Division of Research
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October 1979

EVALUATING THE MANAGEMENT OF INVENTORIES: A STOCK AVAILABILITY MODEL

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Working Paper No. 196
The University of Michigan

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EVALUATING THE MANAGEMENT OF INVENTORIES

A STOCK AVAILABILITY MODEL

HEADNOTE

A key problem faced by distribution managers is the determination of the current effectiveness of established systems for the control of inventories. Are the systems performing at design levels? Are the system parameters realistic given the existing competitive environment? The authors suggest an approach which can provide answers to questions such as these and when so indicated prescribe for improvement in inventory system design or mode of implementation.

INTRODUCTION

There is a large body of literature dealing with various methods of controlling inventories. Some of the writings deal with the development of theoretical bases for inventory control systems[1] while others are concerned with alterations and extensions of the basic economic order quantity (EOQ) model aimed at accommodating volume transportation rates or quantity discounts[2] or the uncertainty associated with variations in supply or demand[3].

[1] Martin K. Starr and David W. Miller, Inventory Control: Theory and Practice (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962)

[2] Donald J. Bowersox, Logistical Management, 2nd ed. (New York: MacMillan Publishing Co., Inc., 1978) p. 161 ff.

[3] Robert B. Fetter and Winston C. Dalleck, Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin, Inc., 1961)

The goal of the inventory manager, regardless of what technique is used, is to provide a competitive level of service to customers at the lowest cost. This level is measured by the time lag between order receipt and delivery of the order, by the percent of items backordered, by the variability of delivery times and by the landed condition of the goods.

The models which have been developed to achieve these goals can, in many cases, be implemented by computer. These approaches have many advantages in that service levels can be maintained or improved while inventory holdings can be kept to a minimum level. One disadvantage to the use of current inventory control methods, however, is the lack of knowledge as to whether or not the system developed to control inventories is performing at the design level or if the present design is suited to the nature of the economic environment in which the firm operates. Another shortcoming of traditional approaches is that they often examine inventories on an item by item basis rather than in their entirety.

What is needed is a method of appraising or evaluating a given inventory system in terms of its impact on the total inventory regardless of its design or mode of implementation. Given such a method, management could make those changes necessary to improve the inventory management function. The suggestion of such a means of evaluating inventory system operation and prescribing for its improvement is the purpose of this paper.

A Stock Availability Model

Let us start the process of developing a model for the evaluation of inventory control policies by considering the availability or non-availability of an individual stock item as a function of time. Of course, at any given point in time an item is either available or it is not available. With the passage of time, however, available items may become out of stock and out of stock items may become available. Describing this phenomenon in mathematical terms we have:

$$(1) \quad S_a + \sum_{T=\phi}^{N} S_d \overset{<}{\underset{\geq}{\text{or}}} \sum_{T=\phi}^{N} AMD + S_o \; ,$$

where

S_a = stock available for issue

S_d = stock received per time period

N = number of time periods

T = time period (usually months)

AMD = average monthly demand

S_o = stock due out (back ordered)

The inequality conditions describe real world situations in which there is an excess of supply over demand or vice-versa. The equality condition is of little interest as it describes a transitory occurrence in which stock in equals stock out for short periods of time. The inequalities may be extended to accommodate special additions to supply or special requirements of demand, but to simplify our discussion such extensions will not be used in this paper.

The application of the above inequality provides a rather simple method of calculating the number of time periods which are required for an out-of-stock item to come into stock or the number of periods required for an in-stock item to move to an out-of-stock condition. The conceptual foundation of the model is that the evaluation of the inventory of a specific item should be viewed not just in terms of what is on hand at a given point of time, but rather in terms of when goods currently out of stock will become available and when goods currently available will be out of stock. Because the inventory holding process is dynamic, the key variable is time; specifically, the amount of time needed for a change of state in inventory availability. When time is used as the underlying measure it then becomes feasible to evaluate the impact of managerial decision not only on individual items, but on the inventory as a whole.

The Stock Availability Distribution

Assuming that the inequality is used to analyze each item type in an inventory, one would discover that over time most items would be out of stock for short periods of time with only a few items being out of stock for long periods of time. Conversely, most items will occasionally be overstocked for short periods of time with only a few items being in an overstock condition for long periods of time.

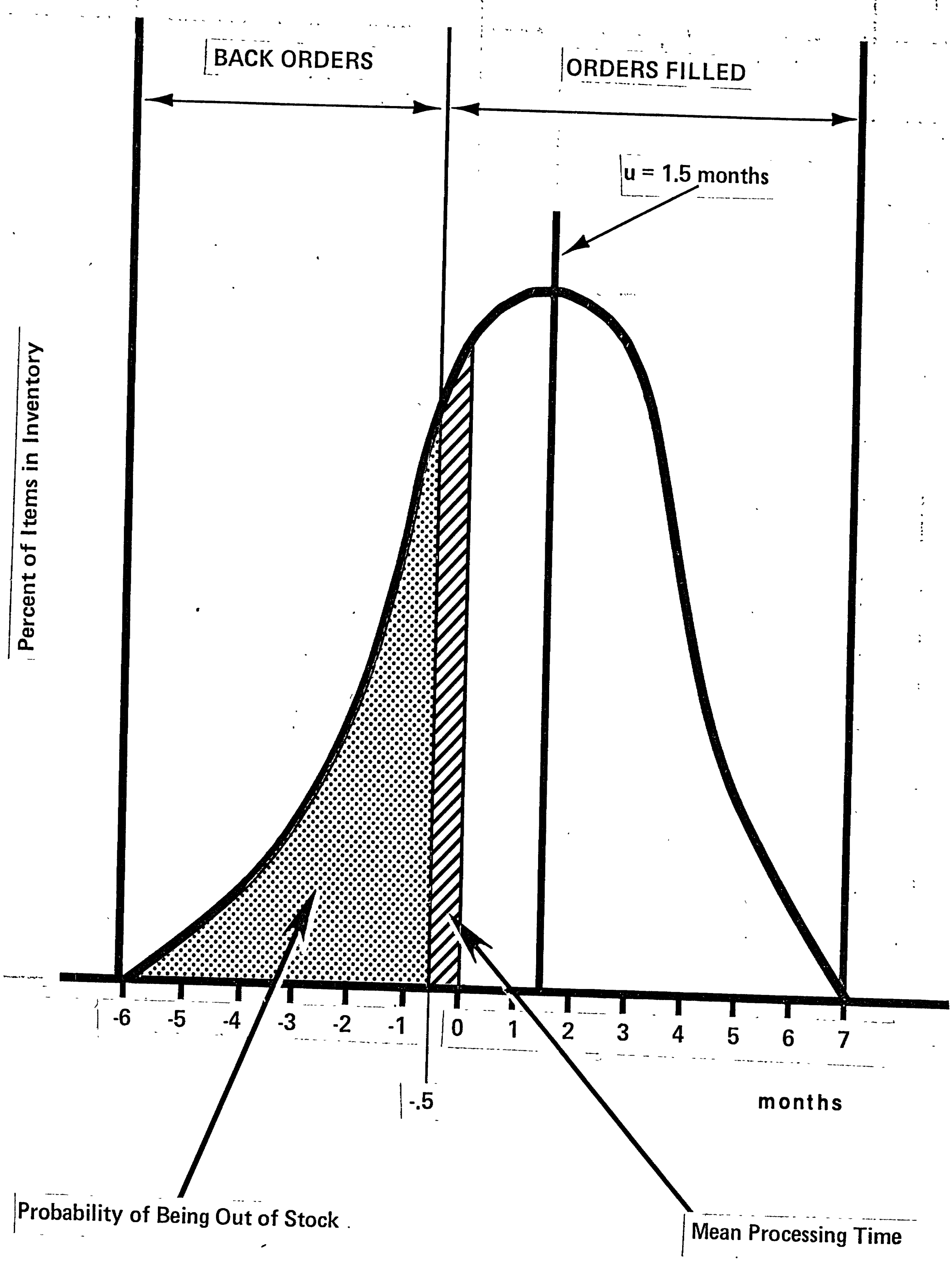
Given such a situation, which is typical for most inventories, one can develop a frequency distribution of the percent of total items in an inventory in terms of number of time periods to a change in stock state. That is, for example, what percent of the items which are in stock at present will still be in stock T months in the future and what percent will have become out of stock. Or, conversely, what percent of items currently out of stock will become in stock T months from the present. Such a distribution which we will call the Stock Availability Distribution (SAD) is illustrated in Figure 1.

Figure 1 illustrates a situation in which the percentages of items in stock are distributed normally with a mean of 15 months. As the zero on the horizontal axis indicates when stock-in exactly equals stock-out, the mean value (u) indicates that the average reserve or buffer time is 1.5 months. Thus extra stocks are being held to cushion against uncertainty in supply or demand. If the mean processing time for an order is 2 weeks after its receipt, then an additional .5 months can be added to the cushion. Back orders will, therefore, occur when items in the left tail of the distribution requiring more than .5 months to be received in stock are ordered by customers. The probability of a backorder is illustrated by the area in the left tail from -.5 months to to $-\infty$. In order to quantify this probability one would need to know the distance of the cut-off point (-.5) from the mean in terms of standard deviations from the mean.

Inasmuch as the probability of a back order is a function of the mean and variance of the distribution, it is obvious that it can be affected by changes in the mean value or the dispersion of the distribution around the mean. The farther the mean value of the distribution is from the zero cut-off point and the "tighter" the distribution is around the mean (low variance), the less will be the probability of a stock-out.

FIGURE 1

A FREQUENCY DISTRIBUTION OF PERCENT OF TOTAL INVENTORY IN TERMS OF TIME OF CHANGE OF STOCKING CONDITION



We have assumed that the frequency distribution of stock availability is normal, but this may not always be the case. The distribution may be skewed as a result of managerial decision to hold safety stock, to make special buys or to ship goods earlier than planned. Regardless of causation, in most cases a Beta distribution can be used to approximate the SAD. What is important is that the actual distribution be approximated by one which when given the required values will allow computation of probability densities.

Inventory Management Policy

Effective inventory management provides the required level of customer service while keeping inventory costs to a minimum. To understand better how the application of the stock availability distribution can be useful, let us review briefly some concepts and terms used by inventory managers.

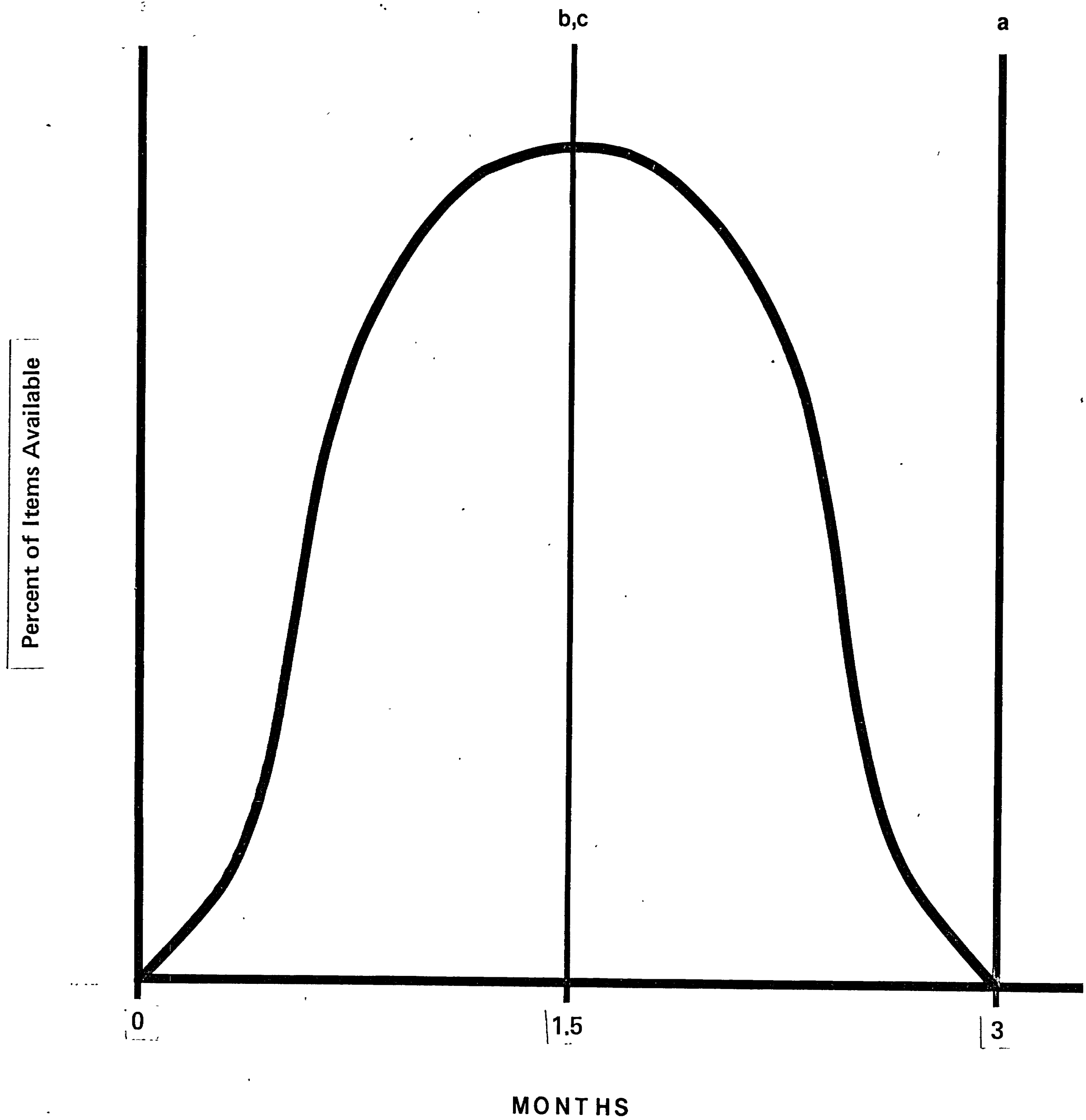
Requirements objective (RO).--This is the sum of items back ordered, items needed for reserves and safety stock, and items needed to cover procurement lead times.

Economic order quantity (EOQ).--This is the quantity which when ordered minimizes the total cost of procuring and holding stock.

Reorder cycle (RC).--This is a period of time between orders for a particular item. It is determined by dividing each items' EOQ by the average monthly demand (AMD) for the item. For example, if the EOQ was 150 units and the AMD was 50 units the reorder cycle would be 3 months.

FIGURE 2

STOCK AVAILABILITY CURVE FOR A THREE MONTH REORDER CYCLE



If an inventory manager were controlling an inventory in which all items had the same EOQ and AMD, then the reorder cycle would be the same for the entire inventory. Such a situation is illustrated by Figure 2.

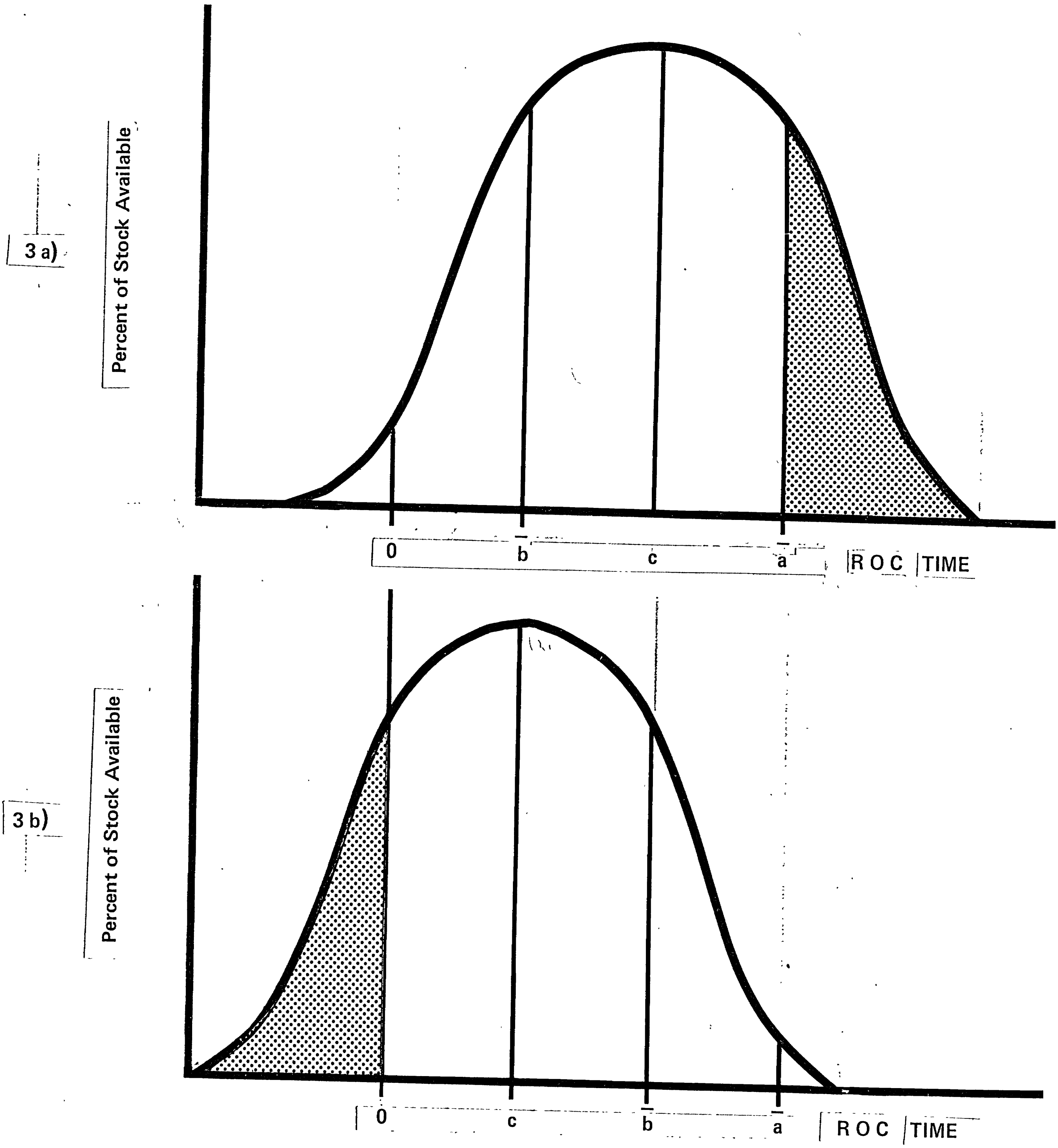
Figure 2 Stock Availability Curve for a Three Months Reorder Cycle

In the above figure "a" is the reorder cycle value of 3 months, "b" is the mid-point in time between 0 and 3 months and "c" is the mean of the SAD. In this example b = c which means that the average of the percent of items available have been in stock for 1.5 months and will be in stock for another 1.5 months.

In terms of inventory management, Figure 2 illustrates an ideal situation. The inventory is well balanced and the tails of the distribution are within the bounds of the reorder cycle (0, 3). Thus there is no excess stock being held and no probability of an out-of-stock condition occurring.

FIGURE 3

TWO SITUATIONS IN WHICH SAD IS SUPERIMPOSED ON REORDER CYCLE TIME FRAME



Unfortunately, the ideal situation is rarely found in practice. Because EOQ's and AMD's vary across the many types of items held, there are a number of reorder cycles. To obtain a usable value we must consider a distribution of the order cycle times encountered and calculate the mean value. Let us call this value $\bar{a}$. By dividing $\bar{a}$ by 2, one can determine $\bar{b}$ the midpoint of the average reorder cycle as measured in time. Now one can superimpose the stock availability distribution on this time frame to see how closely the mean of the distribution (c) is to $\bar{b}$. Let us look at the two conditions illustrated in Figure 3 below.

Figure 3 Two Situations in Which SAD is Superimposed on Reorder Cycle Time Frame

In Figure 3a, the mean value of the midpoint of the reorder cycle distribution ($\bar{b}$) is less than the mean value of the SAD (c) and a large portion of the inventory indicated by the shaded area will be held in quantities greater than required given the average reorder cycle time ($\bar{a}$). In Figure 3b, the opposite case holds. Here $\bar{b}$ is greater than c and insufficient stock is being held to prevent back-orders, also indicated by the shaded area.

Thus, a simple matching of the means of the midpoint of the reorder cycle distribution and the SAD enables the inventory manager to make an important check on the effectiveness of his policies and of how well they are being implemented. If the mean of the SAD is far from $\bar{b}$, then appropriate actions can be taken to increase the convergence thus reducing underages or overages in stock being held.

Of course, a prerequisite to any course of action is a careful review of EOQ's and AMD forecasts as well as the SAD itself to make certain that the values assigned are as accurate reflections of reality during the planning period as can be reasonably obtained.

Corrective Measures

Having been alerted to the need for corrective measures by the divergence of the mean of the SAD from the $\bar{b}$ target several courses of action are possible. First, forecasting errors may be causing the AMD's to be wrong resulting in non-optimal reorder cycles. Incorrect EOQ values can have a similar effect. If the AMD's and EOQ's look good then the problem lies with the mistiming of procurement.

Incorrect procurement lead times are usually at the heart of the problem. These lead times are components of the requirement objectives of groups of items as noted above. The requirements objectives (RO's) can be expressed in terms of times or quantities. The conversion is simple requiring the expression of RO's in months and the multiplication of that time period by the associated AMD's. In the next section we will be discussing the lead time component of the RO as a quantity.

Procurement lead times.--The accurate estimation of a required procurement lead time is essential if one is to have a dependable requirements objective (RO). A faulty RO results in holding the wrong amount of stock. Too long procurement lead times result in too high RO's and thus excessive levels of stock. As the stock levels rise the stock availability distribution (SAD) shifts to the right as does its mean (c). In contrast, too short procurement lead times will shift "c" to the left.

It is obvious that shifts in the SAD require managerial attention if over or under stocking conditions are to be avoided. The shift is a signal that something must be changed and its direction and magnitude provide insight as to the nature of the problem.

The variance of the SAD.--Even if corrective measures bring the mean of the SAD into convergence with the $\bar{b}$ target, the situation may still not be optimal. If, for example, the variance of the SAD is sufficiently large, one or both tails of the SAD may be beyond the limits of the mean reorder cycle. Figure 4 illustrates such a situation.

FIGURE 4

EFFECTS OF VARIABILITY OF SAD WITH MEAN CONVERGENCE WITH TARGET

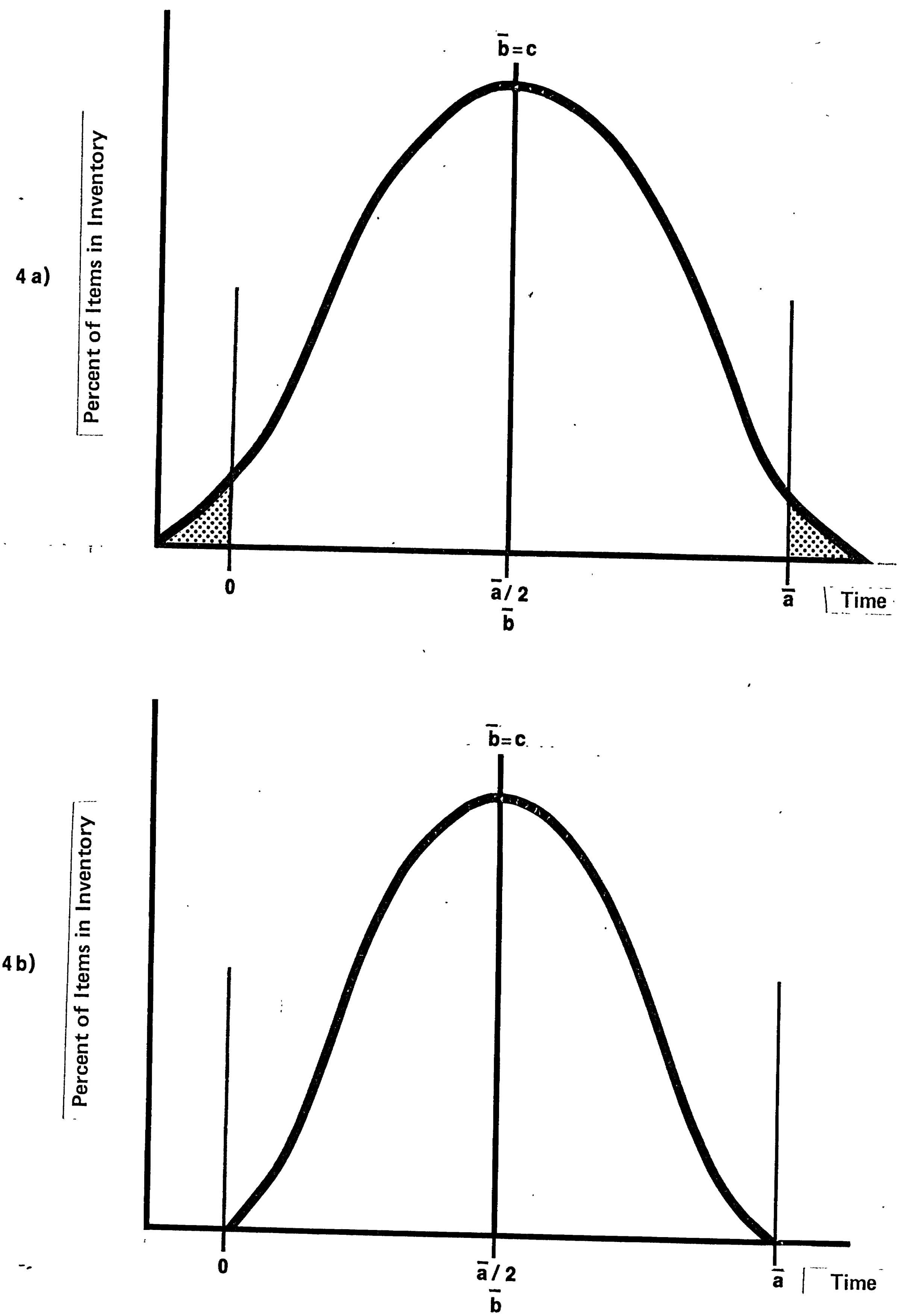


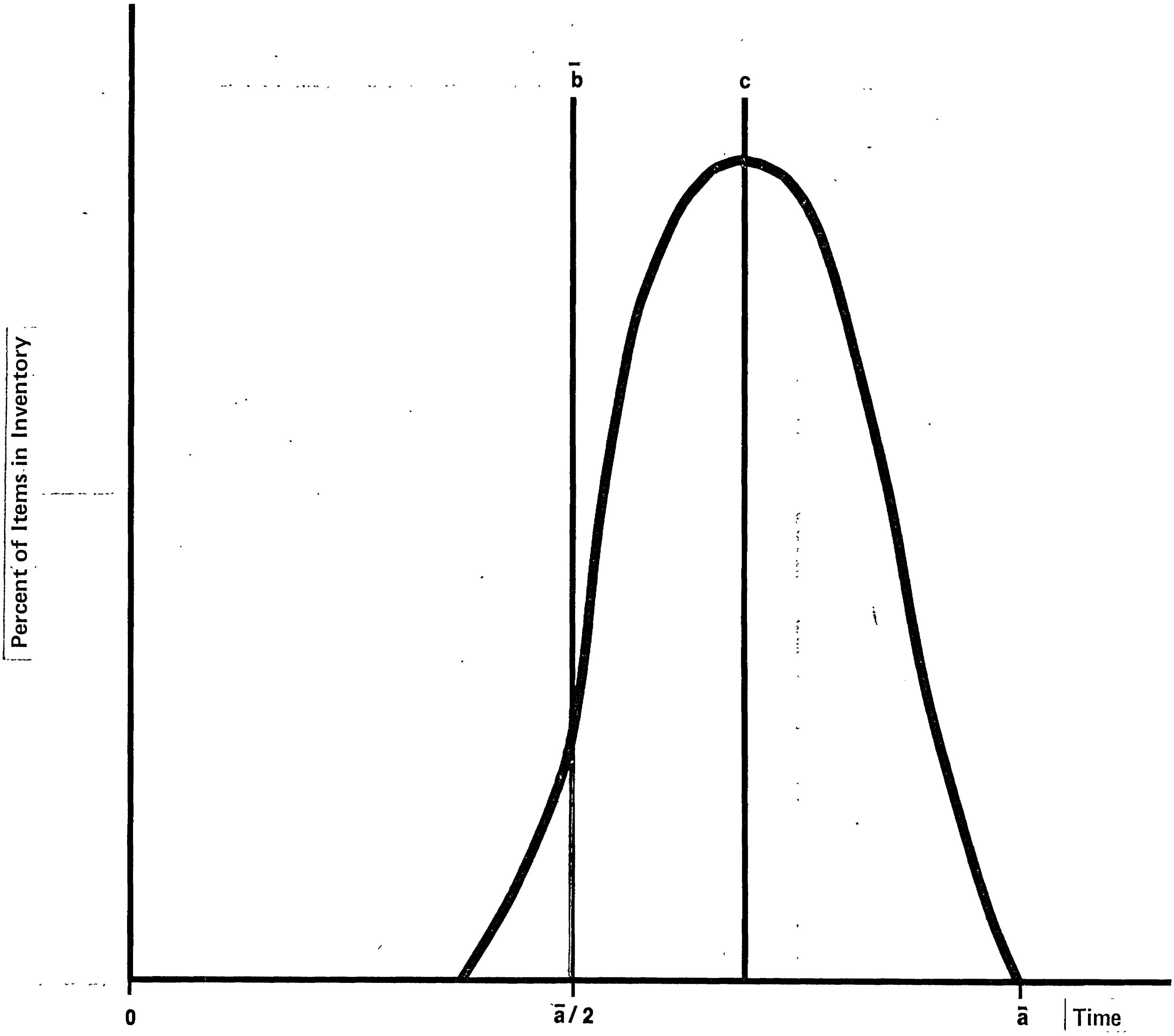
Figure 4 Effects of Variability of SAD with Mean Convergence with Target $\bar{b}$

In Figure 4a although there is convergence of the means, the tails of the stock availability distribution (SAD) fall outside of the boundaries of the average reorder cycle (o, $\bar{a}$). This means that some items will be out of stock and some in over-stock condition during the reorder cycle. Figure 4b illustrates a preferable situation in which the SAD fits within the bounds of the average reorder cycle. In this case the inventory is balanced with no probability of a back order or over stock condition.

A situation in which the variance of the SAD has been further decreased is illustrated by Figure 5. In this case, backorders are minimized and stock is not held too far into the future. The mean of the SAD, however, is still to the right of the target $\bar{b}$ resulting in the holding of more stock than the optimal level. This distribution is an improvement over Figure 4a illustrating that a reduction in

FIGURE 5

SAD WITH LOW VARIANCE AND NON-CONVERGENCE WITH TARGET $\bar{b}$



the variance of the SAD can be an even more effective way of decreasing backorders than gaining greater convergence of the mean of the SAD with the target $\bar{b}$.

Figure 5 SAD with Low Variance and Nonconvergence with Target $\bar{b}$

The above case is, of course, extreme but it illustrates how minimization of the variance of the SAD can compensate for a situation in which the mean of the SAD does not coincide with the midpoint of the average reorder cycle ($\bar{b}$). Noting the advantages accruing from the reduction of the variance of the SAD, the question is raised as to how can management achieve a distribution with greater density around the mean.

What is required as a preliminary to action is an understanding of what causes the SAD to spread out. Basically, it is a matter of timing. The mean of the SAD is a function of the EOQ's but the variance around the mean is determined by the timing of procurement in relation to demand. Thus to minimize the variance, management must upgrade the quality of the demand forecasts and must avoid, whenever, possible delayed or premature buys. Such buys will also alter the quantities purchased and thus the mean of the SAD as well as its variance. In this article we shall attempt to treat the mean and variance separately, although there is a fundamental relationship between them. In actual practice management efforts to reduce the variance will shift the mean and vice versa but the policy maker should be cognizant of the individual roles of the mean and the variance.

Tactics to keep individual items within the limits of the reorder cycle are varied. Purchases can be made automatically when reorder points are reached for those items whose history indicates relatively low variability in demand and delivery times. For other items whose demand is highly variable or whose delivery times are uncertain, periodic reviews of the requirements objectives may be required.

Forecasting the next buy.--An approach to determining when the next buy should be made for those items which are causing trouble can be illustrated by a simple equation.

(2) $\frac{TSA - RO}{AMD} + RC = \text{months until buy}$

where

TSA = total stock available in units (on hand and on order)

RO = requirements objective quantity

AMD = average monthly demand

RC = reorder cycle in months

If the inventory manager at the time of ordering (T = 0) buys the quantity RO and if the RO is equal to TSA then the months until buy would equal RC. Given a situation in which it is assumed that the item has a constant demand and a reorder cycle of 3 months the following table could be drawn:

Table 1 Months until Buy with Constant Demand and Reorder Cycle of 3 Months

As each month passes, the TSA is reduced by one AMD. This process continues until "months until buy" reaches 0 at which time the item is purchased. If we relax the assumption of constant demand then, of course, "months until buy" becomes a forecast.

If, in actual practice, the inventory manager finds that the reorder cycle is consistently longer than forecast he must check the accuracy of the estimates of AMD. If the "months to buy" quantity is larger than the reorder cycle (RC) then the item has been overbought. If it is less than zero then the item has been underpurchased and an order in excess of the EOQ is required.

Table 1 Months until Buy with Constant Demand and Reorder Cycle of 3 Months

Time	$\frac{\text{TSA-RO}}{\text{AMD}}$	+ R.C.	= Mos. Until Buy
T = 0	0	3	= 3
T = 1	-1	3	= 2
T = 2	-2	3	= 1
T = 3	-3	3	= 0

Another way in which the variance about the mean of the SAD can be reduced is to measure the time (in months) greater than the requirements objective (RO) or below the reorder point. Consider the following equations:

(3) If $TSA - RO \geq 0; \frac{TSA - RO}{AMD}$ = MOS OVER RO

(4) If $TSA - RO < 0; \frac{TSA - RO}{AMD} + RC$ = MOS UNDER REORDER POINT (IF A NEG. QUANTITY)

From these equations, the inventory manager can construct a frequency distribution of items under his or her control in terms of how the items stand with reference to the length of time represented by the order cycle. Such a distribution is illustrated in Table 2.

Table 2 Frequency Distribution of Number of Items Above or Below Order Cycle Time by Months

Ideally, all items should fall within the order cycle time period. But fluctuations in demand, in delivery, and errors in forecasting and procurement timing cause many items to be under or over-stocked. Achieving the ideal situation is difficult because it requires an item by item analysis. However, by concentrating on the items that represent

Table 2 Frequency Distribution of Number of Items Above or Below Order Cycle

Time by Months

MOS. > REQUIREMENTS OBJECTIVE | MOS. < REORDER POINT

> 8	4 to 8	3 to 4	2 to 3	1 to 2	0 to 1	WITHIN ORDER CYCLE	0 to 1	1 to 2	2 to 3	3 to 4	4 to 8	> 8
5	5	5	8	10	15	75	15	10	8	5	5	5

NUMBER OF ITEMS | NUMBER OF ITEMS

costly outliers first and by using the tools presented such as the measurement of "months until buy" and "time distance from order cycle period" significant progress can be achieved.

Summary and Conclusions

Simply put, the stock availability model indicates how long it will take an item to reach an out of stock condition if in stock or an in-stock condition if currently out-of-stock. The inventory manager can describe all of the items under his or her control in terms of these times and construct a frequency distribution of these items in terms of what percent of them are in a specific time state with respect to a change in stocking condition. Such a distribution is termed the Stock Availability Distribution or SAD. Given this distribution, the manager can then develop a second distribution composed of the varying reorder cycles for the items to be controlled. One half the mean value of this distribution becomes the target for the mean of the SAD and the time period represented by the average reorder cycle becomes the desired limits of the SAD in terms of its variability.

For example, given a situation such as shown in Figure 6 in which there is considerable divergence between the actual and the "ideal" SAD's, the inventory manager can see immediately that the variance of the actual SAD is too large and that the average time that goods are held in stock is too long. Corrective measures to be investigated include changing the lead time or reorder cycle time to gain greater convergence of the means or improving demand forecasting to reduce the variance of the actual SAD.

FIGURE 6

THE ACTUAL SAD SUPERIMPOSED ON THE "IDEAL" SAD

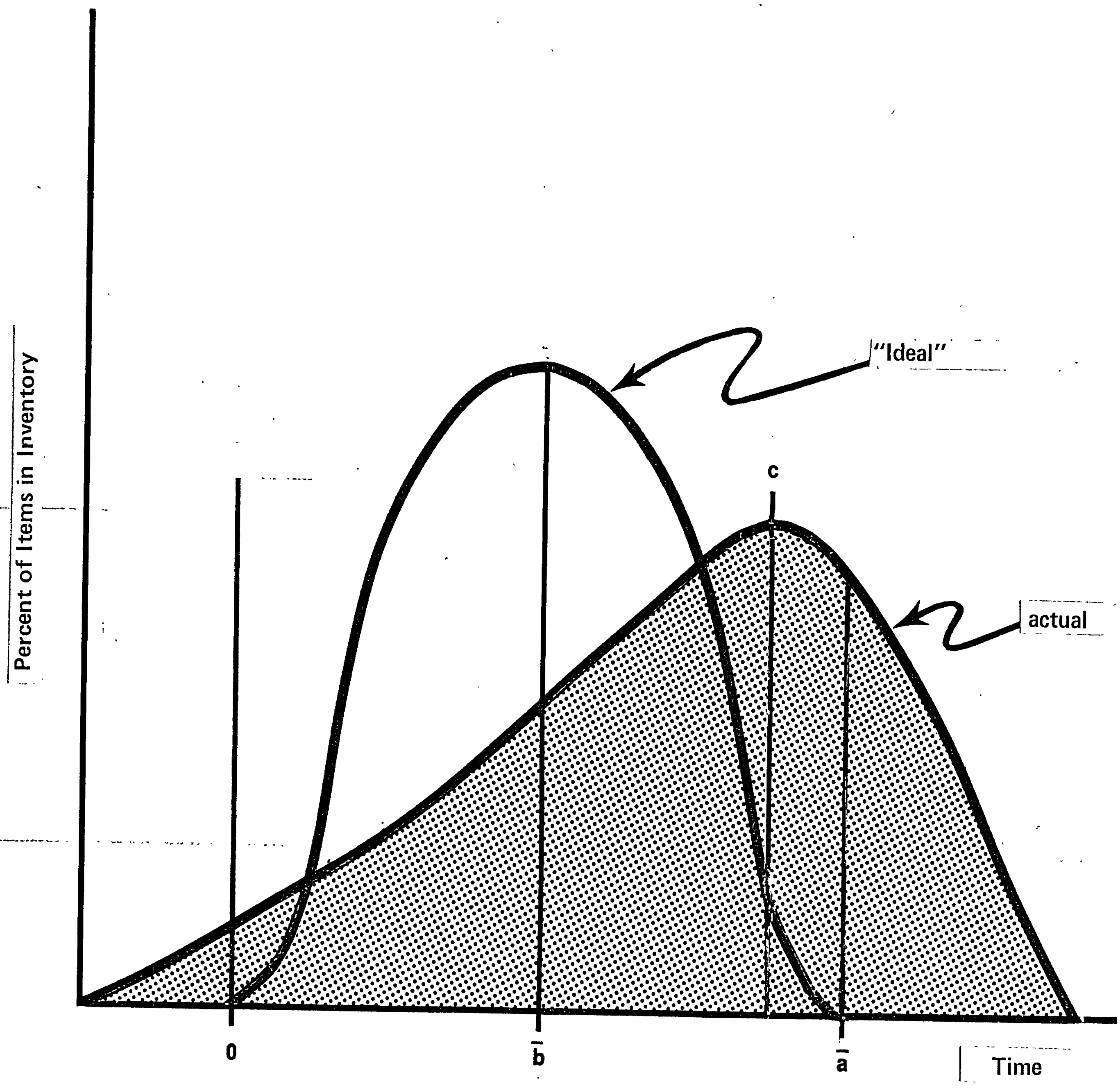


Figure 6 The Actual SAD Superimposed on the "Ideal" SAD

Thus the inventory manager has a method of evaluating the performance of present policies and, if necessary, prescribing for their improvement. The contribution of this paper has been to tie all the pieces of the inventory management task together in such a manner so as to provide an analytic tool for management. This tool builds on familiar concepts, is easy to visualize and to understand intuitively and is readily applicable by managers with modest mathematical skills and limited computer budgets. In addition, the approach allows both an analysis of current practice as well as answers to the "what if" questions which arise when policy change is considered.