

AN ANALYSIS OF FIDELITY PROGRAMS : DOES PARTICIPATION MATTER ?

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The author thanks David Bell, Sushil Bikhchandani, Randolph Bucklin, Donald Morrison, John Riley and Atanu Sinha for their very helpful suggestions.

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Abstract

Mature markets are often characterized by stagnant sales, excess capacity and cut-throat price competition. A panacea to soften price competition and increase profits suggested by researchers, retail industry consultants and managers is the introduction of appropriate "fidelity programs". Fidelity programs are defined as those in which a retailer offers a rebate to its own customers, who are members of its fidelity program, on their future purchases. Other common trade names for fidelity programs are "loyalty reward programs", "rebate programs", "frequency programs", "repeat-buyer programs" and "affinity programs".

Fidelity programs have already become a very popular marketing strategy among retailers and their number is fast growing. Recent evidence from the U.S. market shows that across a variety of industries some companies such as Ford, AT&T and Charles Schwab have discontinued their fidelity programs. This raises the natural question why are fidelity programs successful as marketing strategies in some markets and not profitable in others ? This paper offers program participation as a feasible explanation for the mixed degree of success.

Existing literature offers conflicting advice to academics and practitioners about the profit increasing potential of fidelity programs. From the point of view of competitive advantage, Kotler (1999) states that the benefit of first introduction may disappear when competitors also start offering fidelity programs and may finally result in an added financial burden to all the offering companies. A very different position is taken by Brandenburger (1995) and Brandenburger and Nalebuff (1996) who suggest that fidelity programs could be a way out of the classic Prisoner's Dilemma for price competing retailers. They contend that in a mature market with two price competing retailers (i) if one retailer has a fidelity program, then both retailers can make higher profits in equilibrium, and (ii) if both retailers offer fidelity programs, then in equilibrium both retailers can make even higher profits. Thus, there is no free rider problem and imitation being healthy, fidelity programs are a "win-win" marketing strategy.

This contrasts with myopic market share increasing strategies such as temporary price reductions which could be “lose-lose” strategies when imitated.

This paper attempts to partially resolve the conflict in the existing literature by explaining when and why can fidelity programs be profit increasing marketing strategies. An important difference between the analytical model used in this paper and that of Brandenburger (1995) is the explicit recognition of the fact that enrollment or participation in fidelity programs is not one hundred percent in real world implementation even when all competing retailers offer these programs. Consumers who are not members of any fidelity program are called *infidels*. The impact of infidelity on profitability of fidelity programs in a mature market where retailers compete on price is examined. It is shown that the ability of fidelity programs in mitigating price competition and increasing profits is critically dependent on the proportion of infidels. Analytical and intuitively appealing new conditions for profitability of fidelity programs are derived both for markets where all retailers have such programs as well as for markets with only some retailers having fidelity programs.

When all retailers have fidelity programs, it is shown that the proportion of infidels must be small for profits to increase. The intuition is that the presence of infidels keeps price undercutting profitable as in a market without fidelity programs. Merely increasing the size of the loyalty rebate awarded to fidels is not a solution when the proportion of infidels is not small enough.

When only some retailers offer fidelity programs, it is shown that the proportion of infidels must be small for profits to increase. The intuition is that if the proportion of infidels is large, price undercutting may still remain profitable for retailers who offer fidelity programs. If the proportion of infidels is intermediate, price undercutting remains profitable for retailers who do not offer fidelity programs. Again there are no simple solutions like increasing the size of the rebate given to fidels. Fidelity programs could be a “win-win” marketing strategy since imitation is healthy, there is no free rider problem and any retailer may safely start a fidelity program when the proportion of infidels is small. In the case of intermediate values of the proportion of infidels, there is a potential free rider problem. When the proportion of infidels is high, no retailer should offer a fidelity program.

The key contribution of this paper is in pointing out the fact that infidelity or incomplete enrollment in fidelity programs may affect their profit increasing potential in different ways depending on whether some or all retailers in the market have such programs. Thus, we hope to sound a timely word of caution to those retailers who may be mistakenly engaged in adopting the currently popular “win-win” marketing strategy of fidelity programs. Anecdotal evidence from the airline, automobile and supermarket industries is consistent with the important predictions of the proposed theory.

(Fidelity Programs, Pricing Strategy, Game Theory)

1. Introduction

A 1998 national study sponsored by the Retail Advertising & Marketing Association reported that nearly half (46%) of the U.S. population is a member of at least one fidelity program and these programs include a wide variety of retail activities, from airlines and restaurants, to supermarkets, bookstores, department and specialty stores. Membership in these programs is reported to be growing at the rate of 11% annually. Shapiro and Varian (1999) predict that many more businesses will use fidelity programs as on-line trading grows in popularity “...And keeping track of historical sales of different products will be a lot easier than licking Green Stamps or having your card punched every time you buy a burrito.”

Popular references in the literature offer conflicting advice to academics and practitioners about the profit increasing potential of fidelity programs. Kotler (1999, page 51) states : “Typically, the first company to introduce an FMP [frequency marketing program] gains the most benefit, especially if competitors are slow to respond. After competitors respond, FMPs can become a financial burden to all the offering companies.” A diametrically opposite recommendation is made by Brandenburger (1995) and Brandenburger and Nalebuff (1996) who label these programs as “win-win” strategies since imitation is healthy unlike myopic market share increasing strategies such as temporary price reductions which could be “lose-lose” strategies when imitated. The resolution of this conflict is topical, important and useful for researchers and practitioners in marketing today. This paper tries to resolve the controversy at least partially. In offering explanations for when and why retailers can use fidelity programs to increase their profits, we explicitly recognize the fact that enrollment in fidelity programs is not near one hundred percent when such strategies are implemented in the real world. Thus, even if rebates could be accurately targeted to each competing retailer’s *fidels* (members of its own fidelity program) only, the proportion of *infidels* (non-members of fidelity programs) is shown to be a critical determinant of the profitability of fidelity programs. Further, it is shown that fidelity programs can be a “win-win” marketing strategy only when the proportion of infidels is small, otherwise they may not be

profit increasing or there may be a potential free rider problem. This is the key contribution of this paper.

Common pricing strategies currently used by retailers may be classified into two types - fidelity programs and non-fidelity programs. We define non-fidelity programs as those which offer discounts on current purchases to all consumers (or all program members). Examples of non-fidelity programs include bundling, temporary price reductions, loss leader pricing, and price discrimination devices like coupons and temporal sales. Fidelity programs offer discounts on future purchases at that retailer only to the retailer's own customers, i.e. those customers who have made a previous purchase at the retailer. We prefer to use the term fidelity programs to distinguish between innate loyalty of consumers arising from the strength of their natural preferences and the artificial additional switching cost induced by such programs. In this paper, we focus on the profit increasing potential of fidelity programs for retailers recognizing the existence of infidels when such marketing strategies are implemented.

Specifically, we explore the impact of fidelity programs on equilibrium prices and profits in a mature market where demand is largely inelastic and two retailers compete for customers who have little intrinsic loyalty and there are infidels in the market. We use a simple game theoretic model to answer two main research questions :

- Can both retailers earn higher profits if only one retailer offers a fidelity program ?
- How do profits change when both retailers offer fidelity programs ?

The major objective is not to suggest an optimal design of a fidelity program to retailers but to examine the much more fundamental question whether fidelity programs can at all be profit increasing. Thus we do not consider detailed design issues such as nonlinear versus linear rewards or hard (discounts, points) versus soft (recognition, preferential treatment) rewards. We focus only on linear rewards in the form of discounts on future purchases to a retailer's existing customers who are members of its fidelity program.

The remainder of the paper is organized as follows. The second section briefly reviews the conventional wisdom embodied in past research to show more clearly the research gaps and potential contributions of this paper. The third section outlines a model of a mature market in which two

symmetric retailers of an identical product compete on price for customers who have very little intrinsic store loyalty. We find the equilibrium prices and profits for our benchmark model without any fidelity programs. In section four we examine the equilibrium prices and profits if one or both retailers offer fidelity programs. Section five contains a summary of the important insights and section six concludes with a discussion of anecdotal evidence from different industries, managerial implications and directions for future research.

2. Background Literature

The switching costs literature in economics offers the insight that loyalty rebates create artificial switching costs for customers and thus allow firms to charge higher prices than they could in the absence of these rebates (Klemperer 1987). This insight by itself does not imply that competing retailers will be able to increase profit by introducing fidelity programs because the latter increases costs to the firms as well. Also, if customers realize that they eventually pay higher prices under a fidelity program, they may prefer to shop at a retailer that does not have such a program. A survey paper of the switching costs literature (Klemperer 1995) shows that there has been little research on markets with endogenous switching costs like fidelity programs. Banerjee and Summers (1987), and Caminal and Matutes (1990) are two important exceptions. Banerjee and Summers (1987) use a two-period, homogeneous-product duopoly model to explore the effects of artificial loyalty-inducing economic arrangements such as frequent-flyer programs. Assuming a leader-follower price setting structure, they show that such programs can enable firms to achieve prices as if they were colluding. Caminal and Matutes (1990) use a two-period, differentiated-product duopoly model with preference uncertainty of consumers to examine the effects of two types of pricing strategies. They show that equilibrium profits decline if firms precommit to a second-period price for their loyal customers but increase if the precommitment is for a discount to loyal customers. The research questions posed in this paper have not been examined previously in the setting in which retailers compete on price (setting prices simultaneously) explicitly

recognizing the fact that there are infidels in the market. This is the major research gap the paper seeks to fill at least partially.

It is tempting to reason that a loyalty rebate is a price reduction on future purchases made by existing customers, so if all stores offer loyalty rebates on a product with inelastic demand, each store's profits will decrease. Brandenburger (1995), in looking at competition between two sellers of a single product argues that this intuition is wrong. Suppose stores A and B charge equal prices to customers with little intrinsic store loyalty. Neither store can attempt to increase prices unilaterally without losing customers. Further, both stores have a strong incentive to undercut the other. If only store A issues targeted repeat purchase rebates to its existing customers then undercutting is not profitable for store B. Store A can safely raise price appropriately without losing its previous customers who have a loyalty rebate usable only at store A. This allows store B to raise its price without losing its customers. Store A can again safely raise its price a bit and the process continues till both stores settle on higher prices and profits in equilibrium. Further, it has been shown that if both stores issue loyalty rebates to their existing customers, then the equilibrium prices and profits are even higher. Thus, there is no free rider problem and any retailer may be inclined to start a fidelity program.

Brandenburger and Nalebuff (1996) build on the foregoing intuition and label fidelity programs as "win-win" strategies since imitation by one retailer of the other retailer's strategy is healthy. This is in contrast to myopic market share increasing strategies like temporary price reductions which could be "lose-lose" strategies when imitated. They draw the parallel between loyalty rebate programs like the GM and Ford Cards, and frequent flyer programs of airlines as win-win strategies which prevent price wars by being based on the same underlying principle - treat your own customers better than your rival's customers. If this logic is correct then fidelity programs are indeed a *panacea* in mature markets and should be widely adopted as a marketing strategy.

Some relevant developments in the business world are instructive. Several industry consultants are currently busy designing fidelity programs for numerous clients eager to start fidelity programs or fine tune existing ones. Executives of client companies privately report mixed success of these programs in increasing profits. In 1998, after about five years since starting it as an imitation of the

General Motors' program, Ford discontinued its loyalty rebate card program. In October, 1999 the GM rebate card program is still active. Airline prices and profits have been steadily increasing over the last few years. Worldwide about seventy airlines have frequent flyer programs with three trillion miles as accrued rewards on their books (The New York Times Magazine, March 8, 1998). Companies such as AT&T and Charles Schwab have discontinued their programs. The plausible reasons for this mixed degree of success of fidelity programs could range from faulty design to implementation problems or even industry specific idiosyncratic factors. Recent research such as Chen (1997) and Nalebuff (1998) have concentrated on the question : Is it better to charge lower prices to your own customers (loyals) or to your competitor's customers (switchers) ? This is a comparison of fidelity programs versus switcher reward programs (rebates for new customers) like those offered by long distance phone companies. Other researchers are trying to examine the most efficient forms of loyalty rewards offered through fidelity programs - cash discount versus free products. Research issues regarding the optimal form of loyalty rewards are not the focus of this paper.

The intuition embodied in Brandenburger (1995), and Brandenburger and Nalebuff (1996) as to why fidelity programs can be win-win strategies is appealing but overlooks the fact that even in mature markets there could exist a proportion of infidels. Infidels are not members of either fidelity program that one or both competing retailers are offering and do not possess loyalty rebates for any retailer. In mature markets in the real world, unit sales or dollar sales may be flat but exactly the same set of consumers do not make repeated purchases over time. Infidels could represent "newcomers" or first time buyers in a market. Repeat buyers who due to high opportunity cost of time or any other transaction cost reasons do not sign up as members of any fidelity programs are also infidels. The number of consumers could remain constant over time if the inflow of newcomers is equal to the outflow of old customers who will not buy again in the same market. For example, in durable consumer good industries such as the automobile industry the proportion of newcomers (first time buyers) is significant. Even for department stores, supermarkets, hotels, restaurants, car rental agencies, pizzerias and salons, which are currently offering fidelity programs, there is always a proportion of infidels.

There is a tradeoff which retailers must make in their pricing decision since infidels may make price undercutting profitable to retailers even when all of them offer fidelity programs. Undercutting to attract infidels results in higher share of infidels but also lowers the price charged and hence profit margin made from fidels. On the other hand, one might argue that the size of the rebate given to fidels can be decreased to lower costs of the fidelity program. But the fidelity program itself may not be able to soften price competition by artificially raising switching costs adequately if rebate sizes are arbitrarily reduced. Arbitrarily small sized rebates can never make fidelity programs succeed in softening price competition - the rebate must be of a minimum size depending on the characteristics of the market. It is non-obvious, interesting and important to find out whether there at all exist any equilibrium outcomes wherein fidelity programs can be profitable with reasonable sized rebates, in the presence of infidels. Thus, this paper gets back to the basics by re-examining the fundamental premise for the profit increasing potential of fidelity programs, through game theoretic analysis of a formal model, which allows for the possible presence of infidels.

We assume that two retailers (hereafter called stores interchangeably) of an identical product are price competitors and want to maximize profits. We model the stores' profits for several different states of the market in a one period game. The states of the market we examine are (i) no store has a fidelity program, (ii) only one store has a fidelity program, and (iii) both stores have fidelity programs. By ranking the different states of the market according to the stores' expected profits, we can conclude which state is the most desirable outcome from the stores' perspective. We do not seek to advise retailers on *how* to get from one state of the market to another. However, we identify and explain the states of the market favorable to the retailers.

3. A Model of Retail Price Competition

Our objective is to first set up a model of price competition between two retailers in a mature market without fidelity programs in order to provide a benchmark for evaluating the impact of fidelity programs offered by one and by both retailers. Further, we seek only qualitative insights about the role

nothing. Thus, consumers have an outside option of spending their budget on products not sold by either store if the prices are too high. Assume that r is the reservation price of all consumers.

Let the prices charged by stores A and B be denoted by p^A and p^B respectively. Assume that prices are costlessly well-advertised so that all consumers know both prices before deciding which store to make a trip to, if at all. Further, assume that there are no capacity constraints so that each store has the capacity to serve the entire market⁴.

A consumer located at the point x decides whether to purchase one unit of the product from store A, or from store B, or not to purchase at all, as follows. The utility derived by a consumer located at x , from the available alternatives of not shopping at all and buying one unit of the product from store A or B are respectively given by :

$$\begin{aligned} U(0,y,x) &= y && \text{if consumer does not buy} \\ U(1,y,x) &= r - p^A - tx + y && \text{if consumer buys from A} \\ U(1,y,x) &= r - p^B - t(1-x) + y && \text{if consumer buys from B} \end{aligned} \quad (1)$$

where the income y may be spent by the consumer on goods not sold by either of stores A and B. Thus the consumer will buy from store A if $r \geq p^A + tx$ and $p^B + t(1-x) > p^A + tx$. The consumer will not buy at all if

$$r < \min[p^A + tx, p^B + t(1-x)]. \quad (2)$$

Given any price profile, to calculate profits of each store the location of the marginal consumer, who is just indifferent between shopping at store A or B, has to be determined. If x^* is the location of the marginal consumer then store A will have as its customers all consumers located in $[0, x^*)$ while store B gets those located in $(x^*, 1]$. Assume that both stores charge prices so that all consumers participate in the market. The marginal consumer is indifferent between choosing store A and store B, so we have,

$$r - p^A - tx^* + y = r - p^B - t(1-x^*) + y \Rightarrow x^* = \frac{1}{2} + \frac{(p^B - p^A)}{2t} \quad (3)$$

⁴ This assumption ensures intense price competition between the two stores.

Let Π^A and Π^B denote the profits made by stores A and B respectively. It follows that $\Pi^A = (p^A - c)x^*$ and $\Pi^B = (p^B - c)(1 - x^*)$. We now obtain the equilibrium prices and profits as an outcome of price competition in this market.

The two players, stores A and B, play a simultaneous price setting game. In a market without any store loyalty and hence no differentiation between the two stores, the standard Bertrand competition result predicts that in equilibrium the two stores will price at their marginal cost and make zero profits. In our model, since we allow for store loyalty, there is some differentiation already and we would expect in equilibrium that the two stores would be able to charge prices higher (depending on the extent of loyalty) than marginal cost and make strictly positive profit.

Proposition 1 : If neither retailer offers any fidelity program then the unique Bertrand-Nash equilibrium in pure strategies is (p_*^A, p_*^B) , where $p_*^A = p_*^B = c + t$. At these prices, each retailer has a market share of $\frac{1}{2}$ and makes profit $\Pi^* = \Pi^A = \Pi^B = \frac{t}{2}$.

Proof : See Appendix.

Thus, the symmetric equilibrium price for the product is its marginal cost plus the marginal trip cost, and each store makes a profit of one-half times the marginal trip cost. As t increases, each store competes less strenuously for “the same consumers” and the neighboring clientele of a store becomes more captive, giving the store greater “monopoly power” which allows it to increase its price. When $t = 0$, there is no loyalty among the consumers, that is, all consumers can go to either store for the same zero trip cost and we have, in equilibrium, the Bertrand competition result of marginal cost pricing by both stores and zero profits.

4. Markets With Fidelity Programs

Decrease in loyalty increases the intensity of price competition as the stores become less differentiated. Conversely, increase in loyalty softens price competition as the stores become more differentiated. One way that stores can increase loyalty in a given market is by introducing purely artificial switching costs through fidelity programs which offer repeat purchase discounts. In a market

with such fidelity programs, competing stores that are ex ante alike become, after a purchase is made at one of them, ex post differentiated. Thus both stores may have an opportunity to increase their prices.

The interesting question is whether introduction of fidelity programs by one or both stores will enable them to make higher profits above the benchmark profit level of $t/2$ without any loyalty rebates in a sustainable equilibrium. If prices can be increased when loyalty rebates are offered by the stores, the highest price that can be charged is the monopoly price for the product. So we first obtain the monopoly price and examine the conditions that must be satisfied such that consumers would be willing to pay and stores would want to sell the product at this price.

The purely collusive outcome, with the entire market being served, would lead to the stores splitting the customers equally, that is, store A serving customers in $[0, 1/2)$ and store B serving customers in $(1/2, 1]$. For a consumer located at $1/2$ in the linear city (Figure 1) the trip cost is $t/2$. The maximum price that can be charged by the two stores so that all customers are served is $(r - \frac{t}{2})$. We assume that $r - t \geq c$. This is the most interesting case with maximal competition between the stores for customers because each store could still make positive profit if it sold to a consumer located right next to the other store. So there are no captive customers for either store, which implies there is intense price competition. If $r - t < c$, then each store has some captive customers and competes only for a subset of the remaining customers. If fidelity programs can soften price competition in the case of maximal competitive intensity then these programs will be even more effective in situations of less intense price competition. This is so because fidelity programs artificially increase the degree of differentiation between retailers irrespective of what it was before the introduction of such programs. Thus, we assume $r - t \geq c$ and test the profit increasing potential of fidelity programs in a scenario of intense price competition which is most unfavorable to the success of such programs.

We now examine whether fidelity programs would allow both stores to charge, in equilibrium, price higher than the competitive price (i.e., $c + t$, as shown in Proposition 1) and up to the monopoly price (i.e., $r - t/2$, as shown above). Our focus is on a mature market in which demand is largely inelastic and consumers have very little intrinsic store loyalty. This requires that we consider a situation where t

is very small compared to the potential profit margin of $r-c$. Thus we assume that the competitive price, $c + t$, is less than the monopoly price $r - t/2$. This is equivalent to assuming that $2(r-c) > 3t$.

We consider both one-sided and two-sided fidelity programs in the language of the switching cost literature. This captures the fact that in the real world there are both types of markets - those where some retailers offer loyalty rewards and those in which all retailers offer them. We allow a proportion, α , of “infidels” in the market. Infidels are consumers who do not have loyalty rebates from any retailer. This accounts for the fact that not all consumers may enroll in fidelity programs even when they are offered and there might also be some turnover in a market.

4.1 Both retailers have fidelity programs

First, consider a market with two-sided fidelity programs. Suppose both retailers offered loyalty rebates through fidelity programs which are “costlessly” and “instantaneously” distributed to those consumers who sign up for them. The assumption of costless distribution of the loyalty rebates is a mere analytical convenience and would not change any of our qualitative results. Of course, the rebate value itself is a cost which we account for in our subsequent calculations. The assumption of instantaneous distribution of the rebates to a store’s own fidels is also an analytical convenience so that we can avoid “time” and “information” issues in our model. With time being kept outside of the model, it is sufficient to have a one period model since we are not endogenizing the decision to start a fidelity program by the competing retailers. Information issues are also not considered in this paper. It is possible to think of fidelity programs as “screening” devices which retailers may use to screen types of consumers, for example “fidels” and “infidels”. Such asymmetric information game models may be good for future research.

Let α be the proportion of infidels, i.e. consumers who are not members of a fidelity program. The proportion of fidels is $(1 - \alpha)$. Thus, the total mass of consumers is one and hence the profits in this scenario are comparable with that in the benchmark scenario. We assume that the infidels of mass α are also distributed as a continuum over the linear city. This is a reasonable assumption to make as the infidels may vary in their preferences for the two differentiated stores just like the fidels. The only distinguishing characteristic between the two types of consumers is the fact that fidels have loyalty rebates at one store and infidels do not have a rebate at all.

In this market, with both stores offering loyalty rebates of value L to their respective fidels, the locations of the marginal consumers among infidels and fidels, denoted by x_{α}^* and $x_{1-\alpha}^*$ respectively, determine the market shares of each store. The equilibrium prices and profits are as follows :

Proposition 2 : If both retailers issue a loyalty rebate of value L , $L > \text{Max}\left\{\frac{(r-c)}{2}, t\right\}$, to their respective fidels, that is, store A gives the loyalty rebate to its fidels in $[0, 1/2)$ and store B to its fidels in $(1/2, 1]$, then a symmetric Bertrand-Nash equilibrium in pure strategies is (p_*^A, p_*^B) , where

$p_*^A = p_*^B = \text{Min}\left\{c + \frac{t}{\alpha}, r - \frac{t}{2}\right\}$. If $\alpha > \bar{\alpha} = \frac{2t}{2(r-c)-t}$ then the equilibrium price is

$p_*^A = p_*^B = c + \frac{t}{\alpha}$ and both retailers make equal profit $\Pi^* = \Pi_*^A = \Pi_*^B = \frac{t}{2\alpha} - (1-\alpha)\frac{L}{2}$.

Proof : See Appendix.

This is not a unique equilibrium but the one obtained by restricting the retailers to raising prices to the ceiling of the previously obtained monopoly prices so that no consumers are left out of the market.

Without this condition, there could be other equilibria in which some customers around the point $1/2$ in the linear city (Figure 1) may be left out of the market due to prices being higher than $(r-t/2)$. We restrict our attention only to the symmetric equilibrium prices and profits shown in Proposition 2 above. The result is very intuitive. Unless the size of the rebate is large enough, viz. $L > \text{Max}\left\{\frac{(r-c)}{2}, t\right\}$, undercutting to attract each others' fidels remains profitable. Moreover, unless the proportion of infidels, α , is large enough, viz. $\alpha > \bar{\alpha} = \frac{2t}{2(r-c)-t}$, the stores will find it more profitable to raise prices above $r - t/2$. This condition on α ensures that $c + \frac{t}{\alpha} < r - \frac{t}{2}$. As the proportion of infidels, α , is close to one, the prices and profits approach those in the no-fidelity-program case in Proposition 1. As α decreases the prices and profits in Proposition 2 increase towards the monopoly level. Increasing the value of the rebate, L , decreases profits since it is an explicit cost element. Retailers would like to offer the minimum sufficient amount which will help them increase their profits over the benchmark competitive scenario without fidelity programs. As shown later in section five, when the proportion of infidels is small, that is $\alpha < \frac{t}{L}$, the profits made by the retailers by both offering fidelity programs is greater than the profit they would make without such programs.

4.2 One retailer has a fidelity program

Consider the scenario in which only retailer A offers a fidelity program while B does not. Let (p_*^A, p_*^B) be a Nash equilibrium in pure strategies. We first explain our conjecture regarding possible pure strategy equilibria in Figure 2, which shows the range of p_*^B relative to p_*^A . If p_*^B lies in Region 1, then A gets all fidels and some infidels. On the other hand, if p_*^B is in Region 2, then the two retailers fully separate out the two types of consumers and A gets all the fidels while B serves all the infidels. If p_*^B lies in Region 3, then A gets some fidels while B serves all infidels and some fidels. Intuitively, given a value of p_*^A , the amount by which B would want to undercut depends on the proportion of infidels. Also, the price level p_*^A that A would like to set depends on the proportion of infidels. When α is high, we would expect the equilibrium (p_*^A, p_*^B) to be in Region 1. For low values of α , it is intuitive to conjecture that p_*^B lies in Region 3. For intermediate values of α , the outcome might be in Region 2.

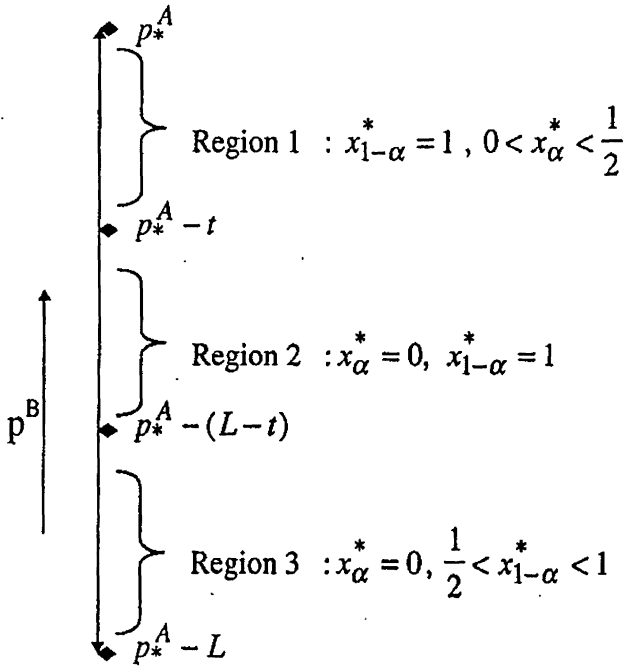


Figure 2

If there is a pure strategy Nash equilibrium in Region 2, then it would be one in which the two stores separate out the two types of consumers completely, namely store A which has a fidelity program serves all the fidels and store B which has no such program serves all the infidels. But there can be no pure

strategy equilibria in Region 2, that is we cannot have $p_*^A - t \geq p_*^B \geq p_*^A - (L - t)$. The reason is that market shares remain unchanged in the entire Region 2, and hence each store is always able to increase its profit by raising its price. The only two possible pure strategy equilibria can be in Regions 1 and 3, that is $p_*^A \geq p_*^B > p_*^A - t$ and $p_*^A - L \leq p_*^B < p_*^A - (L - t)$, respectively, depending on the level of α . We prove our conjecture and obtain both equilibria in the two propositions which follow. First consider Region 1 for which we have :

Proposition 3 : If retailer A issues a loyalty rebate of value $L > 2t$ to all fidels and the proportion of infidels is large, namely $0.6 < \alpha \leq 1$, then a Bertrand-Nash equilibrium in pure strategies is (p_*^A, p_*^B) , where

$$p_*^A = c + t \frac{(4 - \alpha)}{3\alpha}, p_*^B = c + t \frac{(\alpha + 2)}{3\alpha}, \text{ and profits of the two retailers are}$$

$$\Pi_*^A = t \frac{(4 - \alpha)^2}{18\alpha} - L(1 - \alpha), \text{ and } \Pi_*^B = t \frac{(2 + \alpha)^2}{18\alpha}.$$

Proof : See Appendix.

The above equilibrium is quite intuitive. When the proportion of infidels is large, namely $0.6 < \alpha \leq 1$, retailer B who does not have a fidelity program gets a part of the infidels while A takes all fidels plus some infidels. Observe that the proportion of infidels has to be larger than 0.6 for this equilibrium. If it is smaller then retailer B will deviate and try to undercut to get a share of fidels. Now consider Region 3 for which we have :

Proposition 4 : If retailer A issues a loyalty rebate of value $L > 2t$ to all fidels and the proportion of infidels is small, namely $0 < \alpha \leq 0.4$, then a Bertrand-Nash equilibrium in pure strategies is (p_*^A, p_*^B) , where

$$p_*^A = c + L + t \frac{(3 - \alpha)}{3(1 - \alpha)}, p_*^B = c + t \frac{(3 + \alpha)}{3(1 - \alpha)}, \text{ and profits are}$$

$$\Pi_*^A = t \frac{(3 - \alpha)^2}{18(1 - \alpha)} \text{ and } \Pi_*^B = t \frac{(3 + \alpha)^2}{18(1 - \alpha)}.$$

Proof : See Appendix.

This equilibrium is also intuitive. When the proportion of infidels is small, namely $0 < \alpha \leq 0.4$, retailer A who has a fidelity program gets a part of the fidels while B takes all infidels plus some fidels at the lower equilibrium price. Note that there is no pure strategy Nash equilibrium for intermediate values, namely $0.4 < \alpha < 0.6$, of the proportion of infidels. The intuition is that retailer B who does not offer a

fidelity program will find it profitable to deviate and attract fidels from retailer A. Explicit comparisons of the profits in Propositions 3 and 4 with those in Propositions 1 and 2 to find the conditions when profits may increase and the existence of a potential free rider problem are presented in section five.

5. Results

From Proposition 1, the benchmark case when neither retailer has a fidelity program, both retailers make the same profit $\Pi_1 = \frac{t}{2}$. From Proposition 2, the case when both retailers have fidelity programs and $\alpha > \bar{\alpha}$, both retailers make the same profit $\Pi_2 = \frac{t}{2\alpha} - (1-\alpha)\frac{L}{2}$. Note that in one extreme case of high proportion of infidels, as $\alpha \rightarrow 1$, $\Pi_2 \rightarrow \Pi_1$. Thus introduction of fidelity programs by all retailers in markets where the proportion of infidels is high may not be profit increasing. In the other extreme case of very low proportion of infidels, as $\alpha \rightarrow 0$, we could have Π_2 considerably greater than Π_1 . Thus introduction of fidelity programs by all retailers in markets where the proportion of infidels is very low may be very profitable. For intermediate values of the proportion of infidels, we have the following general result by comparing the equilibrium profits in Propositions 1 and 2 :

Result 1 : Fidelity program introduction by both retailers can help soften price competition and increase profits provided the proportion of infidels is small enough, that is $\alpha < \frac{t}{L}$.

Increasing the size of the loyalty rebate is not desirable from the viewpoint of the competing retailers. The larger the rebate, the lower the bound below which the proportion of infidels must be for fidelity programs to be profit increasing. Also as L increases, the profit Π_2 in Proposition 2 decreases. Thus, we have shown that the proportion of infidels does matter and result 1 is a significant modification of the established result in Brandenburger (1995) and in spirit echoed in Brandenburger and Nalebuff(1996) that fidelity programs being offered by all competing retailers is always a “win-win” strategy. When the proportion of infidels is high, the statement of Kotler (1999) that when all competing retailers offer fidelity programs they may not be profitable is correct, even though the reason is very different. However, Kotler’s statement is incorrect when the proportion of infidels is low.

From Proposition 3, the case when only retailer A has a fidelity program (B does not) and the proportion of infidels is large, namely $0.6 < \alpha \leq 1$, the profits of retailers A and B are

$$\Pi_3^A = t \frac{(4-\alpha)^2}{18\alpha} - L(1-\alpha) \text{ and } \Pi_3^B = t \frac{(2+\alpha)^2}{18\alpha} \text{ respectively. Using a similar model without infidels}$$

Brandenburger (1995) shows that (a) there is a free rider problem since retailer B's profit is higher, but (b) both retailers make higher profits than in a regime without fidelity programs. We now test whether these results are affected by the presence of infidels as in our model. This entails comparing profits Π_3^A and Π_3^B from Proposition 3, with the benchmark profit $\Pi_1 = \frac{t}{2}$, from Proposition 1.

$$\text{Algebraic manipulation yields, } \Pi_3^A > \Pi_1 \Leftrightarrow \frac{(16-\alpha)}{18\alpha} > \frac{L}{t} > 2 \Leftrightarrow \alpha < 0.43, \text{ which is never true}$$

since $0.6 < \alpha \leq 1$. So retailer A is never better off if it is the only one offering a fidelity program and the proportion of infidels is greater than 0.6. Also note that $\Pi_3^B > \Pi_1 \Leftrightarrow (1-\alpha)(4-\alpha) > 0$, which is always true. So retailer B is always better off than in the no fidelity program scenario. Thus we have the following result which is a significant departure from Brandenburger (1995):

Result 2: When the proportion of infidels is large, namely $0.6 < \alpha \leq 1$, fidelity program introduction by only one retailer reduces its profit while the competing retailer not offering a fidelity program free rides and increases its profit.

From Proposition 4, the case when only retailer A has a fidelity program and the proportion of infidels is small, namely $0 < \alpha < 0.4$, the profits of the two retailers are $\Pi_4^A = t \frac{(3-\alpha)^2}{18(1-\alpha)}$ and

$$\Pi_4^B = t \frac{(3+\alpha)^2}{18(1-\alpha)}. \text{ We again test the robustness of the results in Brandenburger (1995) by comparing}$$

profits Π_4^A and Π_4^B from Proposition 4, with the benchmark profit $\Pi_1 = \frac{t}{2}$, from Proposition 1. Clearly

$\Pi_4^B > \Pi_4^A > \Pi_1$ is always true. Thus we have the following result which is consistent with Brandenburger (1995):

Result 3: When the proportion of infidels is small, namely $0 < \alpha < 0.4$, fidelity program introduction by only one retailer can increase the profits of both retailers and the competing retailer not offering a fidelity program free rides.

In the case when the above result 3 applies, the very interesting natural question then arises whether retailer B can increase its profit even more by also offering a fidelity program. If the answer is in the affirmative then imitation is indeed healthy, there is no free rider problem and any one retailer would be inclined to start a fidelity program. The answer to this question lies in the outcome of a comparison of Π_4^B and Π_2 .

$$\begin{aligned}\Pi_4^B < \Pi_2 &\Leftrightarrow t \frac{(3+\alpha)^2}{18(1-\alpha)} < \frac{t}{2\alpha} - (1-\alpha) \frac{L}{2} \\ &\Leftrightarrow (9 - 18\alpha - 6\alpha^2 - \alpha^3) > \frac{L}{t} 9\alpha(1-\alpha)^2 > 18\alpha(1-\alpha)^2 \\ &\Leftrightarrow 19\alpha^3 - 30\alpha^2 + 36\alpha - 9 < 0 \\ &\Leftrightarrow 0 < \alpha < 0.32\end{aligned}$$

Hence we have the following two results :

Result 4 : When the proportion of infidels is small, namely $0 < \alpha < 0.32$, imitation is healthy and there is no free rider problem.

Result 5 : When the proportion of infidels is in the mid-ranges, namely $0.32 < \alpha < 0.4$, imitation is not healthy and there is a free rider problem.

We have shown that imitation being healthy, fidelity programs are a “win-win” marketing strategy is true only when the proportion of infidels is small. In such cases, any of the two retailers may safely start a fidelity program without fear of retaliation by the competitor. However, when the proportion of infidels is in the mid-ranges, namely $0.32 < \alpha < 0.4$, there is a potential free rider problem. This is a significant modification of the proposition of Brandenburger (1995) that fidelity programs are a “win-win” strategy.

6. Discussion

Existing literature provides conflicting views about the profitability of fidelity programs. Among those in favor, Brandenburger (1995) and Brandenburger and Nalebuff (1996) contend that in a mature market with two price competing retailers, (i) if only one retailer offers a fidelity program, then both retailers can make higher profits in equilibrium, and (ii) if both retailers offer fidelity programs, then in equilibrium both retailers can make even higher profits. Thus, imitation is healthy and this may be a

reason why fidelity programs are currently a very popular marketing strategy. In a departure from previous research, in this paper we recognized that infidels are present when fidelity programs are implemented in real markets. We have shown that the ability of fidelity programs in mitigating price competition and increasing profits is critically dependent on the proportion of infidels.

When all retailers offer fidelity programs, we have demonstrated through our model that the proportion of infidels must be small for profits to increase. The intuition is that a sizeable presence of infidels keeps price undercutting profitable as in a market without fidelity programs. Merely increasing the size of the loyalty rebate awarded to fidels is not a solution when the proportion of infidels is not small enough.

When only some retailers offer fidelity programs, we have shown that the proportion of infidels must be small for profits to increase. The intuition is that if the proportion of infidels is large, price undercutting may still remain profitable for retailers who offer fidelity programs. If the proportion of infidels is intermediate, price undercutting remains profitable for retailers who do not offer fidelity programs. Again there are no simple solutions like increasing the size of the rebate given to fidels. Fidelity programs could be a “win-win” marketing strategy since imitation is healthy, there is no free rider problem and any retailer may safely start a fidelity program when the proportion of infidels is small. In the case of intermediate values of the proportion of infidels, there is a potential free rider problem. When the proportion of infidels is high, no retailer should offer a fidelity program.

The key contribution of this paper is in pointing out the fact that infidelity or incomplete enrollment in fidelity programs may affect their profit increasing potential in different ways depending on whether some or all retailers in the market have such programs. Thus, we hope to sound a timely word of caution to those retailers who may be mistakenly engaged in adopting the currently popular “win-win” marketing strategy of fidelity programs.

We have proposed a theoretical rationale for why retailers in certain industries might have successful fidelity programs. The essence of our arguments has been that it is only under certain market conditions, price competing retailers find it mutually beneficial to move away from myopic market share grabbing strategies (like temporary price reductions and loss leader pricing) and instead

adopt other marketing strategies like fidelity programs which help retailers increase profits. This could be one explanation for the currently observed increasing use of fidelity programs by retailers in some industries. There may be other explanations for this proliferation. At least one such explanation may be that retailers consider fidelity programs as another myopic market share increasing strategy. If this were true, then after the introduction of a fidelity program by a retailer, we would expect prices to either remain at the same level as before the introduction of such programs or even decrease further. On the contrary, if our explanation indeed applies then we would expect prices and profits to increase after the introduction of fidelity programs by competing retailers.

All major U.S. airlines have frequent flyer programs which are in essence the type of fidelity programs examined in this paper. It is a well accepted fact that prices and profits in this industry have been increasing over the years (The Wall Street Journal, November 3, 1997). The airlines industry has experienced four consecutive years of record profits from 1995 to 1998 (The New York Times, March 16, 1999). The National Airline Quality Rating study of 1999 shows that customer complaints against airlines to the Transportation Department were up 26 percent in 1998, after rising 20 percent in 1997 (The New York Times, April 20, 1999). Currently there is an initiative in the Senate to pass legislation to make airlines accountable to passengers for certain services. On March 15, 1999 the major airlines announced the second increase in airfares this year resulting in a 7% increase in restricted fares and 3% increase in unrestricted fares in 1999 (USA Today, March 16, 1999). By the beginning of the summer, the third increase in airfares for 1999 had already taken place (The Detroit News, June 2, 1999). The New York Times Magazine (March 8, 1998) reports : "Today 47% of the passengers on any given flight are earning frequent flyer miles." On the busy business route flights the proportion of passengers who are members of that airline's frequent flyer program is even higher. This is consistent with our prediction that in markets in which all competing retailers have fidelity programs and the proportion of infidels is low, prices and profits may increase. There could be various other reasons like the state of the U.S. economy, or increase in concentration in the industry for the increase in airfares and airline profits which cannot be ruled out. Still the overall consistency with our model predictions is reassuring.

On the other hand, consider the evidence from the U.S. automobile industry where the proportion of infidels is much higher than in the airline industry. In September, 1992, with much fanfare General Motors started its fidelity program - the GM/Household Bank card. In February, 1993, Ford retaliated (imitated) with the Ford/Citibank card. These specific fidelity programs were applauded in Brandenburger and Nalebuff (1996) as the way out of price wars which held lessons for other industries too. In the automobile market the proportion of infidels is high primarily because "newcomers" are to be expected for such a durable good. The average American car buyer gets a new car every five to seven years. But of course there are a large number of first time buyers at all times. From January 1, 1998 Ford had discontinued its fidelity program while GM continues with its program even in October, 1999. Ford and GM dealers in Michigan, where the enrolment in these programs is much higher than the national average, indicate that only 10-20% of cars were being sold to program members. This is again consistent with our prediction that irrespective of whether some or all retailers offer fidelity programs, when the proportion of infidels is high fidelity programs do not increase profits. It is also possible that consistent with our results Ford is free riding on GM which continues its fidelity program. However, it should be noted that GM rebates redeemed worldwide in 1997 were \$656 million while rebates available for future redemption amounted to \$3.5 billion. Ford has discontinued its rebate program but its previously issued rebate liability amounted to \$4 billion in 1997. Given that between Ford and GM they have a lion's share of the U.S. auto market, their head-to-head competition and different behavior regarding fidelity programs provides anecdotal support for our model findings.

Supermarkets have been trying to increase their thin profit margins by cutting back on sales and advertising and having fidelity programs (The Wall Street Journal, May 29, 1997). The recommendations to members and the actions of the Food Marketing Institute (FMI) are consistent with our model predictions that participation in fidelity programs is a critical determinant of their profitability. In 1997, the FMI published a report entitled "Guide to planning frequent shopper programs" and in 1998 published the sequel "Loyalty marketing : After the card is issued". These reports provide detailed implementation guidelines to retailers. The first and very important step is "Building Card Use" or ensuring high percentage of program enrolment. A national survey (summary in

Table 1) conducted in 1997 by A.C.Nielsen indicates that across major cities of the U.S. there is considerable variation in the percentage of households enrolled in frequent shopper card programs of supermarkets. Supermarket card program participation was highest in Chicago at 94% followed by Charlotte at 87% and very low at cities like Miami at 9% and St. Louis at 6%. Cities such as Houston at 50% and Minneapolis at 46% had moderate levels of participating households. Without getting into the reasons why there is considerable geographic variation in supermarket program participation, our theory is useful for supermarkets in setting managerial expectations for the performance of fidelity programs in different metropolitan areas. For example, Kroger, one of the largest supermarket chains, recently launched a major drive to get customers in the midwest to sign up for the Kroger Plus Shopper's Card. To build card enrollment and use, Kroger gave two cards and two key chain tags to all customers who signed up. Supermarkets do appear to have drawn their lessons from the varied experience in other industries such as airlines and automobiles mentioned earlier.

This paper, being only a first cut at an important, under-researched area, has several limitations which also open up many interesting possibilities for future research. The proportion of infidels has been treated as exogenous to the model. This is a reasonable assumption when infidels primarily represent newcomers or first time buyers and there is no customer heterogeneity in preferences for types of fidelity programs. Incorporating such customer heterogeneity and endogenizing at least a part of the participation parameter may be a good area for future research. We restricted our focus to hard, linear loyalty rebates to be consistent with the models used by past researchers so that our results could be compared with the existing results. Exploring other forms of rewards especially the soft type which is very popular across several industries would be a useful direction to pursue for future research. Further, by allowing for heterogeneity among consumers on other behavioral dimensions (e.g. light and heavy shoppers, frequent and infrequent shoppers, low and high inventory holding cost consumers, etc.) one can investigate other important design issues in the search for "optimal" fidelity programs. Clearly the airline industry with almost two decades of experience with such programs has not yet found it. Currently major airlines are debating whether to change to "dollars paid" from "miles flown" as the new

basis of accrual of rewards. Much remains to be explored about the accrual basis and the redemption of rewards before the elusive “optimal” design can be understood, if it at all exists.

APPENDIX

Proof for Proposition 1 : Let p^A, Π^A and p^B, Π^B denote the prices and profits of retailers A and B respectively. Location of marginal consumer is $x^* = \frac{1}{2} + \frac{1}{2t}(p^B - p^A)$. Profits are

$$\Pi^A = x^*(p^A - c) = \left\{ \frac{1}{2} + \frac{(p^B - p^A)}{2t} \right\} (p^A - c) \text{ and } \Pi^B = (1 - x^*)(p^B - c) = \left\{ \frac{1}{2} + \frac{(p^A - p^B)}{2t} \right\} (p^B - c).$$

The first order profit maximizing conditions yield the two reaction functions as follows :

$$\frac{\partial \Pi^A}{\partial p^A} = 0 \Rightarrow 2p^A - p^B = c + t \quad \dots (A1)$$

$$\frac{\partial \Pi^B}{\partial p^B} = 0 \Rightarrow 2p^B - p^A = c + t \quad \dots (A2)$$

The unique solution to (A1) and (A2) is $p^A = p^B = c + t$.

The second order conditions are satisfied as $\frac{\partial^2 \Pi^A}{(\partial p^A)^2} = \frac{\partial^2 \Pi^B}{(\partial p^B)^2} = -\frac{1}{t} < 0$.

At $p^A = p^B = c + t$, $x^* = \frac{1}{2}$ and $\Pi^* = \Pi^A = \Pi^B = \frac{t}{2}$. \square

Proof for Proposition 2 : Let $p_*^A = p_*^B = p_*$ be a symmetric Bertrand-Nash equilibrium in pure strategies.

Observe that $p_*^A = p_*^B = p_* \Rightarrow x_\alpha^* = \frac{1}{2}$, $x_{1-\alpha}^* = \frac{1}{2}$. Let Π^* denote the equilibrium profit made by each retailer. Then, $\Pi^* = \Pi_*^A = \Pi_*^B = \frac{\alpha}{2}(p_* - c) + \frac{(1-\alpha)}{2}(p_* - L - c)$.

The equilibrium price p_* is obtained by setting up the conditions so that retailer A's profit cannot increase above Π^* by its deviations. Small deviations affect the market shares of the infidels only while large deviations affect the market shares of both the infidels and fidels. Due to symmetry it is adequate to consider price deviations by only one retailer. Retailer A can undercut by ε , $0 < \varepsilon \leq t$, to gain $\frac{\varepsilon}{2t}$ market share of infidels only or undercut by z , $L < z \leq L + t$, to get all infidels together with $\frac{(z-L)}{2t}$ increase in market share of fidels. First consider small undercutting by A to gain share of infidels only.

After undercutting by ε , $0 < \varepsilon \leq t$, let retailer A's price and profit be p_ε^A and Π_ε^A respectively. Then $p_\varepsilon^A = p_* - \varepsilon$ and

$$\begin{aligned}\Pi_\varepsilon^A &= \alpha \left(\frac{1}{2} + \frac{\varepsilon}{2t} \right) (p_* - \varepsilon - c) + \frac{(1-\alpha)}{2} (p_* - \varepsilon - L - c) \\ &= \Pi^* - \varepsilon^2 \left(\frac{\alpha}{2t} \right) - \frac{\varepsilon}{2} + \varepsilon \left(\frac{\alpha}{2t} \right) (p_* - c)\end{aligned}$$

Let $f(\varepsilon) = \Pi_\varepsilon^A - \Pi^* = -\varepsilon^2 \left(\frac{\alpha}{2t} \right) - \frac{\varepsilon}{2} + \varepsilon \left(\frac{\alpha}{2t} \right) (p_* - c)$. As $f(\varepsilon)$ is concave and $f(0)=0$, a sufficient condition for $f(\varepsilon) \leq 0$ is $\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0$.

$$\begin{aligned}\frac{df}{d\varepsilon} &= -\varepsilon \left(\frac{\alpha}{t} \right) - \frac{1}{2} + \left(\frac{\alpha}{2t} \right) (p_* - c) \\ \left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0 &\Rightarrow p_* \leq c + \frac{t}{\alpha} \quad \dots (A3)\end{aligned}$$

Similarly, considering a small price increase by A the condition for it to be not profit increasing is

$$p_* \geq c + \frac{t}{\alpha} \quad \dots (A4)$$

Combining (A3) and (A4) we have $p_* = c + \frac{t}{\alpha}$

Further imposing the condition $p_* \leq r - \frac{t}{2}$, so that the entire market is served (otherwise infidels around $\frac{1}{2}$ will not participate) by the two retailers, we have

$$p_* = \min \left\{ \left(c + \frac{t}{\alpha} \right), \left(r - \frac{t}{2} \right) \right\} \quad \dots (A5)$$

At the equilibrium price in (A5) above, if the ceiling has not been attained, both retailers make the same profit given by

$$\begin{aligned}\Pi^* = \Pi_*^A = \Pi_*^B &= \frac{\alpha}{2} (p_* - c) + \frac{(1-\alpha)}{2} (p_* - L - c) \\ &= \frac{t}{2\alpha} - (1-\alpha) \frac{L}{2} \quad \dots (A6)\end{aligned}$$

We now check that the equilibrium price in (A5) above is such that A cannot increase its profit by large undercutting to gain share of fidels also.

After undercutting by z, $L < z \leq L + t$, let retailer A's price and profit be p_z^A and Π_z^A respectively. Then $p_z^A = p_* - z$ and

$$\begin{aligned}\Pi_z^A &= \alpha (p_* - z - c) + (1-\alpha) \left(\frac{1}{2} + \frac{(z-L)}{2t} \right) (p_* - z - L - c) \\ &= \Pi^* - z^2 \left(\frac{1-\alpha}{2t} \right) - z \frac{(1+\alpha)}{2} + z \left(\frac{1-\alpha}{2t} \right) (p_* - c) + \frac{\alpha}{2} (p_* - c) - L \left(\frac{1-\alpha}{2t} \right) (p_* - L - c)\end{aligned}$$

$$\begin{aligned} \text{Let } f(z) &= \Pi_z^A - \Pi^* \\ &= -z^2 \left(\frac{1-\alpha}{2t} \right) - z \frac{(1+\alpha)}{2} + z \left(\frac{1-\alpha}{2t} \right) (p^* - c) + \frac{\alpha}{2} (p^* - c) - L \left(\frac{1-\alpha}{2t} \right) (p^* - L - c) \end{aligned}$$

As $f(z)$ is concave, sufficient conditions for $f(z) \leq 0, \forall z > L$, are $f(L) \leq 0$ and $\left. \frac{df}{dz} \right|_{z=L} \leq 0$.

$$f(L) \leq 0 \Rightarrow \alpha(p^* - c) \leq L(1+\alpha)$$

$$\Rightarrow p^* \leq c + (1+\alpha) \frac{L}{\alpha}$$

Substituting the equilibrium price in (A5) in the above condition we get

$$c + \frac{t}{\alpha} \leq c + (1+\alpha) \frac{L}{\alpha}$$

$$\Rightarrow \alpha \geq \frac{t-L}{L} \text{ which is always satisfied since } L > t \text{ and } 0 < \alpha < 1.$$

We now check the second sufficient condition.

$$\left. \frac{df}{dz} \right|_{z=L} = -z \left(\frac{1-\alpha}{t} \right) - \frac{(1+\alpha)}{2} + \left(\frac{1-\alpha}{2t} \right) (p^* - c)$$

$$\left. \frac{df}{dz} \right|_{z=L} \leq 0 \Rightarrow p^* \leq c + 2L + t \frac{(1+\alpha)}{(1-\alpha)} \quad \dots (A7)$$

We now verify that the equilibrium price in (A5) satisfies (A7). From (A5), the maximum value of the equilibrium price is $(r - \frac{t}{2})$. Since $L > \frac{(r-c)}{2}$, (A7) is always satisfied. \square

Proof for Proposition 3 : Suppose (p_*^A, p_*^B) is a Bertrand-Nash equilibrium in pure strategies. Then from Region 1 of Figure 2, $x_{1-\alpha}^* = 1$, $x_\alpha^* = \frac{1}{2} + \frac{1}{2t}(p_*^B - p_*^A)$ and

$$\Pi_*^A = \alpha \left\{ \frac{1}{2} + \frac{1}{2t}(p_*^B - p_*^A) \right\} (p_*^A - c) + (1-\alpha)(p_*^A - L - c)$$

$$\Pi_*^B = \alpha \left\{ \frac{1}{2} + \frac{1}{2t}(p_*^A - p_*^B) \right\} (p_*^B - c)$$

To solve for p_*^A and p_*^B the only deviations which need to be considered are (1) small deviations by either retailer which affects their shares of infidels only and price charged to all, and (2) large deviation by B to attract fidels.

First we set up the condition such that small deviations by retailer A to attract infidels is not profit increasing. Suppose retailer A increases price by ε , $0 < \varepsilon \leq t$, and let retailer A's price and profit be p_ε^A and Π_ε^A respectively. Then $p_\varepsilon^A = p_*^A + \varepsilon$ and

$$\begin{aligned}\Pi_{\varepsilon}^A &= \alpha \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A) - \frac{\varepsilon}{2t} \right\} (p_*^A + \varepsilon - c) + (1 - \alpha) (p_*^A + \varepsilon - L - c) \\ &= \Pi_*^A - \varepsilon^2 \left(\frac{\alpha}{2t} \right) + \varepsilon \alpha \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A) \right\} - \varepsilon \left(\frac{\alpha}{2t} \right) (p_*^A - c) + \varepsilon (1 - \alpha)\end{aligned}$$

Let $f(\varepsilon) = \Pi_{\varepsilon}^A - \Pi_*^A = -\varepsilon^2 \left(\frac{\alpha}{2t} \right) + \varepsilon - \frac{\varepsilon \alpha}{2} + \varepsilon \left(\frac{\alpha}{2t} \right) (p_*^B - 2p_*^A + c)$. As $f(\varepsilon)$ is concave and $f(0)=0$, a sufficient condition for $f(\varepsilon) \leq 0$ is $\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0$.

$$\begin{aligned}\frac{df}{d\varepsilon} &= -\varepsilon \left(\frac{\alpha}{t} \right) + 1 - \frac{\alpha}{2} + \left(\frac{\alpha}{2t} \right) (p_*^B - 2p_*^A + c) \\ \left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} &\leq 0 \Leftrightarrow p_*^B - 2p_*^A \leq t - c - \frac{2t}{\alpha}\end{aligned} \quad \dots (A8)$$

Similarly, considering a small price decrease by retailer A we have the condition

$$p_*^B - 2p_*^A \geq t - c - \frac{2t}{\alpha} \quad \dots (A9)$$

For (p_*^A, p_*^B) to be a Bertrand-Nash equilibrium in pure strategies, small deviations by retailer A must not be profit increasing. Thus (p_*^A, p_*^B) must satisfy both (A8) and (A9) above. Combining (A8) and (A9) we have the condition

$$p_*^B - 2p_*^A = t - c - \frac{2t}{\alpha} \quad \dots (A10)$$

Next we set up the condition such that small deviations by retailer B to attract infidels is not profit increasing. Suppose retailer B increases price by ε , $0 < \varepsilon \leq t$, and let retailer B's price and profit be p_{ε}^B and Π_{ε}^B respectively. Then $p_{\varepsilon}^B = p_*^B + \varepsilon$ and

$$\begin{aligned}\Pi_{\varepsilon}^B &= \alpha \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^A - p_*^B) - \frac{\varepsilon}{2t} \right\} (p_*^B + \varepsilon - c) \\ &= \Pi_*^B - \varepsilon^2 \left(\frac{\alpha}{2t} \right) + \varepsilon \alpha \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^A - p_*^B) \right\} - \varepsilon \left(\frac{\alpha}{2t} \right) (p_*^B - c)\end{aligned}$$

Let $f(\varepsilon) = \Pi_{\varepsilon}^B - \Pi_*^B = -\varepsilon^2 \left(\frac{\alpha}{2t} \right) + \frac{\varepsilon \alpha}{2} + \varepsilon \left(\frac{\alpha}{2t} \right) (p_*^A - 2p_*^B + c)$. As $f(\varepsilon)$ is a concave function and $f(0)=0$, a sufficient condition for $f(\varepsilon) \leq 0$ is $\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0$.

$$\begin{aligned}\frac{df}{d\varepsilon} &= -\varepsilon \left(\frac{\alpha}{t} \right) + \frac{\alpha}{2} + \left(\frac{\alpha}{2t} \right) (p_*^A - 2p_*^B + c) \\ \left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} &\leq 0 \Leftrightarrow p_*^A - 2p_*^B \leq -c - t\end{aligned} \quad \dots (A11)$$

Similarly, considering a small price decrease by retailer B we have the condition

$$p_*^A - 2p_*^B \geq -c - t \quad \dots (A12)$$

For (p_*^A, p_*^B) to be a Bertrand-Nash equilibrium in pure strategies, small deviations by retailer B must not be profit increasing. Thus (p_*^A, p_*^B) must satisfy both (A11) and (A12) above. Combining (A11) and (A12) we have the condition

$$p_*^A - 2p_*^B = -c - t \quad \dots (A13)$$

So (p_*^A, p_*^B) , a Nash equilibrium in pure strategies, must satisfy (A10) and (A13) so that small deviations by either retailer is not profit increasing. The unique solution of (A10) and (A13) is

$$p_*^A = c + t \frac{(4 - \alpha)}{3\alpha}, \quad p_*^B = c + t \frac{(\alpha + 2)}{3\alpha} \quad \dots (A14)$$

At these prices the equilibrium profits of the two retailers are

$$\Pi_*^A = t \frac{(4 - \alpha)^2}{18\alpha} - L(1 - \alpha) \text{ and } \Pi_*^B = t \frac{(2 + \alpha)^2}{18\alpha}$$

Note that (p_*^A, p_*^B) as given in (A14) above must satisfy $p_*^A \geq p_*^B \geq p_*^A - t$. The imposition of this condition yields restrictions on α , the proportion of infidels, for the Nash equilibrium given by (A14) to hold.

$$p_*^A \geq p_*^B \Leftrightarrow c + t \frac{(4 - \alpha)}{3\alpha} \geq c + t \frac{(\alpha + 2)}{3\alpha} \Leftrightarrow \alpha \leq 1$$

$$p_*^B \geq p_*^A - t \Leftrightarrow c + t \frac{(\alpha + 2)}{3\alpha} \geq c + t \frac{(4 - \alpha)}{3\alpha} - t \Leftrightarrow \alpha \geq \frac{2}{5}$$

Thus we have a restriction that the proportion of infidels must satisfy

$$\frac{2}{5} \leq \alpha \leq 1 \quad \dots (A15)$$

We now check that the equilibrium prices in (A14) above are such that retailer B cannot increase its profit by large undercutting to gain share of fidels. The first step is to find the minimum amount by which retailer B must deviate from the equilibrium price in (A14) to gain any share of fidels. A fidel located at the point 1 in the linear city would face net prices of $p_*^B = c + t \frac{(\alpha + 2)}{3\alpha}$ at store B and

$p_*^A - L + t = c + t \frac{(4 - \alpha)}{3\alpha} - L + t = c - L + 2t \frac{(2 + \alpha)}{3\alpha}$ at store A. Observe that the difference in net prices is $p_*^B - (p_*^A - L + t) = L - t \frac{(2 + \alpha)}{3\alpha}$. Hence to gain any fidels, retailer B must undercut by at least

$z > L - t \frac{(2 + \alpha)}{3\alpha}$. Suppose retailer B undercuts by such an amount z and after undercutting let its price and

profit be p_z^B and Π_z^B respectively. Then B gets all infidels and $\frac{1}{2t} \left\{ z - L + t \frac{(2 + \alpha)}{3\alpha} \right\}$ market share of

fidels and charges price $p_z^B = p_*^B - z$ to all customers. Its profit is given by

$$\Pi_z^B = \alpha \left(p_*^B - z - c \right) + (1 - \alpha) \frac{1}{2t} \left(z - L + t \frac{(2 + \alpha)}{3\alpha} \right) \left(p_*^B - z - c \right)$$

$$\text{Recall that } \Pi_*^B = \alpha \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^A - p_*^B) \right\} (p_*^B - c)$$

$$\text{Let } f(z) = \Pi_z^B - \Pi_*^B$$

$$= \alpha \left(p_*^B - z - c \right) + \left(\frac{1 - \alpha}{2t} \right) \left\{ z - L + t \frac{(2 + \alpha)}{3\alpha} \right\} \left(p_*^B - z - c \right)$$

$$- \frac{\alpha}{2} \left(p_*^B - c \right) - \left(\frac{\alpha}{2t} \right) \left\{ c + t \frac{(4 - \alpha)}{3\alpha} - p_*^B \right\} \left(p_*^B - c \right)$$

substituting the value of p_*^A from (A14).

As $f(z)$ is concave, sufficient conditions for $f(z) \leq 0$, $\forall z > L - t \frac{(2 + \alpha)}{3\alpha}$,

$$\text{are } f(z) \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} \leq 0 \text{ and } \frac{df}{dz} \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} \leq 0.$$

$$f(z) \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} = -L\alpha + t \frac{(\alpha + 2)(10\alpha - 1)}{18\alpha}$$

$$f(z) \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} \leq 0 \Leftrightarrow L \geq t \frac{(\alpha + 2)(10\alpha - 1)}{18\alpha^2}$$

$$\Leftrightarrow \frac{(\alpha + 2)(10\alpha - 1)}{18\alpha^2} < 2 \text{ since } L > 2t.$$

$$\Leftrightarrow 26\alpha^2 - 19\alpha + 2 > 0$$

Let $g(\alpha) = 26\alpha^2 - 19\alpha + 2$. The nature of the function $g(\alpha)$ is illustrated at a few points over the range of feasible values of α in the table below :

| | | | | | | | | | | | | |
|-------------|---|------|-------|-------|-------|-----|-----|------|------|------|------|---|
| α | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | .605 | 0.7 | 0.8 | 0.9 | 1 |
| $g(\alpha)$ | 2 | 0.36 | -0.76 | -1.36 | -1.44 | -1 | 0 | 0.02 | 1.44 | 3.44 | 5.96 | 9 |

Thus we have the condition $0 \leq \alpha \leq 0.1$ or $0.6 < \alpha \leq 1$

... (A16)

$$\frac{df}{dz} = -\alpha - z \left(\frac{1-\alpha}{t} \right) + L \left(\frac{1-\alpha}{2t} \right)$$

$$\frac{df}{dz} \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} = -\alpha - L \left(\frac{1-\alpha}{2t} \right) + \frac{(1-\alpha)(2+\alpha)}{3\alpha}$$

$$\frac{df}{dz} \Big|_{z=L-t \frac{(2+\alpha)}{3\alpha}} \leq 0 \Leftrightarrow L \geq 2t \left\{ \frac{(2+\alpha)}{3\alpha} - \frac{\alpha}{(1-\alpha)} \right\} \quad \dots (A17)$$

Let $g(\alpha) = \left\{ \frac{(2+\alpha)}{3\alpha} - \frac{\alpha}{(1-\alpha)} \right\}$. Note that $g(\alpha)$ is a decreasing function of α . The nature of the function

is illustrated at a few points over the range of feasible values of α in the table below :

| α | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.45 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|-------------|----------|------|------|------|------|------|------|-------|-------|-------|-------|-----------|
| $g(\alpha)$ | ∞ | 6.89 | 3.42 | 2.13 | 1.33 | 1 | 0.66 | -0.05 | -1.05 | -2.83 | -7.93 | $-\infty$ |

Since $L > 2t$, the condition (10) above is always satisfied for

$$0.45 < \alpha \leq 1 \quad \dots (A18)$$

Recall that in (A15) we had an earlier restriction on α , the proportion of infidels. The intersection of the restrictions (A15), (A16) and (A18) is $0.6 < \alpha \leq 1$. \square

Proof of Proposition 4 : Suppose (p_*^A, p_*^B) is a Bertrand-Nash equilibrium in pure strategies. Then from Region 3 of Figure 2, $x_\alpha^* = 0$, $x_{1-\alpha}^* = \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A + L)$ and

$$\Pi_*^A = (1-\alpha) \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A + L) \right\} (p_*^A - L - c)$$

$$\Pi_*^B = \alpha (p_*^B - c) + (1-\alpha) \left\{ \frac{1}{2} - \frac{1}{2t} (p_*^B - p_*^A + L) \right\} (p_*^B - c)$$

To solve for the equilibrium prices p_*^A and p_*^B the only deviations which need to be considered are (1) small deviations by either retailer which affects their market shares of fidels only and the price charged to all, and (2) large deviation by A to attract infidels.

First we set up the condition such that small deviations by retailer A to attract fidels is not profit increasing. Suppose retailer A increases price by ε , $0 < \varepsilon \leq t$, and let retailer A's price and profit be p_ε^A and Π_ε^A respectively. Then $p_\varepsilon^A = p_*^A + \varepsilon$ and

$$\begin{aligned}\Pi_{\varepsilon}^A &= (1-\alpha) \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A + L) - \frac{\varepsilon}{2t} \right\} (p_*^A + \varepsilon - L - c) \\ &= \Pi_*^A - \varepsilon^2 \left(\frac{1-\alpha}{2t} \right) + \varepsilon(1-\alpha) \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - 2p_*^A + 2L + c) \right\}\end{aligned}$$

Let $f(\varepsilon) = \Pi_{\varepsilon}^A - \Pi_*^A = -\varepsilon^2 \left(\frac{1-\alpha}{2t} \right) + \varepsilon \frac{(1-\alpha)}{2} + \varepsilon \left(\frac{1-\alpha}{2t} \right) (p_*^B - 2p_*^A + 2L + c)$. As $f(\varepsilon)$ is concave,

a sufficient condition for $f(\varepsilon) \leq 0$ is $\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0$.

$$\frac{df}{d\varepsilon} = -\varepsilon \left(\frac{1-\alpha}{t} \right) + \frac{(1-\alpha)}{2} + \left(\frac{1-\alpha}{2t} \right) (p_*^B - 2p_*^A + 2L + c)$$

$$\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0.$$

$$\Leftrightarrow \frac{(1-\alpha)}{2} + \left(\frac{1-\alpha}{2t} \right) (p_*^B - 2p_*^A + 2L + c) \leq 0$$

$$\Leftrightarrow 2p_*^A - p_*^B \geq c + t + 2L \quad \dots \text{(A19)}$$

Similarly, considering a small price decrease by retailer A we have the condition

$$2p_*^A - p_*^B \leq c + t + 2L \quad \dots \text{(A20)}$$

For (p_*^A, p_*^B) to be a Bertrand-Nash equilibrium in pure strategies, small deviations by retailer A must not be profit increasing. Thus (p_*^A, p_*^B) must satisfy both (A19) and (A20) above. Combining (A19) and

(A20) we have the condition

$$2p_*^A - p_*^B = c + t + 2L \quad \dots \text{(A21)}$$

Next we set up the condition such that small deviations by retailer B to attract fidels is not profit increasing. Suppose retailer B increases price by ε , $0 < \varepsilon \leq t$, and let retailer B's price and profit be p_{ε}^B and Π_{ε}^B respectively. Then $p_{\varepsilon}^B = p_*^B + \varepsilon$ and

$$\begin{aligned}\Pi_{\varepsilon}^B &= \alpha (p_*^B + \varepsilon - c) + (1-\alpha) \left\{ \frac{1}{2} - \frac{1}{2t} (p_*^B - p_*^A + L) - \frac{\varepsilon}{2t} \right\} (p_*^B + \varepsilon - c) \\ &= \Pi_*^B - \varepsilon^2 \left(\frac{1-\alpha}{2t} \right) + \varepsilon \frac{(1+\alpha)}{2} - \varepsilon \left(\frac{1-\alpha}{2t} \right) (2p_*^B - p_*^A + L - c)\end{aligned}$$

Let $f(\varepsilon) = \Pi_{\varepsilon}^B - \Pi_*^B = -\varepsilon^2 \left(\frac{1-\alpha}{2t} \right) + \varepsilon \frac{(1+\alpha)}{2} - \varepsilon \left(\frac{1-\alpha}{2t} \right) (2p_*^B - p_*^A + L - c)$. As $f(\varepsilon)$ is a concave

function and $f(0)=0$, a sufficient condition for $f(\varepsilon) \leq 0$ is $\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} \leq 0$.

$$\frac{df}{d\varepsilon} = -\varepsilon \left(\frac{1-\alpha}{t} \right) + \frac{(1+\alpha)}{2} - \left(\frac{1-\alpha}{2t} \right) (2p_*^B - p_*^A + L - c)$$

$$\begin{aligned}
\left. \frac{df}{d\varepsilon} \right|_{\varepsilon=0} &\leq 0. \\
\Leftrightarrow \frac{(1+\alpha)}{2} - \left(\frac{1-\alpha}{2t} \right) (2p_*^B - p_*^A + L - c) &\leq 0 \\
\Leftrightarrow 2p_*^B - p_*^A &\geq c - L + t \frac{(1+\alpha)}{(1-\alpha)} \quad \dots \text{(A22)}
\end{aligned}$$

Similarly, considering a small price decrease by retailer B we have the condition

$$2p_*^B - p_*^A \leq c - L + t \frac{(1+\alpha)}{(1-\alpha)} \quad \dots \text{(A23)}$$

For (p_*^A, p_*^B) to be a Bertrand-Nash equilibrium in pure strategies, small deviations by retailer B must not be profit increasing. Thus (p_*^A, p_*^B) must satisfy both (A22) and (A23) above. Combining (A22) and

(A23) we have the condition

$$2p_*^B - p_*^A = c - L + t \frac{(1+\alpha)}{(1-\alpha)} \quad \dots \text{(A24)}$$

So (p_*^A, p_*^B) , a Nash equilibrium in pure strategies, must satisfy (A21) and (A24) so that small deviations by either retailer is not profit increasing. The unique solution of (A21) and (A24) is

$$p_*^A = c + L + t \frac{(3-\alpha)}{3(1-\alpha)}, \quad p_*^B = c + t \frac{(3+\alpha)}{3(1-\alpha)} \quad \dots \text{(A25)}$$

At these prices the equilibrium profits of the two retailers are

$$\Pi_*^A = (1-\alpha) \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A + L) \right\} (p_*^A - L - c) = t \frac{(3-\alpha)^2}{18(1-\alpha)}$$

$$\text{and } \Pi_*^B = \alpha (p_*^B - c) + (1-\alpha) \left\{ \frac{1}{2} - \frac{1}{2t} (p_*^B - p_*^A + L) \right\} (p_*^B - c) = t \frac{(3+\alpha)^2}{18(1-\alpha)}.$$

Note that (p_*^A, p_*^B) as given in (A25) above must satisfy $p_*^A - (L-t) \geq p_*^B \geq p_*^A - L$. The imposition of this condition yields restrictions on α , the proportion of infidels, for the Nash equilibrium given by (A25) to hold.

$$\begin{aligned}
p_*^A - (L-t) \geq p_*^B &\Leftrightarrow c + L + t \frac{(3-\alpha)}{3(1-\alpha)} - L + t \geq c + t \frac{(3+\alpha)}{3(1-\alpha)} \Leftrightarrow \alpha \leq \frac{3}{5} \\
p_*^B \geq p_*^A - L &\Leftrightarrow c + t \frac{(3+\alpha)}{3(1-\alpha)} \geq c + L + t \frac{(3-\alpha)}{3(1-\alpha)} - L \Leftrightarrow \alpha \geq 0
\end{aligned}$$

Thus we have a restriction that the proportion of infidels must satisfy

$$0 \leq \alpha \leq \frac{3}{5} \quad \dots \text{(A26)}$$

We now check that the equilibrium prices in (A25) above are such that retailer A cannot increase its profit by large undercutting to gain share of infidels. The first step is to find the minimum amount by which retailer A must deviate from the equilibrium price in (A25) to gain any share of infidels. An infidel located at the point 0 in the linear city would face net prices of $p_*^B + t = c + t \frac{(3+\alpha)}{3(1-\alpha)} + t$ at store B and

$p_*^A = c + L + t \frac{(3-\alpha)}{3(1-\alpha)}$ at store A. Observe that the difference in net prices is

$p_*^A - p_*^B - t = L - t \frac{(3-\alpha)}{3(1-\alpha)}$. Hence to gain any infidels, retailer A must undercut by at least

$z > L - t \frac{(3-\alpha)}{3(1-\alpha)}$. Suppose retailer A undercuts by such an amount z and after undercutting let its price and

profit be p_z^A and Π_z^A respectively. Then A gets all fidels and $\frac{1}{2t} \left\{ z - L + t \frac{(3-\alpha)}{3(1-\alpha)} \right\}$ market share of

infidels and charges price $p_z^A = p_*^A - z$. Its profit is given by

$$\begin{aligned} \Pi_z^A &= (1-\alpha) \left(p_*^A - z - L - c \right) + \frac{\alpha}{2t} \left(z - L + t \frac{(3-\alpha)}{3(1-\alpha)} \right) \left(p_*^A - z - c \right) \\ &= -z^2 \frac{\alpha}{2t} - z(1-\alpha) + z \frac{L\alpha}{t} - \frac{\alpha}{2t} \left[L^2 - \left\{ t \frac{(3-\alpha)}{3(1-\alpha)} \right\}^2 \right] + t \frac{(3-\alpha)}{3} \end{aligned}$$

Recall that

$$\begin{aligned} \Pi_*^A &= (1-\alpha) \left\{ \frac{1}{2} + \frac{1}{2t} (p_*^B - p_*^A + L) \right\} (p_*^A - L - c) \\ &= t \frac{(3-\alpha)}{6} + t \frac{\alpha(3-\alpha)}{9(1-\alpha)} \end{aligned}$$

Let $f(z) = \Pi_z^A - \Pi_*^A$

$$= -z^2 \frac{\alpha}{2t} - z(1-\alpha) + z \frac{L\alpha}{t} - \frac{\alpha}{2t} \left[L^2 - \left\{ t \frac{(3-\alpha)}{3(1-\alpha)} \right\}^2 \right] + t \frac{(3-\alpha)(3-5\alpha)}{18(1-\alpha)}$$

As $f(z)$ is concave, sufficient conditions for $f(z) \leq 0, \forall z > L - t \frac{(3-\alpha)}{3(1-\alpha)}$,

are $f(z) \Big|_{z=L-t \frac{(3-\alpha)}{3(1-\alpha)}} \leq 0$ and $\frac{df}{dz} \Big|_{z=L-t \frac{(3-\alpha)}{3(1-\alpha)}} \leq 0$.

$$f(z) \Big|_{z=L-t \frac{(3-\alpha)}{3(1-\alpha)}} = -L(1-\alpha) + t \frac{(3-\alpha)}{3} + t \frac{(3-\alpha)(3-5\alpha)}{18(1-\alpha)}$$

$$f(z)\Big|_{z=L-t\frac{(3-\alpha)}{3(1-\alpha)}} \leq 0 \Leftrightarrow L \geq t \left\{ \frac{(3-\alpha)}{3(1-\alpha)} + \frac{(3-\alpha)(3-5\alpha)}{18(1-\alpha)^2} \right\}$$

$$\Leftrightarrow \frac{(3-\alpha)}{3(1-\alpha)} + \frac{(3-\alpha)(3-5\alpha)}{18(1-\alpha)^2} < 2 \text{ since } L > 2t$$

$$\Leftrightarrow (5\alpha - 3)^2 > 0 \text{ which is satisfied for all values of } \alpha.$$

$$\frac{df}{dz} = -z \left(\frac{\alpha}{t} \right) - (1-\alpha) + \frac{\alpha}{t} L$$

$$\frac{df}{dz}\Big|_{z=L-t\frac{(3-\alpha)}{3(1-\alpha)}} = \frac{\alpha(3-\alpha)}{3(1-\alpha)} - (1-\alpha)$$

$$\frac{df}{dz}\Big|_{z=L-t\frac{(3-\alpha)}{3(1-\alpha)}} \leq 0 \Leftrightarrow \frac{\alpha(3-\alpha)}{3(1-\alpha)} \leq (1-\alpha)$$

$$\Leftrightarrow 4\alpha^2 - 9\alpha + 3 \geq 0 \quad \dots \text{ (A27)}$$

Let $g(\alpha) = 4\alpha^2 - 9\alpha + 3$. Note that $g(\alpha)$ is a decreasing function of α . The nature of the function is illustrated at a few points over the range of feasible values of α in the table below :

| | | | | | | | | | | | |
|-------------|---|------|------|------|-----|------|-------|-------|-------|-------|----|
| α | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| $g(\alpha)$ | 3 | 2.14 | 1.36 | 0.66 | 0 | -0.5 | -0.96 | -1.34 | -1.64 | -1.86 | -2 |

The condition (A27) above is always satisfied for

$$0.4 \geq \alpha \geq 0 \quad \dots \text{ (A28)}$$

Recall that in (A26) we had an earlier restriction on α , the proportion of infidels. The restriction (10) is stricter than (A26). \square

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