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THE EFFECTS OF STOCHASTIC CASH FLOWS ON THE INCOME SMOOTHING HYPOTHESIS

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Many academicians and practitioners have hypothesized that flexible accounting rules allow corporate managements to "smooth" the reported incomes of their firms. As it is generally presented, smoothing is an attempt to reduce the variation of reported accounting income around some constant or time trending expectation. Although smoothing has been a popular research topic, little attention has been given to the possibility that smoothing through the manipulation of accounting rules might be a futile exercise. This paper demonstrates that the feasibility of alleged smoothing practices depends on the time series behavior of the firm's net cash flows. Net cash flow is defined to equal the net amount of cash that flows between the firm and its security holders.

The accounting literature contains numerous suggestions that corporate managements do, and in fact should, apply accounting income-measurement rules in a manner that will soften the effect of "hard times" and "good times" on reported incomes. For example, Hepworth [12], Gordon [9, 10], Gordon, Horwitz, and Meyers [11], Schiff [13], Copeland [5], Copeland and Licastro [6], Cushing [7], White [14], and Barefield and Comiskey [2] all investigate the issue of income smoothing. In these studies, the time series of "income" is generally assumed to be generated by a stochastic process whose expectation is either constant or a deterministic function of time. For example, Hepworth observed in 1953 that [12, p. 35]:

> Of course, more direct action in the direction of income smoothing may be accomplished by arbitrarily accumulating current expense in deferred charge accounts during bad years, together with liberal amortization of such deferred costs during periods of high revenue.

Gordon, after suggesting that management should utilize accounting rules to smooth income, made the following prediction [9, p. 262]:

> The prediction is that management does smooth income and the rate of growth in income. To elaborate, insofar as two or more alternative bases of valuation are allowed for a transaction, management will make the choice as follows. If the choice is just relevant for the current year's income, the choice will be the one that raises (lowers) income if it is below (above) the trend value for the year. If the valuation choice will influence income for a number of years in the future, the prediction is that a corporation with a high (low) rate of growth in income will make the choice that lowers (raises) the rate of growth in income.

Gordon also wrote [10, p. 223]:

> If the variation of the observations around the curve are smaller [when accountants adopt a specific accounting practice], income smoothing has been the consequence....

Schiff concluded that [13, p. 66]:

> It can be suggested that we now have a "homeostasis of earnings per share" and that the application of generally accepted accounting principles facilitates the reporting of earnings per share in a constant or rising pattern....

Copeland suggested that [5, p. 101]:

> One manipulating goal widely attributed to management is the desire to smooth reported income. Smoothing moderates year-to-year fluctuations in income by shifting earnings from peak years to less successful periods. This will lower the peaks and support the troughs, making earnings fluctuations less volatile.

Copeland also provided the following definition [5, p. 102]:

> Income smoothing involves the repetitive selection of accounting measurement or reporting rules in a particular pattern, the effect of which is to report a stream of income with a smaller variation from trend than would otherwise have appeared.

These studies suggest that "good" and "bad" times are symmetrically distributed around a constant mean or trend. That is, the smoothing literature assumes that "good times" (positive deviations from a mean or time-determined trend) are on average offset by "bad times" (negative deviations from the mean or time-determined trend), and hence, flexible accounting rules can be utilized to produce a constant or time trending reported income series with little or no variation from the constant mean or trend.

The possibility that "good times" may not be offset by "bad times" has generally been ignored in the smoothing literature.[1] The discussion that follows demonstrates how crucial a stationary underlying stochastic time series[2] is to the smoothing hypothesis. Specifically,

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[1]The single exception to this statement that I know of is Ball and Watts [1].

[2]A strictly stationary stochastic time-series is one whose properties are unaffected by a change in the time origin. To illustrate, suppose that the actual values of successive net cash flows (NCF_t^*'s) are described by a strictly stationary process with a level equal to NCF plus a random component e_t^* which is distributed as $N(0,\sigma_{e^*}^2)$. Given this generating process, the joint probability distribution associated with NCF_t^*, NCF_{t+1}^*, ..., NCF_{t+n}^* will be the same as the joint probability distribution associated with NCF_{t+j}^*, NCF_{t+1+j}^*, ..., NCF_{t+n+j}^*, for any j. With a strictly stationary process, the parameter NCF is the mean about which the process varies. If the process were nonstationary, the parameter NCF would have no specific meaning except as a reference point for the level of the process. See Box and Jenkins [4] for an excellent discussion of stationary and nonstationary processes.

it shows that if the stochastic process that is generating a firm's net cash flow series is stationary, smoothing, as generally described, is conceptually possible. Conversely, if a firm's net cash flow generating process is nonstationary, smoothing attempts will be exercises in futility. Hopefully the discussion that follows will persuade smoothing empiricists that in some instances management will not be able to produce a smooth accounting income sequence by manipulating flexible accounting rules. In particular, when a firm's net cash flow time series is nonstationary in its expected levels, it makes little sense to empirically test for the existence of smoothing attempts by looking at the stability of reported accounting income measurements over time.

Net Cash Flow Defined

The discussion of how the feasibility of smoothing relies on the behavior of corporate net cash flows requires that we first examine the relationship between a firm's reported accounting income measurements and the firm's net cash flows. To facilitate establishment of this relationship, we begin by closely examining the definition given to net cash flow throughout this paper.

In this study actual net cash flow during any period t (NCF_t^*) is defined as the aggregate net amount of cash which is flowing from the firm during period t to its debt and equity holders. Specifically, actual net cash flow during any period t can be written as:

$$NCF_t^* = DR_t^* - NC_t^*$$
$$= D_t^* + IN_t^* - \Delta n_t^* P_{t+1}^* - \Delta m_t PB_{t+1}^* \tag{1}$$

where

NCF_t^* = the actual aggregate net amount of cash flowing from the firm at the end of period t to debt and equity security holders of record at the start of period t.

DR_t^* = the actual total distributions (liquidating and non-liquidating dividend and interest payments) paid by the firm at the end of period t to its debt and equity security holders of record at the start of period t.

NC_t^* = the actual amount of new debt and equity capital (cash) obtained by the firm at the end of period t at the actual ex-distribution closing prices.

D_t^* = the actual total liquidating and nonliquidating dividends paid by the firm at the end of period t to its equity holders of record at the start of period t.

IN_t^* = the actual liquidating and nonliquidating interest payments made by the firm at the end of period t to debt holders of record at the start of period t.

Δn_t^* = the actual number of new equity securities sold at the end of period t at the actual ex-dividend closing price (P_{t+1}^*).

Δm_t^* = the actual number of new debt securities sold at the end of period t at the actual ex-interest closing price (PB_{t+1}^*).

Equation (1) describes NCF_t^* as the net amount of cash flowing between the firm and its security holders. Thus, from the firm's point of view, NCF_t^* can be interpreted as a measure of the firm's financing (as opposed to operating) activities during period t. For example, if NCF_t^* is negative, it denotes that the firm was a net acquirer of debt and equity funds during period t. Conversely, if NCF_t^* is positive, it signifies that the firm was a net distributor of debt and equity funds during period t.

For present discussion purposes it will be desirable to equate NCF_t^* (the measure of the firm's financing activities during period t) to a measure of the firm's operating activities during period t. To accomplish this, let

$$X_t^* - I_t^* = (\text{Operating cash flow})_t \, , \qquad (2)$$

where

X_t^* = the actual cash operating income earned by the firm at the end of period t. Cash operating income is here defined to equal accrual accounting income before interest and dividend payments plus or minus noncash revenues and expenses. Cash operating income thus ignores receivables, payables, prepayments, inventories, depreciation and other similar noncash items which are generally taken into consideration in the determination of accrual accounting income. Cash operating income is simply the algebraic sum of cash receipts from sales and cash outlays for operating expenses.

I_t^* = the actual net amount of cash invested by the firm at the end of period t in assets such as (1) depreciable plant and equipment, (2) land, (3) marketable securities, and (4) the asset "cash balances."

X_t^* minus I_t^* is clearly an operating (as opposed to financing) measurement of the firm's activities during period t. Notice, however, that X_t^* minus I_t^* does not measure the change in the firm's cash account during period t. In fact, such changes in cash balances are deemed to be investments of cash and are included in the I_t^* term.[3]

Given the definitions contained in equations (1) and (2), we can now proceed to equate financing net cash flows (NCF_t^*) with operating net cash flows (X_t^* minus I_t^*). This is easily accomplished by expressing the amount of new debt and equity capital (NC_t^*) raised by the firm during period t in terms of its components. Since the amount of new debt and equity capital (NC_t^*) required is directly related to the net cash outlay for firm investments (I_t^*), the cash outlay for dividend and interest payments (DR_t^*), and the net cash inflow from operations (X_t^*), we can write,

$$NC_t^* = I_t^* + DR_t^* - X_t^* \quad . \tag{3}$$

Then, by rearranging terms and noting from equation (1) that $NCF_t^* = DR_t^* - NC_t^*$, we write

$$NCF_t^* = DR_t^* - NC_t^* = X_t^* - I_t^* \; . \tag{4}$$

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[3]For excellent discussions of the concept of cash flow summarized in equation (2), see Bodenhorn [3] and Fama and Miller [8, pp. 87-8].

Thus, for any period t there is an identity between the firm's financing net cash flows ($NCF_t^* = DR_t^* - NC_t^*$) and the firm's operating net cash flows ($X_t^* - I_t^*$).

Relationship between Reported Income and Net Cash Flow

The measurement of accounting income can be thought of as being accomplished in two phases: (1) the recognition and measurement of revenues and gains, and (2) the recognition and measurement of expenses and losses. Many conceptual and practical problems arise in determining the "proper" periodic accounting income measurements. In conventional accounting systems based on historical cost, one of the most crucial problems encountered in measuring income is selecting the time at which to recognize revenues, gains, expenses, and losses. Often individual judgment is necessary, and alternative rules are available.

Although alternative recognition rules are central to the smoothing hypothesis, the significant point to be recognized is that reported accounting income and net operating cash flows can differ from each other only in timing. For example, conventional accounting rules dictate that the amount to be shown as revenue in any period is the amount ultimately collectible in cash. Similarily, all expenses are ultimately related to cash expenditures. With both revenues and expenses, however, uncertainty and differing circumstances create measurement and timing problems for the periodic determination of accounting income. If, for example, revenue is recognized in the accounting statements prior to the collection of cash, estimates of future credit losses must be made and deducted from gross revenues in computing accounting income. Likewise, if a cash expenditure benefits several periods, estimates are required to allocate the cash costs of services to the benefitted periods. In any event, with conventional accounting rules and procedures based on historical cost measurements, it must be true that over the life-span of a firm aggregate accounting incomes exactly equal aggregate net cash flows. Thus, it is permissible to write[4]

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[4]An example of a limiting case of equation (5) may help persuade the reader of its validity. Suppose a firm with a limited life-span of four years sells $1,000 of debt and equity securities at the end of year 1. Assume further that the firm does nothing with this $1,000 during its four-year life, and that at the end of year 4, it simply returns the $1,000 to its debt and equity security holders. Thus, at the end of year 1,

$$NC_1^* = \$1,000.$$

Since this $1,000 is simply put into a cash account, we must have

$$I_1^* = \$1,000.$$

Conversely, at the end of year 4,

$$DR_4^* = -I_4^* = \$1,000,$$

and

$$\begin{aligned}\sum_{j=0}^{3} (X_{1+j}^* - I_{1+j}^*) &= \sum_{j=0}^{3} (DR_{1+j}^* - NC_{1+j}^*) \\ &= -\$1,000 + 0 + 0 + \$1,000 \\ &= \text{zero}.\end{aligned}$$

From our knowledge of conventional accounting we know that

$$\begin{aligned}\sum_{j=0}^{3} Y_{1+j}^* &= 0 + 0 + 0 + 0 \\ &= \text{zero}.\end{aligned}$$

Thus, equation (5) holds in this limited example.

$$\sum_{j=0}^{N-1} Y^*_{t+j} = \sum_{j=0}^{N-1} (X^*_{t+j} - I^*_{t+j})$$

$$= \sum_{j=0}^{N-1} (DR^*_{t+j} - NC^*_{t+j}) \qquad (5)$$

$$= \sum_{j=0}^{N-1} NCF^*_{t+j}$$

where Y^*_t denotes the actual accounting income before interest and dividends reported in period t; N denotes the life-span of the firm, and the remaining terms in equation (5) are as defined in equations (1) and (2).

Special notice should be given to the fact that Y^*_t is defined as reported accounting income (1) prior to deductions for debt interest payments and accruals, and (2) prior to dividend payment and declaration deductions. Reported accounting income (Y^*_t) is defined in this manner to establish the equality expressed in equation (5). Throughout this study, creditors and stockholders are deemed to hold contracts for the future delivery of dollars from the firm.[5] Ex-post, this series of delivered dollars is represented by (DR^*_t, DR^*_{t+1}, ...) and consists of interest and dividend distributions. Thus, interest and dividends are viewed here as distributions of income and not as deductions from income.

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[5]Creditor and equity contracts can, of course, differ in their terms. Generally, debt contracts are written in terms of a specific DR^*_t, DR^*_{t+1}, ... sequence; whereas, equity contracts are expressed in terms of a residential DR^*_t, DR^*_{t+1}, ... sequence. Although these differing terms create risk differences, the basic nature of the two types of contracts are quite similar. In both cases, moneys are initially paid into the firm (i.e., the sequence NC^*_t, NC^*_{t+1}, ...) in return for the expected future delivery of dollars from the firm (i.e., the sequence DR_t, DR_{t+1}, ...).

Equation (5) establishes a relationship between accounting incomes and net cash flows. What remains to be accomplished is to determine how the feasibility of smoothing the Y_t^*, Y_{t+1}^*, ... sequence depends on the nature of the net cash flow time series.

The Feasibility of Smoothing

The actual net cash flow (NCF_t^*) a firm realizes in any period t can be divided into two elements: the expected component and the unexpected component. If we denote these two elements as NCF_t and e_t^*, respectively, the firm's actual net cash flow during any period t can be represented by the following model:

$$
\begin{aligned}
& NCF_t^* = NCF_t + e_t^* \\
& E(e_t^*) = 0 \\
& E(NCF_t^*) = NCF_t \qquad (6) \\
& \sigma^2(e_t^*, e_{t+j}^*) = 0 \text{ for all } j \neq 0 \\
& \sigma^2(e_t^*) = \sigma^2 \text{ for all } t .
\end{aligned}
$$

Equation (6) describes NCF_t^* as the sum of the expected value of net cash flow (NCF_t) plus a random component (e_t^*) which is assumed to have a constant variance over time merely to simplify exposition.

Equations (5) and (6) reveal the essence of the smoothing hypothesis as it is generally presented. Equation (5) shows that flexible accounting rules can be used for smoothing purposes only by shifting the random element (e_t^*) of actual net cash flows from period t to other periods. Such shifting of e_t^*, however, can produce a constant or time-trending Y_t^*, Y_{t+1}, ... series if, and only if, the expected element (NCF_t) of equation (6) is constant or trending. Therefore, it follows that the literature on income smoothing implicitly assumes that the expectation sequence NCF_t, NCF_{t+1}, ... is constant or a deterministic function of time. The validity of the smoothing hypothesis thus rests on the validity of an untested assumption.[6] This point will be clarified by showing that a "perfect" smoothing device works only when the NCF_t, NCF_{t+1}, ... sequence is stationary.

Copeland studied various smoothing devices and concluded that [5, p. 102].

> A perfect smoothing device must possess all of the following characteristics:
>
> A. Once used, it must not commit the firm to any particular future action.
>
> B. It must be based on the exercise of professional judgment and be considered within the domain of "generally accepted accounting principles."
>
> C. It must lead to material shifts relative to year-to-year differences in income.
>
> D. It must not require a "real" transaction with second parties, but only a reclassification of internal account balances.
>
> E. It must be used, singularly or in conjunction with other practices, over consecutive periods of time.

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[6] A pilot study of eight large firms was conducted by the author, and it was found that most of them had net cash flow sequences that were nonstationary.

If we allow ourselves to ignore the characteristic mentioned by Copeland in item B above, it would follow that the arbitrary deferral procedure, suggested by Hepworth [12, p. 35], is a "perfect" smoothing device. In periods when actual net cash flows (NCF_t^*) are less than expected net cash flows (NCF_t), the arbitrary deferral device would permit charging the difference e_t^* to a balance sheet account with an offsetting credit to income. The reported accounting income during period t (Y_t^*) would thus be bolstered by the amount e_t^*.

If the deferral device is adopted and the current period random components of actual net cash flows are accumulated in deferred balance sheet accounts during "bad" periods (i.e., when the e_t^*'s are negative), together with reductions in such deferred accounts during "good" periods (i.e., when the e_t^*'s are positive), the reported accounting income series Y_t^*, Y_{t+1}^*, ... has an expected path equal to the path followed by the series NCF_t, NCF_{t+1}, ...[7] The term "expected path" is used because the random component series e_t^*, e_{t+1}^*, ... is deemed to

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[7]In this paper, the arbitrary deferral device is applied only to the e_t^*, e_{t+1}^*, ... random component sequence. The application of the deferral device is restricted in this manner because smoothing through accounting manipulations can be accomplished only by interperiod transfers of random components (e_t^*).

There are, of course, other types of arbitrary deferral manipulations possible. For example, it is possible through deferral manipulations to convert a smooth, nongrowing, accounting-income sequence into a smooth, growing, accounting-income sequence. In this example, the smooth, nongrowing, accounting-income sequence must be the same as the smooth, nongrowing, NCF_1, NCF_2, ..., NCF_N sequence. However, the smooth, growing, accounting-income sequence would be:

be distributed as $N(0,\sigma^2_{e^*})$. Hence, if the life-span of the firm is finite, the series e^*_t, e^*_{t+1}, ... is finite, and it is thus conceivable that

$$\sum_{j=0}^{N-1} e^*_{t+j} \neq 0. \tag{7}$$

If equation (7) holds for a particular firm, adjustments to the deferral device described above would be required, causing the path of Y^*_t, Y^*_{t+1}, ... to deviate from the NCF_t, NCF_{t+1}, ... sequence. It is true, however, that if the e^*_t, e^*_{t+1}, ... sequence is distributed as $N(0,\sigma^2_{e^*})$ for reasonably large N, the following expectation will hold:

$$E\left(\sum_{j=0}^{N-1} e^*_{t+j}\right) = 0. \tag{8}$$

Thus, we can concentrate on the expected path of Y^*_t, Y^*_{t+1}, ...

To illustrate the "smoothness" of the reported accounting income sequence Y^*_t, Y^*_{t+1}, ... produced by the above-described deferral

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Footnote 7 continued.

$$Y^*_1 = NCF - \theta_0 + \alpha$$

$$Y^*_2 = NCF - \theta_0 + 2\alpha$$

$$\vdots$$

$$Y^*_N = NCF - \theta_0 + N\alpha$$

where Y^* is accounting income, $NCF = NCF_1 = NCF_2 = \ldots = NCF_N$ (the smooth sequence of expected net cash flows); θ_0 denotes the deferral made at time t=0, and α represents the per-period amortization of θ_0.

Notice that deferral manipulations which do not involve the e^*_t, e^*_{t+1}, ... random component sequence do not involve smoothing. Accordingly, such deferral manipulations are not considered in this paper.

device, let us examine the implications of two classes of stochastic net cash flow generating models.

A <u>constant-expectation, finite-variance process</u> is the first generating model to be considered. If a firm's time series of net cash flows is described by this model, the series will possess the following properties:[8]

$$E(NCF_t^*) = NCF, \text{ a constant for all } t$$

$$NCF_t^* = NCF + e_t^*$$

$$\sigma^2(NCF_t^*) = \sigma^2 \tag{9}$$

$$\rho(NCF_t^*, NCF_{t-1}^*) = 0 \text{ for all } t$$

$$\rho[(NCF_{t+1}^* - NCF_t^*), (NCF_t^* - NCF_{t-1}^*)] = \frac{-\sigma^2}{2\sigma^2} = -1/2$$

where again NCF_t^* denotes actual net cash flow and NCF represents expected net cash flows. With this model, the expected net cash flows during any

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[8]Notice that ρ, the first order autocorrelation coefficient, takes on the value of zero in the original NCF^* time series and a value of -1/2 in the first differences of this series. Higher order autocorrelation coefficients will be zero in both the original and first difference series. The shape of the graph of these autocorrelation coefficients of increasing order (lag) for a particular series and process is commonly referred to as the theoretical autocorrelation function.

In empirical work, the mapping of sample autocorrelation coefficients of increasing order is denoted as the sample autocorrelation function of the observed series. A comparison of this sample autocorrelation function with various theoretical autocorrelation functions then becomes a useful way to identify the underlying generating model of the observed series. In effect, this procedure tells the researcher which class of stochastic generating models are worthy of further consideration in the parameter estimation process that generally follows the identification step.

period t ($NCF_t = NCF$) are unaffected by past observed deviations ($e^*_{t-j} = NCF^*_{t-j} - NCF$) from the expectation. The actual observed sequence NCF^*_j, NCF^*_{t+1}, ... can be considered the result of combining two components: the "true" process or generating model and the "noise." The constant-expectation, finite-variance process is a strictly stationary function which is the significant characteristic of the observed data. Since here the noise component is assumed to be distributed as $N(0,\sigma^2_{e^*})$, the observed sequence NCF^*_t, NCF^*_{t+1}, ... will fluctuate randomly around NCF (the true process or the true mean of the observed sequence), and the variance of the observed sequence will equal $\sigma^2_{e^*}$. That is, if we are standing at time t+j, the expected values of NCF^*'s that have yet to occur is NCF and the expectation of those NCF^*'s that already have happened is the value they have actually realized.[9]

Suppose that a constant-expectation, finite-variance process does describe the net cash flow series of a particular firm. Suppose

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[9]Although in practice the true mean and variance are not known, they can be estimated from recent observations. If the true generating model is a constant-expectation, finite-variance process, it is permissible to estimate the true process (mean) by,

$$\overline{NCF}^* = \frac{1}{N} \sum_{j=0}^{N-1} NCF^*_{t+j} ,$$

and the variance of the process by,

$$\hat{\sigma}^2_{e^*} = \frac{1}{N} \sum_{j=0}^{N-1} (NCF^*_{t+j} - \overline{NCF}^*)^2.$$

further that this firm's management utilizes the previously described deferral device in an attempt to produce a smooth Y^*_t, Y^*_{t+1}, ... sequence. Will management's attempts at smoothing be successful? Earlier it was established that the deferral device will yield a sequence of reported accounting income measurements (Y^*_t, Y^*_{t+1}, ...) with an expected path equal to the path followed by the series NCF_t, NCF_{t+1}, ... Now, since the sequence NCF_t, NCF_{t+1}, ... is generated by a constant-expectation, finite-variance process, it follows that $NCF_t = NCF_{t+1} = \ldots = NCF$, and accordingly, the expected sequence $E(Y^*_t, Y_{t+1}, \ldots)$ will follow a constant path equal to NCF. Hence, one would expect to observe a smooth series of reported accounting income measurements if:

1. the underlying process of net cash flows is strictly stationary, and
2. management uses flexible accounting rules such as the deferral device in an attempt to produce a smooth Y^*_t, Y^*_{t+1}, ... sequence.

Notice that the stationarity of the underlying sequence of net cash flows is what made smoothing feasible.[10] Had the NCF_t, NCF_{t+1}, ... process been nonstationary, attempts at smoothing would not have been successful. To illustrate, let us next examine

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[10]Note that the process level (NCF_t) need not be constant for the process to be stationary. So long as NCF_t can be expressed as a deterministic function of time, the sequence NCF_t, NCF_{t+1}, ... can be detrended to yield a process that is strictly stationary. Hence, our conclusion about the feasibility of smoothing holds for any process that is a deterministic function of time, or is constant.

the implications of a generating process that produces a nonstationary sequence of corporate net cash flows. The process we will look at here can be denoted as a martingale constant-finite-variance process.[11] A time series of net cash flows generated by this process will possess the following properties:

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[11]The martingale is a limiting case of the more general process commonly referred to as a submartingale process. Usually these models only make statements about the expected values of a time series--no statements are made about either the nature or scedasticity of the series' probability density function. In general, a submartingale process can be described as follows:

$$E(NCF_t^*) = NCF_t$$

$$= \alpha_t NCF_{t-1}^* \text{ for all t and where } \alpha \geq 1 \text{ for all t}$$

$$NCF_t^* = NCF_t + e_t^*$$

$$= \alpha_t NCF_{t-1}^* + e_t^*$$

$$= \alpha_t (NCF_{t-1} + e_{t-1}^*) + e_t^*$$

$$= \prod_{j=1}^{t} \alpha_j NCF_0 + \sum_{j=1}^{t} \left(\prod_{i=1}^{j} \alpha_{t+1-i} e_{t-j}^* \right) + e_t^* .$$

If $\alpha_t = 1$ for all t (so that the expected changes in net cash flows are zero), then the time sequence follows a martingale process. And, if in addition, the probability density function remains constant for all t, then the sequence follows a random walk.

$$\begin{aligned}
E(NCF_t^*) &= NCF_t \\
&= NCF_{t-1}^* \text{ for all } t \\
NCF_t^* &= NCF_t + e_t^* \\
&= NCF_{t-1}^* + e_t^* \\
&= NCF_{t-1} + e_{t-1}^* + e_t^* \\
&= NCF_0 + \sum_{j=0}^{t} e_j^* \\
\sigma^2(NCF_t^*) &= \sigma^2 \\
\rho(NCF_t^*, NCF_{t-1}^*) &\to 1 \text{ as } t \to \infty \\
\rho[(NCF_{t+1}^* - NCF_t^*), (NCF_t^* - NCF_{t-1}^*)] &= 0
\end{aligned} \tag{10}$$

where NCF_t^* denotes the actual net amount of cash flowing from the firm at the end of period t to debt and equity security holders of record at the start of period t. The expected portion of NCF_t^* is equal to NCF_t; whereas, e_t^* represents the unexpected component of NCF_t^*.

If a firm's net cash flow sequence is being generated by a martingale constant-finite-variance process, one would expect failure in attempts to produce a smooth Y_t^*, Y_{t+1}^*, ... sequence. The principal reason for this expectation is that the expected levels of the process (i.e., the NCF_t, NCF_{t+1}, ... sequence) are not constant through time nor are they a deterministic function of time. That is, at any time t+j, our knowledge of the future behavior of the process is that it will change from its present level (NCF_{t+j}) in accordance with

$$\Delta NCF = NCF_{t+j+k} - NCF_{t+j} = \sum_{i=0}^{k-1} e^*_{t+j+i}, \tag{11}$$

whose expectation at time t+j is zero, and whose behavior cannot be predicted. At the beginning of time t+j the level of the process is NCF_{t+j} (or, $NCF^*_{t+j-1} = NCF_{t+j-1} + e^*_{t+j-1}$). As soon as the observation NCF^*_{t+j} becomes available (i.e., as soon as the time origin changes to the beginning of period t+j+1), the level of the process will be updated to $NCF_{t+j+1} = NCF_{t+j} + e^*_{t+j}$.

We now turn to the question: can a smooth sequence of reported accounting income measurements (i.e., Y^*_t, Y^*_{t+1}, ...) be produced when the underlying process (NCF_t, NCF_{t+1}, ...) is being generated by a martingale constant-finite-variance model? Application of the previously described deferral device will always yield a Y^*_t, Y^*_{t+1}, ... sequence whose expected path follows the NCF_t, NCF_{t+1}, ... series. When a martingale model is generating the net cash flow sequence, the deferral device will yield a sequence of accounting income measurements whose expected path [$E(Y^*_t, Y^*_{t+1}, \ldots)$] will lag the sequence NCF^*_t, NCF^*_{t+1}, ... by one period. Specifically, the deferral device will produce

$$\begin{aligned} \text{the } E(Y^*_t, Y^*_{t+1}, \ldots) \text{ sequence} &= \text{the } NCF_t, NCF_{t+1}, \ldots \text{ sequence}, \\ &= \text{the } NCF^*_{t-1}, NCF^*_t, \ldots \text{ sequence}. \end{aligned} \tag{12}$$

But, since the NCF_t, NCF_{t+1}, ... sequence is nonstationary, the expected Y^*_t, Y^*_{t+1}, ... sequence must also be nonstationary. We must thus conclude that the deferral device will fail to produce a smooth sequence of reported accounting income measurements when a martingale constant-finite-variance model describes the time series behavior of the underlying net cash flow sequence.

One could be tempted initially to argue that ex-post the NCF_t, NCF_{t+1}, ... sequence does have an arithmetic mean and perhaps a nonzero least squares trend; hence, it should be possible to smooth out the deviations around this mean or trend. But this argument overlooks the fact that the smoothing hypothesis is not an issue once the life-span of the firm has ended. Instead, the smoothing hypothesis argues that flexible accounting rules allow management to smooth the reported sequence of accounting income measurements during the life-span of their firms. It is thus the existence of an ex-ante mean or trend value that makes the smoothing hypothesis tenable. By definition, however, a nonstationary net cash flow series has neither a fixed level (mean) nor a fixed slope (trend). Hence, when the underlying net cash flow sequence is nonstationary, future means or trends cannot be predicted and smoothing becomes an untenable hypothesis.

Throughout this paper, time series behavior of net cash flows has been viewed as an exogenously determined phenomenon because the analysis was confined to examining the smoothing potential of accounting rules and techniques. Introducing the possibility that management deliberately follows an investment policy that yields a stationary net cash flow sequence introduces real transactions with second parties into the smoothing hypothesis. Although this possibility suggests some interesting theoretical and empirical questions, it was excluded from consideration so that the smoothing hypothesis could be preserved as an accounting issue.

Summary and Conclusions

Smoothing involves using accounting rules and techniques to convert an unsmooth sequence of net cash flows into a smooth sequence of reported income measurements. The smoothing hypothesis that has persisted in the literature suggests that flexible accounting rules do permit management to smooth the reported accounting income sequence of their firms. This paper has shown that the smoothing hypothesis is tenable only if the underlying sequence of corporate net cash flows is stationary. Conversely, the smoothing hypothesis can be rejected *a priori* when the underlying sequence of corporate net cash flows is nonstationary. What these findings suggest is that those who are concerned about the flexible accounting rules and/or smoothing should study the time series properties of corporate net cash flows before they empirically test for the existence of smoothing or argue for restricting accounting alternatives in an attempt to eliminate the potential for smoothing.

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