Division of Research
Graduate School of Business Administration
The University of Michigan

July 1979

PARAMETER INSTABILITY IN THE SINGLE FACTOR MARKET MODEL

Working Paper No. 185

Arthur D. Warga

The University of Michigan

Introduction

The purpose of this paper will be to discuss model parameter instability over time, especially in the context of switching regressions, and to illustrate the discussion with an application to a financial markets model. Topics discussed are motivated by two interests developed during the research stage. The first involves a class of independent, linear, and unbiased residuals from which test statistics may be derived. The second interest is in the application, which deals with a simple regression for which we anticipated two regime switching(i.e. the model parameters are stable but change value once at some point in time).

Section one of the paper will discuss techniques for testing for the presence and location of coefficient instability. We present a discussion of recursive residuals, which are factors common to, or strongly related to, each of the testing procedures discussed. We then present several procedures useful for testing for the presence(not point) of switching or instability. We conclude with a procedure to aid in detecting the point in time of a possible switch. In section two, we introduce a market model and discuss specific mechanisms that might cause model parameters to shift. We then proceed to apply some of the techniques of section one to the model, and discuss some of the shortcomings relative to the application.

Recursive Residuals

We shall now examine recursive residuals, in an attempt to gain a perspective about them. Based on the model $Y=X\beta+U$ under classical assumptions,

we define $$w_r = \frac{y_r - x_r b_{r-1}}{(1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r)^{1/2}} = \frac{y_r - x_r(X'_{r-1}X_{r-1})^{-1}x'_{r-1}y_{r-1}}{(1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r)^{1/2}}$$

$r = k+1,\ldots,T$ and where;

$X'_{r-1} = (x_1,\ldots,x_{r-1})$

$Y'_{r-1} = (y_1,\ldots,y_{r-1})$

b_{r-1} = O.L.S. estimator of β

k = number of independent variables in the model

The w_r are obviously normal mean zero and with variance σ^2 (say). The numerator is the L.S. predictor with variance $\sigma^2(1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r)$. Additionally we have $E(w_r w_s) = 0$ (a straight forward exercise in expanding terms) which suffices to establish independence in normally distributed (mean zero) random variables. These residuals are easily computed since simple recursive formulae exist for updating both b_j and $(X'_jX_j)^{-1}$. We can view the set of w_r's as a transformation from a T-dimensional space (of the T dependent variables y_r) to a T-k dimensional space (of the w_r). Letting H denote the transformation, U the vector of u_r's, and W the vector of w_r's, we have

$HY = HX\beta + HU = W$

$E(W) = \underline{0}$, which implies that we can choose H so that $HX = \underline{0}$, and so

$HY = HU = W$, which shows that we can also express the w_r as a linear transformation on the u_r. This formula (as presented in (1)) is

$$w_r = (u_r - x_r(X'_{r-1}X_{r-1})^{-1}\sum_{j=1}^{r-1} x_j u_j)(1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r)^{-1/2}$$

Let us now examine the matrix H with the simple model $y_r = \alpha + u_r$ (where α is a constant). In this case we have (recalling $K=1, x_r=1, X_{r-1}$ is an r-1 by 1 vector of 1's)

$$w_2 = \frac{y_2 - y_1}{\sqrt{2}} = \frac{u_2 - u_1}{\sqrt{2}}$$

$$w_3 = \frac{-y_1 - y_2 + 2y_3}{\sqrt{6}} = \frac{-u_1 - u_2 + 2u_3}{\sqrt{6}}$$

$$\vdots$$

$$w_T = \frac{-y_1 - y_2 - \cdots - y_{T-2} + (T-1)y_{T-1}}{\sqrt{T(T-1)}}$$

$$\text{or} \quad H = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ -1/\sqrt{6} & -1/\sqrt{6} & 2/\sqrt{6} & \cdot & \cdot & \cdot & 0 & 0 \\ \vdots & & & & & & \vdots & \vdots \\ -1/\sqrt{T(T-1)} & \cdot & \cdot & \cdot & & \cdot & -1/\sqrt{T(T-1)} & \frac{(T-1)}{\sqrt{T(T-1)}} \end{bmatrix} \longrightarrow (T-1)\times T$$

This matrix is known as Helmert's transformation. Kendall and Stuart, (2) for example, use it to prove the independence of the sample mean and variance in samples from normal populations. The sums of squares of elements in any row is one, the sum of cross products of elements in any two rows is zero, and it has Jacobian equal to unity. Thus, at least for this special case we have $HH' = I_{T-1}$.

The transformation H has similar properties in the general setting too. Since $HU = W$, and $E(WW') = \sigma^2 I_{T-K}$, we have

$$E(WW') = E(HUU'H') = \sigma^2 I_{T-K} \quad \text{or} \quad HH' = I_{T-K}$$

An even more interesting property of H exhibits itself when we examine H'H. We write

$$E(W'W) = (T-K)\sigma^2 = E(U'H'HU) = E(\mathrm{tr}UU'H'H) = \mathrm{tr}E(UU'H'H) = \sigma^2 \mathrm{tr}(H'H) \text{ or}$$

$$\mathrm{tr}(H'H) = T-K$$

Also $(H'H)(H'H) = H'I_{T-K}H = H'H$ so H'H is idempotent. One matrix with the same two properties as H'H which we have seen before, is the matrix $M = I_T - X_T(X_T'X_T)^{-1}X_T'$ used in the definition of least squares residuals. Theil(3) shows that for H, and in fact for any transformation yielding Linear Unbiased residual vectors with Scalar covariance matrix (LUS), we get H'H=M. We may therefore write three useful relationships:

(i) $MU = (H'H)U = H'(HU) = H'W = e$

(ii) $HY = HU = He = W$

(iii) $W'W = Y'H'HY = Y'MY = Y'M'MY = e'e$

Theil produces a member of the LUS class which he refers to as BLUS, or Best LUS. This is the LUS which produces the residual vector having minimum expected squared length of the error vector. BLUS residuals are independent and yield statistics with distributions equally tractable as those produced by recursive residuals. What they apparently lack is a simple interpretation under parameter instability. Another way to contrast BLUS and recursive residuals is to view BLUS as a hybrid of LUS bred to minimize error, and recursive residuals as the hybrid bred to yield a simple intuitive and visual interpretation (via plots) under parameter instability.

Cusum and Cusum Squares Test

One of the first tests to use recursive residuals in testing for non-stable parameters was the cusum test developed by Brown, Durbin, and Evans (BDE). Beside providing a test in the statistical sense, this approach has the added appeal of providing a visual aid to detecting model instability.

BDE (1) construct scaled, running sums of recursive residuals called CUSUMS;

$W_r = \frac{1}{\hat{\sigma}} \sum_{K+1}^{r} w_j$, $r = K+1,\ldots,T$ which are examined under the null hypothesis H_o of stable coefficients, and where $\hat{\sigma}$ is an estimate of the common standard deviation of the w_j. BDE originally used $\left[\sum_j w_j^2/(T-K)\right]^{1/2}$ as their estimate of $\hat{\sigma}$, but in the comments to (1), Harvey pointed out that the alternative $\hat{\sigma}^* = \left[\sum_j (w_j-\bar{w})^2/(T-K-1)\right]^{1/2}$ would improve the procedure, as the cusum will tend to be larger in absolute value under the alternative hypothesis. An added advantage of this new $\hat{\sigma}^*$ is the fact that $W_T\hat{\sigma}^{*-1}/(T-K)$ has a t-distribution under H_o (see Harvey and Collier(4)). The $\{W_r\}$ form a sequence of asymptotically normal variables such that $E(W_r)=0, V(W_r)=r-K$ and $Cov(W_r,W_s)=\min(r,s)-K$, to a good approximation. By approximating W_r with a continuous Gaussian process, BDE develop "confidence bands" around plots of W_r for $r=K+1,\ldots T$. Anderson (1) points out that cusums will yield a slightly conservative test, due to the continuous approximation. We reject H_o if the plot of cusums "strays" outside the bands. The behavior of the plot may reveal information regarding coefficient movement independent of the testing procedure. BDE emphasize that the function of the "band lines" is to provide a yardstick against which to asses the observed behavior of the plot.

When coefficient instability takes the form of a switch from one stable regime to another, it is easy to see how the systematic nature of change will cause the cusums W_r to deviate away from zero (in one direction) until they cross a certain confidence band. On the other hand, a haphazard departure from constancy may not permit the cusums to "build" away from zero,but may simply cause fluctuations. To cope with this problem, BDE suggest plotting and developing a test on CUSUM SQUARES;

$$s_r = \sum_{j=k+1}^{r} w_j^2 \Big/ \sum_{j=k+1}^{T} w_j^2 = \frac{S_r}{S_T} \qquad r=K+1,\ldots T$$

Under H_o , s_r has a beta distribution with mean $(r-K)/(T-K) = \mu_r$. A plot of the means μ_r forms a line with slope $1/(T-K)$, and BDE develop a technique for placing parallel lines about this plot so that the probability of the sample path crossing one or both lines is α , a specified significance level.

Harvey's Sequential Chow Test

Harvey (5) develops a sequence of independent Chow tests to test for instability, using partial cusum squares. By using the recursive relationships (developed in BDE and elsewhere) ;

$$RSS_j = RSS_{j-1} + w_j^2$$
$$RSS_n = \sum_{j=k+1}^{n} w_j^2$$

, where RSS_i denotes the RSS obtained from running a regression on the first i observations only, and w_j is the recursive residual as before, Harvey notes that the standard Chow test can be written

$$* \qquad F = \frac{(RSS_2 - RSS_1)/m}{(RSS_1)/n-K} = \frac{\sum_{j=n+1}^{n+m} w_j^2/m}{\sum_{j=k+1}^{n} w_j^2/(n-K)}$$

, where RSS_1 applies to the first n observations and RSS_2 applies to n+m observations. Under the null hypothesis H_o of stability developed before, which implies the w_j are independent and share common variance, it is obvious that the right hand side of * is the ratio of two independent chi squares divided by their respective degrees of freedom. Working backward to the left hand side of * provides an "alternative" proof that the Chow statistic has an F distribution.

With * as a model, Harvey suggests breaking up a full set of T observations available into p mutually exclusive groups with m_i ($>K$) observations in each group. Defining $n_i = m_1 + m_2 + \cdots + m_i$ for $i = 1,2,\ldots,p$ and $n_0 = K$, recursive residuals are used to define

$$Z_i = \frac{1}{\sigma^2} \sum_{j=n_{i-1}+1}^{n_i} w_j^2 \qquad i=1,2,\ldots p$$

which are independent and distributed chi square. We can now form the p-1 statistics

$$F_i = \frac{Z_i/m_i}{\sum_{j=1}^{i-1} Z_j/(n_{i-1} - K)} = \frac{\sum_{j=n_{i-1}+1}^{n_i} w_j^2/m_i}{\sum_{j=k+1}^{n_{i-1}} w_j^2/(n_{i-1} - K)} \qquad i=2,3,\ldots p$$

which may be regarded as a sequence of Chow tests. The F_i's are mutually independent under H_o (a lemma due to Hogg and Tanis is required) , and so a variety of tests involving the probability that a particular number or combination of F_i rejects the null hypothesis when it is true are available. Harvey also indicates a method for computing the probability A_r that no run of r significant values occurs under the null hypothesis.

Varying Parameter Regression (VPR)

In this section we will discuss two tests based on linear time varying parameter models where the regression parameters follow a simple random walk.

Using the notation found in Sant(6) , suppose the scalar y_t is generated by the model $y_t = x_t\beta_t + \varepsilon_t$ for $t=1,2,\ldots,T$, and where x_t is a K-dimensional row vector of exogenous variables at time t , and the K-dimensional column vector β_t evolves according to the structure $\beta_t = \beta_{t-1} + u_t$ where ε_t and u_t are unobserved error terms with mean zero, and the following relationships hold:

$$E(\varepsilon_i \varepsilon_j) = \delta_{ij} \sigma_\varepsilon^2$$

$$E(u_i u_j') = \delta_{ij} \sigma_u^2 P$$

$$E(\varepsilon_i u_j) = 0$$

δ_{ij} = Kronecker delta, and P is assumed known. Letting $\hat{\beta}_t(t')$ be an estimate of β_t using observations 1 through t', and $\sigma^2 R_t(t')$ be the covariance matrix of $\hat{\beta}_t(t')$, the Kalman filter is a sequential algorithm for estimating $\hat{\beta}_t(t)$ given by

$$\hat{\beta}_t(t) = \hat{\beta}_t(t-1) + K_t(y_t - x_t \hat{\beta}_t(t-1))$$

where

$$K_t = R_t(t-1)x_t'(x_t R_t(t-1)x_t' + 1)^{-1}$$

$$R_t(t) = R_t(t-1) - K_t x_t R_t(t-1)$$
$$\hat{\beta}_t(t-1) = \hat{\beta}_{t-1}(t-1)$$
$$R_t(t-1) = R_{t-1}(t-1) + P$$

If P = 0 (i.e. stable coefficients) it can be shown that $\hat{\beta}_t(T)$ is equal to its ordinary least squares analogue, and the scalar stochastic innovations $e_t = y_t - x_t \hat{\beta}_t(t-1)$ "driving" the filter are exactly the recursive residuals of BDE (modulo a scaling factor). A recursive algorithim suggested in BDE to compute efficiently the $\hat{\beta}_t$'s is also identical to the Kalman algorithm when P=0.

Garbade (7) develops a test which appears useful for the case of one explanatory variable. If the parameter vector β_t has dimension n=1, the likelihood statistic

$$-2\text{LOG}\,\lambda = -2(L^*(P_o) - L^*(\hat{P})) \quad \text{where;}$$

$L^*(\cdot)$ is the concentrated log likelihood function

$\hat{P}$ is the maximum likelihood estimate of P

P_o is an hypothesized value of $P > 0$

is asymptotically $\chi^2(1)$ under a true null hypothesis. When P=0 however, $-2\log\lambda$ will be more concentrated than $\chi^2(1)$. In simulations run on (i) a random walk with zero drift model, (ii) a discrete jump(two stable regimes with a discrete shift), and (iii) a stable Markov process, Garbade found that the VPR test was much more powerful than cusum squares or cusums (with cusum squares out performing cusums) in rejecting a false null hypothesis of stable coefficients.

LaMotte and McWhorter (8) have recently developed an exact test for the presence of instability. Their test involves the development of a statistic which appears to be a generalized version of the one derived by Harvey above (actually the inverse of Harvey's). Beginning with the VPR model, they introduce a transformation on $y=(y_1,y_2,\ldots,y_T)'$ resulting in residuals sharing all the proprties of the LUS residuals, except for a possibly non-scalar covariance matrix. They form z=Hy, where

$z \sim N(0, \sigma_\varepsilon^2 I + \sigma_u^2 HVH')$, and where V is the variance-covariance matrix of y, and such that

$Hx=0$, $HH'=I_{T-p}$, $H'H=M$ and they then show that

$z'z = y'My = SSE = \sum_{i=1}^{k} Q_i$ where $Q_i = z'P_iP_i'z$

and $(\sigma_\varepsilon^2 + \lambda_i \sigma_u^2)^{-1} Q_i \sim \chi^2_{r_i}$

The $\lambda_1,\ldots,\lambda_K$ are the distinct eigenvalues for H'VH with multiplicities $r_1,\ldots,r_K$ respectively, and the P_i are (T-p) by r_i matrices whose columns are orthonormal eigenvectors of H'VH. LaMotte and McWhorter (LM) prove that the Q_i are mutually independent.

We may now define

$$S_g = \sum_{i=1}^{g} Q_i \quad , \qquad n_g = \sum_{i=1}^{g} r_i$$

Under $H_o: \sigma_u^2 = 0$, it follows that

$$** \quad F_g = (S_g/n_g)/((SSE-S_g)/(T-p-n_g))$$

is distributed $F_{n_g, T-p-n_g}$. We may rewrite ** to aid in comparing it to Harvey's work;

$$F_g = \left(\sum_{i=1}^{g} Q_i / \sum_{i=g+1}^{k} Q_i\right)((T-p-n_g)/n_g)$$

which is a generalization of the inverse of Harvey's F_i statistics. Now, whereas Harvey proceeds to look at tests based on all the F_i, LM proceed to produce guidelines for choosing that partition g which yields a test F_g with the largest power.

Quandt Procedure

Quandt (9) introduced a maximum likelihood procedure for finding an abrupt shift from one stable parameter regime to another. Under the assumption of normal errors in a regression model, the likelihood under dual regimes takes on a simple form, and the likelihood ratio $\lambda_{t^*} = L(\hat{w})/L(\hat{\Omega})$, where $L(\hat{w})$ is the likelihood under the assumption of one regime, and $L(\hat{\Omega})$ is the maximum likelihood achieved by partitioning the data time-wise as two regimes over the entire estimable range, reduces nicely to

$$\lambda_{t^*} = \hat{\sigma}_1^{t^*} \cdot \hat{\sigma}_2^{T-t^*} / \hat{\sigma}^T$$

Here T is the number of time units, t^* the time at which an optimal partition occurs, $\hat{\sigma}_i$ the estimated standard error of regime i, and $\hat{\sigma}$ the estimated standard error of the regression over all the data. While this procedure is feasible for spotting the location of a possible shift (it can easily be implemented on a computer for even large data sets), it does not offer a means for statistically corroborating the existence of a shift. It was originally thought that functions of λ (particularly $-2\log\lambda$) would provide known distributions (possibly asymptotic). Unfortunately the discrete time frame we are working in lacks properties necessary to derive feasible statistics. The Quandt procedure survives primarily as a tool to aid in spotting shift location, although plots of λ_t may be used to "shed light" on the stability of the regression, and to indicate whether changes have occurred as an abrupt transition or gradually.

Market Models

Since 1964 one of the foremost models produced to explain asset returns has been the Capital Asset Pricing Model (CAPM). The most commonly referred to version is attributed primarily to Sharpe (10), and slightly later to Lintner (11), and Mossin (12).

As stated in an article by Bicksler (13), the basic assumptions underlying the model are:

1. All investors are risk averse and choose portfolios in a manner consistent with maximizing expected utility of single-period terminal wealth
2. Portfolio investment opportunities can be described solely in terms of means and variances (or standard deviations) of the ex-ante distribution of one-period portfolio returns.
3. Investors have homogeneous expectations regarding means, variances, and covariances of returns for all securities in the investment opportunity set and, in addition, all investors have identical investment opportunity sets.
4. Capital markets are efficient in the sense that borrowing and lending rates are equal. There are no restrictions to short sales, no taxes, no transactions costs, and capital assets are perfectly divisible, et cetera.
5. The supply of all capital assets is given.

The fundamental result is:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

where $E(R_m)$ is the expected return on the market portfolio (of all risky assets)

$E(R_i)$ is the expected return on an indivdual security for the single period being considered

R_f is the riskless rate of return (for both borrowing and lending)

$\beta_i = \text{Cov}(R_i, R_m)/\text{var}(R_m)$ is the systematic risk of the ith security

Breen and Lerner (14) first suggested that Beta values are not stable over long periods (i.e. a firm's risk characteristic changes over time, probably due to strategic and operational management decisions). Pettway carried out an empirical examination (15) in which he concluded that in the period between 1971 and 1976, some instability (for at least a year) did exist in the electric utility industry.

Other major criticisms of the model are that the borrowing and lending assumptions are too restrictive (and do not reflect reality) , and that it is necessary to use a proxy (such as the New York Stock Exchange index) for the market return R_m. Ross (16) has derived a market model which results in a "generalized" CAPM in that CAPM is a specific case of it. With little of the "excess baggage" in the way of CAPM assumptions, Ross reproduces the basic form of CAPM in a model which also allows us to use a market index in a context where it is truly a determinant of a firm's ex-ante expected return. We present the basic idea behind this model, known as the Arbitrage Pricing Theory (APT), and reinterpret the modes' analogue to R_f. We do this primarily as a means of justifying the estimation procedure we use, which involves using a composite index of market returns in place of R_m.

As presented in the Ross paper, APT is based on the following arguments: Suppose asset returns are generated by some stochastic relation

$$x_i = E_i + \beta_i \delta + \varepsilon_i \quad ; \; i=1,2,\ldots,n$$

where E_i is a constant term representing the ex-ante expected return

δ is a mean zero common factor (we do not need to specify it, but if CAPM were indeed true it would represent the deviations of the market return from its trend)

$\varepsilon = (\varepsilon_1,\ldots,\varepsilon_n)$ is a mean zero noise vector

We now form an arbitrage portfolio η of the n assets. Here η is a 1 by n vector of asset proportions with the property that $\eta' e = 0$, where e is the n by 1 vector $(1,1,\ldots,1)'$. In this portfolio, the wealth invested in long assets is exactly balanced by the amount borrowed from short sales and, net, the portfolio uses no wealth. The portfolio is also chosen in a well diversified manner to permit us to use the law of large numbers to approximately eliminate the noise term $\eta'\varepsilon$, and in such a fashion that it eliminates sysyematic risk as well ($\eta'\beta = 0$).

Thus the portfolio return R is:

$$R = \eta' x = (\eta' E) + (\eta' \beta)\delta + \eta' \varepsilon$$
$$\cong (\eta' E) + (\eta' \beta)\delta$$
$$= \eta' E \quad \text{where } E \text{ is the vector of ex-ante mean returns}$$

The model is named Arbitrage because in place of the primary assumptions of CAPM, it simply requires that arbitrage opportunities not exist, i.e. the ability to make money from a zero wealth portfolio be disallowed. Since η is a zero wealth portfolio we must have

$$R = \eta' E = 0, \text{ or}$$

$\eta' \beta = \eta' e = 0 \Rightarrow \eta' E = 0$, and this in turn implies that E must be a linear combination of e and β. So we can write

$$E_i = E_o + a\beta_i$$

By forming the market portfolio α_m on E we get

$$\alpha_m E = E_m = E_o + a \quad (\text{having normalized } \alpha_m \beta = 1)$$

so $a = E_m - E_o$ and $E_i = E_o + (E_m - E_o)\beta_i$,

which is the arbitrage equivalent of CAPM.

Assuming this could all be achieved among firms listed in the market return index we use below, then E_m can be interpreted as that index. Also E_o, which is analogous to R_f in CAPM, can be interpreted as the return which any zero beta portfolio α_p (i.e. $\alpha_p \beta = 0$) will yield. If an R_f does exist, then Ross shows that $R_f = E_o$.

Empirical Results

We examine below the stability of the parameters of the model $R_{ce} = a + bR_m$

where R_{ce} = Daily returns (adjusted for dividends) of Consolidated Edison of New York

R_m = Daily market returns (based on volume weighted index)

Data was obtained from CRSP tapes (Center for Research on Security Prices-University of Chicago), for the interval between January 9, 1974 and October 24, 1974. During this interval ,treasury bill rates (and rates of other relatively riskless notes) were in a period of sharp ascent. We do not account for this in our model, but treat this as something to "look for" in our stability tests. We expect these rates to affect the stability of the intercept. We emphasize that the spirit of this investigation was primarily in the interest of observing the behavior of the test procedures, and not with an eye toward estimation. Also during this period, Consolidated Edison announced for the first time in ninety four years it would not pay out a quaterly dividend (April 18) . We felt that this may have signalled information to the market concerning the riskiness of the firm, and that this may affect the level of b, which is a measure of firm risk (under CAPM or APT)

Three of the tests described in Part I were readily available through the Troll software package. These were CUSUM, CUSUMSQUARE, and the QUANDT LIKELIHOOD RATIOS (plots of $\log_{10} \lambda_t$, which yield time of optimal partition of the data at the minimum plot value). We present below a brief description and analysis of each of four sequences of plots of these tests, in the order mentioned above. The four sequences correspond to

(i) Tests run on the full set of data

(ii) Tests run on full data set minus thirty observations beginning at and several weeks after the April 18 dividend announcement (to permit a stabilization period) - Cases 1-70 and 100-202

(iii) Tests run on cases 1-70

(iv) Tests run on cases 100-202

Labeled plots will be found in the stated order at the end of the paper. The significance bands in the cusum and cusumsq plots correspond to tests at the 5% level of significance.

Before we proceed to describe the results, let us recall the model we are using. Under CAPM interpretation we have

$E(R_{ce}) = R_f + \beta E(R_m - R_f)$ which we estimate with O.L.S. from $R_{ce} = a + bR_m$ where $a = R_f(1-b)$. Thus, in a period of rising β, "a" may not exhibit extreme behavior because of offsetting effects (we suspect β of rising within the range of from a level of 0.3 to as much as 0.9). Therefore instability of treasury bill rates and other notes related to R_f will exhibit themselves best in periods when β is relatively constant. These effects should be about the same under APT interpretation of "a". We also note that there was no serial correlation present in any of the time periods we examined (using DW at 1%)

SEQUENCE-I

Cusums deviate from zero but return by the end. Inconclusive test at 5%, but instability suspected.

Cusumsq yields significant test for instability, and the plot travels past both confidence bands. It is unclear to us what the significance of this might be.

The Quandt plot reveals a monotonically decreasing sequence of values until t=56 or so. Varible values between -55 and -68 appear until t=75 (Con Ed made their announcement at t=70). After t=75 we note a sharp discontinuous change denoted by # signs which indicates an abrupt shift in the likelihood function when the data is partitioned after t=75 (a plot of actual returns revealed a large outlier at t=73).

We conclude that parameter shifting of some nature has occurred, and that "b" is responsible for at least some of the evidence, due to its suspected link with the April 18 dividend decision.

SEQUENCE-II

Because the data is daily, any shifting process due to an abrupt change in the environment (the dividend decision) will probably require some time to work itself out. Present theories about the market point toward very efficient (time-wise) mechanisms at work. We felt that by eliminating time cases 71-99 (corresponding to April 19 to May 30) we should greatly decrease any problems that might arise in this area. We will see in the plots that t=58 to t=99 may have been a better choice. By not accounting for the possibility of erratic behavior before April 18, we ignore the possibility of an information leak to selective portions of the market before the actual public announcement.

The cusum plot follows the same pattern as before but does not deviate as far.

Cusumsq indicates significant departure from stability as before, however the elimination of the thirty data points has caused the plot to stray outside of just one band-not both.

The Quandt plot shows a well defined minimum with no "local competition" at about t=56. It is again quite apparent that a shift has occurred. What is at question still is parameter characteristics in each regime. What we suspect,of course, is a rising intercept and a relatively constant slope of different value in each regime.

SEQUENCE-III (cases 1-70)

As mentioned above, we felt that this period should be stable relative to the slope, but not the intercept. However, we did not consider the possible effects on the slope before April 18 (case 70), and this seems to have caused the following plot behavior;

Cusums decrease gradually but not past a critical point. We were unable to infer anything from this behavior.

Cusumsq indicates the possibility of a switch and rejects H_o.

The Quandt plot shows an extreme value (boxed in) at the bottom left hand corner indicating a best partition would be to assume one regime. However, with a bias toward forming two regimes brought about by the cusumsq test, we see that another good choice is at t=57 or so. This reinforces our belief that the mechanism at work shifting the slope requires the interval t=58 to t=99 to "iron itself out".

SEQUENCE-IV (cases 100-202)

Cusums just manage to pass the critical band on the positive side.

While cusumsq begin to deviate from zero, they easily stay within the critical bands (for once!) and indicate no shift.

The Quandt plot indicates a cluster of low values at and near the beginning (bottom left)-indicating no shift-with an erratic but uptrending pattern afterwards.

Our conclusions for this portion of the data are that it is a relatively stable period for the slope, but not the intercept. This is felt because cusumsq and Quandt indicate constancy, but cusum rejects it with positive deviation-which could be accounted for by a rising intercept (and constant slope) , as mentioned above.

Further analysis would now take the form of repartitioning the data as suggested above, and examining the model with tests aimed at detecting shifts in just the slope or just the intercept (see Farley and Hinich (17) for an example).

REFERENCES

1. Brown,R.L.,Durbin,J. and Evans,JM, "Techniques for Testing the Constancy Of Regrssion Relationships Over Time", with Discussion. Journal of the Royal Statistical Society, Series B, 37, 1975, 149-192

2. Kendall,M.G. and Stuart,A.S., The Advanced Theory of Statistics Vol.1, p.250 (London:Hafner, 1963)

3. Theil,H., Principles of Econometrics, Chapter 5, (Wiley, 1971)

4. Harvey,A.C. and Collier,P.C. (1975) "Testing for Functional Misspecification in Regression Analysis", Paper presented at 3rd World Congress of the Econometric Society, Toronto, August 1975

5. Harvey,A.C., "An Alternative Proof and Generalization of a Test for Structural Change" The American Statistician Vol.30,No.3,August 1976 p122-123

6. Sant,D.T., "Generalized Least Squares Applied to Time Varying Parameter Models" Annals of Economic and Social Measurement, 6/3,1977 pp301-302

7. Garbade,K., "Two Methods for Examining the Stability of Regression Coefficients" Journal of the American Statistical Association, March 1977,Vol.72,No.357 pp54-63

8. LaMotte,L.R. and McWhorter,A., "An Exact Test for the Presence of Random Walk Coefficients in a linear Regression Model", Journal of the American Statistical Association, Dec.1978,Vol.73, No.364 pp816-820

9. Quandt,R.E., "The Estimation of the Parameters of a Linear Regression SYstem Obeying Two Seperate Regimes", Journal of the American Statistical Association Vol.53, 1958 pp873-880

10. Sharpe,W., "A Simplified Model For Portfolio Analysis", Management Science, Vol.9, January 1963

11. Lintner,J., "Security Prices, Risk, and Maximal Gains from Diversification" Journal of Finance, Vol. 20, December 1965

12. Mossin,J., "Equilibrium in a Capital Asset Market", Econometrica, Vol.34, Oct.1966

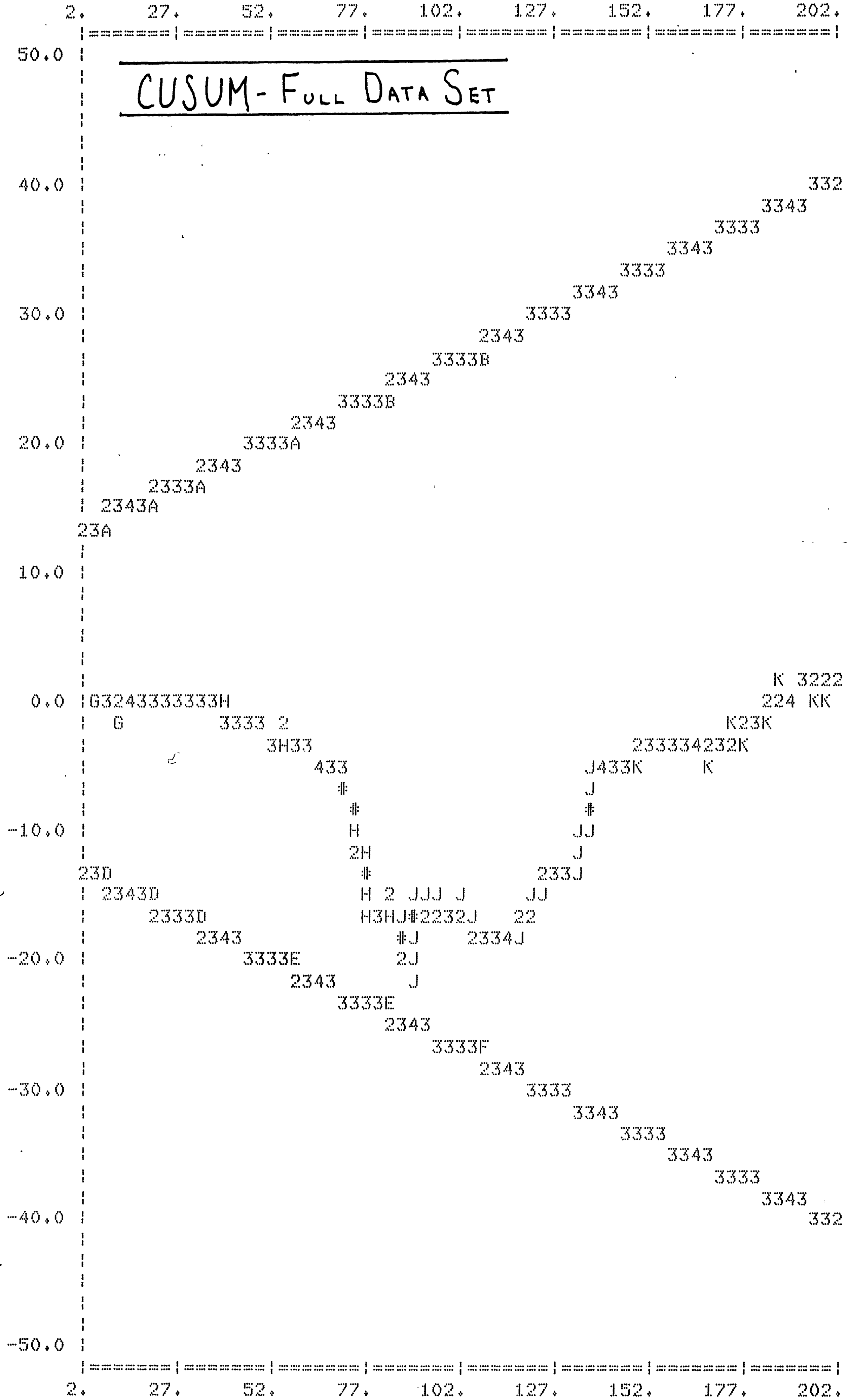
13. Bicksler,J.L., "The Usefullness of Beta Risk for Estimating the Cost of Capital" in: I.Friend and J.Bicksler, eds. Risk and Return in Finance(Ballinger,Cambridge)197

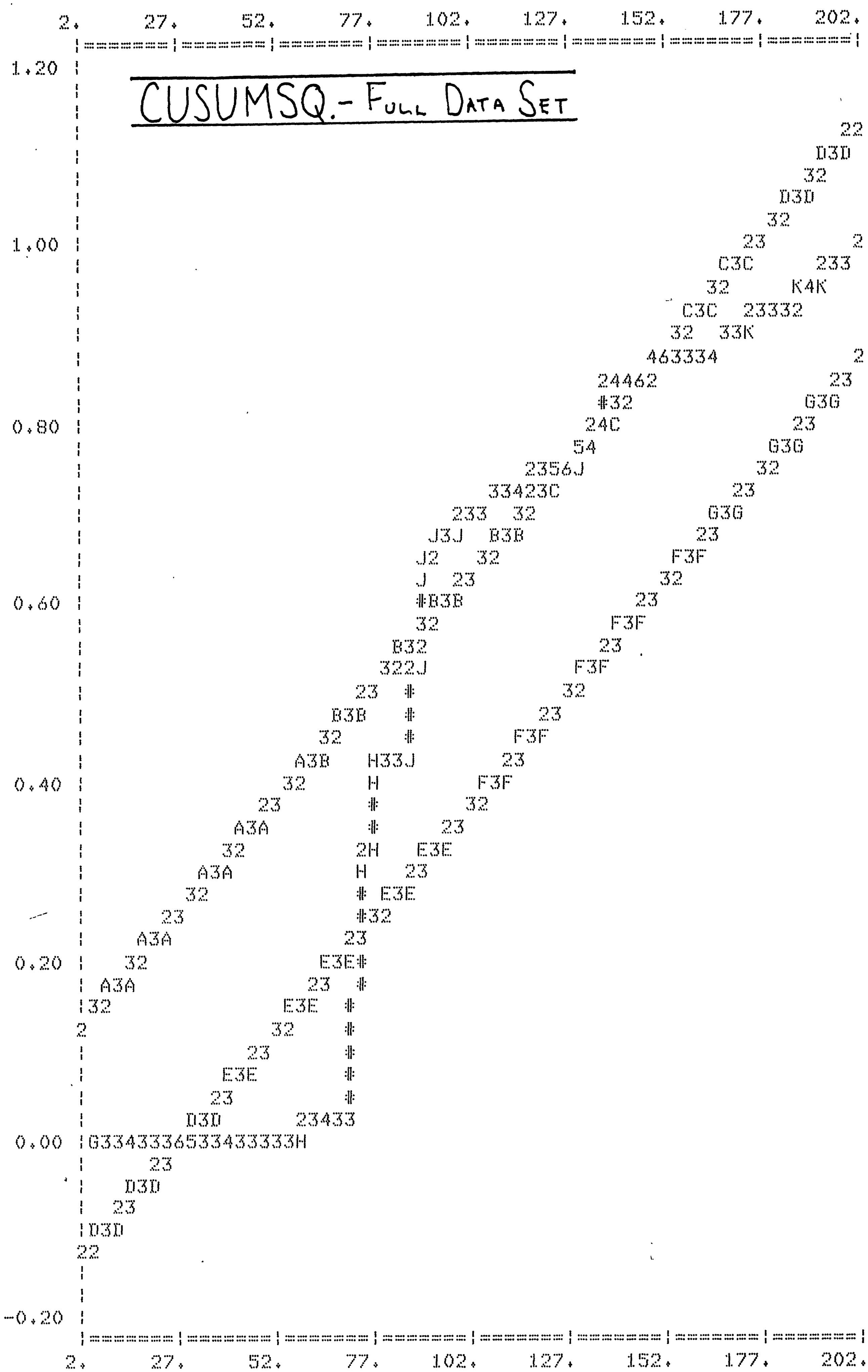
14. Breen,W.J, and Lerner,E.M., "On the Use of Beta in Regulatory Proceedings", The Bell Journal of Economics and Management Science, Vol.3 No.2 (Autumn 1972)

15. Pettway,R.H., "On the Use of Beta in Regulatory Proceedings, an Empirical Examination", The Bell Journal of Economics (Spring 1977)

16. Ross,S.A., "Return, Risk, and Arbitrage", in:I.Friend and J.Bicksler eds., Risk and Return in Finance, (Ballinger,Cambridge),1977

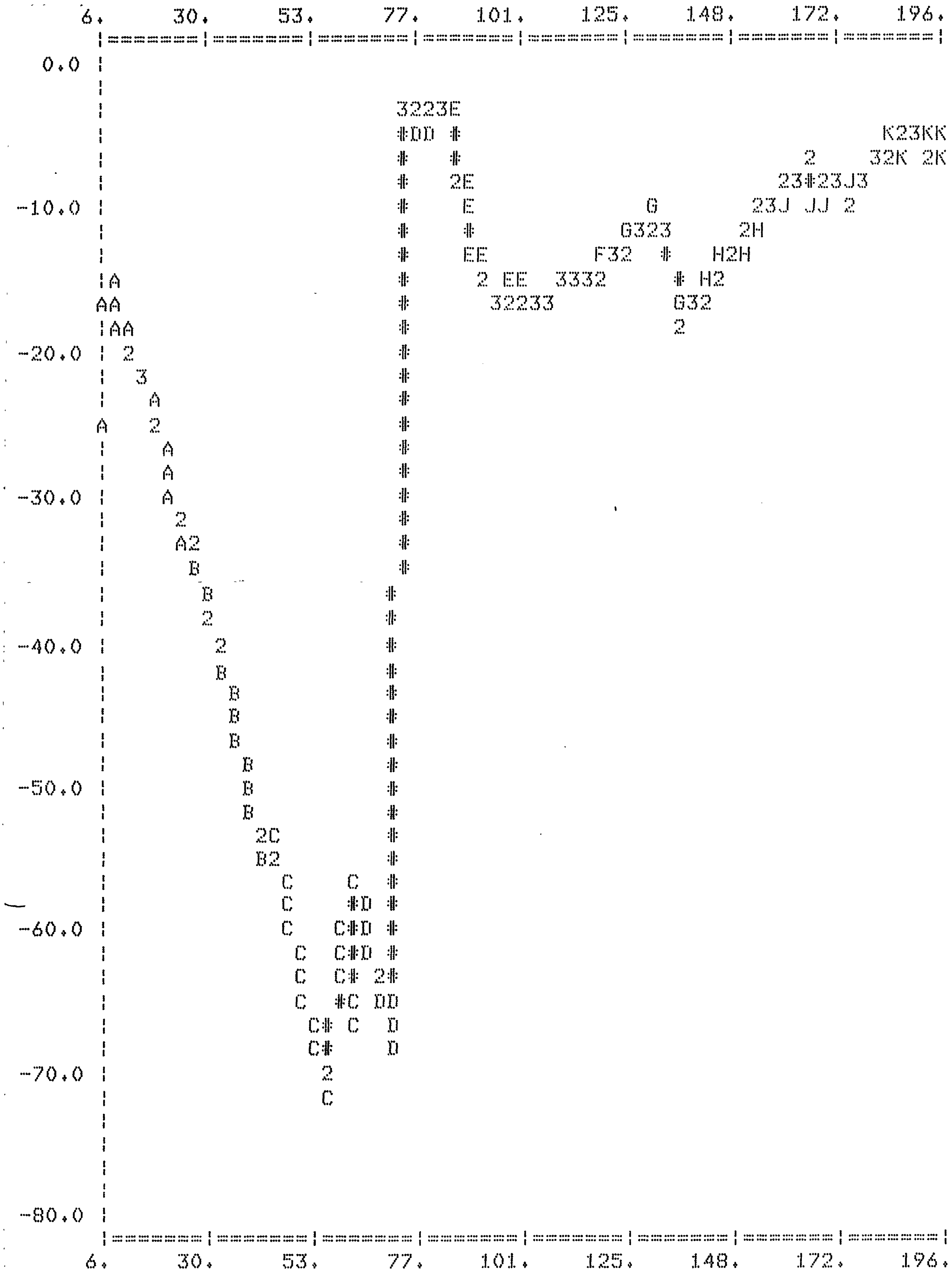
17. Farley,J.U. and Hinich,M.J., "A Test for a Shifting Slope Coefficient in a Linear Model", Journal of the American Statistical Association,65,1970,1320-1329


CUSUM- Full Data Set
2. 27. 52. 77. 102. 127. 152. 177. 202.
50.0
40.0
30.0
20.0
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0


CUSUMSQ.- Full Data Set
2. 27. 52. 77. 102. 127. 152. 177. 202.
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20

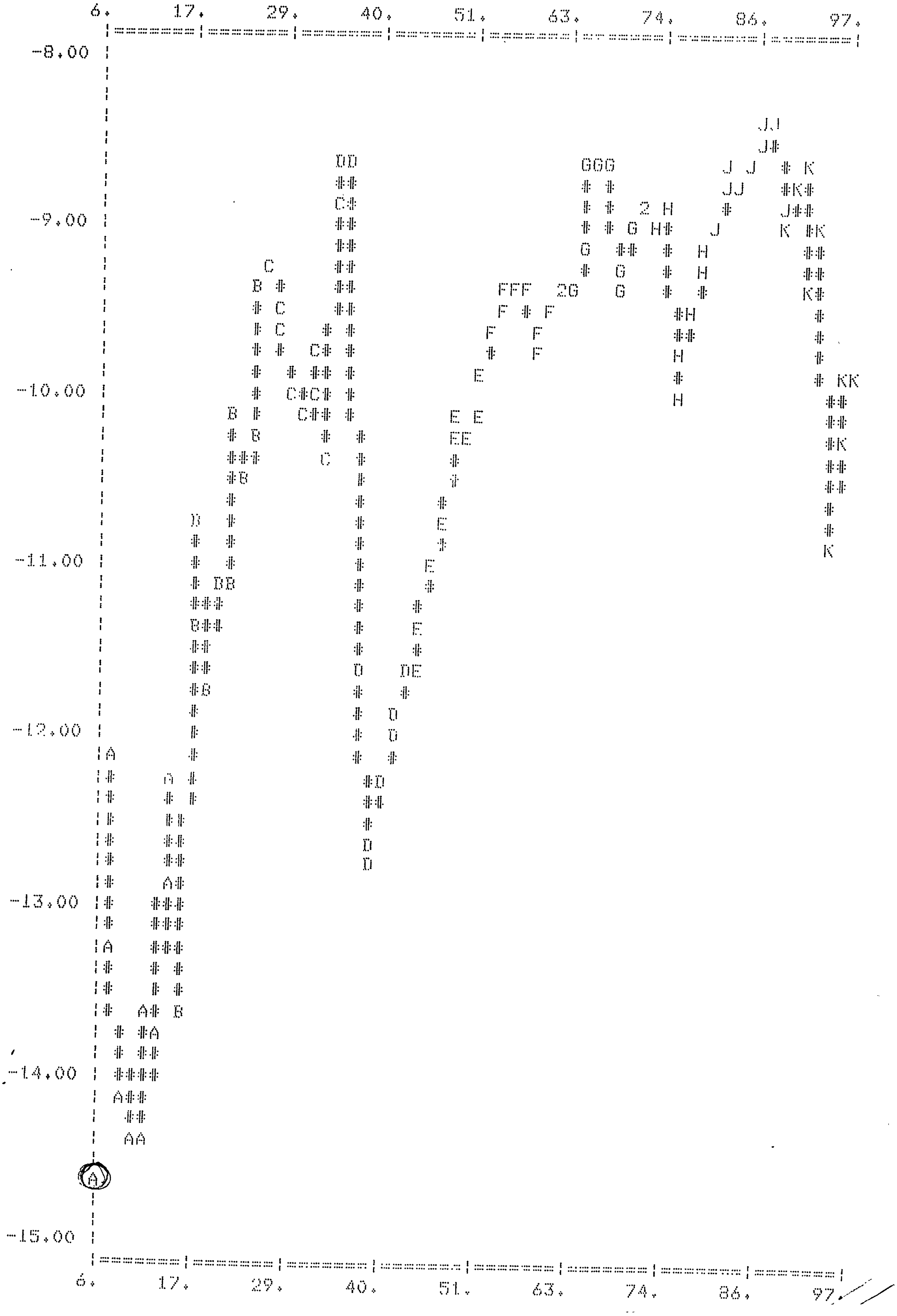
QUANDT- Full Data Set

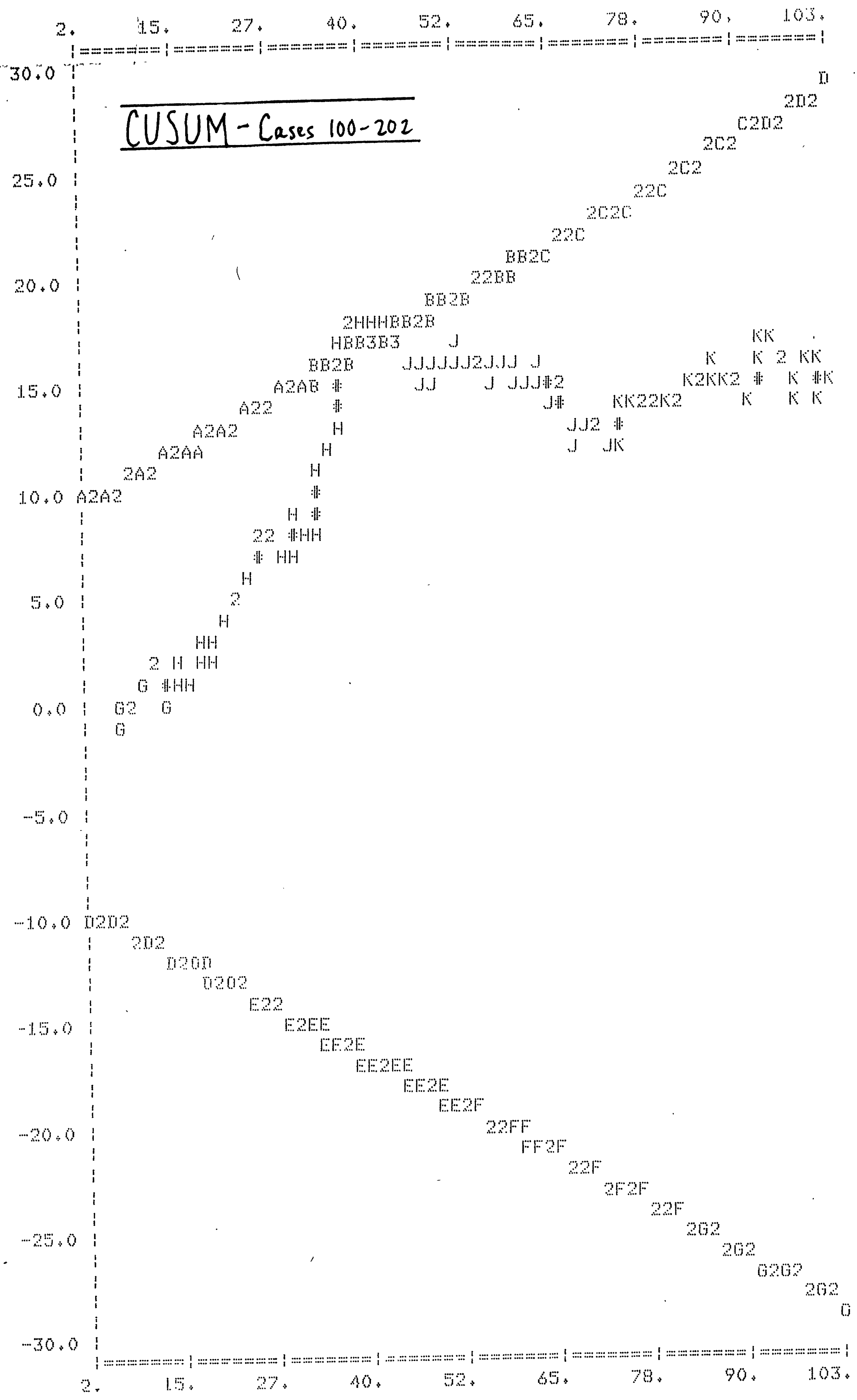
QUANDT LIKELIHOOD RATIOS



QUANDT LIKELIHOOD RATIOS

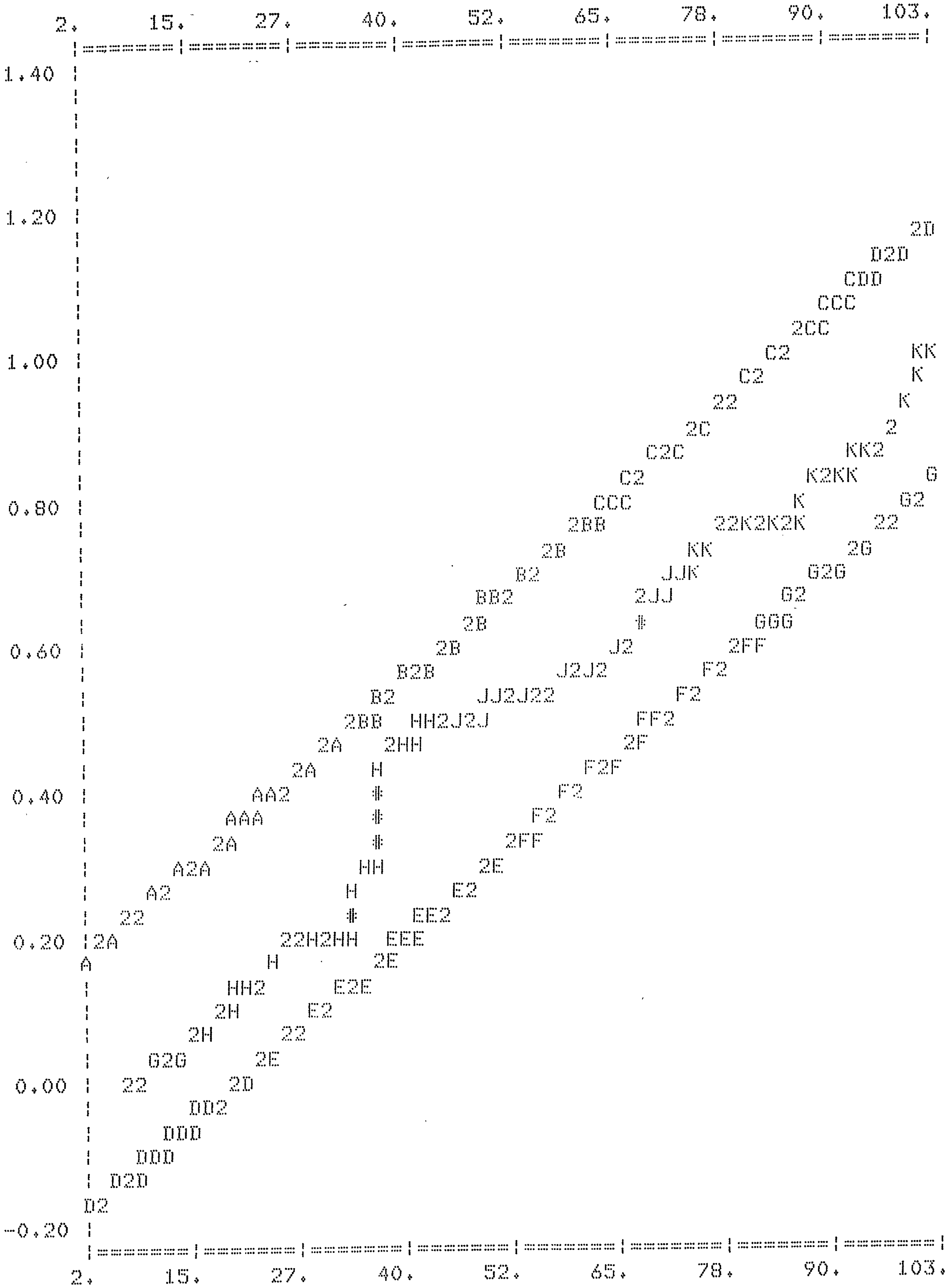
QUANDT- Cases 100-202



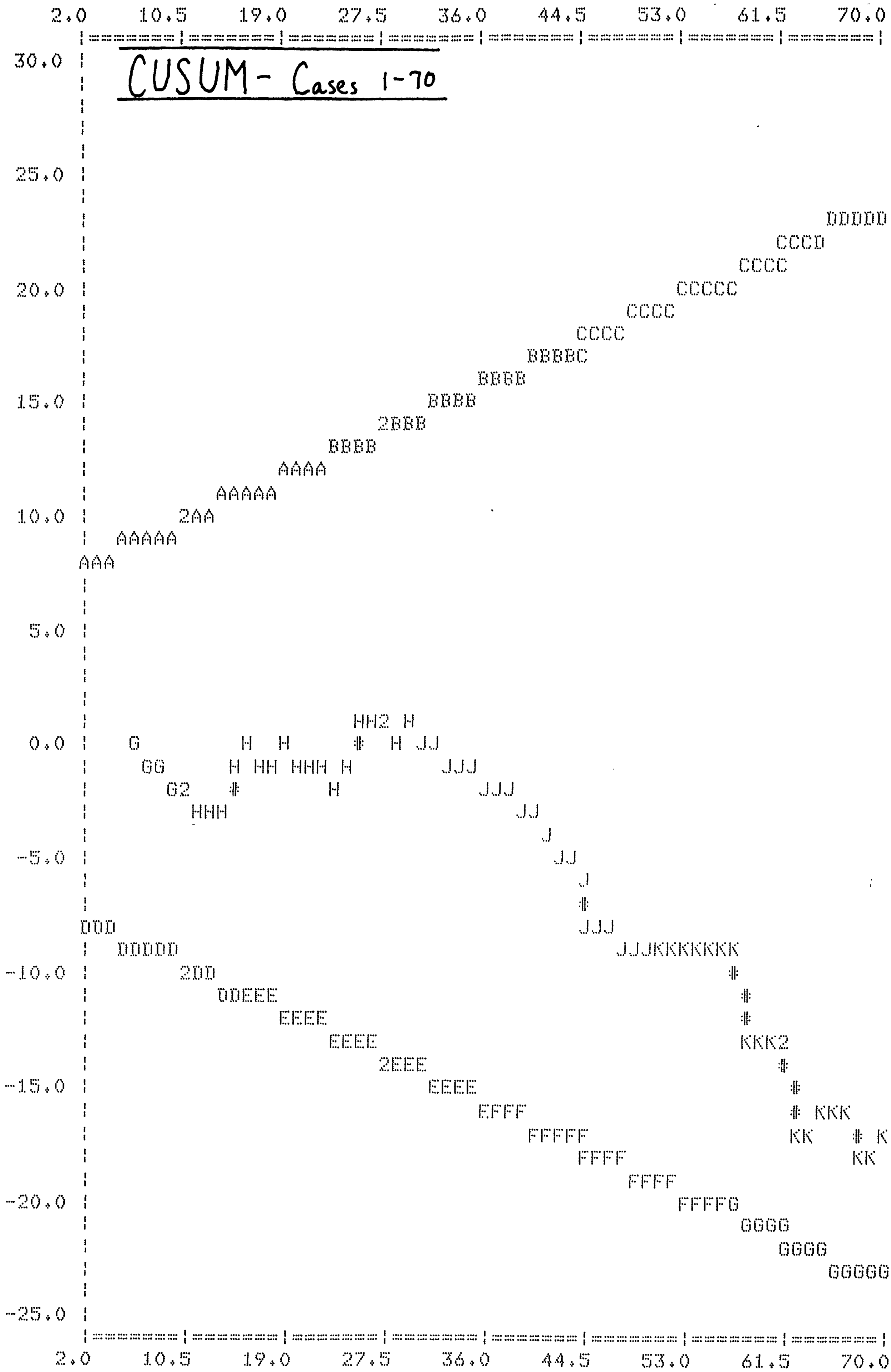

CUSUM - Cases 100-202
2. 15. 27. 40. 52. 65. 78. 90. 103.
30.0
25.0
20.0
15.0
10.0
5.0
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0

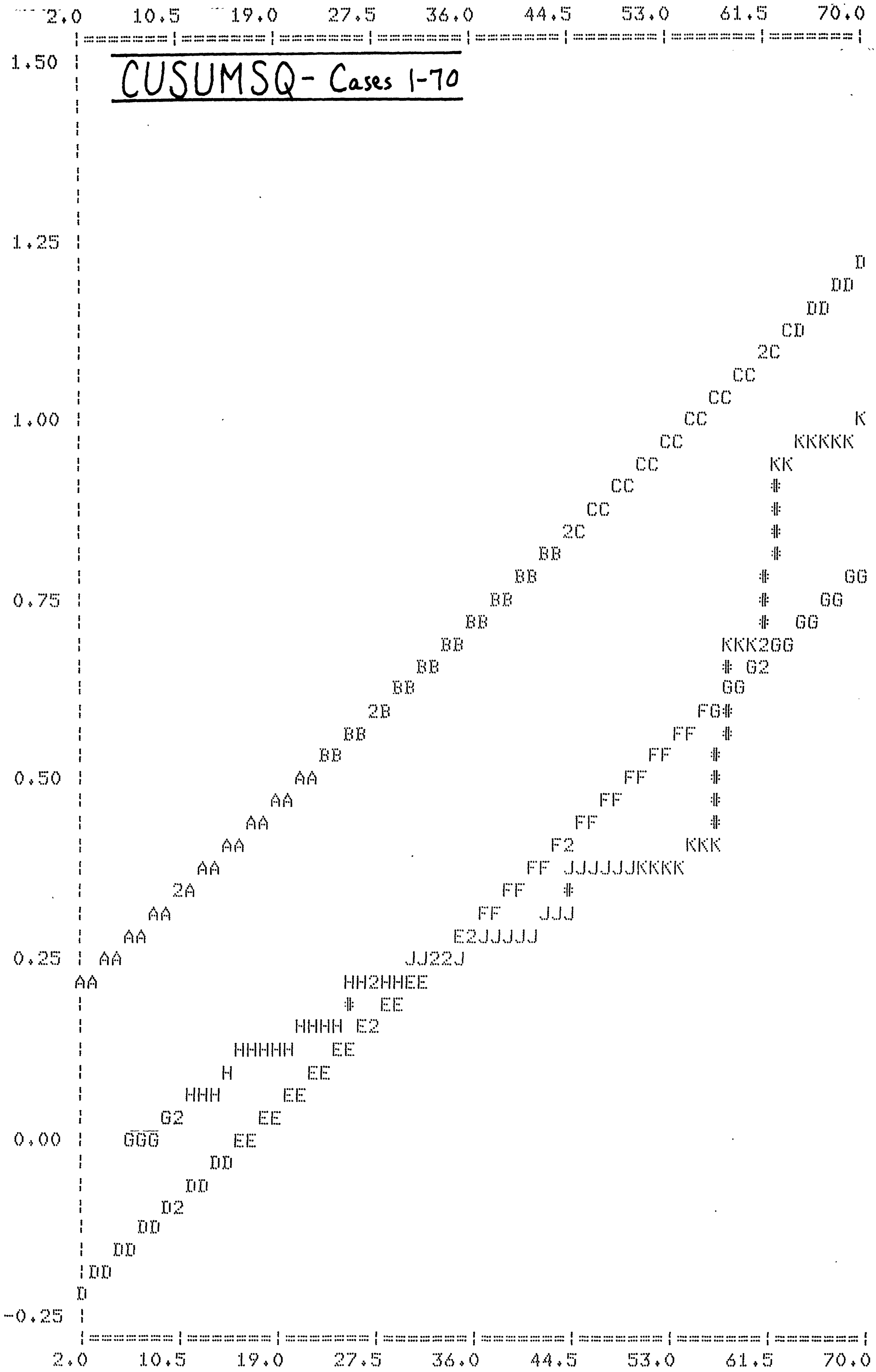
FORWARD CUSUMSQ DISPLAY AT CRITICAL VALUE .17715 (SIGNIFI

CUSUMSQ.- Cases 100-202



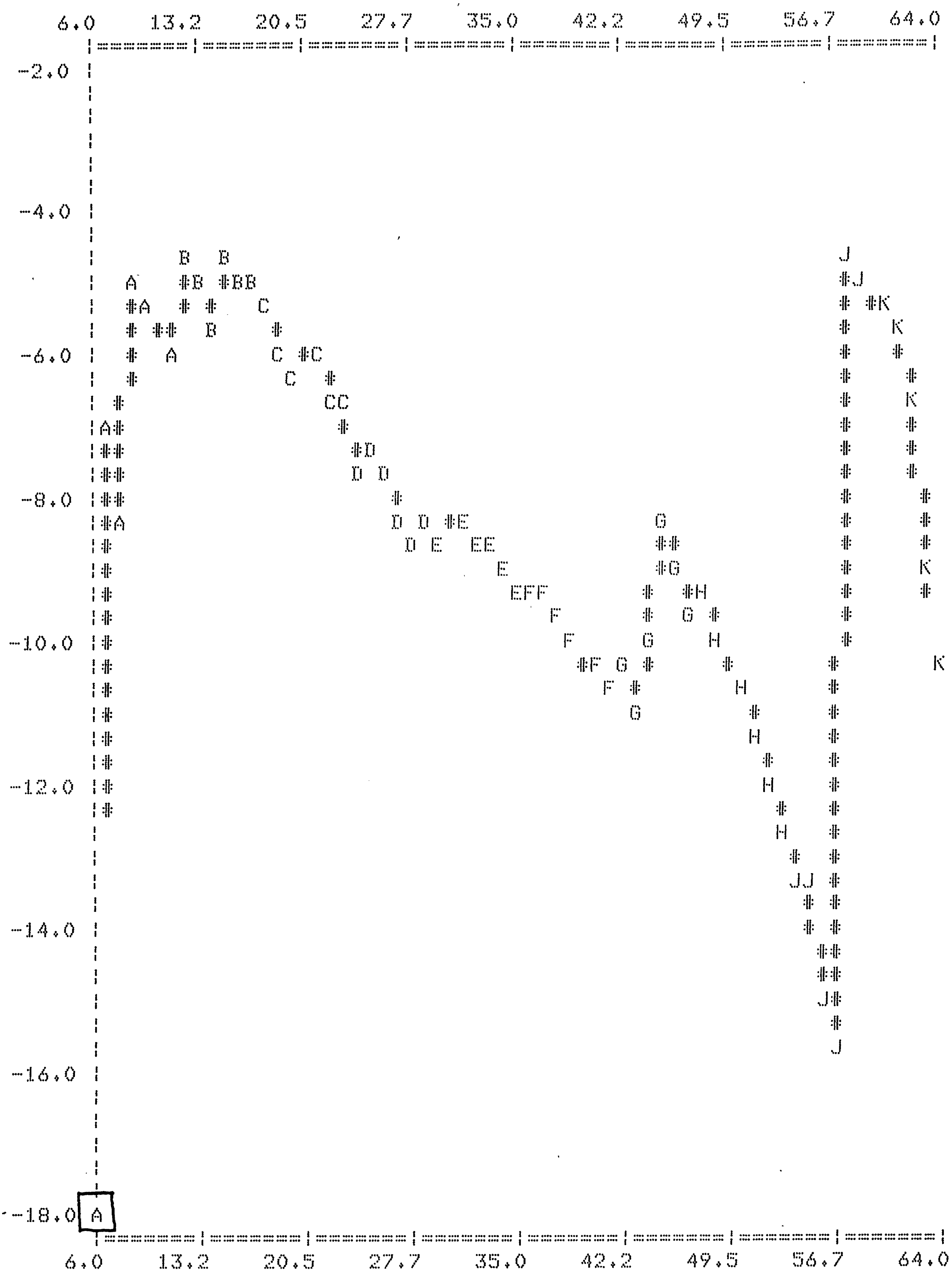
CUSUM- Cases 1-70



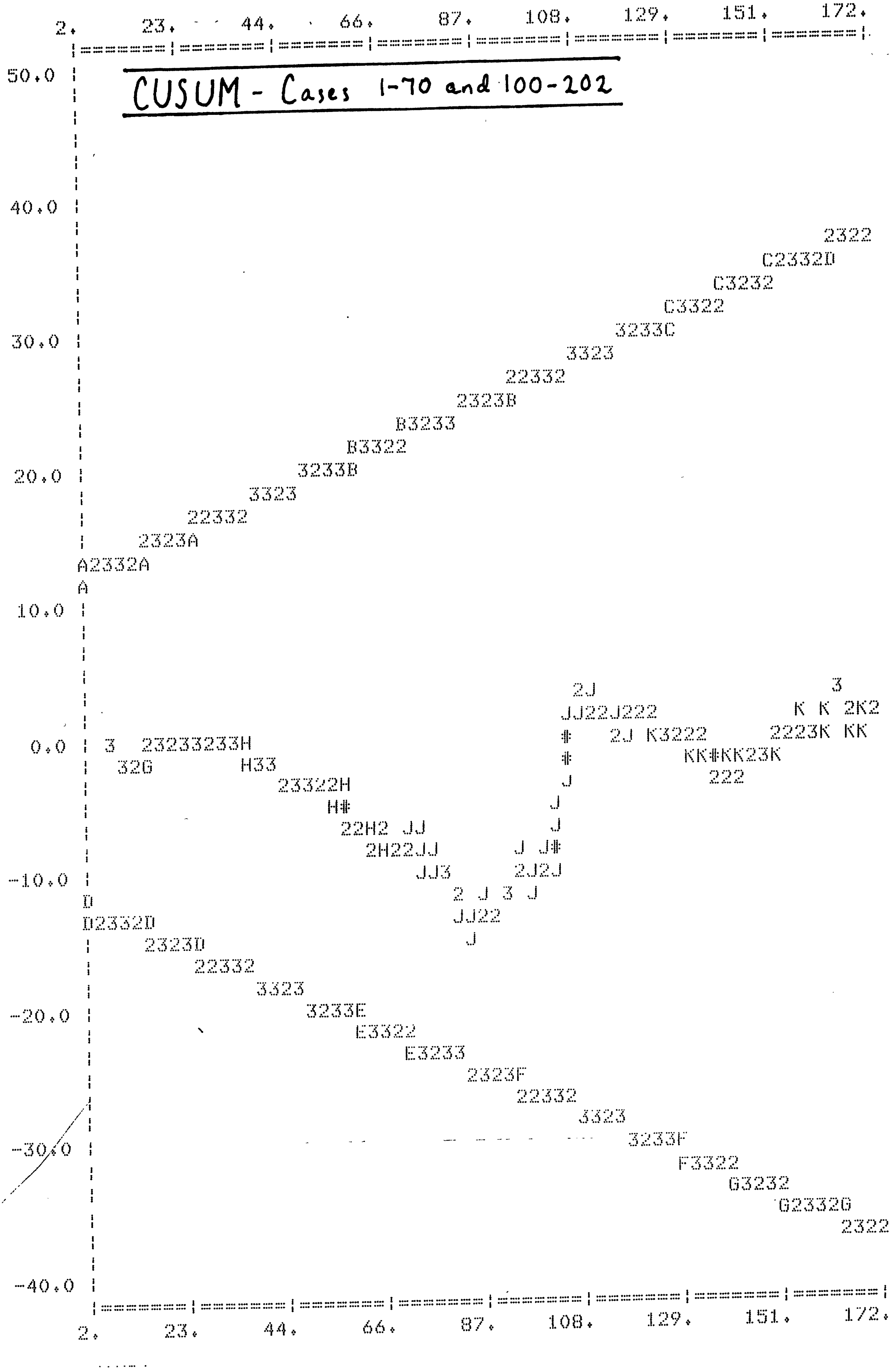

CUSUMSQ- Cases 1-70
2.0
10.5
19.0
27.5
36.0
44.5
53.0
61.5
70.0
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25

QUANDT - Cases 1-70

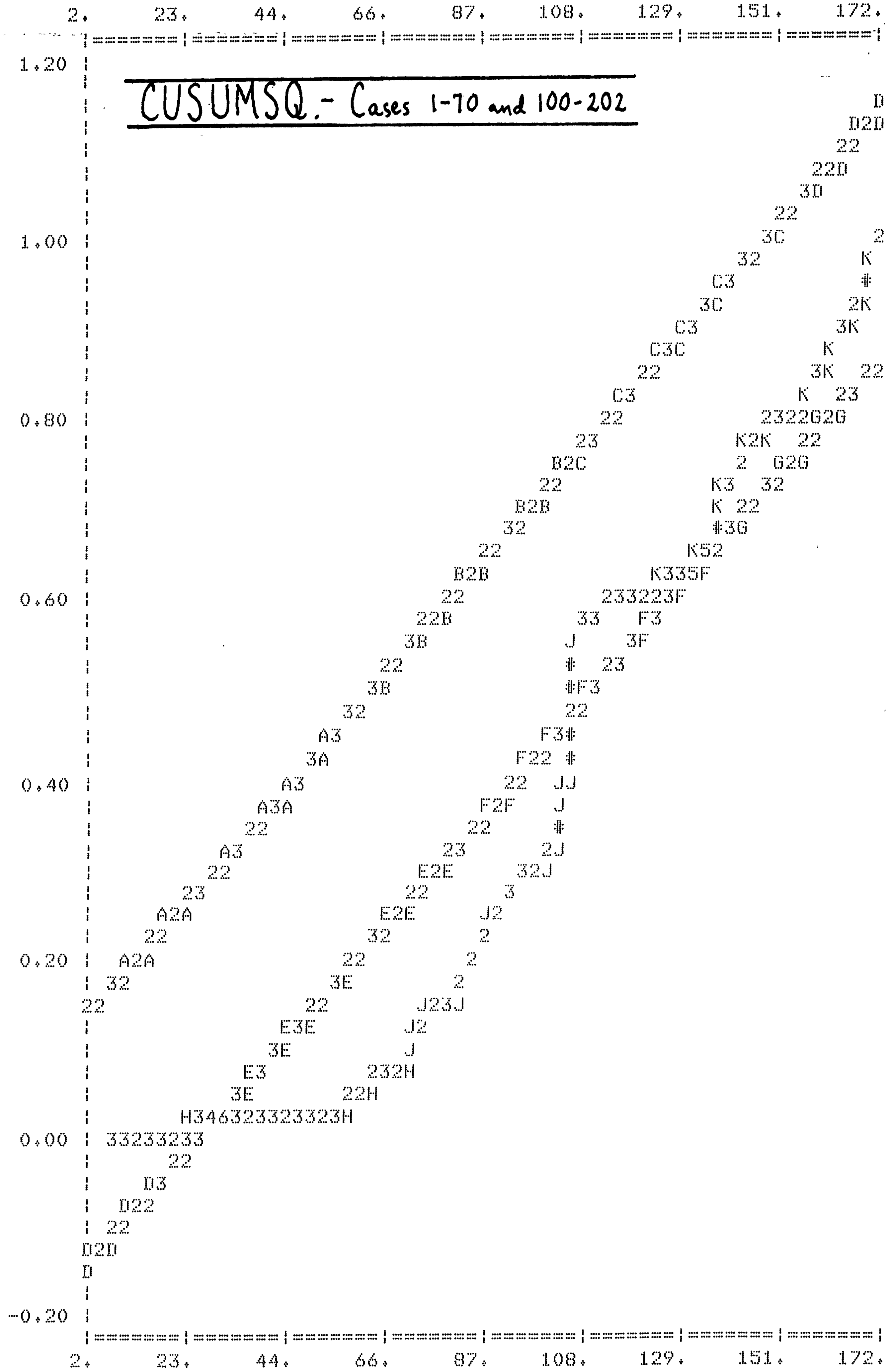
QUANDT LIKELIHOOD RATIOS

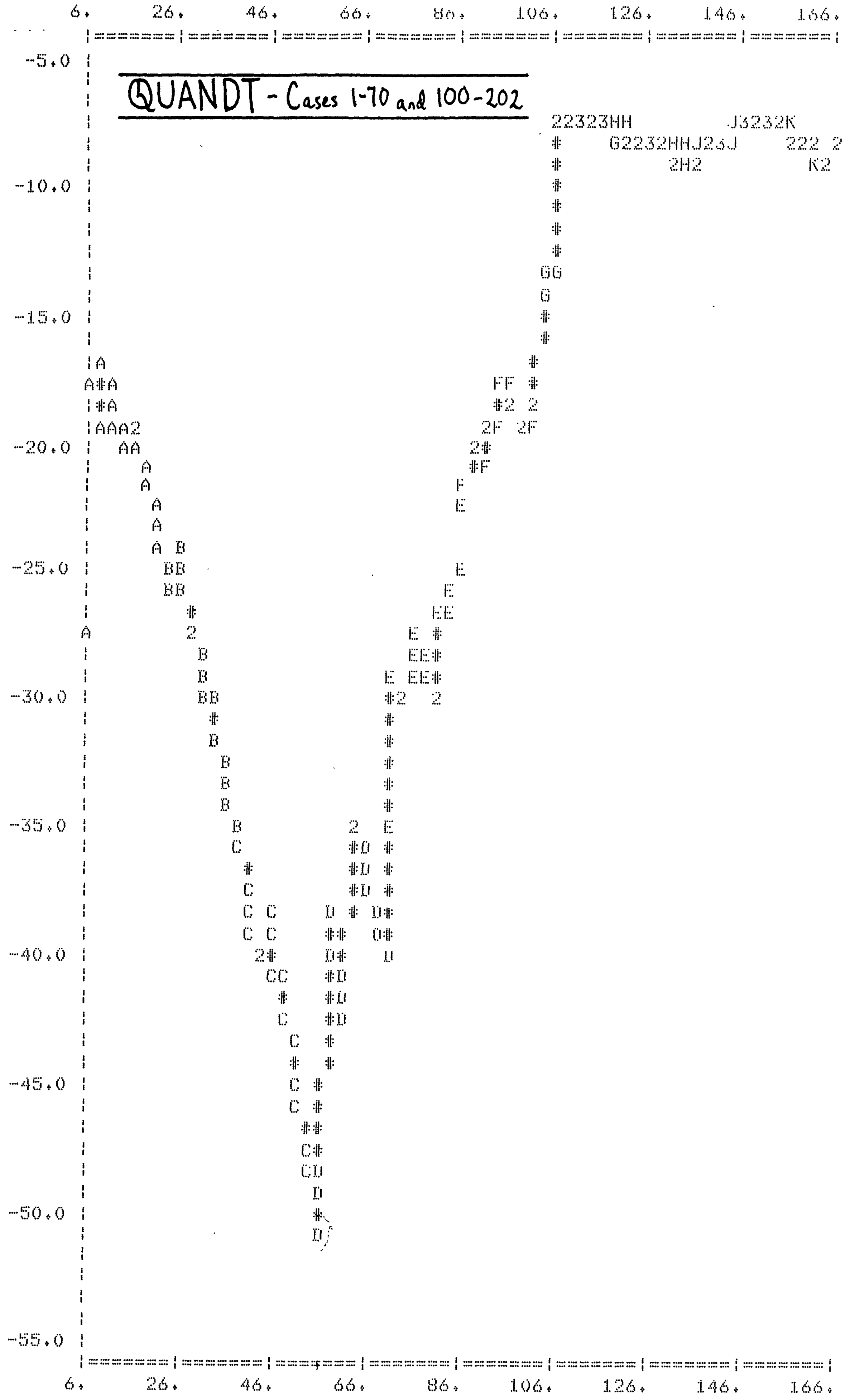


CUSUM - Cases 1-70 and 100-202



CUSUMSQ.- Cases 1-70 and 100-202




QUANDT - Cases 1-70 and 100-202
6. 26. 46. 66. 86. 106. 126. 146. 166.
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-40.0
-45.0
-50.0
-55.0