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RESTORING THE PRINCIPLE OF MINIMUM DIFFERENTIATION

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ABSTRACT

RESTORING THE PRINCIPLE OF MINIMUM DIFFERENTIATION IN PRODUCT POSITIONING

Most research on product positioning supports the idea of differentiation. Yet, in many markets, standardization appears to be common practice. Since Harold Hotelling's 1929 paper presenting "The Principle of Minimum Differentiation," we have a seen a steady stream of research trying to explain the conditions of differentiation vs. standardization. Much of this research has found Hotelling's principle flawed and differentiation is typically considered superior. In the case where support has been found for standardization, the principle of minimum differentiation has been restored but only with very limiting assumptions.

In this paper, we attempt to more fully restore the case for standardization by using more realistic assumptions than what has previously been done. Our basic finding is that standardization is optimal when consumers exhibit sufficient choice variation. Under four different game theoretic scenarios, we show that our results are, nevertheless, consistent with past research because with little or no choice variation (an assumption of previous research), we too find differentiation appropriate.

INTRODUCTION

Perhaps the most fundamental decision in product positioning is that of differentiation vs. standardization. The latter refers to a position similar to competition; the former is the offering of uniqueness. Clearly, the decision to position a product close to competition (or away from it) predicates many other decisions, and has significant consequences for production, marketing and the long-term market performance of the product. Even though the issue is extensively treated in textbooks (e.g., Porter, 1980; Kotler, 1988) and is an important element in marketing models of competition (e.g., Hauser & Shugan, 1983; Carpenter, 1986; Schmittlein & Sarigollu, 1987; Hauser, 1988; Moorthy, 1988), it remains a hotly debated topic. When should a differentiation strategy be pursued? Under what conditions would standardization be better?

Obviously, the issue is not merely an empirical question. Advancement in theory is also called for. This is not to suggest that the problem of differentiation/standardization is devoid of theoretical approaches. In fact, the opposite is true. Ever since the publication of Harold Hotelling's paper "Stability in Competition" in 1929, there has been a steady stream of research in economics on the nature of competition in terms of standardization/differentiation. Recently, marketing scholars have also made contributions (Hauser & Shugan, 1983; Carpenter, 1986; Schmittlein & Sarigollu, 1987; Moorthy, 1988). Most of this work has followed a game theoretic approach.

We aim to continue this tradition. We also hope to add to it by incorporating somewhat more realistic assumptions than has previously been the case and by examining several different situations. Instead of simply assuming stationary consumer choice, we take choice variations within and across consumers into account as well. Moreover, we do not confine the analysis to a symmetric case (on either price or position), nor to a one-stage game. We examine cases where position is given and price is not; where price is given and position is not; where price and position are simultaneously determined and where position is determined in the first stage and price in the second stage. We find standardization optimal in all cases when consumers exhibit sufficient choice variation. When consumer choices are consistent and stationary, on the other hand, our findings are consistent with past research (using deterministic consumer choice models) that differentiation is appropriate (d'Aspremont, Gabszewicz, & Thisse, 1979; Hauser & Shugan, 1983; Moorthy, 1988; Economides, 1989).

Our findings may well fly in the face of typical textbook advice that consumer heterogeneity should be met by differentiation or segmentation. However, we will provide formal theoretical reasoning as well as an intuitive explanation for our findings. We will also examine the extent to which our findings are a result of the assumptions made.

BACKGROUND

The essence of Hotelling's result is the "Principle of Minimum Differentiation" which suggests a long-run equilibrium in which competitors position at the center of the market with an infinitesimal distance between them. What makes Hotelling's model interesting even today is that it provides a good starting point for further analysis because: (1) the physical locations of stores that Hotelling was concerned about can be generalized to any attribute of interest (Hotelling himself used sour and sweet cider as an example), (2) the positions of consumers can be thought of as ideal points in a Coombsian sense (Coombs, 1950), and (3) the assumptions in the model are quite explicit and as such they have been popular targets in many subsequent studies. Over the years, the original assumptions have been reduced, altered, and relaxed.

Judging from recent work in economics, the general conclusion is that the principle of minimum differentiation is flawed and fails to hold under most circumstances (e.g., d'Aspremont, Gabszewicz & Thisse, 1979). Positioning in distinct segments is often considered optimal because of reduced risk of price cutting (for a review of past research, see Moorthy, 1985).

Carpenter (1986) poses an interesting question: "If maximum differentiation is optimal in a wide variety of cases (Economides, 1981; 1989; Hauser, 1988; Moorthy, 1988), why is minimum differentiation so frequently observed?" Eaton and Lipsey (1975) find that standardization is optimal in a duopoly with no price competition, but there is no equilibrium with three firms (even with no price competition). Carpenter (1986) also finds that minimum differentiation can be optimal as long as price competition is limited and preferences are unimodal. His model is not restricted to a single dimension and incorporates advertising/promotion as well as positioning and price. However, it assumes that demand is symmetrically distributed (unimodal). The problem with this is that such a distribution probably affects optimal positioning, prices, and promotions, and thus confounds the issue in question: differentiation vs. standardization. Shilony (1981) and Neven (1986) prove that increasing densities of consumers towards the center raise firm's incentive to move close to the center.

With respect to the "Defender Model" (Hauser & Shugan, 1983; Hauser & Gaskin, 1984; Hauser, 1986a; 1986b; Shugan, 1986; Hauser & Wernerfelt, 1988), originally formulated with the (implicit) assumption of perfect foresight for an entering brand with respect to incumbent brands' response strategies, Hauser (1988) has recently analyzed a variety of equilibrium implications. Only under the assumption of symmetrical and constant prices (and no "leapfrogging" of positions) does he find support for standardization. Under all other conditions investigated, the Defender model argues in favor of differentiation.

In general, deterministic consumer choice has been assumed in previous work (Hotelling, 1929; Eaton & Lipsey, 1975; d'Aspremont, Gabszewicz & Thisse, 1979; Economides, 1986). Consumers are assumed to be heterogeneous only in terms of different tastes. The heterogeneity within a customer (e.g., over time), however, has not been considered, and choice variation due to this type of heterogeneity has been excluded from the model.

As Eliashberg and Chatterjee (1985) point out, marketing is in a position to incorporate more insightful consumer behavior perspectives based on analysis at the individual level. Rather than oversimplified behavioral assumptions for the sake of analytical simplicity, more elaborate models are necessary for providing a more meaningful basis for examining positioning and segmentation strategy (Eliashberg & Chatterjee, 1985). Clearly, it is more realistic to consider heterogeneity both within and across tastes. Different consumers may consider different attributes and have different weights on those attributes, and may exhibit different choices even though they have identical tastes with respect to considered attributes in a model. Further, within an individual consumer, there may be choice variations over time and situations, (Belk, 1975; McAlister & Pessemier, 1982; McAlister, 1982; Pessemier & Hendelsman, 1984; Kahn, Kalwani, & Morrison, 1986). Because of choice variations within and across individuals, a deterministic model of choice has obvious limitations.

Let us therefore turn our attention to stochastic brand choice (Bass, Jeuland & Wright, 1976; Farquhar & Rao, 1976; Givon, 1984), as it has been used in the analysis of differentiation (Carpenter, 1986; de Palma, Ginsburgh, Papageorgiou & Thisse, 1985; de Palma, Ginsburgh & Thisse, 1987). Carpenter (1986) applies a random utility choice model, but his model is not explicitly analyzed and has no effect on the equilibrium results. de Palma et al. (1985) use a probabilistic choice model (logit) for consumer choice variation, and claim to have restored the principle of minimum differentiation. They argue for an agglomerated equilibrium at the center of the market, given sufficient choice variation, in the general n firm case. They also find that, in a duopoly, the firms will always position at the center of the market, regardless of choice variation.

Although this is an important result (recently also found by Schmittlein and Sarigollu, 1987), de Palma et al.'s (1985) model is limited in the sense that they consider a single-stage game of <u>either</u> position <u>or</u> price. Even when both position and price are considered, decisions on both strategic variables are modeled to be simultaneous and independent in a single-stage game. The assumption of simultaneous and independent decisions on position and price is so restrictive that the managerial implication from their finding has limited scope. Further, even though a deterministic choice model is a special case of the probabilistic choice model with zero variation, their model does not show any link between the results from deterministic vs. probabilistic choice assumptions. For example, in a duopoly, their model suggests standardization even with choice variation close to zero, while other studies using a deterministic choice suggest differentiation (d'Aspremont, Gabszewicz & Thisse, 1979; Economides, 1986).

In a more recent study examining three firm competition, de Palma et al. (1987) find that as consumer choice variation increases, optimal positioning moves from differentiation toward standardization. However, only symmetric positioning is considered; price is exogenously determined, and the analysis is confined to a single stage game. From a marketing viewpoint, it is not very attractive to treat price as an exogenous variable. Further, assuming symmetric positioning for the sake of analytical simplicity also restricts the scope of the research and the validity of the equilibrium.

In this paper, we will demonstrate the effect of consumer choice variations within and across individuals. Similar to de Palma et al. (1985; 1987), we will use a random utility model, but we do not confine the analysis to a symmetric case with respect to position or price, nor to one-stage game. In order to compare our results with those in other studies, four cases of duopoly competition will be considered: (a) the positions are given and each firm determines price, (b) prices are given and firms compete on positions, (c) firms determine prices and positions simultaneously in a one-stage game, and (d) firms determine positions in the first stage and prices in the second stage.

Our findings are as follows. With positions given, optimum prices are affected by positional advantages. When firms position identically or symmetrically, they have an identical optimum price. When product position is the strategic variable (with prices given), optimum positioning is always at the center of the market, regardless of price.

When firms determine prices and positions simultaneously in a single stage game, they position at the center of the market with identical price. We also find that, in a two-stage game framework, Hotelling's principle of minimum differentiation holds, provided there is sufficient consumer choice variation. However, insufficient choice variation leads to differentiation. As consumer choice variation increases, the optimal position moves towards the center of the market. It is important to realize that the restoration of the principle of minimum differentiation is obtained by working with assumptions such as probabilistic choice, non-symmetry with respect to position and price, and a two-stage approach in which a competitor can react to another's strategy. We suggest that these assumptions, albeit not without limitations, move us toward a more realistic setting.

We begin by describing the consumer model. Then we turn our attention to modelling competition. Next, we analyze the four cases of competition mentioned above and present the equilibrium for each. Finally, we consider managerial implications of the results and suggest future research directions.

MODELLING THE CONSUMER

In view of the fact that most analytical models support differentiation and that the few that find standardization optimal have very restrictive assumptions, the question of why so many markets exhibit similar products remains unanswered. In this paper, we will return to the simplifying assumption of duopoly and a single dimension. On the other hand, we will not assume away price competition and we will not impose symmetry a priori (neither for price nor position).

We assume that each consumer purchases a single unit of the product per unit of time and has a finite ideal point on one dimensional attribute, [0, 1]. Different tastes across consumers on the attribute are represented in terms of different ideal points. The attribute (or perceptual dimension) represents common attributes to all brands in question. It is also assumed that there is a continuum of consumers differing in tastes which are uniformly distributed on the attribute.

$$f(Y_j) = 1, \qquad 0 \le Y_j \le 1$$

where $f(Y_j)$ is the taste distribution and Y_j the j-th consumer's ideal point on the attribute. While such a distribution is rarely found empirically, it may represent the most general case and has the advantage of eliminating the effect of non-uniformity of tastes distribution as a possible explanation of product positioning (Moorthy, 1988). A non-uniform taste distribution such as unimodal or bimodal distribution may in itself lead to results of standardizing or differentiating (Shilony, 1981; Neven, 1986), and confounds the effect of competition, which is what we wish to analyze.

Instead of always patronizing the brand with maximum utility derived from the considered attribute in the model, consumers are assumed to follow a probabilistic choice model (e.g , a random utility model) to incorporate consumer choice variations within and across consumers. Specifically, the j-th consumer surplus from consuming the i-th product, S_{ij}, is the difference between the utility from the i-th product, U_{ij}, and what s/he is asked to pay for, P_i. The utility is assumed to come from two sources: One is from the considered attribute, U^*_{ij}, and the other from unspecified remaining factors such as consumer-specific attributes, different weights on attributes, variety-seeking, situational factors, and so forth, e_{ij}. When we assume that U^*_{ij} is a linearly decreasing function of the distance between the consumer's ideal and the product position, the consumer surplus is as follows:

$$\begin{aligned} S_{ij} &= U_{ij} - P_i, \qquad i = 1,2,\ldots,m, \qquad j = 1,2,\ldots n, \\ &= U^*_{ij} + \sigma e_{ij} - P_i \\ &= U^m_j - c\left|X_i - Y_j\right| + \sigma e_{ij} - P_i \end{aligned}$$

where U_j^m is the maximum utility level which the j-th consumer can obtain from consuming the ideal product, and X_i the position of the i-th product. The utility from remaining factors is assumed to have a probability distribution with mean zero and unit variance, and the effect of this utility on choice is represented by σ, $\sigma > 0$. As σ increases, a consumer becomes more dependent upon other factors. c is the diminishing rate of utility, $c > 0$, which is analogous to a "transportation" cost.

The probability $Pr_i(Y_j)$ that consumer at Y_j will purchase a product from firm i is

$$Pr_i(Y_j) = Pr\left(S_{ij} \geq S_{i\sim j};\ \text{all } i\sim \neq i\right)$$

$$= Pr\left(c\left\{ \left|X_i - Y_j\right| - \left|X_{i\sim} - Y_j\right| \right\} + P_i - P_{i\sim} - \sigma e_{ij} + \sigma e_{i\sim j} \leq 0;\ \text{all } i\sim \neq i\right)$$

The purchase probability has a monotonically increasing relationship to the utility derived from the considered attribute. This monotonic relationship is assumed to follow a logit model based on Luce's (1959) choice axiom. This type of logit model has been shown to be a utility function of a representative consumer with variety-seeking behavior (Anderson, de Palma, & Thisse, 1988). The interpretation is that a logit model may represent both the behavior of a population of consumers with diverse tastes at a given time (consumer heterogeneity within a taste) and that of a given consumer who tries different brands over a sufficiently long period of time (consumer variety seeking). The evidence for such probabilistic choice is more compelling than that for a deterministic type of model (Schmalensee & Thisse, 1988). Here, we do not differ from the approaches of Carpenter (1986) or de Palma et al. (1985; 1987).

MODELLING COMPETITION

We assume two, non-cooperative, risk-neutral firms (producing one brand each) that maximize their profits. These firms are differentiated with respect to position and price. Without loss of generality, firm 1 is assumed to be positioned to the left of firm 2.

If firm 1 is positioned at X_1 and firm 2 at X_2, where $X_1 \leq X_2$, the intervals $[0, X_1]$, $[X_1, X_2]$, and $[X_2, 1]$, are named regions 1, 2, and 3, respectively. It is also assumed that both firms have the same production costs. This assumption allows us to rule out a trivial explanation of product differentiation, namely, technological differences between the firms (Moorthy, 1988). Non-negative prices are assumed throughout (otherwise we would have negative profit and eliminate the competition).

Given probabilistic choice, and the assumption that the e_{ij} terms have an identical and independent Weibull distribution (as in a logit model), the probability $Pr_i^k(Y_j)$ that a consumer at Y_j in region k will purchase a product from firm i is as follows:

$$Pr_1^1(Y_j) = \frac{1}{1 + EXP\left[\frac{P_1 - P_2}{\sigma} + \frac{c}{\sigma}(X_1 - X_2)\right]},$$

$$Pr_1^2(Y_j) = \frac{1}{1 + EXP\left[\frac{P_1 - P_2}{\sigma} + \frac{c}{\sigma}(2Y_j - X_1 - X_2)\right]},$$

$$Pr_1^3(Y_j) = \frac{1}{1 + EXP\left[\frac{P_1 - P_2}{\sigma} + \frac{c}{\sigma}(X_2 - X_1)\right]},$$

and

$$Pr_2^1(Y_j) = \frac{1}{1 + EXP\left[\frac{P_2 - P_1}{\sigma} + \frac{c}{\sigma}(X_2 - X_1)\right]} = 1 - Pr_1^1(Y_j)$$

$$Pr_2^2(Y_j) = \frac{1}{1 + EXP\left[\frac{P_2 - P_1}{\sigma} + \frac{c}{\sigma}(X_1 + X_2 - 2Y_j)\right]} = 1 - Pr_1^2(Y_j)$$

$$Pr_2^3(Y_j) = \frac{1}{1 + EXP\left[\frac{P_2 - P_1}{\sigma} + \frac{c}{\sigma}(X_1 - X_2)\right]} = 1 - Pr_1^3(Y_j).$$

Figure 1 depicts the probability in three regions. Note that the probability of purchasing from firm 1 or 2 is invariant of Y_j and is constant over region 1 and 3. Since $\frac{\partial Pr_1^{\,2}(Y_j)}{\partial Y_j} < 0$ and $\frac{\partial Pr_2^{\,2}(Y_j)}{\partial Y_j} > 0$, Figure 1 shows that the probability is monotonically decreasing for firm 1 and increasing for firm 2 in region 2. Further,

$$\text{Sign}\,\frac{\partial^2}{\partial Y_j^2} Pr_1^{\,2}(Y_j) = \text{Sign}\left[EXP\left[\frac{P_1 - P_2}{\sigma} + \frac{c}{\sigma}\left(2Y_j - X_1 - X_2\right)\right] - 1\right].$$

The inflexion point of $Pr_1^{\,2}(Y_j)$ (the same as that of $Pr_2^{\,2}(Y_j)$) is at

$$Y^* = \frac{X_1 + X_2}{2} - \frac{P_1 - P_2}{2c},$$

$Pr_1^{\,2}(Y_j)$ $\left(Pr_2^{\,2}(Y_j)\right)$ is strictly convex (concave) over $X_1 < Y < Y^*$, and strictly concave (convex) over $Y^* < Y < X_2$. A consumer at the inflexion point has the same probability (1/2) of purchasing from firm 1 as from firm 2.

Insert Figure 1 here

As a firm raises price, the probability of purchasing from the firm decreases and the probability of purchasing from the other firm increases (see Appendix A). Further, increasing the price of firm 1 moves the inflexion point towards the left while increasing the price of firm 2 moves it towards the right since

$$\frac{\partial Y^*}{\partial P_1} < 0 \text{ and } \frac{\partial Y^*}{\partial P_2} > 0.$$

Figure 2 shows the case when firm 1 increases price with positions fixed and a fixed price of firm 2. As firm 1 raises price, the probability shifts downward and the inflexion point moves toward the position of the firm.

Insert Figure 2 here

Since $\frac{\partial Pr_1(Y_j)}{\partial \sigma} \gtrless 0$ for $Y \gtrless Y^*$, and the inflexion point is invariant of the change in choice variation, the probability curve becomes flat at the .5 probability with the increase of choice variation. When there is no choice variation, $\sigma = 0$, consumers always buy the product with maximum utility on the common attribute. Hence, all consumers to the left of the inflexion point patronize firm 1 and all consumers to the right of the inflexion point patronize firm 2 (as in Hotelling's original model). As σ increases, the effect of choice variation increases. In the perfectly random choice case, $\sigma = \infty$, the firms cannot differentiate their products. The probability is 1/2 everywhere and the firms split the market. Figure 3 describes this effect of consumer choice variation on the purchase probability. When $\sigma = 0$, all consumers to the left of Y^* choose firm 1 and all consumers to the right of Y^* select firm 2. As consumer choice variation increases, the probability curve becomes flat at 0.5 probability and both firms obtain half of the market, regardless of prices and positions.

Insert Figure 3 here

We write the expected demand as

$$Q_i = \int_0^1 Pr\left(S_{ij} \geq S_{i\sim j};\ i \neq i\sim\right) f(Y_j)\, dY$$

$$= \int_0^1 Pr\left(S_{ij} \geq S_{i\sim j};\ i \neq i\sim\right) dY$$

because the ideal points are assumed to be uniformly distributed on the line segment [0, 1], $f(Y_j) = 1, 0 \le Y_j \le 1$. Thus, the expected demand is

$$Q_1 = \int_0^{X_1} Pr_1^{\,1}(Y_j)\,dY + \int_{X_1}^{X_2} Pr_1^{\,2}(Y_j)\,dY + \int_{X_2}^{1} Pr_1^{\,3}(Y_j)\,dY$$

$$= \frac{X_1}{1 + EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]} + \frac{1 - X_2}{1 + EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]}$$

$$+ X_d - \frac{\sigma}{2\,c}\ln\frac{1 + EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]}{1 + EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]}$$

$$Q_2 = \int_0^{X_1} Pr_2^{\,1}(Y_j)\,dY + \int_{X_1}^{X_2} Pr_2^{\,2}(Y_j)\,dY + \int_{X_2}^{1} Pr_2^{\,3}(Y_j)\,dY$$

$$= \frac{EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]}{1 + EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]}X_1 + \frac{EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]}{1 + EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]}(1 - X_2)$$

$$+ X_d - \frac{\sigma}{2\,c}\ln\frac{EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]\left[1 + EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]\right]}{EXP\left[\frac{P_d}{\sigma} - \frac{c\,X_d}{\sigma}\right]\left[1 + EXP\left[\frac{P_d}{\sigma} + \frac{c\,X_d}{\sigma}\right]\right]}$$

$$= 1 - Q_1$$

where $P_d = P_1 - P_2$ and $X_d = X_2 - X_1$. Since the total demand of the market is normalized to 1, the expected demand is the same as market share.

Hence, the expected profits are

$$\pi_1 = P_1 Q_1 \text{ and } \pi_2 = P_2 Q_2.$$

It is worth noticing that expected profit is a continuous function of price and position over the strategy set $[X_1, X_2, P_1, P_2]$ as long as $\sigma > 0$.

A firm is assumed to have perfect information with respect to the other firm's position and price in a non-cooperative game. The best response of a firm is established with consideration of all possible strategies of the other firm. Both firms aspire to maximize their profits, independently and simultaneously, but one firm's decision is affected by the other firm's profit maximization and vice versa. For each quadruple of positions and prices, we can construct a non-cooperative game $G(X_1, X_2, P_1, P_2)$ in which the firms simultaneously choose positions and prices.

When c is zero, position is not a strategic variable because the market shares of the firms are completely determined by price differentials. The firms maximize profits by charging optimal prices. As σ goes to infinity, the expected demand of the firms is constant (each firm gets half of the market), regardless of positions and prices. Hence, position and price are no longer strategic variables. Only price determines profit. The higher the price, the higher the profit and the equilibrium is at a positive infinite price (or a regulated maximum price, if any).

Case 1: Positions Given, Price to be Determined

First, we consider the case when the positions of both firms are exogenously given and each firm decides on price. The objective functions of both firms are as follows:

$$\begin{aligned} \underset{P_1}{\text{Max}} \ \pi_1 &= P_1 Q_1 \\ &= g_1(P_1 \mid P_2^*, X_1, X_2) \end{aligned}$$

subject to $0 \leq P_1$,

$$\max_{P_2} \pi_2 = P_2 Q_2$$

$$= g_2\left(P_2 \mid P_1^*, X_1, X_2\right)$$

subject to $0 \le P_2$,

where X_1 and X_2 are given positions and P_1* and P_2* are the optimal prices determined by the competing firms.

PROPOSITION 1: Given the positions of the firms, X_1 and X_2, the equilibrium prices have the following relationship:

$$P_1^* Q_2^* = P_2^* Q_1^* \qquad \text{or} \frac{Q_1^*}{P_1^*} = \frac{Q_2^*}{P_2^*} \quad , \quad \frac{P_1^*}{P_2^*} = \frac{Q_1^*}{Q_2^*} \, ,$$

where Q_1* and Q_2* are the expected demands of the firms at the profit maximizing prices, P_1* and P_2*.

PROOF: See Appendix B.

Hence, both firms have an identical ratio of expected demand to price. Since, P_1^* $Q_2^* = P_2^* Q_1^*$,

$$P_1^* \left(1 - Q_1^*\right) = P_2^* Q_1^*, \qquad \text{because } Q_2 = 1 - Q_1,$$

$$P_1^* = \left(P_1^* + P_2^*\right) Q_1^*,$$

expected demand (market share) of the firms at equilibrium is

$$Q_1^* = \frac{P_1^*}{P_1^* + P_2^*} \qquad \text{and} \qquad Q_2^* = \frac{P_2^*}{P_1^* + P_2^*}$$

and the equilibrium profits are

$$\pi_1^* = \frac{P_1^{*2}}{P_1^* + P_2^*} \qquad \text{and} \qquad \pi_2^* = \frac{P_2^{*2}}{P_1^* + P_2^*}.$$

Figure 4 shows a hypothetical example. Two firms are positioned at X_1 and X_2 (which are given) and compete on price. Since firm 2 has a positional advantage, it can increase its price. Firm 1, however, with a disadvantage in position should lower price in order to increase market share. When Q_1/P_1 is equal to Q_2/P_2, both firms can maximize their profits. Too much increase in P_2 (or decrease in P_1) causes severe competition and reduces profits. At equilibrium, firm 1 retains a low price and a relatively small market share and profit, while firm 2 enjoys a large market share and profit (at a higher price). The inflexion point is closer to firm 2's position. If given positions are symmetric or identical, the firms have identical prices, $P_1^* = P_2^* = 2\sigma$, market shares, $Q_1^* = Q_2^* = 1/2$, and profits, $\pi_1^* = \pi_2^* = \sigma$ (see Appendix C).

Insert Figure 4 here

Proposition 1 confirms the finding of the price model in de Palma et al. (1985): identical prices and positions (which are given). We add to this that symmetric positions also produce the same results. When the firms have different positions, the positional advantage can affect prices, demands, and profits, and identical price is not the optimal strategy.

Case 2: Prices Given, Position to be Determined

When price is exogenously given, the firms will use positioning for profit maximization. The objective functions become:

$$\max_{X_1} \ \pi_1 = P_1 Q_1$$
$$= g_3\left(X_1 \mid X_2^*, P_1, P_2\right)$$

subject to $0 \leq X_1 \leq X_2^*$,

$$\max_{X_2} \ \pi_2 = P_2 Q_2$$

$$= g_4\left(X_2 \mid X_1^*, P_1, P_2\right)$$

subject to $X_1^* \leq X_2 \leq 1$,

where P_1 and P_2 are the given positive prices and X_1^* and X_2^* are the optimal positions determined by competing firms.

PROPOSITION 2: At any given positive prices, P_1 and P_2, the equilibrium positions of both firms are always at the center of the market, $X_1^* = X_2^* = 1/2$.

PROOF: see Appendix D.

It is worth noticing that regardless of price (the given prices need not be the same), the equilibrium positions are at the center of the market. Further the equilibrium positions represent a unique Nash equilibrium. The expected demands (market share) of the firms at equilibrium are

$$Q_1^* = \frac{1}{\left[1 + \mathrm{EXP}\left[\frac{P_d}{\sigma}\right]\right]} \text{ and}$$

$$Q_2^* = \frac{\mathrm{EXP}\left[\frac{P_d}{\sigma}\right]}{\left[1 + \mathrm{EXP}\left[\frac{P_d}{\sigma}\right]\right]} = 1 - Q_1^*,$$

and the profits are

$$\pi_1{}^* = P_1Q_1{}^* = \frac{P_1}{\left(1 + \text{EXP}\left[\frac{P_d}{\sigma}\right]\right)} \text{ and}$$

$$\pi_2{}^* = P_2Q_2{}^* = \frac{P_2 \,\text{EXP}\left[\frac{P_d}{\sigma}\right]}{\left[1 + \text{EXP}\left[\frac{P_d}{\sigma}\right]\right]}.$$

When firm 1 charges a higher price than firm 2, firm 1 loses customers , and its market share is less than half of the market. Its profit decreases as well. Firm 2, on the other hand, gains market share and profit because of its price advantage. When P_1 and P_2 are identical at P* (e.g., if price is regulated), both firms have half the market and identical profits,

$$Q_1{}^* = Q_2{}^* = 1/2, \qquad \text{and } \pi_1{}^* = \pi_2{}^* = P^*/2.$$

Proposition 2 confirms the findings of de Palma et al. (1985), Carpenter (1986), and Schmittlein and Sarigollu (1987). de Palma et al. argue that there exists an equilibrium at the center of the market with the assumption of given identical prices. However, an identical price is not the condition that produces the equilibrium at the center of the market. Any given positive price will always lead to this equilibrium. In fact, the only condition required is that price competition is limited and that positioning is the only strategic variable. Identical prices produce identical demand and profit. When prices are not identical, demands and profit will not be identical either.

Case 3: Price and Position Determined Simultaneously

We now extend the analysis to the case of two strategic variables: position and price. These variables can be taken into account simultaneously or consecutively. In some product categories, both price and position can be easily and independently adjusted (although this is more common for price), and considered simultaneously. First, we analyze the case when position and price are taken into account simultaneously.

When position and price are determined simultaneously (in a single stage game), the optimal responses are determined by the conditions of proposition 1 and 2. The objective functions of the firms are

$$\underset{X_1,P_1}{\text{Max}} \quad \pi_1 = P_1 Q_1$$
$$= h_1\left(X_1, P_1 \mid X_2^*, P_2^*\right)$$

subject to $0 \le X_1 \le X_2^*$ and $0 \le P_1$,

$$\underset{X_2,P_2}{\text{Max}} \quad \pi_2 = P_2 Q_2$$
$$= h_2\left(X_2, P_2 \mid X_1^*, P_1^*\right)$$

subject to $X_1^* \le X_2 \le 1$, and $0 \le P_2$,

where X_1^* and X_2^* are optimal positions, and P_1^* and P_2^* are optimal prices determined by competing firms.

PROPOSITION 3: There is unique Nash equilibrium at the center of the market with identical prices,

$$X_1^* = X_2^* = 1/2,$$

$$P_1^* = P_2^* = 2\sigma.$$

PROOF: see Appendix E.

The equilibrium price is proportional to the choice variation. As consumer choices become more independent upon situation-specific and consumer-specific attributes, the firms can charge higher prices. The equilibrium expected demands (market share) are identical at half the market and the profits are proportional to the choice variation.

$$Q_1^* = Q_2^* = 1/2, \text{ and}$$

$$\pi_1^* = \pi_2^* = \sigma.$$

Proposition 3 is consistent with the result of the location-price model in de Palma et al. (1985). Both models call for decisions on position and price in a single stage game and produce identical results.

Case 4: A Two-Stage Game

In most differentiation studies using deterministic choice models following Hotelling (1929), decisions on position and price have been modeled in a two-stage game. Two firms position simultaneously in the first stage, and after observing each other's position, they determine price (in the second stage). The reasoning is that price can easily be adjusted and is dependent on product positions. On the other hand, it is often difficult to change product position. We are not aware of any previous research that examined a two-stage game of price and position using a probabilistic choice model.

Since prices are determined in the second stage, given positions, the game, then, has a subgame (pricing game), and needs a subgame perfect Nash equilibrium (or equilibria) as a solution. In this framework, equilibrium prices depend on both firms' positions, and the method of backward induction is applied.

The objective functions of the firms in the first stage are

$$\begin{aligned} \underset{X_1}{\text{Max}} \ \pi_1 &= P_1 Q_1 \\ &= h_3\left(X_1 \mid X_2^*, P_1^*, P_2^*\right) \end{aligned}$$

subject to $0 \leq X_1 \leq X_2^*$

$$\max_{X_2} \ \pi_2 = P_2 Q_2$$

$$= h_4\left(X_2 \mid X_1^*, P_1^*, P_2^*\right)$$

subject to $X_1^* \leq X_2 \leq 1$

where P_1* and P_2* are optimal equilibrium prices depending on positions, and X_1* and X_2* are the optimal positions determined by competing firms.

The objective functions of the firms in the second stage (subgame) are

$$\max_{P_1} \ \pi_1 = P_1 Q_1$$

$$= h_5\left(P_1 \mid X_1, X_2, P_2^*\right)$$

subject to $0 \leq P_1$

$$\max_{P_2} \ \pi_2 = P_2 Q_2$$

$$= h_6\left(P_2 \mid X_1, X_2, P_1^*\right)$$

subject to $0 \leq P_2$

where X_1 and X_2 are positions from the first stage and P_1* and P_2* are the optimal prices determined by competing firms. Equilibrium pricing is a function of the positions because optimal prices are dependent upon positions. With positions given, the first order conditions with respect to prices should be satisfied.

$$\frac{\partial \pi_1}{\partial P_1} = Q_1 + P_1 \frac{\partial Q_1}{\partial P_1} = 0$$

$$\frac{\partial \pi_2}{\partial P_2} = Q_2 + P_2 \frac{\partial Q_2}{\partial P_2} = 0$$

Two non-linear simultaneous equations of the first order conditions produce the optimal prices of both firms. Let $\hat{P}_1$ and $\hat{P}_2$ be equilibrium prices from the solution of the non-linear simultaneous equations.

$$\hat{P}_1 = f_1(X_1, X_2),$$
$$\hat{P}_2 = f_2(X_1, X_2)$$

where X_1 and X_2 are the given positions. Hence, the expected profits of the firms can be determined by their positions.

$$\pi_1 = h_3(X_1 \mid X_2^*, \hat{P}_1, \hat{P}_2)$$
$$= h_7(X_1 \mid X_2^*)$$
$$\pi_2 = h_4(X_2 \mid X_1^*, \hat{P}_1, \hat{P}_2)$$
$$= h_8(X_2 \mid X_1^*)$$

Therefore, the first order conditions with respect to positions provide the equilibrium positions as well as equilibrium prices (which are determined by the equilibrium positions).

$$\frac{\partial \pi_1}{\partial X_1} = \hat{P}_1 \left[\frac{\partial Q_1}{\partial \omega_1} - \frac{\partial Q_1}{\partial P_1} \frac{\partial \hat{P}_2}{\partial X_1} \right]$$

$$\frac{\partial \pi_2}{\partial X_2} = \hat{P}_2 \left[\frac{\partial Q_2}{\partial \omega_2} - \frac{\partial Q_2}{\partial P_2} \frac{\partial \hat{P}_1}{\partial X_2} \right]$$

where $\omega_1 = f_5(X_1) = X_1$

$$\omega_2 = f_6(X_2) = X_2$$

$$\frac{\partial \hat{P}_2}{\partial X_1} = \frac{\dfrac{\partial Q_1}{\partial \omega_1}\dfrac{\partial Q_1}{\partial P_1} + \left(\hat{P}_1 + \hat{P}_2\right)\dfrac{\partial Q_1}{\partial \omega_1}\dfrac{\partial^2 Q_1}{\partial {P_1}^2} - \left(\hat{P}_1 + 2\hat{P}_2\right)\dfrac{\partial Q_1}{\partial P_1}\dfrac{\partial^2 Q_1}{\partial P_1 \partial \omega_1}}{3\left(\dfrac{\partial Q_1}{\partial P_1}\right)^2 + \left(\hat{P}_1 - \hat{P}_2\right)\dfrac{\partial Q_1}{\partial P_1}\dfrac{\partial^2 Q_1}{\partial {P_1}^2}}$$

$$\frac{\partial \hat{P}_1}{\partial X_2} = \frac{\dfrac{\partial Q_2}{\partial \omega_2}\dfrac{\partial Q_1}{\partial P_1} - \left(\hat{P}_1 + \hat{P}_2\right)\dfrac{\partial Q_2}{\partial \omega_2}\dfrac{\partial^2 Q_1}{\partial {P_1}^2} - \left(\hat{P}_2 + 2\hat{P}_1\right)\dfrac{\partial Q_1}{\partial P_1}\dfrac{\partial^2 Q_2}{\partial P_2 \partial \omega_2}}{3\left(\dfrac{\partial Q_1}{\partial P_1}\right)^2 + \left(\hat{P}_1 - \hat{P}_2\right)\dfrac{\partial Q_1}{\partial P_1}\dfrac{\partial^2 Q_1}{\partial {P_1}^2}}$$

$$\frac{\partial^2 Q_1}{\partial P_1 \partial \omega_1} = -\frac{A(1-A)}{(1+A)^3}\frac{cX_1}{\sigma^2} + \frac{B(1-B)}{(1+B)^3}\frac{c(1-X_2)}{\sigma^2} - \frac{A}{\sigma(1+A)^2}$$

$$+\frac{(B+A)(1+A)(1+B) - (B-A)^2}{2\sigma(1+A)^2(1+B)^2}$$

$$\frac{\partial^2 Q_2}{\partial P_2 \partial \omega_2} = \frac{A(1-A)}{(1+A)^3}\frac{cX_1}{\sigma^2} - \frac{B(1-B)}{(1+B)^3}\frac{c(1-X_2)}{\sigma^2} + \frac{B}{\sigma(1+B)^2}$$

$$-\frac{(B+A)(1+A)(1+B) - (B-A)^2}{2\sigma(1+A)^2(1+B)^2}$$

$$\frac{\partial Q_1}{\partial P_1} = \frac{\partial Q_2}{\partial P_2} = -\frac{A}{(1+A)^2}\frac{X_1}{\sigma} - \frac{B}{(1+B)^2}\frac{(1-X_2)}{\sigma} - \frac{B-A}{2c(1+A)(1+B)}$$

$$\frac{\partial Q_1}{\partial \omega_1} = -\frac{A}{(1+A)^2}\frac{cX_1}{\sigma} + \frac{B}{(1+B)^2}\frac{c(1-X_2)}{\sigma} + \frac{B-A}{2(1+A)(1+B)}$$

$$\frac{\partial Q_2}{\partial \omega_2} = -\frac{A}{(1+A)^2}\frac{cX_1}{\sigma} + \frac{B}{(1+B)^2}\frac{c(1-X_2)}{\sigma} - \frac{B-A}{2(1+A)(1+B)}$$

$$A = \mathrm{EXP}\left[\frac{\hat{P}_1 - \hat{P}_2}{\sigma} - \frac{c(X_2 - X_1)}{\sigma}\right]$$

$$B = \mathrm{EXP}\left[\frac{\hat{P}_1 - \hat{P}_2}{\sigma} + \frac{c(X_2 - X_1)}{\sigma}\right]$$

(see Appendix F).

The complexity of this problem makes it impossible to find analytical solutions. We therefore resort to numerical computation. For simplicity, we restrict the value of c to one (not only because our major concern is the effect of consumer choice variation, but because we can easily infer the effect of c). The expected profits of the firms are computed with a grid size of 10^{-2} for $0.01 \leq \sigma \leq 2.00$.[1]

The following results were obtained.

PROPOSITION 4: When c = 1.0,

(a) If $0 < \sigma < 0.75$, there is not center equilibrium, but symmetrically dispersed equilibria do exist.

(b) If $0.75 \leq \sigma < 1.50$, there are both center and symmetric dispersed equilibria, but the center-equilibrium is unstable.

(c) If $\sigma \geq 1.50$, there is an equilibrium at the center of the market.

Insert Figure 5 here

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[1]In each case of σ, optimal prices are calculated with a grid size of 10^{-3} for $0.000 \leq X_1 \leq X_2$ and $X_1 \leq X_2 \leq 1.000$ using IMSL subroutines in double precision. The first derivatives of profits with respect to X_1 and X_2 are obtained with optimal prices. An equilibrium is obtained through checking the changes of the first derivatives. When $\sigma < 0.03$, the numerical computation cannot produce a solution because the inner values in exponential terms become extremely large. However, large numbers in exponential terms cause an equilibrium near the center of the market.

Figure 5 depicts these three intervals. In the first interval, there are symmetrically dispersed equilibria only.[2] As σ increases in the interval, $0.30 \leq \sigma \leq 0.75$, symmetric equilibria converge at the center of the market. This means that as the choice variation increases, optimal positioning moves toward the center of the market and standardization becomes optimal. At $\sigma = 0.30$, there is maximum differentiation (see Figure 5).

When $\sigma \leq 0.30$, symmetric equilibria head to the center as σ decreases to zero. This is probably a consequence of our assumption of a linear utility function, and may represent a link to other product differentiation research using a deterministic choice model. Economides (1986) proved that the degree of product differentiation in Hotelling's model is determined by the curvature of the utility function with respect to the distance between the ideal and a firm's position, confirming the principle of maximum differentiation in d'Aspremont, Gabszewicz, and Thisse (1979): As the utility function becomes more linear, equilibrium positions move toward the center. However, as the utility function becomes more quadratic, maximally differentiated products are optimal. Our findings are consistent with these results.. With the assumption of a linear utility function, Figure 5 suggests standardization where σ is infinitesimal. As σ increases, differentiation becomes optimal until σ reaches .30. As σ increases further, standardization again becomes the optimal strategy. With a quadratic utility function, there would be more differentiation as σ goes to zero. Because the effect of the curvature of the utility function on product differentiation occurs in a rather small range of σ near to zero, we stand by our general claim, that as σ increases, standardization becomes optimal.

In interval 2, both center as well as symmetrically dispersed equilibria exist. The possibility of the center equilibrium, however, is very limited because the equilibrium at the

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[2]There are two symmetrically dispersed equilibria for the firm 1's position to the left and the right of firm 2.

center is very unstable.[3] This equilibrium can exist only under the condition that there is zero probability of a non-center position. Hence, the center equilibrium in the interval, $0.75 \leq \sigma < 1.50$, is not a "perfect" equilibrium (Selten, 1975). However, at $\sigma \geq 1.50$, there is one equilibrium only and it is at the center of the market. As σ increases, symmetric equilibria converge at the center of the market.

Figure 6 and 7 show the optimal price and profit in equilibria. In equilibrium, both firms have identical prices, profits, and market shares. As σ increases, the firms charge higher prices and obtain higher profits. As $\sigma \geq 1.50$, optimal price and profit will increase linearly. At the center equilibrium, the firms charge identical prices at 2σ, and make identical profits of σ with the same market share. When $0.75 \leq \sigma \leq 1.50$, the symmetric equilibrium produces higher prices and profits than the center equilibrium. Prices and profits are their lowest at $\sigma = 0.16$. When σ is less than 0.16, optimal prices and profits increase as σ goes to zero. Again, this is probably due to our assumption of a linear utility function.

Insert Figures 6 and 7 here

We can easily infer the effect of c on the equilibrium. c is the degree of utility reduction due to the distance between consumer ideal and a firm's position. For a small value of c, consumers pay little attention to this distance. As a result, consumer choice variation has relatively large effects. As c increases, this effect becomes smaller, and symmetric equilibria will slowly converge to the center-equilibrium. Each firm, then, may enjoy a local monopoly in equilibrium.

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[3]The equilibrium at the center is a subgame perfect Nash equilibrium because when two firms position at the center of the market, there is no incentive to move. Moving away from the center will reduce profit if the competitor remains at the center.

In contrast to the findings of de Palma et al. (1985), we find symmetrically dispersed equilibria at small degrees of variation. de Palma et al. (1985) argue that there always exists an agglomerated equilibrium at the center of the market in a duopoly. Their finding is probably a consequence of limiting the analysis to a one-stage game. Our findings, on the other hand, are consistent with the research in economics using a deterministic choice model. The deterministic choice model is a special case of random utility model when $\sigma = 0$. We show that differentiation is optimal when consumers consistently choose the product with highest utility. As choice variation enters the picture, standardization become optimal.

SUMMARY AND DISCUSSION

Hotelling introduced a spatial dimension in order to avoid Betrand style price competition. His finding suggests product standardization, or what has become known as "the principle of minimum differentiation." Discontinuity of demand, however, eliminates the continuity of profit functions and the existence of a Nash equilibrium in a pure strategy (see Dasgupta & Maskin, 1986a; 1986b for a Nash equilibrium in mixed strategy). Over the years, it has been shown that optimal as well as actual positions are often dramatically different from Hotelling's principle. Positioning in distinct segments is often optimal because isolated competitors can reduce price cutting. Nevertheless, there may be cases where standardization rather than differentiation is, in fact, appropriate.

There have been several attempts (Eaton & Lipsey, 1975; Carpenter, 1986; de Palma et al. 1985; 1987; Schmittlein & Sarigollu, 1987) to account for standardization, but, as we have discussed, they have all relied upon rather limiting assumptions. In view of the fact that most past research suggest that the principle of minimum differentiation is flawed, we view our contribution as one of more fully restoring the continuity of demand and the principle of minimum differentiation in a duopoly framework. When price competition is limited, both firms will position at the center of the market, regardless of price. These results confirm Carpenter's (1986), Schmittlein and Sarigollu's (1987), and de Palma et al.'s (1985) findings. However, identical price is not a condition for standardization. If there is choice variation (and price competition is limited), standardization at the center of the market is always optimal. This may explain why we observe such a degree of standardization under price regulation. Deregulation have forced airlines (Bauer, 1987), motor carriers (Mentzer & Gomes, 1986) and the telecommunication companies (Marks, 1988) to differentiate their offerings. When positional competition is limited, positional advantages affect firms' prices, market shares, and profits. The firm with a positional advantage can increase price, and have higher market share and profit. The other firm should decrease price in order to keep its market share. Identical prices are not always optimal under positional advantage. Only when both firms position identically or symmetrically will this be the case.

When there are two strategic variables, position and price, determined simultaneously, a unique Nash equilibrium in a pure strategy is found at the center of the market with positive identical prices. That is, both firms position at the center of the market and charge the same price. Each firm has half the market and positive profits. As choice variation increases, the firms can charge higher prices and obtain higher profits. This result is, no doubt, a result of the assumptions of inelastic demand and no entry, however.

Even when position and price are determined sequentially in a two-stage game, standardization is optimal under sufficient choice variation. In contrast to the finding of de Palma et al.'s (1985) duopoly case (where an agglomerated equilibrium at the center is always optimal), differentiation is optimal at a lower degree of the variation (even in a duopoly). This may represent a link to other product differentiation research using a deterministic choice model (e.g., d'Aspremont, Gabszewicz & Thisse, 1979; Economides, 1986). That is, when consumer choices are consistent and stationary, differentiation or segmentation is optimal. However, as choice variation increases, the optimal strategy moves toward standardization. This equilibrium may explain why frequently purchased products often are positioned closer to each other ("me-too products") as compared to durable goods.

Our research is limited in several respects. First, our analysis is confined to competition in duopoly. Drawing upon de Palma et al.'s (1987) results, we conjecture that sufficient choice variation within and across consumers has the same effect when there are more than two firms in the market. Second, we assume a variety model of consumer utility in a Coombsian sense (1950) following the modelling tradition of Hotelling (1929). The effect of choice variation could also be analyzed when consumers have a quality (e.g., Gabszewicz & Thisse, 1979; Shaked & Sutton, 1982; Moorthy, 1988), or a combined type utility model (e.g., Katz, 1984; Hansen & Narasimhan, 1988). Even though the recent research with the quality model suggests product differentiation (Gabszewicz & Thisse, 1979; Shaked and Sutton, 1982; Moorthy, 1988), we conjecture that sufficient choice variation would force competing firms to produce similar products. Third, we assume a linear market, a uniform distribution of consumers, and a perfectly inelastic demand. Increasing densities of consumer toward the center of the market would provide another incentive for standardization (Shilony, 1981; Neven, 1986), but price elastic demand would lead competing firms to move away from each other (Smithies, 1941). Further research should be focused on extensions of these assumptions.

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APPENDIX A

As a firm raises price, the probabilities that consumers in all regions purchase a product from the firm decrease because of higher price,

$$\frac{\partial Pr_1^{1}(Y_i)}{\partial P_1} < 0,\ \frac{\partial Pr_1^{2}(Y_i)}{\partial P_1} < 0, \text{ and } \frac{\partial Pr_1^{3}(Y_i)}{\partial P_1} < 0$$

$$\frac{\partial Pr_2^{1}(Y_i)}{\partial P_2} < 0,\ \frac{\partial Pr_2^{2}(Y_i)}{\partial P_2} < 0, \text{ and } \frac{\partial Pr_2^{3}(Y_i)}{\partial P_2} < 0$$

In contrast, the probability of purchasing from the other firm increases due to relatively lower price,

$$\frac{\partial Pr_1^{1}(Y_i)}{\partial P_2} > 0,\ \frac{\partial Pr_1^{2}(Y_i)}{\partial P_2} > 0, \text{ and } \frac{\partial Pr_1^{3}(Y_i)}{\partial P_2} > 0$$

$$\frac{\partial Pr_2^{1}(Y_i)}{\partial P_1} > 0,\ \frac{\partial Pr_2^{2}(Y_i)}{\partial P_1} > 0, \text{ and } \frac{\partial Pr_2^{3}(Y_i)}{\partial P_1} > 0.$$

APPENDIX B

In the price model, the objective functions of both firms should be satisfied simultaneously with respect to their prices in the non-cooperative game with positions given. Hence, the best responses of both firms are determined by simultaneous first order conditions with respect to prices. The first order conditions are as follows:

$$\frac{\partial \pi_1}{\partial P_1} = Q_1 + P_1 \frac{\partial Q_1}{\partial P_1} = 0$$

$$\frac{\partial \pi_2}{\partial P_2} = Q_2 + P_2 \frac{\partial Q_2}{\partial P_2} = 0$$

The first derivatives of Q_1 and Q_2 with respect to prices are

$$\frac{\partial Q_1}{\partial P_1} = -\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]^2}\frac{X_1}{\sigma} - \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]^2}\frac{(1 - X_2)}{\sigma}$$

$$-\frac{1}{2c}\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right] - \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]}$$

$$\frac{\partial Q_2}{\partial P_2} = -\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]^2}\frac{X_1}{\sigma} - \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]^2}\frac{(1 - X_2)}{\sigma}$$

$$-\frac{1}{2c}\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right] - \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]} = \frac{\partial Q_1}{\partial P_1}$$

Hence $\frac{\partial Q_1}{\partial P_1} = \frac{\partial Q_2}{\partial P_2}$

Then, $\frac{Q_1{}^*}{P_1{}^*}$ should be equal to $\frac{Q_2{}^*}{P_2{}^*}$, and $P_1{}^* \, Q_2{}^* = P_2{}^* \, Q_1{}^*$.

Q.E.D.

APPENDIX C

With positions given, the equilibrium should satisfy the following relationship (from proposition 1)

$$Q_1 = \frac{P_1}{P_1 + P_2}$$

When both firms' positions are identical ($X_1 = X_2$) or symmetrical ($X_1 = 1 - X_2$), Q_1 is (at least monotonically) decreasing while $\frac{P_1}{P_1 + P_2}$ is strictly increasing with respect to $P_1 - P_2$. A unique solution can be obtained at $P_1 = P_2 = 2\sigma$, where $Q_1 = Q_2 = 1/2$ and $\pi_1 = \pi_2 = \sigma$.

APPENDIX D

In the positioning model, the objective functions of the firms should be satisfied simultaneously with respect to positions in the non-cooperative game with prices given. Hence, the best responses of both firms are determined by simultaneous first order conditions with respect to positions. The first order conditions are as follows:

$$\frac{\partial \pi_1}{\partial X_1} = P_1 \frac{\partial Q_1}{\partial X_1} = 0$$

$$\frac{\partial \pi_2}{\partial X_2} = P_2 \frac{\partial Q_2}{\partial X_2} = 0$$

Since P_1 and P_2 are positive, $\frac{\partial Q_1}{\partial X_1}$ and $\frac{\partial Q_2}{\partial X_2}$ should be zero 0,

$$\frac{\partial Q_1}{\partial X_1} = -\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]^2} \frac{cX_1}{\sigma} + \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]^2} \frac{c(1 - X_2)}{\sigma}$$

$$+ \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right] - \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{2\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]}$$

$$\frac{\partial Q_2}{\partial X_2} = -\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} - \frac{cX_d}{\sigma}\right]\right]^2} \frac{cX_1}{\sigma} + \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]}{\left[1 + \mathrm{Exp}\left[\frac{P_d}{\sigma} + \frac{cX_d}{\sigma}\right]\right]^2} \frac{c(1 - X_2)}{\sigma}$$

$$-\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]-\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]}{2\left[1+\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]\right]\left[1+\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]\right]}$$

Therefore

$$\frac{\partial Q_2}{\partial X_2}=\frac{\partial Q_1}{\partial X_1}+\frac{\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]-\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]}{\left[1+\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]\right]\left[1+\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]\right]}=0$$

And, because $\frac{\partial Q_1}{\partial X_1}=0$ and

$1+\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]$ and $1+\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]$ are greater than 1,

$$\mathrm{Exp}\left[\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}\right]-\mathrm{Exp}\left[\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}\right]=0$$

Then $\frac{P_d}{\sigma}-\frac{cX_d}{\sigma}=\frac{P_d}{\sigma}+\frac{cX_d}{\sigma}$, and then $\frac{cX_d}{\sigma}=0$.

Since $\sigma > 0$ and $c > 0$, $X_d = X_2 - X_1 = 0$.

Therefore $X_2 = X_1$.

$$\text{Then } \frac{\partial Q_1}{\partial X_1}=-\frac{c(2X_1-1)\left[\mathrm{EXP}\left[\frac{P_d}{\sigma}\right]\right]}{\sigma\left[1+\mathrm{EXP}\left[\frac{P_d}{\sigma}\right]\right]^2}=0.$$

Since $c > 0$, $\sigma > 0$, and $\mathrm{EXP}\left[\frac{P_d}{\sigma}\right]$ and $\left[1+\mathrm{EXP}\left[\frac{P_d}{\sigma}\right]\right]^2$ are positive,

$2X_1 - 1 = 0.$

Therefore, $X_1^* = X_2^* = 1/2$ Q.E.D.

APPENDIX E

In the position-price game in a single stage, the equilibrium positions exist at the center of the market, regardless of price (see proposition 2). When $X_1 = X_2$,

$$\frac{\partial \pi_1}{\partial P_1} = \frac{1}{\sigma\left(1 + \left[\frac{P_d}{\sigma}\right]\right)^2} \left[\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right) - P_1 \, \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right] = 0$$

Since $\sigma > 0$ and $\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right)^2 > 1$, then

$$\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right) - P_1 \, \mathrm{Exp}\left[\frac{P_d}{\sigma}\right] = 0$$

Then $P_1 = \dfrac{\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right)}{\mathrm{Exp}\left[\frac{P_d}{\sigma}\right]}$.

And $$\frac{\partial \pi_2}{\partial P_2} = \frac{\mathrm{Exp}\left[\frac{P_d}{\sigma}\right]}{\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right)^2} \left(\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right) - P_2\right) = 0$$

Since $\mathrm{Exp}\left[\frac{P_d}{\sigma}\right] > 0$, $P_2 = \sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right)$

Therefore $\quad \mathrm{Exp}\left[\frac{P_d}{\sigma}\right] = \frac{P_2}{P_1}$

and $$P_1 = \frac{\sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right)}{\mathrm{Exp}\left[\frac{P_d}{\sigma}\right]} = \sigma \frac{P_1}{P_2}\left[1 + \frac{P_2}{P_1}\right]$$

$$P_2 = \sigma\left(1 + \mathrm{Exp}\left[\frac{P_d}{\sigma}\right]\right) = \sigma\left[1 + \frac{P_2}{P_1}\right]$$

Since $\mathrm{Exp}\left[\frac{P_d}{\sigma}\right] = \frac{P_2}{P_1}$, $\qquad \frac{P_2}{P_1} = \frac{\sigma}{P_1 - \sigma}$,

and $\mathrm{Exp}\left[\frac{P_d}{\sigma}\right] = \mathrm{Exp}\left[\frac{P_1}{\sigma}\left[1 - \frac{\sigma}{P_1 - \sigma}\right]\right]$,

$$\mathrm{Exp}\left[\frac{P_1}{\sigma}\left[1 - \frac{\sigma}{P_1 - \sigma}\right]\right] = \frac{\sigma}{P_1 - \sigma}.$$

When we draw the two functions,

$$f_1(P_1) = \mathrm{Exp}\left[\frac{P_1}{\sigma}\left[1 - \frac{\sigma}{P_1 - \sigma}\right]\right],$$

$$f_2(P_1) = \frac{\sigma}{P_1 - \sigma},$$

we can easily see that there exists a unique solution, which is a Nash equilibrium (see Figure 8),

$$P_1{}^* = P_2{}^* = 2\sigma.$$

Q.E.D.

Insert Figure 8 here

APPENDIX F

The first order conditions with respect to prices in the subgame provide equilibrium prices, which are functions of given positions.

$$\frac{\partial \pi_1}{\partial P_1} = Q_1 + P_1 \frac{\partial Q_1}{\partial P_1} = 0 \tag{1}$$

$$\frac{\partial \pi_2}{\partial P_2} = Q_2 + P_2 \frac{\partial Q_2}{\partial P_2} = 0 \tag{2}$$

Let $\hat{P}_1$ and $\hat{P}_2$ be equilibrium prices from the simultaneous non-linear equations of the first order conditions.

$$\hat{P}_1 = f_1(X_1, X_2)$$

$$\hat{P}_2 = f_2(X_1, X_2)$$

When we substitute $\hat{P}_1$ and $\hat{P}_2$ with P_1 and P_2 in the profit functions, the expected profits of the firms are functions of positions only.

$$\begin{aligned} \pi_1 &= g_1\left(\hat{P}_1, \hat{P}_2, X_1, X_2\right) \\ &= h_1(X_1, X_2) \end{aligned}$$

$$\begin{aligned} \pi_2 &= g_2\left(\hat{P}_1, \hat{P}_2, X_1, X_2\right) \\ &= h_2(X_1, X_2) \end{aligned}$$

Then, the first order conditions with respect to positions are

$$\frac{\partial \pi_1}{\partial X_1} = \frac{\partial \hat{P}_1}{\partial X_1} Q_1 + \hat{P}_1 \frac{\partial Q_1}{\partial X_1} = 0 \tag{3}$$

$$\frac{\partial \pi_2}{\partial X_2} = \frac{\partial \hat{P}_2}{\partial X_2} Q_2 + \hat{P}_2 \frac{\partial Q_2}{\partial X_2} = 0 \qquad (4)$$

Since $Q_1 = f_3\left(\hat{P}_1, \hat{P}_2, X_1, X_2\right)$

$$Q_2 = f_4\left(\hat{P}_1, \hat{P}_2, X_1, X_2\right)$$

and when we introduce new variables, ω_1 and ω_2

$$\omega_1 = f_5(X_1) = X_1, \qquad \omega_2 = f_6(X_2) = X_2$$

$$\frac{\partial Q_1}{\partial X_1} = \frac{\partial Q_1}{\partial P_1}\frac{\partial \hat{P}_1}{\partial X_1} + \frac{\partial Q_1}{\partial P_2}\frac{\partial \hat{P}_2}{\partial X_1} + \frac{\partial Q_1}{\partial \omega_1}\frac{\partial \omega_1}{\partial X_1}$$

$$= \frac{\partial Q_1}{\partial P_1}\left(\frac{\partial \hat{P}_1}{\partial X_1} - \frac{\partial \hat{P}_2}{\partial X_1}\right) + \frac{\partial Q_1}{\partial \omega_1}$$

because $\frac{\partial Q_2}{\partial P_2} = \frac{\partial Q_1}{\partial P_1}$, $\quad \frac{\partial Q_1}{\partial P_2} = -\frac{\partial Q_2}{\partial P_2} = -\frac{\partial Q_1}{\partial P_1}$ and $\frac{\partial \omega_1}{\partial X_1} = 1$

$$\frac{\partial Q_2}{\partial X_2} = \frac{\partial Q_2}{\partial P_1}\frac{\partial \hat{P}_1}{\partial X_2} + \frac{\partial Q_2}{\partial P_2}\frac{\partial \hat{P}_2}{\partial X_2} + \frac{\partial Q_2}{\partial \omega_2}\frac{\partial \omega_2}{\partial X_2}$$

$$= \frac{\partial Q_2}{\partial P_2}\left(\frac{\partial \hat{P}_2}{\partial X_2} - \frac{\partial \hat{P}_1}{\partial X_2}\right) + \frac{\partial Q_2}{\partial \omega_2}$$

because $\frac{\partial Q_2}{\partial P_1} = -\frac{\partial Q_1}{\partial P_1} = -\frac{\partial Q_2}{\partial P_2}$ and $\frac{\partial \omega_2}{\partial X_2} = 1$

When we substitute $\frac{\partial Q_1}{\partial X_1}$ and $\frac{\partial Q_2}{\partial X_2}$ into the equation (3) and (4),

$$\frac{\partial \pi_1}{\partial X_1} = \frac{\partial \hat{P}_1}{\partial X_1} Q_1 + \hat{P}_1 \frac{\partial Q_1}{\partial X_1}$$

$$= \frac{\partial \hat{P}_1}{\partial X_1} Q_1 + \hat{P}_1 \left[\frac{\partial Q_1}{\partial \omega_1} + \frac{\partial Q_1}{\partial P_1} \left(\frac{\partial \hat{P}_1}{\partial X_1} - \frac{\partial \hat{P}_2}{\partial X_1} \right) \right]$$

$$= \frac{\partial \hat{P}_1}{\partial X_1} \left[Q_1 + \hat{P}_1 \frac{\partial Q_1}{\partial P_1} \right] + \hat{P}_1 \left[\frac{\partial Q_1}{\partial \omega_1} - \frac{\partial Q_1}{\partial P_1} \frac{\partial \hat{P}_2}{\partial X_1} \right]$$

$$= \hat{P}_1 \left[\frac{\partial Q_1}{\partial \omega_1} - \frac{\partial Q_1}{\partial P_1} \frac{\partial \hat{P}_2}{\partial X_1} \right], \text{ because } Q_1 + \hat{P}_1 \frac{\partial Q_1}{\partial P_1} = \frac{\partial \pi_1}{\partial P_1} = 0.$$

$$\frac{\partial \pi_2}{\partial X_2} = \frac{\partial \hat{P}_2}{\partial X_2} Q_2 + P_2 \frac{\partial Q_2}{\partial X_2}$$

$$= \frac{\partial \hat{P}_2}{\partial X_2} Q_2 + \hat{P}_2 \left[\frac{\partial Q_2}{\partial \omega_2} + \frac{\partial Q_2}{\partial P_2} \left(\frac{\partial \hat{P}_2}{\partial X_2} - \frac{\partial \hat{P}_1}{\partial X_2} \right) \right]$$

$$= \frac{\partial \hat{P}_2}{\partial X_2} \left[Q_2 + \hat{P}_2 \frac{\partial Q_2}{\partial P_2} \right] + \hat{P}_2 \left[\frac{\partial Q_2}{\partial \omega_2} - \frac{\partial Q_2}{\partial P_2} \frac{\partial \hat{P}_1}{\partial X_2} \right]$$

$$= \hat{P}_2 \left[\frac{\partial Q_2}{\partial \omega_2} - \frac{\partial Q_2}{\partial P_2} \frac{\partial \hat{P}_1}{\partial X_2} \right], \text{ because } Q_2 + \hat{P}_2 \frac{\partial Q_2}{\partial P_2} = \frac{\partial \pi_2}{\partial P_2} = 0.$$

Because the first order conditions with respect to prices should always be satisfied, regardless of positions, the partial derivatives of profit with respect to prices and positions should be zero.

$$\frac{\partial^2\pi_1}{\partial P_1\partial X_1} = \frac{\partial^2\pi_1}{\partial P_1^2}\frac{\partial\hat{P}_1}{\partial X_1}+\frac{\partial^2\pi_1}{\partial P_1\partial P_2}\frac{\partial\hat{P}_2}{\partial X_1}+\frac{\partial^2\pi_1}{\partial P_1\partial\omega_1}\frac{\partial\omega_1}{\partial X_1}=0.$$

$$\frac{\partial^2\pi_2}{\partial P_2\partial X_1} = \frac{\partial^2\pi_2}{\partial P_1\partial P_2}\frac{\partial\hat{P}_1}{\partial X_1}+\frac{\partial^2\pi_2}{\partial P_2^2}\frac{\partial\hat{P}_2}{\partial X_1}+\frac{\partial^2\pi_2}{\partial P_2\partial\omega_1}\frac{\partial\omega_1}{\partial X_1}=0.$$

$$\frac{\partial^2\pi_1}{\partial P_1\partial X_2} = \frac{\partial^2\pi_1}{\partial P_1^2}\frac{\partial\hat{P}_1}{\partial X_2}+\frac{\partial^2\pi_1}{\partial P_1\partial P_2}\frac{\partial\hat{P}_2}{\partial X_2}+\frac{\partial^2\pi_1}{\partial P_1\partial\omega_2}\frac{\partial\omega_2}{\partial X_2}=0.$$

$$\frac{\partial^2\pi_2}{\partial P_2\partial X_2} = \frac{\partial^2\pi_2}{\partial P_1\partial P_2}\frac{\partial\hat{P}_1}{\partial X_2}+\frac{\partial^2\pi_2}{\partial P_2^2}\frac{\partial\hat{P}_2}{\partial X_2}+\frac{\partial^2\pi_2}{\partial P_2\partial\omega_2}\frac{\partial\omega_2}{\partial X_2}=0.$$

Since $\frac{\partial\omega_1}{\partial X_1}=1$ and $\frac{\partial\omega_2}{\partial X_2}=1$

$$\frac{\partial\hat{P}_2}{\partial X_1}=\frac{\frac{\partial^2\pi_2}{\partial P_1\partial P_2}\frac{\partial^2\pi_1}{\partial P_1\partial\omega_1}-\frac{\partial^2\pi_1}{\partial P_1^2}\frac{\partial^2\pi_2}{\partial P_2\partial\omega_1}}{\frac{\partial^2\pi_1}{\partial P_1^2}\frac{\partial^2\pi_2}{\partial P_2^2}-\frac{\partial^2\pi_2}{\partial P_1\partial P_2}\frac{\partial^2\pi_1}{\partial P_1\partial P_2}}$$

$$\frac{\partial\hat{P}_1}{\partial X_2}=\frac{\frac{\partial^2\pi_1}{\partial P_1\partial P_2}\frac{\partial^2\pi_2}{\partial P_2\partial\omega_2}-\frac{\partial^2\pi_2}{\partial P_2^2}\frac{\partial^2\pi_1}{\partial P_1\partial\omega_2}}{\frac{\partial^2\pi_1}{\partial P_1^2}\frac{\partial^2\pi_2}{\partial P_2^2}-\frac{\partial^2\pi_2}{\partial P_1\partial P_2}\frac{\partial^2\pi_1}{\partial P_1\partial P_2}}$$

Since

$$\frac{\partial^2\pi_1}{\partial P_1^2} = 2\frac{\partial Q_1}{\partial P_1}+\hat{P}_1\frac{\partial^2 Q_1}{\partial P_1 2}$$

$$\frac{\partial^2 \pi_1}{\partial P_1 \partial P_2} = \frac{\partial Q_1}{\partial P_2} + \hat{P}_1 \frac{\partial^2 Q_1}{\partial P_1 \partial P_2} = -\frac{\partial Q_1}{\partial P_1} - \hat{P}_1 \frac{\partial^2 Q_1}{\partial P_1{}^2}$$

$$\text{because } \frac{\partial^2 Q_1}{\partial P_1 \partial P_2} = -\frac{\partial}{\partial P_1}\left(\frac{\partial Q_2}{\partial P_2}\right) = -\frac{\partial}{\partial P_1}\left(\frac{\partial Q_1}{\partial P_1}\right) = -\frac{\partial^2 Q_1}{\partial P_1{}^2}$$

$$\frac{\partial^2 \pi_1}{\partial P_1 \partial \omega_1} = \frac{\partial Q_1}{\partial \omega_1} + \hat{P}_1 \frac{\partial^2 Q_1}{\partial P_1 \partial \omega_1}$$

$$\frac{\partial^2 \pi_1}{\partial P_1 \partial \omega_2} = \frac{\partial Q_1}{\partial \omega_2} + \hat{P}_1 \frac{\partial^2 Q_1}{\partial P_1 \partial \omega_2} = -\frac{\partial Q_2}{\partial \omega_2} + \hat{P}_1 \frac{\partial^2 Q_2}{\partial P_2 \partial \omega_2} \text{ because } \frac{\partial Q_1}{\partial \omega_2} = \frac{\partial(1 - Q_2)}{\partial \omega_2} = -\frac{\partial Q_2}{\partial \omega_2}$$

$$\frac{\partial^2 \pi_2}{\partial P_1 \partial P_2} = \frac{\partial Q_2}{\partial P_1} + \hat{P}_2 \frac{\partial^2 Q_2}{\partial P_1 \partial P_2} = -\frac{\partial Q_1}{\partial P_1} + \hat{P}_2 \frac{\partial^2 Q_1}{\partial P_1{}^2} \text{ because } \frac{\partial Q_2}{\partial P_1} = \frac{\partial(1 - Q_1)}{\partial P_1} = -\frac{\partial Q_1}{\partial P_1}$$

$$\frac{\partial^2 \pi_2}{\partial P_2^2} = 2\frac{\partial Q_2}{\partial P_2} + \hat{P}_2 \frac{\partial^2 Q_2}{\partial P_2{}^2} = 2\frac{\partial Q_1}{\partial P_1} - \hat{P}_2 \frac{\partial^2 Q_1}{\partial P_1{}^2}$$

$$\text{because } \frac{\partial^2 Q_2}{\partial P_2{}^2} = \frac{\partial}{\partial P_2}\left(\frac{\partial Q_2}{\partial P_2}\right) = \frac{\partial}{\partial P_2}\left(\frac{\partial Q_1}{\partial P_1}\right) = \frac{\partial^2 Q_1}{\partial P_1 \partial P_2} = -\frac{\partial^2 Q_1}{\partial P_1{}^2}$$

$$\frac{\partial^2 \pi_2}{\partial P_2 \partial \omega_1} = \frac{\partial Q_2}{\partial \omega_1} + \hat{P}_2 \frac{\partial^2 Q_2}{\partial P_2 \partial \omega_1} = -\frac{\partial Q_1}{\partial \omega_1} + \hat{P}_2 \frac{\partial^2 Q_1}{\partial P_1 \partial \omega_1}$$

$$\text{because } \frac{\partial Q_2}{\partial \omega_1} = \frac{\partial(1 - Q_1)}{\partial \omega_1} = -\frac{\partial Q_1}{\partial \omega_1}$$

$$\frac{\partial^2 \pi_2}{\partial P_2 \partial \omega_2} = \frac{\partial Q_2}{\partial \omega_2} + \hat{P}_2 \frac{\partial^2 Q_2}{\partial P_2 \partial \omega_2},$$

$$\frac{\partial\hat{P}_2}{\partial X_1} = \frac{\frac{\partial Q_1}{\partial \omega_1}\frac{\partial Q_1}{\partial P_1} + \left(\hat{P}_1 + \hat{P}_2\right)\frac{\partial Q_1}{\partial \omega_1}\frac{\partial^2 Q_1}{\partial P_1{}^2} - \left(\hat{P}_1 + 2\hat{P}_2\right)\frac{\partial Q_1}{\partial P_1}\frac{\partial^2 Q_1}{\partial P_1 \partial \omega_1}}{3\left(\frac{\partial Q_1}{\partial P_1}\right)^2 + \left(\hat{P}_1 - \hat{P}_2\right)\frac{\partial Q_1}{\partial P_1}\frac{\partial^2 Q_1}{\partial P_1{}^2}}$$

$$\frac{\partial\hat{P}_1}{\partial X_2} = \frac{\frac{\partial Q_2}{\partial \omega_2}\frac{\partial Q_1}{\partial P_1} - \left(\hat{P}_1 + \hat{P}_2\right)\frac{\partial Q_2}{\partial \omega_2}\frac{\partial^2 Q_1}{\partial P_1{}^2} - \left(\hat{P}_2 + 2\hat{P}_1\right)\frac{\partial Q_1}{\partial P_1}\frac{\partial^2 Q_2}{\partial P_2 \partial \omega_2}}{3\left(\frac{\partial Q_1}{\partial P_1}\right)^2 + \left(\hat{P}_1 - \hat{P}_2\right)\frac{\partial Q_1}{\partial P_1}\frac{\partial^2 Q_1}{\partial P_1{}^2}}$$

where

$$\frac{\partial Q_1}{\partial P_1} = \frac{\partial Q_2}{\partial P_2} = -\frac{A}{(1+A)^2}\frac{X_1}{\sigma} - \frac{B}{(1+B)^2}\frac{(1-X_2)}{\sigma} - \frac{(B-A)}{2c(1+A)(1+B)}$$

$$\frac{\partial Q_1}{\partial \omega_1} = -\frac{A}{(1+A)^2}\frac{cX_1}{\sigma} + \frac{B}{(1+B)^2}\frac{c(1-X_2)}{\sigma} + \frac{(B-A)}{2(1+A)(1+B)}$$

$$\frac{\partial Q_2}{\partial \omega_2} = -\frac{A}{(1+A)^2}\frac{cX_1}{\sigma} + \frac{B}{(1+B)^2}\frac{c(1-X_2)}{\sigma} - \frac{(B-A)}{2(1+A)(1+B)}$$

$$\frac{\partial^2 Q_1}{\partial P_1 \partial \omega_1} = -\frac{A(1-A)}{(1+A)^3}\frac{cX_1}{\sigma^2} + \frac{B(1-B)}{(1+B)^3}\frac{c(1-X_2)}{\sigma^2} - \frac{A}{\sigma(1+A)^2}$$

$$+ \frac{(A+B)(1+A)(1+B) - (B-A)^2}{2\sigma(1+A)^2(1+B)^2}$$

$$\frac{\partial^2 Q_2}{\partial P_2 \partial \omega_2} = \frac{A(1-A)}{(1+A)^3}\frac{cX_1}{\sigma^2} - \frac{B(1-B)}{(1+B)^3}\frac{c(1-X_2)}{\sigma^2} + \frac{B}{\sigma(1+B)^2}$$

$$- \frac{(A + B)(1 + A)(1 + B) - (B - A)^2}{2\sigma(1 + A)^2(1 + B)^2}$$

$$A = \mathrm{Exp}\left[\frac{\hat{P}_1 - \hat{P}_2}{\sigma} - \frac{c(X_2 - X_1)}{\sigma}\right]$$

$$B = \mathrm{Exp}\left[\frac{\hat{P}_1 - \hat{P}_2}{\sigma} + \frac{c(X_2 - X_1)}{\sigma}\right].$$

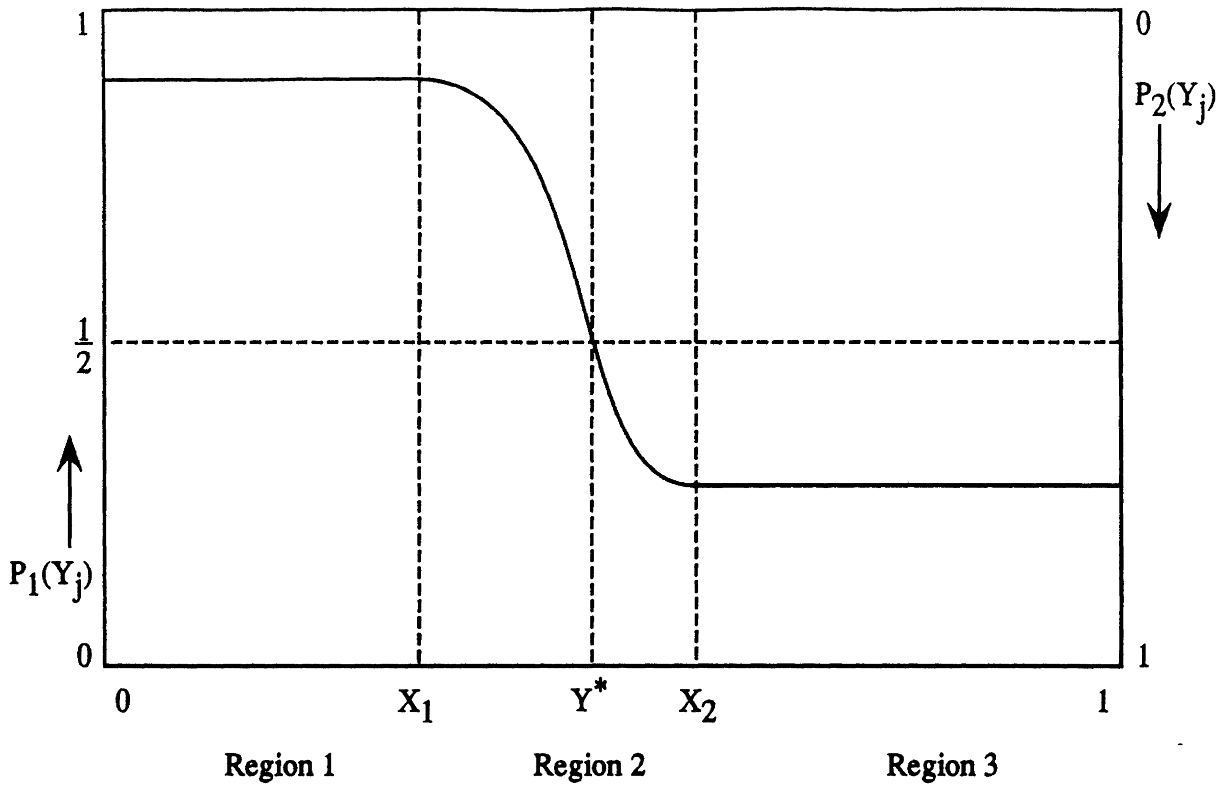

Figure 1: The probability that a consumer in each region will purchase a product from firm 1 or 2

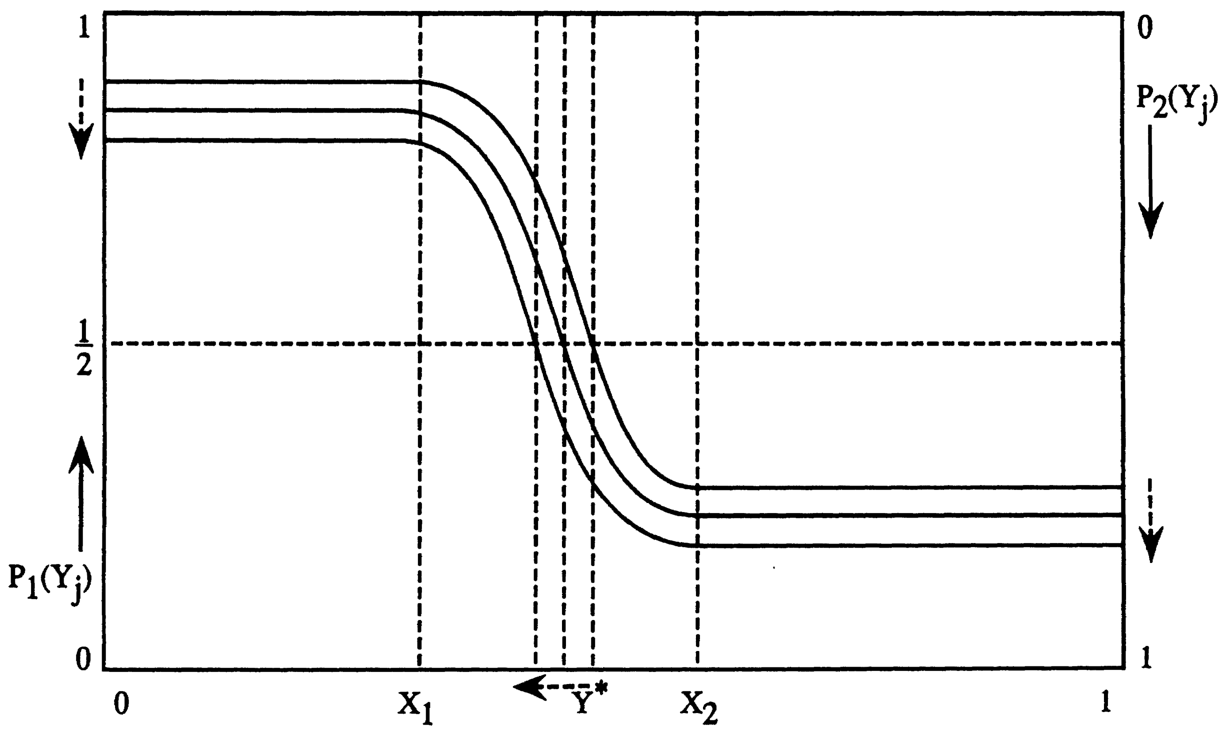

Figure 2: The shift in probability when firm 1 raises price (with positions fixed and a fixed price of firm 2)

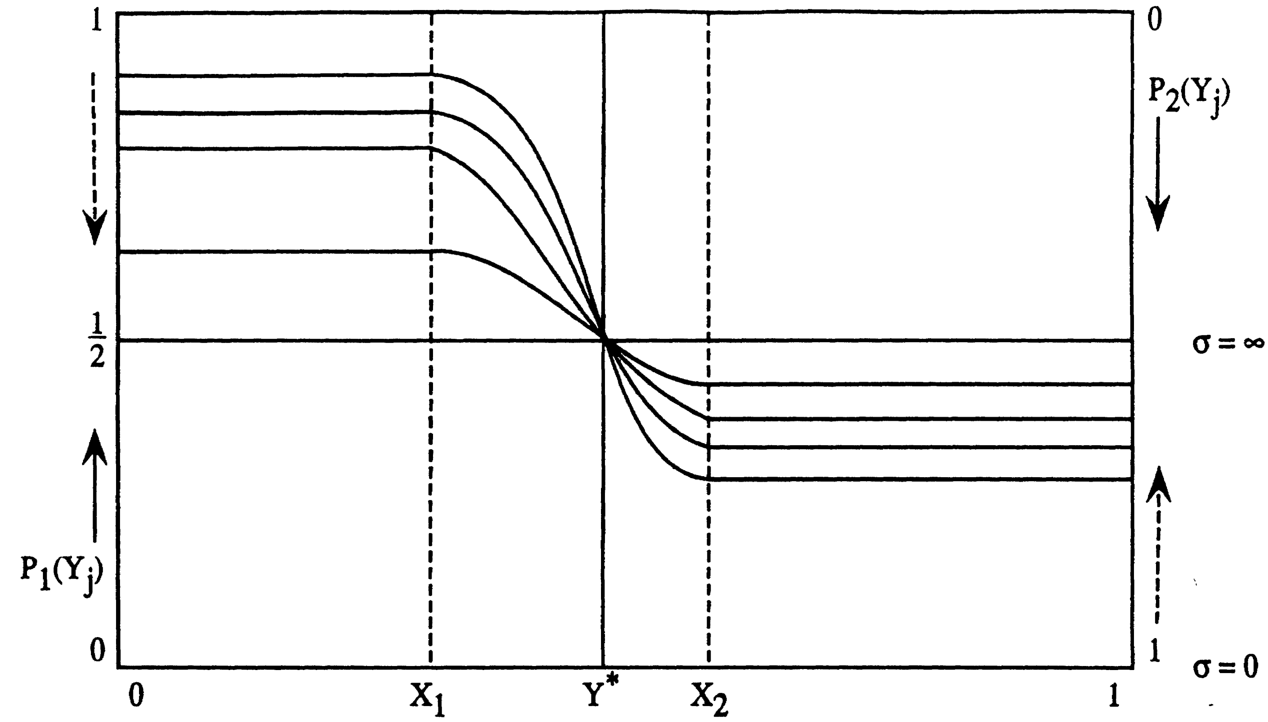

Figure 3: The effect of consumer choice variation on purchase probability

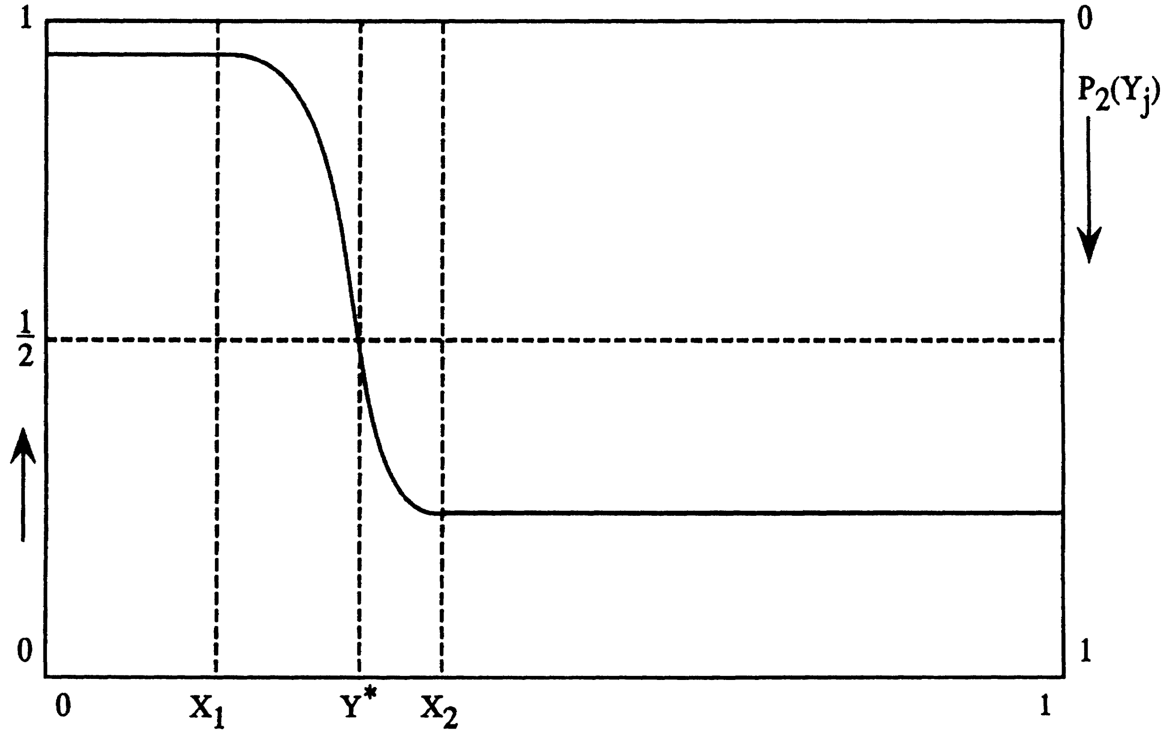

Figure 4: A hypothetical case of price competition with positions fixed

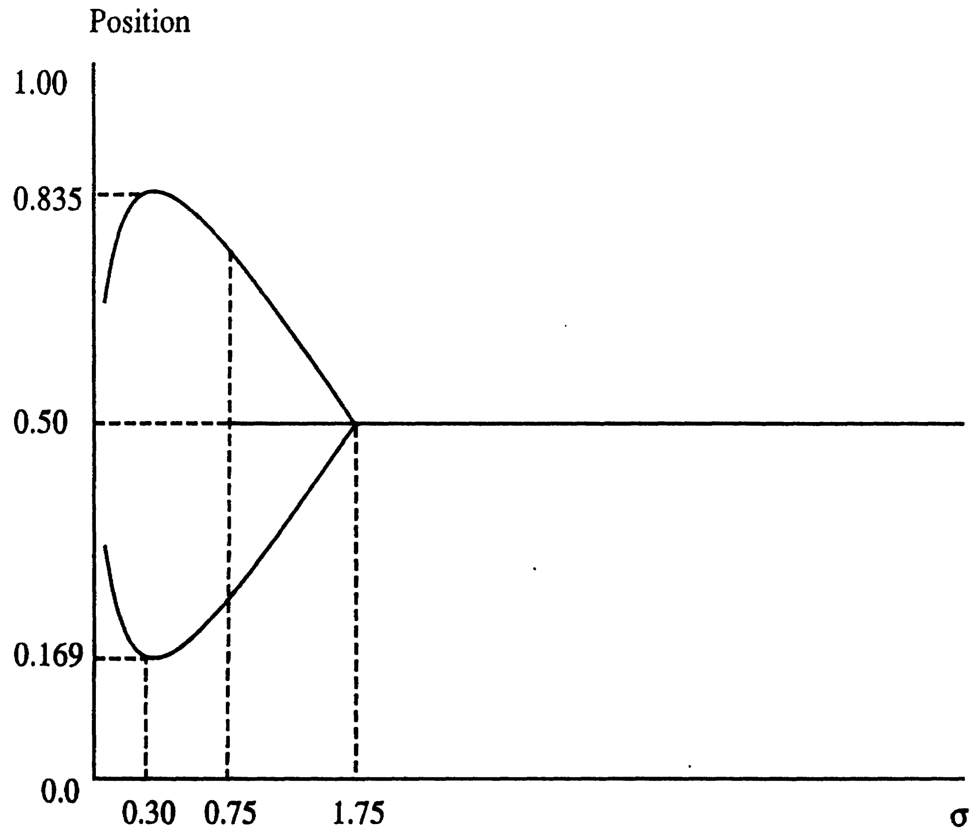

Figure 5: Equilibrium positions with respect to σ

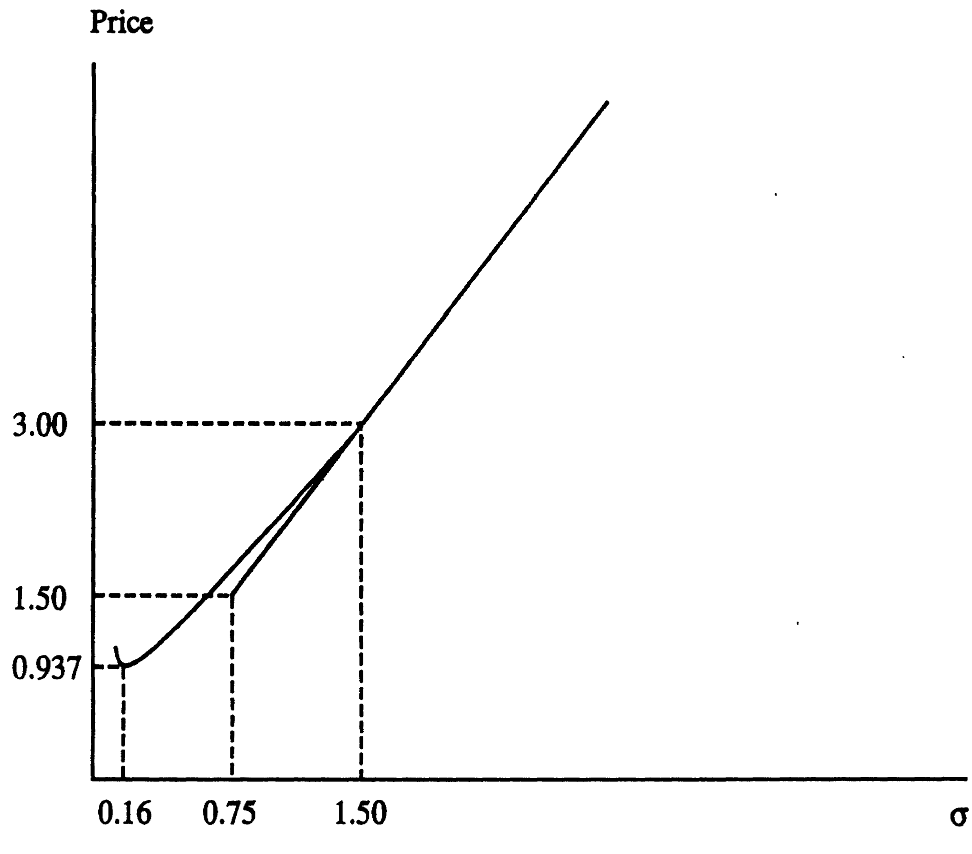

Figure 6: Equilibrium price level with respect to σ

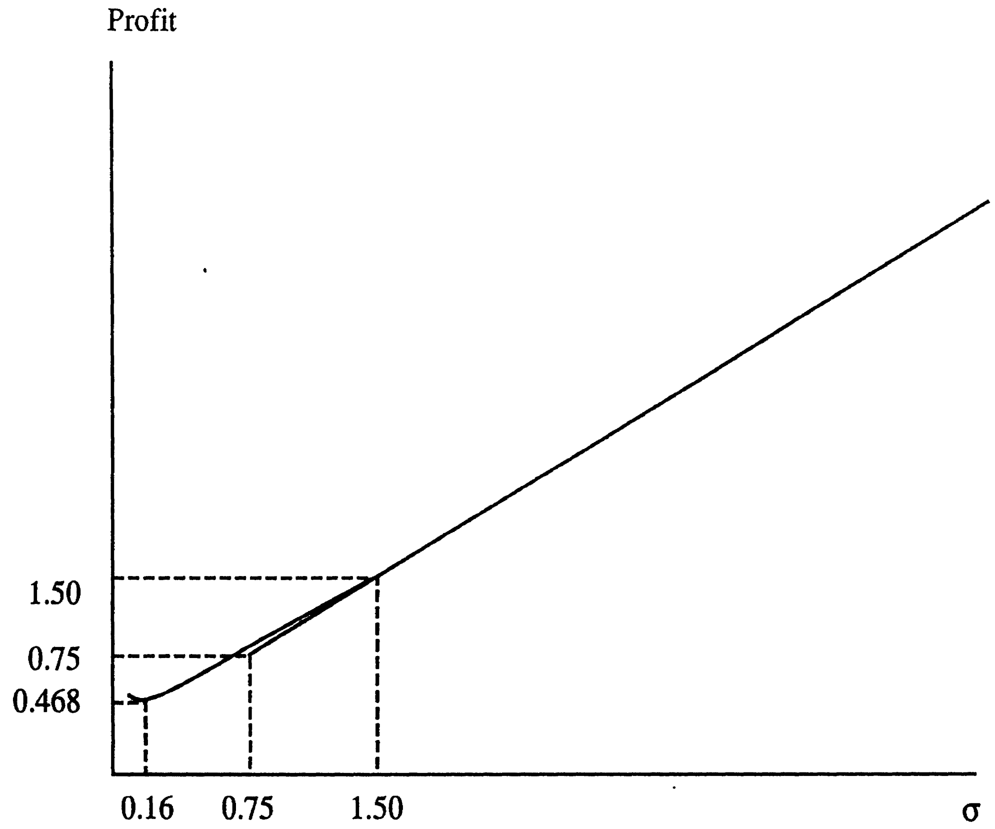

Figure 7: Equilibrium profit level with respect to σ

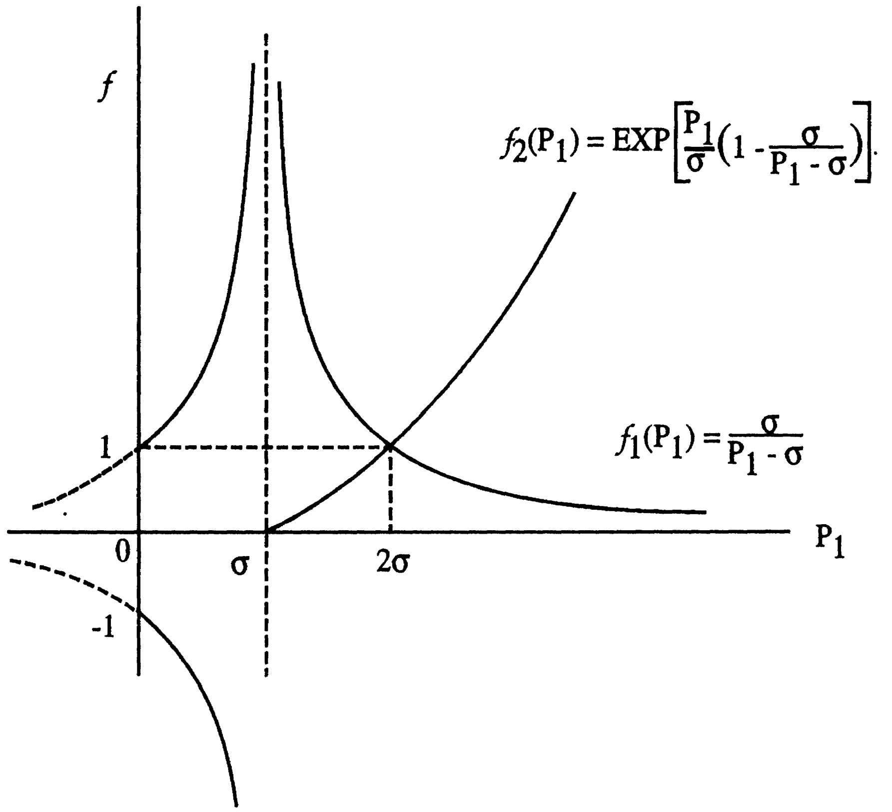

Figure 8: The existence of the unique equilibrium in one-stage position-price game