose coefficients α, β, and γ depend on θ. If we use (5.8) and (5.9) these coefficients are found to be $\alpha(\theta) = p_o^2 + p_o$

$(1 - p_o)/n$, $\beta(\theta) = -2(\lambda p_o^2 + p_o\ (p_1 - \lambda p_o)/n)$, and $\gamma(\theta) = \lambda^2 p_o^2 +$ $(p_1 - \lambda p_o)(p_o + \lambda p_o)/n$. Thus, the value a* of a which minimizes this average weighted mean square error criterion is given by

$$a^* = \int_\Theta \beta(\theta)\omega(\theta)d\theta / 2 \int_\Theta \alpha(\theta)\omega(\theta)d\theta. \qquad (5.11)$$

Applying the criterion in (5.10) to the special case in which the weighting function $\omega(\theta) = 1$ is chosen leads to the stationary value $a^* = 0.5$ as the minimizing value, and the corresponding canonical estimator in the family (5.7) is

$$\hat{p}_1^* = (m_1 + 0.5m_o)/n = \hat{p}_1^*(0.5). \qquad (5.12)$$

This equal allocation of the responses in one partially informative sampling category to the two population categories is of special interest in brand preference analysis, and it has been discussed in Odesky [5] without quantitative statistical justification.

Using the data previously discussed in which 20 respondents from a sample of $n = 100$ reported a preference for Brand A, 48 for Brand B, and 32 did not reveal their preferences, the population proportion p_1 preferring Brand A to Brand B would be estimated by $\hat{p}_1 = 0.360$ using (5.12) rather than by $\hat{p}_1 = 0.294$ when the "natural" estimator (5.1) is used.

6. Squared Bias Comparisons of $\hat{p}_1$ and $\hat{p}_1(a^*)$

Since for large n, $Var(\hat{p}_1)$ and $Var(\hat{p}_1(a^*))$ for $a^*=0.5$ both approach zero while their biases do not, we compare the squared biases $B(\hat{p}_1)$ and $B(\hat{p}_1(a^*))$ of these two estimators. Thus we define $R(a,p_1,p_o,\lambda)$

as the ratio of the squared bias of $\hat{p}_1(a)$ to the squared bias of $\hat{p}_1$, namely,

$$R(a,p_1,p_o,\lambda) = B(\hat{p}_1(a))/B(\hat{p}_1) = (\lambda-a)^2(1-p_o)^2/(\lambda-p_1)^2. \qquad (6.1)$$

Figures 1-5 give the contour R=1 of $R(a^*,p_1,p_o,\lambda)$ for a*=0.5 and p_1 = 0.2,0.4,0.5,0.6, and 0.8 in the admissable (p_o,λ)- parameter space. It is interesting to note that in each of these figures, the proportion of the admissable (p_1,λ)- parameter space in which the squared bias of the estimator $\hat{p}_1(a^*)$ for a*=0.5 is smaller than the squared bias of the "natural estimator" (namely, where R<1) is greater than the proportion of the admissable region where the "natural" estimator $\hat{p}_1$ is preferred to $\hat{p}_1(a^*)$ for a* = .05 in terms of its squared bias (namely, where R>1).

In fact, for any value of p_1, let $A(p_1)$ denote the percentage of the area in the admissable (p_o,λ)- parameter space where the estimator $\hat{p}_1(a^*)$ for a* = 0.5 is preferred to $\hat{p}_1$ on the basis of smaller squared bias. Figure 6 gives a graph of $A(p_1)$. Thus, in terms of smaller squared bias, one finds that the "canonical" linear estimator $\hat{p}_1(a^*)$ is more frequently preferred in terms of smaller squared bias than the "natural" estimator for values of p_1 between 0.19 and 0.81.

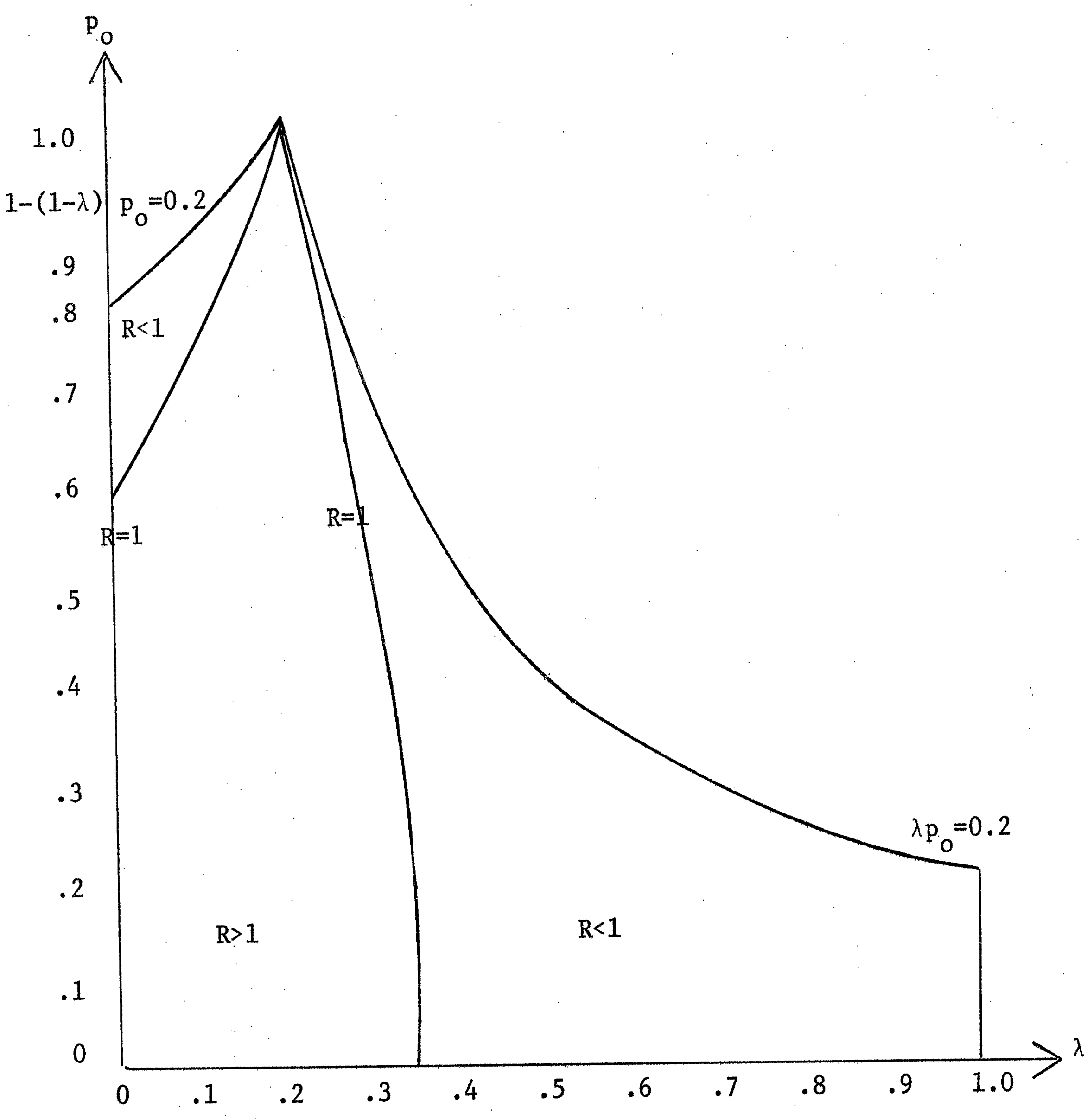


Figure 1: $R(.5,.2,p_o,\lambda)$

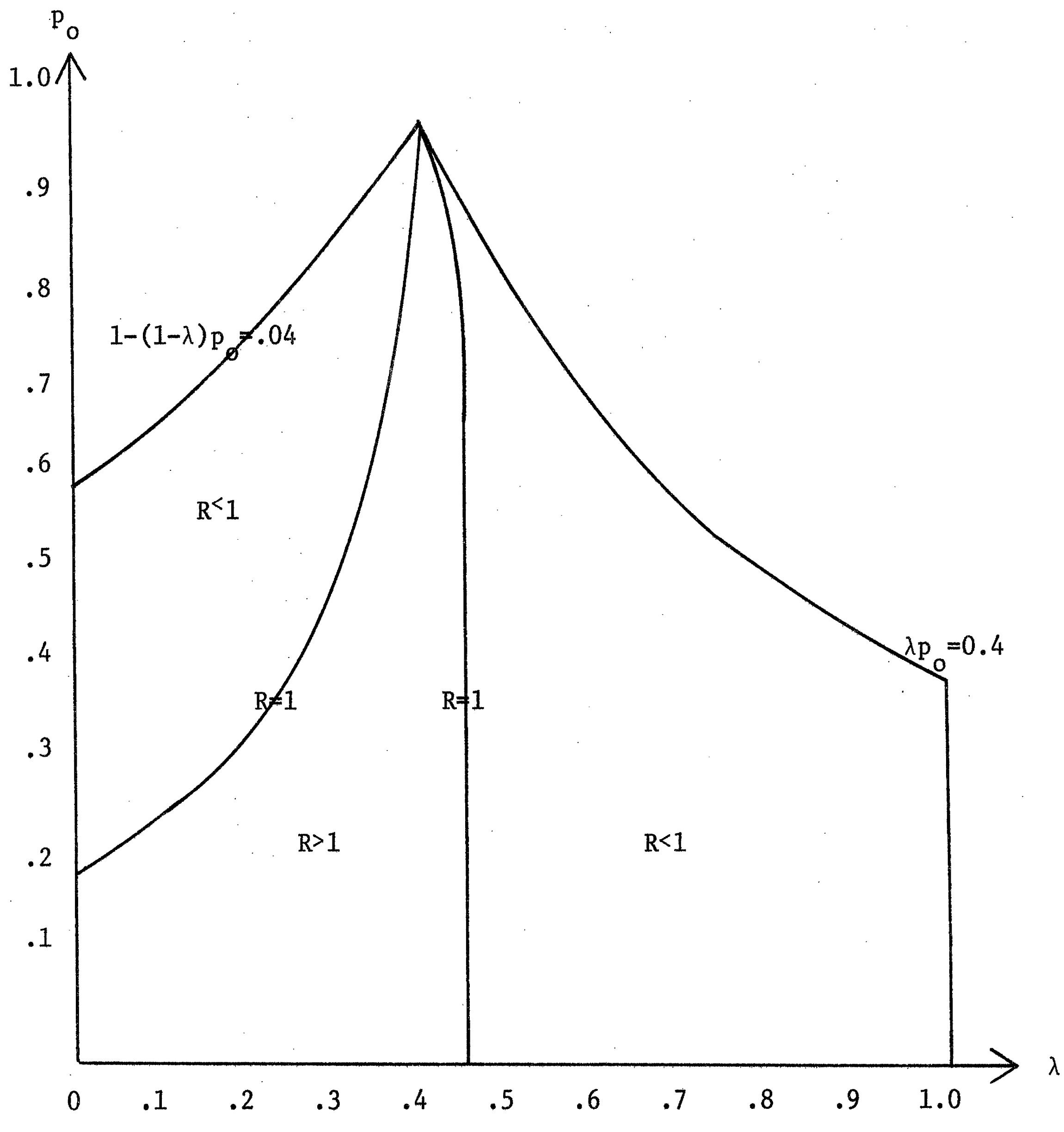


Figure 2: $R(.5,.4,p_o,\lambda)$

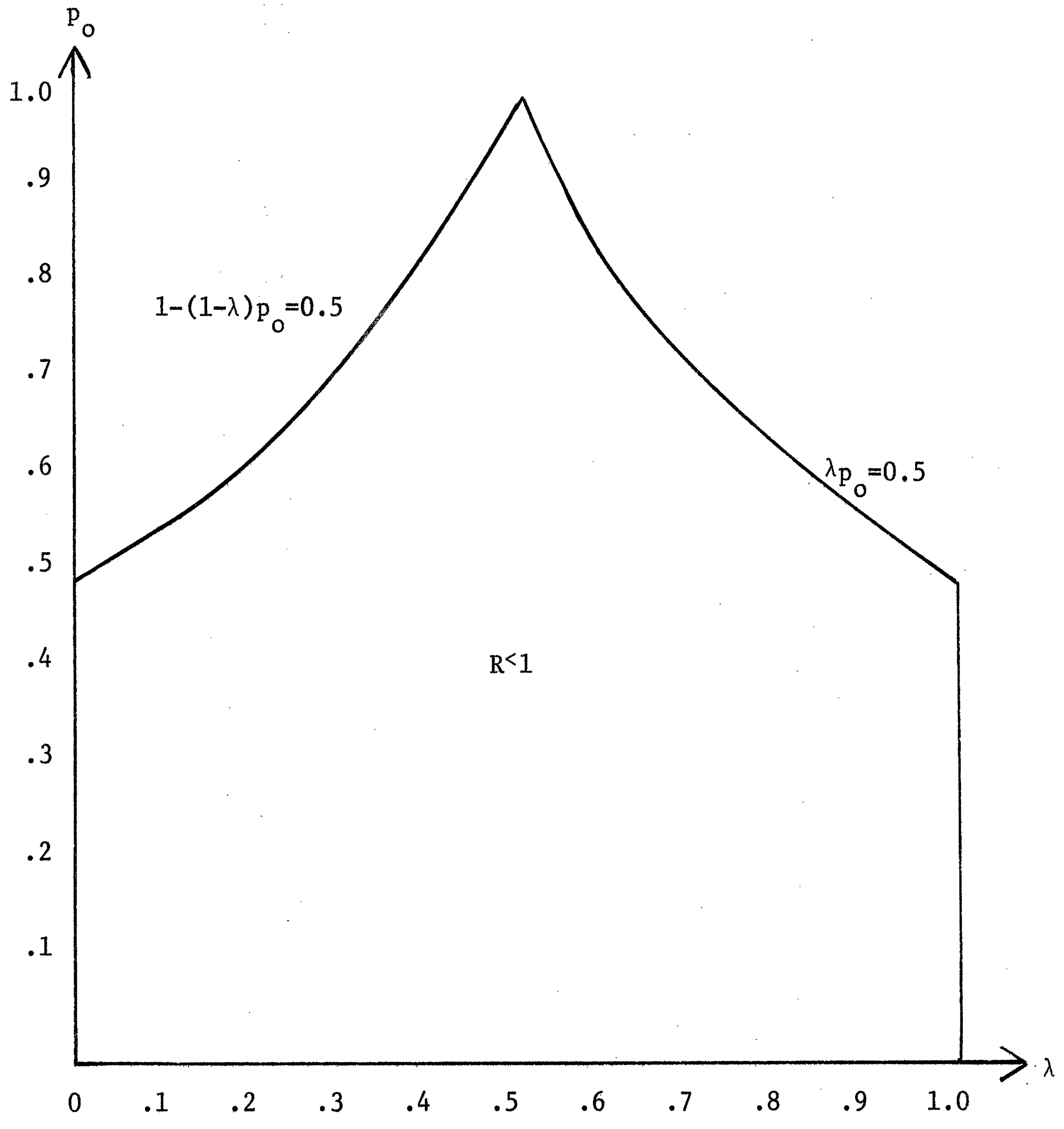


Figure 3: $R(.5,.5,p_o,\lambda)$

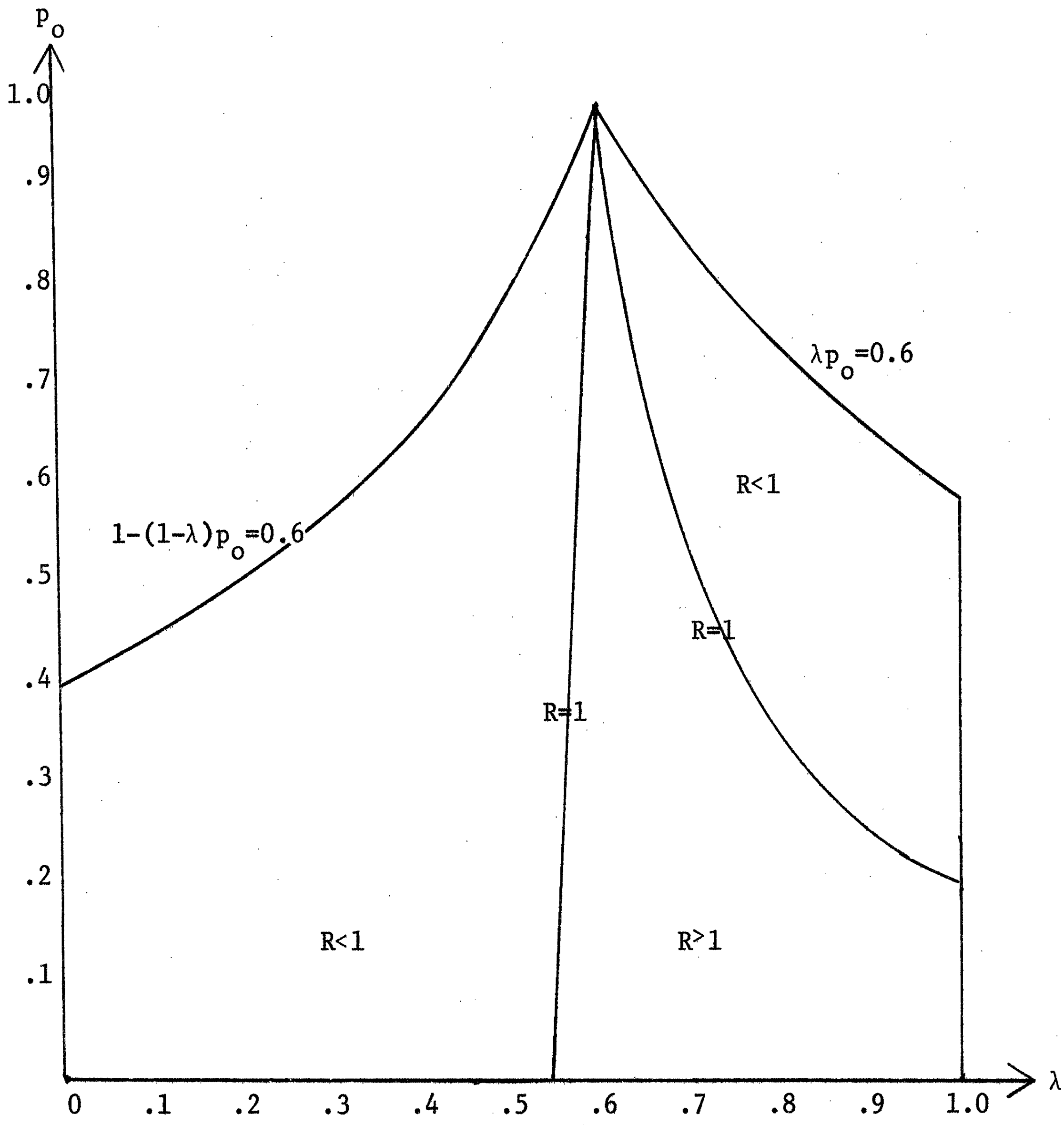


Figure 4: $R(.5,.6,p_o,\lambda)$

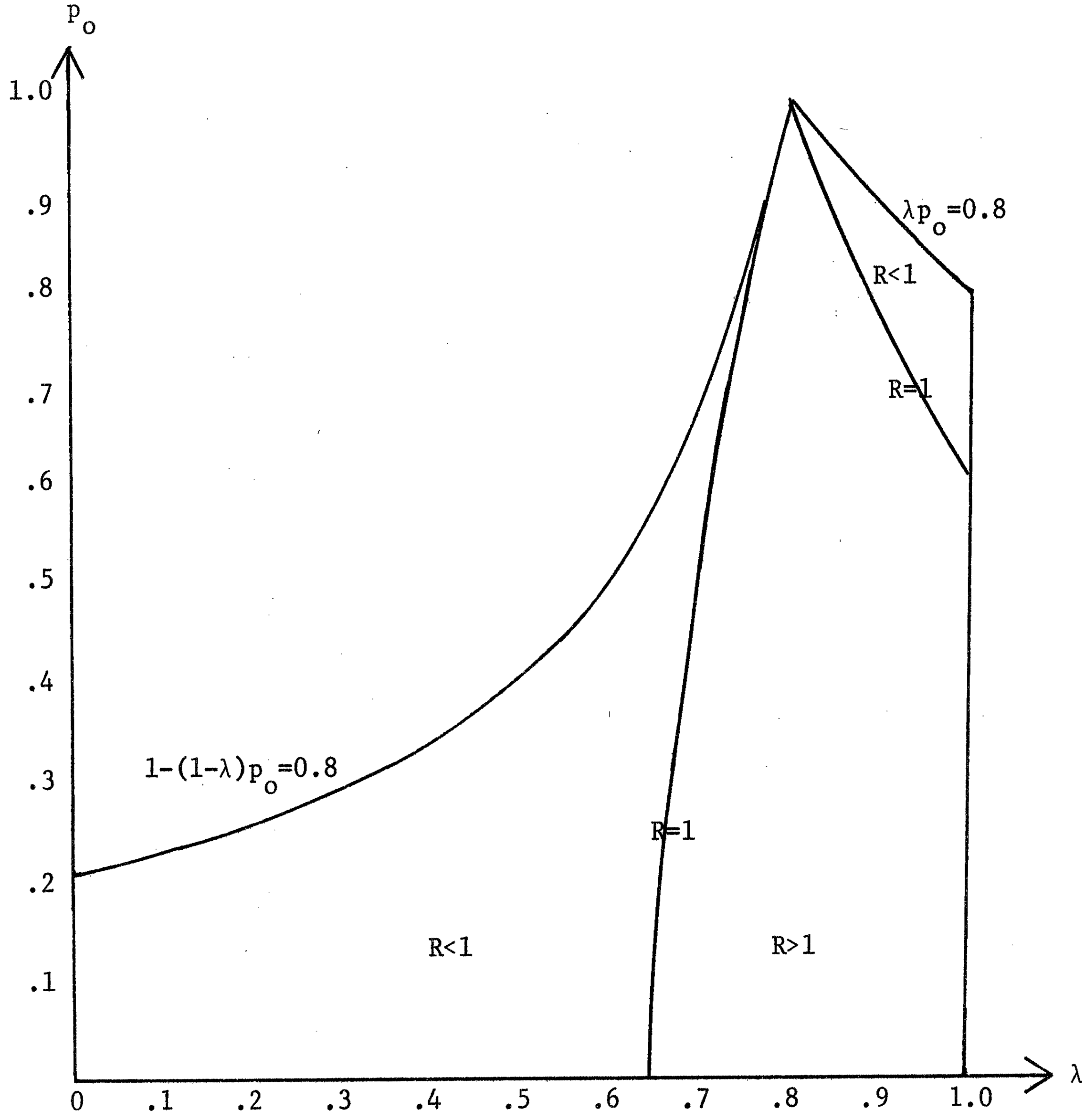
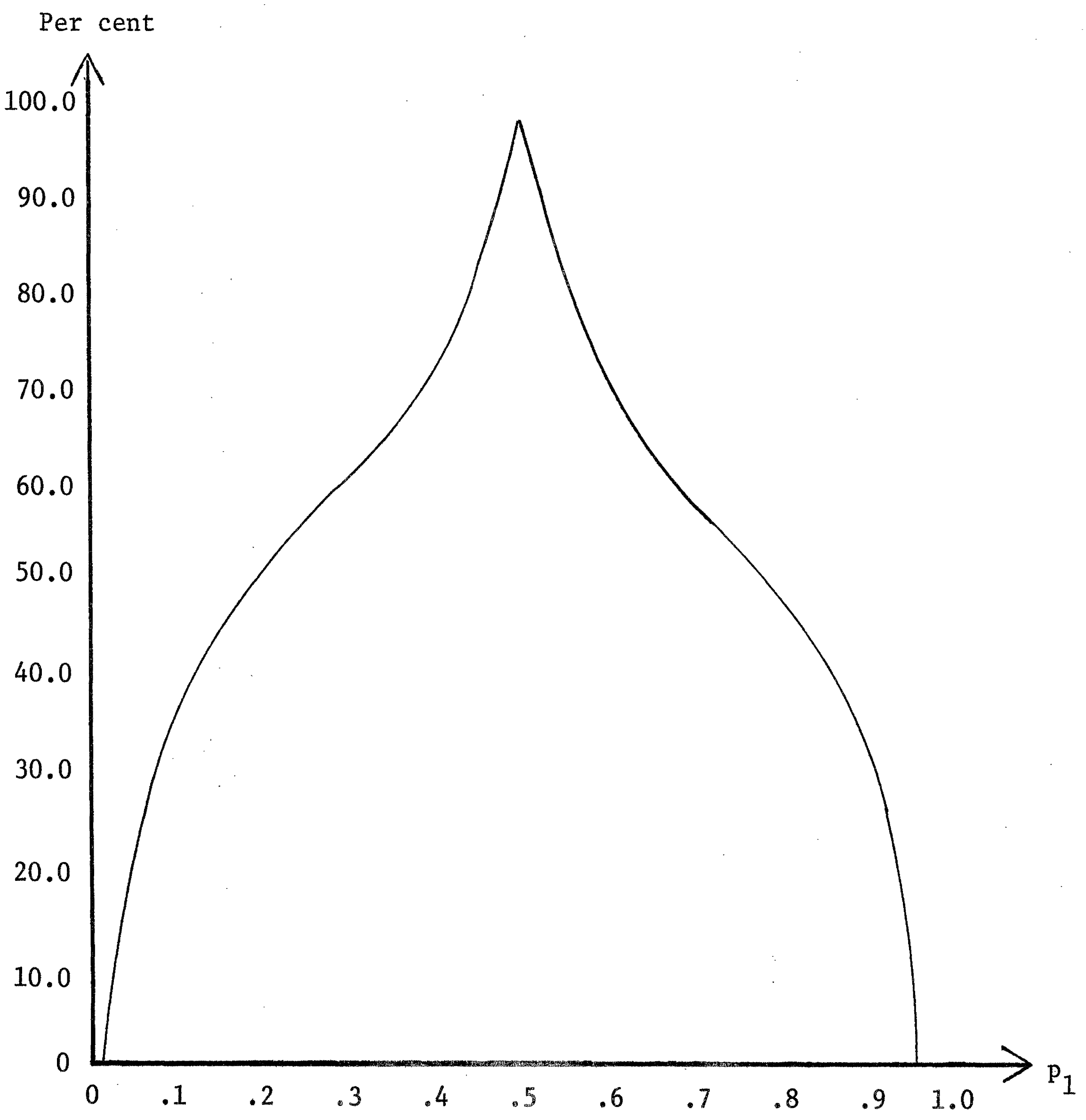


Figure 5: $R(.5,.8,p_o,\lambda)$



Per cent
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
p_1

Figure 6: $A(p_1)$

7. References

1. Blumenthal, S. "Multinomial Sampling with Partially Categorized Data." Journal of the American Statistical Association, Vol. 63, No. 322 (June, 1968).

2. Bryson, M. R. "Error of Classification in a Binomial Model." Journal of the American Statistical Association, Vol. 60, No. 309 (March, 1965).

3. Draper, N. R.; Hunter, W. G.; Tierney, D. E. "Which Product is Better?" Technometrics, Vol. 2, No. 2 (May, 1969).

4. Mendenhall, W., and Lehman, E. H., Jr. "An Approximation to the Negative Moments of the Positive Binomial Useful in Life Testing." Technometrics, Vol. 2, No. 2 (May, 1960).

5. Odesky, S. H. "Handling the Neutral Vote in Paired Comparison Product Testing." Journal of Marketing Research, Vol. 4, No. 2 (May, 1967).

WORKING PAPERS

Working Paper Number	
1	"Reflections on Evolving Competitive Aspects in Major Industries," by Sidney C. Sufrin
2	"A Scoring System to Aid in Evaluating a Large Number of Proposed Public Works Projects by a Federal Agency," by M. Lynn Spruill
3	"The Delphi Method: A Systems Approach to the Utilization of Experts in Technological and Environmental Forecasting," by John D. Ludlow
4	"What Consumers of Fashion Want to Know," by Claude R. Martin, Jr.--Out of print. To be published in a forthcoming issue of the <u>Journal of Retailing</u>.
5	"Performance Issues of the Automobile Industry," by Sidney C. Sufrin, H. Paul Root, Roger L. Wright, and Fred R. Kaen--Out of print. To be published as a future Michigan Business Paper.
6	"Management Experience with Applications of Multidimensional Scaling Methods," by James R. Taylor
7	"Profitability and Industry Concentration," by Daryl Winn
8	"Why Differences in Buying Time? A Multivariate Approach," by Joseph W. Newman and Richard Staelin--Out of print. To be published in a forthcoming issue of the <u>Journal of Marketing Research</u>.
9	"The Contribution of the Professional Buyer to the Success or Failure of a Store," by Claude R. Martin, Jr.
10	"An Empirical Comparison of Similarity and Preference Judgments in a Unidimensional Context," by James R. Taylor
11	"A Marketing Rationale for the Distribution of Automobiles," by H.O. Helmers
12	"Global Capital Markets," by Merwin H. Waterman
13	"The Theory of Double Jeopardy and Its Contribution to Understanding Consumer Behavior," by Claude R. Martin, Jr.
14	"A Study of the Sources and Uses of Information in the Development of Minority Enterprise--A Proposal for Research on Entrepreneurship," by Patricia Braden and H. Paul Root

Working Paper Number	
15	"Program Auditing," by Andrew M. McCosh
16	"Time Series Forecasting Procedures for an Economic Simulation Model," by Kenneth O. Cogger
17	"Models for Cash Flow Estimation in Capital Budgeting," by James T. Godfrey and W. Allen Spivey
18	"Optimization in Complex Management Systems," by W. Allen Spivey
19	"Support for Women's Lib: Management Performance," by Claude R. Martin, Jr.
20	"Innovations in the Economics of Project Investment," by Donald G. Simonson
21	"Corporate Financial Modeling: Systems Analysis in Action," by Donn C. Achtenberg and William J. Wrobleski
22	"Sea Grant Delphi Exercises: Techniques for Utilizing Informed Judgments of a Multidisciplinary Team of Researchers," by John D. Ludlow
23	"The Spanish in Nova Scotia in the XVI Century--A Hint in the Oak Island Treasure Mystery," by Ross Wilhelm
24	Not yet ready for release.
25	"Market Power, Product Planning, and Motivation," by H. Paul Root
26	"Competition and Consumer Alternatives," by H. Paul Root and Horst Sylvester
27	"Stepwise Regression Analysis Applied to Regional Economic Research," by Dick A. Leabo