MATHEMATICAL EXPRESSIONS FOR DROP SIZE DISTRIBUTIONS

IN SPRAYS

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Abstract

A number of mathematical expressions have been proposed
the particle size distributions of crushed solids and of
liquid drops in sprays. The various mathematical forms of three
of the most common of these, the Nukiyama-Tanasawa, Rosin-
Rammler, and logarithmico-normal, are presented in order to compare
their utility for representing distributions in sprays. All
three of the expressions require the evaluation of two constants,
one a function of the mean drop size and the other a function of
the degree of variation in drop size. The methods of plotting
distribution data to obtain straight lines for evaluating the
constants are described. All three equations are fitted to
Kolupaev's data on a pressure-atomized oil spray and all produce
distribution curves that are in reasonable agreement with the data.
The closest agreement is produced by the Rosin-Rammler expression;
this expression is also easier to use than the others in the
cumulative mass-fraction form since it is a relatively simple ex-
ponential function whereas the others contain incomplete Gamma

functions. Expressions for various types of mean drop size have been derived from the three distribution expressions. Numerical values of various mass-mean drop sizes calculated from the three expressions are in close agreement with those obtained by direct integration of Kolupaev's data. Because of the extreme sensitivity to the very small drops, the agreement is not very good for the number means. In fact, the Rosin-Rammler equation is unsatisfactory for calculating number means since it predicts an infinite number of infinitesimal drops unless the constant q is greater than 3.

I. Introduction

One of the important properties of liquid sprays is the frequency of occurrence of the various sizes of droplets, or the size distribution in the spray. A number of mathematical expressions of size distribution have been proposed and used both for liquid sprays and for the similar distributions of particle sizes resulting from the crushing and grinding of solids. No criterion has been established for choosing the preferred distribution function for any given application from among the ones proposed. It is the purpose of this paper to examine and compare critically the most commonly used distribution functions and to determine whether a basis exists for a choice among them.

As is the case for any statistical distribution, there are two common forms of expressing size distributions. One of them is the cumulative form, in which the distribution is expressed as the

fraction by weight or volume of the total drops having a
diameter larger than a given diameter. A typical distribution
is presented in this form in Fig. 1. The second is the differen-
tial form and expresses the relative frequency of occurrence of
drops of any given size. It is simply the derivative of the
cumulative form. The distribution of Fig. 1 is presented in the
differential form in Fig. 2. The area under the differential
curve represents the total quantity of liquid in the spray so that
the ratio of the area under any portion of the curve to the total
area represents the fraction of the spray in that particular drop-
size range. Instead of expressing the drop-size distribution in
terms of the mass of liquid, the distribution is sometimes expressed
in terms of the relative number of drops of each size. This number
distribution is very simply related to the mass distribution by
the geometry and density of the drops.

It is expected that in sprays formed by a single atomizing
process, the number of particles per unit of size interval will have
a maximum value at one single size, and will decrease smoothly as
the size becomes smaller or larger, (see Fig. 2). This is the type
of curve for which a mathematical function is required.

The most satisfactory approach for obtaining a mathematical
expression of any observed phenomenon is by derivation from basic
physical laws. This would be the logical method of obtaining a dis-
tribution function for drop sizes if the mechanism of spray formation
were clearly understood and simple enough to be handled analytically.

Unfortunately, the mechanism is still incompletely described even qualitatively, so that derivation of a distribution function must await further clarification of the theory. In the meantime, mathematical expressions may be fitted empirically to experimental data.

II. Proposed Mathematical Expressions

This empirical procedure was used by Nukiyama and Tanasawa (6) in correlating their extensive data on drop sizes in sprays formed by air atomization. They started with a general equation of the form

$$dN/dx = ax^p e^{-bx^q}$$, or in the mass-fraction form,

$$\frac{dR}{dx} = a'x^{p+3}e^{-bx^q}$$

where N is the number of drops in the spray which have diameters larger than x, R is the mass fraction of drops larger than x, and a, p, b, and q are constants. The value of the constant a depends only on the size of the sample and will be discussed later. After trying values of 0, 1, 2, and 3 for p, Nukiyama and Tanasawa found that a value of 2 was always required for a good correlation. Similarly, a value of 1 was usually required for q; although the value for this constant changed somewhat depending on atomizing conditions. H. C. Lewis and colleagues (5) used the same equation with success in correlating their own data on drop size distribution and also the data on various types of spray clouds reported by several other workers. This appears to be the only distribution function which has been applied to experimental drop size studies in sprays in any work

reported in the literature.

Size distributions of solid particles resulting from crushing and grinding operations have been studied much more frequently and carefully than the size distributions in liquid sprays, and several satisfactory mathematical expressions have been found. Some of these expressions are reviewed by Austin (1). Since the size distribution curves for spray clouds and for crushed solids are similar in shape, the distribution functions developed for solids should also have application to liquid sprays. Two of the most common of these distribution expressions will be compared with the one used by Nukiyama and Tanasawa.

A distribution function for solid particles was developed by Rosin and Rammler (7) on the basis of empirical rules for the breaking of solids. It is a special form of the general equation used by Nukiyama and Tanasawa in which the exponents p and q are related in such a way that integration is readily accomplished. It has the following form:

$$dR/dx = -bqx^{q-1}e^{-bx^q}$$

Another commonly used distribution function for solid particles is the logarithmico-normal distribution. Epstein (2) has recently demonstrated that this distribution law will be approached asymptotically in the crushing of solids if certain assumptions are valid concerning the breaking process. In this case, the distribution as a function of the logarithm of the particle size follows the normal distribution law and may be expressed by the following equation:

$$dR/dx = \frac{1}{(2.3)x\sqrt{2\pi} \log \sigma_g} e^{-(\log x - \log M_g)^2 / 2 \log^2 \sigma_g}$$

where M_g and σ_g are the geometric mean and the standard geometric deviation according to the customary probability nomenclature.

In Table I, the three distribution functions are listed together with the general form of the Nukiyama-Tanasawa and Rosin-Rammler functions. This general form contains three distribution constants. The functions are listed both in terms of R, the fractional mass of drops having diameters larger than x, and of N, the number of drops per unit mass of total spray which have diameters larger than x, and both in the integral or cumulative form and in the differential form.

Each of the three recommended distribution functions contains two constants which determine the distribution. The third constant, a or a', is merely a measure of the size of the sample being considered. Since R and N have been defined on the basis of a unit mass of liquid in the spray, a and a' must have definite values in terms of the other constants; these values have been listed in columns 6 and 7 of Table I.

Of the two distribution constants, q (or σ_g) determines the spread of particle sizes. A high value of q (low value of σ_g) means that a small fractional variation of particle size on either side of the mean is sufficient to embrace a large fraction of the total sample.

The other distribution constant, b or M_g, measures the average particle size. Expressions for b and M_g in terms of the mass median particle size, designated by D_m, are given in the last column of the table and may be substituted for b to give generalized forms of the equations. In the generalized form, the mass cumulative distributions are functions of a single constant and the ratio of the drop size x to the median drop size D_m only. Generalized equations can also be written in terms of the ratio of the drop size x to any other type of mean drop size.

Many of the expressions in Table I involve Gamma functions. For those readers who are not familiar with this type of function, the complete Gamma function is defined as follows:

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx = (n-1)!$$

Tables of complete Gamma functions are available for use when n is not an integer. The incomplete Gamma function is defined as

$$\Gamma_x(n) = \int_0^x x^{n-1} e^{-x} dx$$

Tables have also been published for incomplete Gamma functions but since an additional variable has been introduced, the tables are much more extensive than those required for complete Gamma functions and are not as extensively available. Incomplete Gamma functions can always be evaluated by graphical integration if the tables are not available.

The number of drops, N and the mass fraction, R are defined in such a way that they are at a maximum at X = 0 and are equal to

zero at $x = \infty$. This causes the constants a and a' to have
negative signs. In addition, the cumulative distribution functions
consist of the difference between two terms when they contain
Gamma functions because the Gamma function is the integral from
zero to x instead of from infinity to x.

III. Choice of Experimental Data for Comparing Equations

One way of comparing the three distribution equations is
to apply them all to the same experimental data on drop size dis-
tribution of a spray. For such a comparison, it is important to
select data which are based on a truly representative sample of
a spray, which are accurate with respect to measurement of particle
sizes and numbers, and which cover the size range of the spray as
completely as possible. The experimental results that have been
found to come nearest to satisfying these requirements are those
obtained by Kolupaev (4) on the spray from a 19-gallon per hour
swirl-type pressure-atomizing nozzle. He sampled a spray of
Bunker C fuel oil which had been heated to 224°F (viscosity =
0.216 cm.^{2}per sec.) and atomized with a pressure drop of 140 psi.
The method used was to expose sooted slides for a standard time
interval to various portions of the spray and then to measure and
count the drop traces under a microscope. The positions of
sampling in the spray were chosen so that an average distribution
for the entire spray could be obtained by suitable weighting of
samples. Thousands of drops were counted to determine a single
distribution curve. Checks of the counting technique were made
by weighing the oil collected on some of the slides and comparing
with the weight predicted from the count and by using a special

device to produce drops of a known size.

IV. Determination of Equation Constants from Experimental Data

The most convenient way to evaluate the constants of an equation from experimental data is to rearrange the equation to a form whereby the data may be plotted to produce a straight line. This can only be done if no more than two constants are to be evaluated, which is the case with the three distributions under consideration. In fact, evaluation of more than two constants from experimental data is so difficult that distribution functions containing more than two constants are almost never used. Nukiyama and Tanasawa started with a distribution function containing three constants but they established a value for one of them which was not changed in further applications of the function.

To evaluate the constants of the Nukiyama-Tanasawa equation, it can be converted to the following form:

$$\log\left(\frac{1}{x^5}\frac{dR}{dx}\right) = \log a' - \frac{bx^q}{2.3}$$

or

$$\log\left(\frac{1}{x^2}\frac{dN}{dx}\right) = \log a - \frac{bx^q}{2.3}$$

By plotting $\log\left(\frac{1}{x^5}\frac{dR}{dx}\right)$ against x^q a straight line should be obtained having a slope of $b/2.3$. Generally a value of 1 is satisfactory for q but for some distributions, values less than 1, e.g., 1/2, 1/3, 1/4, etc., are required. The proper value may quickly be determined by trial and error. An example of this technique is shown in Fig. 3 where it is applied to experimental data obtained

by Kolupaev.

The constants for the Rosin-Rammler distribution expression can be obtained by taking logarithms twice of the cumulative mass fraction form of the equation

$$\log (\log 1/R) = \log b/2.3 + q \log x$$

If $\log (\log 1/R)$ is plotted against $\log x$, (loglog vs. log paper
                                                                commercially)
can be obtained/ the points should fall on a straight line of slope q. The value of $\log b/2.3$ is equal to $\log (\log 1/R)$ at the point on the line where x equals one. An example of this type of plot for Kolupaev's data is shown in Fig. 4. A curved line has been drawn through the points to indicate the actual distribution in addition to the straight dotted line which is the best straight line as evaluated by eye through the points from a residue R of 0.01 which is larger than size x, to R = 0.80. This method of plotting magnifies the ends of the distribution greatly, so that the discrepancy between the straight line and the actual points is really not as bad as it appears, as will be seen later. In fitting a straight line to the data, matching along the central portion of the curve is most essential, since that portion represents nearly all of the mass of the spray. The range from 1 per cent to 80 per cent was chosen for matching in this case because the completion of a combustion process is generally limited by the largest drops present and it was believed to be most important to have the greatest accuracy in the fractional distribution of the large drops. In other applications it might be more important to

obtain greatest accuracy in fractional distribution over other
ranges of the curve.

The constants for the logarithmico-normal distribution
function can best be determined by using logarithmic probability
graph paper, which is commercially available. The integration of
the equation has already been performed in preparing this paper
and the coordinates of the abscissa have been adjusted accordingly.
It is merely necessary to plot particle size on the logarithmic
scale against either R or N/N_{total} on the probability scale to ob-
tain a straight line. In this case, the forms of the equations for
both R and N are the same except that of course the constant M_g
has different values. From the straight line, the constant M_g'
may be obtained as the value of x at R = 0.50. The constant σ_G is
equal to the value of x at R = 0.8413 divided by x at R = 0.50 or
X at R = 0.50 divided by x at R = 0.1587. Kolupaev's data are also
used to illustrate this method in Fig. 5. Here again two lines
are drawn on the plot, the curved line following the actual data
and the straight line being the best fit from R = 0.01 to
R = 0.80. As in the case of the Rosin-Rammler plot, the ends of
the distribution are greatly magnified by this method of plotting
so that the agreement between the experimental results and the
straight line is actually much better than it appears.

V. Comparison of the Distribution Functions

In choosing the mathematical expression of size distribu-
tion to be used for any given situation, the first criterion should

naturally be how well the expression can fit the experimental data. None of the distribution functions discussed here can express exactly an actual spray distribution since they all predict the existence of droplets of all sizes from zero to infinity, whereas in any actual spray cloud it is obvious that there is a finite upper limit in particle size. All of the equations, however, predict that the frequency of occurrence of particles approaches zero very rapidly as the particle size is increased beyond the size of greatest probability, and the error introduced in the prediction of oversize particles is insignificant. The use of a function which takes cognizance of the limiting drop size is not warranted since it is much more unwieldy than these relatively simple equations and will not increase the accuracy appreciably.

A comparison of how well the three distribution functions fit the data of Kolupaev has been made using the following equations obtained from Figs. 3, 4, and 5.

Nukiyama and Tanasawa $\dfrac{dR}{dX} = -8.1 \times 10^5\ x^5\ e^{-21.2x}$

Rosin and Rammler $\dfrac{dR}{dX} = -47.9\ x^{1.63}\ e^{-18.2x^{2.63}}$

Logarithmico-Normal $dR/dx = -\dfrac{1.098}{x}\ e^{-\frac{(\log x + 0.56)^2}{0.050}}$

Values of dR/dx as calculated from these equations have been plotted versus x in Fig. 6 together with a smoothed curve representing the

actual data points. Quite striking is the discrepancy between the maximums of the logarithmico-normal and Nukiyama-Tanasawa curves and the maximum of the experimental curve. This would not have been expected from a casual glance at the straight-line correlations of Figs. 3 and 5 because it occurs in an insensitive region of the correlation and because the percentage difference is not very great. The Rosin-Rammler distribution, on the other hand, comes just about as close to fitting the experimental curve in the small drop size region as the other two distribution functions even though from a casual examination of Fig. 4 it would appear to be greatly in error in this region. In the large drop size region, the Rosin-Rammler curve does a remarkably good job of matching the experimental distribution. Although the Rosin-Rammler distribution is somewhat superior to the other two in fitting the experimental data of Fig. 6, all of the expressions produce curves that are acceptably close to the experimental curve, and they would have to be tested with a number of other sets of data before the superiority of any one could definitely be established.

It is possible that there might be a greater difference between the three distribution expressions if they were applied to sprays formed by methods other than a swirl-type pressure atomizing nozzle. A comparison has been made using Houghton's (3) data on an air-atomized water spray. These data were correlated by Lewis (5) et al (5) using the Nukiyama-Tanasawa function. The three curves together with the experimental points are presented in Fig. 7. Since the experimental points are so scattered, this figure

merely indicates the general agreement among the three distribution functions.

It is noteworthy that the Rosin-Rammler distribution predicts too large an amount of very small drops and the logarithmico-normal too small an amount, in both Figs. 6 and 7. These appear to be general characteristics of these distribution functions.

Another basis for choosing among the three functions is the type of data with which they are to be used. If the data are obtained in such a manner that the total quantity of liquid is not known but only the number or mass of drops in certain size ranges, the Nukiyama-Tanasawa function can still be used in a straightforward manner. Values proportional to $\Delta N/\Delta x$ or $\Delta R/\Delta x$ can be used without ever knowing the proportionality constant until the constants of the equation have been established. This is often the case when samples of a spray are collected on slides, and drops of various size ranges are counted, since it is difficult to get accurate counts of the very small and very large drops. On the other hand, if the spray is sampled by a method such as freezing and sieving the drops, it is quite easy to get the total quantity of material and the fraction of material that does not pass through screens of various sizes. For these data, it is more straightforward to use the Rosin-Rammler or logarithmico-normal functions. Actually, these differences in technique are not very important, since the Rosin-Rammler or logarithmico-normal distribution can be obtained by a trial and error process of guessing the total

quantity of liquid if it is not known, and the Nukiyama-
Tanasawa function can always be established by taking slopes
of a cumulative fraction curve.

There is one important advantage to using the Rosin-
Rammler distribution function, if the cumulative mass fraction
form is required. Both of the other distribution functions
contain incomplete Gamma functions when written in this form as
seen in Table I, whereas the Rosin-Rammler function in this form
is a relatively simple exponential.

VI. Mean Drop Sizes

Mean drop sizes are sometimes used to characterize a
spray. There are an infinite number of means which may be cal-
culated by the following formula:

$$\text{mean drop size} = \left(\frac{\int_{o}^{N} x^{n} dN}{\int_{o}^{N} x^{m} dN} \right)^{\frac{1}{n-m}}$$

$$\text{or} = \left(\frac{\int_{o}^{1} x^{n-3} dR}{\int_{o}^{1} x^{m-3} dR} \right)^{\frac{1}{n-m}}$$

Only a few means are of any practical value, however.

Median drop sizes are also occasionally used. Median
sizes are defined as the size which divides the spray into two
halves by number, weight, surface area or other property. The
only median drop size commonly used is the mass median and the
chief justification for its use is that it has very nearly the same

value as the mass mean. The latter mean is the diameter obtained
by giving to each particle diameter a weighting factor proportional
to the mass of the particle; it is one of the more important types
of means (see Table II).

Mean and median drop sizes may be calculated from the
distribution functions. The resultant expressions for the mass
median and four types of means are listed in Table 2. These means
may all be calculated readily from any of the distribution functions
with the exception of the mass median from the Nukiyama-Tanasawa
equation. For this calculation either the use of tables of
incomplete Gamma functions or a graphical integration is required
to determine K as a function of q. After this relationship has
been established, however, the median drop size may be calculated
as easily from this equation as from the others.

It is of interest to note that the ratios of the various
types of means as calculated from any one of the distribution
expressions are functions of the exponent q, or δg, only.

Numerical values of the various means as calculated from
Kolupaev's data are also listed in Table II together with the means
obtained by direct integration of the data. For all of the means
except the two types of number-means, the numerical values are in
very close agreement, with no one function being consistently the
best.

The numerical values of the number means as calculated
from the three equations and as determined directly from the data
differ quite widely. These means are very sensitive to the number

of very small drops present since these small drops represent
by far the greatest fraction of the total number of drops even
though they are a small fraction of the total mass. As would
be expected, therefore, the Nukiyama-Tanasawa equation does the
best job of predicting the number means. The Rosin-Rammler
equation is of no value for predicting number means unless q is
greater than 3 since it predicts an infinite number of infinitesi-
mal drops for all values of q less than 3. The number means are
included in Table 2 principally to demonstrate how sensitive they
are to the very small drops. Since these small drops represent
such a small fraction of the total liquid in the spray, they are
unimportant for most considerations. Consequently, number means
are rarely used.

VII. Conclusions

1. The Nukiyama-Tanasawa, Rosin-Rammler, and logarithmico-
normal distribution expressions all fitted the experimental size
distribution data for an oil spray in an acceptable manner. The
Rosin-Rammler distribution was somewhat superior to the others in
fitting this data.

2. To use any of the three distribution functions, it is
necessary to evaluate two constants, one of which is a function
of the mean drop size and the other a function of the degree of
variation in drop size. The constants for any of the expressions
may be evaluated from experimental distribution data by plotting the
data in a suitable manner to obtain a straight line.

3. In the cumulative mass fraction form, the Rosin-Rammler function is much easier to use than the other two since it is a relatively simple exponential function whereas the others contain incomplete Gamma functions.

4. Mean drop sizes may be calculated readily from any of the three distribution expressions.

5. There was very close agreement between the numerical values calculated for the mean drop sizes of practical interest by the three distribution expressions and the values determined by direct integration of what are believed to be reliable experimental data on distribution.

6. The ratios of the various means as determined by any of the three distribution functions are dependent only on the exponent q or δg.

7. Because of the sensitivity to very small drops, the calculated numerical values of the number-mean drop sizes varied widely. The Rosin-Rammler equation predicts number means of zero for values of q less than 3 since unless q is greater than this the equation predicts an infinite number of infinitesimal drops.

NOMENCLATURE

a	constant, a measure of the size of the sample.
a'	constant, a measure of the size of the sample.
b	constant, a measure of the mean drop size.
D_m	mass median drop diameter
K	constant, defined in Tables 1 and 2
M_g	constant, geometric mean for number distribution
M_g'	constant, geometric mean for mass distribution.
N	number of drops having diameters larger than x per unit total liquid mass in spray.
p	constant
q	constant, a measure of the degree of variation of drop sizes.
R	fractional mass of drops having diameters larger than x.
X	drop diameter.
σ_g	constant, geometric standard deviation, a measure of the degree of variation of drop sizes.
ρ	density of liquid

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TABLE I

MATHEMATICAL FORMS OF DISTRIBUTION EXPRESSIONS

	N	R	dN/dX	dR/dx	a	a'	b or M_g
3 Constant Equation	$\dfrac{a}{qb^{\frac{p+1}{q}}}\left(\Gamma\left(\dfrac{p+1}{q}\right) - \Gamma_{bx^q}\left(\dfrac{p+1}{q}\right)\right)$	$1 - \dfrac{\Gamma_{bx^q}\left(\frac{p+4}{q}\right)}{\Gamma\left(\frac{p+4}{q}\right)}$	$ax^p e^{-bx^q}$	$a'x^{p+3} e^{-bx^q}$	$\dfrac{-6qb^{\frac{p+4}{q}}}{\pi\rho\,\Gamma\left(\frac{p+4}{q}\right)}$	$\dfrac{-qb^{\frac{p+4}{q}}}{\Gamma\left(\frac{p+4}{q}\right)}$	$b = K/D_m^q$ where: $K\left(\dfrac{p+4}{q}\right)= 0.5\,\Gamma\left(\dfrac{p+4}{q}\right)$
Nukiyama-Tanasawa	$\dfrac{a}{qb^{\frac{3}{q}}}\left(\Gamma\left(\dfrac{3}{q}\right) - \Gamma_{bx^q}\left(\dfrac{3}{q}\right)\right)$	$1 - \dfrac{\Gamma_{bx^q}\left(\frac{6}{q}\right)}{\Gamma\left(\frac{6}{q}\right)}$	$ax^2 e^{-bx^q}$	$a'x^5 e^{-bx^q}$	$\dfrac{-6qb^{6/q}}{\pi\rho\,\Gamma(6/q)}$	$\dfrac{-qb^{6/q}}{\Gamma(6/q)}$	$b = K/D_m^q$ where: $K(6/q)= 0.5\,\Gamma(6/q)$
Rosin-Rammler	$\dfrac{a}{qb^{\frac{q-3}{q}}}\left(\Gamma\left(\dfrac{q-3}{q}\right) - \Gamma_{bx^q}\left(\dfrac{q-3}{q}\right)\right)$	e^{-bx^q}	$ax^{q-4} e^{-bx^q}$	$a'x^{q-1} e^{-bx^q}$	$\dfrac{-6bq}{\pi\rho}$	$-bq$	$\dfrac{(2.3)\log 2}{D_m^q}$
Logarithmic Normal	$\log x/M_g > 0$ $\dfrac{a(2.3)\log\sigma_g}{\sqrt{2}}\left(\Gamma_{\log x/M_g}(1/2) - \Gamma(1/2)\right)$ $\log x/M_g < 0$ $\dfrac{a(2.3)\log\sigma_g}{\sqrt{2}}\left(\Gamma_{\log x/M_g}(1/2) + \Gamma(1/2)\right)$	$\log x/M'_g > 0$ $\dfrac{1}{2} - \Gamma_{\frac{\log x/M'_g}{2}}(1/2)$ $\log x/M'_g < 0$ $\dfrac{1}{2} + \Gamma_{\frac{\log x/M'_g}{2}}(1/2)$	$\dfrac{a}{x}e^{-\frac{(\log x - \log M_g)^2}{2\log^2\sigma_g}}$ $M_g = M'_g e^{-3(2.3\,\log\sigma_g)^2}$	$\dfrac{a'}{x}e^{-\frac{(\log x - \log M'_g)^2}{2\log^2\sigma_g}}$	$\dfrac{6\,e^{-\frac{((3)(2.3)\log\sigma_g)^2}{2}}}{M_g^3(2.3)(\rho)(\pi)\sqrt{2\pi}\log\sigma_g}$	$\dfrac{1}{2.3\sqrt{2\pi}\log\sigma_g}$	D_m

TABLE 2

EXPRESSIONS FOR MEAN DROP SIZES

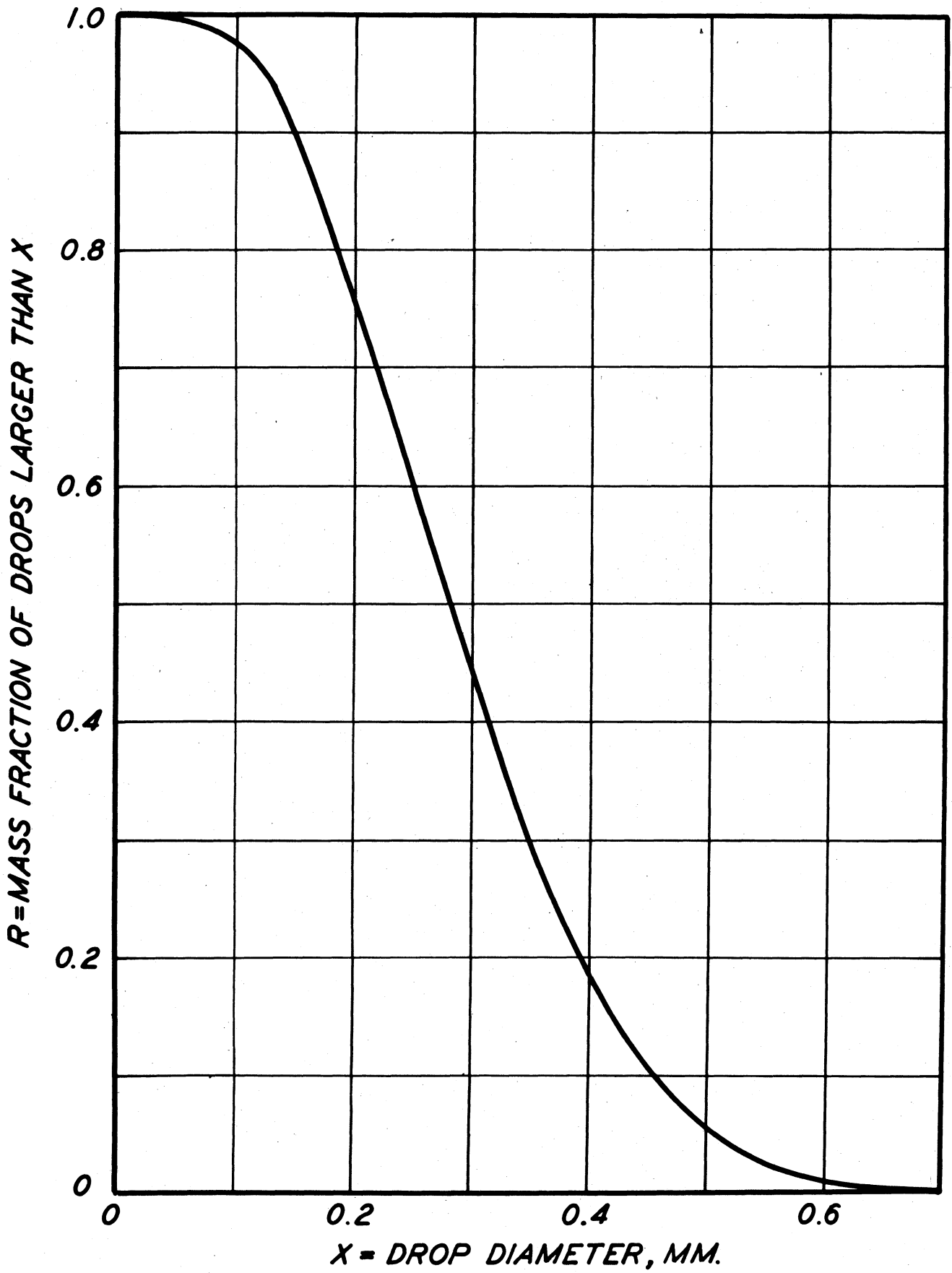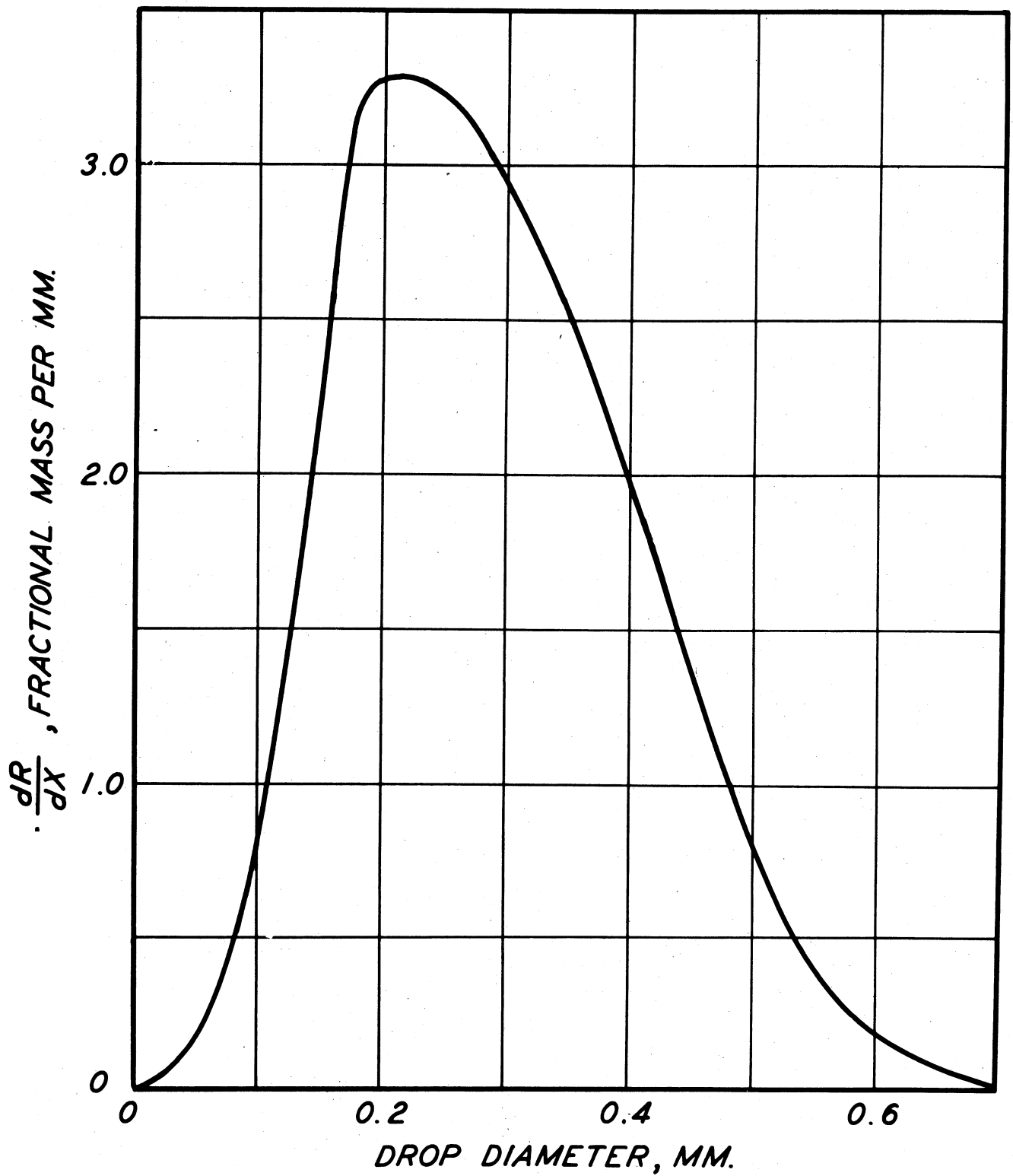| DEFINITION	MASS MEDIAN	Kolupaev Data mm	MASS MEAN	Kolupaev Data mm	SURFACE-TO-VOLUME MEAN	Kolupaev Data mm	NUMBER MEAN	Kolupaev Data mm	VOLUME-NUMBER MEAN	Kolupaev Data mm
(Definition)	$x_R = 0.5$		$\int_0^\infty x\,dR$		$\dfrac{\int_0^\infty dR}{\int_0^\infty x^{-1}\,dR}$		$\dfrac{\int_0^\infty N_x\,dN}{\int N\,dN}$		$\left(\dfrac{\int_0^\infty N_x{}^3\,dN}{\int N\,dN}\right)^{\frac{1}{3}}$	
NUKIYAMA-TANASAWA	$D_n = \left[\dfrac{K}{b}\right]^{\frac{1}{q}}$ where K is defined by: $K\Gamma\left(\dfrac{6}{q}\right) = 0.5\,\Gamma\left(\dfrac{6}{q}\right)$	0.269	$\dfrac{\Gamma\left(\frac{7}{q}\right)}{b^{\frac{1}{q}}\,\Gamma\left(\frac{6}{q}\right)}$	0.283	$\dfrac{\Gamma\left(\frac{4}{q}\right)}{b^{\frac{1}{q}}\,\Gamma\left(\frac{3}{q}\right)}$	0.236	$\dfrac{\Gamma\left(\frac{2}{q}\right)}{b^{\frac{1}{q}}\,\Gamma\left(\frac{1}{q}\right)}$	0.142	$\dfrac{1}{b^{\frac{1}{q}}}\left[\dfrac{\Gamma\left(\frac{6}{q}\right)}{\Gamma\left(\frac{3}{q}\right)}\right]^{\frac{1}{3}}$	0.184
ROSIN-RAMMLER	$\left[\dfrac{2.3\,\log 2}{b}\right]^{\frac{1}{q}}$	0.288	$\dfrac{\Gamma\left(1-\frac{1}{q}\right)}{b^{\frac{1}{q}}}$	0.294	$\dfrac{1}{b^{\frac{1}{q}}\,\Gamma\left(1-\frac{1}{q}\right)}$	0.229	$\dfrac{\Gamma\left(\frac{q-2}{q}\right)}{b^{\frac{1}{q}}\,\Gamma\left(\frac{q-3}{q}\right)}$	0	$\dfrac{1}{b^{\frac{1}{q}}}\left[\dfrac{1}{\Gamma\left(\frac{q-3}{q}\right)}\right]^{\frac{1}{3}}$	0
LOGARITHMICA-NORMAL	$M_g{}'$	0.275	$M_g{}'\,e^{\frac{(2.3\,\log\sigma_g)^2}{2}}$	0.294	$M_g{}'\,e^{-\frac{(2.3\,\log\sigma_g)^2}{2}}$	0.257	$M_g{}'\,e^{-\frac{5(2.3\,\log\sigma_g)^2}{2}}$	0.201	$M_g{}'\,e^{-\frac{3(2.3\cdot\log\sigma_g)^2}{2}}$	0.226
DIRECT INTEGRATION OF KOLUPAEV'S DATA		0.280		0.292		0.239		0.121		0.177

FIGURES

FIG. 1

FIG. 2

FIG.3

FIG.4

FIG. 5

FIG.6

FIG.7