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EFFICIENCY IN THE FOREIGN EXCHANGE MARKET: CONCEPTS AND METHODOLOGICAL IMPROVEMENTS FOR WEAK FORM TESTS

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by

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ABSTRACT

This paper develops the concept of efficiency in the foreign exchange market from a broader and more general perspective than that commonly associated with previous work on efficient markets. Within this framework it is shown that several tests of weak form market efficiency applicable to securities markets may be invalid in the foreign exchange market. The use of filter, or trading rule, tests is examined as an alternative method for testing weak form efficiency and the sampling distribution of filter rule returns is derived so that tests of statistical significance between filter rule and buy-and-hold returns may be made.

1.0 Introduction

Recent research on the behavior of exchange rates has centered around the efficiency of the foreign exchange market. Efficiency in this context has followed the concepts of efficient market theory developed over the past two decades and heretofore applied primarily to research on securities and commodities markets. The basic precept of this theory is that in an efficient market participants utilize a currently available information set to form unbiased estimates of future prices and current prices fully impound the implications for future prices contained in this information set.[1/] The degree of market efficiency is determined by the contents of the information set to which the market reacts efficiently. A market is said to be "weak form" efficient if the information set consists of historical price data; "semi-strong form" efficient if the information set consists of all publicly available information; and "strong form" efficient if the information set consists of all information currently available, both public and private. Thus the existence of weak form efficiency in the foreign exchange market implies that historical price (exchange rate) data cannot be used to form better forecasts of future prices (exchange rates) than those which are already being made by the market and impounded in current exchange rates, and therefore such information cannot be used to establish trading rules which would consistently earn better than average returns. The existence of semi-strong or strong form efficiency would carry the same implications for the profitable use of the respective information sets.

This paper deals with methodological improvements for tests of weak form efficiency in the foreign exchange market. Before expanding upon the theory or discussing the methodology, it is useful to look at foreign exchange market efficiency in a broader context; to understand some of the implications of market inefficiency which provide a rationale for research in this area; and to discuss briefly the assumptions implicit in tests of the efficient market theory described above.

2.0 Efficiency: Some Theoretical Constructs

Efficiency ultimately implies some optimal, or efficient, allocation of scarce resources among alternative uses. We can quickly run into difficulty defining the optimal allocation of resources, but for the purposes of this paper we assume that individuals' utility functions are homogeneous and homothetic, which allows us to define a single social utility or welfare functions.[2] We further assume that individuals have well-defined and unique preferences, that they are utility-maximizers, and that they are risk-averse in a mean-variance sense.[3] We now define the optimal allocation of resources as that allocation which maximizes utility with respect to both goods and risk. The assumption of unique preferences and utility-maximizing individuals is sufficient to ensure a unique optimal allocation of resources, which will be crucial in the development which follows.

Prices are determined simultaneously with the allocation of resources, given market parameters (technological and other--e.g., legal--constraints). The simultaneous determination of prices and resource allocation implies the assumptions of perfect mobility of resources and perfect knowledge of current prices and allocations among all market participants--i.e., it implies cleared markets. We define the efficient set of prices as that set which is simultaneously determined with the optimal allocation of resources. And the existence of a unique optimal allocation of resources implies a unique efficient set of prices.[4]

The optimal allocation of resources among current goods and between current and future uses depends not only on the current endowment of resources, current consumption preferences, and current market parameters, but also on future endowment, preferences, and parameters. More specifically, we assume that future endowment, preferences, and parameters are random variables generated by some stochastic process; that they possess certain expectations (means) and variances; and that they are related to current events. Then the optimal allocation of resources depends on the (true) expectations and variances of future endowment, preferences, and parameters. Carrying this process infinitely far into the future, it is clear that the current optimal allocation of resources and efficient set of prices depend on the (true) expectations of next period's (and all future periods') optimal allocation of resources and efficient set of prices. More correctly, the current optimal allocation of resources and efficient set of prices are simultaneously determined with (true) expectations and variances of future optimal allocations of resources and efficient set of prices.

The actual allocation of resources depends on market participants' forecasts of future endowments, preferences, and parameters, and on their perception of the degree of uncertainty associated with those forecasts, i.e., the perceived forecast variance. And the actual allocation of resources and set of prices are simultaneously determined with forecasts of future optimal allocations of resources and efficient sets of prices.[5/] When market participants' forecasts are equal to true expectations, it follows that the variance of the forecasts will be equal to the true variances, and the allocation of resources and concommitant set of prices will clearly be efficient. It is not immediately obvious that when forecasts are not equal to the true expectations, the current allocation of resources will be suboptimal and, hence, that an inefficient set of prices will obtain. For it might be argued that if, on average, the forecasts of some variables were higher than the true expectations but others lower, the allocation of resources between current and future uses might be unaffected, yielding an optimal allocation of resources among current goods and between current and future uses and, therefore, an efficient set of prices. Such an argument might hold if market participants were risk-neutral. However, we have assumed risk-averse market participants in a mean-variance sense. Biased forecasts (i.e., not equal to true expectations) will necessarily cause the variance of the forecast error (the risk to which participants are averse) to be greater than the true variance.[6/] So, to the extent that market participants are risk-averse, the allocation of resources to current uses will tend to be greater than optimal the greater the difference between the forecasts and the true expectations - even when high and low forecast errors tend to "cancel out."

The necessary and, together, sufficient conditions for forecasts to be equal to true expectations (and therefore the necessary conditions for an efficient set of prices to obtain) are 1) that market participants form their forecasts of future endowments, preferences, and parameters using the same 'model' as the one which generates the true expectations, and 2) that as events occur, forecasters immediately and unbiasedly apply the information content of those events to the correct model. The first of these conditions is, essentially, the theory of rational expectations;[7/] the two conditions together form the crux of efficient market theory. While it is of great interest that the theory of rational expectations and efficient market theory appear to be closely linked, we shall not explore the relationship further in this paper.

Now that an efficient allocation of resources and an efficient set of prices have been defined and the conditions under which they will be attained set forth, the discussion of an efficient market is straightforward. It is clear from the preceding discussion that a market cannot be efficient if any of its submarkets is inefficient, i.e., does not efficiently price and allocate the particular resource traded therein. Of more interest, however, is whether a submarket can be efficient if the greater market of which it is a part is not efficient and, if so, under what conditions. While it is possible for an inefficiency in the greater market to be so far removed from the submarket that there is absolutely no interdependence, this condition seems unlikely. For we originally assumed the simultaneous determination of prices and resource allocation, which implies interdependence. Therefore efficiency in any submarket must be considered as conditional on either 1) assumed efficiency in the rest of the greater market, or 2) assumed imperfections in the rest of the greater market. And it is only when the first condition holds that the submarket can be efficient in a strict sense.

2.1 Efficiency in the Foreign Exchange Market

Exchange rates are, in a sense, prices. But they are unique in that they represent the price of one currency in terms of another rather than the price of a real good in terms of a particular currency. A national currency represents a claim on real resources within a particular country, and from this derives its value. And the risk associated with holding a particular currency arises from the uncertainty as to the size of the future claim on real resources to which that currency will entitle its owner within the country in which it is legal tender and also from the variability of the future claims on real resources in other countries entitled by other currencies for which it may be exchanged.

Exchange rates are simultaneously determined with national prices and with the allocation of resources between and within countries and between current and future uses--given, for each country, resource endowments, consumption preferences, market parameters, and forecasts of future endowments, preferences, and parameters, and given external parameters which define relationships between national markets and forecasts of future external parameters. These external parameters include, among other things, exchange controls, capital controls, transportation costs, and any legal constraints which differentiate residents from non-residents. In addition, the determination of exchange rates, national prices, and resource allocations depends on the uncertainty, or variance of the forecast error associated with both (1) the forecasts of future domestic variables--i.e., endowments, preferences, parameters and (2) the forecasts of external parameters. If actual exchange rates depend on forecasts of future variables while the efficient set of exchange rates depends on the true expectations of future variables then, as in the case of a single market, the set of exchange rates will be efficient if and only if market participants form their forecasts using the same model as the one which generates the true expectations and if they immediately and unbiasedly apply all new information to this model.

There are some important distinctions which must be made between domestic and external markets. Among the more important considerations are:

1. The assumed homogeneity and homotheticity of individual utility functions in a single market make it possible to define a single social utility function and, from that, a unique optimal allocation of resources and efficient set of prices. With heterogeneous consumption preferences among countries as a stimulant to trade and capital flows, however, it may not be possible to define a unique world utility function and, hence, a unique optimal allocation of resources and a unique efficient set of exchange rates.
2. Risk-return relationships, which might be considered fixed within a single country, may be different from one country to another and may not be well defined for external risks.
3. The actions of governments in domestic markets might be considered consistent with other domestic government actions and, perhaps, even consistent with utility maximizing behavior. However, the external actions of governments may be inconsistent with the actions of other governments and with the maximization of 'world utility.'

Defining efficiency in the foreign exchange market clearly poses some conceptually difficult problems. It is useful, therefore, to examine some of the implications of inefficiency in the exchange market, in a 'real world' than a purely theoretical context, which provides a rationale for continued research in this area.

2.2 Implications of Inefficiency in the Foreign Exchange Market

In discussing the implications of inefficiencies in the foregin exchange market, it is expedient to drop some of the rigor necessary for the preceding theoretical development and the following discussion will be at an intuitive level. We start by assuming that each national currency has some definable 'intrinsic value.' The ratios of intrinsic values represent the 'proper' exchange rates. The effects on resource allocation of persistent over- or undervaluation of a currency have been recognized for some time and have, more recently, been taken into account in domestic economic policy decisions.[8/] Briefly, if a currency were overvalued we would expect exports to be "too high" and imports to be "too low" and, more importantly, we would expect that country to be allocated relatively "too much" real investment relative to the rest of the world. Furthermore, we would expect that within the country, relatively "too much" investment would be allocated to traded goods industries and relatively "too little" to home goods industries. The converse would hold for a currency which was undervalued. If over- or undervaluation were to persist over long periods of time, the impact on the efficient allocation of resources is quite clear. And it seems likely that the shorter the duration and the less the magnitude of over- or undervaluation, the less the extent of inefficient resource allocation. Even when there is no persistent over- or undervaluation in the long run, there may exist some non-randomness in the short run. The existence of non-randomness implies that the current exchange rate reflects biased forecasts of future exchange rates and, therefore, is a biased estimate of the true relative intrinsic values of currencies. This, in turn, may imply several things:

1. Sufficient real resources are not being efficiently allocated to obtaining and/or analyzing relevant information which would be useful in making better (unbiased) estimates of the relative intrinsic values.

2. More efficient utilization of such resources could lead to better forecasts of future exchange rates.

3. The duration of periods and the magnitudes of over- and undervaluation is prolonged.

4. Non-randomness may imply greater variance in forecasting errors which, because of increased risk, may lead to reduced and less efficient levels of international trade and capital flows.

From a pragmatic standpoint, the most important issues which may be addressed by furthering the theoretical development and empirical investigation of foreign exchange market efficiency are those which involve considered government economic and exchange policy decisions. The remainder of this paper, however, will be devoted to the application and tests of the efficient markets hypothesis, as it has already been developed, to the foreign exchange market.

3.0 Efficiency, the Efficient Market Hypothesis, and the Foreign Exchange Market

Within the general framework of efficiency developed in sections 2.0 and 2.1, the efficient market hypothesis may be viewed as a partial equilibrium analysis. Specifically, it theorizes the relationship among current prices, current information, and future prices in a given submarket, e.g., securities, commodities, or foreign exchange market. An efficient market as defined by the efficient market hypothesis is a necessary condition for the broader concept of efficiency developed earlier, but not vice-versa. The hypothesis defines efficiency contingent upon conditions existing in the greater market of which the efficient submarket is a part and assumes clearing within the submarket.[10/] That is, it assumes that utility-maximizing market participants will respond to changes in current price, keeping the market for the particular traded resource continuously cleared, and that the allocation of that resource will be pseudo-optimal--i.e., that it will lead to utility maximization in the greater market given the imperfections which may exist there. It is clear that different degrees of imperfection, or inefficiency, in the greater market should lead to different pseudo-optimal allocations of resources within an efficient market[11/] and, therefore, to different pseudo-efficient price sets.

The impact of different types and degrees of inefficiencies in the greater market on intertemporal price behavior within the efficient market is not clear, but it seems possible that different types of intertemporal price behavior might be observed. If so, then it is also possible that the price behavior characteristic of an efficient securities market might not be the same as the price behavior which would characterize an efficient foreign exchange market, for the immediate surroundings in the greater market within which each exists quite likely exhibit different types and degrees of inefficiencies. Therefore, some tests of the efficiency of securities markets may not be directly applicable to the foreign exchange market, or their results may have to be interpreted differently.

We examine here only tests of the weak form efficient market hypothesis: that any information relevant to future price (exchange rate) contained in the set of historical prices (exchange rates) is already fully reflected in the current price (exchange rate).

3.1 Tests of Weak Form Efficiency in the Foreign Exchange Market

The logical place to start in tests of efficiency in the foreign exchange market is with tests of the weak form efficiency. If the market is not weak form efficient, then it cannot, by definition, be semi-strong or strong form efficient. Recent tests of weak form efficiency in the foreign exchange market by Poole (1967); Pippenger and Phillips (1973); Sweeney and Willett (1976); Dooley and Shafer (1975); Cornell and Dietrich (1976); Cummins; Logue, Sweeney, and Willett (1975); Burt, Kaen, and Booth (1976); and Giddy and Dufey (1975) have relied largely on the methodology developed for similar tests of securities markets. The primary methodological tools have been tests of serial correlation, time series analysis, and filter rule or trading rule tests.

Samuelson (1965) and Mandelbrot (1963, 1966) have presented formal proofs that in free and competitive speculative markets, 'correct' forecasts of future prices, i.e., equal to the true expectations, will lead to random (and hence uncorrelated) price fluctuations. Samuelson extended this proof to the case where market participants were risk averse. As noted by Giddy and Dufey, these proofs hold only for free and competitive markets where no single market participant can manipulate prices[12/] and, in addition, they hold only for markets in which all participants are utility-maximizers with the same utility function vis-â-vis risk.

Now the efficient market hypothesis, correctly interpreted, states that in an efficient market future prices are 'fair game' variables with respect to market forecasts.[13/] That is,

$$\text{if } X_{t+1} = \tilde{P}_{t+1} - F(\tilde{P}_{t+1} \mid \emptyset_t) \quad \text{then } E(\tilde{X}_{t+1} \mid \emptyset_t) = 0$$

where $F(\cdot)$ is the market forecast operator; $E(\cdot)$ is the expectations operator; and $\emptyset_t$ is the information set at time t (i.e., historical price data in tests of weak form efficiency). More importantly, it specifically does not state that prices in an efficient market must be serially independent, or even serially uncorrelated. However, the efficient market hypothesis stated in terms of fair game properties is difficult to test empirically, as it makes no assumptions about the forecast operator, $F(\cdot)$. It is important to remember that empirical tests of securities market efficiency, particularly weak form tests, largely preceded rather than followed the theoretical development. Early empirical work found no significant serial correlation in security price changes. The 'fair game' model together with the proofs of Samuelson and Mandelbrot provide the theoretical basis for serial correlation and time series tests of weak form efficiency in securities markets, because the underlying assumptions of the proofs may reasonably be assumed to hold. Thus, in the case of free and competitive markets where participants are utility maximizers with the same utility functions and where no single market participant can manipulate prices, the existence of serial correlation in price changes constitutes prima facie evidence of weak form inefficiency.

Three important caveats must be recognized when using serial correlation and time series analysis tests of weak form efficiency. First, serial correlation tests and most time series analysis tests are tests for linear dependence between variables; they may not detect nonlinear dependencies. Second, the conventional tests of statistical significance assume normality; if the test variables are highly skewed or leptokurtic with respect to the normal distribution, statistically significant dependence may be found, when it does not, in fact, exist. Third, and most important, if the market being studied does not conform with the assumptions of the Samuelson and Mandelbrot proofs, then such tests are inappropriate for verification of the weak form efficient market hypothesis in that market.

It is the contention of this paper that the serial correlation and time series tests originally developed for the examination of weak form efficiency in securities markets may be inappropriate tests of weak form efficiency in the foreign exchange market. It is clearly the case that under the system of fixed exchange rates, á la Bretton Woods, rates were not allowed to fluctuate completely randomly and that a single market participant (government) did manipulate rates. Even under the flexible exchange rate system which has existed since early 1973 governments have been active participants in the foreign exchange market. It has been argued that recent government intervention has been much less predictable as to timing and even direction than it was under the earlier system.[14/] However, governments clearly do not have the same utility functions as other market participants, nor do they act as utility-maximizers under the same criteria as other market participants. Even in the absence of government intervention it is not clear that the Samuelson-Mandelbrot assumptions would be strictly met. It is possible that the existence of exchange and capital controls, among other things, leads to exchange market transactions between participants with significantly different utility functions.[15/]

A more appropriate methodology for tests of weak form efficiency in the foreign exchange market is that of filter rules or, more generally, trading rules based on historical exchange-rate information. Tests of this type have been reported by Sweeney and Willett (1976), Kolhagen (1976), Dooley and Shafer (1975), Logue and Sweeney (1975), Cornell and Dietrich (1976), Cummins et al. (1975), and others. In general, these tests have been viewed as a confirmation of the results of serial correlation tests and the theoretical development has been shallow. If serial correlation and time series analysis tests of weak form efficiency in the foreign exchange market are inappropriate, as is contended here, then increasing reliance must be placed on trading rule tests of weak form efficiency and a more thorough understanding of their application is required.

4.0 Trading (Filter) Rule Tests

A trading rule test of weak form efficiency is one in which a trading (buy/sell) rule is developed on the basis of historical price movements and in which returns generated by the trading rule are compared to the returns generated by a buy-and-hold strategy for the same security. A filter rule is a trading rule in which a security is bought whenever its price increases by x% over the previous low (or over the last period price) and sold short whenever its price drops by y% below the previous high (or below last period price). Alexander (1961, 1964) and Fama and Blume (1966) found evidence that filter rules with low (about 1%) thresholds, used on short-term (daily or intra-daily) price swings in securities markets, could produce returns in excess of buy-and-hold returns on a gross basis but not on a net (of transactions costs) basis.

The primary advantages of trading rule tests over serial correlation tests for our purposes are 1) they do not require the absence of government intervention, and 2) they may capture the subtle and nonlinear dependencies which chartists claim exist. The primary disadvantage is that there are an infinite number of possible trading rules, and each test which supports efficiency rules out only one or, at best, one type of trading rule.

A common flaw of all trading rule tests to date is that they have failed to test the statistical significance of any excess returns generated by trading rules in cases where the trading rule return exceeded the buy-and-hold return. A major contribution of this paper is the theoretical development of the sampling distribution of trading rule returns, which will allow tests of statistical significance to be made and which should help to make trading rule tests of weak form market efficiency more powerful.

4.1 Derivation of Statistical Properties of Trading Rule Returns

In a recent paper, Praetz (1976) attempts to establish that filter tests of the type employed by Alexander and by Fama and Blume are inherently biased in favor of the buy-and-hold strategy. Although the conclusions he draws from his results are incorrect,[16/] he has made an important contribution toward the development of more powerful tests using trading rules. For the purposes of his argument, it was necessary for Praetz to derive only the expected value of the trading rule return. The proof which follows extends the derivation to the sampling distribution of trading rule returns and corrects an error in Praetz' derivation of its expected value.

Following Praetz, we assume that price relatives are identically and independently distributed lognormal. It is recognized that these assumptions hold only as approximations. Empirical evidence indicates that exchange rate changes, like security price changes, may not strictly follow a lognormal distribution.[17/] The development which follows should be viewed as a first, rather than a final, step in the process of establishing the statistical properties of trading rule returns. The derivation has been carried out in such a manner that, as a better understanding of the distributional characteristics of exchange rate changes is developed, better approximations of the true sampling distribution may be found by substitution of the appropriate parameters. The proof is presented in terms of security prices and returns; the extension to the foreign exchange market is given at the end of the proof.

Let r_j be the one period return in period j, i.e., $(P_j - P_{j-1})/P_{j-1}$ where P_j is the price of the security at time j plus any distributions between j-1 and j. Assume the price relative, $P_j/P_{j-1} = (1 + r_j)$ is lognormally distributed with constant mean and variance over all j. Let r be the average one period return such that

(1) $$(1+r)^n = \frac{P_1}{P_o} \cdot \frac{P_2}{P_1} \cdot \ldots \frac{P_n}{P_{n-1}} = P_n/P_o \ .$$

Define e_j such that

(2) $$e_j = \ln(P_j) - \ln(P_{j-1}) = \ln\ (P_j/P_{j-1}) \ .$$

Taking the log of both sides of (1) we have

$$\ln(1+r) = \frac{1}{n}\ln(P_n/P_o) = \frac{1}{n}\,[\ln(P_1/P_o) + \ln(P_2/P_1)\ldots + \ln(P_n/P_{n-1})]$$

$$= \frac{1}{n}\sum_{j=1}^{n} e_j = \Sigma\frac{1}{n}\,e_j \ .$$

Now since (1+r) is lognormally distributed with constant mean and variance, e_j is normally distributed such that

$$e_j \sim N(m,v) \ .$$

It follows that $\Sigma\frac{1}{n}\,e_j$ is also normally distributed and $(1+r) = \exp(\Sigma\frac{1}{n}\,e_j)$ is lognormal.

We wish to find E(1+r) expressed in terms of m and v. We have

$$E(1+r) = E[\exp(\Sigma\frac{1}{n}\,e_j)] \ .$$

From the moment generating function of the multivariate normal distribution (of e_j's) we obtain

(3) $$E[\exp(\Sigma\frac{1}{n}\,e_j)] = \exp(m\,\overset{n}{\Sigma} a_j + \frac{1}{2}\,v\,\overset{n}{\Sigma}\,a_j^2) \quad \text{where,}$$

in this case, $a_j = \frac{1}{n}$. Therefore

(4) $$E(1+r) = \exp(m+\frac{1}{2}\,v/n) \ .$$

Define the return on a long position as

$$(1+RL)^n = P_n/P_o$$

$$RL = (P_n/P_o)^{1/n} - 1.$$

At this point we depart from Praetz' derivation and correct an error in his development. Praetz followed the definition of Fama and Blume[18/]

for the return on a short position (RS), i.e., RS = -RL. Dryden (1969) took exception to Fama and Blume's definition of the return on a short position and defined it as $RS = (2 - P_n/P_o)^{1/n} - 1$. However his argument was not convincing and he failed to resolve adequately the discrepancy between his definition and that of Fama and Blume. The resolution is quite straightforward when it is recognized that the discrepancy arises because the long return is an interest while the short return is a discount. The two definitions are equivalent when returns are continuously compounded. However they are applied in situations where compounding is assumed discreet. If the number of periods is quite large or the variance of returns quite small with respect to the mean, then little difficulty will be encountered, because the two definitions will be approximately equal. It can be shown that the limit of both Dryden's and F&B's RS as the compounding period approaches zero is given by

$$1 + RS = \frac{1}{1 + RL}; \quad RS = (P_o/P_n)^{1/n} - 1$$

and this is the definition of RS which will be used for the remainder of this derivation.

The expected value of RL follows directly from (4)

$$E(1+RL) = \exp(M + \frac{1}{2}\, v/n)$$

$$E(RL) = \exp(m + \frac{1}{2}\, v/n) - 1 .$$

Expanding the RHS we have

$$E(RL) = m + \frac{1}{2}\, v/n + \frac{1}{2}\, (m^2 + mv/n + \frac{1}{4}\, v^2/n^2) + \dots \quad .$$

The variance of RL, V(RL) is clearly equal to V(1+RL). And

$$V(1+RL) = E\{[(1+RL) - E(1+RL)]^2\}$$

$$= E\{(1+RL)^2\} - \{E(1+RL)\}^2 .$$

The first term is

$$(6) \qquad E\{(1+RL)^2\} = E\{[\exp(\sum_j^n e_j/n)]^2\}$$

$$= \exp(2m + 2v/n) ,$$

while the second term is

(7) $$\{E(1+RL)\}^2 = [\exp(m+\frac{1}{2}\,v/n)]^2$$

$$= \exp(2m+v/n) \ .$$

Therefore

$$V(RL) = V(1+RL) = \{\exp(2m+2v/n)\} - \{\exp(2m+v/n)\}$$

Expanding, we obtain

(8) $$V(RL) = v/n + \frac{1}{2}\,(4mv/n + 3v^2/n^2) + \ldots$$

For the expected value and variance of RS we have

$$1 + RS = (P_o/P_n)^{1/n}$$

$$\ln(1+RS) = \frac{1}{n}\ln(P_o/P_n) = -\frac{1}{n}\ln(P_n/P_o)$$

$$= -\frac{1}{n}\sum^{n} e_j = \sum^{n}(-\frac{1}{n})\ e_j$$

and

$$1 + RS = \exp\{\Sigma(-\frac{1}{n})\,e_j\} \ .$$

Therefore

$$E(1+RS) = \exp(-m+\frac{1}{n}\,v/n) \ .$$

Expanding,

(9) $$E(RS) = -m + \frac{1}{2}\,v/n + \frac{1}{2}(m^2-mv/n+\frac{1}{4}\,v^2/n^2) + \ldots \ .$$

Following the procedure used in (6) and (7),

(10) $$V(RS) = V(1+RS) = v/n + \frac{1}{2}(-4mv/n+3v^2/n^2) + \ldots \ .$$

Define the return on a buy-and-hold strategy (RBH) over N periods as

$$(1+RBH)^N = P_n/P_o$$

$$(1+RBH) = (P_n/P_o)^{1/N}$$

The expected value and variance of RBH are given by (5) and (8) with N substituted for n

(11) $$E(RBH) = m + \frac{1}{2} v/N + \frac{1}{2}(m^2 + mv/N + \frac{1}{4} v^2/N^2) + \ldots$$

(12) $$V(RBH) = v/N + \frac{1}{2}(4mv/N + 3v^2/N^2) + \ldots \quad .$$

Define the return on a filter rule (or on any trading rule in which a security is held either long or short) as

(13a) $$(1+RF)^N = (1+RL)^{N_1}(1+RS)^{N_s}$$

(13b) $$(1+RF) = (1+RL)^{1-f}(1+RS)^f$$

$$f = N_s/N; \; 1 - f = N_1/N; \; N_s + N_1 = N \quad .$$

That is, RF is the geometric average return for the trading rule over N periods, during which a long position is taken in N_1 periods and a short position is taken in N_s periods, i.e., a short position is taken for a fraction, f, of the total period.

It makes no difference whether the the short position is taken for N_s consecutive periods or whether it is taken for several shorter nonconsecutive time spans which total up to N_s periods.

The expectations and variances for the periodic return on long positions taken over N_1 periods and on short positions taken over N_s periods are given by

(14) $$E(RL) = m + \frac{1}{2} v/N_1 + \frac{1}{2}(m^2 + mv/N_1 + \frac{1}{4} v^2/N_1^2) + \ldots$$

(15) $$V(RL) = v/N_1 + \frac{1}{2}(4mn/N_1 + 3v^2/N_1^2) + \ldots$$

(16) $$E(RS) = -m + \frac{1}{2} v/N_s + \frac{1}{2}(m^2 - mv/N_s + \frac{1}{4} v^2/N_s^2) + \ldots$$

(17) $$V(RL) = v/N_s + \frac{1}{2}(-4mv/N_s + 3v^2/N_s^2) + \ldots \quad .$$

The expected value of RF is given by

$$E(1+RF) = E[1+RS)^f(1+RL)^{1-f}] \quad .$$

RS and RL are independent, since they have no e_j's in common. Therefore

$$E(1+RF) = \{E[(1+RS)^{f}]\}\{E[(1+RL)^{1-f}]\}$$

$$= \{E[\exp(\sum^{N_s}(-f)e_j)]\}\{E[\exp(\sum^{N_l}(1-f)e_j)]\}$$

$$= \{\exp(-fm+\frac{1}{2}f^2v/N_s)\}\{\exp([1-f]m+\frac{1}{2}[1-f]^2v/N_l)\} \quad .$$

Recalling that $f = N_s/N$ and $1 - f = N_l/N$,

$$E(1+RF) = \{\exp(-fm+\frac{1}{2}fv/N)\}\{\exp([1-f]m + \frac{1}{2}[1-f]v/N\}$$

$$= \exp[(1-2f)m+\frac{1}{2}v/N] \quad .$$

Expanding gives

(18) $$E(RF) = (1-2f)m + \frac{1}{2}v/N + \frac{1}{2}[(1-2f)m+\frac{1}{2}v/N]^2 + \ldots \quad .$$

The variance of RF is given by

$$V(RF) = V(1+RF) = \{E[\exp(\sum^{N_s}(-2f)e_j)]\}\{E[\exp(\sum^{N_l}2(1-f)e_j)]\}$$

$$- \{\exp[1-2f)m+\frac{1}{2}v/N]\}^2$$

$$= \exp[2(1-2f)m+4v/N] - \exp[2(1-2f)+v/N] \quad .$$

Expanding gives

(19) $$V(RF) = 3v/N + \frac{1}{2}(12mv/N+15v^2/N^2) + \ldots \quad .$$

The usefulness of the preceding derivation is twofold. First, it is possible to incorporate the expectation and variance of the return on a trading rule into a test for the verification of the distribution of price changes. For example, we have here derived the theoretical sampling distribution for an assumed lognormal distribution of $(1 + r)$. The distribution is fully determined by four parameters: the mean and variance of changes in log prices (m & v), the total sample size (N), and the fraction of the time the security is held short (f). The assumed distribution may be easily verified or disproved by choosing some arbitrary sample size, determining m and v, choosing some arbitrary fraction over which to hold the security short, and then in repeated tests randomly selecting $N_s = fN$ periods in which to hold a short position. $E(RF)$ is determined given m, v, N, and f. If, say k, repetitions are performed then the hypothesis that $\overline{RF} = E(RF)$ may be easily tested using the standard deviation of the sampling mean $S_{\bar{x}} = \sqrt{V(RF)/k}$. Alternatively, a chi-square or another test for 'goodness-of-fit' might be performed to see how closely the observed distribution of RF's fits the hypothesized distribution.

Second, assuming the hypothesized distribution is verified in the above tests using randomly selected trading periods, the derivation may be used to determine whether trading rule returns generated by a specifically chosen trading rule are significantly greater than the return on a buy-and-hold strategy.

5.0 Specific Applications to the Foreign Exchange Market

There are several ways in which this methodology can be applied to the foreign exchange market. The simplest and most direct method, and the one which most closely follows the use of trading rules for security prices, is to purchase forward exchange for a particular value date whenever the trading rule indicates a 'buy' or a long position and to close that contract out and sell forward exchange whenever the trading rule indicates a 'sell' or a short position. The only data required for this test is a sequence of forward quotations for the same value date, and these are readily available for trades in the International Money Market.

Another way in which this methodology can be used is with spot rates, where the price relatives, P_j/P_{j-1}, are represented by the change in the spot rate plus the interest agio from j-1 to j.

6.0 Conclusions

We have identified some potential problems in establishing the efficiency of national submarkets such as securities and commodities markets and some of the assumptions which are implicit when extending the results of efficient market tests to broader concepts of efficiency. In addition, we have identified some problems which may be unique to the study of efficiency in foreign exchange markets and some reasons why conventional tests of market efficiency developed for securities markets may be inappropriate for foreign exchange markets.

Finally, we have presented some methodological improvements for trading rule tests which may eventually lead to some much more powerful tests of market efficiency.

FOOTNOTES

1. See Fama (1970) for an excellent survey of efficient market theory and empirical evidence.
2. All individuals have the same utility function, and the sum of individual utilities is the same regardless of the distribution of wealth, i.e., regardless of whether all individuals receive the same allocation of goods or some receive a relatively large allocation while others receive a relatively small allocation.
3. Individuals are able to state their preferences, i.e., choose, among risky alternatives solely on the basis of expected value and variance of each alternative. For a more detailed explanation see Mossin (1973). ch. 3.
4. The set of prices includes interest rates, the intertemporal price of money and risk.
5. It is assumed that market participants believe efficiency is the 'natural' state that will prevail in the future, and current decisions are based on forecasts of future efficient resource allocations and prices.
6. A symmetric distribution is implied. For any symmetric distribution, the second moment about the mean is smaller than the second moment about any other point. The variance of the forecast error is the second moment about the forecast. For further discussion see Ball and Watts (1972) p. 664n or Stevenson (1976b).
7. For a more detailed explanation of the theory of rational expectations see Mullineaux (1976) or see Dornbusch (1975) for an example of an application.
8. For more details see Stevenson (1975). This topic was recently raised by Paul McCracken (11/23/76) in a public lecture on current public policy problems.

10. This is a reasonable assumption for a securities or commodities market, but the existence of government intervention in foreign exchange markets may indicate that hte market does not clear among private transactors.
11. Pseudo-optimal or pseudo-efficient indicates the best possible (most utility-maximizing) allocation of resources given the imperfections in the greater market.
12. Giddy and Dufey (1975) p. 5
13. Fama (1970) p. 111
14. Giddy and Dufey (1975) p. 4
15. If individuals in different countries have different utility functions and, for example, the selling of a particular currency is done primarily by the residents of one country while the purchasing of that currency is done primarily by residents of another country.
16. Praetz bases this conclusion on his derivation of expected return, which he shows to be always less than the expected return on a buy-and-hold strategy. However, Praetz <u>assumes</u> the independence of price changes whereas the normal use of filter tests implies the examination of price changes for independence.

17. Empirical evidence is provided by Dooley and Shafer (1976), Giddy and Dufey (1975), Hagerman and Richmond (1973), Logue and Sweeney (1975), and Praetz (1972) among others.

18. Praetz's derivation is not severely impaired by using this definition. The differences between Praetz's derivation and this one show up in the second and succeeding terms in the expansion and will only be significant if either 1) the variance is very large relative to the mean or 2) the number of observations is relatively small.

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IMR: Industrial Management Review
JF: Journal of Finance
JB: Journal of Business
JFQA: Journal of Financial and Quantitative Analysis
JIBS: Journal of International Business Studies
SEJ: Southern Economic Journal
JFE: Journal of Financial Economics