

# Extraction of Sugar from Beets

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This paper presents the results of a previous investigation on the continuous counter-current extraction of sugar beets (3) and its correlations. The relationship between the continuous diffusion and the simple diffusion is discussed from the standpoint of extraction rate.

## ANALYSIS OF PROBLEM

The extraction of beet sugar may be resolved into two steps as far as mass transfer is concerned: (1) diffusion of sugar solution through the permeable membrane of beet cells toward the interface between the beets and the extracting solution, and (2) mass transfer of sugar through a liquid film at the interface into the extracting solution. These mechanisms will be discussed separately.

### Diffusion Through Cell Membrane

A theory of fundamental interest regarding the diffusion phenomenon is that of simple molecular diffusion. The derivation of this theory is comprised of two familiar differential equations of the Fick's Law of Diffusion

$$P = -D \frac{\partial c}{\partial x} \quad (1)$$

and

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \quad (2)$$

Equation (1) gives the rate of permeation per unit area of a medium in terms of the diffusion coefficient and the concentration gradient across the medium ( $-\partial c/\partial t$ ). The second equation, referring to an unsteady diffusion operation, expresses the rate of accumulation of mass at a given point in the medium.

Equation (2) may be solved by the method of boundary value problems under unsteady state of flow. The following assumptions and boundary conditions are specified in the derivation of a formal solution.

1. The extracted material is porous but rigid in its structure and is chemically inert.

2. The extracted material has uniform initial solute concentration.

3. The solute concentration in the liquid surrounding the extracted material is constant and uniform.

4. The liquid-film resistance to mass transfer at the interface is negligible.

5. The equilibrium solute concentration in the extracted material is a function of the solute concentration in the extracting liquid.

When one considers a solid slab of finite thickness  $L = 2a$ , the following boundary values exist

$$\begin{aligned} c &= c(t, x) \\ c &= c_1 \\ \text{for } -a \leq x \leq a & \text{ at } t = 0. \\ c &= c_e \\ \text{for } x = -a & \text{ and } x = a \text{ at all } t. \end{aligned}$$

An integrated solution of Equation (2) is obtained as follows (1)

$$\bar{c} = \frac{1}{2a} \int_{-a}^a c \, dx \quad (3)$$

and

$$\begin{aligned} \bar{c} &= c_e + (c_1 - c_e) \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \\ &\cdot \exp \left[ \frac{-D(2m+1)^2 \pi^2 t}{L^2} \right] \quad (4) \end{aligned}$$

Sherwood (9) and Newman (7), by a simple transformation of Equation (4), obtained the following equation:

$$E = \frac{\bar{c} - c_e}{c_1 - c_e} \quad (5)$$

$$\begin{aligned} E &= \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \\ &\cdot \exp \left[ \frac{-D(2m+1)^2 \pi^2 t}{L^2} \right] \quad (5a) \end{aligned}$$

It may be seen that  $E$  is dimensionless and expresses in a sense the fraction of extractable solute in an extracted ma-

terial. The magnitude of  $E$  ranges from unity to zero; thus  $E = 1.0$  for any unextracted material. The value of  $E$  reduces to a fraction during the course of extraction. It can also be seen that the right hand side of Equation (5a) is a rapidly converging series. If  $D$  and  $L$  are constant,  $E$  will nearly be a linear function of the extraction time on a semilogarithmic coordinate. This was confirmed for systems involving solids of uniform, porous structure and negligible liquid-film resistance (2, 6, 8).

### Mass Transfer through Liquid Film

The mass transfer rate through liquid film at the solid-liquid interface may be expressed by the following equation

$$-\frac{d\bar{c}}{dt} = k_l A (c_e - s) \quad (6)$$

where  $(c_e - s)$  is the concentration gradient across the liquid film. Actually for a material such as sugar beets, the term  $c_e$  in Equation (6) should represent the true sugar concentration in the natural beet juice rather than the sugar concentration in the beets. A term  $c_{ei}$  may therefore be used to denote the true equilibrium sugar concentration in the beet juice under diffusion. Equation (6) reduces to

$$-\frac{d\bar{c}}{dt} = k_l A (c_{ei} - s) \quad (7)$$

If the liquid-film resistance is negligible, the mass transfer coefficient should be substantially independent of the relative velocity of extracting liquid, temperature, solid particle size, etc. This may be regarded as the criterion of a diffusion-controlled solid-liquid extraction process.

The extraction of sugar from beets is generally considered a diffusion-controlled phenomenon. If this can be experimentally confirmed, it is then logical to

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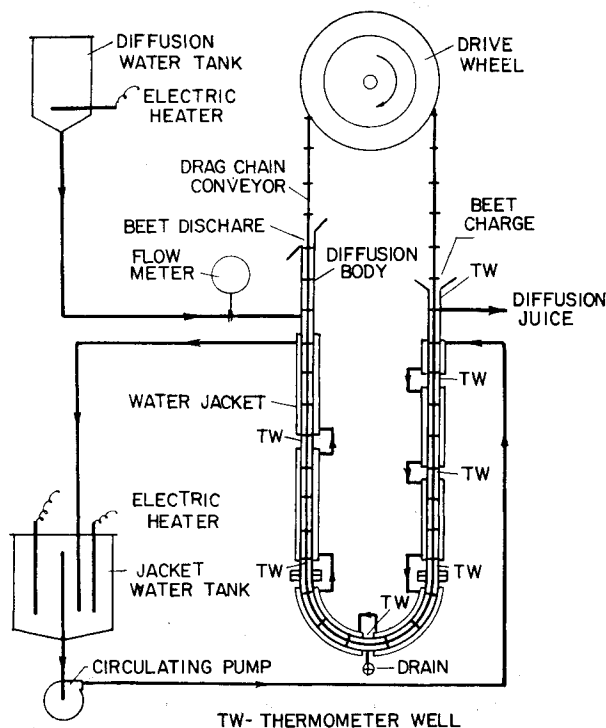


Fig. 1. Flow diagram of continuous diffusion equipment.

correlate the extraction data of beet sugar in terms of  $E$ , as defined by Equation (5), and the extraction time. Such correlation may be made in the following manners.

#### Continuous Countercurrent Diffusion

If the distribution of solute concentration in the extracted solid and in the extracting liquid in the entire diffuser are known, the local values of  $E$  along the diffusion path can be calculated accordingly. Also define the extraction time as the time of traverse for the extracted solid since entering the diffuser. Knowing the speed of the conveyer which transports the extracted solid, therefore, one can determine the distribution of  $E$  in the diffuser as a function of the extraction time. Such an  $E$ - $t$  relationship represents a generalized correlation of mass transfer in a continuous diffuser, since  $E$  is an explicit function of  $c_1$ ,  $c$ , and  $c_e$ .

The rate of change in  $E$  in a continuous diffuser can be obtained by differentiating  $E$  against the extraction time:

$$\frac{\partial E}{\partial t} = \frac{(c_1 - c_e) \frac{\partial \bar{c}}{\partial t} - (c_1 - \bar{c}) \frac{\partial c_e}{\partial t}}{(c_1 - c_e)^2} \quad (8)$$

where  $\partial \bar{c} / \partial t$  represents the rate of change in solute concentration in the extracted solid or the instantaneous extraction rate, and  $\partial c_e / \partial t$  represents the rate of change in equilibrium solute concentration.  $\partial E / \partial t$  may be regarded as the generalized instantaneous extraction rate of a continuous diffusion process.

#### Simple Diffusion

Simple diffusion means that the diffusion process can be carried out under the conditions defined in the derivation of Equations (4) and (5a), particularly item 3. It is actually possible to obtain experimental diffusion data under these conditions. That is a batch of solid may be extracted by a solution with constant solute content. The results thus obtained represent the change in solute concentration in the extracted solid with respect to the diffusion time. Since the solute concentration in the extracting liquid is known, the simple diffusion data can be readily converted into  $E$  as a function of diffusion time.

Differentiation of the  $E$ - $t$  function of simple diffusion also gives the rate of change in  $E$ . Under the conditions of simple diffusion however the equilibrium solute content of the extracted solid is a constant value. We have therefore

$$\frac{\partial E}{\partial t} = \frac{1}{c_1 - c_e} \frac{\partial \bar{c}}{\partial t} \quad (9)$$

The  $E$ - $t$  correlations for the continuous countercurrent diffusion are apparently different from those for the simple diffusion. A continuous diffusion system however may be comprised of infinite number of simple diffusion systems. At each point in a continuous diffuser there may be a simple diffusion system, where the local solute concentration in the extracting liquid is momentarily constant; if so Equation (8) reduces to Equation (9). Thus the local diffusion rate at any point in a continuous diffuser is equal to that of a simple diffusion system under

the same conditions of temperature, solute concentrations, particle size, etc. Therefore the authors postulate that the rate data of simple diffusion may be manipulated to simulate a continuous countercurrent diffusion system. For example the change in  $E$  in a continuous diffuser may be estimated by using the rate data of simple diffusion in the following manner:

$$\Delta E = \int_{t_1}^{t_2} \left( \frac{\partial E}{\partial t} \right) dt \quad (10)$$

Also the required extraction time of a continuous diffusion process may be calculated by

$$\Delta t = \int_{E_1}^{E_2} \frac{1}{\left( \frac{\partial E}{\partial t} \right)} dE \quad (11)$$

In the present investigation Equations (10) and (11) were suggested to interrelate the extraction rates of continuous diffusion and simple diffusion and examine the feasibility of applying the  $E$ - $t$  data to diffusion calculations.

#### EXPERIMENTAL—CONTINUOUS COUNTERCURRENT DIFFUSION

The equipment employed in the present investigation was originally constructed to develop a continuous countercurrent process for the extraction of beet sugar. Figure 1 shows the flow diagram of the diffusion equipment. The diffuser was in the shape of a U tube and was fabricated from a  $\frac{3}{8}$  in. brass sheet. The major part of the diffuser was provided with several sections of hot water jacket. These sections were designed to allow the installation of sampling taps and thermometer wells along the diffusion path. The diffuser body was made square in its cross-section to permit a drag chain conveyer which transported the sugar beets to be extracted. The entire conveyer consisted of a large number of chain units; each was 4 in. in its effective length. Each chain unit was composed of a  $\frac{9}{12}$  in. diam. shaft and a screen plate in its middle. The screen plate was 1-13/32 in. sq. in cross section, providing a total clearance of not more than  $\frac{1}{8}$  in. to the diffuser wall. There were thirty-two evenly spaced perforations of  $\frac{1}{16}$  in. diam. to permit the flow of extracting solution. The extracting solution flowed countercurrent to the sugar beets.

The conveyer was driven at a constant speed of 1.494 in./min. This speed allowed the manual charging of beets into the diffuser. The diffusion water was fed to the diffuser by means of gravity flow. An inverted manometer was available to control the flow rate of diffusion water. The average effective diffusion path was 88.5 in. corresponding to a total diffusion time of 59.25 min.

The experiments of continuous countercurrent diffusion were carried out with the Michigan sugar beets almost exclusively. To imitate commercial operation sugar beets were cut into 0.0383 in. thick,  $\frac{1}{4}$  in. wide, and  $\frac{1}{2}$ -1 in. long V-shaped chips, commercially called the *cosettes*. During

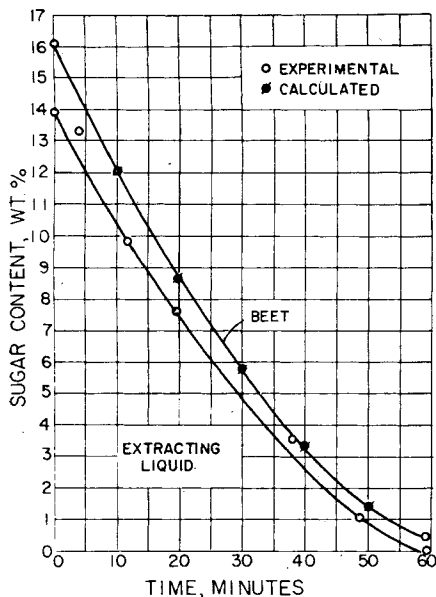


Fig. 2. Typical continuous diffusion curves of Michigan sugar beets.

the experiment cossettes were charged into the diffuser at a rate of 61 g./conveyor unit. This resulted in a diffusion bed about 50% packed, which generally prevails in commercial diffusers. While the cossettes were charged at a constant rate, the diffusion water was fed countercurrently at a relative rate of 1, 1.1, or 1.2 lbs./lb. of fresh cossettes. The diffusion temperature, which was regulated by circulating the jacket water, was kept constant at 65, 70, 75, or 80°C.

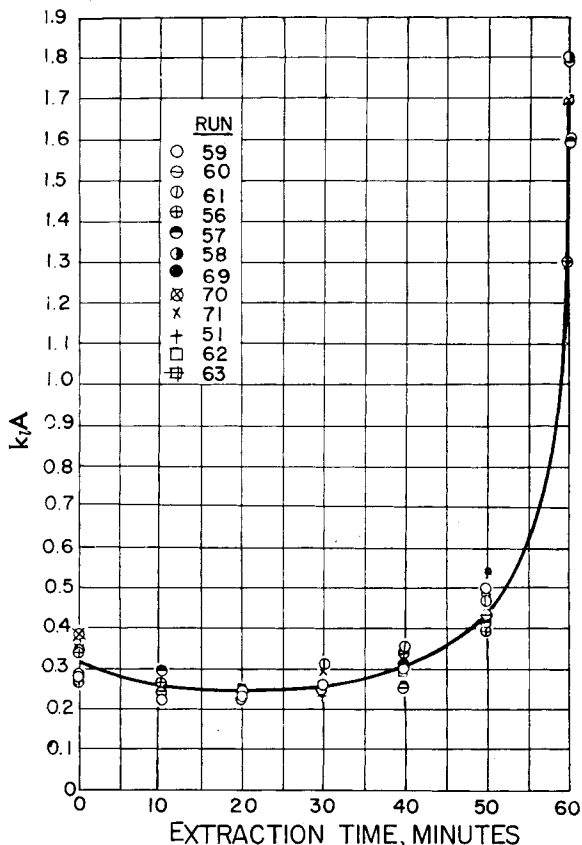


Fig. 4. Correlation of  $k_t A$  in continuous countercurrent diffuser.

At the steady-state of a diffusion test, samples of diffusion juice were withdrawn from the sampling taps along the diffuser. These samples were analyzed for their sugar content by the refractometric method (4). The sugar content of the fresh and exhausted cossettes was determined by the official polarimetric method (4). The sugar content of the fresh and exhausted cossettes was determined by the official polarimetric method (4). The sugar content of the beets is expressed in the weight percentage of the beets rather than in the weight percentage of the natural beet juice. It is known that the weight fraction of inert materials in the beets is generally constant.

A series of twelve experiments was devoted to the study of the continuous diffusion phenomenon. Each experiment was conducted at constant temperature and solvent/feed ratio with cossettes of a uniform size. Table 1 presents the results of these experiments and the data of Runs 5, 7, and 8, made with the California beets.\*

#### CORRELATION OF CONTINUOUS DIFFUSION DATA

In correlating the continuous diffusion data the relative positions of all sampling taps along the diffuser are converted into the extraction time of cossettes by a consideration of the conveyor speed. This establishes the distribution of sugar concentration in the extracting liquid as

\*Tabular material has been deposited as document 5764 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$1.25 for photoprints or \$1.25 for 35-mm. microfilm.

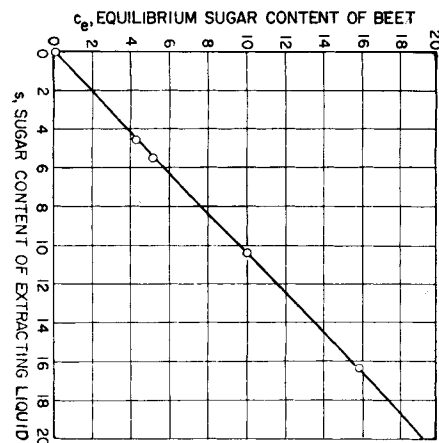


Fig. 3. Equilibrium concentration curve of Michigan sugar beets.

a function of extraction time along the entire diffuser. The sugar content of the cossettes was known at the inlet and outlet of the continuous diffuser only. Its distribution in the diffuser had to be calculated by material balance between the cossettes and the extracting liquid. Figure 2 illustrates a typical concentration-time plot of the continuous countercurrent diffusion experiment, the lower curve representing the distribution of sugar concentration in the diffusion juice throughout the entire diffuser and the upper curve the distribution of sugar concentration in the beets throughout the diffuser. It is plotted with the experimental values of sugar concentration at the inlet and outlet and the calculated sugar concentrations at inter-

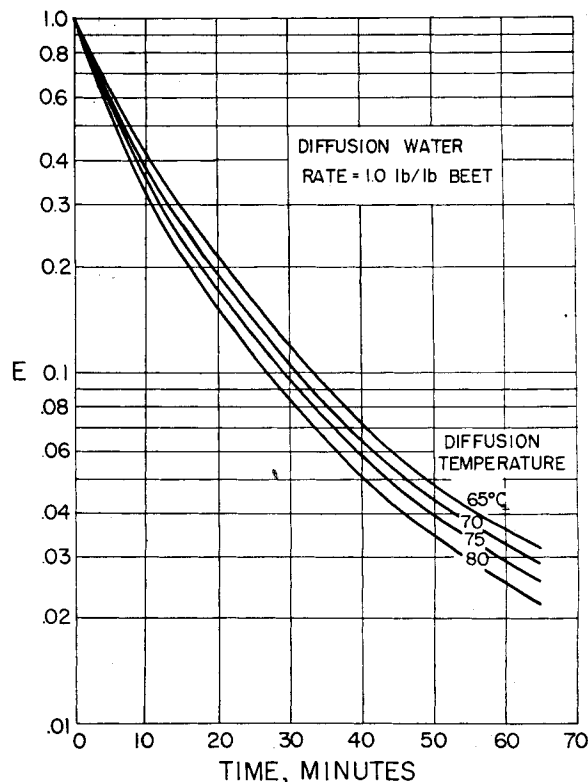


Fig. 5. Generalized plot of continuous diffusion data of Michigan sugar beets.

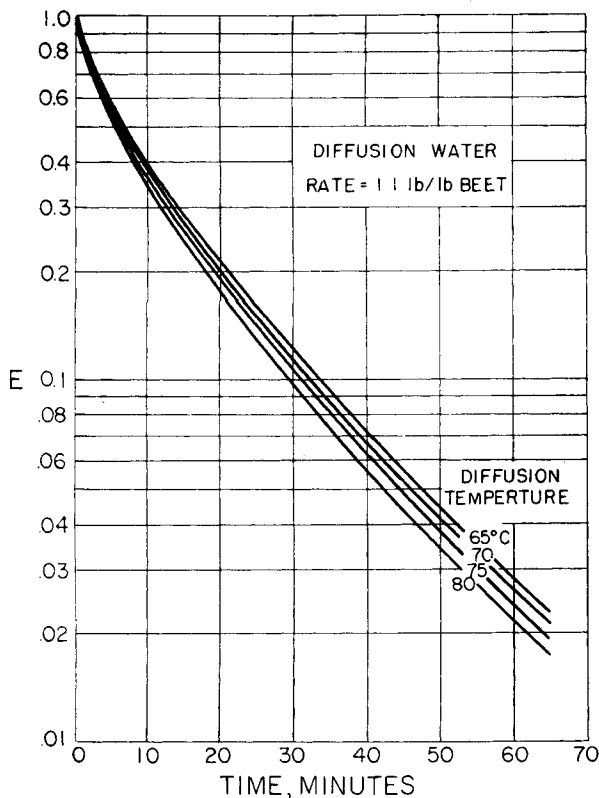


Fig. 6. Generalized plot of continuous diffusion data of Michigan sugar beets.

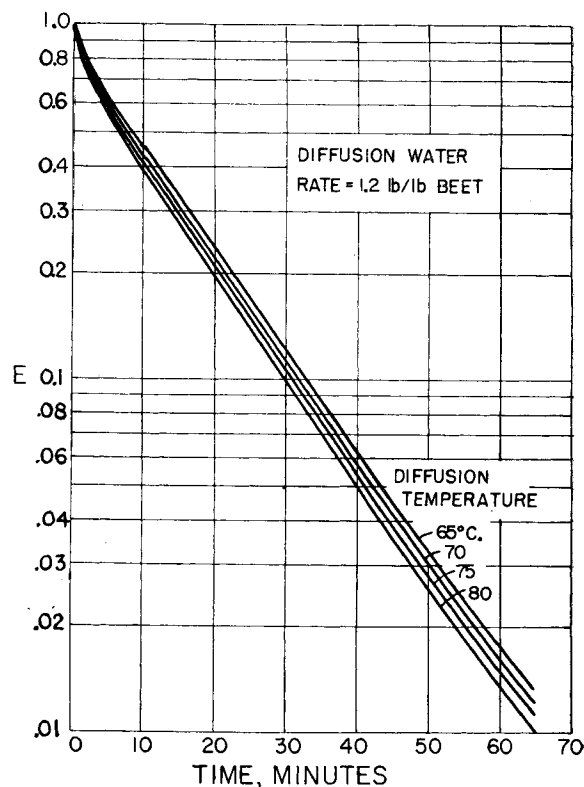


Fig. 7. Generalized plot of continuous diffusion data of Michigan sugar beets.

mediate points of the diffuser. These calculated values of sugar concentration are summarized in the middle part of Table 1 accordingly.

For the purpose of correlation the equilibrium sugar content of Michigan beets was determined experimentally by the simple diffusion method. In Figure 3 the equilibrium sugar content of beets is plotted as a function of the sugar content of the extracting liquid. Applying the sugar concentrations in Table 1, one may readily obtain the corresponding equilibrium sugar concentrations from Figure 3; these values of  $c_e$  are entered in Table 1.

This paper now proceeds to determine whether the extraction of beet sugar is a diffusion-controlled phenomenon by using experimental data. From Equation (7)

$$k_l A = -\frac{d\bar{c}}{dt} \frac{1}{c_e - s} \quad (12)$$

The true equilibrium sugar concentration in the preceding equation can be computed from the experimental values of  $c_e$  in Table 1 according to

$$c_{ei} = \frac{100c_e}{74 + c_e} \quad (13)$$

since the average water content of Michigan sugar beets weighed about 74% (5). The interfacial area ( $A$ ) may be assumed to be constant because the size of cosettes was kept uniform and constant in this investigation. The term  $k_l A$  is therefore indicative of the liquid-film transfer coefficient.

The values of  $k_l A$  calculated according to Equation (12) are plotted against the extraction time in the continuous counter-current diffuser in Figure 4. Almost all experiments tabulated in Table 1 are included in this plot. It is seen that the distribution of  $k_l A$  in the continuous diffuser is not affected by the extraction temperatures and diffusion water rates employed in various runs. Therefore the liquid-film resistance is negligible, and

the extraction of beet sugar is predominantly a diffusion-controlled process.

On the basis of this conclusion the continuous countercurrent diffusion data may be advantageously correlated in terms of the function  $E$  and the extraction time. Thus Equation (5) is applied to evaluate the distribution of  $E$  in the continuous diffuser.

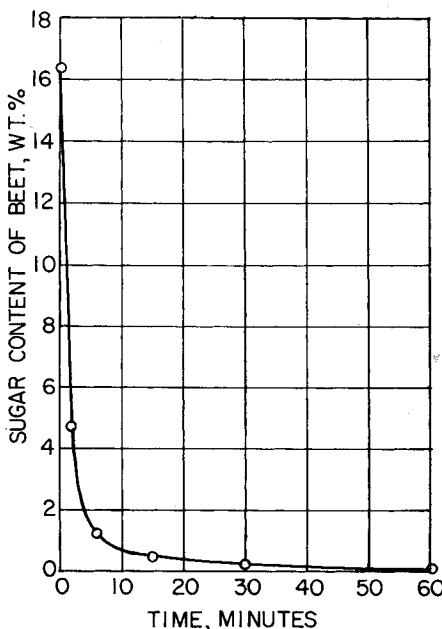


Fig. 8. Typical simple diffusion curve of Michigan sugar beets (Run 67).

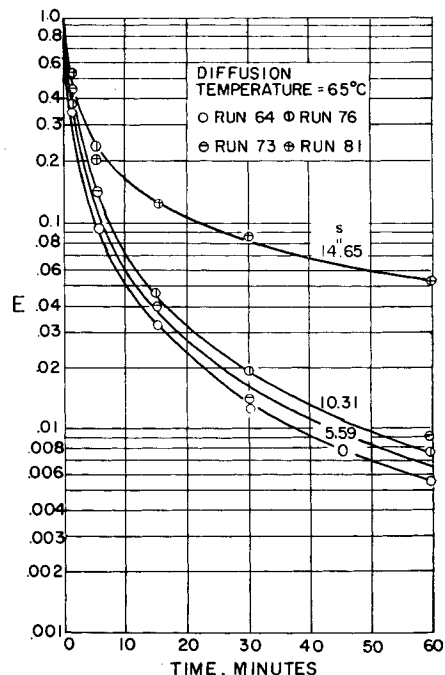


Fig. 9. Correlation of simple diffusion data of Michigan sugar beets.

For example in Table 1, under Run 51,

$$c_1 = 16.65$$

$$\bar{c} = 13.32 \text{ at 10 min. of extraction}$$

$$s = 11.39 \text{ at 10 min. of extraction}$$

From Figure 3 one obtains

$$c_e = 10.90$$

Hence,

$$E = \frac{13.32 - 10.90}{16.65 - 10.90} = 0.421$$

This value is entered in Table 1 along with the other calculated  $E$  values.

The calculated values of  $E$  from Table 1 are plotted in Figures 5, 6, and 7 against the extraction time, with the water/beet ratio and diffusion temperature used as the parameters; these experiments were carried out with cosettes of a uniform size. These correlations were examined with the experimental data of twenty-three continuous diffusion tests (10). The deviation between the calculated and experimental data was found to be about  $\pm 5\%$ . One may conclude that the correlations presented in Figures 5 to 7 have a generalized nature.

#### EXPERIMENTAL—SIMPLE DIFFUSION

The apparatus employed in the study of simple diffusion consisted of a 1 liter beaker with a fitted detachable screen batch. The cosettes to be extracted were placed in the screen batch and packed to roughly the same degree as in the continuous diffuser. Also cosettes of the same size as those employed in the continuous diffusion tests were employed. A sugar solution having a constant sugar concentration was then introduced into the batch through several nozzles and continually overflowed. In a series of simple diffusion experiments cosettes from the same stock were extracted in several batches separately for 2, 6, 15, 30, and 60 min. with a sugar solution of the

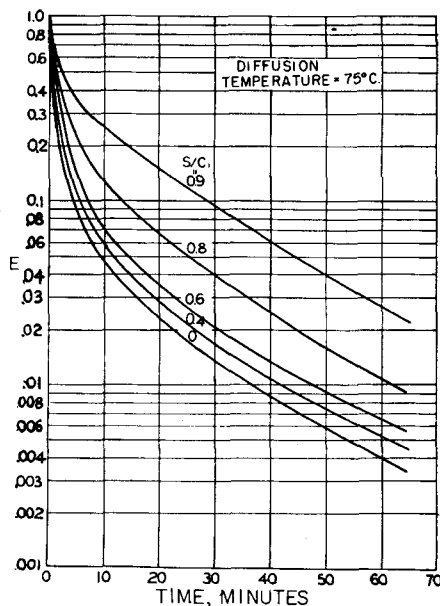


Fig. 11. Generalized plot of simple diffusion data of Michigan sugar beets.

same strength. The extracted batches were then analyzed for their sugar content.

Table 2\* presents the operating conditions and the concentration-time data of the simple diffusion experiments. The sugar content of the fresh cosettes of each run is referred to zero time of extraction, while the equilibrium sugar content relative to the diffusion liquid is referred to infinite time of extraction. A typical plot of the concentration and time data is shown in Figure 8.

A special series of simple diffusion experiments was conducted to determine the equilibrium sugar content of Michigan sugar beets. Each diffusion batch was extracted by a solution of constant sugar content for a period of more than 24 hr. The residual sugar content of the beets was then regarded as the equilibrium sugar

\*See footnote on page 455.

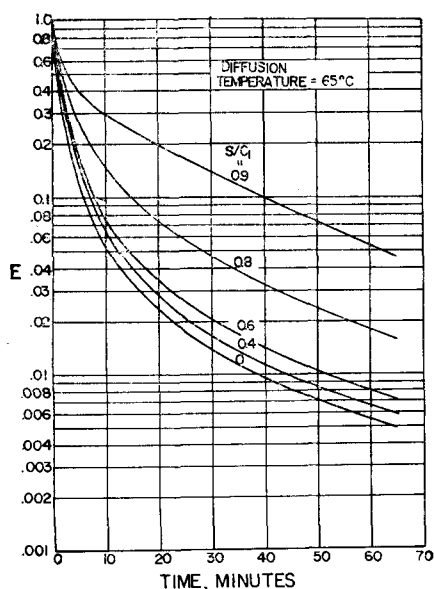


Fig. 10. Generalized plot of simple diffusion data of Michigan sugar beets.

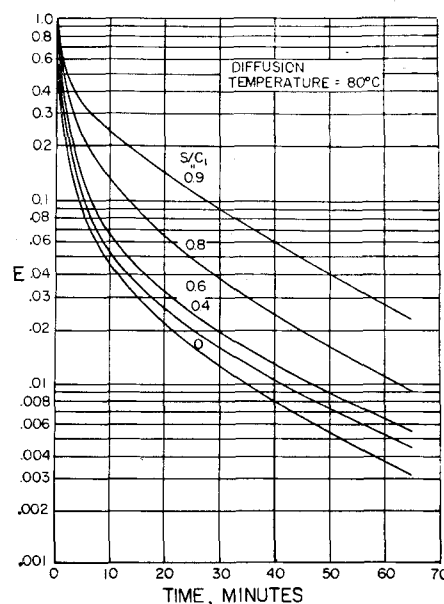


Fig. 12. Generalized plot of simple diffusion data of Michigan sugar beets.

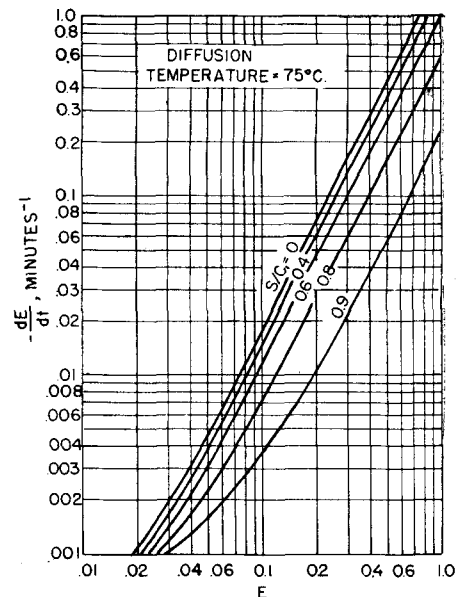


Fig. 13. Generalized plot of instantaneous diffusion rate of Michigan sugar beets.

content. Results of these tests were employed to construct the equilibrium curve in Figure 3.

#### CORRELATION OF SIMPLE DIFFUSION DATA

The correlation of simple diffusion data includes the  $E-t$  relationship and the generalization of simple diffusion rate. Since the sugar concentration in the extracting solution was constant, the equilibrium sugar concentration in the beets in each run was constant. This makes the calculation of  $E$  a rather simple matter.

For example in Table 2, under Run 65

$$c_1 = 18.25$$

$$s = 0$$

$$c_e = 0.04$$

At 6 min. of extraction,

$$\bar{c} = 2.34$$

Therefore

$$E = \frac{2.34 - 0.04}{18.25 - 0.04} = 0.127$$

The calculated values of  $E$  are presented in the lower part of Table 2. These data, when plotted against diffusion time as in Figure 9, scatter around widely. A family of curves are drawn according to the constant sugar concentration in the extracting liquid. This is different from King's correlation (6), where a single  $E-t$  curve is plotted. The plot in Figure 9 indicates that the rate of change in  $E$  decreases with increasing sugar concentration in the extracting solution. One would question why the dimensionless number  $E$ , being a function of the equilibrium sugar concentration and others, is still affected by the sugar

concentration. The following discussion may perhaps be offered.

According to the simple diffusion theory the diffusion coefficient is constant at constant temperature. The value of  $E$  as given in Equation (5a) is a unique function of diffusion time at constant diffusion temperature and constant thickness of the extracted solid slab. In the nonidealized diffusion, such as the extraction of beet sugar, the diffusion coefficient is no longer a constant owing to the nonuniform structure of beet cells. Therefore such a system deviates from Equation (5a) not only in that the  $E-t$  plot is curvilinear but in that  $E$  is dependent on the sugar concentration ( $s$ ) in the extracting liquid.

The value of  $E$  can be affected by the solute concentration in the extracting liquid under certain circumstances. Thus  $E$  will remain at unity in a diffusion system, if the sugar concentration is such that the corresponding equilibrium sugar concentration equals the initial sugar concentration in the natural beet juice. Under such condition of course the net mass transfer by diffusion should be zero. It may be shown mathematically that

$$E = \lim_{\substack{\bar{c} \rightarrow c_1 \\ c_e \rightarrow c_1}} \frac{\bar{c} - c_e}{c_1 - c_e} = \frac{\partial}{\partial c_e} (\bar{c} - c_e) / \frac{\partial}{\partial c_e} (c_1 - c_e) = 1$$

For a nonideal system therefore the effect of solute concentration on  $E$  is pronounced appreciably at a high concentration level (Figure 9).

The diffusion of beet sugar is complicated by the fact that up to 18% of the beet cells are ruptured mechanically in the preparation of cossettes. There is always considerable washing effect in such a diffusion process. This is indicated by the abrupt concentration change of the simple diffusion curve in Figure 9. The diffusion mechanism is further complicated by the phenomenon of fluid flow through a packed bed in the diffuser.

To generalize the correlation as suggested in Figure 9 it is necessary to employ a dimensionless parameter instead of the sugar concentration. The concentration ratio of  $s/c_1$  is therefore introduced for this purpose. The significance of using  $c_1$  here can be derived from the argument that  $E = 1$  at all time when  $c_e$  approaches  $c_1$ . With reference to Figure 3 this prevails when  $s/c_1 = s/c_e = 1.04$  to 1.06 in the high concentration range. If the experimental data are plotted as  $E$  vs.  $s/c_1$  with the extraction time used as the parameter, all curves should converge at  $E = 1$  and  $s/c_1 = 1.04$  to 1.06. It is then possible to extrapolate the experimental data and obtain the  $E-t$  functions at even values of  $s/c_1$ . Such correlations are shown in Figures 10, 11, and 12 for diffusion tests at 65, 75, and 80°C., respectively.

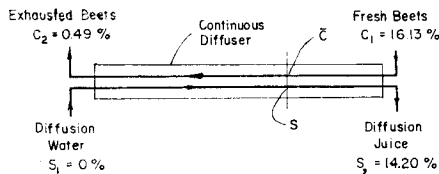


Fig. 14. Terminal conditions of example.

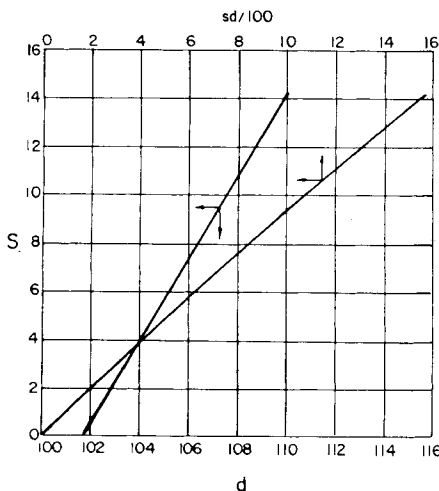


Fig. 15. Example, estimation of distribution of diffusion liquid in a continuous countercurrent diffuser.

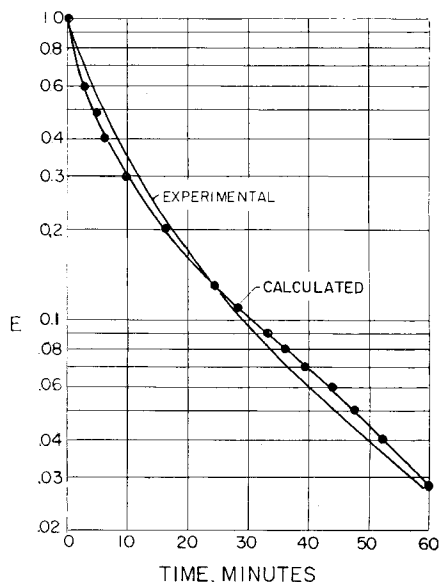


Fig. 16. Example, comparison of experimental and calculated continuous diffusion data.

These simple diffusion curves may be differentiated readily to yield the generalized diffusion rate ( $\partial E/\partial t$ ). Figure 13 shows a plot of  $-\partial E/\partial t$  vs.  $E$  for simple diffusion at 75°C.

#### SIMPLE DIFFUSION AND CONTINUOUS DIFFUSION

As has been suggested the rate data of simple diffusion may be integrated

according to Equation (10) or (11) to simulate a continuous diffusion process. The experimental data of both simple diffusion and continuous countercurrent diffusion can now be employed to verify such a postulation. The following example illustrates the principles of mathematical manipulation and provides an evidence of verification:

#### EXAMPLE

Michigan sugar beets are to be processed in a continuous countercurrent diffuser as shown in Figure 14. The cossettes are 0.0383 in. thick and 1/2 to 1 in. long, so that the correlation in Figure 13 can be applied. The required time of extraction is estimated under the following operating conditions.

- Diffusion temperature = 75°C.
- Diffusion water rate = 1.017 lb. water/lb. fresh beets
- $c_1 = 16.13$  lb. sugar/100 lb. wet beets
- $c_2 = 0.49$  lb. sugar/100 lb. wet beets
- $s_1 = 0$
- $s_2 = 14.20$  lb. sugar/100 lb. extracting solution
- Diffusion juice produced = 1.102 lb./lb. fresh beets

The value of diffusion juice produced often deviates from what it should be according to the material balance of diffusion water rate and amount of sugar extracted. This is assumed to result from the counterdiffusion of water into the beet cells. No correlation regarding the weight loss of the exhausted beets is available.

#### SOLUTION

To simulate the continuous countercurrent diffusion operation it is necessary to establish first the distribution of  $E$  along the entire diffuser. The relative amount of extracting liquid is assumed to increase linearly with its sugar content. From the given conditions the following terminal figures are known.

$$\begin{aligned} s_1 &= 0, & d_1 &= 101.7; \\ s_2 &= 14.20, & d_2 &= 110.2. \end{aligned}$$

A straight line may then be drawn through these values on a  $s-d$  plot in Figure 15.

It can be seen that the value of  $sd/100$  represents the amount of sugar in the extracting liquid in pounds per 100 lb. of fresh beets. The  $s-d$  plot in Figure 15 enables one to calculate  $sd/100$  as a function of  $s$ , which is plotted in the same figure. If the sugar concentrations at two points in the diffuser are known, the corresponding values of  $sd/100$  can be obtained readily from this graph. The difference between two local values of  $sd/100$  will then represent the amount of sugar extracted between the same points.

At a certain point near the inlet of fresh cossettes for instance, let the average sugar content of the beets be 14.0%. The amount of sugar already extracted from the fresh beets since entering the diffuser is approximately equal to

$$16.13 - 14.0 = 2.13 \text{ lb./100 lb. fresh beets.}$$

This material balance is made without converting the concentrations into solute-free basis. It is true only if the beets did not change in weight at a small amount of extraction, and if the counterdiffusion of water equals that of sugar juice. Since

$$s_2 d_2 / 100 = (110.2)(14.20) / 100 = 15.54$$

Hence at the point under consideration

$$\begin{aligned} sd/100 &= s_2 d_2 / 100 - \Delta(sd/100) \\ &= 15.54 - 2.13 \\ &= 13.41 \end{aligned}$$

From the  $s$ - $d$  plot in Figure 15 therefore

$$s = 12.30 \text{ at this point.}$$

From Figure 3  $c_e = 11.55$ . Consequently at this point

$$E = \frac{14.0 - 11.55}{16.13 - 11.55} = 0.535$$

The diffusion rate  $\partial E / \partial t$  at this point may then be obtained in the following manner. Since

$$s/\bar{c} = \frac{12.30}{14.0} = 0.878$$

From Figure 13 at  $E = 0.535$

$$-\frac{\partial E}{\partial t} = 0.075 \text{ min.}^{-1}$$

This procedure is followed by small decrements in the sugar concentration in the remaining part of the diffuser. The distribution of  $E$  and  $\partial E / \partial t$  in the entire diffuser may thus be established. Table 3\* summarizes the results of such evaluation.

At the end of the diffusion process

$$E_2 = \frac{0.49 - 0.04}{16.13 - 0.04} = 0.028$$

The total extraction time may therefore be estimated by applying Equation (11) as

$$\Delta t = \int_{0.028}^1 \left[ \frac{1}{-\frac{\partial E}{\partial t}} \right] dE$$

Graphical integration of this expression was carried out and tabulated in the last column of Table 3. The calculated extraction time was found to be 59.90 min. against an experimental value of 59.25 min. as required for Run 69 (Table 1). The calculated values of  $E$  are plotted against the extraction time in Figure 16 and compared with the experimental  $E$ - $t$  curve transposed from Figure 5. The peculiar shape of the calculated curve is probably owing to the use of sugar concentrations on the wet basis. However the agreement between the calculated and experimental curves is satisfactory.

#### CONCLUSION

1. The extraction of sugar beets is a diffusion-controlled process. As is ex-

pected the liquid-film mass transfer coefficient of the beet-water system is independent of the diffusion temperature, the extracting liquid rate, etc.

2. The continuous diffusion data may be correlated in the form of  $E$  vs. the extraction time by using the diffusion temperature, the extracting liquid rate, etc. as the parameters. Such a correlation, as illustrated in Figures 5 to 7, offers some merits on account of its generalized nature. It is questionable however if this type of correlation would be suitable for the solid-liquid leaching process, where the liquid-film resistance has considerable influence on the extraction rate.

3. The simple diffusion experiments proved to be a valuable technique in the study of solid-liquid diffusion phenomena. This method can be equally applied to determine the effects of alternate variables such as the particle size of the extracted solid and the packing effect of the extracted bed.

4. The  $E$ - $t$  correlation of simple diffusion data of the beet-water system reveals the influence of sugar concentration on the extracting liquid. The rate of change in  $E$  decreases with increasing sugar concentration in the extracting liquid. This effect is considerably pronounced at high sugar concentration close to the sugar concentration in the fresh beets. A dimensionless concentration ratio of  $s/c_1$  is introduced as a parameter in the  $E$ - $t$  correlation.

5. The relationship between the simple diffusion and the continuous diffusion was studied. On the basis of the verification of Equation (11) by numerical calculation in the preceding example, it is feasible to assume that a continuous diffusion system consists of an infinite number of simple diffusion systems operated under the same local conditions. The generalized simple diffusion data may therefore be manipulated to simulate a continuous diffusion system in process or scale up calculations.

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#### NOTATION

$A$  = solid-liquid interfacial area for mass transfer, sq. ft.  
 $a$  = one half thickness of the extracted solid slab, ft.  
 $c$  = solute concentration in the extracted solid or sugar concentration in the beets, lb./100 lb. of wet beets

$c_1$  = initial solute concentration in the extracted solid  
 $c_2$  = final solute concentration in the extracted solid  
 $c_e$  = equilibrium solute concentration in the extracted solid  
 $c_{ei}$  = true equilibrium sugar concentration in the natural beet juice, lb. of sugar/100 lb. of beet juice  
 $\bar{c}$  = average solute concentration in the extracted solid slab as defined by Equation (3)  
 $d$  = relative rate of the extracting liquid, lb./100 lb. of the extracted solid or the fresh beets  
 $d_1$  = relative rate of the extracting liquid when entering a diffuser  
 $d_2$  = relative rate of the extracting liquid when leaving a diffuser  
 $D$  = diffusion coefficient, sq. ft./min.  
 $E$  = fraction of extractable solute of an extracted solid as defined by Equation (5)  
 $k_L$  = liquid-film mass transfer coefficient, lb. of sugar extracted/100 lb. of wet beets/sq. ft./min.  
 $L$  = wet thickness of the extracted solid slab, ft.  
 $m$  = positive integer  
 $s$  = solute concentration in the extracting liquid or sugar concentration in the extracting sugar solution, lb./100 lb. of extracting liquid  
 $s_1$  = initial solute concentration in the extracting liquid  
 $s_2$  = final solute concentration in the extracting liquid  
 $P$  = rate of permeation in a diffusion process, lb./sq. ft./min.  
 $t$  = time of extraction, min.  
 $x$  = distance of diffusion within the solid slab being extracted, ft.

#### LITERATURE CITED

- Barrer, R. M., "Diffusion In and Through Solids," Macmillan, Cambridge (1941).
- Boucher, D. F., J. C. Brier, and J. O. Osburn, *Trans. Am. Inst. Chem. Engrs.* **38**, 967 (1942).
- Brier, J. C., and H. H. Yang, "A Study on the Continuous Countercurrent Diffusion of Beet Sugar," Univ. Mich., *Dept. Eng. Research, Eng. Research Bull.* No. 2322 (July 1955).
- Browne, C. A., and F. W. Zerban, "Physical and Chemical Methods of Sugar Analysis," 3rd ed., John Wiley and Sons, New York (1941).
- Howell, J. L., Michigan Sugar Company, Saginaw, Michigan, private communication.
- King, C. O., D. L. Katz, and J. C. Brier, *Trans. Am. Inst. Chem. Engrs.*, **40**, 5, 533 (1944).
- Newman, A. B., *Trans. Am. Inst. Chem. Engrs.*, **27**, 310 (1931).
- Osburn, J. O., and D. L. Katz, *Trans. Am. Inst. Chem. Engrs.*, **40**, 5, 511 (1944).
- Sherwood, T. K., *Ind. Eng. Chem.* **21**, 12 (1929).

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