Division of Research
Graduate School of Business Administration
The University of Michigan

March 1982

INGRAINING HABITS OF TIME-ECONOMIC CONSUMERS

Working Paper No. 296

Birger Wernerfelt

The University of Michigan

INGRAINING HABITS OF TIME-ECONOMIC CONSUMERS

By

Birger Wernerfelt*

The paper develops a new theory of consumption, in which decisions are made in terms of habits rather than as selected purchases. The consumer searches for and receives offers of alternative brands as point processes with controlled, but costly, intensity. Since the user skills that are assumed to accumulate with experience are not fully transferable, the consumer will demand an increasing price discount to shift brands.

*The paper benefitted from technical comments by P. Protter.

This paper will present a radically new theory of consumers, based on the idea that man is a limited-capacity information processor and has to operate in habits, which are revised only if it is worth the time to do so.

1. INTRODUCTION

This section will present the basic reasoning behind the theory of the time economic consumer by means of an example.

a. An Example

Let us take an intuitive look at the way an agent faced with a time constraint will operate. In doing this, it is helpful to think in terms of a particular example. Suppose that a person from country X, where one always eats a local, nonexported fruit for breakfast, moves to the U.S. and decides to have cereal for breakfast. The first time our cosmopolite goes out to buy cereal, he is totally ignorant of the nature of the different brands and their relative prices. If he has anything like an ordinary income and an ordinary workday, however, it is a trivial conclusion that it does not pay off for him to search out all the existing brands, read their labels, and check their prices.

One way or the other, our cosmopolite makes a decision and brings home a brand about which he knows very little. The problem now is, how is he going to eat the cereal, e.g., with milk? with sugar? with fruit? what fruit? and how much of each of the above? The point is that cereal is merely one input into a process by which he produces a meal (this is the consumer theory from Lancaster [1971]) and that he knows very little about the production function. Here again, our friend will take a shortcut and try out the brand in a particular, if arbitrary, combination. One reason for this is that he logically cannot know how he will like things he has never tasted.

Another reason is that even a trained chemist/biologist would not bother to calculate the perfect mix in terms of moisture, sweetness, spiciness, etc., assuming that this could be done. So the decision made on the first day of use is of a rough and ready nature; it is just not worth it to experiment too much--the train leaves in seven minutes.

Depending on how the meal tasted the first day, the combination can be revised in various ways on the second day; he might, e.g., use less milk and more raisins. Further improvements can be tested on the third day, etc., until the box is empty.

When the cosmopolite goes out to buy his second box of cereal, the situation is fundamentally different from the first time. He can still do what he did the first time: gather information on a few brands, remembering what he can from the previous search, then make a quick decision and bring home a brand for taste experimentation. He can also, however, make the following argument: "The information I found is probably still valid; my criteria have not changed and I have now gained some user skill (production experience, if you want) with the brand I first bought." If he is relatively satisfied with his original purchase, buying the same brand again is clearly easier in a lot of ways; he saves search time, decision time, and experimentation time. His opinion of the first brand therefore has to be pretty low in order to warrant not simply repeating the purchase. It is thus time-optimal to develop a habit, a decision rule, according to which he buys the same cereal every time.

Let us now think about the optimal strategy for searching out candidates for alternative habits. Suppose that you want to take advantage of an offer of a brand of cereal right now; depending on where you read this, it will probably take you at least ten minutes to get to a supermarket and find the shelf with cereal. Once you are at the supermarket, you can, of course, get information about several brands at little additional time cost, but almost surely not about all brands. If you really wanted to investigate the market at a specific point in time, it would probably take all day. If, instead, you, are content to receive fewer, occasional offers--for example, when you are already in the supermarket--this can be done at very low time cost. In fact, many opportunities for easy acquisition of information/offers are encountered every day; when we shop, read newspapers, chat with friends, etc. Other opportunities, such as ordering a furniture catalogue over the telephone, are slightly more time-consuming, but are still very, very cheap compared to a complete search.

In particular, we can compare two searching strategies. In the first, one takes the habits one by one and samples until one is satisfied; in the second, one continually decides, for each habit, how easy a sample opportunity has to be before one takes it. Because the former procedure is a special case of the second, the optimal strategy of the second type will dominate that of the first type. In this view, and contrary to the assumption made in most economic search theory, the agent is to a certain degree alert to search opportunities and uses some of them to check on his habits, the checks most often having the form of price-quality offers. The amount of checking time devoted to various habits depends on the riskiness (see Weizsäcker [1971]) and actuality of the related purchase situations. If, for example, you expect to have to buy a new car soon, you will use more time reading, talking, and thinking about it.

Let us now go back to our cereal consumer. Assume that he has decided on his checking intensity for his cereal habit, and consider the rational response to an offer from a new brand of cereal. Once the offer has been received, the marginal search costs are zero; moreover, the decision to compare it to the brand currently used has also been made a priori. The decision about whether or not to shift brands can therefore be reduced to a weighting of three things: price, quality, and user skills. For the user skills in particular, one has to assess the likely start-up level of using the new brand and compare the learning speeds of both brands.

A particularly simple example, which I will call the case of semidifferentiated products and will analyze more closely in the next section, is the following. All brands are both different (in the sense that user skills obtained on one will not transfer to another) and similar (in the sense that they are expected to perform identically, utilitywise, for similar amounts of user experience). In this case, the challenger brand is preferred only, if it sells at a discount large enough to warrant the loss of accumulated user skills on the currently used brand. (In Appendix C, I will look at the case with partly transferable user skills and partly heterogenous products.)

The ingraining of a habit as it grows "older" is due to the fact that the fixed cost of changing it will grow with the age of the habit, as one's user skills grow with experience. This effect will lead to a decline in checking frequency and the amount of price discount required for change. So learning declines and habits become ingrained, in the sense that it takes more to change them the older they are. (In a more refined analysis, one could see this effect as counteracted by a general diffusion of brand-specific information.)

b. More General Concerns

In the cereal example, it seems reasonable to define the habit in terms of the brand, but the concept extends to many other cases. For some services, such as lawyers, the habit is defined by the supplier, while purchases of most branded goods represent a combination of two interdependent habits, one associated with the store and the other with the brand.

Ones does, however, perform tasks other than shopping-consuming in a habitual way. Consider, for example, what you do between waking up and arriving at work. In this limited period, you have to do a number of things in the "best" possible way; and, if you are like most people, you have developed a set of only rarely changing rituals, habits, which you execute every morning. These habits--e.g., the point in the sequence when you shave--can be analyzed in exactly the same way as the cereal habit in the preceding section. You could shave earlier or later, but doing so would force you to learn how to shave in a more or less dressed, wet, and awakened state; and, unless you change the whole sequence, most alternatives will necessitate an extra trip to the bathroom (assuming that your current "production plan" is efficient).

The example above is merely suggestive; clearly, people also perform their jobs in habitual ways. The learning curve for assembly workers is a result of the ingraining of more and more efficient habits on the part of the individuals involved. The worker begins doing things in a certain order and manner and learns how to execute that plan as he carries it out. Changes in the plan will have to promise a noninfinitesimal improvement in terms of efficiency or ease before they are accepted.

c. Theoretical Antecedents

While brand loyalty is a prominent concept in the marketing literature, it has not so far been given an economic explanation. The economic theory of habit (e.g. Lluch [1974], Spence [1981], and Weizsäcker [1971]) has left the rationality of habit formation unexplained and has not been linked to search theory (e.g., Rothschild [1974]).

The mechanisms used in this paper have close ties with the bounded rationality school (e.g., Barnard [1938], Leibenstein [1976], Lesourne [1977], Radner [1975], Radner and Rothschild [1975], Rothschild [1974], and Simon [1945, 1957]), although the agent here is kept rational through explicit introduction of the time costs resulting from limited information-processing capacity (Bruner [1958], Haines [1974], and Jacoby, Speller, and Kohn [1974]).

Finally, it should be noted that models of this type are the implicit foundation for all differential equation models of sales flow (e.g., Clembout [1971], Phelps and Winter [1971], Wernerfelt [forthcoming 1982a, Ch. 7], and Wernerfelt [forthcoming 1982b].

2. THE MODEL

In this Section, I will look at a formal model of the above reasoning.

2.1 Premises

Let us consider an idealization of the cereal situation described above. Our consumer uses n goods, each of which is available in many "semidifferentiated" brands (or from many "semidifferentiated" suppliers; these two terms will be used interchangeably in the following discussion). As stated above, the "semidifferentiated" brands are assumed to be different, in the sense that user skills obtained on one do not transfer to another, and yet similar, utilitywise, for similar levels of user skills. The consequences of partly transferable user skills and partly heterogeneous brands are mentioned in Appendix C.

I will allow the consumer to use his time in three ways: price scouting, learning of user skills, and price bargaining with his current supplier. The consumer is stupid in the sense that he does not learn from his price-scouting experience; his probability distribution over the prices of suppliers other than his own is constant and unaffected by experience and time. The consumer is furthermore constrained to follow a single cutoff policy in relation to price offers, such that he accepts all offers below a given cut-off value, which depends on the state variables of the problem. (This type of policy need not be optimal, although it most often is.) I am considering a consumer with habitual relationships to suppliers of each of n goods. These relationships are continually checked as the consumer scouts for alternative offers, which, depending on their attractiveness, are either dropped and forgotten or taken up as substitutes for that of the current supplier. In addition, the price paid can be influenced to a certain extent by bargaining with the current supplier. The process governing the prices paid can be written:

$$(1)\ dp_i(t,\omega) = \alpha_i(t,\omega)p_i(t,\omega)dt + dN_i[\lambda_i(t,\omega),p_{\Delta i}(t,\omega)P_i(t,\omega)],\ i = 1, 2, \ldots n,$$

where

$p_i(t,\omega)$ is the price paid for the i'th good at time t for a particular realization ω of the process;

$\alpha_i(t,\omega)$ is a scalar expressing the rate of price change which results from the bargaining at t for a given ω; and

$dN[\lambda_i(t,\omega),\ p_{\Delta i}(t,\omega)P_i(t,\omega)]$ is the increment of the process describing the effect of price scouting.

Reflecting the randomness of search opportunities, offers arrive as in a Poisson type process with intensity $\lambda_i(t,\omega)$, and are accepted if they are below the cutoff $p_{\Delta i}(t,\omega)$. So dN_i causes no change in p_i if an arriving offer is above $p_{\Delta i}$, while an offer below $p_{\Delta i}$ causes p_i to jump to that value.

As soon as the consumer starts a relationship with a new supplier, he engages in efforts directed towards increasing the satisfaction he can get from the particular brand bought from the new supplier. As outlined above, I will here assume user skills to be nontransferable, so that one starts from scratch every time a new price offer is taken up. Thus, the process governing the indices of user skills can be written as:

(2) $d\theta_i(t,\omega) = \eta_i(t,\omega)\theta_i(t,\omega)dt + dM_i[\lambda_i(t,\omega),p_{\Delta i}(t,\omega),\theta_i(t,\omega)]$, $i = 1, 2\ldots n$,

where

$\theta_i(t,\omega)$ is the value of the user skill index of the i'th habit at t for a given ω;

$\eta_i(t,\omega)$ is the positive rate at which learning efforts cause the index θ_i to grow at t for a given ω; and

$dM_i[\lambda_i(t,\omega),p_{\Delta i}(t,\omega)\theta_i(t,\omega)]$ is the process describing the effect on θ_i of the taking up of new price offers. As long as no new offer is taken up, dM_i causes no change in θ_i, while arrival of an offer below $p_{\Delta i}$ causes θ_i to jump to 1.

For simplicity, I assume that the consumer finances his consumption with cash from a checking account which pays a certain interest. So the dynamic budget constraint is

(3) $dS(t,\omega) = [rS(t,\omega - \sum_{i=1}^{n} p_i(t,\omega)y_i(t,\omega)]dt$,

where

$S(t,\omega)$ is the cash balance at time t for a given ω;

r is the interest rate at which S inflates; and

$y_i(t,\omega)$ is the level at which the i'th good is consumed.

Now make the crucial assumption that the workings of the user skill index are valued as purely volume-augmenting, such that the maximant can be written as the expectation of

(4) $\int_0^T e^{-\rho t} U(\bar{y}(t,\omega), S(t,\omega), \lambda(t,\omega), \eta(t,\omega), \alpha(t,\omega))dt + B(\theta(T,\omega), S(T,\omega), p(T,\omega))$,

where

T is the time horizon;

ρ is the positive felicity discount rate;

$\bar{y}(t,\omega)$ is the n vector of the products $y_i(t,\omega)\theta_i(t,\omega)$, namely, the perceived volume equivalent of y_i given θ_i;

$\lambda(t,\omega)$, $\eta(t,\omega)$, and $\alpha(t,\omega)$ are the three n vectors with typical elements λ_i, η_i and α_i;

$U[\cdot]$ is assumed scalar-valued and C^2;

$p(T,\omega)$ is the terminal price vector; and

$B[\cdot]$, the bequest function, is scalar-valued and C^2.

I will assume that the consumer attempts to maximize the expectation of (4), subject to (1), (2), and (3), by way of feedback control of y, p_Δ, λ, η, and α. In particular, the problem is the construct piecewise continuous functions $\hat{y}$, $\hat{p}_\Delta$, $\hat{\lambda}$, $\hat{\eta}$, and $\hat{\alpha}$ of S, p, θ, and t, such that the positive-valued

$$y_i(t,\omega) = \hat{y}_i(S(t,\omega), p(t,\omega), \theta(t,\omega), t) \quad , i = 1, 2\ldots \; n;$$
$$p_{\Delta i}(t,\omega) = \hat{p}_{\Delta i}(S(t,\omega), p(t,\omega), \theta(t,\omega), t) \quad , i = 1, 2\ldots \; n;$$
$$\lambda_i(t,\omega) = \hat{\lambda}_i(S(t,\omega), p(t,\omega), \theta(t,\omega), t) \quad , i = 1, 2\ldots \; n;$$
$$\eta_i(t,\omega) = \hat{\eta}_i(S(t,\omega), p(t,\omega), \theta(t,\omega), t) \quad , i = 1, 2\ldots \; n;$$
$$\alpha_i(t,\omega) = \hat{\alpha}_i(S(t,\omega), p(t,\omega), \theta(t,\omega), t) \quad , i = 1, 2\ldots \; n$$

maximize the expectation of (4) for a given set of initial values S(0), p(0), $\theta(0)$, a given probability space over ω, and diverse constraints.

In the following I will, for simplicity of notation, suppress explicit dependence on t and ω.

Assume that $U[\cdot]$ and $B[\cdot]$ are nonnegative, bounded, and concave. Using subscripts to denote partial derivatives, it is furthermore assumed that

$U \to 0$ for $S \to 0^+$;

$U_z > 0$ for $z = \bar{y}, S, \alpha, -\lambda, -\eta$;

$U_y \to \infty$ for $y \to 0^+$,

$U_\alpha \to \infty$ for $\alpha \to \bar{\alpha}^+$ where $-\infty < \bar{\alpha} < 0$;

$U_z \to 0$ for $z = \lambda, \eta \to 0^+$;

$U_{zz} < 0$ for $z = \bar{y}, S, \lambda, \eta, \alpha$;

$U_{z_1 z_2} < 0$ for z_1, z_2 being any two different arguments of $\lambda, \eta, -\alpha$;

$B_z > 0$ for $z = \theta, S, -p$; and

$B_{zz} > 0$ for $z = \theta, S, p$.

In this case, the optimal policy[2] must satisfy the dynamic programming conditions below[3].

2.2 Necessary Conditions

The value function $W(t,\theta,S,p)$ satisfies

$$
(5)\quad 0 = \underset{(\hat{y},\hat{p}_\Delta,\hat{\lambda},\hat{\eta},\hat{\alpha})}{\mathrm{MAX}} \{U(\bar{y},S,\lambda,\eta,\alpha) + \dot{W}_s[rS - \sum_{i=1}^{n} y_i p_i] + \sum_{i=1}^{n} W_{p_i}\alpha_i p_i
$$

$$
+ \sum_{i=1}^{n} W_{\theta_1}\eta_1\theta_1 + W_t - \rho W - \sum_{i=1}^{n} \lambda_i W
$$

$$
+ \sum_{i=1}^{n} \lambda[\sum_{\bar{p}_i \le p_{\Delta i}} W(t,\theta_1,\ldots\theta_{i-1},1,\theta_{i+1}\ldots\theta_n,S,p_1,\ldots
$$

$$
p_{i-1},\bar{p}_i,p_{i+1}\ldots p_n)f_i(\bar{p}_i)
$$

$$
+ \sum_{\bar{p}_i > p_{\Delta i}} W f_i(\bar{p}_i)]\},
$$

where $f_i(\bar{p}_i)$ is the probability that a received offer has value $\bar{p}_i$. The equation (5) is constrained by the initial conditions $(\theta(0), S(0), p(0))$, the terminal conditions on W,

(6) $W(T,\theta,S,p) = B(\theta,S,p)$,

the dynamics (1), (2), (3), and the need for the control variables to stay positive. I will now assume the latter problem to be academic and look at the first-order conditions for internal maxima. The first-order conditions are, for i = 1, 2, ...n are;

(7) $U_{y_i} = W_s p_i$,

(8) $W(t,\theta_1,\ldots\theta_{i-1},1,\theta_{i+1}\ldots\theta_n,S,p_1,\ldots p_{i-1},p_{\Delta i},p_{i+1}\ldots p_n) = W$,

(9) $U_{\lambda_i} = \int_0^{p_{\Delta i}} W_{p_i}(t,\theta_1,\ldots\theta_{i-1},1,\theta_{i+1}\ldots\theta_n,S,p_1,\ldots p_{i-1},\bar{p}_i,p_{i+1},\ldots p_n)F_i(\bar{p}_i)d\bar{p}_i$,

(10) $U_{\eta_i} = -W_{\theta_i}\theta_i$, and

(11) $U_{\alpha_i} = -W_{p_i}p_i$,

where $F_i(\bar{p}_i)$ is the probability that an offer received is below $\bar{p}_i$.

Let me first look at p_Δ. If one defines the variables $\check{p} \equiv \frac{p}{\theta}$, the constraints can be rewritten as

(1,2') $d\check{p}_i = (\alpha_i - \eta_i)\check{p}_i dt + d\check{N}_i\ (\lambda_i, \check{p}_{\Delta_i}\check{p}_i)$,i = 1, 2... n, and

(3) $dS = [rS - \sum_{i=1}^{n} \bar{y}_i \check{p}_i]dt$.

In this formulation, all traces of the variables θ, p have disappeared and search, as described by $d\check{N}$, can be seen as search for lower $\check{p}$. So the original complex problem has been reformulated into a very familiar one, with the trivial decision rule that all offers below the current $\check{p}$ should be taken. So <u>the optimal value of p_Δ</u>, call p^*_Δ, is <u>$\frac{p}{\theta}$</u>.

If we next consider λ, (9) implies that an exogenous downward shift in the domain of $F(\bar{p})$, leading to higher values of F, will cause λ to rise. So search increases, if perceived potential increases.

Further comparative static reasoning, based on the assumed shape of U[•], shows that a lower value of n, i.e., fewer goods, will lead to faster learning, as measured by λ, η, $-\alpha$.

If we interpret the model as being concerned with a mass product consumer, the price effect of bargaining, αp, is typically zero, since the prices in the supermarket are given to the buyer. Apart from this case, where no bargaining takes place, αp may be zero as a result of mutually counter-balancing efforts from both buyer and seller. If we specialize our model by thus setting α equal to zero, a couple of well-known effects appear. First, in the absence of habit change, search will decline as the upper limit of integration in (9) falls.

Finally, if we think of one of the goods, say the k'th, as representing all new, not yet consumed products, such that $\theta_k = 0$, $y_k = 0$, we find that, ceteris paribus, novelty seeking will grow over time, as search in other directions declines. (This, of course, requires slightly different properties of U_y than those stated above).

3. CONCLUSION

This paper is an attempt to introduce a new dynamic theory of consumption inspired by the brand loyalty phenomenon as used by marketers. While the model shows the behavioral realism of bounded rationality models, it has the additional attractive feature of portraying the agent as rational.

Since the model can be used as a theoretical foundation for differential equation models of market-share flows, it might be a useful building block for further work in dynamic economic theory.

The University of Michigan

APPENDIX A

THE ALLOCATION OF WORK TIME

If one thinks of jobs as sets of acts performed by sets of habits, one can modify the model slightly to consider work habits. We will interpret

y_i as the daily level at which the i'th act is performed;

p_i as the time cost of each performance;

θ_i as the performance skill;

λ_i as the intensity with which one searches for better ways of performing the act;

η_i as the speed with which one learns to execute the habit better; and

α_i as the negative of the force with which one bargains with one's employer for lower standards.

The only problem is that the dynamic constraint (3) fails to apply, so I will look at λ, η, $-\alpha$ as direct measures of the respective time uses, remove them from the utility function, and introduce a static time constraint. I now want to

$$(12) \qquad \max_{(\hat{y},\hat{p}_\Delta,\hat{\lambda},\hat{\eta},\hat{\alpha})} E[\int_0^T e^{-\rho t} U(\bar{y},S)dt + B(\theta(T),S(T),p(T))],$$

subject to the previous constraints, e.g. (1), (2), and

$$(13) \quad \sum_{i=1}^{n} (\lambda_i + \eta_i - \alpha_i) \leq L,$$

where L is some positive scalar.

In this case, the optimal policy is bang/bang, i.e., all effort is allocated to one of the λ, η, $-\alpha$, depending on the element

$$\mathrm{MAX}\{-\int_0^{p_{\Delta i}} W_{p_i}(\bar{p}_i,1,\cdot),F_i(\bar{p}_i)d\bar{p}_i, W_{\theta_i}\theta_i, -W_{p_i}p_i \mid i = 1, 2, \ldots n\}.$$

Therefore, in this model time/effort uses can be ordered on a scale and we always pick from the top of the list. If the top element loses its position by our attention, we can be said to "put out fires," whereas if its position is enforced, we "stay with a winner."[4] If we set $\alpha = 0$ and use the assumptions made on U [•] above, we are in the "putting out fires" case and the fires get smaller and smaller, reflecting declining learning speed.

APPENDIX B

AN EXPLICIT SOLUTION

Let us look at

$$(14)\quad \underset{(\hat{y},\hat{p}_\Delta,\hat{\lambda},\hat{\eta},\hat{\alpha})}{\text{MAX}} E\{\int_0^\infty e^{-\rho t} -\prod_{i=1}^{n} [(y_i\theta_i)^{a_i}\lambda_i^{b_i}\eta_i^{c_i}\alpha_i^{d_i}]dt\},$$

subject to

$$(1)\ dp_i = \alpha_i p_i dt + dN_i(\lambda_i, p_{\Delta i}) \quad, i = 1, 2, \ldots, n,$$

$$(2)\ d\theta_i = \eta_i\theta_i dt + dM_i(\lambda_i, p_{\Delta i}) \quad, i = 1, 2, \ldots, n, \text{ and}$$

$$(3)\ dS = [rS - \sum_{i=1}^{n} p_i y_i]dt,$$

where the constants (r, ρ, a, b, c, d) satisfy

$$0 < r < \rho,$$

$$a, d < 0, \text{ and}$$

$$1 < b, c,$$

and where all control and state variables are required to be positive at all times. The initial values of p, θ, S are also assumed to be positive, and prices can never go below $p_0 > 0$.

I will now use the theorems from note 3 in sufficiency (that is, verification) form. The shift to infinite time horizon is made for expositional ease only.

For a candidate policy to be optimal, the value function should satisfy

$$(15)\quad 0 = \underset{(\hat{y},\hat{p}_\Delta,\hat{\lambda},\hat{\eta},\hat{\alpha})}{\text{MAX}} \{-\prod_{i=1}^{n} (y_i\theta_i)^{a_i}\lambda_i^{b_i}\eta_i^{c_i}\alpha_i^{d_i} + W_s(rS - \sum_{i=1}^{n} y_i p_i)$$

$$+\sum_{i=1}^{n} W_{p_i}\alpha_i p_i + \sum_{i=1}^{n} W_{\theta_i}\eta_i\theta_i - \rho W - \sum_{i=1}^{n} \lambda_i \int_{p_0}^{p_{\Delta i}} W_{p_i}(\bar{p}_i, 1, \cdot)F_i(\bar{p}_i)d\bar{p}_i\},$$

where

(16) $E\{e^{-\rho t}W[\cdot]\}\to 0$ for $t\to\infty$

is a transversality condition, replacing the terminal condition (6) on the finite horizon problem.

If one inserts the candidate policies (7) through (11) into (15), one gets the following partial differential equation in $W(S,p,\theta)$:

$$(15)\quad 0 = K_1 \prod_{i=1}^{n} [W_s p_i^{\frac{1}{\theta_i}})^{a_i}(\int_{p_0}^{p_i^{\frac{1}{\theta_i}}} W_{p_i} F_i(\bar{p}_i)d\bar{p}_i)^{b_i}(W_{\theta_i}\theta_i)^{c_i}(W_{p_i}p_i)^{d_i}]^{\frac{1}{a_0}}$$

$$-\rho W + W_s rS,$$

where $a_0 = \sum_{i=1}^{n} a_i$ and K_1 is a negative constant.

To solve this, we decide to look for a solution in the form $W(S,p,\theta) \equiv W^0(p,\theta)S^{a_0}$. In this case, W^0 shall make the following expression constant:

$$\prod_{i=1}^{n} [(p_i^{\frac{1}{\theta_i}})^{a_i}(\int_{p_0}^{p_i^{\frac{1}{\theta_i}}} W^0_{p_i} F_i(\bar{p}_i)d\bar{p}_i)^{b_i}(W^0_{\theta_i}\theta_i)^{c_i}(W^0_{p_i}p_i)^{d_i}].$$

One may verify that this is the case for

$$(18)\quad W^{0*} \equiv K_0 \prod_{i=1}^{n} [\int_{p_0}^{p_i} \theta_i \bar{p}_i^{\frac{1}{\theta_i}x_i}(\int_{p_0}^{\bar{p}_i} \check{p}_i^{x_i} F_i(\check{p}_i)d\check{p}_i)^{-z_i} d\bar{p}_i],$$

where K_0 is a constant, $x_i = -\frac{a_i+c_i+d_i}{c_i+d_i}$, and $z_i = \frac{b_i}{b_i+c_i+d_i}$ S_0 $W^* \equiv W_0^{0*}S^{a_0}$ solves (17). Furthermore, the upper bound on the utility function, we can make the transversality condition (16) stick, such that W^* inserted into (7) through (11) actually gives an optimal policy.

We thus find

$$y_i^* = \beta_y \frac{S}{p_i} \qquad i = 1, 2, \ldots, n,$$

$$p_{\Delta i}^* = p_i \frac{1}{\theta_i} \qquad i = 1, 2, \ldots, n,$$

$$\lambda_i^* = \beta_\lambda [\int_{p_0}^{p_i^{\frac{1}{\theta_i}}} \bar{p}_i^{x_i} F_i(\bar{p}_i) d\bar{p}_i]^{z_i - 1} \int_{p_0}^{p_i^{\frac{1}{\theta_i}}} \bar{p}_i^{x_i} (\int_{p_0}^{\bar{p}_i} \check{p}_i^{x_i} F_i(\check{p}_i) d\check{p}_i]^{-z_1} d\bar{p}_i$$

$$i = 1, 2, \ldots, n,$$

$$\eta_i^* = \beta_\eta (p_i (p_i^{\frac{1}{\theta_i}})^{-x_i - 1} [\int_{p_0}^{p_i^{\frac{1}{\theta_i}}} \bar{p}_i^{x_i} F_i(\bar{p}_i) d\bar{p}_i]^{z_i} \int_{p_0}^{p_i^{\frac{1}{\theta_i}}} \bar{p}_i^{x_i} (\int_{p_0}^{\bar{p}_i} \check{p}_i^{x_i} F_i(\check{p}_i)$$

$$i = 1, 2, \ldots, n, \text{ and}$$

$$\alpha_i^* = \beta_\alpha \frac{1}{\beta_\eta} \eta_i^* \qquad i = 1, 2, \ldots, n,$$

where β_y, β_λ, β_η, β_α are four constants.

APPENDIX C

TRANSFERABLE SKILLS AND HETEROGENEOUS PRODUCTS

If we allow user skills to be partly transferable between brands, such that a switch to a new brand causes θ_i to jump to

$$\theta_i^0 = (\theta_i - 1)\nu_i + 1 \qquad ,0 \leq \nu_i \leq 1,$$

then the value of p_Δ is given by

$$p_i \frac{\theta_i^0}{\theta_i}.$$

If we generalize to partly heterogeneous brands, such that

$$\bar{y}_i = y_i \theta_i \mu_i$$

where μ_i is a positive quality index which takes different values for each brand, the value of p_Δ is given by

$$\frac{p_i}{\theta_i} \frac{\mu_i^*}{\mu_i},$$

where μ_i^* is the value of the quality index for the candidate brand.

FOOTNOTES

1. This simple form causes no loss of generality compared to $y_i(t,\omega)$ $f_i[\theta;(t,\omega)]$, where $f_i' > 0$, but I need the linearity in $y_i(t,\omega)$.

2. The existence of such a policy cannot be guaranteed at the moment, although progress is being made for similar types of problems. See, e.g., Goor [1973, thrm. 2.2] and Jacod and Protter [1982].

3. See Stone [1973, thrm. 4.5] and Boel and Varaiya [1977, thrm. 5.6].

4. Terminology is taken, from Radner [1975].

REFERENCES

[1] Barnard, C. I. The Functions of the Executive. Cambridge, Mass.: Harvard University Press, 1938.

[2] Bauer, R. A. "Consumer Behaviour as Risk Taking," in Risk Taking and Information Handling in Consumer Behaviour, ed. by D. F. Cox. Boston: Division of Research, Harvard Business School, 1967.

[3] Boel, R., and P. Varaiya. "Optimal Control of Jump Processes." SIAM Journal on Control and Optimization 15 (1977), 92-119.

[4] Bruner, J. S. "Social Psychology and Perception." In Readings in Social Psychology, ed. by E. F. Maccoby, T. M. Newcomb, and E. L. Hartly. New York: Holt, Rinehart and Winston, 1958.

[5] Clemhout, S., G. Leitmann, and H. Y. Wan, Jr. "A Differential Game Model of Duopoly." Econometrica 39 (1971), 911-938.

[6] Goor, R. M. "Existence of an Optimal Control for Systems with Jump Marker Disturbances." SIAM Journal on Control and Optimization 11 (1973), 187-201.

[7] Haines, G. H., Jr. "Process Models of Consumer Decision Making." In Buyer-Consumer Information Processing, ed. by G. D. Hughes and M. L. Ray. Chapel Hill, N.C.: University of North Carolina Press, 1974.

[8] Jacoby, J., D. E. Speller, and C. A. Kohn. "Brand Choice Behaviour as a Function of Information Load." Journal of Marketing Research 11 (1974), 63-69.

[9] Jacod, J., and P. Protter. "Quelques Remarques sur un nouveau type d' equations differentielles stochastiques." In Séminarie de Probabilities XVI. Heidelberg: Springer Lecture Notes in Mathematics, forthcoming, 1982.

[10] Lancaster, K. Consumer Demand, A New Approach. New York: Columbia University Press, 1971.

[11] Leibenstein, H. Beyond Economic Man. Cambridge, Mass.: Harvard University Press, 1976.

[12] Lesourne, J. A Theory of the Individual for Economic Analysis, Vol. 1. Amsterdam: North-Holland Publishing Co., 1977.

[13] Lluch, C. "Expenditure, Savings and Habit Formation." International Economic Review 15 (1974), 786-797.

[14] Phelps, E. S., and S. G. Winter, Jr. "Optimal Price Policy under Atomistic Competition." In Microeconomic Foundations of Employment and Inflation Theory, ed. by E. S. Phelps. London: Macmillan and Co., 1971.

[15] Radner, R. "A Behavioural Model of Cost Reduction." Bell Journal of Economics 6 (1975), 196-215.

[16] Radner, R., and M. Rothschild. "On The Allocation of Effort." Journal of Economic Theory 10 (1975), 358-376.

[17] Rothschild, M. "Further Notes on the Allocation of Effort." In Proceedings of the Symposium on Adaptive Economics. Madison, Wis.: 1974.

[18] Rothschild, M. "Searching for the Lowest Price When the Distribution of Prices Is Unknown." Journal of Political Economy 82 (1974), 689-711.

[19] Simon, H. A. Administrative Behaviour. New York: The Free Press, 1945.

[20] Simon, H. A. Models of Man. New York: John Wiley and Sons, 1957.

[21] Spence, M. A. "The Learning Curve and Competition." Bell Journal of Economics 12 (1981), 49-70.

[22] Stone, L. D. "Necessary and Sufficient Conditions for Optimal Control of Semimarkov Jump Processes." SIAM Journal on Control and Optimization 11 (1973), 187-201.

[23] von Weizsäcker, C. C. "Notes on Endogenous Change of Tastes." Journal of Economic Theory 3 (1971), 345-372.

[24] Wernerfelt, B. Time Economic Man, forthcoming 1982a.

[25] Wernerfelt, B. "Consumers with Differing Reaction Speeds, Scale Advantages, and Industry Structure," forthcoming 1982b.