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A THEORY OF INVESTMENT BANKING CONTRACT CHOICE

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ABSTRACT

Firms issuing stock in the U.S. almost always use either a firm commitment or a best efforts contract. This paper presents a model of the contract choice decision in which there are two classes of investors: informed and uninformed. With a firm commitment offering, uninformed investors face an adverse selection problem in the allocation of shares, which they must be compensated for via underpricing. This required underpricing is an increasing function of the ex ante uncertainty about share value. With a best efforts offering, the issuing firm precommits to withdraw the issue if demand from informed investors is not forthcoming. Consequently, the issuing firm does not have to compensate uninformed investors for adverse selection risk.

I. Introduction

Firms issuing common stock typically contract with an investment banker on either a best efforts or a firm commitment basis. For new issues of seasoned common stock, Booth and Smith (1986) find that 97.4 percent of offerings use a firm commitment contract, while for unseasoned new issues, only 45.7 percent do so. I present a model that explains this pattern.[1]

With a best efforts contract, the issuing firm and investment banker negotiate an offering price and the investment banker then uses its "best efforts" to sell the issue at that price. If there is insufficient demand for the issue at the offering price, the offering is withdrawn. With a firm commitment contract, an investment banker guarantees the proceeds to the issuing firm; if the issue is not fully subscribed at the agreed-upon offering price, the investment banker lowers the price on the unsold shares until the market clears. Whether or not the firm commitment offering is fully subscribed, the issuing firm receives the net proceeds that were guaranteed by the investment banker. With both types of contracts, once an offering price has been set it cannot be increased if demand for the issue is unexpectedly strong.

My analysis of the contract choice decision is based upon informational asymmetries among investors. Thus, it builds upon Rock's (1986) model of the underpricing of initial public offerings. In line with Baron (1979, 1982), Baron and Holmstrom (1980), Beatty and Ritter (1986), Guenther (1987), Mandelker and Raviv (1977), Parsons and Raviv (1985), and Rock (1986), I assume that the issuing firm is uncertain about the value that the market will assign to it. Among potential investors some (the informed) have superior information about this value, and some (the uninformed) don't. Because informed investors will only submit purchase orders when the shares are

underpriced, uninformed investors face a potential adverse selection problem. In this paper, the contract choice decision focuses on the adverse selection problem facing uninformed investors.

In particular, I show that the adverse selection problem facing uninformed investors causes issuing firms to underprice (in an expected value sense) firm commitment offerings in a manner that is an increasing function of the ex ante uncertainty about share value. This underpricing imposes a cost on the shareholders of the issuing firm. With a best efforts contract, uninformed investors do not face an adverse selection problem because issues for which informed demand is not forthcoming are withdrawn. Consequently, uninformed investors do not have to be compensated for getting "stuck" with overpriced issues. Issuing firms are taking a risk, however, in that if the issue is withdrawn, no money is raised. It is worth taking this risk only if a firm commitment offering would have had to be severely underpriced. Consequently, the model predicts that issuing firms for which there is a low level of ex ante uncertainty about their value will use a firm commitment contract, while those for which there is a high level of ex ante uncertainty will use a best efforts contract.

A number of other papers have also modeled the contract choice decision. Mandelker and Raviv (1977) focus on differential risk aversion between the (shareholders of the) issuing firm and its investment banker. They view the contract choice decision as one of optimal risk sharing. Baron (1979, 1982) and Baron and Holmstrom (1980) base their analyses upon informational asymmetries between the issuing firm and investment banker.[2] Cho (1987) focuses on informational asymmetries between the issuing firm and potential investors, with the investment banker taking a largely passive role.

Both Parsons and Raviv (1985) and Rock (1986) present models of the underpricing of firm commitment offerings, but do not address the contract choice decision. The model of the underpricing of firm commitment offerings of this paper is based directly on Rock's model. Guenther (1987) models the underpricing of best efforts offerings, but does not address the contract choice decision.

The organization of the remainder of this paper is as follows. In Section II, the optimal offering prices for firm commitment and best efforts offerings are derived. Based upon these offering prices, Section III derives the expected post-offering value per share for the two contract types, and states this paper's proposition regarding the contract choice decision. Section IV summarizes the paper and discusses the empirical predictions.

II. The Pricing of Firm Commitment and Best Efforts Offerings

A. Assumptions

To analyze the contract choice decision for firms issuing equity securities, I assume that each firm is endowed with a project requiring a lump-sum investment which has a payoff per dollar invested of x, where x is uniformly distributed on [g, h]. With no loss of generality, I assume that the one-period discount rate is zero. Consequently, values of x in excess of 1 correspond to positive net present value investments. If g is restricted to be greater than or equal to 1, then negative NPV projects do not have to be worried about. This investment opportunity is the only asset of the firm, and it evaporates if it is not undertaken.

To finance this investment, the firm must issue n_i shares at an offering price of OP_i, subject to the constraint that the proceeds equal the required investment:

$$I = n_i OP_i \tag{1}$$

where the subscript refers to the contract chosen, firm commitment (f) or best efforts (b). The initial shareholders, who own n_o shares, are thus faced with the decision problem of choosing a contract type and an offering price (which, given equation (1), determines the number of shares to be sold) so as to maximize the expected utility of their (post-offering) wealth. The more shares that must be sold, the lower is the fraction of the firm retained by the initial shareholders.

For analytical simplicity, I assume that there are no costs of issuing securities other than the implicit cost of underpricing, and the opportunity cost of foregoing positive net present value investments if a best efforts offering is withdrawn. To further simplify the contract choice analysis, I assume that the initial shareholders are risk-neutral, so that the objective function reduces to maximizing the expected value of terminal wealth. Since n_o, the number of shares owned by the original shareholders, is given, maximizing expected terminal wealth corresponds to maximizing the expected value per share, which is

$$E(v_i) = \frac{IE(x \mid \text{success})}{n_o + n_i} \cdot [\text{prob. of success}_i] + 0 \cdot [1 - \text{prob. of success}_i] \tag{2}$$

where [prob. of success_i] is the probability that the project will be undertaken. For a firm commitment offering, $E(x \mid \text{success}) = E(x)$, since the probability that the offering will be withdrawn is zero. Through equation (2), the greater is the number of shares issued (n_i), the lower is the original shareholders' expected terminal wealth, for a given probability of a successful offering. Ceteris paribus, the original shareholders seek to minimize the dilution of their ownership.

All investors, including the original shareholders in the firm and the underwriter, are assumed to know the ex ante distribution of a firm's value. The uniform density function for the per share value of a firm is given by

$$f(v_i) = \frac{1}{b-a} = \frac{n_o + n_i}{I} f(x) = \frac{n_o + n_i}{I} \frac{1}{h - g} \text{ on } [a, b] \tag{3}$$

where

$$a = \frac{Ig}{n_o + n_i} \text{ and } b = \frac{Ih}{n_o + n_i} \tag{4}$$

and where the subscripts on v_i and n_i are due to their dependence on the contract chosen.

I assume that, for a cost c, an investor can improve his or her information from knowledge of $f(v_i)$ to knowledge of v_i. Investors who incur this cost are termed informed investors. Those who don't are termed uninformed investors. The initial shareholders and the investment banker are explicitly precluded from being informed.

While the dichotomy of informed and uninformed investors is clearly artificial, this is a simple way of modeling differential information. The requirement that the issuing firm and its investment banker are among the uninformed is merely a way of modeling that the sellers don't know the true value of the securities that they are selling. The objectionable assumption is <u>not</u> that the issuing firm and investment banker don't know the true firm value. The objectionable assumption is that some outside investors do. But this is just a simple method, albeit extreme, of creating a situation where there is differential information among outside investors, which results in some investors facing an adverse selection problem. Because all potential investors possess information at least as good as that of the issuing firm, the offering price does not signal information to potential investors. This results in a simpler model than if investors had to condition their demand upon information signaled by the offering price.

B. The Optimal Offering Price for a Firm Commitment Offering

Given the above assumptions, this section derives the optimal offering price for a firm commitment offering using the framework introduced by Rock (1982, 1986).

Informed investors, each of whom has investable wealth of $W - c$, will submit purchase orders only if the offering is underpriced ($v_f > OP_f$). This behavior by informed investors creates an adverse selection problem for uninformed investors. For underpriced issues ($v_f > OP_f$), both informed and uninformed investors will submit purchase orders, and uninformed investors will be allocated only some of the shares that trade at a premium in the aftermarket. For overpriced issues ($v_f < OP_f$), however, only uninformed investors submit purchase orders, so the uninformed are allocated 100 percent of all the issues that trade at a discount in the aftermarket. Consequently, if an uninformed investor is allocated shares in a new issue, there is a greater than usual chance that the issue will start trading at a discount in the aftermarket. This is the "winner's curse" problem. In other words, for an uninformed investor, the expected return conditional upon being allocated shares is less than the expected return conditional upon submitting a purchase order. But an uninformed investor will participate in the market only if the expected return conditional upon being allocated shares is non-negative. This can only happen if, on average, issuers underprice their shares.

The aggregate demand from uninformed investors obviously depends upon the offering price of an issue, for a given distribution of a firm's value. If certain conditions are met,[3] the aggregate demand curve of uninformed investors will be negatively sloped, so that the issuing firm is faced with a tradeoff between the amount of expected underpricing and the fraction of an offering that will be subscribed by uninformed investors. If the offering price is set at a sufficiently large discount from $E(v_f)$, uninformed investors will fully subscribe an offering by themselves. An issuing firm would never want to set an offering price lower than this, for in doing so it would be increasing the dilution suffered by the original shareholders with no offsetting gain in the fraction of offered shares demanded by uninformed investors.

Since the vast majority of firm commitment offerings are fully subscribed in practice, I will assume that the issuing firm's desired offering price is such that aggregate uninformed demand is sufficient to fully subscribe an issue, i.e., equal to $n_f \cdot OP_f$. Rock (1986, p. 198) refers to this offering price as the "full subscription" price.[4]

Given risk-neutrality on the part of investors and an endogenous number of informed investors, there are two equilibrium conditions that determine the optimal offering price. These two conditions are (i) zero expected profits for informed investors and (ii) zero expected profits for uninformed investors. The first condition is satisfied when the aggregate costs of becoming informed equal the expected gross profits of the informed:

$$N \cdot c = \frac{N(W - c)}{N(W - c) + OP_f \cdot n_f} \int_{OP_f}^{b} n_f(v_f - OP_f) f(v_f) dv_f \qquad (5)$$

where N is the number of informed investors, c is the cost per investor of becoming informed, (W - c) is the investment per informed investor (no borrowing or short-selling is allowed), OP_f is the offering price, n_f is the number of shares, b is the upper limit of integration for the uniformly distributed aftermarket price, and v_f is the aftermarket price. The left-hand side is the aggregate cost of becoming informed. The right-hand side is the proportion of each underpriced issue that will be allocated to informed investors, assuming that rationing is done on a pro rata basis, multiplied by the gross profits on underpriced issues. The product of these gives the gross profits earned by informed investors.

The second equilibrium condition, zero expected profits for the uninformed, occurs when the aggregate losses on overpriced issues (the uninformed get all of the losing issues) equal the uninformed's share of the gross profits on underpriced issues:

$$\int_a^{OP_f} n_f(OP_f - v_f)f(v_f)dv_f = \frac{OP_f \cdot n_f}{N(W - c) + OP_f \cdot n_f} \int_{OP_f}^{b} n_f(v_f - OP_f)f(v_f)dv_f \qquad (6)$$

Performing the integrations in equations (5) and (6), substituting equations (1), (3), and (4) in, and solving for OP_f results in a quadratic equation for the issuing firm's optimal offering price:

$$OP_f = \frac{I}{n_o}\,[E(x) - 1 + (C - \tfrac{1}{2})(h-g) + \sqrt{2CE(x)(h-g) - C(h-g)^2 + C^2(h-g)^2}] \qquad (7)$$

where $C \equiv c/(W - c)$.[5] C is the cost of becoming informed as a fraction of the investable wealth of the informed. Equation (7) applies when there is sufficient uncertainty so that N, the number of informed investors, is positive. If the parameter values are such that no investor chooses to become informed, a pooling equilibrium results in which $OP_f = E(v_f)$ since there is no adverse selection against the uninformed. Because $OP_f \geq 0$, equation (7) only applies for parameter values that result in a non-negative offering price.

To facilitate the interpretation of equation (7), it is convenient to multiply both sides by n_o. Then the left-hand side is $OP_f \cdot n_o$, which is the valuation on the shares owned by the original shareholders. On the right-hand side, E(x) - 1 can be interpreted as the expected net present value per dollar invested in the project, and E(x) - 1 multiplied by the investment I gives the expected net present value (NPV). Normally, all of this NPV would accrue to the original shareholders, so that $OP_f \cdot n_o$ would equal $I[E(x) - 1]$. However, to compensate uninformed investors for the adverse selection that they face, the original shareholders must share this NPV with new shareholders. This is accomplished by selling the new shares at a discount from their expected value. The remainder of the bracketed term to the right of $E(x) - 1$ is the discount. The size of the discount depends upon the cost of becoming informed, C, and the level of uncertainty, $(h - g)$, as well as $E(x)$, the expected payoff per dollar invested.

C. The Optimal Offering Price for a Best Efforts Offering

With a best efforts offering, an offering price is set and potential investors are then solicited by the investment banker. If a minimum number of the shares being offered are not subscribed within a set period of time, usually 90 days from the start of the selling period, the offering is withdrawn and potential investors receive their money back (which had been placed in an escrow account). In practice, the minimum number of shares is usually in the range of 50-75 percent of the maximum, although a "best efforts, all or none" offering is also common.

In common with the assumption for the analysis of a firm commitment contract, the required investment I is assumed to be a lump sum, so a best efforts, all or none, contract is appropriate to analyze. As before, investors can choose to become informed about the true value per share by incurring a cost c. Informed demand will be forthcoming whenever $OP_b < v_b$, where v_b is the true price per share (conditional upon the investment being undertaken). I assume that the issuing firm restricts uninformed demand so that uninformed demand by itself is insufficient to subscribe an issue. Consequently, issues will be subscribed only if there is informed demand augmenting uninformed demand. For modeling purposes, I assume that if an offering is withdrawn, the underlying project that was being financed evaporates.

Some discussion of the assumption that the issuing firm intentionally restricts demand for its shares is called for.[6] By setting a threshold level of demand that is sufficiently high so that the only offerings that are subscribed are those for which both uninformed and informed investors submit purchase orders, the issuing firm does not have to compensate uninformed investors for the adverse selection that they would otherwise face.

One can analyze a best efforts contract in terms of the contingent claims being issued. A best efforts contract can be viewed as a firm commitment contract in which the issuing firm gives uninformed investors the right to sell the shares back to the firm at the offering price. This put option will be exercised whenever $v_b < OP_b$, leaving the issuing firm with no proceeds with which to undertake the desired investment.

Obviously, uninformed investors would be willing to submit purchase orders at higher offering prices if they have the right to put the shares back to the issuer than if they don't have this option. In fact, if there are no costs of submitting a purchase order and exercising the put option, uninformed investors will submit purchase orders no matter what the offering price is, as long as there is some positive probability that $v_b > OP_b$. This shift in the demand curve of uninformed investors with a best efforts offering relative to a firm commitment offering is what makes a best efforts offering potentially attractive to an issuer. Because of this altered behavior by uninformed investors, the issuing firm can set a higher OP_b than would be possible using a firm commitment contract. The issuer is risking a withdrawn offering, however.

In this scenario, since only underpriced offerings are subscribed, uninformed investors do not face any adverse selection risk, and consequently the expected return for an uninformed investor submitting a purchase order is positive, unlike with a firm commitment offering. In equilibrium, investors seeking these profits will compete away the surplus by incurring costs, such as generating large commissions, in order to be offered shares. This process will occur until the expected profits of uninformed investors, net of all costs, fall to zero.

Note that it is the issuing firm that finds it optimal to set a threshold level of demand that is sufficiently high relative to uninformed demand so that uninformed demand by itself will be inadequate to prevent an offering from being withdrawn. Because of this, the issuers do not have to compensate uninformed investors for the adverse selection that they would otherwise face.

The actions of the issuing firm and its investment banker in a best efforts offering can be interpreted in the following manner. The investment banker agrees to contact a fixed number of potential investors, offering them the opportunity to purchase shares. This number is large enough so that if they all subscribe, they will buy the maximum number of shares offered for sale. If they do not all subscribe, the offering will be withdrawn. (In practice, investment bankers have a limited amount of time in which to either consummate the offering or withdraw it. If the offering is not closed within 90 days, a refiling with the S.E.C. is required. If it is assumed that a fixed number of potential investors are contacted per day, then the time limit corresponds to a limit on the number of potential investors who are apprised of the opportunity. Also, in practice, investors who have submitted purchase orders start to cancel their purchase orders if the offering is not closed within a certain period of time, because they realize a lengthy selling period indicates weak demand.)

By submitting purchase orders only when the offer is underpriced, informed investors are creating an externality. Their behavior, in conjunction with only a limited number of investors being offered the opportunity to buy shares in the offering, prevents overpriced offerings from being consummated. Uninformed investors benefit from this behavior. (In a firm commitment offering, the behavior of informed investors also creates an externality for uninformed investors, but a negative externality). Since informed investors are generating a positive externality with best efforts offerings, issuing firms and their investment bankers will want to create incentives for potential investors to become informed. One simple mechanism creating this incentive is the opportunity to apprise more offerings than would an uninformed investor. The exact number of investors who choose to become informed is not important, as long as enough do so that when their demand is added to the demand of uninformed investors, the aggregate demand is sufficient to fully subscribe underpriced offerings. In equilibrium, the expected profits of informed investors, net of all costs, will be equal to those of uninformed investors.

Since the offering is withdrawn if demand from informed investors is not forthcoming, the issuing firm is faced with a situation where there is a tradeoff between the offering price and the probability that the offering is subscribed. The higher is the offering price, the less is the dilution if the offering is not withdrawn, but the higher is the probability that the offering will be withdrawn.

The decision problem facing the firm in this scenario is to choose an offering price to maximize the expected value per share, where the firm will be worthless if the offering fails. This optimal offering price involves a tradeoff between dilution of the original shareholders' interests and the probability of subscription. The issuing firm's decision problem is

$$\max_{OP_b} \int_{OP_b}^{b} v_b f(v_b) dv_b \tag{8}$$

subject to equations (1), (3), and $a \leq OP_b \leq b$, or, equivalently, $\frac{I}{n_o}(g - 1) \leq OP_b \leq \frac{I}{n_o}(h - 1)$.[7] The non-negativity constraint of $OP_b \geq 0$ is also present.

After performing the integration in equation (8), the LaGrangean for the maximization problem is:

$$L = \frac{h^2 I^2 OP_b - OP_b(n_o^2\, OP_b^2 + 2In_o\, OP_b + I^2)}{2I(h - g)(OP_b n_o + I)}$$

$$+ \lambda_1 \left[OP_b - \frac{I}{n_o}(g - 1)\right] + \lambda_2 \left[-OP_b + \frac{I}{n_o}(h - 1)\right] \tag{9}$$

The Kuhn-Tucker conditions for an optimum are

$$\frac{\partial L}{\partial OP_b} = \left\{\frac{(OP_b n_o + I)[I^2h^2 - 3OP_b^2 n_o^2 - 4In_o OP_b - I^2] - n_o OP_b[I^2h^2 - (n_o OP_b + I)^2]}{2I(h - g)\ (OP_b n_o + I)^2}\right.$$

$$\left. + (\lambda_1 - \lambda_2)\right\} \leq 0 \qquad OP_b \geq 0 \quad \text{and } OP_b \frac{\partial L}{\partial OP_b} = 0 \tag{10a}$$

$$\frac{\partial L}{\partial \lambda_1} = OP_b - \frac{I}{n_o}(g - 1) \geq 0 \qquad \lambda_1 \geq 0 \quad \text{and} \quad \lambda_1 [OP_b - \frac{I}{n_o}(g - 1)] = 0 \tag{10b}$$

$$\frac{\partial L}{\partial \lambda_2} = -OP_b + \frac{I}{n_o}(h - 1) \geq 0 \qquad \lambda_2 \geq 0 \quad \text{and} \quad \lambda_2 [OP_b - \frac{I}{n_o}(h - 1)] = 0 \tag{10c}$$

Since in the neighborhood of b, $E(v_b)$ (continuously) approaches zero as OP_b increases towards b from below, the constraint that $OP_b \leq b$ will never be binding, so λ_2 will always equal zero. Two cases are then left. When $\lambda_1 > 0$, which occurs when $OP_b = a$, the optimum offering price is given by

$$OP_b = \frac{I}{n_o}(g - 1) \tag{11}$$

When $\lambda_1 = 0$, the optimum offering price is the solution to (10a) with $\lambda_1 = \lambda_2 = 0$. Since it is the numerator of the first term of equation (10a) that equals zero, letting $z \equiv n_o OP_b$, a cubic equation results, the unique real root of which gives the optimal offering price for a best efforts offering:

$$0 = z^3 + \frac{5}{2} Iz^2 + 2I^2 z + I^3 \left(\frac{1 - h^2}{2}\right) \tag{12}$$

Using Cardan's formula (see Fine [1961, pp. 483-4]), the unique real root is found to be

$$z = [-\frac{5}{6} + A^{1/3} + B^{1/3}]I \tag{13}$$

where

$$A \equiv \frac{1}{216} + \frac{h^2}{4} + \frac{h}{4} \sqrt{h^2 + \frac{1}{27}}$$

$$B \equiv \frac{1}{216} + \frac{h^2}{4} - \frac{h}{4} \sqrt{h^2 + \frac{1}{27}} .$$

Substituting for A and B in equation (13) and solving for OP_b results in

$$OP_b = \frac{I}{n_o}[-\frac{5}{6} + (\frac{1}{216} + \frac{h^2}{4} + \frac{h}{4} \sqrt{h^2 + \frac{1}{27}})^{1/3} + (\frac{1}{216} + \frac{h^2}{4} - \frac{h}{4} \sqrt{h^2 + \frac{1}{27}})^{1/3}] \tag{14}$$

subject to $a \leq OP_b \leq b$. Upon making the further substitution,

$$h = \frac{h + g}{2} + \frac{h - g}{2} = E(x) + 1/2\ (h - g) \tag{15}$$

OP_b can then be expressed in terms of its fundamental determinants.

The optimal offering price for a best efforts offering is given by equation (14), except when the parameter values are such that the resulting value of OP_b would be less than a. In this case, equation (11) applies.

III. Expected Post-offering Wealth for the Contract Types

In the previous section I derived expressions for the optimal offering prices for firm commitment and best effort offerings. In this section, I derive the expressions for the expected post-offering wealth of the original shareholders. I then derive the major theoretical result of this paper--that the expected post-offering wealth is greater for firm commitment than best efforts contracts for low levels of ex ante uncertainty, but greater for best efforts than firm commitment contracts for high levels of ex ante uncertainty.

The optimal offering price for a firm commitment offering is given by equation (7) of Section II, providing that (h - g) is sufficiently large to induce a non-zero number of investors to become informed, and sufficiently small so that $OP_f \geq 0$. If the number of investors who choose to become informed is zero, then there is no need for underpricing, and

$$E(v_f) = \frac{I}{n_o} [E(x)-1] \tag{16a}$$

If there is a positive number of informed investors, then equation (7) allows one to solve for the expected post-offering price per share, $E(v_f)$. Since the required investment, I, is equal to the gross proceeds raised, $n_f \cdot OP_f$, one finds that for a firm commitment offering,

$$E(v_f) = \frac{IE(x)[E(x)-1 + (C-1/2)(h-g) + \sqrt{2E(x)C(h-g) - C(h-g)^2 + C^2(h-g)^2}]}{n_o[E(x) + (C-1/2)(h-g) + \sqrt{2E(x)C(h-g) - C(h-g)^2 + C^2(h-g)^2}]} \tag{16b}$$

Multiplying this expression by n_o gives the expected post-offering wealth of the original shareholders. If equation (7) implies a negative offering price, the issuing firm can always set it at zero. In this case,

$$E(v_f) = 0 \tag{16c}$$

Consequently, the expected wealth per share for a firm commitment offering is given by:

(i) equation (16a) when N = 0, where N is the (endogenously determined) number of informed investors

(ii) equation (16b) when N > 0 and equation (7) implies $OP_f > 0$

(iii) equation (16c) when N > 0 and equation (7) implies $OP_f \leq 0$

It is also possible to compute the expected post-offering wealth of the original shareholders if a best efforts contract is used. Since

$$E(v_b) = \int_{OP_b}^{b} v_b f(v_b) dv_b \tag{17}$$

one can perform the integration to find that, for the case of $OP_b > a$,

$$E(v_b) = \frac{I\{\frac{1}{24} - \frac{4}{3} h^2 + (h^2 + \frac{1}{6})[A^{1/3} + B^{1/3}] + \frac{1}{2}[A^{2/3} + B^{2/3}]\}}{2n_o(h - g)[\frac{1}{6} + A^{1/3} + B^{1/3}]} \tag{18a}$$

where A and B are given after equation (13) and $h = E(x) + \frac{h - g}{2}$. Multiplying this equation by n_o gives the expected post-offering wealth of the original shareholders. Note that it is being assumed that if the offering fails ($v_b < OP_b$), the firm becomes worthless. Thus $E(v_b)$ is a weighted average of the expected value, conditional upon having a successful offering, and zero, the value if the offering fails.

If $OP_b = a$, then

$$E(v_b) = \frac{IE(x)}{n_o} \frac{(g - 1)}{g} . \tag{18b}$$

For low values of (h - g), $E(v_b)$ is given by equation (18b), providing that $g > 1$. As (h - g) increases, equation (18a) becomes the expression for $E(v_b)$.

Insert Figure 1 Here

In Figure 1, the relation between $E(v_i)$ and ex ante uncertainty is graphed for both firm commitment and best effort contracts. The fundamental determinants of $E(v_f)$ are E(x), (h - g), C, I, and n_o. The fundamental determinants of $E(v_b)$ are E(x), (h - g), I and n_o. The cost of becoming informed enters the expression for $E(v_f)$ but not $E(v_b)$ because only in the analysis of the firm commitment contract do the total costs of becoming informed result in adverse selection losses for uninformed investors. In the best efforts contract, the issuing firm's precommitment to withdraw an undersubscribed offering ameliorates this problem. Consequently, the effect of a change in the ex ante uncertainty on $E(v_i)$ differs for the two contract types. In particular, for a firm issuing securities it is optimal to use a firm commitment contract when there is low ex ante uncertainty. As the ex ante uncertainty increases, at some point it becomes optimal to switch to a best efforts contract. This is the major theoretical result of this paper, and it is formalized in the following proposition.

<u>Proposition 1</u>: If maximizing expected ex post wealth of the firm's initial shareholders is the objective function for a firm issuing equity securities, then (i) firms for which there is a low level of ex ante uncertainty about their value will use firm commitment offerings, and (ii) firms for which there is a high level of ex ante uncertainty will use best efforts offerings.

The proof of Proposition 1 is contained in the Appendix.

The intuition behind Proposition 1 is straightforward. With firm commitment offerings, as ex ante uncertainty increases, more and more money must be left on the table to compensate uninformed investors for the adverse selection problem that they face, increasing the dilution of the original owners' interest. At some point, the issuing firm is better off switching to a best efforts offering where the adverse selection problem is avoided. The disadvantage of a best efforts offering, however, is that some positive net present value (NPV) projects will be foregone as the issuing firm trades off the probability of subscription versus the dilution facing the original owners if the offering is not withdrawn. It is optimal to forego some positive NPV projects by using a best efforts contract only if the dilution from using a firm commitment contract is sufficiently great.[8]

IV. Summary and Conclusions

This paper develops a model in which the contract choice decision is motivated by informational asymmetries among potential investors. With a firm commitment contract, uninformed investors are subject to an adverse selection problem due to the behavior of informed investors. As the degree of ex ante uncertainty about a firm's value increases, the adverse selection problem intensifies. If uninformed investors are to remain in the market, the degree of underpricing required to compensate them for this adverse selection problem grows as ex ante uncertainty increases. This underpricing can become so severe that an issuer may choose to instead use a best efforts contract, in which it is risking that the issue will fail. The advantage of a best efforts contract is that less underpricing is required since uninformed investors do not face being allocated a disproportionate number of shares in overpriced offerings. This is because with best efforts offering, the issuing firm precommits to withdraw undersubscribed offerings.

In a best efforts contract, the issuing firm raises I dollars with a certain probability, equal to the probability that $v_b > OP_b$. If the issue is withdrawn, the issuing firm loses the net present value of the foregone investment. By using a firm commitment contract, the issuer is essentially purchasing an insurance contract--it is guaranteed sufficient demand to consummate the offering, but it must pay an "insurance premium" in the form of an offering price that compensates uninformed investors for adverse selection risk. This insurance premium is an increasing function of ex ante uncertainty. Those issuers who are faced with a small premium find it optimal to pay for insurance, and use a firm commitment contract. Those issuers who are faced with a large premium find it optimal to be uninsured, and use a best efforts contact.

The model produces the empirical prediction that issuers for which there is a high degree of ex ante uncertainty about their value will use best efforts contracts. This prediction about the contract choice decision is consistent with the relative frequency with which best efforts contracts are used among seasoned and unseasoned equity issues. For a new issue of seasoned equity, there is relatively little uncertainty about the aftermarket price because the pre-offering market price is readily observable. For an unseasoned new issue, however, this information is not available. Consequently, I would predict that a much larger percentage of unseasoned offerings use a best efforts contract than do seasoned equity offerings.

Booth and Smith (1986, Table 1) report that, for 1977-82, only 2.6 percent of seasoned equity offerings used a best efforts contract. This is in contrast to their finding that 54.3 percent of initial public offerings used a best efforts contract for this same time period. Furthermore, in Ritter (1987), I present empirical evidence for initial public offerings of common stock that is consistent with this paper's prediction that those issues for which there is a high level of ex ante uncertainty are more likely to use a best efforts contract. I find that firms whose stock prices are more volatile once they start trading were disproportionately likely to have used a best efforts contract in going public.

FOOTNOTES

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1. While the analysis of this paper applies to issuing securities other than common stock, the information issues appear to be more important in equity offerings. Consequently, the terminology will refer to equity issues.

2. Muscarella and Vetsuypens (1987) test Baron's model by examining the initial returns on 37 investment banking firms going public during 1970-July 1987. They find that these offerings are underpriced by the same order of magnitude as other offerings, which they interpret as being inconsistent with Baron's model.

3. As Rock (1982, 1986) discusses, it is possible to show that the aggregate demand curve is positively sloped due to the winner's curse problem if the probability density function f(v) is of a certain form. The assumption of a uniform distribution for x here ensures that the demand curve is negatively sloped.

4. The desired offering price and the extent of subscription by uninformed investors would depend in general on the utility function of the firm's original shareholders. Since there is no advantage to the issuing firm from setting an offering price less than the full subscription price, the issuer will never set a lower price. If the issuer is sufficiently risk-adverse, it can be shown that the full subscription price is optimal. Since, for analytical simplicity, I have assumed that the original shareholders are risk-neutral, there is an inconsistency in my modeling. For the more general optimization problem, see Rock (1982, 1986). In Rock (1982, Chapter 2), the number of informed investors is determined endogenously, as is done here. In Rock (1986), the number of informed investors is exogenously specified.

5. The offering price given in equation (7) is unique, providing that C is sufficiently small so that at least some investors choose to become informed. (If C is so large or if (h - g) is so small that no investor chooses to become informed, then a pooling equilibrium results involving, on average, no underpricing.) The other root of the quadratic equation for which equation (7) is a root has the property that it is below the lowest possible value of v_f, resulting in all offerings being underpriced. If this were the case, there would be no adverse selection against the uninformed. Consequently, this other root is not economically meaningful.

6. French and McCormick (1987) discuss other circumstances in which the winner's curse problem results in a seller finding it optimal to restrict demand for an item being auctioned.

7. Technically, $f(v_b) = 0$ for $v_b < a$ and $v_b > b$. Economically, the maximization occurs on [a, b] because for an offering price less than a, the probability of subscription is unity, so that there is no tradeoff between dilution and the probability of subscription. For an offering price greater than b, the probability of subscription is zero, so the value of the objective function (8) is zero. Consequently, the value of OP_b that maximizes the expected value per share must be in the set [a, b].

8. Introducing risk aversion on the part of the issuing firm's securityholders has the effect of reducing the desirability of a best efforts contract. Thus, with risk aversion on the part of the issuer, the "switching point" in Figure 1 would be to the right of X_2, but the qualitative conclusions would remain unaffected. If an alternatiave source of funds is available to issuing firms in the event that a best efforts offering is withdrawn, this "safety net" would have the effect of making a best efforts offering more attractive, moving the switching point in Figure 1 to the left of X_2.

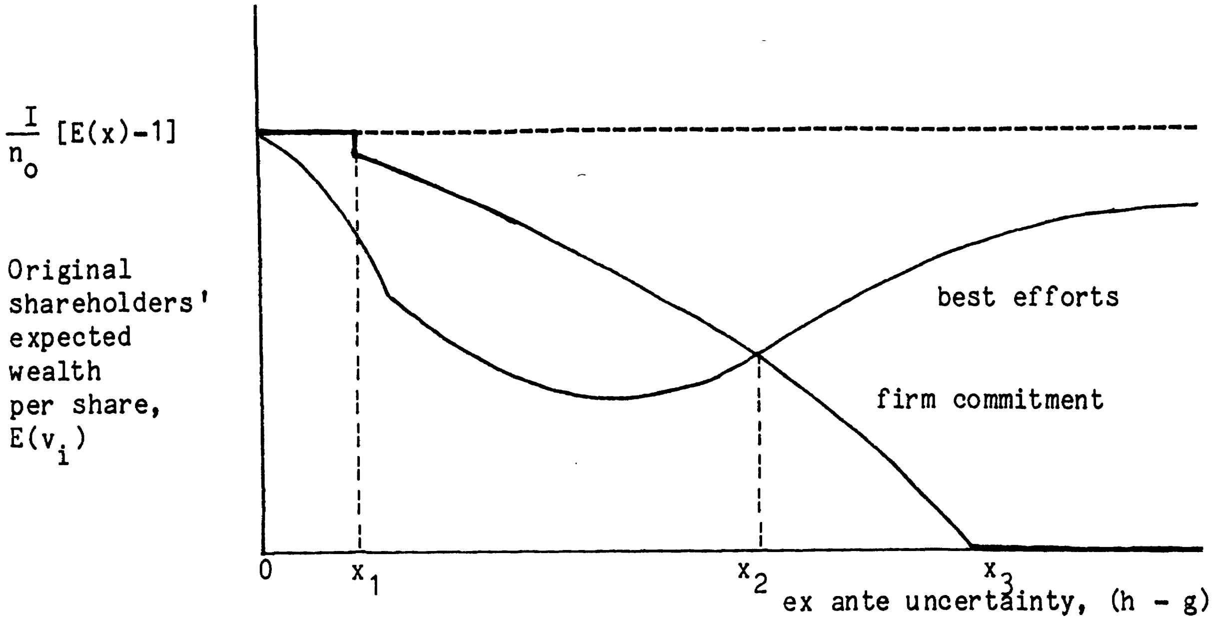

Figure 1—The relation between the original shareholders' expected wealth per share, $E(v_i)$, and ex ante uncertainty, $(h - g)$, for firm commitment and best efforts offerings. From 0 to x_1, for firm commitment offerings, $(h - g)$ is so small that no investors choose to become informed, and thus there is no need for underpricing, so $E(v_f)$ is given by equation (16a). From x_1 to x_3, the function graphed is that of equation (16b) in the text. For $(h-g) > x_2$, equation (16c) is graphed. For the best efforts contract, the functions graphed are equations (18a) and (18b) in the text. For low levels of $(h-g)$, providing that $g > 1$, equation (18b) is plotted. As $(h-g)$ increases, equation (18a) takes over.

Appendix

Proposition 1: If maximizing expected ex post wealth of the firm's original owners is the objective function for a firm going public, then (i) firms for which there is a low level of ex ante uncertainty about their value will use firm commitment offerings, and (ii) firms for which there is a high level of ex ante uncertainty will use best efforts offerings.

Proof: To demonstrate that the functions $E(v_f)$ and $E(v_b)$ are as graphed in Figure 1, with $E(v_b) < E(v_f)$ for $0 < (h-g) < x_2$, and $E(v_b) > E(v_f)$ for $(h-g) > x_2$, it is sufficient to show that:

(i) at $(h-g) = 0$, $E(v_f) = E(v_b)$;

(ii) for very low values of $(h-g)$, $E(v_f)$ is constant, while $\frac{\partial E(v_b)}{\partial (h-g)} < 0$;

(iii) for low values of $(h-g)$, for which $E(v_b)$ is given by equation (18b), the value of $E(v_f)$ given by equation (16b) is greater than this value of $E(v_b)$; and

(iv) equation (16b) is decreasing and concave, while equation (18a), the expression for $E(v_b)$ when $(h-g)$ is high, is either increasing, or decreasing and convex.

Part (i) is straightforward. If there is no ex ante uncertainty, there is no need to sell shares at a discount using either type of contract. At $(h-g) = 0$, both equations (16a) and (18b) give $E(v_f) = E(v_b) = \frac{I}{n_o}[E(x)-1]$,

since $g = E(x)$ when $(h-g) = 0$.

Part (ii) follows from the fact that, as long as the number of informed investors is zero, there is no adverse selection against the uninformed, so

$E(v_f)$ is constant. Differentiation of equation (18b) demonstrates that, after making the substitution $g = E(x) - \frac{h-g}{2}$,

$$\frac{\partial E(v_b)}{\partial(h-g)} = \frac{-IE(x)}{2n_o[E(x) - \frac{(h-g)}{2}]^2} < 0$$

Part (iii) can be demonstrated by noting that, for equation (18b), the probability of subscription is 100 percent, as it is for a firm commitment offering. Consequently, $E(v_f) > E(v_b)$ whenever $OP_f > OP_b$. Equations (7) and (11) are the relevant expressions for OP_f and OP_b. Equation (11) can be expressed as

$$OP_b = \frac{I}{n_o}[E(x) - \frac{h-g}{2} - 1].$$

Canceling the common $\frac{I}{n_o}$ terms in equations (7) and (11), and then subtracting $E(x)-1$ from each, it is easy to show that $OP_f > OP_b$ whenever

$$C(h-g) - \frac{h-g}{2} + \sqrt{2CE(x)(h-g)-C(h-g)^2 + C^2(h-g)^2} > -\frac{h-g}{2}$$

which is derived from equations (7) and (11) by writing $(C - \frac{1}{2})(h-g)$ as $C(h-g) - \frac{h-g}{2}$. Since $C(h-g)$ and the square root term are both positive, the above inequality is always satisfied. Thus, $E(v_f) > E(v_b)$ whenever equation (18b) is the relevant expression for $E(v_b)$.

Part (iv) is much more tedious to demonstrate. First, I will show that expression (16b) for $E(v_f)$ is monotone decreasing and concave. Equation (16b) can be written as

$$E(v_f) = \frac{IE(x)}{n_o}[1 - M^{-1}]$$

where

$$M \equiv E(x) + (C - \frac{1}{2})(h-g) + \sqrt{2CE(x)(h-g)-C(h-g)^2+C^2(h-g)^2}$$

which is positive for the parameter values for which equation (16b) is relevant. Differentiating $E(v_f)$ with respect to (h-g) results in

$$\frac{\partial E(v_f)}{\partial(h-g)} = \frac{IE(x)}{n_o}\frac{M'}{M^2}$$

$$= \frac{IE(x)}{n_o} \frac{[C - \frac{1}{2} + \frac{CE(x) - C(h-g) + C^2(h-g)}{\sqrt{2CE(x)(h-g) - C(h-g)^2 + C^2(h-g)^2}}]}{[E(x) + (C - \frac{1}{2})(h-g) + \sqrt{2CE(x)(h-g) - C(h-g)^2 + C^2(h-g)^2}]^2}$$

The proof that this expression is negative follows immediately from the analogous proof in Beatty and Ritter [1986, p. 231-2] if one substitutes E(x) for E(v) and (h-g) for (b-a).

The second derivative of $E(v_f)$ with respect to (h-g) is

$$\frac{\partial^2 E(v_f)}{\partial(h-g)^2} = \frac{IE(x)}{n_o M^3}[M\,M'' - 2\,[M']^2]$$

where

$$M'' = \frac{-C^2[E(x)]^2}{\sqrt{[2CE(x)(h-g) - C(h-g)^2 + C^2(h-g)^2]^{3/2}}}$$

which is negative whenever the denominator is positive. Since

$$2CE(x)(h-g) - C(h-g)^2 + C^2(h-g)^2$$

can be rewritten as

$$C(h-g)\,[2g + C(h-g)]$$

which is always positive for non-negative values of g (which, by assumption, the analysis is restricted to), M" is negative, and thus

$$\frac{\partial^2 E(v_f)}{\partial (h-g)^2} < 0.$$

This completes the proof of the concavity of the $E(v_f)$ schedule given by equation (16b).

The second part of the proof involves demonstrating that $E(v_b)$, as given by equation (18a), is either increasing, or decreasing and convex. Equation (18a) can be expressed as

$$E(v_b) = \frac{IF[h^2 - (1+F)^2]}{2n_o\ (h-g)\ (1+F)} = \frac{I[h^2F - F^3 - 2F^2 - F]}{2n_o(h-g)\ (1+F)} \tag{A1}$$

where $F \equiv -\frac{5}{6} + A^{1/3} + B^{1/3}$, since, as direct calculation demonstrates, $A^{1/3}B^{1/3} = \frac{1}{36}$ and $A + B = \frac{1}{108} + \frac{h^2}{2}$. Note that F is a function of (h-g), although for simplicity of notation, this functional relation has been suppressed.

Differentiating equation (A1) results in

$$\frac{\partial E(v_b)}{\partial (h-g)} = \frac{I\{F(F+1)[h(h-g)-h^2+(F+1)^2] - (h-g)\ \frac{\partial F}{\partial (h-g)}\ [(2F+1)(F+1)^2-h^2]\}}{2n_o(h-g)^2(1+F)^2}$$

Since the denominator is always positive, the sign of the numerator determines whether $E(v_b)$ is increasing or decreasing. Since $(2F+1)(F+1)^2 = h^2$, the second term of the numerator is zero. Consequently,

$$\frac{\partial E(v_b)}{\partial (h-g)} = \frac{I}{2n_o(h-g)^2\ (1+F)}\ \{F[h(h-g)-h^2+(F+1)^2]\} \tag{A2}$$

When this is positive, $E(v_b)$ is rising, resulting in a unique intersection of $E(v_b)$ and the monotonicly decreasing $E(v_f)$. Thus, the convexity of $E(v_b)$ must be demonstrated only for parameter values such that the numerator of equation (A2) is negative.

Taking the second derivative of $E(v_b)$ with respect to (h-g) yields

$$\frac{\partial^2 E(v_b)}{\partial(h-g)^2} = \frac{-2}{(h-g)} \frac{\partial E(v_b)}{\partial(h-g)} + \frac{I}{2n_o(h-g)\ (F+1)^2} \left\{\frac{1}{2} F(F+1) + \frac{F}{(h-g)} h\right\} \qquad \text{(A3)}$$

Since this derivative is of interest only when $\frac{\partial E(v_b)}{\partial(h-g)}$ is negative, the first term on the right hand side will be assumed to be positive. The denominator of the second term is also positive. Consequently, a sufficient condition for $E(v_b)$ to be convex when it is decreasing is that

$$\frac{1}{2} F(F+1) + \frac{\partial F}{\partial(h-g)} h > 0.$$

From equation (14), $F > 0$, so if $\frac{\partial F}{\partial(h-g)} > 0$, equation (A3) is positive.

Straightforward differentiation of $F \equiv -\frac{5}{6} + A^{1/3} + B^{1/3}$ yields

$$\frac{\partial F}{\partial(h-g)} = \frac{1}{3}(e+f)^{-2/3}(c+d) + \frac{1}{3}(e-f)^{-2/3}(c-d) \qquad \text{(A4)}$$

where

$$c \equiv \frac{h}{4}$$

$$d \equiv \frac{1}{8}\left(h^2 + \frac{1}{27}\right)^{1/2} + \frac{h^2}{8}\left(h^2 + \frac{1}{27}\right)^{-1/2}$$

$$e \equiv \frac{1}{216} + \frac{h^2}{4}$$

$$f \equiv \frac{h}{4}\left(h^2 + \frac{1}{27}\right)^{1/2}$$

Equation (A4) is positive if, since $c-d < 0$ whenever $h > 1$ (as is the case whenever any project has a positive net present value),

$$(e+f)^{-2/3} (c+d) > (e-f)^{-2/3} (d-c) \tag{A5}$$

We want to demonstrate that equation (A5) holds.

Since both sides of this inequality are positive, cubing both sides and rearranging yields

$$\frac{(c+d)^3}{(e+f)^2} > \frac{(d-c)^3}{(e-f)^2}$$

which can be rewritten as

$$(e^2+f^2)(c^3+3cd^2) - 2ef(d^3+3c^2d) > 0$$

Substituting for c, d, e, and f, and multiplying by 432 results in

$$[\frac{1}{108} + 2h^2 + 54h^4] [\frac{13}{256} h^3 + \frac{h}{2304} + \frac{3}{256} h^5 (h^2+\frac{1}{27})^{-1}]$$

$$- \frac{[h+54h^3]}{512} [(h^2 + \frac{1}{27})^2 + 15h^2 (h^2 + \frac{1}{27}) + 15h^4 + h^6 (h^2 + \frac{1}{27})^{-1}] > 0$$

Dividing by h, multiplying by 256, and then multiplying by $h^2 + \frac{1}{27}$ results in $\frac{1}{78732} > 0$, so that the inequality is satisfied. This complete the proof that $E(v_b)$ is convex when it is decreasing.

REFERENCES

Baron, David, "The Incentive Problem and the Design of Investment Banking Contracts," Journal of Banking and Finance 3 (1979), 157-75.

Baron, David, "A Model of the Demand for Investment Banking Advisement and Distribution Services for New Issues," Journal of Finance 37 (September 1982), 955-76.

Baron, David, and Bengt Holmstrom, "The Investment Banking Contract for New Issues Under Asymmetric Information: Delegation and the Incentive Problem," Journal of Finance 36 (December 1980), 1115-38.

Beatty, Randolph, and Jay Ritter, "Investment Banking, Reputation, and the Underpricing of Initial Public Offerings," Journal of Financial Economics 15 (January/February 1986), 213-32.

Booth, James R. and Richard L. Smith, "Capital Raising, Underwriting and the Certification Hypothesis," Journal of Financial Economics 15 (January/February 1986), 261-81.

Cho, Sung-Il, "An Alternative Model to Resolve the Underpricing Puzzle of Initial Public Offerings," unpublished University of Michigan working paper, 1987.

Fine, Henry Burchard, 1961, College Algebra. New York: Dover Publications.

French, Kenneth R., and Robert E. McCormick, "Sealed Bids, Sunk Costs, and the Process of Competition," Journal of Business 57 (October 1984), 417-42.

Guenther, Ann E., "The Pricing of Best Efforts New Issues," unpublished University of Minnesota working paper, 1987.

Mandelker, Gershon, and Artur Raviv, "Investment Banking: An Economic Analysis of Optimal Underwriting Contracts," Journal of Finance 32, (June 1977), 683-94.

Muscarella, Chris J., and Michael R. Vetsuypens, "A Simple Test of Baron's Model of IPO Underpricing," unpublished Southern Methodist University working paper, 1987.

Parsons, John E., and Artur Raviv, "Underpricing of Seasoned Issues," Journal of Financial Economics 14 (September 1985), 377-97.

Ritter, Jay R., "The Costs of Going Public," forthcoming in the Journal of Financial Economics, 1987.

Rock, Kevin F., "Why New Issues Are Underpriced," unpublished University of Chicago Ph.D. dissertation, 1982.

Rock, Kevin F., "Why New Issues Are Underpriced," Journal of Financial Economics 15 (January/February 1986), 187-212.