Division of Research
School of Business Administration

August 1987
Revised February 1988

A FLEXIBLE APPROACH TO PART TYPE SELECTION USING PART MIX RATIOS IN FLEXIBLE FLOW SYSTEMS

Working Paper #531-b

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ABSTRACT

The short-term production planning function for setting up a flexible manufacturing system (FMS) prior to production has to be developed so as to interact well with the operation of the system over time. During FMS operation, planning for system set-up has to be performed somewhat periodically, for example, when the part mix is changed, or when the production requirements are finished for some part type, or when a machine breaks down. Since all cutting tools required for all part types that need to be machined cannot usually fit in the machines' limited capacity tool magazines, decisions need to be made concerning which part types will be produced next.

This paper suggests a flexible approach to short-term production planning. A description of when either a batching or a flexible approach is appropriate is provided. An existing mathematical programming procedure that determines balanced (or unbalanced) part mix ratios for part types to balance machine workloads is adapted to solve another planning problem of selecting part types to be machined together over the upcoming time period. A simulation model is developed to demonstrate the implementation of both the suggested flexible approach and batching on a realistic flexible flow system (FFS) having pooled machines of unequal sizes.

The research results indicate that: (1) the flexible approach usually leads to better system utilization and makespan than batching; (2) using the flexible approach enables the system to be utilized more constantly; and (3) the flexible approach requires many fewer fixtures dedicated to particular part types than batching to complete the production requirements of all part types. Computational results are presented to demonstrate the performance of the part type selection approaches. Further research needs are also discussed.

1. INTRODUCTION

An FMS in the metal-cutting industry consists of computer numerically controlled machine tools capable of performing multiple functions. The machine tools have automatic tool interchange capabilities and are linked together with automatic material handling equipment. All components are hierarchically computer-controlled. The type of systems examined in this paper are Flexible Flow Systems (FFSs), where the routing of parts is unidirectional.

The attractive combination of automation and flexibility necessitates an improvement in efficiency of production planning in dynamic situations. This is because production management becomes more complex and a goal is to cope well with dynamic situations so as to attain the potential FMS or FFS efficiency. Production planning should be developed to help managers to set up an existing system for subsequent efficient production.

The short-term production planning function for setting up an FMS prior to production should be implemented so as to interact well with the operation of the system over time. FMS production planning problems are defined as system set-up decisions that have to be made before an FMS can begin to produce part types. According to Stecke [1983], the production planning problems are decomposed into five:

(1) **Part Type Selection Problem:**
Determine a subset of part types for immediate and simultaneous processing over the upcoming period of time.

(2) **Machine Grouping Problem:**
Partition the machines of similar types into identically tooled machine groups. Each machine in a particular group is then able to perform the same operations.

(3) **Production Ratio Problem:**
Determine the relative part mix ratios at which the selected part types should be produced over time.

(4) **Resource Allocation Problem:**
Allocate the (minimum numbers of) pallets and fixtures of different fixture types to the selected part types to maintain the determined production ratios.

(5) **Loading Problem:**
Allocate the cutting tools of all operations of the selected part types to some machine's limited capacity tool magazine.

There have been several research studies to date dealing with some of these planning problems. Chakravarty and Shtub [1984] apply group technology techniques to identify families of part types that require similar processing requirements. Their aim is to group the part types and tools together so as to increase the production efficiency. Kusiak [1983] suggests a coding system based on the similarity between parts to select part types. This coding system is used to minimize the sum of distances between the part types' attributes. Some mathematical programming and heuristic approaches to part type selection include studies by Whitney and Gaul [1984], Hwang [1986], and Rajagopalan [1986]. They partition the part types having production requirements into distinct batches to be machined one batch at a time. This batching approach aims to minimize the frequency of tool changeovers.

Research on grouping and loading problems usually assumes that either the set of part types to be machined together or their relative mix ratios have already been found (i.e., see Berrada and Stecke [1986], Erschler, Lévêque and Roubellat [1982], Hildebrant [1980], Hitz [1980], and Stecke [1983]). Mathematical programming and queueing networks have been used to address these problems.

This paper presents a flexible approach to short-term production planning, to be implemented over time and in advance of actual production. The use of existing procedures that determine part mix ratios of part types are adapted to also select the part types to be machined together and on a dynamic basis. The part mix *ratios* for a selected set of part types are defined here to be the relative *numbers of parts* of each part type that will be produced over an upcoming period of time.

A simulation model is introduced to show the uses and benefits of the suggested flexible approach on a realistic flexible flow system (FFS) containing pooled machines of unequal sizes. Statistics on blocking, transportation, buffer, cart, and machine utilizations; makespan; fixture requirements of various types; and different workload distributions among the machine types are provided for both flexible and batching approaches to part type selection. Also, sensitivities to transportation times as well as to the number of carts in the system are examined for the flexible approach. The purpose of this latter study is to investigate the effects of limited resources on system performance using the suggested flexible approach.

The suggested concepts and approaches can be appropriate for particular general types of FMSs. This is discussed in §2. The particular implementation examined here is for an FFS. This is so that the flexibility of the approach is not confounded with the flexibility and scheduling issues associated with alternative routing in job shop types of FMSs. In general, there are no scheduling problems for FFSs other than determining the procedure by which parts are input into the system. Methods to determine a part input sequence for FFSs under various assumptions and for various objectives have been developed by Hitz [1980], Erschler et al. [1982], Akella et al. [1985], Afentakis [1986], and Pinedo et al. [1986], for example.

These studies differ in the size of buffers that are allowed, whether or not breakdowns and capacity are considered, and whether or not machines are allowed to be bypassed as parts follow the fixed route through the system. In virtually all of these studies, periodic production requirements (for example, weekly) for all part types are scaled proportionally to a minimal part set (MPS). An MPS defines the part mix ratios that are the *smallest integer multiple* of the production requirements for every part type. A (usually periodic) part input sequence is developed as some permutation of these part mix ratios.

Under ideal conditions (infinite buffer at each machine and no breakdowns), the maximum production is defined by the bottleneck machine. Afentakis [1986] notes that under ideal conditions, *any* part input sequence will maximize production. Under more general conditions, where orders continually arrive and breakdowns are considered, the bottleneck can shift (and hence change the production rate).

Some studies are also concerned with decreasing WIP, which *is* affected by the part sequence. For example, Akella et al. [1985] provide an approach to determine the input of parts that *nearly* meets the production requirements while effectively decreasing WIP. Pinedo et al. [1986] notes that there are scheduling decisions if there is a buffer at each machine and parts can be resequenced. However, most studies assume FCFS in the buffer and in these there are no scheduling problems. For example, the application of Akella et al. [1985] has 30 buffers in front of 4 machines, yet there are no scheduling decisions (other than part input) because FCFS is followed.

This paper differs from previous FFS studies in that issues are addressed other than determining part input into an FFS. Firstly, more flexibility (and hence operating advantages) can be obtained by selecting part mix ratios independent of the production requirements to maximize production rate or utilization. Secondly, advantages of pooling machines has not been adequately addressed in the literature. Thirdly, selecting a subset of parts to be immediately produced has been addressed for general FMSs, but not in FFS studies. Finally, a method is suggested to determine a part input sequence to schedule the FFS although this is not the focus of this study. The part input sequence problem here differs from previous FFS studies in that:

1. Groups of pooled machines are considered;
2. Pallet and fixture limitations are considered;
3. The part types to be input continuously change.

In addition, travel time and finite buffers are considered.

The purpose of this paper, which is an extension of Stecke and Kim [1986], is as follows. Two distinct approaches to FMS part type selection are discussed, one from the literature and one new, each appropriate for different types of FMSs. The implementation of each is demonstrated on an FFS. In particular, a flexible approach is proposed that should be considered in choosing a part type selection procedure for certain FMS types. The flexible and batching approaches are suggested for dynamic systems, where orders arrive over time. The goal is to balance machine workloads to try to maximize system utilization. For static problems, where there is a fixed set of orders for requirements of various part types, different but analogous approaches need to be applied. For these systems, an appropriate system objective may be to minimize makespan.

In addition, this FFS study differs from previous ones in the issues that are addressed. Also, the flexible approach is further demonstrated by investigating the effects of limited numbers of transportation vehicles and different travel times on system performance.

This paper is organized as follows. §**2** describes the flexible approach to implement solutions to the FMS planning problems. Situations in which batching or the flexible approach may be appropriate are discussed. §**3** reviews solution methodology that determines part mix ratios for the operating objectives of balancing or unbalancing workloads. Similar part mix ratio formulations can also be used to help select the part types to be produced simultaneously over the immediate flexible time horizon. §**4** illustrates these procedures to solve the FMS planning problems and provides some computational results. The simulation model is described in §**5.1**. §**5.2** demonstrates the implementation of both the suggested flexible approach and batching with simulation studies. In §**5.3**, simulations with different data sets of travel times and number of carts in the system are performed to further demonstrate the flexible approach and investigate some of these operating issues. Conclusions and future research needs are provided in §**6**.

2. A FLEXIBLE APPROACH TO SHORT-TERM PRODUCTION PLANNING

An FMS is highly capital-intensive. Many FMS users (i.e., Caterpillar Tractor, Fiat Trattori, Vought Aerospace, Celakovice) are concerned with achieving high system utilization. Then one appropriate objective of production planning is to *maximize production* or *system utilization*. The flexible approach to production planning that is suggested here follows these objectives. The algorithms used in this approach should be efficient and allow better integration with subsequent FMS operation.

An example of a flexible approach to part type selection is as follows. When the production requirements for some part type are completed, space in the tool magazines is freed up so some new part type(s) can be introduced into the system if this input can help system balance and system utilization.

A flexible approach to short-term production planning could be able to adapt to dynamic situations as well as help lead to increased system utilization. Then a more flexible FMS operation can help reduce system cost by resulting in a more efficient system utilization.

The flexible approach is implemented by updating the solutions to the FMS production planning problems whenever events such as the following occur:

. The production requirements for some part type(s) are finished;
. Some urgent order(s) arrive;
. Some production order(s) change;
. One or more new part types are to begin production;
. A machine tool goes down and will remain down for a long enough period of time;
. Preventative maintenance is to be performed.

Whenever events occur such as these that change the system environment, the system does not always have to be set up again. Like tool replacement due to breakage and wearing, the cutting tools required by the input of some new part type(s), for some systems, can be changed in a small amount of time without stopping the whole system. Tools for one machine can be changed while other machines are running. If only one or few part types in the mix are changed, this system changeover time can be quite small. (This is called a partial or minor changeover.) In some FMSs, tools can be changed on the fly. Tool changes are done over a period of time, with less idle time than with batching. For these types of FMSs, the flexible approach suggested here may be useful to select which parts enter the system and ratios for these parts that balance machine workloads.

Batching is sometimes performed because it is easier to manage. When the requirements for all of the selected part types are complete, the cutters are in various stages of wear. Some are totally worn or nearly so and others are partly worn. It can be easier to unload all cutters from the magazines and take them all to the tool room for regrinding and calibrating, even those tools that are not very worn. If the tools for the complete batch of part types are changed, the system could be idle for a significant length of time for this changeover. (This is called a total or major changeover.)

There are two distinct general FMS production environments. If the FMS is a subsystem of the factory that produces parts for later assembly, the FMS planning function may receive its production requirements and due dates from the factory-wide production planning system. An example is MRP weekly time buckets. If there are certain part types required in particular relative ratios, then an appropriate FMS operating objective is to start and complete those part types during the same time period in the same batch. These requirements can be met by producing at relative ratios that are proportional to the production requirements. Then all magazines are unloaded and reloaded for the next production batch.

However, if the demand for the part types (or for a subset of the part types) is independent, FMS production planning can be developed in a more flexible manner, to take advantage of available operating options. There can be freedom in determining the relative mix ratios at which a set of part types could be machined together. This freedom, in conjunction with the objective of balancing machine workloads, can be used to help attain a good system utilization. Some FMSs are driven by daily or monthly due dates for various part types that differ over time. Maximizing utilization can help such daily or monthly due date constraints to be met.

The flexible approach is implemented as follows. The algorithm of §3 is applied to select a set of part types and their mix ratios to balance workloads. There is a constant number of parts in the FFS. As a part leaves the system, a new part is input according to the mix ratios (i.e., the relative numbers of parts of each of the selected part types). Production continues until the production requirements of some part type are finished. One or more new part types are selected to be machined with the remaining part types and new part mix ratios are found. Sometimes no new part type is selected, but new optimal part mix ratios are found for the reduced set of part types. Each production run can result in a minor tool changeover as a part type leaves and a new part type enters production. The cutting tools no longer required are unloaded and new cutters are loaded.

Each of these approaches may be appropriate for different FMS types. However, some FMSs will require system-specific part type selection procedures that would not fall into either of these categories. For example, if a system produces parts having individual and specific due dates and unit requirements, the flexible approach cannot be applied. Since workload on different machine types is balanced, the flexible approach as described here cannot be directly applied to FMSs consisting of all identical machines. However, a variant of the flexible approach suggested here could be developed.

To summarize, batching may be appropriate if the demand for some part types is dependent and certain part types are required in relative ratios. However, batching defines the workload and the bottleneck machine. The flexible approach may be appropriate when the demand for the part types is independent. Part types are phased into and out of production. Mix ratios are determined to balance machine workloads to help attain good system utilization.

3. SOLUTION METHODOLOGY

Table I reviews the notation of Stecke [1985]. Given the production and aggregate processing time requirements of each part type on each machine type, the problem to select parts and determine mix ratios is reviewed as the following integer formulation, Program (P1).

(P1) Minimize $$\sum_{k=1}^{K} C_{k1} x_{k1} + \sum_{k=1}^{K} C_{k2} x_{k2}$$

subject to

$$\sum_{i=1}^{N} pw_{ik} a_i - x_{k1} + x_{k2} = W_k, \qquad k=1,\ldots,K \qquad (1)$$

$$a_i \le f_i, \qquad i=1,\ldots,N \qquad (2)$$

$$a_i \ge 0 \text{ and integer}, \qquad i=1,\ldots,N \qquad (3)$$

$$x_{k1}, x_{k2} \ge 0, \qquad k=1,\ldots,K \qquad (4)$$

TABLE I. Notation.

i	part types,	$i=1,\ldots,N$
j	machines,	$j=1,\ldots,M$
k	machine types,	$k=1,\ldots,K$
a_i	part mix ratio of part type i	
r_i	production requirements for part type i	
p_{ijk}	processing time of a part of type i on machine j (of some machine type k)	
m_k	number of machines of type k	
pw_{ik}	average workload required by a part of type i on a machine of type k $= p_{ijk}/m_k$	
W_k	constant value indicating the (un)balanced workload on a machine of type k over time	
x_{k1}	load over (un)balanced, W_k, on machine type k	
x_{k2}	load under (un)balanced, W_k, on machine type k	
C_{k1}	weight assigned to the potential overload (x_{k1})	
C_{k2}	weight assigned to the potential underload (x_{k2})	
f_i	maximum number of fixtures dedicated to part type i	
n	total number of pallets in the system	

The objective function can be changed by weighting the coefficients (C_{k1} and C_{k2}) of the overload and underload on each machine type differently. The coefficients, C_{k1} and C_{k2}, can be selected arbitrarily. This provides alternative sets of optimal ratios that balance workloads.

Constraint **(1)** describes the workload on each machine type. This enables the workload per machine to be balanced (if $W_k = W$, for all k) or unbalanced (for systems configured as pooled machines of unequal sizes). The relative and unbalanced workloads per machine type (W_k) can be calculated using the closed queueing network (CQN) model, CAN-Q (Solberg [1977]). The workloads, W_k, can be scaled arbitrarily. Since the processing time information is input to Program **(P1)**, different scalings for the W_k result in proportional scaling for the a_i. Constraint **(2)** restricts the maximum ratio values (maximum number of parts of each type) to be allowed in the system simultaneously. This could be caused by a limitation on the number of fixtures of each type as well as on the remaining production requirements.

Those part types with near zero ratio values in the optimal solution are not selected to be in the part mix to be machined together over the upcoming period. The zero production ratios indicate that these part types are not compatible with respect to (un)balancing machine workloads among the different machine types.

The following algorithm both selects the subset of part types to be machined together and determines mix ratios over time:

PART TYPE/MIX RATIO SELECTION ALGORITHM

Step 1. Formulate and solve Program **(P1)** for a particular set of parameters W_k, C_{k1}, C_{k2}.

Step 2. For those part types with positive ratio values in the optimal solution (i.e., $a_i \geq 1$), produce at those ratios until some event occurs, such as the completion of the requirements of some part type(s), or a machine will be down for a while, or an urgent order arrives.

Step 3. Update the part mix ratios by introducing the following constraints:

$a_{i_1} \geq 1$, where i_1={previously selected part types that have not yet completed their requirements}

$a_{i_2} = 0$, where i_2={part types that have completed their requirements}

$a_{i_3} \geq 0$, where i_3={part types that have not yet been selected to be machined}.

Step 4. If all requirements for all part types are completed, **STOP**.
Otherwise, go to **Step 1.**

For a static problem, without arriving orders, the algorithm is reiterated until the requirements of all part types are completed. In actual implementation, orders would arrive dynamically and the same procedure is followed. At **Step 2**, the part types with near zero ratio values are not selected to be produced over the upcoming time horizon. **Step 3** updates the part mix as well as their ratios, if the input of one or more new part types allows the workloads to be better balanced. Otherwise, only the mix ratios of the same set of part types are updated. The part types that do not complete their requirements continue production over the next time period without cutting tool changeovers.

If the total remaining processing times required by some of the part types with remaining requirements are small after the completion of requirements of some other part type(s), it could be effective to continue production of the reduced set of part types at updated ratios, rather than introducing new part types. This saves an unnecessary changeover. Then a bound on the remaining processing time of any one part type is suggested here before any cutting tools are changed. If the total processing time required to complete the remaining requirements of any one part type is less than four hours after the completion of requirements of some other part type(s), a tool changeover is not performed until the remaining requirements of that part type are finished. A new part type is *not* introduced. In reality, different bounds could be determined by considering: the ease and time of tool changeovers, number of machine tools affected, the length of shifts, and the short tool changeovers already required for wearing or worn tools. In addition, the maximum ratio values of those part types with small requirements should not be larger than their remaining requirements. In this case, additional constraints similar to constraint (2) are introduced. The algorithm is illustrated in the next section.

4. AN EXPERIMENTAL STUDY OF FLEXIBLE AND BATCHING APPROACHES

In this section, the part mix ratios selected for the flexible and batching approaches are illustrated using a realistic FFS scenario configured as pooled machines of unequal sizes. §4.1 describes the problem sets. In §4.2, computational results on finding part mix ratios are provided.

4.1. FMS Scenarios Investigated

To demonstrate the algorithm of §3 to select part mix ratios, consider the problem sets of Table II. There are twelve part types ordered to be produced on an FMS containing unequally-sized pooled machine groups. In particular, there are pooled drills and VTLs, each group having two identical machines. There is only one mill. A set of processing times and three different sets of production requirements for each part type are provided for this system of three machine types and five machines. Processing times are in minutes.

The problem sets were chosen to cover various realistic scenarios. For example, in Problem 1 of Table II, the total processing times ($\sum_i pw_{ik} r_i$, k=Mill, Drill, VTL) are distributed more to the pooled drills and VTLs than to the mill. In Problem 2, the mill is more heavily loaded. Finally, the workloads are about equally distributed in the third Problem.

In a comparison of unbalancing and balancing objectives, expected production and system utilizations are all higher when unbalancing workloads in systems of groups of pooled machines of unequal sizes. (See Stecke and Solberg [1985].) It is also shown in Stecke and Kim [1987] that: (1) in order to maximize system utilization or production rate, the appropriate number of pallets should be examined for a given system in advance of operation, when either unbalancing or balancing; and (2) unbalanced part mix ratios con-

TABLE II. Processing Times and Production Requirements for Twelve Part Types on Three Machine Types with Five Machines.

Part Type	Mill(1)	Drill(2)	VTL(2)	Production Requirements		
				Problem 1	Problem 2	Problem 3
PT1	11(1)	50	58	35(2)	15	29
PT2	20	40	20	24	29	35
PT3	35	30	10	10	20	20
PT4	25	20	12	14	18	10
PT5	15	18	40	30	40	35
PT6	16	30	20	21	33	28
PT7	30	20	38	14	17	25
PT8	20	10	10	14	28	20
PT9	5	30	34	50	44	30
PT10	7	36	40	40	24	33
PT11	10	52	44	55	30	34
PT12	15	20	30	20	8	10

(1) Processing times are in minutes.
(2) Production requirements are in number of parts.

versely lead to balanced machine utilizations among unequally pooled machine types. System utilization is more sensitive to the number of pallets in the system when unbalancing than when balancing.

The number of parts (pallets) in the system is eight for the problems of Table II. Separate simulation runs were performed to evaluate system performance for various numbers of pallets in the system and eight pallets provided the best system utilizations (Stecke and Kim [1986b]). In reality, this is an important design issue. However, this is not a focus of the present research.

The values of parameters W_k, C_{k1}, and C_{k2} are specified as 100, 1, and 1, respectively, when workloads are balanced. C_{k1} and C_{k2} can be selected arbitrarily. Changing these parameters will provide alternative part types and mix ratios that balance workloads. Different values for W_k will only scale the ratios, a_i. The integer programs **(P1)** are run using LINDO on an AMDAHL 5860.

For the problems of Table II, the system of three groups of 1, 2, and 2 machines each, the unbalanced workloads per machine that provide the maximum expected production (i.e., $[W_1,W_2,W_3]=[84,104,104]$ for $n=8$ –– see Stecke and Solberg [1981]) are used in the integer Program **(P1)** that provides the part mix ratios. These ratios will then unbalance workloads, as the theoretical unbalanced optimum suggests.

There are two cases of fixture limitations. First, the number of fixtures of each type is limited for each part type to be four ($f_i \leq 4$, $i=1,...,12$). Again, determining the appropriate numbers of fixtures of each fixture type is an important design problem that is not addressed in the present study. The limitation of four fixtures per part type used here is arbitrary, to demonstrate the procedure. Other alternatives can be modeled, such as several part types sharing the same fixture type or parts required in certain ratios, for example. In the simulation studies, only eight parts in total can be in the system and also a maximum of four parts of each type can be in the system when the fixture limitations are considered. The second case requires no restriction on this value ($f_i < \infty$, $i = 1, \ldots, 12$). This second case is considered in order to examine the number of fixtures of each type that would be required to finish all requirements of all part types if there were no limitations.

4.2. Analysis of the Selected Part Types and Mix Ratios

In this section, the procedures are demonstrated by running two of the three problems of Table II as Program **(P1)** to find the part mix ratios. The results of the third problem and others are in Stecke and Kim [1986b]. Both the flexible and batching approaches are demonstrated for a given number of pallets in the system.

Batching aims to minimize the frequency of system set-up. Batching is implemented favorably as follows. In particular, whenever the requirements for a part type in a particular batch are completed, new ratios are found for the remaining part types that aim to (un)balance machine workloads as optimally as possible. Therefore, the selected part types in a particular batch are machined continuously until all requirements are completed, by following ratios that are continuously updated as any one part type completes its requirements. We caution that batching can be implemented differently in the literature.

Table III provides the part types selected over time and their part mix ratios as determined by the flexible and batching approaches for the objective of unbalancing workloads. Table III is obtained as follows. Following the flexible approach first, Program **(P1)** is run and part types 3, 8, 9, and 10 are selected to be machined in ratios of 1:1:2:3 over time. The objective function value is 2, which is a very small overload or underload on some machine. A simulation is run to check which part types finish their production requirements first. (In this case, part type 3 finishes first.) These finished part types are deleted, those not finished remain, and **(P1)** is run again to see if new part types should enter the system. (In this case, part types 2 and 12 enter.) Another simulation is run as part types 2, 8, 9, 10, and 12 are machined in ratios of 1:1:1:3:1. The series of Programs **(P1)** and simulations are run until all production requirements of all part types are finished. The details and performance results of the simulation runs are described in §5.

The batching results of Table III are as follows. The first batch is the same as the first flexible batch. The production ratios that balance workloads and the objective function

TABLE III. Integer Optimum Solutions for the Objective of Unbalancing Workloads When Eight Pallets are in the System.

a. PROBLEM 1

Run	Rule	Selected Part Types	Production Ratios	Objective Function Value	CPU Time (seconds)
1	FLEX(a)(1)	**3,8,9,10**	1:1:2:3	2	16.873
	BATCH(a)(2)	**3,8,9,10**	1:1:2:3	2	16.873
	(b)	8,9,10	3:3:2	15	1.700
	(c)	9,10	2:4	58	1.213
2	FLEX(a)	**2**,8,9,10,**12**	1:1:1:3:1	6	10.270
	BATCH(a)	**2,5,6,11**	1:2:1:2	3	6.194
	(b)	2,6,11	1:2:3	34	1.986
	(c)	2,11	3:2	42	1.062
3	FLEX(a)	2,**5**,9,12	2:1:3:1	3	5.328
	BATCH(a)	**1,4**	3:2	15	1.557
4	FLEX(a)	2,5,**11**,12	1:1:2:2	4	1.791
	BATCH(a-1)(3)	**7,12**	1:4	85	1.033
	BATCH(a-2)(4)	**12**	7	56	1.156
5	FLEX(a)	**1**,5,**6**,11	1:1:3:1	4	3.406
	BATCH(a-2)	**7**	3	127	0.996
6	FLEX(a)	1,**4**,5,11	1:2:1:2	15	2.073
7	FLEX(a)	1,5,**7**,11	1:1:1:2	24	1.182
	(b)	1,7,11	1:2:2	21	1.454
	(c)	1,11	2:2	44	1.046

b. PROBLEM 2

Run	Rule	Selected Part Types	Production Ratios	Objective Function Value	CPU Time (seconds)
1	FLEX(a)	**3,8,9,10**	1:1:2:3	2	16.873
	BATCH(a)	**3,8,9,10**	1:1:2:3	2	16.873
	(b)	3,8,9	1:2:4	51	1.692
	(c)	3,8	2:1	165	1.087
2	FLEX(a)	**1**,3,8,9	1:1:1:4	6	3.282
	(b)	1,3,8	3:1:1	20	2.468
	(c)	3,8	2:1	164	1.049
	BATCH(a)	**2,5,6,11**	1:2:1:2	3	6.194
	(b)	2,5,6	3:3:1	43	1.667
	(c)	2,6	2:4	68	1.063
3	FLEX(a)	8,**11,12**	1:3:2	8	2.423
	(b)	8,11	2:4	20	1.065
	BATCH(a)	**1,4**	3:2	15	1.557
4	FLEX(a)	**2,5,6**,11	1:2:1:2	3	2.553
	(b)	2,5,6	3:3:1	43	1.643
	(c)	5,6	3:4	46	1.255

Run	Rule	Selected Part Types	Production Ratios	Objective Function Value	CPU Time (seconds)
	(d-2)	6	7	63	0.969
	BATCH(a-1)	**7,12**	1:4	85	1.033
	(a-2)	**12**	7	56	1.156
5	FLEX(a)	6,7	4:1	89	1.001
	BATCH(a-2)	**7**	3	127	0.966
6	FLEX(a)	**4**,7	1:2	135	0.989

0 indicates the new part types selected to be machined simultaneously over the upcoming time period.
(1) FLEX refers to the suggested flexible approach.
(2) BATCH refers to the batching approach.
(3) specifies the limit of four fixtures of each type.
(4) specifies no fixture limitations.
(5) Unbalancing rule specifies that $W_{mill}=84$, $W_{drill}=104$, and $W_{vtl}=104$.

value are also the same. The simulation indicates that part type 3 finishes its 10 requirements first. To avoid the tool change, the remaining part types 8, 9, and 10 are machined at new ratios of 3:3:2 that balance workloads as much as possible. The ratios are updated until this batch is completed. The second batch is 2, 5, 6, and 11.

The rules labeled (a) in Table III imply that new part types are scheduled to enter the system. These new part types are noted in boldface. The rules labeled as (b), (c), and (d) indicate that the current ratios are updated with no new part type entering, although the requirements of some part type(s) are completed. The label (2) indicates that Program **(P1)** is solved without fixture limitations and that the ratio value for at least one part type is greater than four. Then the label (1) indicates that **(P1)** is again solved after adding the constraint that restricts the maximum ratios values to be no larger than four.

The problems of Table II demonstrated here are static (i.e., orders are not arriving). A series of integer programs is solved, until all requirements of all twelve part types are completed. This is so that both approaches will have identical starting and ending conditions. This would be necessary for comparison purposes. As the approaches are applied over time, different part types are selected in different ratios.

However, this fixed problem setting is not quite appropriate to demonstrate the performance of the two approaches as they should usually be applied in dynamic settings. The typical situation where these approaches are applicable is dynamic, as production orders continuously arrive over time and the finished orders leave. If the FMS or FFS setting were fixed, other methods to solve these static part type selection problems need to be developed. However, the algorithm suggested here is appropriate for dynamic situations.

The objective function value of Program **(P1)** for the last of each series of runs depends on the distribution of the workloads per machine among the three machine types. For the last run, there is no choice as to which part types to select. The last of each set of runs is, hence, not representative of the relative performances of the two approaches. These ending conditions bias the results. For example, in Problem 2 of Table II, the sixth (last) objective function values are very large. This is because the remaining workload to finish the requirements of all twelve part types is much higher on the unpooled mill. This results in large overload values on the mill. This bias in the last run would tend to not occur in the more typical situation, in which random orders arrive over time.

The following observations about selecting part types and mix ratios can be made from Table III. The completion of each simulation run for the flexible approach, as a part type's requirements are finished, can lead to a minor tool changeover (changing a few cutting tools). There are six minor tool reloadings in Problem 1 and five partial changeovers for Problem 2.

There are three or four total changeovers for batching. For Problems 1 and 2, the batches are: {3,8,9,10}, {2,5,6,11}, {1,4}, and {7,12}. The ratios for both problems are also the same. This is because the processing time requirements for all twelve part types are the same. All solutions of **(P1)** for unbalancing workloads suggest various combination of 3–5 part types that are compatible for subsequent simultaneous machining. For Problem 1, these combinations are: {3,8,9,10}, {2,8,9,10,12}, {2,5,9,12}, {2,5,11,12}, {1,5,6,11}, {1,4,5,11}, and {1,5,7,11}. The combinations are different for Problem 2.

The objective function values for batching get worse whenever new mix ratios for the remaining part types in a particular batch are found. This leads to lower processing utilization. This is because new ratios are found as a part type's requirements are finished without the introduction of some new part type(s) that can make the system more highly utilized.

Setting $W = 100$ (a low number) in Program **(P1)** keeps the part mix ratio values small enough to be directly useful in such realistic situations as where there are limited numbers of pallets and dedicated fixtures. These low ratios values are useful to help solve subsequent scheduling problems, such as determining a good part input sequence (see Stecke [1985]). The summation of the ratios for each run of Table III is always less than nine. A change in W usually results only in a proportional change in the ratios. For example, if $W = 1000$ were used, the sums of the ratio values would all be a bit less than 90, which is too large to be able to permute to find an input sequence, for example.

The objective function values often tend to get larger with the number of runs. (This results in a much lower system utilization for the last of each set of runs.) This is because the problems considered here are static, having fixed orders with no new arrivals. In the more typical dynamic situation of orders arriving to the FMS continuously, a better objective function value can be anticipated. With fresh orders arriving, there would usually be a better opportunity to balance or unbalance workloads.

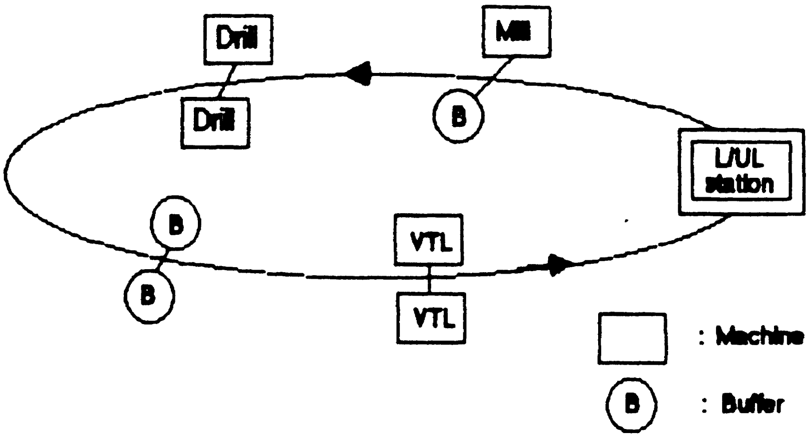
5. DESIGN OF THE SIMULATION EXPERIMENTS

In this section, we present some typical performance evaluation results for the flexible and batching approaches using a realistic simulation model of an FFS containing unequally sized pooled machine groups. §5.1 outlines the various parameters of the simulation model. In §5.2, the simulation results are analyzed. §5.3 presents the simulation studies that vary both the number of carts and travel times in the system.

5.1. The Simulation Model

The simulation model is developed in GPSS/H. The FMS configuration is provided in Figure 1. It is a flow system with unidirectional transportation. There are one mill, two pooled drills, and two pooled VTLs. There are three buffer spaces, one after the mill and two in between the drills and lathes. All part types share the load/unload station having five storages. Other system resources are fixtures of different types, pallets, and carts (wire-guided vehicles). There are five carts. (This may be too many. To study system utilization and blocking, etcetera, we did not want to confound these with cart restrictions. This is relaxed in §5.3, to show the additional effect of not having enough carts.)

FIGURE 1. System Configuration.



This issue of limited resources is also investigated in a related part mix ratio simulation study (Schriber and Stecke [1986]). This study investigates the effects of a limited central buffer of various sizes on a similar but not identical FFS. Transportation times in the system are a linear function of the distance being traveled. Travel times are one minute between all links, i.e., between: L/UL-mill; mill-drill; buffer-drill; drill-buffer;

buffer-VTL; VTL-L/UL.

A fixed number of parts (representing the pallet limitations) of mixed types having nonzero ratio values is always in the system. When the production requirements of some part type are finished, a current simulation run is terminated. When one or more new part types are selected to be input into the system, new ratios are found to begin the next run. Otherwise, if no new part type is to enter, the current simulation run continues. However, new *optimal* ratios are found for the reduced set of part types.

There was no precise algorithm that we could find in the literature to determine the part input sequence into a flow shop having pooled machines. The part input sequence into the flow shop is determined here by using a combination of a modified Johnson's algorithm (see Campbell, Dudek, and Smith [1970]) and the current part mix ratios as follows.

PART INPUT SEQUENCE ALGORITHM

Step 1. All part types that have production requirements are ordered according to a modified Johnson's algorithm.

Step 2. Whenever new part mix ratios are found, the part input sequence follows the new part mix ratios exactly, in the order for part types as specified at **Step 1.**

This algorithm is intuitively appropriate to balance workloads and performs well in our studies to date. This input sequencing method is followed because, on average, it will provide the desired unbalanced or balanced workloads over time. However, further research is required to determine a good part input sequence and to compare the algorithm suggested here to other methods.

In the simulations, the selected part types are always input to the system according to the following sequence: 9, 5, 10, 11, 1, 2, 7, 12, 6, 3, 4, and 8. This ordering of part types comes from the modified Johnson's algorithm as applied to the production data of Table II. When a machine and cart become available, a part can be moved. When two or more parts wait for the machine, FCFS is used.

5.2. Experimental Findings

In this section, we present simulation results of the performance of the flexible and batching approaches. These are the results of the simulation studies that are discussed in §4.2, where the implementation of the approaches is demonstrated. The number of pallets in the system is eight and there are fixture limitations for each part type for Problems 1 and 2 of Table II. The part mix ratios provided in Table III are used in the series of simulations.

The measures of system performance are as follows. *Processing* (*transportation*, *blocking) utilizations* are found for each machine type. These indicate the proportions of total processing (transportation, blocking) times to total makespan. *Processing utilization* is calculated as the ratio of total actual machining time to total makespan, for each machine type: Mill, Drill, and VTL. *Machine utilization* is expressed as the sum of processing, transportation, and blocking utilizations, for each machine type. *System utilization* is a weighted average of the *processing* utilizations of the three machine types and is a measure of overall system usage. System utilization is equal to the sum of the Mill processing utilization, twice the Drill processing utilization, and twice the VTL processing utilization, and divided by five.

The machine and system utilizations in the subsequent Figures are average values. These are cumulative utilizations and calculated as requirements are completed after each simulation run. The difference between machine and system utilizations provides the average amount of time spent in transportation and blocked. The *all machines utilization* is a weighted average of the utilizations of the machines and is calculated as the sum of the Mill *machine utilization*, twice the Drill machine utilization, and twice the VTL machine utilization, and divided by five.

Results on makespan are presented. However, these are not representative as the part type selection approaches presented here are developed for dynamic situations. The ending conditions of finishing a few remaining requirements will decrease system performance here.

Tables IV and V provide results for the first two Problems of Table II on the machine, processing, and system utilizations as well as makespan. The higher utilizations and lower makespans are noted in boldface. Figures 2 and 3 show the *cumulative* machine and system utilizations for each of the distinct simulation runs required to finish the requirements of all part types for the two cases, with and without fixture limitations for each part type. Table V provides the average utilizations both for all runs *and* for all runs except for the last run.

The following observations can be made from Tables IV and V and Figures 2 and 3.

(1) For Problem 1, the *flexible approach results in higher system utilization than batching* (82.9% and 80.5%: The difference is 2.4%). With no fixture limitations, the difference is 2.6%. This is consistent with the decrease in makespan for the flexible approach. In almost all of the problems examined, the flexible approach performed better.

(2) It can be seen in Figure 3 of Problem 2 that the system utilization for the flexible approach is better than batching until the sixth and last simulation run. The system utilization for the flexible approach for the last run is poorer because of the

TABLE IV. Simulation Results of Table III After the Completion of All Production Requirements of All Twelve Part Types for Problem 1.

Comparison	Four Fixtures		No Limitations	
	FLEX	BATCH	FLEX	BATCH
Makespan (minutes)	**6486**	6678	**6476**	6689
Mill Utilization	**.892**	.826	.901	**.918**
-Processing Utilization	**.683**	.663	**.684**	.662
– –Transportation Utilization	.076	.064	.076	.070
– –Blocking Utilization	.133	.099	.141	.186
Drill Utilization	**.901**	.859	**.902**	.868
-Processing Utilization	**.854**	.829	**.855**	.828
– –Transportation Utilization	.029	.029	.030	.030
– –Blocking Utilization	.018	.001	.017	.010
VTL Utilization	**.916**	.890	**.917**	.902
-Processing Utilization	**.878**	.852	**.879**	.851
– –Transportation Utilization	.038	.037	.038	.039
– –Blocking Utilization	.000	.001	.000	.012
System Utilization	**.829**	.805	**.830**	.804
Average Buffer Utilization	.466	.290	.478	.415
Cart Utilization	.060	.054	.060	.057
Number of Dedicated Fixtures	42	43	50	72
CPU Time (seconds)	1.798	1.542	1.733	1.543

ending conditions (see Table III). These ending conditions result in most of the machine and processing utilizations in Table V to be better from batching. With the last run of Problem 2 (the ending conditions) deleted, the flexible approach provides better system performance. See the parenthetical values of Table V.

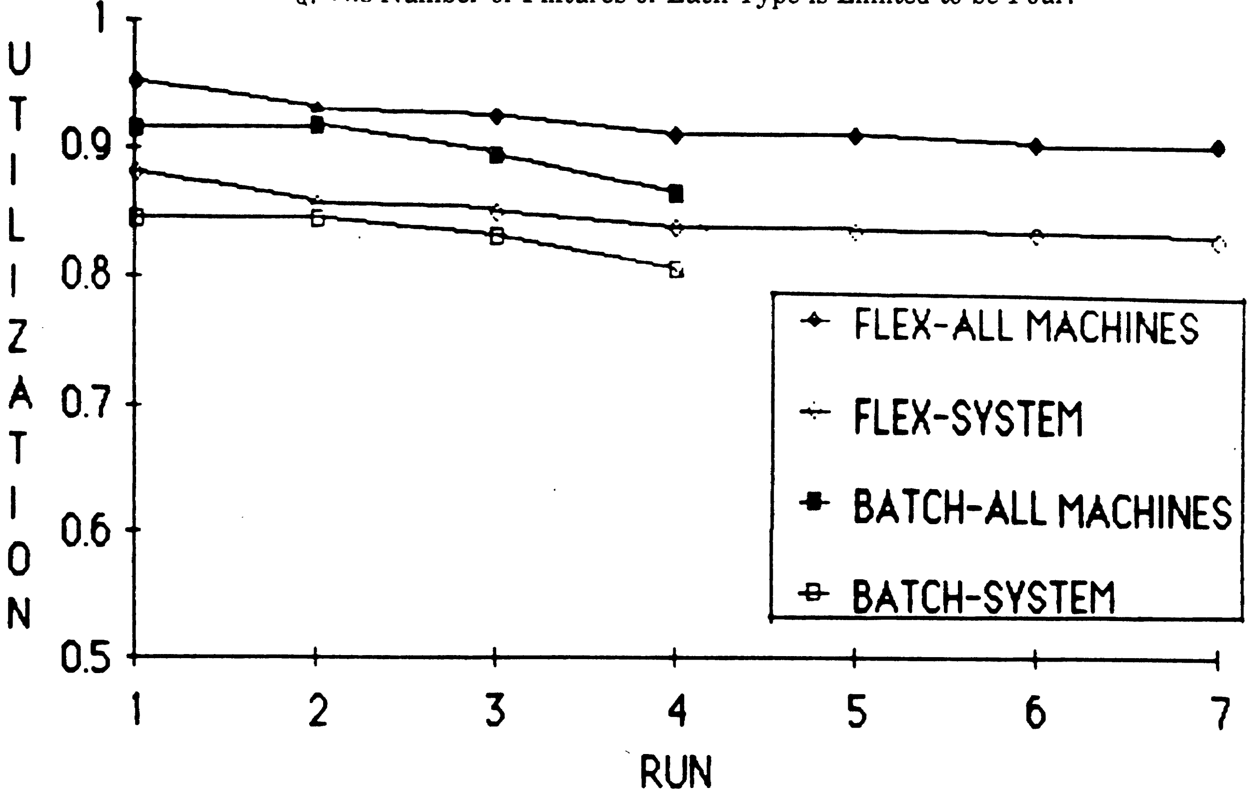
(3) When there are no fixture limitations, the flexible approach requires many fewer fixtures dedicated to each part type than batching. This is because when all requirements of the part types in a particular batch except for one part type are completed, batching has only the remnants of that single part type having remaining requirements to process. These few remaining requirements then require additional fixtures for that part type to be finished.

(4) The utilizations decrease quicker with the number of runs in Problem 2 than in

FIGURE 2.

Cumulative Utilizations of the Flexible and Batching Approaches for Problem 1.

a. The Number of Fixtures of Each Type is Limited to be Four.



b. No Fixture Limitations.

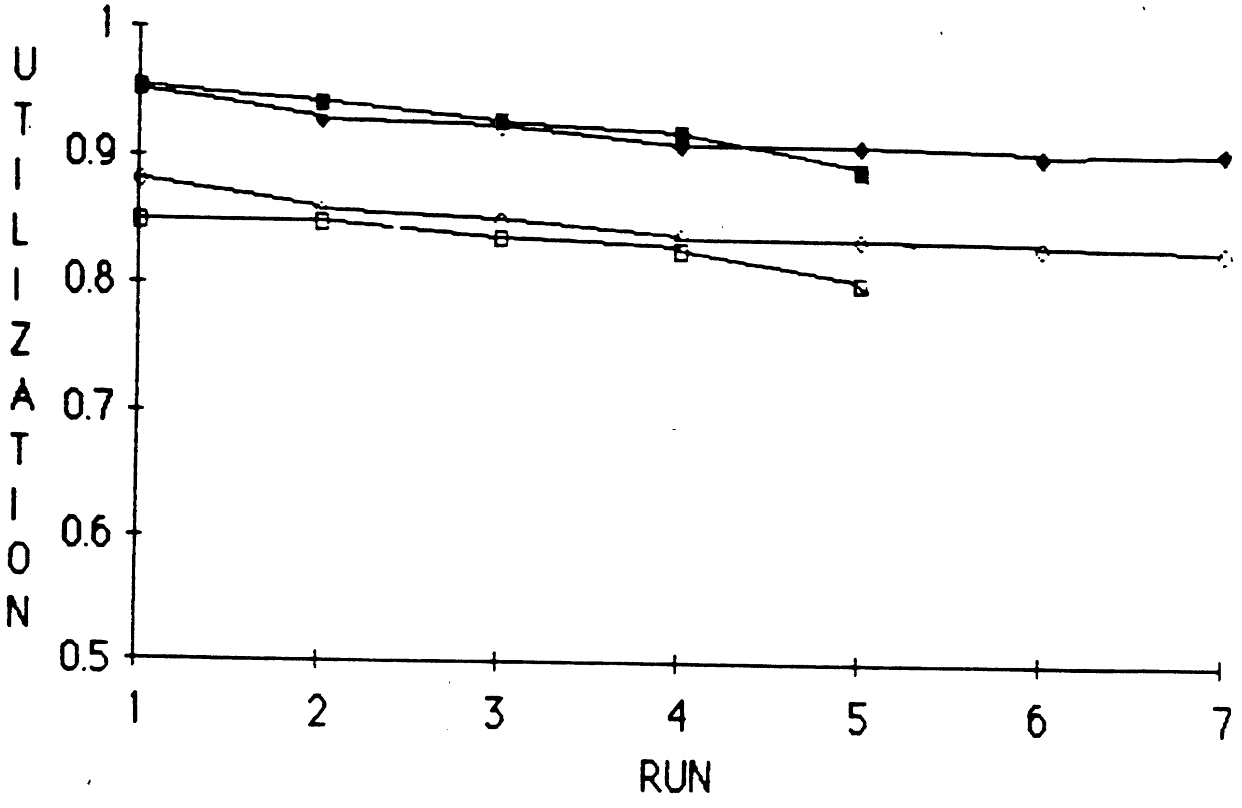


TABLE V. Simulation Results of Table III After the Completion of All Production Requirements of All Twelve Part Types for Problem 2.

Comparison	Four Fixtures		No Limitations	
	FLEX	BATCH	FLEX	BATCH
Makespan (minutes)	6046	**5945**	6037	**5998**
Mill Utilization	.937 (.933)	**.959** (**.965**)	.949 (.947)	**.961** (**.968**)
-Processing Utilization	.810 (.790)	**.824** (**.816**)	.811 (.792)	**.817** (**.811**)
— —Transportation Utilization	.064 (.069)	.064 (.069)	.066 (.071)	.067 (.071)
— —Blocking Utilization	.063 (.074)	.071 (.080)	.072 (.084)	.077 (.086)
Drill Utilization	.786 (**.865**)	**.798** (.857)	.787 (**.863**)	**.793** (.846)
-Processing Utilization	.752 (**.828**)	**.765** (.822)	.754 (**.826**)	**.758** (.810)
— —Transportation Utilization	.030 (.032)	.032 (.033)	.029 (.032)	.032 (.033)
— —Blocking Utilization	.004 (.005)	.001 (.002)	.004 (.005)	.003 (.003)
VTL Utilization	.795 (**.859**)	**.809** (.831)	**.841** (**.858**)	.823 (.830)
-Processing Utilization	.749 (**.814**)	**.762** (.782)	.751 (**.814**)	**.755** (.778)
— —Transportation Utilization	.043 (.045)	.041 (.042)	.041 (.044)	.040 (.042)
— —Blocking Utilization	.003 (.000)	.006 (.007)	.049 (.000)	.028 (.010)
System Utilization	.762 (**.815**)	**.776** (.805)	.764 (**.814**)	**.769** (.797)
Average Buffer Utilization	.222 (.263)	.207 (.223)	.239 (.271)	.222 (.241)
Cart Utilization	.057 (.061)	.057 (.060)	.057 (.061)	.058 (.061)
Number of Dedicated Fixtures	47	48	69	75
CPU Time (seconds)	1.520	1.400	1.545	1.438

( ) indicates cumulative utilizations minus the last run.

Problem 1. This is because the workloads are distributed more to the pooled drills and VTLs in Problem 1. This allows the optimal objective function value for Problem 1 to be maintained better for each run.

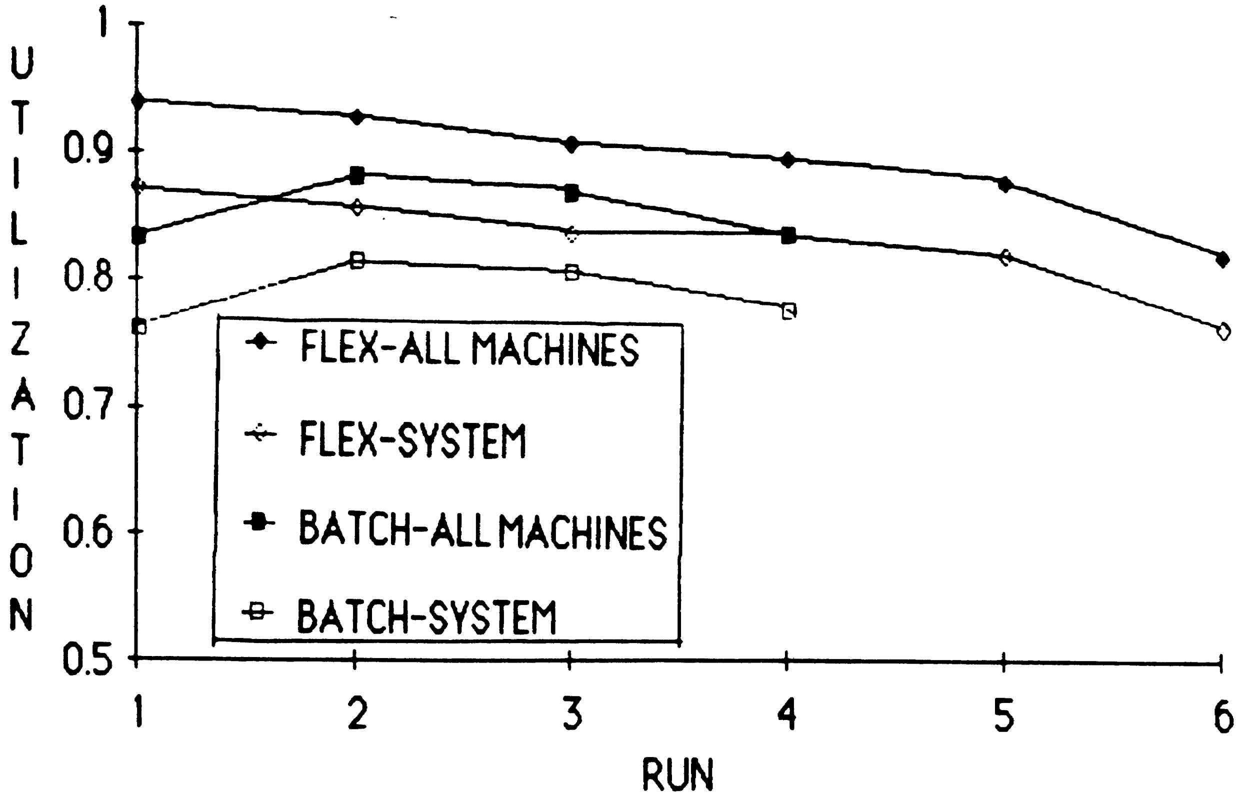
(5) In Problems 1 and 2, the makespans for batching are longer when there are no fixture limitations than when there is the limit of four fixtures for each part type. This is also because there is more blocking. The amount of blocking can increase significantly with this batching approach (see Table IV).

(6) In all Figures, the decreasing slopes on the cumulative utilizations of the batching approach are steeper than those of the flexible approach. This indicates that the use of the flexible approach enables the system to be utilized more constantly to

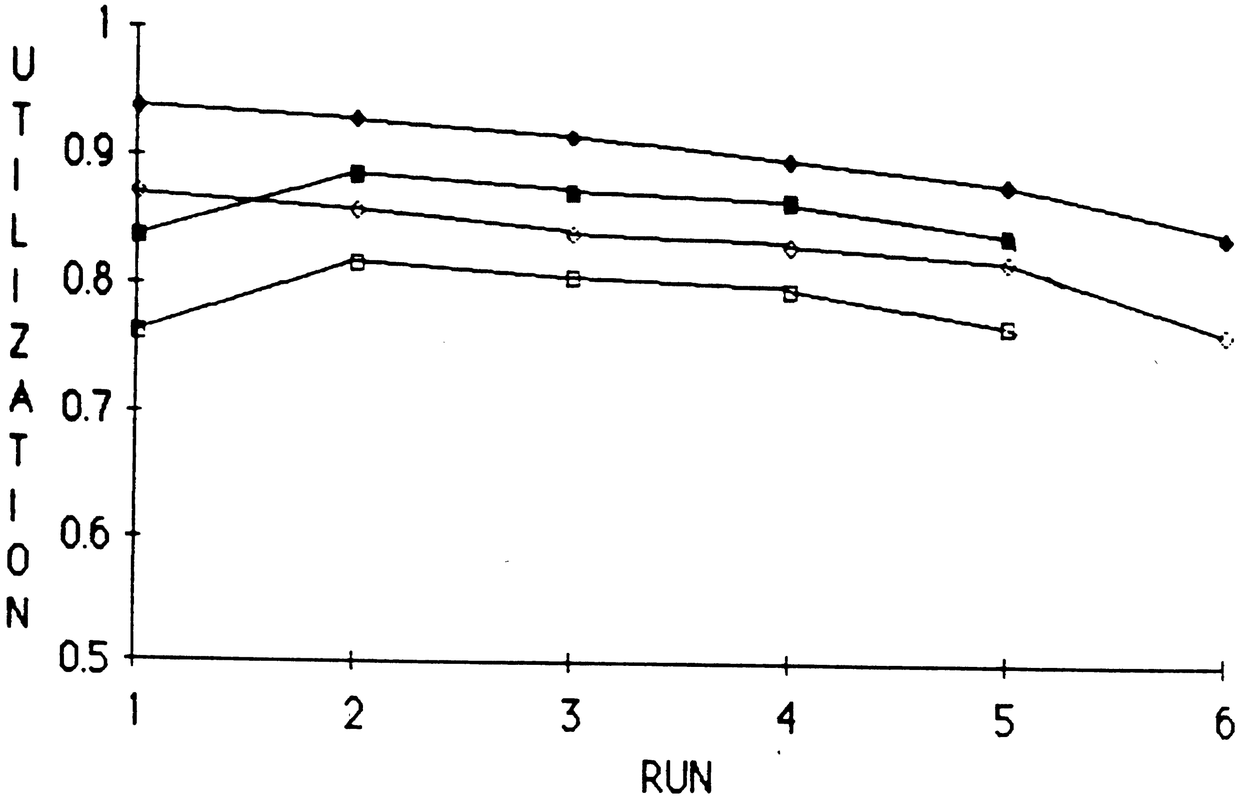
FIGURE 3.

Cumulative Utilizations of the Flexible and Batching Approaches for Problem 2.

a. The Number of Fixtures of Each Type is Limited to be Four.



b. No Fixture Limitations.



finish all requirements of all part types.

(7) For Problem 1, the buffers are used nearly 50% of the time. This indicates that perhaps more buffers could help increase system performance. A breakdown of buffer usage according to location would be required. Carts have very low utilization, as intended.

We can conclude that for the problems presented here, the flexible approach tends to increase overall system utilization, at least in the situations examined to date. The Figures show the flexible approach to be better than batching until the last run. These last run ending conditions would not occur in reality, in the types of FFSs for which the approaches suggested here are appropriate. In these, orders would continuously arrive to the system. Some of these observations are particular to these problem sets while most are general.

The tendencies demonstrated here are validated in other simulation studies. These studies vary the number of parts, number and groups of machines, processing times, workload distributions, and production requirements, all for FFSs. Balanced and unbalanced workloads per machine are examined. These results can be found in Stecke and Kim [1986b].

5.3. Studies that Vary the Number of Carts and Travel Times

In this section, the results of simulations are reported that vary both the number of transportation vehicles and the travel times for the same FFS of Figure 1. These studies use the part mix ratios of Table III selected by the flexible approach for Problem 1. This study is performed in order to investigate the effects of having both cart restrictions and different travel times on system performances as well as to further demonstrate some implications of using the flexible approach.

The numbers of carts in the system are varied as two, three, four, and five. Two different travel times between all links are considered: one and two minutes. A representative sample of the simulation results are provided in Table VI, from which the following observations can be made. These are representative of similar results obtained by varying the many problem parameters listed at the end of §5.2.

(1) Decreasing the number of carts significantly lowers *system utilization.* Related to this, makespan increases.

(2) Increasing the number of carts leads to lower *machine utilizations* for the mill and VTLs, both when there is a limit of four fixtures of each type and when there is no fixture limitation. However, processing utilizations increase with the number of carts for all machine types. This is because the amount of time spent in transportation and blocking decreases as more carts are available.

TABLE VI. Simulation Results Varying Both the Number of Carts and Travel Times for Problem 1.

Comparison (*Four Fixtures*)	2 Carts		3 Carts		4 Carts		5 Carts	
	T=1(1)	T=2(2)	T=1	T=2	T=1	T=2	T=1	T=2
Makespan	8140	10191	7004	7812	6652	7091	6486	6816
Mill Utilization	.904	.916	.896	.903	.896	.897	.892	.893
-**Processing Utilization**	.544	.434	.632	.567	.666	.624	.683	.650
— —Transportation Utilization	.246	.380	.172	.291	.116	.204	.076	.158
— —Blocking Utilization	.114	.102	.092	.045	.114	.069	.133	.085
Drill Utilization	.909	.909	.886	.872	.897	.881	.901	.890
-**Processing Utilization**	.680	.543	.790	.709	.832	.781	.854	.812
— —Transportation Utilization	.090	.150	.067	.122	.045	.083	.029	.063
— —Blocking Utilization	.139	.216	.029	.041	.020	.017	.018	.015
VTL Utilization	.944	.947	.934	.940	.919	.916	.916	.915
-**Processing Utilization**	.699	.558	.813	.729	.856	.803	.878	.835
— —Transportation Utilization	.111	.179	.083	.147	.061	.107	.038	.080
— —Blocking Utilization	.134	.210	.038	.064	.002	.006	.000	.000
System Utilization	.660	.527	.768	.689	.808	.758	.829	.789
Average Buffer Utilization	.721	.769	.512	.501	.461	.413	.466	.434
Cart Utilization	.359	.573	.182	.320	.102	.183	.060	.112
Number of Dedicated Fixtures	41	42	42	45	42	43	42	42

(1) "T=1" means that the travel time between all links is one minute.
(2) "T=2" means that the travel time between all links is two minutes.

TABLE VI (CONTINUED). Simulation Results Varying Both the Number of Carts and Travel Times for Problem 1.

Comparison (*No Limitations*)	2 Carts		3 Carts		4 Carts		5 Carts	
	T=1(1)	T=2(2)	T=1	T=2	T=1	T=2	T=1	T=2
Makespan	8206	10322	6994	7855	6650	7100	6476	6812
Mill Utilization	.911	.924	.905	.909	.905	.902	.901	.902
-Processing Utilization	.539	.429	.633	.564	.666	.624	.684	.650
– –Transportation Utilization	.250	.390	.173	.295	.117	.202	.076	.159
– –Blocking Utilization	.122	.105	.099	.050	.122	.076	.141	.093
Drill Utilization	.912	.912	.886	.871	.898	.879	.902	.891
-Processing Utilization	.675	.536	.792	.705	.833	.780	.855	.813
– –Transportation Utilization	.092	.151	.064	.122	.045	.082	.030	.063
– –Blocking Utilization	.145	.225	.030	.044	.020	.017	.017	.015
VTL Utilization	.946	.949	.937	.940	.921	.914	.917	.916
-Processing Utilization	.694	.551	.814	.725	.856	.802	.879	.836
– –Transportation Utilization	.113	.177	.084	.150	.062	.106	.038	.080
– –Blocking Utilization	.139	.221	.039	.065	.003	.006	.000	.000
System Utilization	.655	.521	.769	.685	.809	.758	.830	.790
Average Buffer Utilization	.741	.799	.536	.516	.473	.423	.478	.445
Cart Utilization	.364	.579	.182	.323	.103	.183	.060	.123
Number of Dedicated Fixtures	48	50	50	49	50	51	50	51

(1) "T=1" means that the travel time between all links is one minute.
(2) "T=2" means that the travel time between all links is two minutes.

(3) The increase in travel times, from one to two minutes between all links, leads to longer makespans as well as increased cart utilizations. *Machine utilizations* increase in general for the mill and VTL, while *processing utilizations* decrease for all machine types.

(4) The cart and buffer utilizations are higher when there is no fixture limitation than when the number of fixtures of each type is limited to be four.

(5) Having no limitation on the number of fixtures of each type (except for the case of five carts in the system) does not lead to either a better makespan or a better system utilization when the travel time between all links is two minutes.

(6) The total numbers of dedicated fixtures that are required for the different travel times and different numbers of carts in the system are similar.

(7) Increasing the number of carts results in a smaller amount of time spent in blocking for the drills and VTLs, but blocking for the mill is minimized when three carts are utilized.

One might suggest from this study that the appropriate number of carts for this system might be three. This is because the largest marginal improvement in system utilization is attained for three carts. When transportation time is not large with respect to processing time, the appropriate number of carts might be much less than the number of machine tools in the system. However, as the number of carts increases to five, system utilization does increase significantly. Even though cart utilization is extremely low, the additional production and resultant decrease in idle time due to transportation time, waiting time, and blocking may make five carts the most desirable choice. An economic evaluation comparing the costs of AGVs or other types of transportation vehicles and the additional production would be required to analyze these trade-offs.

6. SUMMARY AND CONCLUSIONS

This paper presents a flexible approach to short-term FMS production planning. This paper shows how existing decision procedures that determine the relative part mix ratios that balance workloads of ordered part types contribute also to selecting the set of part types to be machined simultaneously in an FMS that manufactures independent part types. This paper also demonstrates how these same part mix ratios can be useful in determining a part input sequence that will tend to balance workloads over time.

For the types of systems that machine independent part types with varying numbers of production requirements, the objectives of balancing or unbalancing workloads is applied to select part types and determine mix ratios dynamically over time. Those part types with near zero ratio values in the optimal solutions to Program **(P1)** are not selected to be in the part mix to be machined together over the immediate and flexible time period.

Both flexible and batching approaches to part type selection, each appropriate in different types of FMSs, are demonstrated. The approaches as presented here are appropriate for systems in which orders arrive continuously and for systems that need to attain a high utilization. We caution that different flexible or batching approaches are required for systems that need to minimize makespan.

The research results show that the use of the flexible approach to short-term production planning can help the system attain a high utilization as well as to be utilized more constantly over time. It is also observed that a system using the flexible approach requires many fewer dedicated fixtures than batching. Use of a flexible approach results in a decrease in, and smoothing of, tool changeover time. This is because when a part type finishes production only those few tools affected need to be changed, which can be a minor changeover. The reduction in tool changeover time can also enable the use of such a flexible approach to lead to higher FMS productivity.

The problems that have been demonstrated here, as well as others (see Stecke and Kim [1986b]), are typical of the sizes of real FMSs. Most FMSs consist of one, two, or three types of machines (some combination of mill, drills, machining centers, lathes, and head indexers). In Program **(P1)**, it is the number of machines that dictates the size of the integer program. Also, in most FMSs, the number of part types is much larger than the number of machine types. This means that usually there will be many sets of optimal ratios that balance workloads. Varying the objective function coefficients directs the search to a different set of part types with optimal ratios.

Also, all of the problems here were run to integer optimality, for demonstration purposes. However, Program **(P1)** could be run as a linear program by deleting the requirement that a_i be integer. Nonlinear ratio values could be interpreted as follows. If $a_i = .5$, for example, this means that a part of type i is input into the system in every other cycle of part types. Although, in general, the integer programs will not be too large, they could also be run as linear programs.

Machine breakdowns can be handled. Program **(P1)** would be run again, for one less machine of the type that was down. New optimal ratios would be found that balance workloads on the reduced set of machines. The methods proposed here are quick and efficient.

There are further research needs along these lines. The studies reported here are for a flow shop type of system. Alternative routing is allowed through the availability of pooled machine groups, but the routing is still unidirectional. Similar studies should be done in a job shop environment, where parts can follow different routes through the system. Other constraints, such as tool magazine capacity and due dates, should also be considered when selecting the most appropriate part mix.

Further research is also required to develop analogous flexible and batching approaches to part type selection that would be appropriate in a more static environment, where there are production requirements for several part types, but new orders are not continuously arriving to supplement the list of requirements. In this environment, minimizing makespan is an appropriate system objective.

Determining the appropriate approach to both selecting part types and determining mix ratios is also needed when the demand for part types is dependent and certain relative output ratios are required. Other issues that need to be addressed regarding part mix ratios are the interactions of these ratios with subsequent FMS planning and operating problems. For example, further research is required to develop procedures to determine a good part input sequence.

ACKNOWLEDGEMENTS

The authors gratefully thank the Referees for their careful reading and thoughtful suggestions which improve the clarity of the presentation.

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