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FORECAST.A
AN INTERACTIVE FORECASTING PROGRAM

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INTRODUCTION

FORECAST.A is a forecasting program intended for users with limited statistical knowledge. Therefore, input and output are deliberately kept simple. The user is guided interactively through the program by prompts and comments. The program uses three forecasting methods:

1. Decomposition
2. Winters' three-parameter exponential smoothing model
3. Trend-line analysis

Forecasts are made up to 12 periods ahead. Although the program was originally designed to handle monthly time series, provisions are made to handle periods of different lengths. A minimum input of 12 and a maximum input of 120 periods can be handled.

TECHNIQUES

Decomposition

The decomposition method used in FORECAST.A is similar to the Census II X-11 method, but simplified to suit the needs of users with limited experience. This part of the program requires a minimum of thirty-six monthly data points; it cannot handle other than monthly data. Options offered are trading day adjustment and cutoff of extreme values. Trading days are days during which business has actually been transacted. A correction for trading days allows the user to take into consideration normal fluctuations in the number of trading days, as well as unusual events like strikes, etc. Trading day adjustment, if desired, takes place before decomposition starts.

Decomposition assumes that a time series consists of four components: trend, cycle, seasonal, and an irregular component. An attempt is made to isolate these components in order to explain the behavior of the time series and to make predictions based on the components. Generally the components can be connected in two different ways, by addition or by multiplication. Census II X-11 offers both possibilities, while FORECAST.A is limited to multiplicative models as illustrated below:

$$X_t = T_t \times C_t \times S_t \times I_t$$

where

X_t = actual observation at time t

T_t = trend factor at time t
C_t = cyclical factor at time t
S_t = seasonal factor at time t
I_t = irregular factor (random element in time series).

Trend and cyclical factors are not separated in this model because a cycle is assumed to be so long that its separation from the trend would not improve its explanatory power.

As a first step FORECAST.A attempts a preliminary separation of the trend-cycle components from the seasonal and irregular components:

$$\frac{X_t}{M_t} = R_t = \frac{T_t \times C_t \times S_t \times I_t}{T_t \times C_t} = S_t \times I_t$$

where

$$M_t = T_t \times C_t$$
$$R_t = S_t \times I_t.$$

This separation is achieved by the use of twelve-month centered moving averages (MA), which eliminate most seasonal and irregular variations from the time series. The ratio of this series (M_t) to the original series (X_t) is the basis for further calculations (R_t).

It is now possible, if so desired, to remove extreme values from the time series. The program will transform the time series into a set of twelve time series, each of which contains the data for a specific month. For example, one series will contain all January data, one will contain all February data, and so forth. For each of these monthly series, extreme values will be removed by a two-step process. In step one, 3 x 3 MA's[1] are calculated. The process

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[1] 3 x 3 MA's are calculated by calculating three-month MA's of the time series and then calculating three-month MA's of those MA's.

results in the loss of two values at the beginning and two values at the end of the series. These four values are replaced by weighted averages calculated from the first and last values, respectively.

As a second step, means and standard deviations for each monthly series (e.g., Januaries, Februaries, etc.) are calculated. All values falling outside of a user determined range will be replaced by the average of the values immediately preceding and following the dropped value. This range will be determined by the user in terms of standard deviations. For example, the user can specify that all values deviating more than two standard deviations from the mean are to be replaced. This option should be used only if it is known that outliers are caused by unusual events such as strikes, catastrophes, etc., whose effects have not already been removed by the use of the trading day adjustment option. Careless use of this option will lead to a loss of information.

At this point in the procedure, the time series is twelve values shorter than the original series because six values were lost from the beginning and six from the end by the calculation of the twelve-month centered MA's. Before new preliminary seasonal factors can be calculated, these values have to be replaced. Replacement values are the corresponding values of the second year for the first six values of the time series and the corresponding values of the next to last year for the last six values. The original series will then be divided by this adjusted series. At this point the time series is actually a series of ratios, and in order to determine preliminary seasonal factors this series will be normalized so that the mean for every year equals 100. Then 3 x 3 MA's will be calculated for every month as previously described. The results are preliminary seasonal factors, and the division of the original series by these factors yields a preliminary seasonally adjusted time series:

$$PS_t = \frac{X_t}{S_t} = \frac{T_t \times C_t \times S_t \times I_t}{S_t} = T_t \times C_t \times I_t$$

where

PS_t = preliminary seasonally adjusted value in time t.

The time series of these preliminary seasonally adjusted values (PS_t's) is the basis for the application of thirteen-month Henderson moving average weights (see Table 1) to eliminate random fluctuations.[1]

$$M'_t = \frac{PS_t}{I_t} = \frac{T_t \times C_t \times I_t}{I_t} = T_t \times C_t$$

where

$M'_t = T_t \times C_t$.

In order to determine the final seasonal-irregular factor ($S_t \times I_t$), the original time series (X_t) is divided by the M'_t. Extreme values in the resulting series of ratios will again be removed, if so desired, by the procedure mentioned previously. The following series will result:

$$FSI_t = \frac{X_t}{M'_t} = \frac{T_t \times C_t \times S_t \times I_t}{T_t \times C_t} = S_t \times I_t$$

where

$FSI_t = S_t \times I_t$.

In order to obtain the final seasonal factors (S_t) from this series (FSI_t), the irregular factor (i.e., random fluctuations) must be removed. This will be achieved by the application of 3 x 3 MA's to each month in the manner described above:

$$S_t = \frac{FSI_t}{I_t} = \frac{S_t \times I_t}{I_t} .$$

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[1] Shiskin, J., Young, A. H., Musgrave, J. G. "The X-11 Variant of the Census Method II Seasonal Adjustment Program." Bureau of the Census, Technical Paper No. 15, p. 63.

TABLE I

13-Term Henderson Weights

Henderson MA for month	Weight for Month												
	N-12	N-11	N-10	N-9	N-8	N-7	N-6	N-5	N-4	N-3	N-2	N-1	N
N..........	0	0	0	0	0	0	-.092	-.058	.012	.012	.244	.353	.421
N-1........	0	0	0	0	0	-.043	-.038	.002	.080	.174	.254	.292	.279
N-2........	0	0	0	0	-.016	-.025	.003	.068	.149	.216	.241	.216	.148
N-3........	0	0	0	-.009	-.022	.004	.066	.145	.208	.230	.201	.131	.046
N-4........	0	0	-.011	-.022	.003	.067	.145	.210	.235	.205	.136	.050	-.018
N-5........	0	-.017	-.025	.001	.066	.147	.213	.238	.212	.144	.061	-.006	-.034
N-6........	-.019	-.028	0	.066	.147	.214	.240	.214	.147	.066	0	-.028	-.019

Source: Shiskin, J., Young, A. H., Musgrave, J. C. "The X-11 Variant of the Census Method II Seasonal Adjustment Program," Bureau of the Census, Technical Paper No. 15, p. 63.

Division of the original series by the final seasonal factors yields a series which is seasonally adjusted:

$$FA_t = \frac{X_t}{S_t} = \frac{T_t \times C_t \times S_t \times I_t}{S_t} = T_t \times C_t \times I_t$$

where

FA_t = final seasonally adjusted value in time t.

Elimination of random variations (I_t) from this time series yields the final trend-cycle component of the time series. It is obtained by applying thirteen-month Henderson MA's to the series of the final seasonally adjusted values (FA_t):

$$TC_t = \frac{FA_t}{I_t} = \frac{T_t \times C_t \times I_t}{I_t} = T_t \times C_t$$

where

TC_t = final trend-cycle value in time t.

Separate forecasts are made for the final trend-cycle component (TC_t) and the seasonal factor (S_t) for twelve periods into the future. Random fluctuations (I_t) are unpredictable, and therefore I_t will be ignored in forecasts. The seasonal factors are predicted as the expected values of the seasonal factors for the same months of the previous two years. The trend-cycle component is predicted by trend-line extrapolation. In order to capture non-linear developments in the trend-cycle component, two attempts at extrapolation are made, one using the linear equation

$$Y = a + bX,$$

the other using the nonlinear equation

$$Y = e^{a+bX},$$

where

X = time variable
Y = predicted variable
a, b = parameters to be estimated.

For the linear equation, the slope b will be estimated by the equation:

$$b = \frac{\Sigma Y \times \Sigma X}{n} - \Sigma(X \times Y) \qquad \frac{(\Sigma X)^2}{n} - \Sigma X^2$$

and the intercept a will be estimated by:

$$a = (\Sigma Y - b\Sigma X)/n$$

where

n = number of observations.

The coefficient of correlation (r) is obtained by the equation:

$$r = b \sqrt{\left(\Sigma X^2 - \frac{(\Sigma X)^2}{n}\right) \Big/ \left(\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}\right)}.$$

The parameters a and b and the coefficient of correlation are estimated in similar manner after $y = e^{a+bX}$ is transformed into a linear equation using the natural logarithm:

$$Y = e^{a+bX} \rightarrow \ln Y = a + b \times X.$$

The final forecast is the combination of the predicted values of the linear and non-linear trend-cycle components with the predicted seasonal factor:

$$FF_t = TC_{T+\tau} \times S_{T+\tau}$$

where

FF = final forecast
τ = forecasted period, $\tau = 1, \ldots, 12$
T = last available period.

Several measures of forecast accuracy are offered. The program provides the mean squared error (MSE), the standard deviation (SD), the average error (AE), and the coefficient of correlation (r).

The MSE is the variance of the predicted value from the actual value. The program calculates the MSE by starting with year 2, predicting the trend-cycle component (TC), combining the TC with the seasonal factors for the respective period of the final forecast, and comparing this forecast with the actual values for available periods. This measure is labeled "variance" in the program output. The standard deviation is the square root of the MSE. The average error is the average deviation of the predicted from the actual value. The MSE and SD are measures of the magnitude by which the prediction deviates from the true value; the average provides information as to whether the program is, on the average, predicting correctly or is consistently over- or under-estimating the true value.

$$MSE_m = \frac{\sum_{i=2}^{r} (X_{m,i} - FF_{m,i})}{r - 2}$$

$$SD_m = MSE_m$$

$$AE_m = \frac{\sum_{i=2}^{r} (X_{m,i} - FF_{m,i})}{r - 1}$$

where

MSE = mean squared error
SD = standard deviation
AE = average error
X = actual value
FF = final forecast
r = number of years input
m = predicted month, m = 1,...,12.

The coefficient of correlation for each trend line is also presented. This number is a measure of the goodness of fit of the estimated trend line to the data points.

Winters' Three Parameter Model of Exponential Smoothing

Unlike the decomposition method Winters' model is capable of handling periods of different lengths. Every period length from yearly data to weekly data can be handled. The minimum input requirement is four years; thus, for monthly data, at least forty-eight values must be available. Basic equations for Winters' model are:

$$S_t = \alpha \frac{X_t}{I_{t-L}} + (1 - \alpha)\ (S_{t-1} + b_{t-1})$$

$$b_t = \gamma\ (S_t - S_{t-1}) + (1 - \gamma)\ b_{t-1}$$

$$I_t = \beta \frac{X_t}{S_t} + (1 - \beta)\ I_{t-L}$$

$$F_{t+m} = (S_t + B_t \times m)\ I_{t-L+m}$$

where

S_t = simple exponentially smoothed value

b_t = trend adjustment factor

I_t = seasonal adjustment factor

F_{t+m} = forecast for period m

X_t = original observation

$T = 1,\ldots,T$ = observed time

$m = 1,\ldots,M$ = predicted time

α = smoothing constant for S_t

β = smoothing constant for b_t

γ = smoothing constant for I_t

L = number of periods per cycle

The weights α, β, and γ can either be determined by the user or by the program. If the latter option is chosen the program selects the optimal combination of weights. The optimization criterion is the MSE. The weights are chosen so that the average MSE of the forecast for the first and sixth period is minimized. The routine used for optimizing α, β, and γ is EXPLORE[1] based on an algorithm developed by Keefer and Gottfried.[2]

The user also has to determine how many years are to be used for initialization of S_t, b_t, and I_t. It is recommended that not more than half of the available years be used but the minimum requirement is at least two years.

Initial value for S will be the simple average of the observations for the first year.

Initial value for b_t will be the difference between the means of the last year and the first year used for initialization divided by the number of observations used for initialization less the number of periods in one cycle.

$$b_1 = \frac{\text{Mean 2} - \text{Mean 1}}{(P-1)xL)}$$

where

Mean 1 = mean of the observations for the first year used for initialization

Mean 2 = mean of the observations for the last year used for initialization

P = total number of cycles used for initialization.

Under the assumption of a linear trend, b is the monthly increase or decrease per period due to the trend. For the seasonal component (I_t), starting values have to be calculated for all periods used for initialization. Starting values for I_t will be the ratios of actual observations to seasonally adjusted values of the same periods for all periods used for initialization. The seasonal factors for the same months of succeeding years will be averaged to yield the starting values for the seasonal factors.

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[1]Becker, J. R. "EXPLORE, A Computer Code for Solving Nonlinear, Continuous Optimication Problems." Computer Application No. 10, Division of Research, Graduate School of Business Administration, The University of Michigan.

[2]Keefer, D. L., and Gottfried, B. S. "Differential Constraint Dealing in Penalty Function Optimization." American Institute of Industrial Engineers Transactions 2 (1970): 281-89.

$$I_i = \frac{1}{p} \sum_{k=1}^{p} \left(\frac{X_{(k-1) \times L+i}}{S_p - \left(\frac{L+1}{2} - i\right) \times b_i} \right)$$

where

$$i = 1,\ldots,L.$$

Finally, these seasonal factors will be normalized so that their average is one. These initial values will be used as starting values for the original equations given above, which will be applied to the observations used for initialization. The values for S, b, and I obtained in this step will be the final starting values.

After the starting values have been calculated, the program proceeds with the basic equations of Winters' model as they are given above. Starting with the second year after the last year used for initialization, forecasts will be made and compared to the actual values. The output shows the forecast for the last available year together with the actual data for the same year, as well as the standard deviation of the forecasted from the actual data which allows the user to evaluate the quality of the forecast. A final forecast is then made for twelve periods in the future.

Trend-Line Extrapolation

If the number of observations is too small for either decomposition or for Winters' method, but exceeds twelve, a trend-line extrapolation can still be performed. This is done by using the same routine used to perform trend-li extrapolation of the cycle-trend component in decomposition. The linear an nonlinear extrapolations are also made using the equations

$$Y = a + bX$$

and

$$Y = e^{a+bX}$$

as previously described. The only measure of forecasting quality provided by this routine is the coefficient of correlation.

APPENDIX A

FORECAST.A PROGRAM USER GUIDE

FORECAST.A PROGRAM

Location: N735:FCAST.OBJECT

Contents: The object module of the FORECAST.A program.

Purpose: To forecast time series.

Use: This program is invoked by a $SOURCE N735:FORECAST.A command. MTS file run contains the following command:

```
$RUN  K45V:EXPLR.O+N735.OBJECT
     5=INPUT  6=*DUMMY* 10=*MSOURCE* SCARDS=*MSOURCE*
     SPRINT+*MSINK*  T=5
```

Logical I/O units Referenced:

SCARDS - The source of commands to the FORECAST program (defaults to *SOURCE*).

SPRINT - Messages to user (defaults to *SINK*).

- Input data for optimization algorithm used by Winters' method. The user does not have to be concerned about this unit.

- Output of the optimization algorithm used by Winters' method. The user may reassign the unit (defaults to *DUMMY*).

Commands: The FORECAST commands are described in the following pages.

COMMAND DESCRIPTIONS

This appendix contains a detailed description of each FORECAST command. The command descriptions are presented alphabetically with each command description starting on a new page.

Abbreviated portions of each command may be used as long as they include enough of the command to be distinguished from other commands. The following standard notation conventions are used in the command prototype descriptions:

1) Command prototype fields appearing in lower case are generic terms which are to be replaced by an item supplied by the programmer. Command prototype fields appearing in upper case are fields which are to be repeated verbatim in the command.

2) Brackets ([ ]) indicate that a particular field is optional.

3) An ellipsis (...) indicates that the preceding field may be repeated in the command.

4) Positional parameters must always follow the command name. However, other parameters options may be specified in any order.

   Example: In the command

   READ FDATA NUMVAR=12 NUMDAT=120

   "FDATA" must follow "READ," while "NUMVAR=12" can be placed before or after "NUMDAT=120."

5) Character strings that are parameter values must be enclosed within single quotes.

A few of the generic terms which appear within the command descriptions require explanation:

1) "keyword" means any keyword option available for that command.

2) "FDname" means the name of a user MTS file.

FORECAST COMMAND SUMMARY

DECOMPOSITION	- Forecasts by means of the decomposition method.
MTS	- Returns to MTS mode.
PLOT	- Plots the original and forecasted time series.
READ	- Reads the time series data.
RTRDAY	- Reads data about trading day adjustments.
STOP	- Terminates program execution.
TREND	- Forecasts by means of trend-line extrapolation method.
WINTERS	- Forecasts using Winters' three-parameter exponential smoothing model.
$cmd	- Immediately executes MTS command "cmd".

DECOMPOSITION

Prototype: DECOMPOSITION [keyword=...].

Purpose: To forecast using the decomposition method.

Notes: This command can only be issued after the time series data have been read using the READ command. This method is limited to use with monthly data only.

Options: Keyword options, their default values, and possible values for reassignment are the following:

CUTOFF = integer value DEFAULT = no cutoff
Extreme cutoff values stated in terms of number of standard deviations around the mean. All time series data that fall outside the specified limit are replaced by the average value of the preceding and following values.. This parameter must be used very carefully, e.g., only in cases where it is known that the outliers exist because of abnormal environmental conditions.

Example: DECOMPOSITION
The time series is forecast using the decomposition method.

MTS

Prototype: MTS [MTS command]

Purpose: This command returns control to MTS.

Notes: An optional MTS command may be specified. To restart execution of the FORECAST.A program, a $RESTART should be invoked.

Example: MTS
  would return control to the MTS command mode.

PLOT

Prototype:	PLOT
Purpose:	To plot the original and forecasted time series.
Notes:	This command plots the forecasted time series data for all the methods that have been used before issuing this command.
Example:	PLOT A plot of the original and forecasted time series is produced

READ

Prototype: READ FDname [key word=...].

Purpose: To read the time series data points.

Options: Keyword options, their default values, and possible values for reassignment are the following:

FMT = 'character string'
Format specification of data in FDname to be read. Note that the format specification must be enclosed within brackets and then parentheses.

NUMDAT = integer value
Total number of data points to be read from the FDname.

NUMVAR = integer value Default = 1
Number of data points in each line of FDname.

Example: READ FDATA NUMVAR=12 NUMDAT=120 FMT='(12(F10.1,1X))'
The time series data points are read from the MTS file FDATA. Each line of FDATA contains 12 data points, there are 120 data points and the format of each line is 12(F10.1,1X).

RTRDAY

Prototype: RTRDAY FDname [keyword=...]

Purpose: To read trading day adjustment data.

Notes: This command should be issued after the READ command. Notice no option is provided to specify total number of data points, since they must be equal to the number specified in the READ command.

Options: Keyword options, their default values and possible values for reassignment are the following:

FMT = 'character string'
Format specification of data in FDname to be read. Note that the format specification must be enclosed within brackets and then parentheses.

NUMVAR = integer value   Default = 1
Number of data points in each line of FDname.

Example: RTRDAY FDATA FMT='(F10.2)'
Data for trading day adjustment is read from file FDATA with format specification F10.2.

STOP

Prototype: STOP

Purpose: This command terminates processing and returns control to the MTS command mode.

Notes: Execution of the FORECAST. A program cannot be restarted because the program is unloaded.

Example: STOP

TREND

Prototype: TREND

Purpose: To forecast using the trend-line extrapolation method.

Notes: This command can only be issued after the READ command has been executed.

Example: TREND

The time series is a forecast using the trend line extrapolation.

WINTERS

Prototype: WINTERS [keyword=...]

Purpose: To forecast using the Winters three-parameter exponential smoothing model.

Notes: This command can only be issued after READ command has been executed. Further, a minimum of four years cycles of data must be available for using this method. Example

Options: Keyword options, their default values, and possible values for reassignment are the following:

FPART = integer value   Default = Half the time series
First part of the time series used for initialization by the Winters' method. This value must be specified in terms of number of years of the time series.

LCYCLE = integer value
Length of cycle of time series in terms of number of periods.

ALPHA = real value
BETA = real value   Default = Optimal weights found by
GAMMA = real value   the program

Smoothing parameters for Winters' method. Default option is taken if all the three weights are not explicitly specified by the user.

Example: WINTERS FPART=3 LCYCLE=12
The time series is forecast using the Winters' method. The first three years of the time-series are used for initialization; each year consists of twelve periods.

APPENDIX B

AN EXAMPLE

The dataset used in the example contained the monthly Business Week Index for 495 months. Before the actual printout of the sample run of FORECAST.A is shown, this file is listed. Disregarding the line number, every line of the file contains the following information:

Column	Context
1-2	Month
4-5	Year
7-11	Business Week Index

```
'list -bwj
>     1      01.39.021.7
>     2      02.39.021.1
>     3      03.39.020.7
>     4      04.39.019.7
>     5      05.39.019.4
>     6      06.39.020.9
>     7      07.39.021.4
>     8      08.39.022.2
>     9      09.39.024.1
>    10      10.39.025.3
>    11      11.39.025.9
>    12      12.39.026.2
>    13      01.40.025.7
>    14      02.40.023.6
>    15      03.40.022.4
>    16      04.40.021.9
>    17      05.40.023.4
>    18      06.40.025.8
>    19      07.40.026.7
>    20      08.40.026.8
>    21      09.40.027.5
>    22      10.40.028.3
>    23      11.40.029.5
>    24      12.40.029.7
>    25      01.41.030.4
>    26      02.41.031.3
>    27      03.41.032.6
>    28      04.41.031.7
>    29      05.41.033.5
>    30      06.41.034.7
>    31      07.41.035.3
>    32      08.41.035.1
>    33      09.41.035.3
>    34      10.41.035.4
>    35      11.41.035.7
>    36      12.41.036.2
>    37      01.42.037.3
>    38      02.42.038.8
>    39      03.42.039.8
>    40      04.42.040.8
>    41      05.42.040.9
>    42      06.42.041.5
>    43      07.42.042.1
>    44      08.42.043.1
>    45      09.42.043.9
>    46      10.42.044.7
>    47      11.42.045.6
>    48      12.42.046.0
>    49      01.43.046.6
>    50      02.43.047.4
>    51      03.43.048.1
>    52      04.43.048.8
>    53      05.43.048.9
>    54      06.43.048.7
>    55      07.43.049.3
>    56      08.43.049.7
>    57      09.43.049.9
>    58      10.43.050.3
>    59      11.43.050.4
>    60      12.43.049.8
>    61      01.44.049.9
>    62      02.44.050.0
>    63      03.44.049.8
```

>	64	04.44.050.0
>	65	05.44.049.6
>	66	06.44.049.1
>	67	07.44.049.1
>	68	08.44.048.9
>	69	09.44.048.3
>	70	10.44.048.2
>	71	11.44.048.2
>	72	12.44.048.3
>	73	01.45.048.3
>	74	02.45.047.9
>	75	03.45.048.4
>	76	04.45.048.3
>	77	05.45.047.3
>	78	06.45.046.0
>	79	07.45.045.0
>	80	08.45.040.5
>	81	09.45.035.5
>	82	10.45.033.3
>	83	11.45.034.8
>	84	12.45.036.0
>	85	01.46.034.2
>	86	02.46.031.4
>	87	03.46.035.3
>	88	04.46.034.8
>	89	05.46.033.5
>	90	06.46.035.5
>	91	07.46.037.5
>	92	08.46.038.3
>	93	09.46.038.2
>	94	10.46.038.4
>	95	11.46.038.7
>	96	12.46.038.6
>	97	01.47.038.7
>	98	02.47.038.7
>	99	03.47.040.3
>	100	04.47.039.8
>	101	05.47.039.0
>	102	06.47.039.3
>	103	07.47.038.8
>	104	08.47.039.0
>	105	09.47.039.5
>	106	10.47.039.5
>	107	11.47.039.3
>	108	12.47.040.0
>	109	01.48.040.8
>	110	02.48.040.1
>	111	03.48.040.4
>	112	04.48.040.5
>	113	05.48.040.8
>	114	06.48.041.3
>	115	07.48.041.9
>	116	08.48.041.9
>	117	09.48.040.4
>	118	10.48.041.5
>	119	11.48.040.5
>	120	12.48.040.8
>	121	01.49.040.5
>	122	02.49.039.6
>	123	03.49.039.9
>	124	04.49.040.7
>	125	05.49.039.3
>	126	06.49.039.2
>	127	07.49.038.7
>	128	08.49.039.0
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#END OF FILE
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 $SOURCE N735:FORECAST.A
#$SET ECHO=OFF

&*** Forecast.A Program ***

&
&READ -BWI NUMDAT=495 FMT='(6X,F5.1)'
&TREND

 FINAL FORECAST USING REGRESSION ANALYSIS, ROW 1: Y=A+B*X, ROW 2: Y=A*B**X, X= TIME

 COEFFICIENT OF CORRELATION:0.96742

       137.      138.      138.      138.      138.      139.      139.      139.      139.      140.      140.      140.

 COEFFICIENT OF CORRELATION:0.97218

       158.      158.      159.      160.      160.      161.      161.      162.      162.      163.      163.      164.
&
```

DECOMPOSITION
CUTOFF=0,

TREND-CYCLE FORECASTS ARE MADE USING THE REGRESSION EQUATIONS: 1. Y=A+B*X 2. Y=A*B**X

ORIGINAL SERIES

22.	21.	21.	20.	19.	21.	21.	22.	24.	25.	26.	26.
26.	24.	22.	22.	23.	26.	27.	27.	28.	28.	30.	30.
30.	31.	33.	32.	34.	35.	35.	35.	35.	35.	36.	36.
37.	39.	40.	41.	41.	42.	42.	43.	44.	45.	46.	46.
47.	47.	48.	49.	49.	49.	49.	50.	50.	50.	50.	50.
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73.	72.	73.	73.	72.	72.	73.	74.	74.	74.	73.	74.
76.	76.	75.	76.	76.	78.	78.	77.	79.	79.	78.	79.
80.	80.	80.	82.	81.	82.	83.	83.	84.	83.	84.	85.
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95.	96.	97.	98.	99.	99.	99.	99.	100.	101.	97.	101.

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139.	139.	141.	142.	143.	144.	144.	145.	146.	147.	147.	148.
149.	150.	151.	152.	152.	152.	151.	151.	151.	151.	150.	149.

SEASONAL FACTORS

0.95166	0.96322	1.01102	1.01075	0.99861	1.01406	1.02002	1.02650	1.01986	1.01436	0.99006	0.97988
0.95774	0.97023	1.01010	1.01092	1.00025	1.01138	1.01571	1.02111	1.01358	1.00947	0.99293	0.98658
0.96995	0.98101	1.00825	1.01060	1.00201	1.00654	1.00855	1.01308	1.00595	1.00355	0.99534	0.99514
0.98593	0.99249	1.00672	1.01104	1.00355	0.99949	0.99899	1.00458	1.00021	0.99850	0.99469	1.00380
1.00100	1.00061	1.00664	1.01370	1.00469	0.99126	0.98834	0.99802	0.99968	0.99647	0.98905	1.01053
1.01097	1.00320	1.00805	1.01816	1.00629	0.98458	0.97979	0.99378	1.00185	0.99629	0.98156	1.01548
1.01531	1.00162	1.00961	1.02146	1.00808	0.98200	0.97657	0.99121	1.00384	0.99739	0.97581	1.01709
1.01529	0.99810	1.00963	1.02042	1.00950	0.98437	0.98182	0.98978	1.00340	0.99837	0.97502	1.01430
1.01621	0.99709	1.00902	1.01528	1.00991	0.99088	0.99130	0.98902	1.00165	1.00015	0.97367	1.00582
1.01567	0.99780	1.00881	1.01015	1.01113	0.99750	1.01040	0.98959	1.00073	0.99848	0.98313	0.99934
1.03472	1.00710	1.01046	1.00091	1.00927	1.01233	1.00551	0.98843	0.99789	1.00727	0.95096	0.97516
1.00057	0.99884	1.01407	1.01302	1.02222	1.00278	0.99524	0.95855	0.98261	0.99734	0.99661	1.01816
1.01249	1.01281	1.02270	1.00548	0.97789	0.98311	0.98298	0.98327	0.99939	1.00226	1.00335	1.01427
1.00068	0.98200	0.96269	0.95496	0.99307	1.03182	1.02607	1.01601	1.02012	1.01674	0.98925	1.00660
1.01116	1.00241	1.01250	1.03228	1.02780	1.01568	1.01631	0.98135	0.97904	0.97945	0.96592	0.97612
0.98186	1.00139	1.01902	1.01554	0.98989	0.98615	0.97964	0.98169	1.01100	1.01500	1.00485	1.01397
1.00655	1.00451	1.00161	1.01454	0.99653	0.99993	1.00304	0.99923	1.01568	0.99393	0.97801	0.98645
1.00105	1.00063	0.99882	0.96542	0.98673	0.99953	1.00265	0.99158	1.01488	1.02082	1.00840	1.00949
1.00683	0.98900	1.00513	1.03050	1.03475	1.03223	1.01707	0.99283	0.98420	0.97942	0.96138	0.95646

0.94202	0.95561	0.99813	1.00364	1.01380	1.01251	1.01154	1.00700	1.02838	1.01341	1.00141	1.01255
1.01999	1.02096	1.02913	1.00696	0.98045	0.96596	0.95251	0.95633	1.01634	1.02524	1.01245	1.01368
1.00825	1.00038	0.99772	1.01681	1.02670	1.03125	1.02088	0.99330	0.98795	0.97838	0.96940	0.96898
0.98482	0.98593	0.99381	1.01452	1.01066	1.00894	0.98681	0.99273	1.01281	1.00271	0.99846	1.00780
1.00235	0.99425	0.98571	0.99986	1.00507	1.00967	0.99758	0.98795	0.99942	1.01567	1.00609	0.99639
0.99549	0.99652	1.01069	1.00549	0.99533	1.00451	1.00844	0.99316	0.99805	1.00404	0.99370	0.99458
1.00637	0.99574	0.99834	1.00543	0.99891	1.00304	0.98560	0.99199	0.99607	1.00842	1.00458	1.00551
1.00463	0.99734	0.99869	1.00943	1.00251	0.99058	0.98946	0.98388	0.98893	1.00958	1.00845	1.01654
1.00815	1.01194	1.00285	1.00273	0.99637	1.00461	1.01465	0.97787	1.00985	1.00098	0.98832	0.98167
0.99373	0.99992	0.99387	0.98663	1.00524	0.99806	0.99349	1.00736	1.01303	1.00495	1.00376	0.99994
0.99907	1.00937	1.00956	1.00528	0.99279	0.98985	0.99170	0.99824	0.99827	0.99905	0.99866	1.00818
1.00278	0.99324	1.00190	1.00088	1.00984	1.00959	1.00874	1.00176	0.99437	0.98606	0.99162	0.99922
1.00194	1.00635	1.00616	1.01296	1.01553	0.99738	0.97402	0.97298	0.99141	1.00766	1.00553	1.00810
1.00851	1.01187	1.00798	0.99724	0.98451	0.99160	0.99476	0.99562	0.99660	1.00452	1.00259	1.00419
1.00493	1.00090	0.99728	0.99669	0.99409	0.99836	1.00352	0.99909	0.99661	0.99924	1.00456	1.00475
1.00184	1.00563	1.00326	1.00178	0.99955	1.00254	1.00740	1.00853	0.99970	0.98713	0.98636	0.99628
0.99482	1.00799	1.01351	1.02066	1.02841	1.04330	1.04624	1.01980	0.98230	0.95489	0.94616	0.94192
0.95523	0.97345	0.99315	1.00450	1.01385	1.01639	1.00623	1.00616	1.00259	1.00708	1.01061	1.01074
1.00704	1.00998	1.00581	1.00467	1.00556	0.99973	0.99385	0.99610	0.99952	0.99055	0.98964	0.99755
1.00091	1.00815	1.00974	1.01091	1.00222	1.00181	1.00015	0.99864	0.99650	0.98937	0.98711	0.99449
1.00019	1.00176	1.00079	0.99810	0.99865	0.99708	0.99706	0.99736	1.00005	1.00105	1.00238	1.00554
1.00590	1.00628	1.00372	0.99962	0.99581	0.99660	1.00024	0.99517	0.99540	0.99477	1.00237	1.00413

FINAL TREND-CYCLE COMPONENT

20.	20.	21.	22.	23.	24.	25.	25.	25.	25.	24.	24.
24.	24.	25.	26.	27.	27.	28.	29.	30.	30.	31.	32.
33.	34.	34.	35.	35.	35.	35.	36.	36.	37.	39.	40.
41.	41.	42.	42.	43.	44.	45.	45.	46.	47.	47.	48.
48.	49.	49.	49.	50.	50.	50.	50.	50.	50.	50.	50.
49.	49.	49.	49.	49.	49.	49.	49.	49.	49.	48.	48.
48.	47.	45.	43.	40.	38.	36.	35.	34.	34.	34.	34.
34.	35.	35.	37.	38.	38.	39.	39.	39.	39.	39.	39.
39.	39.	39.	39.	39.	39.	40.	40.	40.	40.	40.	41.

43.	44.	45.	46.	47.	47.	48.	48.	49.	49.	49.	49.
49.	49.	49.	49.	49.	49.	49.	49.	49.	50.	50.	50.
50.	51.	51.	52.	52.	53.	53.	54.	55.	55.	56.	57.
57.	57.	57.	57.	57.	57.	56.	56.	55.	55.	54.	54.
54.	53.	53.	54.	54.	54.	55.	55.	56.	57.	58.	59.
60.	61.	62.	62.	63.	63.	63.	63.	63.	63.	63.	63.
63.	63.	63.	63.	63.	63.	63.	64.	64.	64.	64.	64.
64.	64.	64.	63.	63.	62.	61.	61.	60.	59.	59.	58.
58.	58.	58.	58.	59.	60.	61.	61.	62.	63.	64.	65.
65.	66.	66.	67.	67.	67.	67.	67.	67.	67.	67.	67.
66.	66.	66.	65.	65.	65.	65.	65.	65.	65.	65.	66.
66.	67.	68.	69.	70.	70.	71.	71.	72.	72.	72.	73.
73.	73.	73.	73.	73.	73.	74.	74.	74.	75.	75.	76.
76.	77.	77.	77.	78.	78.	79.	79.	79.	80.	80.	81.
81.	82.	82.	83.	83.	84.	84.	85.	85.	86.	86.	87.
87.	88.	89.	89.	90.	91.	91.	92.	93.	94.	95.	96.
97.	98.	98.	99.	99.	99.	99.	100.	100.	99.	99.	99.
99.	99.	100.	100.	100.	101.	101.	102.	103.	103.	104.	105.
105.	106.	106.	107.	107.	107.	108.	108.	109.	109.	109.	110.
110.	111.	111.	111.	111.	111.	111.	111.	111.	110.	110.	109.
109.	108.	108.	107.	107.	107.	107.	107.	107.	107.	107.	108.
108.	108.	109.	109.	110.	111.	111.	112.	113.	114.	115.	116.
117.	118.	119.	120.	121.	122.	123.	124.	125.	126.	127.	128.
128.	129.	129.	130.	130.	130.	131.	131.	131.	131.	131.	131.
131.	130.	130.	129.	128.	126.	125.	123.	122.	120.	119.	118.
117.	117.	117.	117.	118.	119.	121.	122.	123.	124.	125.	126.
127.	128.	129.	129.	130.	131.	131.	132.	132.	133.	134.	135.
135.	136.	137.	137.	138.	139.	139.	140.	140.	141.	141.	142.
142.	143.	144.	144.	145.	146.	147.	148.	148.	149.	150.	150.
151.	151.	151.	151.	151.	151.	151.	150.	150.	149.	149.	148.

FORECAST OF SEASONAL FACTORS

1.00875	1.00855	1.00518	1.00039	0.99439	0.99636	1.00183	0.99407	0.99307	0.99163	1.00236	1.00342

FORECAST OF TREND-CYCLE COMPONENT

138.	138.	138.	138.	139.	139.	139.	139.	140.	140.	140.	140.

COEFFICIENT OF CORRELATION:0.96800

FINAL FORECAST 12 PERIODS AHEAD

139.	139.	139.	138.	138.	138.	139.	138.	139.	139.	140.	141.

FORECASTING ERROR: ROW 1: VARIANCE, ROW 2: STANDARD DEVIATION, ROW 3: AVERAGE ERROR

84.	88.	90.	93.	97.	101.	105.	106.	110.	114.	118.	123.
9.	9.	10.	10.	10.	10.	10.	10.	10.	11.	11.	11.
4.	4.	4.	4.	4.	4.	4.	4.	4.	4.	5.	5.

FORECAST OF TREND-CYCLE COMPONENT

157.	158.	158.	159.	159.	160.	160.	161.	162.	162.	163.	163.

COEFFICIENT OF CORRELATION:0.97156

FINAL FORECAST 12 PERIODS AHEAD

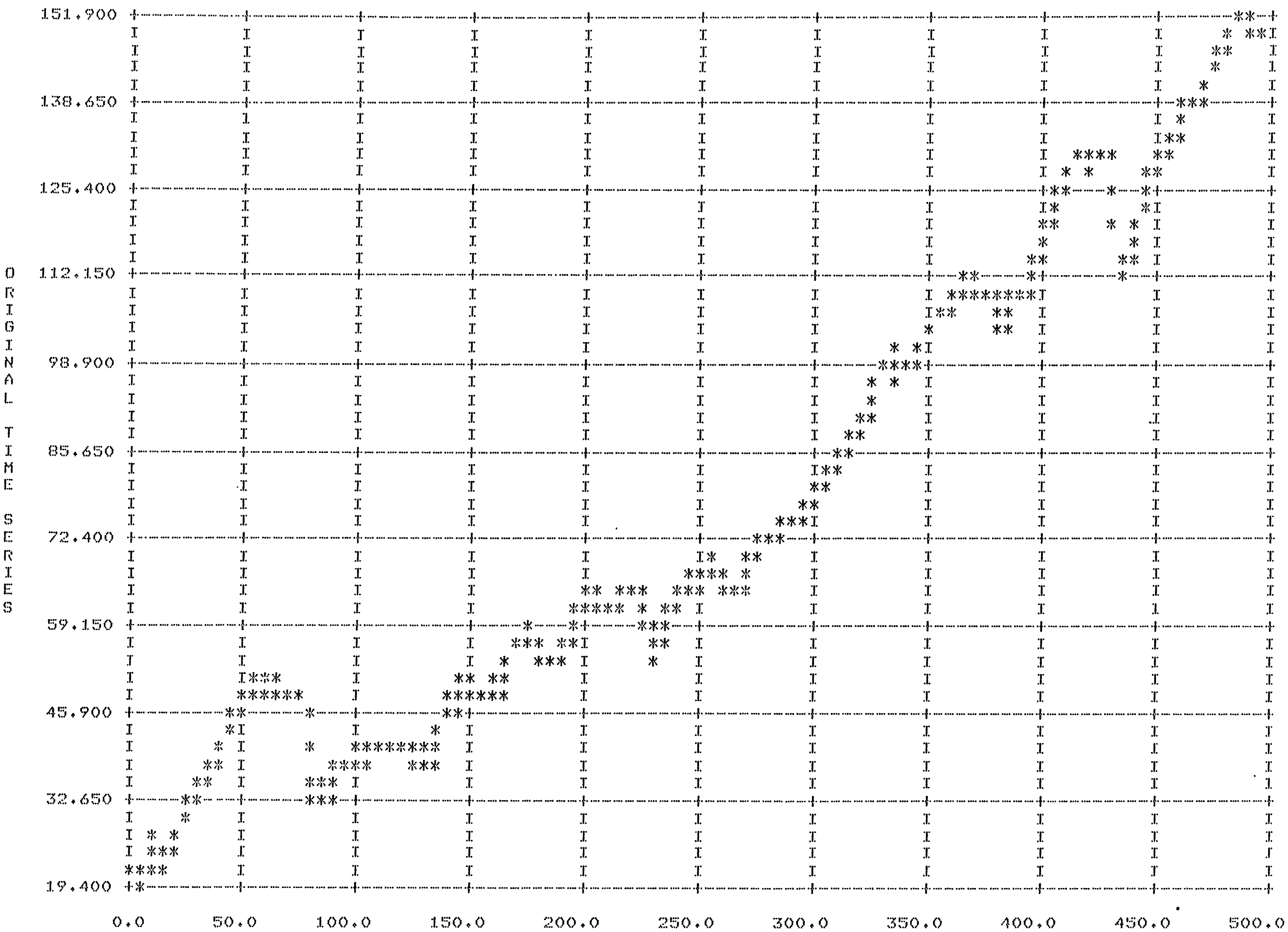
159.	159.	159.	159.	159.	159.	161.	160.	160.	161.	163.	164.

FORECASTING ERROR: ROW 1: VARIANCE, ROW 2: STANDARD DEVIATION, ROW 3: AVERAGE ERROR

44.	47.	50.	53.	58.	62.	67.	70.	75.	81.	85.	93.
7.	7.	7.	7.	8.	8.	8.	8.	9.	9.	9.	10.
-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.

&

PLOT


ORIGINAL TIME SERIES
151.900
138.650
125.400
112.150
98.900
85.650
72.400
59.150
45.900
32.650
19.400
0.0
50.0
100.0
150.0
200.0
250.0
300.0
350.0
400.0
450.0
500.0

WINTERS FPART=5 LCYCLE=12

*** FORECASTING BY WINTERS' METHOD:

Optimization option used:
Alpha = 0.801 Beta = 0.982 Gamma = 0.006

Comparison of ACTUAL DATA and FORECASTS over the last cycle of data points provided:
Note: Forecasting is done from the cycle before the last one.

Period #	Trend Factor	Seasonal Factor	Smoothed Data	Actual Data	Forecasted Data
1	0.362	0.978	155.111	151.500	152.073
2	0.362	0.984	155.111	151.900	153.364
3	0.362	0.987	155.111	151.800	154.099
4	0.362	0.988	155.111	151.400	154.703
5	0.362	0.989	155.111	150.900	155.141
6	0.362	0.989	155.111	150.900	155.545
7	0.362	0.985	155.111	151.100	155.328
8	0.362	0.979	155.111	149.800	154.609
9	0.362	0.971	155.111	149.200	153.712
10	0.362	0.965	155.111	148.500	153.110
11	0.362	0.967	155.111	149.100	153.781
12	0.362	0.974	155.111	148.900	155.225

Standard deviation of forecasting error = 4.29

Forecasts for next cycle:

Period #	Forecasts
1	150.13573
2	151.29927
3	151.99022
4	152.52345
5	152.93259
6	153.41063
7	153.34914
8	152.44614
9	151.67331
10	151.00334
11	151.64029
12	152.72676

&&&* Forecast.A Program ***

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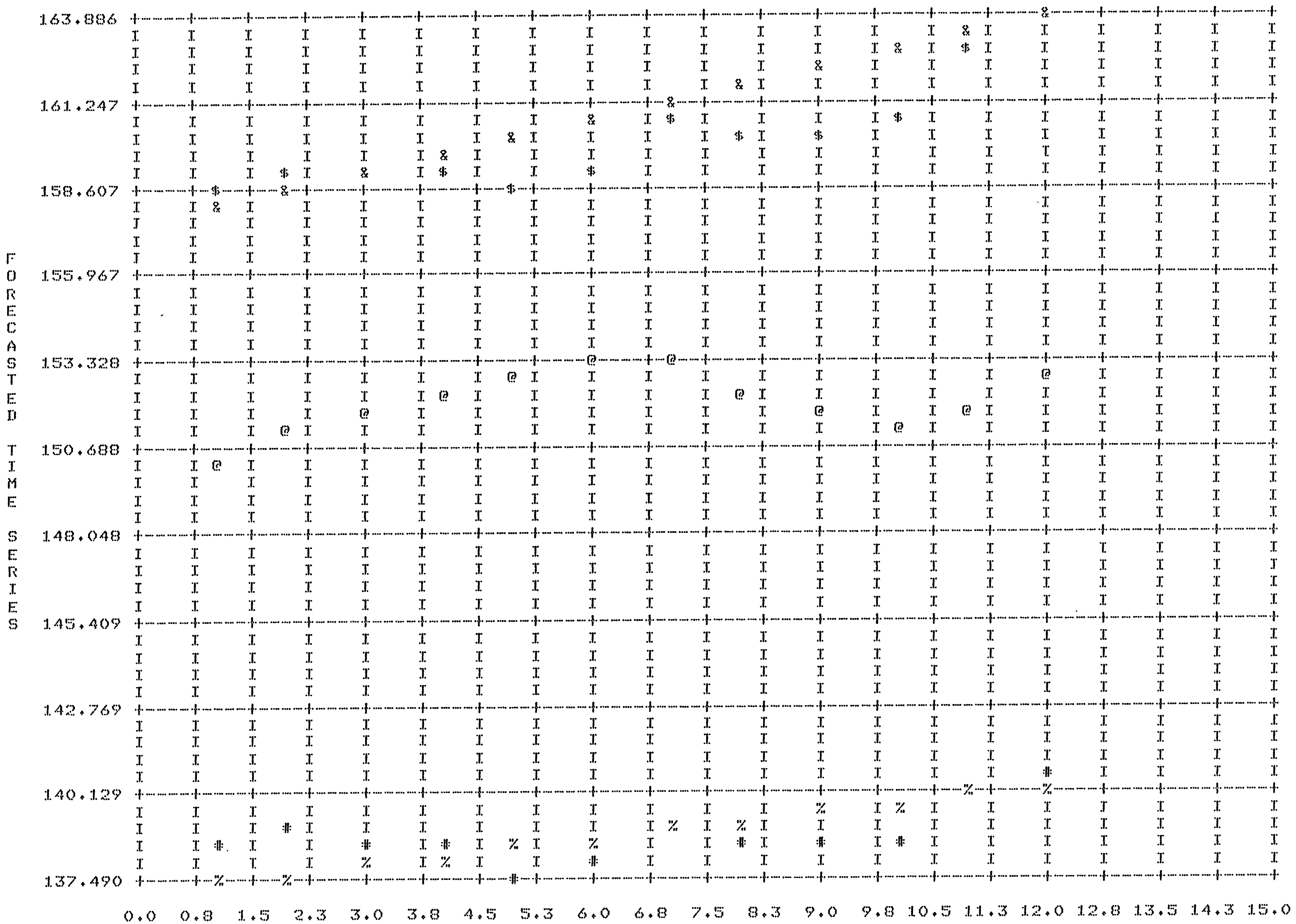
&



KEY: @ Winters' # Decomposition (Linear) $ Decomposition (Non-linear) % Trend (Linear) & Trend (Non-linear)

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1) Becker, J. R. "EXPLORE, A Computer Code for Solving Nonlinear, Continuous Optimization Problems." Computer Applications No. 10. Ann Arbor: The University of Michigan, Division of Research, Graduate School of Business Administration, 1974.

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4) Shiskin, J., Young, A. H.; and Musgrave, J. C. "The X-11 Variant of the Census Method II Seasonal Adjustment Program." Technical Paper No. 15. Washington D.C.: Bureau of the Census, n.d.

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