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# WEIGHTED AVERAGE: MAXIMIZATION OF VALUE VS. MINIMIZATION OF COST OF CAPITAL

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## I. INTRODUCTION

In research, teaching, and application we have found it convenient for some time now to compute the cost of capital as a weighted average of the costs of the various sources of capital. This entire notion, though, that the true cost of capital can be computed as a weighted average of the component costs has recently been questioned. Reilly and Wecker [9] remind us that the conditions under which Solomon [10] first formally developed the weighted-average approach to the cost of capital are rather restrictive. Linke and Kim [6] and Ezzell and Porter [2] argue that these restrictions as to cash-flow patterns are not necessary and that a weighting approach to computing the cost of capital is valid as long as the firm's leverage position is maintained.

In addition to these discussions on the general validity of the weighting procedure as a computational device, other arguments have focused on the validity of the particular formulations utilized. Specifically, three interrelated issues have been raised. First, Haley and Schall [3, pp. 298-304] imply and Arditti [1] states flatly that the traditional method of weighting in an after-tax cost of debt is incorrect. Secondly, these same authors feel that once the cost of debt is correctly specified on a before-tax basis, the resultant weighted-average cost of capital, as a function of the firm's capital structure, is not minimized at the point of value maximization. Finally, Ezzell and Porter [2], Arditti [1, p. 1004, footnote 4], and Haley and Schall [3, pp. 304-313] imply that the weighted-average figure utilizing the so-called correctly specified (before-tax) cost of debt may not be usable as a cutoff rate in making investment decisions. It is the contention of this paper that the arguments offered in support of these criticisms are misleading.

Most of the confusion generated by this literature results from the failure to recognize that even with a generally accepted definition of the true cost of capital, innumerable specifications of this definition, all equally valid, are possible. Two such specifications of the true cost of capital definition are developed in Section II. For each specification, two corresponding weighted-average costs of capital are formulated. One is less general than the other in the sense that it requires conditions necessary for Modigliani and Miller's [7] capital structure propositions.[1] An important conclusion of this section is that as weighted-average formulations of a single definition, there is no basis upon which to identify any one formulation as being superior to any other. Arditti's formulation with the cost of debt unadjusted for taxes is, as a specification of a definition, no more or no less useful than the traditional formulation in which the cost of debt is adjusted for taxes.[2] On the other hand, important distinctions between the

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[1] The most relevant conditions involve the existence of competitive capital markets and riskless debt and the absence of transaction costs.

[2] For an example of the traditional formulation, see Weston and Brigham [11, Chapter 11].

specifications are possible in terms of their usefulness in making financial decisions. Section III considers the issue of selecting an optimal capital structure. A proof is presented showing that for the traditional specification, the weighted-average cost of capital is minimized at the same capital structure that maximized the value of the firm. This is not the case for the alternative specification.

The third major issue is settled in Section IV by restating and proving an axiom that has apparently been forgotten--namely, that any one of a whole set of weighted-average cost of capital formulations is usable for making investment decisions as long as the cash flows are consistently specified.

In general, it is our feeling that little has been gained by criticizing the traditional weighted-average formulation. Our conclusions are summarized in Section V and an example is given in the appendix in hope of further clearing up what has unnecessarily become a clouded issue.

## II. THE BASIC FORMULATIONS

Modigliani and Miller [7] and Solomon [10] suggest a definition for the true cost of capital that has gained general acceptance. They define it as the discount rate that equates the capitalized value of a firm's expected future cash flows to the firm's value. To clarify this definition, it is necessary to specify in more detail what is meant by the firm's cash flows and what is meant by the firm's value. Since we are working only with a definition, we may specify these variables in any arbitrary manner desired.[3] Generally, firm value is taken to be the sum of the market value of the firm's debt and equity. The literature contains no such generally accepted specification of the firm's cash flows. However, there are two cash-flow concepts that dominate the literature. One is the after-tax net operating income and the other identifies cash flows in terms of the ultimate flow of funds to the investors in the firm, debt, and equity holders. Each of these cash-flow conceptualizations implies its own specification of the definition of the firm's cost of capital. The after-tax net operating income cash-flow concept yields the following:[4]

$$k_o = \frac{\bar{X}(1 - t)}{V} \tag{1}$$

where $\bar{X}$ = expected net operating income
$t$ = firm's marginal tax rate
$V$ = market value of the firm
$k_o$ = firm's overall cost of capital.

The specification resulting from an alternate cash-flow concept is

$$k_o^* = \frac{(\bar{X} - \bar{F})(1 - t) + \bar{F}}{V} \tag{2}$$

where $F$ = $k_i B$, expected interest payments to the bondholders
$k_i$ = investor's expected return on debt
$B$ = market value of the firm's debt
$k_o^*$ = firm's overall cost of capital
$(\bar{X} - \bar{F})(1 - t)$ = expected earnings available to equity holders.

[3] Again, this is not to say that all such arbitrary specifications are equally useful in making financial decisions, but merely to point out that one may define terms in any way he wishes.

[4] Cash flows are assumed to be perpetuities for algebraic convenience only. Linke and Kim [6] and Ezzell and Porter [2] show that this assumption does not play a crucial role in any other sense.

The first term in the numerator identifies expected cash flows accruing to the stockholders and the second term identifies expected cash flows accruing to the bondholders. In order to contrast this specification with the previous one, equation (2) is rewritten as:

$$k_o^* = \frac{\overline{X}(1 - t)}{V} + \frac{tk_iB}{V} = k_o + \frac{tk_iB}{V} .$$

Therefore, in general, because different cash-flow concepts are used in each specification, $k_o \neq k_o^*$. However, since we are still dealing with alternative specifications of a definition, there is no basis upon which to argue, as Arditti does, that one is superior to the other.

Weighted-Average Formulations.

To develop the weighted-average formulation for $k_o$, add and subtract $k_iB(1 - t)/V$ to the right hand side of equation (1) and rearrange:

$$k_o = \frac{(\overline{X} - k_iB)(1 - t)}{V} + \frac{k_i(1 - t)B}{V} .$$

If we multiply the first term by S/S, we have

$$k_o = \frac{(\overline{X} - k_iB)(1 - t)}{S} \frac{S}{V} + \frac{k_i(1 - t)B}{V}.$$

where S = market value of the firm's equity.

Since $k_e$, the equity investor's expected rate of return (i.e., the cost of equity) is defined as $(\overline{X} - k_iB)(1 - t)/S$ and letting $\theta = B/V$, the ratio of debt to total value, $k_o$ can be rewritten as

$$k_o = k_e(1 - \theta) + k_i(1 - t)\theta. \qquad (1a)$$

Thus, equation (1a), which we refer to as the traditional weighted-average cost of capital formulation because it weights the debt on an after-tax basis, represents a valid computational device as long as the cost of capital definition is specified as in equation (1).

Arditti, on the other hand, prefers the "alternate" specification of the firm's overall cost of capital, $k_o^*$. To develop the weighted-average formulation for $k_o^*$, multiply the first term on the right-hand side of equation (2) by S/S and write $\bar{F}$ as $k_i B$:

$$k_o^* = \frac{(\bar{X} - \bar{F})(1 - t)}{V} + \frac{\bar{F}}{V} \tag{2}$$

$$k_o^* = \frac{(\bar{X} - k_i B)(1 - t)}{S}\frac{S}{V} + \frac{k_i B}{V}.$$

By substituting $k_e$ amd $\theta$ we have

$$k_o^* = k_e(1 - \theta) + k_i\theta. \tag{2a}$$

This alternate weighted-average formulation is perfectly valid as long as the cost of capital definition is specified as in equation (2). The confusion engendered by Arditti is the insistence that the alternate formulation in (2a) is in some sense correct, while the traditional formulation in (1a) is invalid. We submit that a definition cannot be declared valid or invalid.

Less General Formulations.

Somewhat less general formulations of $k_o$ and $k_o^*$ as weighted-averages can be developed by assuming that the securities markets are competitive and that firms and individuals can issue and/or purchase riskless debt. As Modigliani and Miller [8] have shown most clearly in a reply to Heins and Sprenkle [4], these conditions are conducive to an operation yielding arbitrage profits unless

$$V = V_u + tB \tag{3}$$

where $V_u = \dfrac{\bar{X}(1-t)}{k_u}$,

and $k_u$ = cost of capital for an unlevered firm,

$$tB = \frac{tk_i B}{k_i} = \frac{t\bar{F}}{k_i},$$

or the capitalized value of the tax savings on the interest payments. In this context $k_i$ is the investor's expected rate of return on riskless debt.[5] Substitution of the equilibrium relationship represented by equation (3) into the two specifications of the definition of the cost of capital yields two further weighted-average cost of capital formulations. For the specification corresponding to the after-tax net

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[5]Haley and Schall [3, p. 298], with no explanation, indicate that equation (3) is also valid in a world of risky debt. Modigliani and Miller [8, pp. 594-595] explain that equation (3) is valid in a world of riskless debt or, failing that, it is valid if "....individuals as well as corporations are able effectively to limit their liability." Lewellen [5, Chapter 4] also extends, without any adjustment, the applicability of equation (3) to a world with risky debt. This issue, though, is not the central concern of our paper.

operating income concept, substitute equation (3) into equation (1):[6]

$$k_o = \frac{\bar{X}(1 - t)}{V_u + tB}$$

$$k_o = k_u(1 - \theta) + k_u(1 - t)\theta. \qquad (1b)$$

Lewellen [5, pp. 43-45] develops this same formulation somewhat differently and refers to $k_u$ as the cost of equity and $k_u(1 - t)$ as the cost of debt. The cost of equity for an all-equity firm is $k_u$ and $k_u(1 - t)$ is the cost of debt for an all-debt firm but they are not the proper component costs for a firm consisting of both debt and equity. Under the conditions that make the use of equation (3) valid, equations

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[6]Actually, to develop equation (1b) start with equation (1):

$$k_o = \frac{\bar{X}(1 - t)}{V}$$

$$k_o = \frac{\bar{X}(1 - t)}{V \cdot \frac{V_u}{V_u}}$$

Use equation (3) to rewrite one of the $V_u$'s:

$$k_o = \frac{\bar{X}(1 - t)}{V \cdot \frac{V_u}{V - tB}}$$

$$= \left(\frac{\bar{X}(1 - t)}{V_u}\right)\left(\frac{V - tB}{V}\right)$$

$$= k_u(1 - t\theta).$$

Adding and subtracting $k_u\theta$ and rearranging yields

$$k_o = k_u(1 - \theta) + k_u(1 - t)\theta. \qquad (1b)$$

(1a) and (1b) are not only consistent formulations of the firm's cost of capital, they also are computationally identical. In this situation, one is just an algebraic rewriting of the other. Traditionalists may be confused by referring to $k_u$ as the cost of equity and $k_u(1 - t)$ as the cost of debt, yet computing $k_o$ from these component costs does yield an identical value to computing $k_o$ with $k_e$ as the cost of equity and $k_i(1 - t)$ as the cost of debt as long as the equilibrium relationship of equation (3) is valid. This conclusion follows since only equations (1) and (3) are involved in the formulating of (1a) and (1b).[7]

Just as equation (1b) is a weighted-average formulation for one cash-flow concept when equation (3) is valid, so can another formulation for the alternate cash-flow concept be developed.

Haley and Schall [3, pp. 298-300] specify the cost of capital in terms of the ultimate flow of funds to investors just as in equation (2). Rewriting (2) as:

$$k_o^* = \frac{\bar{X}(1 - t)}{V} + \frac{tk_iB}{V}$$

and then substituting the equilibrium condition of (3) into (2) for V in the first term yields:[8]

[7] In another part of his work [5, pp. 47-48], Lewellen proposes a computational method for arriving at a $k_u$ value. The procedure is based on the assumption that cash flows can be represented by a perpetuity. Although it is not germane to the immediate concern of this paper, it must be noted this Lewellen errs further on in his development [5, pp. 98-104] when he applies this conputational technique to the problem of computing $k_u$ for a growth firm.

[8] To see how $\bar{X}(1 - t)/V = k_u(1 - t\theta)$, see footnote 6.

$$k_o^* = k_u(1 - t\theta) + tk_i\theta.$$

This can be rewritten as

$$k_o^* = k_u - t(k_u - k_i)\theta. \qquad (2b)$$

As with (1a) and (1b), this formulation is not only consistent with (2a), but yields a $k_o^*$ value identical to that computed by way of (2a) as long as $V - V_u + tB$.

To this point, we have utilized two specifications of a single cost of capital definition to develop four weighted-average cost of capital formulations. The specifications are distinguished by alternative specifications of the cash flows. Equations (1a) (traditional) and (1b) (Lewellen) are derived from cash flows specified as after-tax net operating income to the firm, $\bar{X}(1 - t)$. Equation (1a) is more general than (1b), but it yields $k_o$ values identical to it whenever conditions are such that $V = V_u + tB$.

Equations (2a) (Arditti) and (2b) (Haley and Schall) are derived from cash flows specified as the sum of the proceeds available to stockholders and bondholders, $(\bar{X} - \bar{F})(1 - t) + \bar{F}$. Equation (2a) is more general than (2b), but it yields $k_o^*$ values identical to it whenever conditions are such that $V = V_u + tB$.

All four formulations are equally valid as alternative specifications of a definition. However, they are not equally attractive for purposes of investigating financial and investment policies. Before turning to these issues in the following sections, the development of still another specification of the definition of the true cost of capital is undertaken. The value of this final specification should become obvious in the next section. To distinguish this specification from the others, a third cash-flow concept is used. Cash flows are specified on a before-tax basis:

$$k = \frac{\bar{X}}{V}. \tag{4}$$

where k = before-tax cost of capital

The weighted-average formulation derived from this definition is[9]

$$k = (k_e/(1 - t))(1 - \theta) + k_i\theta. \tag{4a}$$

The significance of this before-tax formulation for this paper is that one of the after-tax formulations, (1a), can be written in terms of it:

$$k_o = (1 - t)k. \tag{5}$$

Table 1 presents a list of the weighted-average formulations developed in this section.

## III. COST MINIMIZATION VS. VALUE MAXIMIZATION

In the preceding section we saw that, contrary to the statements of some authors, there is no single correct way to formulate a firm's overall weighted-average cost of capital. In fact, five equally valid formulations of the definition of cost of capital were developed.[10]

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[9] First, add and subtract $\bar{F}/V$ to the right hand side. Then multiply the first two terms on the right hand side by $(1 - t)S/(1 - t)S$, and write the third term as $k_iB/V$.

[10] Two of the formulations, (1b) and (2b), are less general than the others in that they are valid only if $V = V_u + tB$.

TABLE 1

SUMMARY OF THE WEIGHTED-AVERAGE COST OF CAPITAL FORMULATIONS

| | SPECIFICATION OF THE DEFINITION | MARKET CONDITIONS | FORMULATIONS |
|---|---|---|---|
| After-Tax | $k_o = \frac{\overline{X}(1 - t)}{V}$ (Equation 1)[a] | General | $k_o = k_e(1 - \theta) + k_i(1 - t)\theta$ (Traditional; 1a) |
| | | Perfect Markets[d] | $k_o = k_u(1 - \theta) + k_u(1 - t)\theta$ (Lewellen; 1b) |
| | $k_o^* = \frac{(\overline{X} - \overline{F})(1 - t) + \overline{F}}{V}$ (Equation 2)[b] | General | $k_o^* = k_e(1 - \theta) + k_i\theta$ (Arditti; 2a) |
| | | Perfect Markets[d] | $k_o^* = k_u - t(k_u - k_i)\theta$ (Haley and Schall; 2b) |
| Before-Tax | $k = \frac{\overline{X}}{V}$ (Equation 4)[c] | General | $k = (k_e/(1 - t)(1 - \theta) + k_i\theta$ (Before-tax; 4a) |

(a) Cash flows are specified as expected after-tax net operating income to the firm.
(b) Cash flows are specified as total expected flow of funds to investors in the firm.
(c) Cash flows are specified as expected before-tax net operation income to the firm.
(d) Under perfect markets, $V = V_u + tB$.

However, not all five are equally useful for purposes of selecting the firm's optimal capital structure in the sense that the capital structure that minimizes the firm's weighted-average after-tax cost of capital is not always the value maximizing capital structure. However, both Arditti and Haley and Schall have contributed to the confusion surrounding this issue by failing to restrict their comments to the relevant specifications. In reference to $k_o^*$ as computed in (2a), Arditti [1, p. 1007] says that "...the after-tax weighted-average cost of capital should be ignored, since its minimization will lead to a non-optimal capital structure." Arditti proves this and the statement is quite correct in a narrow sense. The error, or at least the confusion, comes from Arditti's insistence that $k_o^*$ is the only valid after-tax weighted-average cost of capital. However, we now know that as a definition $k_o$ as represented in (1a) is just as valid as $k_o^*$. Even though the minimization of $k_o^*$ does not maximize the value of the firm, $k_o$ need not be ignored for purposes of making capital structure decisions. Rewriting equation (4) as $V = \bar{X}/k$ it follows that V is maximized where k is minimized. Since equation (5) shows that $k_o$ is minimized where k is minimized, it follows that minimizing $k_o$ maximizes the value of the firm.

Given the alternative cash-flow specifications that lead to $k_o$ and $k_o^*$ as given in (1a) and (2a) respectively, it should not surprise us that $k_o$ is useful for making capital structure decisions while $k_o^*$ is not. Rewriting equations (1) and (2) as

$$V = \frac{\bar{X}(1 - t)}{k_o} \qquad (1')$$

$$\text{and } V = \frac{X(1-t) + tk_i B}{k_o^*} \qquad (2')$$

highlights the fact that in (1'), if value is affected by varying the capital structure, it works only through $k_o$ so that when $k_o$ is minimized, V must be maximized. On the other hand, equation (2') indicates that value is affected by varying the capital structure not only through $k_o^*$, but also through the tax savings on the interest payments which is included in the numerator. So minimizing $k_o^*$ does not necessarily maximize V since the increase in $k_o^*$ from moving beyond the cost minimizing capital structure may be more than offset by an increase in tax savings from employing additional leverage.

Another way to make the same point is by looking at (1a) and (2a) themselves. In equation (2a), $k_o^* = k_e(1 - \theta) + k_i\theta$ where leverage affects $k_o^*$ only by way of tradeoffs between $k_e$ and $k_i$. The tax advantage of debt is included elsewhere in the valuation formulation. However, for $k_o = k_e(1 - \theta) + k_i(1 - t)\theta$, the tax advantage is included in the cost of capital formulation.

Haley and Schall add to the confusion in much the same manner as Arditti. Without distinguishing between $k_o$ as given in (1b) and $k_o^*$ as given in (2b), they state that[11]

[11] See Haley and Schall [3, p. 304]. The italics are theirs. Haley and Schall are well aware of the fact that alternative specifications of the cost of capital may be perfectly valid; see [3, footnote 6, p. 298]. So their error is only one of omission. By failing to explain that this quote applies only to $k_o^*$ and not to $k_o$, they merely add to the confusion surrounding the issue. Arditti's error is more serious in that he claims that $k_o^*$ is the only valid formulation.

> "It means that, with firm taxes, neither in theory nor in practice can the optimal capital structure of the firm, be generally specified as the point of minimum cost of capital. Instead, direct attention must be paid to the impact of debt financing on the value of the firm. The use of the cost-of-capital model for this purpose is not in general appropriate."

This statement is, of course, correct only as it relates to $k_o^*$ as given in (2b) (or (2a) for that matter, although Haley and Schall are referring to the (2b) formulation). To see that Lewellen's $k_o$ (equation (1b)) is minimized at the value maximizing capital structure, we first identify the value maximizing $\theta$ by rewriting equation (3)

$$V = V_u + tB \tag{3}$$

$$V/V = V_u/V + tB/V$$

$$V_u/V = 1 - t\theta$$

$$V = V_u/(1 - t\theta) \tag{3a}$$

As expected, because (3) is Modigliani and Miller's equilibrium condition, V is maximized for $\theta = 1$; i.e., for an all-debt firm. Lewellen's $k_o$, (1b), is also minimized at $\theta = 1$. Lewellen's $k_o$ is, in general, appropriate for making capital structure decisions while Haley and Schall'a $k_o^*$ is not. Formulations of $k_o$ and $k_o^*$ are equally valid specifications of the definition of the cost of capital but only $k_o$ is directly applicable for making capital structure decisions.

## IV. COST OF CAPITAL AS A CUTOFF RATE

Haley and Schall [3, pp. 306-313] add to the confusion surrounding the weighted-average cost of capital by shifting from $k_o^*$ to $k_o$ for purposes of making investment decisions. The unwritten implications is that $k_o^*$ is not valid as a cutoff rate in the evaluation of capital projects. As will be shown, such an implication is unwarranted.[12] Ezzell and Porter's error is more direct in that they actually claim that $k_o^*$ is not correct for capital budgeting purposes.[13] Arditti is confusing on this matter only in the sense that he chooses to ignore it completely.

Our contention, as might be expected by now, is that $k_o$, $k_o^*$, or k could all serve as cutoff rates as long as the cash-flows generated by the projects are consistently specified. The cutoff rate on an incremental investment of dI dollars is defined as that rate of return that must be expected to be earned on the dI dollars so that the market value of the firm remains unchanged; i.e., dV/dI = 1. If we specify V as in (1'), then

$$\frac{dV}{dI} = \frac{d\bar{X}(1 - t)/dI}{k_o} - \frac{(dk_o/dI)\bar{X}}{k_o^2} .$$

If we assume that the firm's overall cost of capital is unaffected by the

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[12] Again, Haley and Schall's error is one of ommission since they never directly indicated that $k_o^*$ could not be used for capital budgeting purposes. However, one can only wonder why they switch from $k_o^*$ to $k_o$. (Actually, it is a switch from specifying cash-flows as $\bar{X}(1 - t) + tk_iB$ to specifying them as $\bar{X}(1 - t)$.)

[13] See Porter and Ezzell [2, p. 10].

investment in this project, (i.e., if $dk_o/dI = 0$),[14] then the criteria for accepting the project may be stated as

$$\frac{dV}{dI} = \frac{d\bar{X}(1 - t)/dI}{k_o} \geq 1, \text{ or}$$

$$\frac{d\bar{X}(1 - t)}{dI} \geq k_o.$$

In this case, where cash-flows are computed as $\bar{X}(1 - t)$, the rate of return expected on the project must be greater than $k_o$; i.e., $k_o$ is a valid cutoff rate.

On the other hand, if we specify V as in (2'), then the criteria becomes

$$\frac{dV}{dI} = \frac{d\bar{X}(1 - t)/dI}{k_o^*} + \frac{dtk_iB/dI}{k_o^*} \geq 1,$$

or

$$\frac{d\bar{X}(1 - t)}{dI} + \frac{tk_i dB}{dI} \geq k_o^* .$$

Therefore, contrary to the implications of Haley and Schall, Ezzell and Porter, and Arditti, $k_o^*$ is a valid cutoff rate as long as the tax savings on the interest on the new bonds is included in the calculation of the cash flows from the project. Once again, $k_o$ includes the tax savings on the debt, so there is no need to add it to the cash flows. The $k_o^*$ as given in (2a) and (2b) does not include these savings, yet $k_o^*$ is still a valid cutoff rate as long as the cash-flows from the project do include the tax savings due to the issuance of new bonds to help finance the

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[14] As Arditti points out [1, p. 1004, footnote 4], this assumption is hardly ever valid. This issue, though, is beyond the scope of this paper.

project. It also follows that $k = (k_e/(1 - t))(1 - \theta) + k_i\theta$ is a valid cutoff rate as long as cash-flows from a project are calculated as $d\bar{X}$. In short, just as there exists a whole set of valid specifications of the definition of the true cost of capital depending on how cash-flows are specified, there also exists a whole set of valid cutoff rates for capital budgeting purposes depending on how the project's cash-flows are computed.

## V. CONCLUSIONS

The major conclusion of this paper is that the so-called traditional weighted-average cost-of-capital concept is valid for determining the optimal capital structure that maximizes the value of the firm. In addition, it is also valid for use as a cutoff rate in the evaluation of capital projects.

We have shown that $k_o$, $k_o^*$ and $k$ are all valid cutoff rates for evaluating capital projects as long as the cash-flows are consistently or accurately specified. However, minimization of $k_o^*$, the weighted-average return to stock and bondholders or the alternate specification of cost of capital, will not maximize the value of the firm. The reason $k_o^*$ fails this test is because it does not include the tax advantage of debt in its specification. The tax advantage of debt is included in the numerator of the valuation equation, i.e., in the basic specification of the firm's cash-flows.

## APPENDIX

This appendix illustrates the concepts discussed in this paper with a hypothetical company. The Cartell Company has the following financial characteristics:

| | | |
|---|---|---|
| Bonds (B) = | \$ 2 mil. | 20% = $\theta$ = B/V |
| Stocks (S) = | 8 mil. | 80% = $(1 - \theta)$ = S/V |
| Value (V) = | \$10 mil. | 100% |

| | |
|---|---|
| Net Operating Earnings | $\bar{X}$ = \$2.00 mil. |
| Interest to Bondholders $k_iB$ | $\bar{F}$ = .16 mil. |
| (where $k_i$ = 8% before tax cost of debt) | |
| Earnings before taxes $(\bar{X} - \bar{F})$ | = \$1.84 mil. |
| Marginal tax rate t = 50% | .92 mil. |
| Earnings available to stockholders $(\bar{X} - \bar{F})(1 - t)$ | = \$ .92 mil. |

The traditional formulation, equation (1), is computed as follows:

$$k_o = \frac{\bar{X}(1 - t)}{V} = \frac{\$2 \text{ mil.}(1 - .5)}{10 \text{ mil.}} = .10 \qquad (1)$$

$$k_o = 10\%.$$

The cost of equity capital is calculated as follows:

$$k_e = \frac{(\bar{X} - \bar{F})(1 - t)}{S} = \frac{(2 \text{ mil.} - .16 \text{ mil.})(1 - .5)}{8 \text{ mil.}} = .115$$

$$k_e = 11.5\%.$$

Using equation (1a) we find $k_o$ as follows:

$$k_o = k_e(1 - \theta) + k_i(1 - t)\theta \qquad (1a)$$

$$k_o = .115(.80) + .08(1 - .5)(.20) = .10$$

$$k_o = 10\%.$$

Equation (2), the specification of an alternative cash-flow concept by Haley and Schall and Arditti, is calculated as follows:

$$k_o^* = \frac{(\overline{X} - \overline{F})(1 - t) + \overline{F}}{V} \tag{2}$$

$$k_o^* = \frac{(2 \text{ mil.} - .16 \text{ mil.})(1 - .5) + .16 \text{ mil.}}{10 \text{ mil.}} = .108$$

$$k_o^* = 10.8\%.$$

Thus, we see that $k_o^* \neq k_o$. However, they are equivalent as follows:

$$k_o^* = k_o + \frac{t\overline{F}}{V} = .10 + \frac{(.5)(.16 \text{ mil.})}{10 \text{ mil.}} = .108 - 10.8\%.$$

Equation (2a), the $k_o^*$ formulation by Arditti, is calculated as follows:

$$k_o^* = k_e(1 - \theta) + k_i\theta \tag{2a}$$

$$k_o^* = .115(.80) + .08(.20) = .108 = 10.8\%.$$

Equation (3) allows us to calculate the firm value for an unlevered position as follows:

$$V = V_u + tB \tag{3}$$

$$\$10 \text{ mil.} = V_u + (.5)(2 \text{ mil.})$$

$$V_u = \$9 \text{ mil.}$$

Since $V_u = \frac{\overline{X}(1 - t)}{k_u}$ we can solve for

$$k_u = \frac{\overline{X}(1 - t)}{V_u} = \frac{\$2 \text{ mil.}(1 - .5)}{9 \text{ mil.}} = .1111$$

$$k_u = 11.1\%.$$

Using (1b) we can calculate $k_o$ as follows:

$$k_o = k_u(1 - \theta) + k_u(1 - t)\theta \tag{1b}$$

$$k_o = .111(.80) + .111(1 - .5)(.20) = .10 = 10\%.$$

Lewellen refers to $k_u$ as the cost of equity and $k_u(1 - t)$ as the cost of debt, but this is obviously in error since $k_i = 8\%$ and we have calculated $k_e = 11.5\%$. If the firm was all equity, then $k_u = k_e$; if the firm was all debt, $k_u(1 - t) = k_i$.

Equation (2b), Haley and Schall's formulation of $k_o^*$, is calculated to be:

$$k_o^* = k_u - t(k_u - k_i)\theta \tag{2b}$$

$$k_o^* = .111 - .5(.111 - .08).20 = .108 = 10.8\%.$$

Equation (4) utilizes a third cash flow, a third cost of capital specifications, and a before-tax cost of capital. It is calculated as:

$$k = \frac{\overline{X}}{V} = \frac{\$\ 2\text{ mil.}}{10\text{ mil.}} = .20 = 20\%. \tag{4}$$

If weighted-average formulation is:

$$k = \frac{k_e}{(1 - t)}(1 - \theta) + k_i\theta \tag{4a}$$

$$k = \frac{.115}{(1 - \theta)}(.80) + (.08)(.20) = .20 = 20\%.$$

The after-tax formulation is related to the before-tax formulation according to equation (5) as follows:

$$k_o = (1 - t)k = (1 - .5).20 = .10 = 10\%. \tag{5}$$

In summary, we can rewrite an overview of all the preceding (after-tax) equations as follows:

$$V = \frac{\overline{X}(1 - t)}{k_o} = \frac{\overline{X}(1 - t)}{k_u} + \frac{t\overline{F}}{k_i} = \frac{\overline{X}(1 - t) + t\overline{F}}{k_o^*}$$

$$V = \frac{\$1\text{ mil.}}{.10} = \frac{\$1\text{ mil.}}{.111} + \frac{\$(.5)(.16)}{.08} = \frac{\$1\text{ mil.} + (.5)\$(.16)}{.108}$$

$$V = \$10\text{ mil.} = \$9\text{ mil.} + \$1\text{ mil.} = \$10\text{ mil.}$$

In addition, we can rewrite all the weighted-average cost of capital (after-tax) equations as follows:

$$k_o = k_e(1 - \theta) + k_i(1 - t)\theta = (11.5)(.8) + (.8)(1 - .5)(.2) = 10\%$$

$$k_o = k_u(1 - \theta) + k_u(1 - t)\theta = (11.11)(.8) + (11.11)(1 - .5)(.2) = 10\%$$

$$k_o^* = k_u - t(k_u - k_i)\theta = (11.11) - .5(11.11 - .08).2 = 10.8\%$$

$$k_o^* = k_e(1 - \theta) + k_i\theta = 11.5(.8) + .08(.2) = 10.8\%.$$

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