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DURABLE GOODS MONOPOLY UNDER PRIVATE INFORMATION

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by

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Abstract

In the durable goods monopoly literature it is standard to assume perfect foresight over prices on the part of consumers. The typical result that the monopolist's market power is gradually lost is a function of this full information requirement. This paper employs a two period model to examine the effect of asymmetric information on the seller's equilibrium price path and on society's welfare. I show that the firm can regain some of its lost market power, depending on consumer's expectations. Also, there is an expected profit loss compared to the full information case.

I. Introduction

In the durable goods monopoly literature it is standard to assume perfect foresight over prices on the part of consumers. One way to relax this full information assumption is to recognize consumers' uncertainty regarding the monopolist's costs. In this paper a two-period model is used to analyze the problem of a monopolized durable goods industry in which only the monopolist knows its costs and how these costs change with time. The purpose of the study is to investigate the effect on profits of private information and analyze the implications for price dynamics.

Consumers are assumed to know only the distribution of costs in each period. Consumers have rational expectations over second period costs and can infer information about costs and second period price from the price charged in the first period. One option open to the firm with private information over costs is to attempt to "commit" to a lower second period output level by claiming that its costs are high. The low cost firm accomplishes this by charging the same price in the first period as would a high cost firm. If this action can be taken in equilibrium, consumers may be unable to infer the second period price with certainty.

Whether the price path under private information differs significantly from the full information price path depends on consumers' expectations of the monopolist's true cost structure and how these expectations respond in equilibrium to varying price signals. The way in which expectations vary is endogenous to the model. The exogenous element behind the results is the statistical dependence of costs over time.

Section II discusses the framework for the model and presents examples. In Section III a more general case is developed. Conclusions are given in Section IV.

II. General Model with Two Cost Types

A. The Game

The game under consideration can be described in four stages. In the first stage Nature draws a random variable, determining the firm's "type," τ. In the second stage the firm learns its type. Firm type could be determined by any one of a number of firm-specific variables. The actual units and range of $\tau \in [\tau^-, \tau^+]$ will vary depending on whether τ delineates, for instance, costs or capacity. At stage 3 the firm chooses a first period strategy. A strategy for the firm is a function from its type to first period sales price, P_1. The monopolist is assumed to have no means of commiting to a given strategy. The range of types in conjunction with the specification of demand will determine the minimum and maximum observable P_1. Define these to be P_1^- and P_1^+, respectively.

Consumers in Stage 4 must draw an equilibrium inference about the sales price in period two, P_2, from the information contained in P_1. A strategy for consumers is a function g: $[r^-, r^+] \times [P_1^-, P_1^+] \rightarrow [0, 1]$ where r can be thought of as reservation price and "0" ("1") denotes that the consumer does not (does) buy in the first period. We are assuming that risk-neutral consumers are distributed continously along the rental demand curve and that they each buy one unit of the good. Consumers willing in principle to pay P_1 must decide whether to buy the good in the first or second period. This decision is based on the expectation of the capital loss on the durable good, $P_1 - \delta P_2$, where δ is the rate of discount common to consumers and the firm. Consumers form expectations of second period price that are confirmed in equilibrium. Second period sales will be determined directly from the demand curve.

Implicitly, the game is one of competition between types.[1] A low cost monopolist may wish to hide behind its accounting figures in the first period and claim that it is a high cost producer. On the other hand, it may choose to charge that P_1 that reveals it to be low cost. If the former action is a feasible one for a low cost firm, the high cost monopolist may in turn have an incentive to charge a P_1 that distinguishes its true type in the eyes of consumers. These possibilities for strategic behavior must be taken into account by the consumer. A monopoly is known to exist, but its type may not be completely revealed by P_1. There is only one firm in the industry for any given realization of the game.

The interesting case occurs when consumers cannot directly infer P_2 by observing P_1. This imposes the requirement on the model that the private information be multi-dimensional, for example, period one and period two costs. If P_1 were a function of only one unknown, consumers could theoretically invert the function that relates P_1 to the unknown variable and infer P_2.[2]

B. Private Information with Two Cost Types

Let c_1 and c_2 be the constant levels of marginal cost in periods one and two, respectively.[3] Start with the case where c_1 and c_2 each take on only two values, $c_1 = (\underline{c}_1, \bar{c}_1)$ and $c_2 = (\underline{c}_2, \bar{c}_2)$. In this paper it is assumed that consumers gain no information about the monopolist's true type by observing first period price. That is, an equilibrium condition is imposed that requires a high or low cost firm to charge the same price in the first period. The purpose of this restriction is to separate the welfare effect of imposing private information from the signalling effect of P_1.

Assume a firm is low cost $(\underline{c}_1, \underline{c}_2)$ or high cost $(\bar{c}_1, \bar{c}_2)$ in both periods. Costs in the first period are uniformly distributed. A probability λ is attached to $\underline{c}_2$ and $(1 - \lambda)$ to $\bar{c}_2$. The uniform distribution assumption over c_1

implies that the likelihood, λ, of a firm's second period costs being $\underline{c}_2$ or $\bar{c}_2$ does not depend on the observed P_1. The values that c_1 and c_2 take on will be determined by the equilibrium condition mentioned above.

Consumers try to infer the true cost structure of the firm after P_1 is announced. The definition of the user cost of capital and rational expectations over price imply the following relationship between sales price (P) and implicit rental prices (R): $P_1 = R_1 + \delta R_2$. First period buying decisions are based on the expectation of P_2 given P_1, $E(P_2|P_1)$. The demand relation for first period stock, Q_1, as a function of rental price is $Q_1 = Q_1(R_1)$. Noting that in a two period model $R_2 = P_2$, we can write this as $Q_1(P_1 - \delta E(P_2|P_1))$.

Assume a linear demand curve: $\tilde{Q}_2 = \alpha - \beta\tilde{P}_2$. The symbol "~" is used to denote variables under the private information regime. In period two the monopolist chooses P_2 to maximize period-two flow profits:

$$\tilde{P}_2^* = \underset{P_2}{\text{argmax}}\ (P_2 - c_2)[Q_2(P_2) - Q_1(P_1 - \delta E(P_2|P_1)]$$

(1) $$\Rightarrow \tilde{P}_2^* = \frac{\alpha - \tilde{Q}_1 + \beta c_2}{2\beta}.$$

Note that

(2) $$E(\tilde{P}_2^*|\tilde{P}_1) = \lambda\tilde{P}_2^*\Big|_{\underline{c}_2} + (1-\lambda)\tilde{P}_2^*\Big|_{\bar{c}_2}$$

$$= \frac{\alpha - \tilde{Q}_1 + \beta\hat{c}_2}{2\beta},$$

where $E(c_2|\tilde{P}_1) \equiv \hat{c}_2$, and,

(3) $$\tilde{Q}_1 = \alpha - \beta(\tilde{P}_1 - \delta E(\tilde{P}_2|\tilde{P}_1)).$$

Substituting equation (3) into (2) we can eliminate the expectation and derive a reduced form for $\tilde{Q}_1$ as a function of $\tilde{P}_1$:

$$(4) \qquad \tilde{Q}_1(\tilde{P}_1) = \alpha - \frac{2\beta}{2+\delta}\tilde{P}_1 + \frac{\beta\delta c_2}{2+\delta}.$$

To find the optimal $\tilde{P}_1^*$ under private information miximize the present discounted value of profits,

$$\tilde{P}_1^* = \underset{P_1}{\operatorname{argmax}}\ (P_1 - c_1)Q_1(P_1) + \delta[(P_2 - c_2)(Q_2(P_2) - Q_1(P_1))],$$

subject to the perfectness constraint (4).

In general, the perfectness constraint alters the first order condition for P_1 by the addition of a term $\partial P_2/\partial P_1$. We define this term as the <u>price updating rule</u>. A non-zero value for $\partial P_2/\partial P_1$ implies that sales prices are linked over time. Consumers foresee a set relationship between any given Q_1 and P_2. The monopolist incorporates this relationship into the first period problem. This establishes the link: consumers' beliefs about P_2 will influence the P_1 they are willing to pay and thus the P_1 the monopolist can optimally charge. Note that the best of all possible worlds for the durable goods seller would be $(\partial P_2/\partial P_1) = 0$.

Before proceeding, the question of existence must be addressed. The low cost firm has two choices: either choose a $\tilde{P}_1$ that will reveal the firm to be low cost (a "separating" strategy), or choose the $\tilde{P}_1^*$ that the high cost firm would set under private information (a "matching" strategy). The high cost firm also has two choices: either charge $\tilde{P}_1^*$. or charge $\tilde{P}_1^* + \varepsilon$ and reveal to consumers that it is high cost. If the low cost firm finds it more profitable to reveal its true type, an equilibrium with the same properties as the full information case will exist. If the low cost firm mimics the high cost firm when there are only two cost types there is non-existence. The high cost firm will always want to raise price by ε and let consumers know its type. If there were a continuum of types, as in the case that follows, one firm raising its price by ε will only be matching another firm's price and an equilibrium will still be possible. The two cost types case is useful nonetheless in providing intuition for the cases to follow.

Heuristically, the outcome of the model can be seen with the aid of Figures 1 and 2. Figure 1 represents the case for a low cost producer. The low cost firm under private information can afford to charge a higher price because the effective demand curve it faces is shifted out relative to the public information case. With a non-zero probability that the monopolist could be high cost, consumers will attach a positive weight to both potential second period price outcomes. This lowers the expected capital loss given the first period price. Buyers will demand more at any given price as is shown in Figure 1. That is, demand has shifted out. While consumers are on their demand curve in expected price, they are off the curve ex-post. The opposite effect (Figure 2) holds if the firm is a true high cost producer. In this case consumers may be afraid that the firm will be revealed to have low costs and charge a second period price much lower than expected. Uncertainty on the part of consumers as to the identity of the monopolist imposes a negative externality and decreases revenues for the high cost monopolist.[4]

To find the price updating rule substitute equation $\tilde{Q}_1(\tilde{P}_1)$ from equation (4) into the first order condition for second period price (equation (1)) and differentiate:

$$\frac{\partial \tilde{P}_2^*}{\partial \tilde{P}_1} = \frac{1}{2 + \delta}. \qquad (5)$$

This price updating rule is identical to that under full information. If expectations of second period cost are not a function of P_1, imposing private information on the model has no effect on price dynamics. No leverage is gained over consumers when P_1 does not signal commitment to a cost type. Changing the assumptions of the model to allow c_1 and c_2 to be imperfectly correlated, as in Section III, drives a wedge between the public and private information cases.

III. The Effect of Private Information with a Continuum of Types

We now turn to the equilibrium properties and welfare implications of the model when there is a continuum of potential types. The analysis begins by using the illustrative case of two cost types from the previous section and applies it first to the case where c_1 is a continuum distributed independently of c_2 and second to the case of costs which are correlated over time. Again, the equilibrium studied is that which occurs when the consumers cannot unravel the monopolist's true type by observing P1.

A. Independent Costs Over Time

Let $c_1 \varepsilon (c_1^- , c_1^+)$ be a uniformly distributed variable and let $c_2 = (\underline{c}_2, \bar{c}_2)$ as before. There is now a range of first period prices that could be charged either by a $\underline{c}_2$ or $\bar{c}_2$ firm, depending on the associated c_1. There must be some $\underline{c}_1^i$ and $\bar{c}_1^j$ for which $(\underline{c}_1^i, \underline{c}_2)$ and $(\bar{c}_1^j, \bar{c}_2)$ lead to the same P_1. These are depicted in Figure 3 as $\underline{c}_1^*$ and $\bar{c}_1^*$. Unknown to the consumers is whether their state of the world is the upper or lower line segment of Figure 3.

Given the support of $\underline{c}_1$ and $\bar{c}_1$, defined by $(\underline{c}_1^{\ell}, \underline{c}_1^{u})$ and $(\bar{c}_1^{\ell}, \bar{c}_1^{u})$, respectively, the marginal densitites, $Pr(\underline{c}_1)$ and $Pr(\bar{c}_1)$, are uniform. In order to isolate the welfare effect of private information, the supports of c_1

are constructed so that the conditional densities are uniform. In particular, when one takes a horizontal slice of Figure 3, the probability of c_2 being $\underline{c}_2$ or $\bar{c}_2$ is invariant as the slice moves up or down. Changes in P_1 provide no additional information on the probability that the monopolist is high or low cost. λ remains a constant. Further, Figure 3 demonstrates that the case under study is actually made up of a continuum of two cost cases.

The first result focuses on expected profits under private and full information.

Theorem 1: With linear rental demand, expected profits under private information are less than those under public information when the two firm types choose the same first period action.

Proof: Given in Appendix A.

Theorem 1 implies that while the low cost firm gains at the expense of the high cost firm, *on average* profits are *lower* under private information than under public information. The equations in Appendix A point out that the difference between $E\tilde{\pi}$ and $E\pi$ turns on the covariance of quantity with second period costs. In the private information case quantity covaries too little with costs, resulting in a misallocation of costs across types.

It is not surprising that in expected value the firm is worse off under private information. Given that consumers do not update their inferences about costs upon observing prices, firms would like to reveal who they are.

Also, given that consumers can substitute out of Q_1 into Q_2 they will not bear all of the social losses. The production misallocation represents the share of this loss borne by the firm. If Q_2 were independent of second period price (for instance, if the demand curve were highly kinked), then this nonlinearity would reverse the result. In this case, Q_2 would not covary with c_2 and the second period expected profit difference would disappear. Net first period profit gain would remain. In the linear case, however, the direct expected gain from fooling consumers in the first period is swamped by the monopolist's share of the social losses in the second period.

An important final remark concerns the price updating rule. Note that equation (5), which defines the price updating rule, still applies for the case where c_1 is continuous but independently distributed of c_2. P_1 provides no information that consumers can use to readjust their prior beliefs about $\hat{c}_2$. In this respect, the case of perfectly correlated costs that are private information for the firm and the case of given costs that are common knowledge are not dissimilar. Although the price levels will be different, the rate of change of prices is unaltered. There is no change in the price updating rule that the monopolist uses. The perfectness constraint binds with as much force as in the full information case.

B. Associated Costs Over Time

Again assume that there is a continuum of possible marginal costs in the first period and two possible levels of marginal cost the second period. This section relaxes the independence assumption and allow costs to be associated over time. Differing price signals in the first period will alter consumers' estimates of the probability that the monopolist is high or low cost. If consumers knew that costs were positively associated over time a high P_1 might be used to infer that the likelihood of the firm being a true low cost type is small. No assumptions on the sign of the covariance between first and second period costs are made.

Let $\underline{c}_1(P_1)$ be that $\underline{c}_1$ which leads to P_1 if $\underline{c}_2$ is the state of the world in period two. Similarly define $\bar{c}_1(P_1)$. Denote the joint density of c_1 and c_2 by $f(c_1, c_2)$. While the marginal densities of $\underline{c}_1$ and $\bar{c}_1$ are uniform, as before, the density of c_1 conditional on c_2 is not. Thus,

$$\Pr(\underline{c}_2|P_1) = \frac{f(\underline{c}_1(P_1), \underline{c}_2)}{f(\underline{c}_1(P_1), \underline{c}_2) + f(\bar{c}_1(P_1), \bar{c}_2)}, \tag{6}$$

where the numerator is the probability of the particular event $(\underline{c}_1, \underline{c}_2)$ and the denominator is the probability of the firm being low or high cost. Defining $\Pr(\bar{c}_2|P_1)$ in a similar fashion and taking the expectation yields:

$$E(c_2|P_1) \equiv \hat{c}_2(P_1) = \frac{f(\underline{c}_1(P_1), \underline{c}_2)\underline{c}_2 + f(\bar{c}_1(P_1), \bar{c}_2)\bar{c}_2}{f(\underline{c}_1(P_1),\underline{c}_2) + f(\bar{c}_1(P_1),\bar{c}_2)}. \tag{7}$$

Equations (1) and (2) remain the same while (4) is changed to

$$\tilde{Q}_1(\tilde{P}_1) = \alpha - \frac{2\beta}{2 + \delta}\tilde{P}_1 + \frac{\beta\delta\hat{c}_2(\tilde{P}_1)}{2 + \delta}. \tag{8}$$

Going through the analogous first period maximization a new equilibrium $\tilde{P}_1$ is found for the case where the expectation of costs itself depends on $\tilde{P}_1$. In general, $\tilde{P}_1^* = \tilde{P}_1^*(\alpha, \beta, \delta, c_1, c_2, \hat{c}_2, \hat{c}_2')$, where

$$\hat{c}_2' \equiv \frac{\partial\hat{c}_2}{\partial\tilde{P}_1} = \frac{\partial E(c_2|\tilde{P}_1)}{\partial\tilde{P}_1}. \tag{9}$$

It is $\hat{c}_2'$ that emerges as the driving force behind the dynamic price path. The sign of $\hat{c}_2'$ will determine whether the perfectness constraint is more or less binding. Equations (7) and (9) together define a first order differential equation in $\hat{c}_2$. Without solving this equation we can learn something about the equilibrium and the properties of $\hat{c}_2'(P_1)$. Using (9), one can define $c_1(P_1)$ explicitly (taking advantage of the linear nature of the problem):

$$c_1(P_1) = c(\alpha, \beta, \hat{c}_2, \hat{c}_2', P_1) + \frac{\delta}{2} c_2. \tag{10}$$

Let $\bar{f}$ be shorthand notation for $f(\bar{c}_1(P_1), \bar{c}_2)$ and similarly for $\underline{f}$. Then the rational expectations identity (7) can be written as:

$$\hat{c}_2(\underline{f} + \bar{f}) = \underline{c}_2\underline{f} + \bar{c}_2\bar{f}$$

$$\Longleftrightarrow \quad \hat{c}_2(1 + \frac{\bar{f}}{\underline{f}}) = \underline{c}_2 + \bar{c}_2(\frac{\bar{f}}{\underline{f}}). \tag{11}$$

From (11) we claim that sgn $\hat{c}_2' =$ sgn $\partial(\bar{f}/\underline{f})/\partial P_1$. This observation leads to the following theorem.

<u>Theorem 2</u>: If the likelihood of observing $\bar{c}_2$ decreases in c_1, i.e., if $\frac{\partial}{\partial c_1} \ln (\frac{\bar{f}}{\underline{f}}) < 0$, then $\hat{c}_2' < 0$. If the likelihood of observing $\bar{c}_2$ increases in c_1, then $0 < \hat{c}_2' < \frac{2}{\delta}$.

<u>Proof</u>: See Appendix B.

The sign of the likelihood function with respect to an increase in c_1 relates to the covariance of costs over the two periods. Suppose the production process involves learning by doing. Then costs will covary positively over time. As c_1 increases, consumers are less surprised to see $\bar{c}_2$ next period and more surprised if $\underline{c}_2$ occurs: $\frac{\partial}{\partial c_1} \ln (\frac{\bar{f}}{\underline{f}}) > 0$. At the same time this positive covariance implies that if a high P_1 is announced, consumers reason that such a P_1 is more likely to have come from a relatively high cost firm both periods than a relatively low cost firm: $\hat{c}_2'$ will be positive. The two inferences on expectations and the likelihood ratio go hand in hand given the covariance of costs with time.

Taking positive covariance as the more likely case, the conclusion follows that the perfectness constraint becomes less binding. The derivative of $\tilde{P}_2^*$ in equation (1) with respect to P_1 is

$$\frac{\partial \tilde{P}_2^*}{\partial \tilde{P}_1} = \frac{1}{2 + \delta} - \frac{\delta}{2(2 + \delta)} \hat{c}_2'$$

$$< \frac{1}{2 + \delta} \quad \text{if } \hat{c}_2' > 0.$$

But, $\frac{1}{2 + \delta}$ is the value of $\partial P_2/\partial P_1$ both in the perfect certainty case and in the private information case when costs are perfectly correlated over the two periods. A lower value for $\partial \tilde{P}_2^*/\partial \tilde{P}_1$ reflects two properties of the equilibrium. First, it implies that the function relating $\tilde{P}_1$ to $\tilde{P}_2$ is flatter, or rather, the dynamic characteristics of the price path have changed. Second, it shows that the link between $\tilde{P}_1$ and $\tilde{P}_2$ has been weakened. In other words, the incentive constraint on the monopolist is relaxed with imperfectly correlated costs and private information.

This in turn implies that the externality under private information with imperfectly (positively) correlated costs is diminished. When costs have a positive association over time, the monopolist, despite the uncertainty, can convince consumers that a minimum expected capital loss is more likely. An extremely high first period price will lead consumers to believe that an extremely low second period price is very unlikely and this lowers the expected capital loss. Rational expectations over second period costs guarantees that consumers will be right. This guarantee is a means of commitment for the monopolist no matter what his cost level.

IV. Conclusion

This paper has emphasized the role of private information in the durable goods monopoly problem. Consumers must infer second period price from the price charged in the first period, given that they do not know the monopolist's true cost structure. Under these conditions, a low cost firm may have an incentive to charge the same price as would a high cost firm. If costs are unassociated over time, first period price is an uninformative signal. When price in period one does not signal commitment to a particular cost type the characteristics of the dynamic price path are unchanged relative to the full information case. Second, imposing private information leads to an expected profit loss, again relative to the full information case. Abstracting from the effect on profits of consumers basing their decisions on expected second period price, there is a misallocation of production across types under private information. In the linear demand case this misallocation outweighs the direct benefit to the monopolist of influencing expected second period sales price.

Positively associated costs over time reduces the tendency for the monopolist to lose profits on average when consumers cannot infer true costs from first period price. The nature of the private information alters the commitment problem for the monopolist. In many situations private information makes it more difficult to have binding contracts. In the durable goods monopoly model credible revelation continues to be a problem for the monopolist. Positive association of costs over time, however, allows the monopolist to "commit" in expected value to a price strategy. The classic incentives constraint faced by the durable goods monopolist is relaxed.

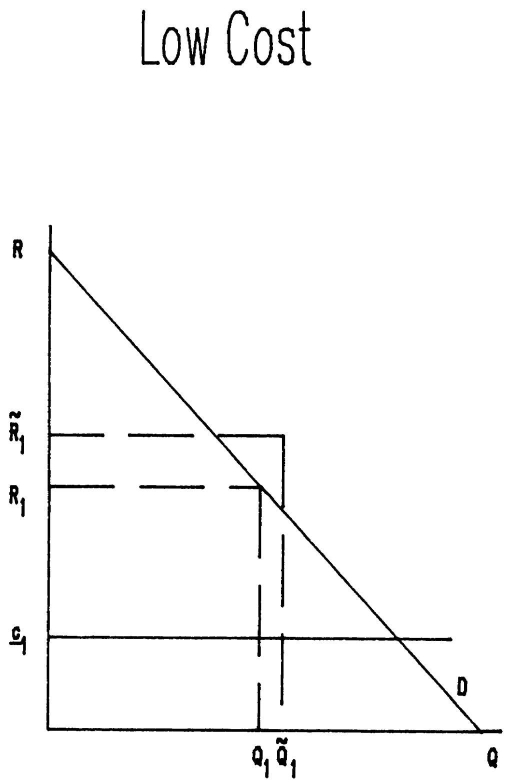

Figure 1

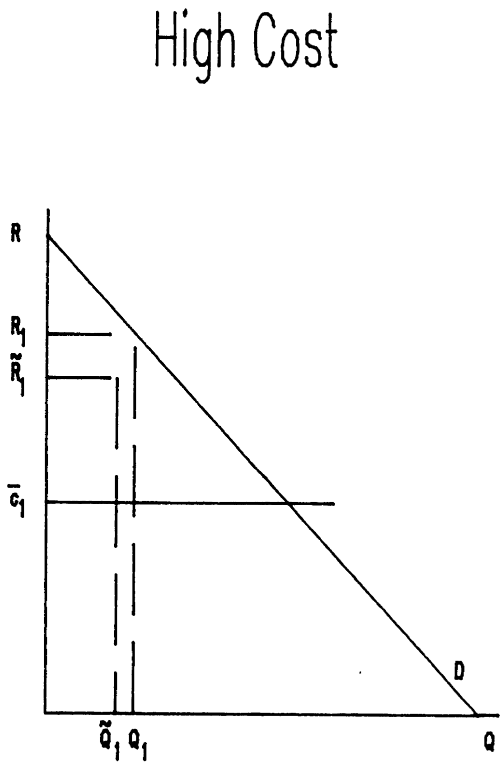

Figure 2

Continuum of Types with Independent Costs

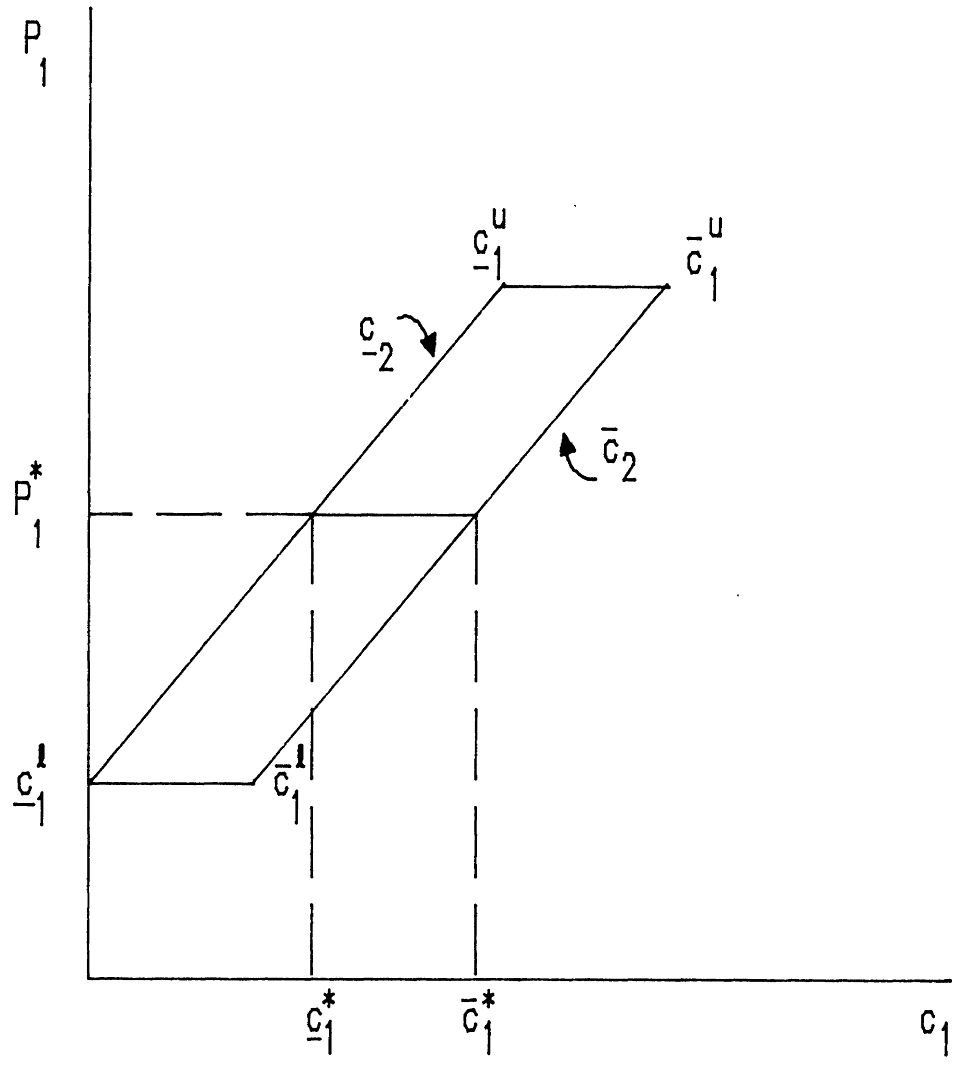

Figure 3

Appendix A

Proof of Theorem 1

Both firm types will charge an identical $\tilde{P}_1$ if

$$\underline{c}_1 = \bar{c}_1 + \frac{\delta}{2}(\bar{c}_2 - \underline{c}_2). \tag{A.1}$$

Using the assumption of perfect correlation for the two cost case one can further assume $\bar{c}_1 = k\hat{c}_2 - \frac{\delta}{2}\,\bar{c}_2$ which implies $\underline{c}_1 = k\hat{c}_2 - \frac{\delta}{2}\,\underline{c}_2$ from (A.1). Letting k be continuously valued between 0 and 1 will map out all the possible c_1 values. Subtracting profits in the two information regimes yields:

$$\begin{aligned}\tilde{\pi} - \pi = {} & -\alpha(c_2 - \hat{c}_2)\frac{k_3}{K} + P_1(c_2 - \hat{c}_2)[-k_5 + \frac{\beta k_3}{K} + \delta\beta k_4] \\ & - P_2(c_2 - \hat{c}_2)[\frac{\delta\beta k_3}{K} + \delta\beta k_4 - \delta k_5 + \delta(1 + \delta)\beta k_4] + c_1(c_2 - \hat{c}_2)k_5 \\ & + c_2(c_2 - \hat{c}_2)[\delta\beta k_4 - \delta k_5] + (c_2 - \hat{c}_2)^2[\frac{k_3k_5}{K} - \delta\beta k_4^2 + \delta\ k_4k_5],\end{aligned} \tag{A.2}$$

where $k_3 = \frac{2\beta\delta}{(2 + \delta)^2}$, $k_4 = \frac{\delta}{2(4 + \delta)}$, $k_5 = \frac{2\beta\delta}{(2 + \delta)(4 + \delta)}$, and $K = \frac{2\beta(4 + \delta)}{(2 + \delta)^2}$.

Taking the expectation with respect to c_2 yields

$$E(\tilde{\pi} - \pi) = \sigma^2_{c_2}\beta\delta^2[\frac{-(16 + 20\delta + 8\delta^2 + \delta^3)}{4(2 + \delta)^2(4 + \delta)^2}] < 0, \tag{A.3}$$

where $\sigma^2_{c_2} = \mathrm{Var}\,(c_2)$.

Q.E.D.

The magnitude of the difference in expected profits in the two information regimes depends on the discount rate, the slope of the demand curve for rental services, and the variance of second period costs.

More insight into this result is possible if one uses the reduced form representation of $\tilde{Q}_1$ given by equation (4) in the text to write present discounted profits as

$$\tilde{\pi} = (\tilde{P}_1 - c_1)(\alpha - \frac{2\beta}{2+\delta}\tilde{P}_1 + \frac{\beta\delta}{2+\delta}\hat{c}_2) + \delta[\tilde{P}_2 - c_2)(\tilde{Q}_2 - \tilde{Q}_1)]$$

$$= [\tilde{P}_1 - c_1)(\alpha - \frac{2\beta}{2+\delta}\tilde{P}_1)] + \delta[\tilde{P}_2 - c_2)(\tilde{Q}_2 - \tilde{Q}_1) + \frac{1}{\delta}(\frac{\beta\delta}{2+\delta})(P_1 - c_1)\hat{c}_2]$$

$$= \tilde{\pi}^1 + \delta\tilde{\pi}^2,$$

letting $\tilde{\pi}^1$ and $\tilde{\pi}^2$ represent the first and second bracketed terms, respectively.

Define π^1 and π^2 similarly for the full information case. Consider first the comparison of $E\tilde{\pi}^1$ with $E\pi^1$. Using the relationships set out in equation (iv) of footnote 4, one can write $\tilde{P}_1$ as a function of P_1, and $(c_2 - \hat{c}_2)$. In calculating $E(\tilde{\pi}^1 - \pi^1)$, after making this transformation, the P_1 terms will cancel, unless they are multiplied by $(c_2 - \hat{c}_2)$. Upon manipulation of the remaining terms, one finds that

$$\text{(A.4)} \qquad E(\tilde{\pi}^1 - \pi^1) = -\sigma^2_{c_2} \frac{\partial Q_1}{\partial \Delta c_2} \left(\frac{\partial \tilde{P}_1}{\partial c_2} + \frac{\partial \tilde{P}_1}{\partial \Delta c_2} \right) < 0$$

where $\Delta c_2 = (c_2 - \hat{c}_2)$ and the signs of the derivatives have already been incorporated in the equation. This representation comes from rewriting $\tilde{P}_1$ given by equation (5) as a function of the two terms c_2 and Δc_2. Equation (A.4) describes a deadweight triangle. There is misallocation across types under private information when first period profits are accounted on an accrual basis (where the expected capital loss in the first period is moved into second period profits). The dampened "covariance" of production and costs under private information plus the linearity of the problem drive the result.

Following the same procedure for the second period profits we have:

$$E(\tilde{\pi}^2 - \pi^2) = \sigma^2_{c_2} \left\{ -\frac{\partial \tilde{P}_2}{\partial \Delta c_2} \left[\frac{1}{2} \left(\frac{\partial \tilde{Q}_1}{\partial \tilde{P}_1} \frac{\partial P_1}{\partial c_2} + \frac{\partial Q_1}{\partial c_2} + \frac{\partial Q_1}{\partial P_1} \right) \right.\right.$$

$$\left.\left. + \left(\frac{\partial \tilde{Q}_2}{\partial \Delta c_2} - \frac{\partial \tilde{Q}_1}{\partial \Delta c_2} \right) \right] + \frac{1}{\delta} \frac{\partial \tilde{P}_1}{\partial \Delta c_2} \frac{\partial \tilde{Q}_1}{\partial \hat{c}_2} \right\}.$$

Or,

$$E(\tilde{\pi}^2 - \pi^2) = \sigma_{c_2}^2 \left\{ - \left(\frac{8 + \delta^2}{2 + \delta} \right) \frac{\partial \tilde{P}_2}{\partial \Delta c_2} \frac{\partial \tilde{Q}_2}{\partial \Delta c_2} + \frac{1}{2} \frac{\partial \tilde{Q}_1}{\partial \tilde{P}_1} \frac{\partial \tilde{P}_1}{\partial \Delta c_2} \right\} < 0,$$

after performing several substitutions.

In the second period there are two effects to a change in $(c_2 - \hat{c}_2)$. The first can be thought of as a shift in the second period demand curve. Ex-post, consumers are not where they thought they would be on the demand curve. The second effect is a movement along the first period demand curve. If consumers see an increase in Δc_2 they substitute into Q_1.

Putting the two pieces of the profit comparison back together yields

$$E(\tilde{\pi} - \pi) = -\sigma_{c_2}^2 \left\{ \frac{\partial \tilde{Q}_1}{\partial \Delta c_2} \left(\frac{\partial \tilde{P}_1}{\partial c_2} + \frac{\partial \tilde{P}_1}{\partial \Delta c_2} \right) \right.$$

$$\left. + \left(\frac{8 + \delta^2}{2 + \delta} \right) \frac{\partial \tilde{P}_2}{\partial \Delta c_2} \frac{\partial \tilde{Q}_2}{\partial \Delta c_2} - \frac{\partial \tilde{Q}_1}{\partial \tilde{P}_1} \frac{\partial \tilde{P}_1}{\partial \Delta c_2} \right\} < 0.$$

Q.E.D

Appendix B

Proof of Theorem 2

Differentiate equation (11) with respect to P_1:

$$\text{(B.1)} \qquad \hat{c}_2'(1 + \frac{\bar{f}}{\underline{f}}) + \hat{c}_2(\frac{\bar{f}}{\underline{f}})' = \bar{c}_2(\frac{\bar{f}}{\underline{f}})', \text{ where } \frac{\bar{f}'}{\underline{f}} = \frac{\partial(\bar{f}/\underline{f})}{\partial P_1}$$

$$=> \hat{c}_2'(1 + \frac{\bar{f}}{\underline{f}}) = (\frac{\bar{f}}{\underline{f}})' \; (\bar{c}_2 - \frac{\underline{f}\,\underline{c}_2 + \bar{f}\,\bar{c}_2}{\underline{f} + \bar{f}}), \text{ using (7).}$$

Or,
$$\hat{c}_2{}' = \frac{1}{(1 + \bar{f}/\underline{f})^2} \; (\frac{\bar{f}}{\underline{f}})' \; (\bar{c}_2 - \underline{c}_2).$$

Furthermore,

$$\text{(B.2)} \qquad (\frac{\bar{f}}{\underset{\sim}{f}})' = \frac{\partial c_1}{\partial \tilde{P}_1} \frac{1}{\underline{f}} (\frac{\partial \bar{f}}{\partial c_1} - \frac{(\partial \underline{f}/\partial c_1)\bar{f}}{\underline{f}}) = \frac{\bar{f}}{\underline{f}} \frac{\partial c_1}{\partial \tilde{P}_1} [\frac{\partial}{\partial c_1} \ell \quad \frac{\bar{\ }}{\underline{f}}$$

Using the first order condition for $\tilde{P}_1$, $\pi(\tilde{P}_1, c_1, c_2) = 0$, and totally differentiating with respect to $\tilde{P}_1$ and c_1 yields:

$$\text{(B.3)} \qquad \frac{dc_1}{d\tilde{P}_1} = \frac{\pi_{\tilde{P}_1^2}}{\pi_{\tilde{P}_1, c_1}} = - \frac{\pi_{\tilde{P}_1^2}}{\frac{\beta}{2 + \delta}(2 - \delta\hat{c}_2')} = - \frac{\frac{2 + \delta}{\beta}\pi_{\tilde{P}_1^2}}{2 - \delta\hat{c}_2'}.$$

$$=> \text{sgn}\, \frac{dc_1}{d\tilde{P}_1} = \text{sgn}\, (2 - \delta\hat{c}_2{}'),$$

since $\pi_{\tilde{P}_1^2} < 0$ by the second order conditions for profit maximization. More explicitly:

$$c_2' > \frac{2}{\delta} \quad <=> \quad \frac{dc_1}{d\tilde{P}_1} < 0,$$

and

$$c_2' < \frac{2}{\delta} \quad <=> \quad \frac{dc_1}{d\tilde{P}_1} < 0.$$

Substituting (B.2) into (B.1) yields an equation that relates the sign of changes in the likelihood ratio with respect to c_1, to the sign of changes in expectations with respect to $\tilde{P}_1$:

$$\text{(B.4)} \qquad \hat{c}_2' = \frac{\bar{f}/\underline{f}}{(1 + \bar{f}/\underline{f})^2} \frac{\partial c_1}{\partial \tilde{P}_1} \left[\frac{\partial}{\partial c_1} \ell n \left(\frac{\bar{f}}{\underline{f}}\right) \right] (\bar{c}_2 - \underline{c}_2).$$

The parameter $\hat{c}_2'$ can assume a value in one of three intervals: $\hat{c}_2' \varepsilon \{\hat{c}_2' < 0, 0 < \hat{c}_2' < \frac{2}{\delta}\ \hat{c}'_2 > \frac{2}{\delta}\}$. One of these intervals can be plausibly eliminated by reference to the reduced form demand curve $\tilde{Q}_1(\tilde{P}_1)$. Note that

$$Q_1'(\tilde{P}_1) = \frac{-2\beta}{2 + \delta} + \frac{\beta\delta}{2 + \delta} \hat{c}_2'$$

is less than zero if and only if $\hat{c}_2' < \frac{2}{\delta}$. Thus, if the demand for the first period stock, which is identical to the first period flow, is to be decreasing in $\tilde{P}_1$, $\hat{c}_2'$ cannot be greater than $\frac{2}{\delta}$. If $\frac{\partial}{\partial c_1} \ell n \left(\frac{\bar{f}}{\underline{f}}\right) < 0$ then $\hat{c}_2'$ must be less than zero since it appears on both sides of equation (B.4) via equation (B.3). The opposite case holds when $\frac{\partial}{\partial c_1} \ell n \left(\frac{\bar{f}}{\underline{f}}\right) > 0$. Q.E.D.

Notes

[1] This idea of competition between types is similar to that used in Rogerson [1981] and is distinct from the Milgrom and Roberts [1982] model in which the firm can commit to a strategy as a function of type.

[2] Let $c = (c_1, c_2)$ be the vector of variables uncertain to the consumer. Let $\pi = \pi_1 + \delta\pi_2$ be discounted monopoly profits. Totally differentiate the first order conditions for P_2 and P_1, $\frac{\partial\pi_2}{\partial P_2}(P_2, P_1, c_1, c_2) = 0$ and $\frac{\partial\pi}{\partial P_1}(P_1, c_1, c_2) = 0$, respectively, with respect to P_1, P_2, c_1 and c_2:

$$\begin{vmatrix} \frac{\partial^2\pi_2}{\partial P_2 \partial c_1} & \frac{\partial^2\pi_2}{\partial P_2 \partial c_2} \\ \frac{\partial^2\pi}{\partial P_1 \partial c_1} & \frac{\partial^2\pi}{\partial P_1 \partial c_2} \end{vmatrix} \begin{vmatrix} dc_1 \\ dc_2 \end{vmatrix} = \begin{vmatrix} \frac{\partial^2\pi_2}{\partial P_2 \partial P_1} & \frac{\partial^2\pi_2}{\partial P_2^2} \\ \frac{\partial^2\pi}{\partial P_1^2} & 0 \end{vmatrix} \begin{vmatrix} dP_1 \\ dP_2 \end{vmatrix}$$

The 2 x 2 matrix on the left-hand side of the equation must have full rank if there is to be a unique (c_1,c_2) pair that determines prices.

[3] It is easy to show that making first and second period costs the operative privately known variables for the firm satisfies the requirements of equilibrium (the matrix on the left hand side of the above equation has rank 2).

[4] Solving for the optimal first period price given linear demand yields:

$$\text{(i)} \qquad \tilde{P}_1^* = \frac{\alpha + \frac{2\beta}{2+\delta} c_1 - \frac{\beta\delta}{2+\delta} c_2 + \frac{2\beta\delta}{(2+\delta)^2}\hat{c}_2}{\frac{2\beta(4+\delta)}{(2+\delta)^2}}.$$

Formulas for P_1^* and P_2^*, the optimal prices when costs are public information, are found by setting $\hat{c}_2 = c_2$:

(ii) $$P_2^* = \frac{\alpha - Q_1 + \beta c_2}{2\beta}.$$

(iii) $$P_1^* = \frac{\alpha + \frac{2\beta}{2+\delta} c_1 - \frac{\delta^2 \beta}{(2+\delta)^2} c_2}{\frac{2\beta(4+\delta)}{(2+\delta)^2}}.$$

Writing the private information prices and quantities in terms of their public information counterparts yields:

$$\tilde{P}_1^* = P_1^* - \frac{\delta}{4+\delta}(c_2 - \hat{c}_2)$$

$$\tilde{P}_2^* = P_2^* - \frac{\delta}{2(4+\delta)}(c_2 - \hat{c}_2)$$

(iv) $$E(\tilde{P}_2^* | \tilde{P}_1) = P_2^* - \frac{1}{2+\delta}(c_2 - \hat{c}_2)$$

If the monopolist were high cost, $\bar{c}_2 - \hat{c}_2$ would be positive causing $\tilde{P}_1^*$ and $\tilde{P}_2^*$ given $\bar{c}_2$ to be less than P_1^* and P_2^*, respectively.

References

Aiyagari, S. R. and Mohring, H. [1979], "A Note on 'Lease Only'," Center for Economic Research, University of Michigan, mimeo., October.

Bulow, J. I. [1982], "Durable Goods Monopolists," JPE, Vol. 90, No. 2 (April), pp. 314-332.

Coase, R. H. [1972], "Durability and Monopoly," JLE, Vol. 15 (April), pp. 143-149.

Kreps, D. [1981], "Sequential Equilibria," Institute for Mathematical Studies in the Social Sciences, the Economics Series, Technical Report. No. 340, Stanford University, July.

Milgrom, P. and J. Roberts [1982], "Limit Pricing and Entry Under Incomplete Information: An Equilibrium Analysis," Econometrica, Vol. 50 (March), pp. 443-460.

Stokey, N. L. [1979], "Intertemporal Price Discrimination," QJE, (August), pp. 355-371.

Stokey, N. L. [1981], "Rational Expectations and Durable Goods Pricing," BJE, Vol. 12 (Spring), pp. 112-128.

Suslow, V. [1984], "Monopoly Pricing with Intertemporal Complications," unpublished PhD dissertation, Stanford University.