Division of Research
Graduate School of Business Administration

July 1987

# EFFECTS OF DELAYED PAYMENTS ON ECONOMIC ORDER QUANTITY

Working Paper #516

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# Effects of Delayed Payments on Economic Order Quantity

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We address the problem of determining Economic Order Quantity when delayed payments are possible. We derive an exact cost expression for the problem and show that the earlier studies on this problem provide analytical approximations. Inconsistency between the results obtained by the earlier authors are explained.

## Introduction

Recently Goyal[2] analyzed and devised a general procedure for determining economic order quantity in situations when i) a grace period is provided by the vendor for settling the account and ii) interest cost for financing the inventories ($I_c$) is different from the return on investment opportunities ($I_d$) available to the buyer. Chand and Ward[1] analyzed the same problem under the assumption that $I_c = I_d$. The results obtained by Chand and Ward[2] do not agree with Goyal's[1] results. In this note we establish that the inconsistencies in their results are due to the fact that i) the average cost analysis is only an approximation to the fundamentally rigorous Net Present Value Analysis in most situations and ii) Goyal's[2] attempts to intermingle revenue effects with inventory decisions. It is assumed that the reader is familiar with Chand and Ward[1], Goyal[2] papers. We use the same notation.

## NPV Analysis

Both authors do not explicitly recognize the fact that the basic reasoning in decision-making is to maximize the net present value of the owner (or shareholder). Consider the case when return on investment opportunities is same as the cost financing inventories--i.e., $I_d = I_c = r$. Revenue inflows are irrelevant because the revenue inflow rates and timing are not influenced by the inventory decisions. Whatever be the order quantity, net present value of revenue stream remains the same. In the current context, this translates to minimizing the net present value of all relevant cash outflows. Let NPV(T) represent the net present value of all future cash outflows for a reorder interval of T.

$$NPV(T) = S + \int_0^T hD(T - \ell)e^{-r\ell}d\ell + DTpe^{-rt} + e^{-rT}\, NPV(T) \tag{1}$$

First term on the right hand side (RHS) is the present value of order cost for the first order. Second term is the present value of the holding costs (exclusive of interest charges) for the first order. Third term on RHS is the present value of the first payment made to the vendor. Note that this term reflects both purchase price and discounting for time value of the money (thus the interest charges). Last term on RHS indicates the net present value of all future expenses beyond T. Rearranging the terms in (1),

$$NPV(T) = \frac{S}{1 - e^{-rT}} + \frac{1}{1 - e^{-rT}} \int_0^T hD(T - \ell)e^{-r\ell}d\ell + \frac{DTpe^{-rt}}{1 - e^{-rT}} \tag{2}$$

(2) indicates the net present value of all future cash outflows if we decide to order the items at periodic intervals of T. However, in order to compare with conventional average cost analysis, we use annualized cost--equivalent uniform cash stream that generates the same NPV. Let ANN(T) represent annualized cost for a reorder interval of T. When we are considering infinite horizon (from the basics of discounting),

$$ANN(T) = r*NPV(T)$$

Hence (2) can be rewritten as

$$ANN(T) = \frac{Sr}{1 - e^{-rT}} + \frac{r}{1 - e^{-rT}} \int_0^T hD(T - \ell)e^{-r\ell}d\ell + \frac{DTpre^{-rt}}{1 - e^{-rT}} \tag{3}$$

With a little algebraic manipulation, (3) can be rewritten as

$$ANN(T) = \frac{Sr}{1 - e^{-rT}} + hD \frac{rT - 1 + e^{-rT}}{r(1 - e^{-rT})} + \frac{rpDTe^{-rt}}{1 - e^{-rT}} \tag{4}$$

For small values of rT above expression can be approximated as

$$ANN(T) \approx \frac{S}{T} + \frac{hDT}{2} + pe^{-rt}D + \frac{pe^{-rt}DTr}{2}$$

$$\approx \frac{S}{T} + \frac{1}{2}(h + rpe^{-rt})DT + pe^{-rt}D \tag{5}$$

The reader may note that, if we set $t = 0$, expression (5) reduces to the average cost per period. This is the cost expression used in determining economic ordering quantity. Thus average cost analysis used in classical analysis is an approximation to annualized cost, ANN(T).

## Chand and Ward[1] Analysis

Consider the situation when vendor provides a grace period for the payment. Solving the approximate annualized cost function (5) for optimal T*, we obtain

$$T^* \approx \sqrt{\frac{2S}{D\{h + pe^{-rt}r\}}} \tag{6}$$

Or optimal order quantity is

$$Q^* \approx \sqrt{\frac{2SD}{h + pe^{-rt}r}} \tag{7}$$

Above expression can be approximated as

$$Q^* \approx \sqrt{\frac{2SD}{h + \frac{pr}{1 + rt}}} \tag{8}$$

Chand and Ward[1] used (8) for determining the optimal order quantity when delayed payments are permitted. Hence the formulation provided by Chand and Ward[1] is an approximate solution for (4). However, the real value of r (nominal interest rate less the inflation rate) tends to be small and hence their approximation is adequate for all practical purposes.

## Goyal's[2] Analysis

Goyal[2] provided an analysis for determining optimal order quantity when delayed payments are possible. His analysis does not appear to be precise since it attempts to intermingle revenue effects with inventory decisions. Under the assumptions of classical EOQ, revenue stream remains the same irrespective of inventory decisions. Consider the case where $I_c = I_d = r$ and $T \geq t$ (Figure 1).

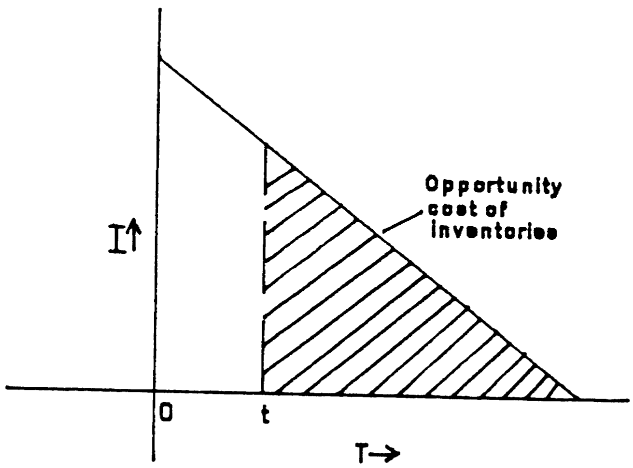


Figure 1

Goyal[2] values the benefits of delayed payment as equal to $Dpt^2I_d/2$. This is in error. Opportunity benefit of delayed payments is DpTtr since the amount

DpT which otherwise would have been paid to vendor at time 0 is currently available to use for a period of t in each cycle. Thus average cost per cycle is given by

$$\text{Average cost per cycle} = S + \frac{1}{2}\,Dp(T - t)^2 r + \frac{1}{2}\,hDT^2 - (DpTtr) \qquad (9)$$

Note that the effect of delayed payments are captured under average cost analysis in two ways: opportunity benefits of delayed payments (as per the expression in parenthesis in (9)) and reduced opportunity costs of financing inventories. Goyal's analysis fails to recognize the fact that the inventory carrying costs measured in average cost analysis are <u>opportunity</u> costs and not necessarily <u>out-of-pocket</u> costs (Hadley and Whitin[3], page 13). (9) can be rewritten as

$$\text{Average cost per year} = \frac{S}{T} + \frac{1}{2}\,\frac{Dp(T - t)^2 r}{T} + \frac{1}{2}\,\frac{hDT^2}{T} - (DpTr/T) \qquad (10)$$

Note that the difference between our expression (10) and Goyal's[2] expression (1) with $I_c = I_d = r$ is in the determination of <u>opportunity</u> benefits of delayed payments. It is evident from our expression (10) that opportunity benefits per year of delayed payments is invariant with respect to order quantity. This is a limitation of average cost analysis. It fails to exactly account for time value of money. Solving (10),

$$Q^* = \sqrt{\frac{D(2S + rpDt^2)}{h + pr}} \qquad (11)$$

Under average cost anaysis, (11) provides optimal order quantity if $T \geq t$. This is, in fact, same as the expression derived by Goyal when $I_d = 0$. When $t > T$, under average cost analysis, the average cost per cycle is given by

$$\text{Average cost per cycle} = S + \frac{1}{2} hDT^2 - (pDTtr) \tag{12}$$

$$\text{Average cost per year} = \frac{S}{T} + \frac{1}{2} hDT - (pDtr) \tag{13}$$

Optimal order quantity is given by

$$Q^* = \sqrt{\frac{2SD}{h}} \tag{14}$$

Note that the solution derived using expressions (11) and (14) is not the same as (8). This is primarily due to the fact that Chand and Ward[2] analysis is approximate (partly average cost and partly discounting) and our modification of Goyal's procedure based on average cost criterion is yet another approximation to the solution of expression (4). These differences result in different order quantities.

## CONCLUDING REMARKS

In this note we reconciled the inconsistent results obtained by Goyal[2] and Chand and Ward[1]. We showed that both derivations are approximations to the exact Net Present Value Analysis. The differences in results are due to different degrees of approximations made by authors and Goyal's[2] attempts to intermingle revenue effects with inventory decisions. Further, it is suggested that in teaching classical EOQ, it is better to point out the fact that we wish to minimize the NPV of cost streams and the average cost analysis is only a good approximation for it.

## REFERENCES

1. S. Chand and J. Ward (1987). A note on "Economic Order Quantity Under Conditions of Permissible Delay in Payments." J. Opl. Res. Soc. 38, 83-84.

2. S. K. Goyal (1985). Economic order quantity under conditions of permissible delay in payments. J. Opl. Res. Soc. 36, 335-338.

3. G. Hadley and T. M. Whitin (1963). Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, New Jersey, USA.