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PROCEDURES TO DETERMINE PART MIX RATIOS FOR INDEPENDENT DEMANDS IN FLEXIBLE MANUFACTURING SYSTEMS

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ABSTRACT

Five production planning problems have been defined to address system set-up decisions for flexible manufacturing systems (FMSs). This paper focuses on one of these planning problems that has not been sufficiently addressed either in practice or in the literature. The problem is to determine the part mix ratios at which a set of part types should be produced over the next production period. These ratios specify the *relative numbers of parts* of each part type that will be produced for an upcoming time period.

Integer programming is used to solve this problem, with computations based on machine utilizations. Several objectives are discussed, each applicable in different FMS situations. However, this paper illustrates one objective, balancing machine workload to eliminate bottlenecks.

The ratios provide input into more detailed models that can be used to evaluate these part mix ratios. The following three complementary models are suggested: (1) a stochastic, multiclass, closed queueing network model, providing pessimistic, aggregate, and steady state performance evaluation results; (2) a deterministic, timed Petri net model, and its associated algebraic representation, which provides optimistic results; (3) simulation, which provides the detailed modeling capabilities necessary to evaluate the performance of the ratios for research purposes. Either of the first two models is adequate to evaluate the ratios in practice.

The part mix ratios can be useful to help solve other planning and operating problems. For example, they can establish guidelines to help in determining appropriate part input sequences or they can be used to select the next part types for production. They can reduce subsequent scheduling problems by decreasing the set of feasible alternatives.

Another purpose of this paper is to propose simple, yet effective solution procedures to a particular FMS planning problem. These procedures can be the basis of a new approach to operating an FMS in a more productive manner than usual. Computational tests reported indicate that the procedures suggested are fast and accurate enough to be of considerable practical value.

1. INTRODUCTION

A metal-cutting flexible manufacturing system (FMS) consists of a set of computerized numerically controlled machine tools connected via an automated material handling system. A high level of automation allows efficient and flexible simultaneous machining of a variety of part types in unit batch sizes. The operation of these systems is different from the traditional assembly line or job shop. The FMS planning, scheduling, and control problems have sometimes similar, but often different counterparts in the conventional manufacturing systems.

Five production planning problems were defined in Stecke [1983] to help an FMS manager set up a system in an efficient and productive manner prior to the start of production. These planning problems are addressed and implemented periodically, and in advance of the start of production of a new or different part mix. Suri and Whitney [1984] refer to such problems as "second level decisions," that should be addressed over a time period of several days or weeks.

1. Part types that are to be produced next, and simultaneously over the upcoming time period, are selected.
2. Within each machine type, machines may be partitioned into identically-tooled machine groups. Each machine within a pooled group can perform the same operations.
3. The part mix ratios at which the selected part types should be produced over time are determined.
4. The minimum numbers of pallets and fixtures of different fixture types required to maintain the part mix ratios are determined.
5. The cutting tools of all operations of the selected part types have to be loaded into a machine's (one or more) limited capacity tool magazine in advance of production. This determines which machines each operation can be performed on during the real-time production of parts.

Several of these problems have been addressed previously at various levels of detail. This paper assesses problem three.

The purposes of this paper are to present the concept of part mix ratios and a simple but effective solution approach to help avoid bottlenecks and also to demonstrate a variety of beneficial uses for the ratios. They can be used to link many of the planning and subsequent scheduling problems.

The FMS scenario is described as follows. There are production requirements, defined as the number of parts required of each part type. These requirements may change over time and are derived either from some forecast of demand, or actual customer orders, or to maintain an inventory. Depending on many factors such as system capacity (time or tool magazine), or due dates usually only a subset of the part types may be selected for simultaneous production over the upcoming time period. When the requirements for some part types are finished, space is then available in the tool magazines. Either one or more part types can be input into the system (if space for all cutting tools can be found) or perhaps only the reduced set of part types can be machined (it may be that more machines can be pooled into the same group--see Stecke and Kim [1989, 1991]). Alternative heuristics to select the part types to be produced next are suggested by Whitney and Gaul [1985], Hwang [1986], and Rajagopalan [1986]. Their approaches partition all part types having requirements into distinct batches, then machine one batch at a time.

The grouping and loading problems (problems 2 and 5) have been treated at several levels of detail. Queueing networks have been used to characterize appropriate solutions to these and other FMS problems at an aggregate level of detail. They are used to provide qualitative or operational insights in Buzacott and Shanthikumar [1980], Cavaillé and Dubois [1982], Dallery and David [1983], Dubois [1983], Hildebrant [1980], Solberg [1977, 1979], Shanthikumar and Stecke [1986], Stecke and Solberg [1985], Suri [1983], and Suri and Hildebrant [1984], for example. At a detailed level, the planning problems have been addressed using mathematical programming (Arbib et al. [1991], Afentakis [1986], Kiran and Tansel [1986], Hwang [1986], Stecke [1983], Berrada and Stecke [1986], Stecke and Kim [1991]), or heuristics (Mazzola et al. [1989], Stecke and Talbot [1983], Whitney and Gaul [1985], Rajagopalan [1986]). Many of these studies assume that the part mix problems have already been solved.

The problem addressed here, the third planning problem, is the following: Given the production and processing time requirements on each machine type, determine the part mix ratios of the part types that have been selected to be produced over the next time period. The part mix ratios are the relative ***numbers of parts*** of each type that will be produced in a cyclical manner during the next time period.

The approach to determine mix ratios is to balance machine workloads to avoid bottlenecks. This in turn helps machine utilization. The approaches developed in this paper can also be used to help address the problem of meeting daily (or monthly) due dates. Both the part types that must be produced and their requirements can vary daily. One appropriate means to help meet such daily deadlines is to try to increase machine utilization by balancing machine workload. Utilizing the FMS as much as possible is one aid to help meet daily due dates, especially if a breakdown occurs or a rush part type arrives, causing workloads to shift. An Iveco-Torino FMS operates with this objective (Stecke [1989]).

These procedures can be applied as follows. Given the day's order for some particular part types and their requirements, an *initial subset* of these can be selected, subject to tool magazine capacity constraints, and mix ratios determined to balance workloads. Production of these selected part types can commence in a *cyclical* manner following the calculated *mix ratios*, *until* the requirements for some part type are finished. The tools required for this completed part type can be taken out of the magazines to free space for perhaps a new part type or more. Then new mix ratios that balance workloads are calculated for both those part types having *remaining requirements* to satisfy and any new part types that may be selected to enter. Manipulating the mix ratio of any part type can guarantee that its requirements will be met. The dynamics of such a flexible approach to FMS operation over time is the subject of ongoing research. (See Stecke and Kim [1988, 1991].)

Many FMSs produce the same part types continuously, for inventory purposes. Maximum utilization is a stated objective of many users (i.e., Boeing, Vought Aeroproducts, Caterpillar Tractor, John Deere, Renault, Olomouc, Celakovice, Fiat Trattori, and Iveco-Bourbon Lancy). Although some of these systems are driven by daily or monthly due dates, these due dates can sometimes be viewed as constraints. A means *to help* attain such due dates is to maximize system utilization, i.e., machine cutting time.

However, due dates are not explicitly considered here. The procedures are not proposed to address the classical job shop due date problem, where there are orders for part types and their requirements that are due on future dates. The preceding discussion aims to motivate a consideration of the use of the part mix formulations proposed here to drive FMS production.

The plan of the paper is as follows. §2 begins by presenting two objectives to determine part mix ratios. Each is appropriate for different FMS types. We describe situations in which each is better. Then the operational advantages of following each objective are presented. Each objective is intimately related to a particular approach to select the part types to be produced next and this relationship is discussed. The remainder of the paper focuses on the second objective. §3 suggests solution approaches to determine mix ratios of a selected set of part types to balance machine workload. Additional uses of the determined ratios are suggested and discussed in §4. For many FMSs, the ratios can be used directly. For complex systems (with parts visiting many machines for example), the ratios may need to be revised. In §5, three models are suggested to evaluate the ratios. Future research needs are identified in §6.

2. OBJECTIVES IN DETERMINING PART MIX RATIOS

There are different objectives in determining mix ratios. The appropriate objective is a function of the FMS type. For an FMS with dependent demand, where several part types are *required* in relative ratios, the output should be *proportional* to the requirements. The output can be directly translated into appropriate mix ratios (see Stecke [1985]). Other FMSs produce part types with independent demands. The demand for one part type is independent of the demand for another. The output does not have to be proportional to the requirements. In this case, the mix ratios can be calculated to balance workloads, to help attain a good utilization. Two distinct and relevant objectives can be used to determine the mix ratios for the two scenarios of dependent and independent demand. One objective is to begin and finish all requirements of all selected parts during the same time period. The second objective is to balance workloads.

The objective of processing all requirements of a batch of part types during the same time can be applicable in various situations. For example, several part types may be required in predetermined ratios to be fed to downstream workstations, say, for subsequent assembly purposes. In another example, suppose that several part types are designated for the same customer. Then the orders should be shipped together when completed to keep freight costs down. Also, there may be only a small area to store finished goods inventory. Then batching customer requirements will help minimize finished goods inventory.

This objective may be appropriate in an MRP environment (dependent demand), where predetermined quantities (obtained by exploding the bills of materials) of several components must be manufactured within a particular time bucket. Within each time bucket are the requirements of several part types, all to be produced during the same time period.

There are reasons for operating a system in this manner. In particular, this objective tends to minimize the frequency of tool changes. Yet this approach unfortunately defines the workload, which is usually unbalanced, and the bottleneck machine type. In general, it neither maximizes production nor utilization. It also requires a longer tool changing time. However, we have indicated when this may be an appropriate approach for some FMSs, for example, if demand for some part types is dependent and the part types are required in certain relative output ratios.

Each objective to determine mix ratios is intimately related to an approach to select the part types to be produced next. Largely because of tool magazine capacity constraints, all part types having production requirements cannot usually be machined simultaneously.

The usual approach taken by researchers and industry to select part types is batching (Whitney and Gaul [1985], Hwang [1986], Rajagopalan [1986]). All part types are partitioned into distinct batches and each batch of part types is machined one at a time, consecutively. Before production begins, each required cutting tool for the batch has to find a place in the tool magazines. In a particular batch, all part types are produced until all requirements are finished, addressing the first objective. Then all tools are taken out of the tool magazines and new tools are put in for the next batch. This system set-up can take up to a shift of person-hours to perform. As described earlier, there are reasons to follow this approach.

The second objective of balancing workloads is appropriate for those systems that produce independent part types. There is freedom to determine the relative ratios in which independent part types are produced. A subset of part types are selected and mix ratios that balance workloads are calculated by the methods suggested in this paper. FMS production begins and continues cyclically according to the mix ratios until the requirements of a part type are completed. Then either the reduced set of part types (having remaining requirements) are produced in new ratios that balance workloads or new part types are selected to supplement the others and ratios for all are found to balance workloads. Since only the tools for the few finished part types are changed, the system setup time is short. Tool changing occurs more often, but takes less time. Also, this flexible approach smooths the use of the set-up people, who change the cutting tools. They work continuously, rather than intermittently in between batches. Another advantage is that while some tools are changed on a machine, the other machines can still cut metal. This can't occur when batching, because there is no work to be done until all tools are changed: the requirements of the current batch are finished.

For example, suppose that different components are to be machined in fixed ratios but for different assemblies. The ratios of final assemblies can be determined to balance workloads.

A study compares seven approaches to FMS part type selection from the literature (six batching and a flexible approach) (Stecke and Kim [1988]). The flexible approach provides better machine utilizations, makespan, and system utilization than others. Many simulation studies of realistic FMSs demonstrate that by determining mix ratios to balance workloads, idle time tends to decrease, the amount of buffer space required is less than it would be otherwise, less lead time is required, and on-hand inventory requirements can be smaller. Some theoretical justification of the latter observation is provided by Shanthikumar and Stecke [1986].

The purpose of this discussion of part type selection procedures over time is to motivate a consideration of mix ratio approaches. Although mix ratios can be found to begin and end production of several part types together (see Stecke [1985]), it is sufficient to produce parts in proportion to their requirements. This first objective is not considered further.

The focus is on methods to determine mix ratios that balance workloads. In §3, this problem is formulated as a parametric integer program. Examples demonstrate the usefulness and further research needs regarding the use of these ratios.

3. BALANCING WORKLOAD PER MACHINE

Given the production and processing time requirements of each of a selected set of part types on each machine type, the problem is to determine relative ratios at which the part types should be machined cyclically, so as to keep the workloads on the machine types balanced. Notation is provided in Table I. The following integer formulation, Problem (P1), provides these ratios. This is the simplest version of such formulations, for the purpose of explaining the nature of the balancing problem.

(P1) Minimize $\sum_{k=1}^{K} C_{k1} X_{k1} + \sum_{k=1}^{K} C_{k2} X_{k2}$

subject to

$$\sum_{i=1}^{N} a_i \, pw_{ik} - X_{k1} + X_{k2} = W_k, \qquad k = 1,\ldots,K$$

$$a_i \leq f_i, \qquad i = 1,\ldots,N$$

$$a_i \geq 1, \text{ (} a_i \text{ are integer)} \qquad i = 1,\ldots,N$$

$$X_{k1}, X_{k2} \geq 0, \qquad k = 1,\ldots,K$$

The first constraint defines the target workload over time. If $W_k = W$, for all k, then the target workloads are balanced. The W_k can be unbalanced target workloads for systems with groups containing unequal numbers of machines. When allocating workload to such groups, an unbalanced (rather than balanced) workload per machine maximizes expected production (Stecke and Solberg [1985] and Stecke and Kim [1989]). We note that this generalizes the notion of balancing in the traditional sense. The second constraint is a fixture constraint that limits the maximum number of parts of each type that can be on the system simultaneously.

The objective function balances workloads as much as possible by minimizing weighted machine overloads and underloads. The C_{k1} and C_{k2} can be chosen arbitrarily as long as they are nonnegative. They serve no function other than determining alternative sets of optimal production ratios that balance workloads. It would be useful to somehow set the constants as costs that reflect, for example, material cost or imminent due dates. However, some care should be taken in setting the W_k parameter. This is discussed further in §4. Subscripts j (machines) and k (machine types) can be used interchangeably, depending on the FMS application.

The decision variables are the mix ratios, a_i, the relative numbers of parts of each type to be produced cyclically on the FMS during a time period of length W, if workloads are balanced. The number of cycles is a function of the mix of part types, their processing times, and their requirements. In operation, parts are produced cyclically according to these ratios, until the

TABLE I. Notation.

i	part types, $i = 1,\dots,N$
j	machines, $j = 1,\dots,M$
k	machine types, $k = 1,\dots,K$
a_i	part mix ratio of part type i
r_i	production requirements for part type i
p_{ij}	processing time of a part of type i on machine j
m_k	number of machines of type k
pw_{ik}	average workload required for a part of type i on a machine j of type $k = p_{ij}/m_k$.
n_i	number of pallets required for part type i
n	total number of pallets required = $\sum_{i=1}^{N} n_i$
W_k	constant value, the aggregate (un)balanced workload (time) on machine type k over time
X_{k1}	load over the (un)balanced workload (W_k) on machine type k
X_{k2}	load under the (un)balanced workload on machine type k
C_{k1}	weight assigned to the potential overload (X_{k1})
C_{k2}	weight assigned to the potential underload (X_{k2})
f_i	maximum number of fixtures dedicated to part type i

requirements of some part type are finished. Then new ratios are found using (P1), either for the remaining part types, or perhaps with a new type included with those remaining. For example, it may be beneficial to include a new part type to complement the remaining types, to require more time on an underutilized machine type, if there is one.

In general, there are many more part types than machine types. This indicates that there could be many sets of optimal ratios that balance workloads. Different sets of ratios can be found by parametrically varying the coefficients of overload and underload in the objective function.

Formulation (P1) can be complicated in various ways, depending on the particular problem of interest. For example, due date constraints can be included to address other FMS part type selection problems; tool magazine capacity constraints can be added to consider the FMS loading problem. We return to these issues later. Our purpose here is to show the versatility and usefulness of this simple formulation in addressing the mix ratio problem.

Problem (P1) does not account for some real-time considerations, such as travel time, waiting time, or congestion. These factors are considered in §5, where ratios are input into other models to evaluate performance. Simulation experiments show that the ratios rarely need to be changed (Stecke and Kim [1988]). Most often the ratios found by Problem (P1) are the actual ratios to implement.

We now demonstrate the use of formulation (P1) to provide optimal or near optimal sets of ratios. One way is to vary the parameters W, C_{k1}, C_{k2}, k = 1,...,K. LINDO is used to provide both LP and IP optimum solutions to Program (P1) for the four part type, three machine type example of Table II for varying parameters W, C_{k1}, C_{k2}. (Results for a ten part type problem are provided subsequently.) Table II can be read as follows: part type 2 requires 20 minutes on a mill, 10 minutes on a drill, and 5 minutes on a VTL (Vertical Turret Lathe).

All of the examples were run to integer optimality for several reasons. First, more can be observed and discussed here about the ratios by running to integer optimality. Secondly, all integer solutions were obtained within three seconds of CPU time.

TABLE II. Processing Times in Minutes for Four Part Types on Three Machine Types.

	Mill	Drill	VTL
PT_1	10	40	50
PT_2	20	10	5
PT_3	10	30	20
PT_4	15	20	40

Since LINDO uses binary rather than integer variables, the following constraints are added to Problem (P1):

$$a_i = a_{i1} + 2a_{i2} + 4a_{i3} + 8a_{i4} + 16a_{i5}, \qquad i = 1,...,N$$

$$a_{im} = 0 \text{ or } 1, \qquad i = 1,...,N \text{ and } m = 1,...,5.$$

In some problems, the constraints $a_i \geq 1$ are deleted, to demonstrate the potential use of Problem (P1) to address the FMS part type selection problem also. Those part types that have a_i equal to zero in the optimal solution are incompatible with the other part types selected (w.r.t. workload balance) and hence are not selected for the upcoming production period.

The results are provided in Table III. CPU time is given in seconds. Case 1 of Table III is a solution obtained from manually solving the problem (see Stecke [1985]). Cases 4-7 are from one LINDO run of (P1) with the workload parameter W = 500 minutes. LINDO first provides an LP optimum, then an initial integer solution (Case 5), then a second integer solution (Case 6), before converging to the integer optimum (Case 7). Similarly, Cases 8-10, 11-13, 14-16, and 17-19 are from one LINDO run each.

Cases 2-13 and 17-19 all include the constraints that $a_i \geq 1$, for all i. Cases 14-16 require only that $a_i \geq 0$. Cases 2-10 and 14-16 have all C_{k1} and C_{k2} equal to 1, while Cases 11-13 have different C_{km}. Finally, Cases 17-19 allow the workload W to be a decision variable.

These various cases demonstrate some uses of Problem (P1). We make the following observations from Table III and subsequently discuss the implications of these in §4.

1. Perfect balance among the workloads is obtained only with the LP optimal solutions of Cases 4, 11, 14, and 17. The remaining LP solutions and all of the IP solutions (except Case 19), both initial and optimal solutions, are unbalanced, with overloads or underloads on one or more of the machine types.

2. In four of the six IP Optimum cases, the optimal integer solution is the rounding of the LP solution. Only Cases 7, 18, and 19 provide substantially different ratios. In three of the five Initial IP Solution cases, even the first IP solution is quite similar to, or a rounding of, the LP solution.

3. Varying W from 100 to 500 to 1000 provides different optimal sets of ratios.

4. We suggest that the sum of the ratios can provide the number of pallets, n, in a cycle. From Table IV, we see that n increases about linearly with W. Notice, for example, that the three integer solutions for W = 100 (Cases 3, 15, and 16) all sum to six.

TABLE III.

Linear and Integer Optimum Solutions with Varying Parameters for the Problem of Table III.

Case	W	C_{11}	C_{12}	C_{21}	C_{22}	C_{31}	C_{32}	Objective Function	a_1	a_2	a_3	a_4	CPU Time	
1	100							0.	.585	3.6	.739	1.		No Objective Function
2	100	1	1	1	1	1	1	48.75	1.	3.25	1.	1.	.188	LP Optimum
3	100							50	1	3	1	1	.350	**IP Optimum**
4	500							0.	1.	16.982	5.382	6.436	.186	LP Optimum
5	500							20	3	19	3	5		First IP Solution
6	500							15	3	18	3	5		Second IP Solution
7	500							10	6	20	1	2	.926	**IP Optimum**
8	1000							40.3125	1.	31.	12.625	13.563	.186	LP Optimum
9	1000							60	1	31	13	13		First IP Solution
10	1000							55	1	31	12	14	.993	**IP Optimum**
11	1000	50	0	50	0	1	1	0.	1.	31.	28.75	5.5		LP Optimum
12	1000							25	1	31	25	8		First IP Solution
13	1000							5	1	31	28	6		**IP Optimum**
14	100	1	1	1	1	1	1	0.	1.296	4.074	0.0	.37	.201	LP Optimum
15	100							40	2	4	0	0		First IP Solution
16	100							15	1	4	0	1	.606	**IP Optimum**
17	155.7							0.	1.	5.7	1.	1.43	.204	LP Optimum
18	240							5	2	9	1	2		First IP Solution
19	390							0	2	14	3	4	.616	**IP Optimum**

TABLE IV. Number of Pallets as a Function of Workload.

W	100	500	1000
n	6	29	66

5. Varying C_{k1} and C_{k2} also provide substantially different sets of ratios.
6. Letting $a_i \geq 0$ in Cases 14-16, a_3 is always zero. This suggests that part type three is the "least compatible," and that if one has a choice, it should not be produced with the other part types.
7. By letting the parameter W vary, even the IP optimum is balanced.
8. The CPU times (in seconds) are less than one second for all 19 cases. The times reported for the IP optimum all include the time to reach the LP optimum, since LINDO uses the LP optimum as a lower bound (see Schrage [1981]).

A larger example is now provided to demonstrate additional possible uses of formulation (P1). Table V provides processing time and demand information for a ten part type, three machine type, and five machine problem.

Table VI provides both linear and integer solutions to (P1) both for a variable W and for W = 100, 500, and 1000. These W define four LINDO runs, resulting in the 26 cases, 20-45.

TABLE V. Processing Times for Ten Part Types on Three Machine Types with Five Machines.

	Mill (1)*	Drill (2)	VTL (2)	r_i
PT_1	10	20	50	50
PT_2	15	20	40	100
PT_3	20	10	30	70
PT_4	10	20	20	100
PT_5	10	10	20	200
PT_6	10	30	20	150
PT_7	20	10	10	100
PT_8	15	20	30	50
PT_9	25	10	20	150
PT_{10}	5	40	40	200

*The number of machines of each type is in parentheses.

The following can be observed from the results listed in Table VI.

1. The pairs of Cases 24 and 25, 28 and 29, and 35 and 36 all provide two different LP solutions. Rounding these does not cause a significant unbalance.
2. As W increases from 100 to 500 to 1000, Σa_i increases linearly, approximately from 8 to 40 to 80, as Table IV shows for the smaller problem. This provides some insight into how to select W. We recommend that W (or W_k) be small so that the Σa_i is small for other uses, which are discussed in §4.
3. All of the IP optima are balanced.
4. Many of the intermediate IP solutions provide nearly balanced workloads (Cases 21, 22, 26, 31, 32, 33, and 44). Any of these could be used to suggest compatible part types and appropriate ratios to follow.
5. Out of a candidate set of 10 part types, the solutions suggest various combinations of 3-7 part types that are compatible for possible simultaneous machining.
6. All integer optima are found within 3 CPU seconds, including the time to obtain the LP optimum.

Notice that all of the IP optima are perfectly balanced for the ten part type problem, but only one (out of six) is balanced for the four part type problem. This is because four are too few to choose from. In general, FMSs produce the larger number of part types.

4. BENEFITS AND USES OF THE PRODUCTION RATIOS

The purpose of the two examples of §3 is to demonstrate potential uses of the ratios. The implications of our observations from these examples are now explored. The observations are representative of those of many examples. The potential benefits from using these ratios for various operating situations are outlined.

Integer Versus LP Solutions

An FMS usually consists of only one, two, or three different machine types. The *size* of the problems, in terms of the number of constraints, is a *function of the number of machine types* and is small. However, since there are usually many more part types, there could be many sets of optimal ratios that balance workloads. All problems that we have run took less than 4 seconds of CPU time to reach *integer* optimality and are of a typical FMS size. The integer solutions provide alternative optima.

TABLE VI.

Linear and Integer Optimum Solutions for a Ten Part Type, Three Machine Type, Five Machine Example for the Objective of Balancing Workloads.

Case	W*	Objective Function	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	CPU Time	No. of $a_i \geq 0$	Σa_i		
20	230	0.	1.	1.	1.	1.	1.	10.	1.	1.	1.	1.	.242	All	19	LP Optimum	$a_i \geq 1$
21	365	10	2	1	2	1	1	15	2	1	2	3			30	First IP Solution	$a_i \geq 1$
22	325	5	2	1	2	1	1	15	2	1	1	1			27	Second IP Solution	$a_i \geq 1$
23	350	0	2	1	2	1	2	15	1	1	2	2	1.174		29	**IP Optimum**	$a_i \geq 1$
24	100	0.	.625		1.875			5.625					.207	3	8	LP Optimum	$a_i \geq 0$
25	100	0.		2.632				4.737			.26			3	8	LP Optimum	⋮
26		10		2		2		4						3	8	First IP Solution	
27		0		2		1		3			1	1	1.333	5	8	**IP Optimum**	
28	500	0.	3.125		9.375			28.125					.204	3	40	LP Optimum	
29	500	0.		3.	1.65	32.		9.3	.45					5	45	LP Optimum	
30		120				16			8		4	16		4	44	First IP Solution	
31		15	1	1	1				8		8	20		6	39	Second IP Solution	
32		0					1		8	2	8	20		5	39	Third IP Solution	
33		5	1	1		1			8	1	8	19		7	39	Fourth IP Solution	
34		0	2	1		1			8		8	19	1.456	6	39	**IP Optimum**	
35	1000	0.	6.25		18.75				56.25				.205	3	81	LP Optimum	
36	1000	0.		20.42				27.16			13.68	16.		4	78	LP Optimum	
37		125	7			1	2			3	32	36		6	81	First IP Solution	
38		125	7	4						1	32	36		5	80	Second IP Solution	
39		120	7							3	32	37		4	79	Third IP Solution	
40		120	11				1				32	36		4	80	Fourth IP Solution	
41		110	11			1					32	36		4	80	Fifth IP Solution	
42		110	9							1	32	37		4	79	Sixth IP Solution	
43		105	10								32	37		3	79	Seventh IP Solution	
44		30	16					28	2		18	16		5	80	Eighth IP Solution	
45		0	16				1	31			18	14	2.795	5	80	**IP Optimum**	

Cases 20-23 specify that all $a_i \geq 1$ and allow W to be a variable. The remaining cases specify that $a_i \geq 0$ and fix W to be either 100, 500, or 1000.

In most cases, the integer optimum is close to the LP optimum. This suggests that it may be sufficient to round the LP solutions, especially if the problems are large. Obtaining perfect balance from an integer optimum is not necessary. The LP solution can be sufficient because:

(a) Rounding the fractions does not change the balance much;

(b) The LP solutions provide the best balance;

(c) These are aggregate ratios. When secondary criteria, delay and travel times, and real-time congestion are considered, "optimum" is less important. The balanced or almost-balanced ratios have been sufficient from the computational experiments to date.

Secondary Criteria

Program (P1) can be run parametrically as demonstrated in Tables III and VI by varying the objective function coefficients to provide alternative sets of ratios. Then secondary criteria, such as flow time or due dates, can be used to select the most appropriate set. We expand on this issue shortly.

All Selected Part Types Have to be Produced

Cases 2-13 and 17-19 in Table III and Cases 20-23 in Table VI were run with $a_i \geq 1$. If all of the part types <u>have</u> to be selected (i.e., for assembly or due date purposes or the batching of a customer's orders), then all a_i should be greater than one. Otherwise, if an LP were run, a fractional solution could be rounded up to one. On the other hand, as another use of Program (P1), an $a_i = .5$ could specify that one part i is introduced every other cycle.

Part Type Selection

Program (P1) can be used to determine a compatible set of part types to be machined together as follows. All candidate a_i are required to be greater than or equal to zero. Those i such that a_i is equal to zero in the optimal solution are excluded from the selected part types. For example, Cases 14-16 indicate that part type 3 is incompatible with the others and should be omitted from the current mix if possible. Cases 24-45 suggest various sets of compatible part types and different ratios for each. Other criteria could be used to select one of these sets.

Given Production Ratios

Given relative ratios of two or more part types (required in the case of assembly, for example) are easily included in the constraints of Program (P1).

Part Input Sequence

The calculated mix ratios might be permuted to determine a part input sequence. See Stecke [1985] and Stecke and Kim [1991]. The input sequence is repeated cyclically. Further research is required to specify precisely how this should be done. For an FMS dedicated to the production of fewer part types, a periodic input sequence developed from a permutation of the ratios should tend to maintain workload balance over time.

Refixturing

For many prismatic parts, after a series of operations are performed, each part is moved off the system to be refixtured. The part is manually reclamped to a different fixture type on a different pallet. The part is then input back into the system and additional cutting and inspection operations are performed on different surfaces of the part. Each refixturing in many respects can be treated as a new part type. However, for each part, the mix ratios of each refixturing may be the same, if there is one part per pallet.

We illustrate with an example. The system described in Table VII consists of two machine types processing two part types, each of which requires a refixturing after passing through the mill and drill. In Table VII, PT_{ij} is part type i with pallet/fixture combination j.

TABLE VII. Processing Times for Two Part Types Requiring Refixturings for Two Machine Types.

	Mill	Drill
PT_{11}	10	40
PT_{12}	10	30
PT_{21}	20	10
PT_{22}	15	20

Aggregating the processing time information of Table VII and substituting into Program (P1), the ratios are: $(a_1, a_2) = (1, 10)$. Maintaining these ratios balances workload. However, we show in §5 that only three rather than ten fixtures for part type 2 are required to produce at these ratios. This determination ignores the refixturing, setup, and transportation times while considering the processing and queueing times. Waiting time and buffer requirements are modeled. When delays due to transportation and fixturing times are accounted for, a few more pallets may be required.

Due Date Criteria

Other factors such as due dates, flowtime, or tool changing can be used to help choose the appropriate ratios. Some relevant due date-based criteria include tardiness, number of late jobs, and earliness (in a JIT environment). Due dates can be considered during production as follows. The due dates should be used to help select the part types to be machined next. From feasible sets of ratios determined from (P1), those that best ensure that the due dates are met can be selected. To determine that the due dates can be met, processing time requirements, transportation, queueing, possible machine breakdown, for example, may need to be considered (see §5).

Some part types may be identified as potentially missing their due dates. The ratios of these part types can be weighted higher in Program (P1) so that they will be produced more often relative to the rest of the mix. To implement this, real-time control procedures should be used to continuously monitor performance relating to the due date of each part type. If possible, appropriate action should be taken, in breakdown situations for example, to change the way the system is operated (to change ratios, for example), so as to meet due dates.

Artificial intelligence techniques are being investigated for real-time monitoring. A rule-based expert system may be useful to propose certain actions to take when the system state changes (i.e., machine breakdown). Such a system may be used to choose, update, or change the ratios as the system changes. For example, a particular set of optimal ratios, among those which balance workloads, may be chosen by an expert system according to the system state.

Setting the Workload Parameter, W

The workload parameter W could be kept small, relative to the processing times, for purposes of determining an input sequence, for example. Then the a_i are small, and a small multiple (α) of the a_i can suggest the number of pallets of each type, as follows: $n_i = \alpha a_i$, where α = 1 or 2. Consider the example of Case 1 of Table III, and let $n_i = a_i$: the suggested distribution of pallets is 1:4:1:1. Then the sum n = 7, which is a reasonable number of pallets. This particular pallet distribution was used to find a good cyclic part input sequence in an example of Stecke

[1985]. Alternatively, W can be allowed to vary. Additional research is required in order to specify precisely what W should be.

Machine Breakdowns

The procedures described here can also be applied directly to machine breakdown situations. For example, Hildebrant's [1980] hierarchical approach assumes that the mix ratios are known for every possible failure state.

The ratios can also help determine the "hedging points" (optimal buffer levels) in the flexible assembly application of Akella, Gershwin, and Choong [1985]. For each part type and each failure state, the hedging point is a function of some parameters such as the routing sequence (higher priority is given to parts that visit more unreliable machines) and the "extent of difficulty if the part is backlogged." For a particular failure state, the ratios provide a measure of the "difficulty if the part is backlogged." Further research is required to define precisely how the procedures developed here can be used to help specify the hedging points.

The models are useful in handling breakdown situations. If a machine breaks down and other machines are tooled to perform the same operations as the down machine, new ratios can be found easily to rebalance workloads.

5. MODELS TO EVALUATE PRODUCTION RATIOS

The ratios found by solving Program (P1) can be input into more detailed evaluative models to verify actual operating ratios. These models are queueing networks, Petri nets, and simulation. The ratios specified by (P1) often perform well when evaluated on realistic systems. For complex FMSs, i.e., where parts may visit many machines along different routes, the ratios found by (P1) should be evaluated and perhaps revised. The first two aggregate models, queueing networks and Petri nets, are recommended for real-time evaluation of an operating FMS. Simulation is recommended to evaluate the ratios for research and experimentation purposes.

One model is a multiclass closed queueing network (CQN), such as MVAQ (see Hildebrant [1980], Cavaillé and Dubois [1982], and Suri and Hildebrant [1984].) For each part type, the input required is the average visit frequency to and the average processing time of an operation at each machine (group). The outputs include the steady-state mean production rate of each part type, machine utilizations, and average queue lengths at each machine (group). The mix ratios that would balance workloads are not known in advance to input into the queueing network model.

MVAQ can also model, as averages and at an aggregate level, load and unload times, refixturing times, queueing times, and transportation times. The ratios found by the methods described in §3 can be used to suggest the numbers of pallets and fixtures of different types required to maintain these ratios. The ratios can help to define the input to a queueing network model. The output (machine utilizations) indicates the system balance when the additional delay factors are included. In addition, the average production rates can help evaluate due dates.

Suri and Hildebrant [1984] indicate that MVAQ is reasonably accurate and is about 10-20% pessimistic in its predictions, as compared to simulations of similar systems allowing more modeling detail. However, MVAQ is more accurate in its *relative* predictions. For example, the ratios of the expected production rates of parts and machine utilizations matched those provided by simulation quite well. It is these relative values that are useful to verify ratios that provide a good balance when system congestion is considered.

The ratios could also be evaluated using operational analysis-based CQN models (Dallery and David [1983]) to maximize the production rates of the various part types. These models require no sequencing assumptions (such as the usual FCFS at each machine (group)).

Another model that accepts the ratios as input to evaluate their performance and also helps find the minimum inventory requirements is a timed Petri net. (See Dubois and Stecke [1983, 1991].) This model complements the queueing network model. It is not aggregate and uses deterministic operation times and part routes. Set-up, transportation times, and queueing times are modeled in detail, unlike a queueing network. Transient effects and finite buffers can be modeled. Finally, it can also be used to help find an appropriate input sequence.

For a certain subclass of timed Petri nets (in particular, decision-free nets), the graphical model can be easily translated into linear state equations in a {max, +}-based algebra. (See Cohen et al. [1983].) Decision-free means that no decisions are made. Operating information is specified in advance, such as part routes and the input sequence. Such a particular Petri net representation can be analyzed very quickly with some algorithms, based in part on Karp's [1978] efficient shortest path algorithm, to provide much information that is useful for performance evaluation.

Some of the output from the model includes the length of the transient period before steady state is reached, the cycle time (hence the production rate), the bottleneck machine, its utilization, and the utilizations of all other machines. Some particularly useful information specifies that production can be increased by either:

i) adding a machine of a particular type; or
ii) inputting another pallet/fixture for a particular part type.

This is because either a machine or a pallet can be a bottleneck.

We show how this information can be used, for example, to determine the minimum inventory requirements to maintain balanced mix ratios. In most of the many problems that were examined (see Stecke [1985]), the ratios found also provided the minimum numbers of pallets and fixtures required. When the aggregate processing time information indicates an unbalanced machine workload (defined here by the workload on each machine when only one part of each type is considered), then the minimum number of pallets required per part type can be much less than that specified by the ratios. We use the example of Table VII to indicate how the Petri net model and the information it provides can be used to determine the minimum inventory requirements.

The information that a Petri net program requires is:

i) the mix ratios for each part type (a_i);
ii) the number of pallets/fixtures dedicated to each part type;
iii) a part input sequence.

For the example described in Table VII, this initial information is:

i) $(a_1, a_2) = (1, 10)$;
ii) $(n_1, n_2) = (1, 4)$; (This is only for demonstration purposes. We know a priori via simulation that the minimum number of pallets required is: (1, 3));
iii) (1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2).

The output from a Petri net program indicates that there are too many pallets/fixtures for part type 2. Changing (n_1, n_2) to (1, 3) in the subsequent run provides the information that: production is maximized; workloads are balanced; there is no idle time; and these are minimum inventory and buffer requirements to maintain the optimal mix ratios of (1, 10).

Note that the ratios are also used to help find a cyclic input sequence. It is useful to know the relative numbers of parts of each type that should be in the system prior to input sequencing.

The model we use to evaluate the performance of the mix ratios is simulation. In several mix ratio evaluation studies, Schriber and Stecke [1987, 1988] simulate a flexible flow system under varying conditions to examine the usefulness of the mix ratios in realistic situations. Ranges on the amount of work-in-process (limited buffers) and on the number of AGVs provide various scenarios. As can be expected, the mix ratios predict an optimistic system performance, since the limited resource requirements, travel time, and congestion had not yet been considered. However, maximum achievable machine utilizations can be realized under realistic FMS operating conditions. Also, ratios as determined by (P1) provide the best system performance and balanced machine utilizations in comparison to ratios determined otherwise. For example, performance from balanced ratios compares favorably to that from ratios that are proportional to the requirements.

6. FUTURE RESEARCH NEEDS

Some considerations when deciding which of the two objectives to follow to determine mix ratios include the following. Tool magazines are changed less often, if all requirements are finished simultaneously. Batching a customer's orders can decrease both finished goods inventory and freight costs. Balancing workloads tends to decrease work-in-process inventory and increase both machine utilization and production rate. Which objective is more important is a function of many parameters: where and how much value is added to the part; how much space is both available and required for in-process and finished goods inventory; the methods and costs of delivering the final products; the nature of the demands on the FMS.

Since there are usually many good solutions for the balancing objective, it may be possible to determine ratios that satisfy both objectives. Further research is required to determine both how to do this as well as to specify a criteria that is satisfactory for both objectives.

The ratios can impact other FMS operating problems. This suggests areas in which further research is required. For example, it has been demonstrated that the ratios can be useful to help determine the minimum inventory requirements in terms of numbers of pallets and fixtures.

The ratios can provide guidelines for determining a good part input sequence (see Stecke and Kim [1988]). However, further research is required to develop a more precise algorithm to find a part input sequence. Some related work along these lines has been done by Hitz [1980] and Erschler, Lévêque, and Roubellat [1982]. These studies determine a periodic part input sequence for flow shops. Alternative routes are not allowed. These studies assume that there is only one pallet or fixture of each type. Both papers also assume that the mix ratios of the part types have previously been determined. The ratios can be suggested by the procedures in §3.

Hitz states that the efficiency of his branch and bound procedure for the permutation flow shop problem is partly a function of the "balance of work among machines." If the workload is balanced, the search is brief, since many descendent nodes can be fathomed immediately as infeasible. It appears that the procedures of both Hitz and Erschler et al. might benefit from using the approaches suggested here to calculate balanced ratios before the use of their algorithms.

Automatic generation and selection of the best mix ratios is needed. Also, implementation of secondary criteria for choosing ratios is needed for situations in which multiple sets of ratios balance the workload. Artificial intelligence techniques may be useful, since the space of feasible and good solutions (sets of ratios) can be reduced a priori by the methods suggested here. This reduces the size of the rule base that an expert system would need to scan. Since all sets of ratios found would tend to balance workloads, a feasible matching in a rule base may be sufficient. This is an area of future research.

Integration of the part mix determination with the other FMS planning problems is required. A framework for this integration is suggested in Stecke and Kim [1991].

The ratios could be used to reduce the size of some other FMS combinatorial problems. The procedures might be used to reduce the size of the batching problems considered by Whitney and Gaul [1985] and Rajagopalan [1986]. The sizes of input sequence problems would be decreased. The size of the binary matrix used by Dar-El and Sarin [1984] to list alternative routing combinations in advance of FMS scheduling could be reduced a priori through the use of these procedures. Whitney and Suri [1984] have formulated a large mixed integer programming problem to help select parts and machines during FMS design. The objective is balancing workloads. The number of candidate part types impacts the problem complexity. The procedures here could be used to reduce the candidate set. In many of these cases, the ratios impact a part type selection problem, which is at a higher level in the hierarchy of FMS problems. In all cases, further research is required to determine how these suggestions might be implemented and how these ratios could be used to simplify subsequent FMS planning and operating problems.

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