Unsteady State Gas-Liquid Slug Flow Through Vertical Pipe

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Macroscopic mass and momentum balances have been used to predict the unsteady behavior of gas-liquid slug flow through vertical pipe. Equations have been developed to predict the time-dependent pressure drop along the pipe as well as the time-averaged pressure drop. The results of the analysis indicate that the gas bubble lengths and frequencies of generation must be known before the pressure drop can be predicted as a function of time. In this work the bubble generation was specified in two ways: by experimental measurement of the bubble lengths and frequencies of generation from an optical technique developed for this purpose, and by assuming a periodic generation of gas bubbles of uniform length. The comparisons between predicted and measured pressure drops (both time dependent and time averaged) are satisfactory and appear to support the theoretical model. An extension of the results obtained in light of the published information on solid-gas and immiscible liquid-liquid systems in slug flow permits the introduction of generalized concepts of slug flow behavior in two-phase systems.

In a large number of engineering operations where vertical, cocurrent, two-phase flow occurs, a particular flow regime termed *slug flow* is observed. The slug flow regime is characterized by the consecutive passage of alternate slugs of liquid and bullet shaped gas bubbles up through the pipe. In contrast to other steady state flow regimes reported in the literature such as annular, bubble, froth, and mist flow, slug flow is characterized by time-dependent pressure drops and phase holdups. The objective of the study reported in this paper is to predict the unsteady state behavior of slug flow.

The literature on two-phase flow is voluminous. Comprehensive literature surveys (*1, 2, 3, 4*) are available covering many facets of two-phase flow phenomena. In the specific area of two-phase slug flow Dumitrescu (*5*) and Davies and Taylor (*6*) have derived analogous expressions for the velocity with which a wakeless bullet shaped bubble rises through a column of stagnant liquid:

$$v_B = C\sqrt{gD} \tag{1}$$

where $C = 0.35$ (Dumitrescu) and $C = 0.33$ (Davies and Taylor). Griffith and Wallis (*7*), Laird and Chisholm (*8*), and Nicklin et al. (*9*) have shown that this expression also predicts the bubble rise velocity for bubbles followed by a wake. Their studies indicated only a slight dependence of bubble-rise velocity on bubble length.

Griffith and Wallis (*7*) have measured and predicted time-averaged pressure drops for the air-water system in ½-, ¾-, and 1-in. pipes. In addition, they have correlated bubble-rise velocities when a continuous upward flow of gas and liquid exists in the pipe by means of the equation

$$v_B = (v_G + v_L) + 0.35\, C_1\sqrt{gD} \tag{2}$$

where C_1 was determined experimentally and found to be a strong function of the water flow rate.

Nicklin et al. (*9*) have also measured and predicted time-averaged pressure drops for the air-water system in a 1-in. pipe. They have similarly determined a correlation for bubble-rise velocities in the form

$$v_B = C_2(v_G + v_L) + 0.35\sqrt{gD} \tag{3}$$

where $C_2 = 1.2$ for the air-water system and is insensitive to the flow rate of either fluid. The expression proposed by Nicklin et al, is more realistic because of the insensitivity of their coefficient to fluid flow rates. Their expression was found to correlate the data taken in connection with this study.

Moissis and Griffith (*10*) have studied entrance effects in two-phase slug flow. They define the concept of fully developed slug flow which exists when the separation distance between any two gas bubbles is large enough so that all bubbles have smoothly rounded heads and rise with a uniform velocity. In addition they have described conditions under which bubbles accelerate and coalesce. Developing slug flow exists obviously at entrance regions where gas and liquid are injected into a pipe. These investigators have experimentally analyzed entrance effects in two-phase slug flow and shown that in the entrance regions the time-averaged pressure drop may be considerably higher than that in the upper regions of the pipe where fully developed flow is assumed to exist.

THEORETICAL ANALYSIS

Let a segment of pipe of length L be taken as the system under consideration. This system and the nomenclature used to describe the gas bubbles and liquid slugs flowing through it are shown in Figure 1. In drawing mass and momentum balances over this macroscopic system the following assumptions are made:

1. The liquid slugs travel at a superficial velocity $(v_G + v_L)$ corresponding to the total volumetric flow rate of fluids.
2. The gas bubbles rise at a velocity $v_B = v_G + v_L + w$. Thus w is the velocity with which the gas bubbles travel with respect to the liquid slugs.
3. The liquid film flowing around the gas bubbles can be assigned a superficial velocity $v^{(f)}$ which is approximated by setting it equal to the bubble-rise velocity. The liquid film is essentially a region of vertically downward flow.
4. The regions of the pipe occupied by the bullet shaped gas bubbles and surrounding liquid film are essentially regions of constant pressure. This assumption is discussed and analyzed by Street and Tek (*11*).
5. The coalescence of the small spherical bubbles into large bullet shaped bubbles is negligible. This assumption

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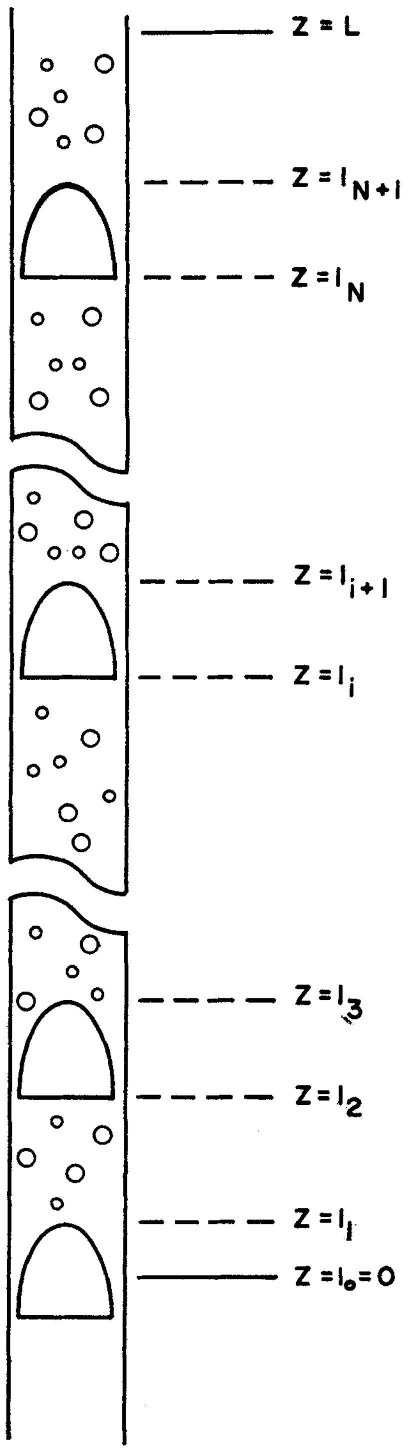

Fig. 1. Macroscopic model of a slugging gas-liquid system.

seems reasonable in the well-developed slug flow regions. It is not valid in the entrance regions of the pipe where the bullet shaped bubbles are formed.

6. The wall shear stress acting on the liquid slugs can be estimated by correlations for a single-phase liquid travelling at a superficial velocity equal to $v_G + v_L$.

7. The effects of the bulk acceleration of the gas bubbles are negligible. The actual terms in the momentum balance which arise when bulk acceleration occurs are derived and discussed in (*12*).

Requirements of brevity preclude complete derivations of the mass and momentum balances. These are presented

Table 1. Range of Independent Variables Covered in the Experimental Work

System: air-water

Pipe diameter	1⅛ in.	2¼ in.
Gas flow rate, v_G	4.0-85 cm./sec.	5.0-88 cm./sec.
Liquid flow rate, v_L	0-52.4 cm./sec.	0-29 cm./sec.
Inlet pressure	2.5-10 lb./sq. in. gauge	3.5-10 lb./sq. in. gauge
Column temperature	approximately ambient	approximately ambient
Reynolds number*	1,140-39,300	2,860-67,000

* Reynolds number $= \dfrac{D(v_G + v_L)\rho_L}{\mu}$.

elsewhere (*12*). Rather, the results are presented in the following form:

$$\alpha = \frac{v_G}{v_G + v_L} - \frac{\overline{l_G}}{l_L}\,\frac{v_L + w/2}{v_G + v_L + w} \tag{4}$$

$$-\Delta P_D(t) = (1-\alpha)\,\frac{L_l(t)}{L} + \frac{4\tau_w}{D\rho_L g}\,\frac{L_l(t)}{L} \tag{5}$$

and

$$-\overline{\Delta P_D} = (1-\alpha)\,\frac{\overline{L_l}}{L} + \frac{4\tau_w}{D\rho_L g}\,\frac{\overline{L_l}}{L} \tag{6}$$

where

$$\Delta P_D \equiv \Delta P/\rho_L L g \tag{7}$$

If $L_{GS}(t)$ is the sum of the lengths of the gas bubbles which have entered the segment L of pipe at time t, then $L_{GS}\,(t - L/v_B)$ is the total length of gas bubbles which have left the system at time t. Thus the length of gas bubbles in the segment L is given by

$$L_g(t) = L_{GS}(t) - L_{GS}(t - L/v_B) \tag{8}$$

and the length of liquid slugs is given by

$$L_l(t) = L - L_g(t) \tag{9}$$

where reference to Figure 1 shows in addition that

$$L_l(t) = (L - l_{N+1}) + \sum_{i=1}^{N/2} (l_{2i} - l_{2i-1}) \tag{10}$$

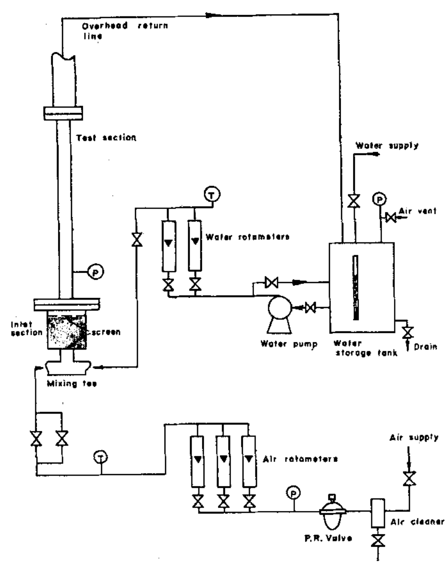

Fig. 2. Apparatus for the experimental investigation of gas-liquid slug flow.

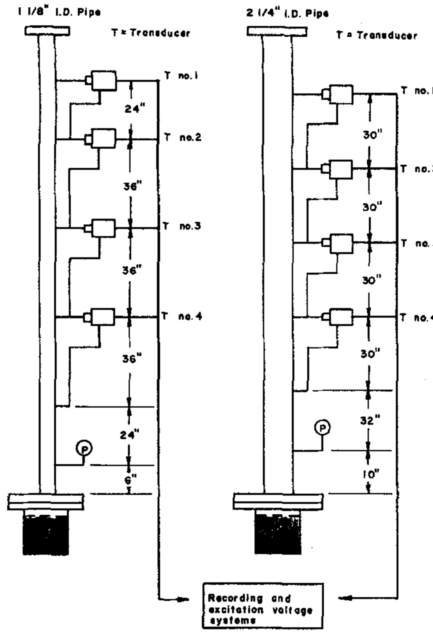

Fig. 3. Transducer arrangement and location on the test section.

and

$$L_g(t) = (l_1 - l_0) + \sum_{i=1}^{N/2} (l_{2i+1} - l_{2i}) \tag{11}$$

In many instances the gas content of the liquid slugs is negligible which implies that $\alpha \to 0$. In this case (4) may be reduced to

$$\overline{l_G} = \overline{l_L}\,\frac{v_G}{(v_G + v_L)}\,\frac{(v_G + v_L + w)}{(v_L + w/2)} \tag{12}$$

Thus for slugging systems in which α is small, the gas bubble lengths can be predicted from a knowledge of the liquid slug lengths.

For the case in which gas bubbles are passed up through a stagnant column of liquid, $v_L = 0$ and (4) reduces to

$$\alpha = 1 - \frac{\overline{l_G}}{\overline{l_L}}\,\frac{w/2}{v_G + w} \tag{13}$$

and as $v_G \to 0$, $\overline{l_G} \to 0$ and $\alpha \to 1$. This result does not agree with the actual physical situation, for as v_G goes to zero, a stagnant column of liquid remains, and α should approach zero. Thus the derivation does not extend in the limit to zero liquid and gas flows. This is due to the approximate value of $v^{(f)}$ used. It can be shown that this approximation is tantamount to setting w equal to a value of the order of $(v_G + v_L)$. However, as v_G and v_L approach zero, it is known experimentally that w approaches $C\sqrt{gD}$. Nevertheless, the values of α predicted by (13) appear to be valid over the range of gas and liquid flow rates covered in the experimental work with the exception of the lowest liquid rates (<8 cm./sec.) in the 2¼-in. pipe.

The prediction of pressure drop as a function of time reduces therefore to the prediction of the bubble and slug input to the pipe segment L. The bubble and slug input may be interpreted as a mathematical boundary condition. This input corresponds to the specification of the density as a function of time at the point $z = 0$ in Figure 1.

The problem of predicting bubble and slug input, that is, predicting the boundary condition at $z = 0$, is a difficult one. The actual formation of gas bubbles depends upon the coalescence of gas at the inlet region where the gas and liquid are initially mixed. Thus the bubble and slug lengths are functions of the particular geometry of the inlet region. In this work a packed section was placed at the inlet to the column. Up to this time no analytical methods exist for the a priori prediction of bubble and slug lengths.

In this work two methods were used to determine the bubble and slug input to the pipe segment L. The first method was experimental in that a light source—photocell system was used to actually record the bubble and slug input to the system. The second method was an approximate, empirical method whereby values of $\overline{l_G}$ and $\overline{l_L}$ were determined by the same experimental method and correlated vs. v_G and v_L for each of the two pipes studied. These average values were then used as input data. This periodic type of boundary condition can be regarded as an idealized boundary condition.

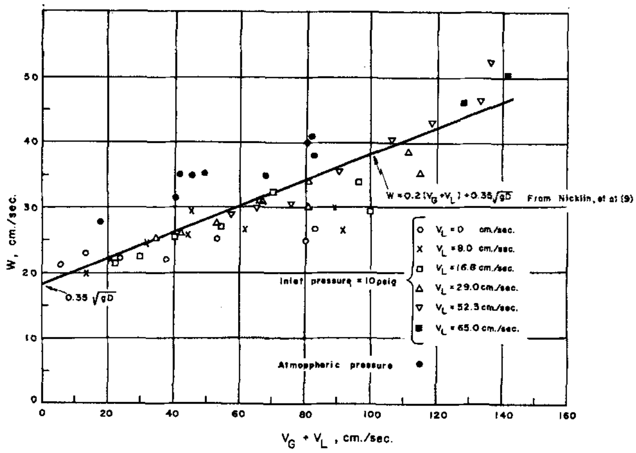

Fig. 4. The velocity of air bubbles through the surrounding water as a function of $(v_G + v_L)$ in a 1⅛-in. I.D. pipe.

EXPERIMENTAL WORK

Experimental measurements have been taken to determine pressure drop, gas bubble-rise velocity, liquid slug lengths, and gas bubble lengths. Two sets of data were taken for a given specification of the independent variables, namely gas-liquid system properties, pipe diameter, gas and liquid flow rates, and inlet pressure and temperature to the column. The first set of data utilized four differential pressure transducers which measured a series of pressure drops over the column as functions of time. The second set of data utilized a light source-photocell system which recorded the light transmission through a flowing gas-liquid system in which the liquid slugs had been dyed. This measurement yielded bubble-rise velocities, liquid slug lengths, and gas bubble lengths.

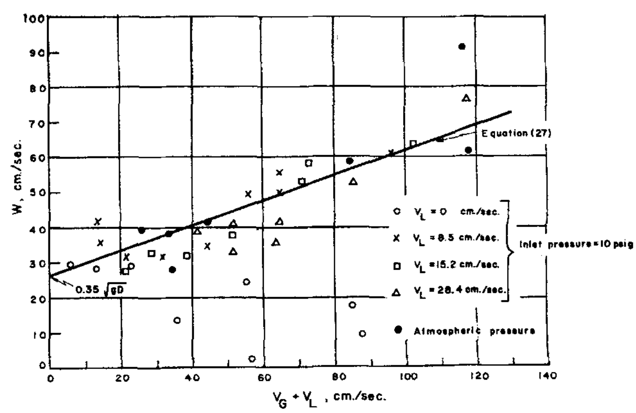

Fig. 5. The velocity of air bubbles through the surrounding water as a function of $(v_G + v_L)$ in a 2¼-in. I.D. pipe.

Table 2. Comparison of the Theoretical and Experimental Values of the Slope of the Pressure Drop-Time Functions Measured Over a Segment of Pipe Partially Filled with Water

v_G, cm./sec.	Slope when $(-\Delta P)$ is increasing, cm./sec.	Slope when $(-\Delta P)$ is decreasing, cm./sec.	w predicted by Nicklin's correlation
5.5	5.0	20.7	19.6
8.0	6.9	21.3	20.1
13.1	13.1	21.7	21.1
19.7	18.8	23.8	22.4
24.3	21.3	23.5	23.4
36.2	35.8	22.6	26.7
54.0	49.0	29.9	29.3

Pipe diameter: 1⅛ in.

The range of the independent variables studied is presented in Table 1. The flow was assumed to take place isothermally at approximately room temperature. For each of two pipes of internal diameter 1⅛ and 2¼ in. respectively, the gas and liquid flow rates, expressed as superficial gas and liquid velocities, were varied over the ranges indicated at an inlet pressure to the test column of 10 lb./sq. in. gauge. In addition, runs were made in each of the two pipes when the column pressure was essentially atmospheric.

The experimental apparatus in which the measurements were made is shown schematically in Figure 2. The arrangement of the transducers and the nomenclature used to describe them are shown schematically in Figure 3. The shortened length of pipe over which transducer No. 1 measures the pressure drop in the 1⅛-in. pipe eliminated an exit effect caused by the enlargement at the test section-overhead return line connection. The shorter lengths over which the pressure drops were measured in the 2¼-in. pipe as compared to the 1⅛-in. pipe were necessitated by the longer length of pipe in which entrance effects were controlling in the larger diameter pipe. The pressure drop was measured as a function of time by a series of four temperature-compensated differential pressure transducers with a pressure range of ± 5 lb./sq. in. gauge and an output of approximately ± 30 mv. full scale. The output from the transducers was recorded.

The pressure drop-time traces, when integrated with respect to time, were plotted in differential form. These data, which are presented fully in (*12*), did indicate that in certain runs some entrance effects existed; that is, an increased value for $\overline{\Delta P_D}$ in transducer No. 4 and to a lesser extent in transducer No. 3 was observed. This was most clearly evident in the data taken in the 2¼-in. pipe. However, the data did indicate that the entrance effects were small and were indeed negligible in transducers No. 1 and No. 2.

The experimental apparatus used to measure bubble velocities and lengths and liquid slug lengths utilized two light source-photocell units which were placed a known distance apart at a known location on the column. The photocells measured the incident light from the sources which were fitted with red filters. The water flowing into the test section was dyed green. The electrical output from the photocells was continuously recorded.

The time intervals for bubbles to traverse the distance between the photocells and for bubbles and slugs to pass a photocell determine the bubble-rise velocity, average gas bubble length, and average liquid slug length, respectively.

RESULTS AND DISCUSSION

The experimental results for w are plotted as a function of $(v_G + v_L)$ in Figures 4 and 5 for the 1⅛- and 2¼-in. pipes respectively. Figure 4 contains the experimental correlation of Nicklin et al. (*9*). This correlation appears to represent the data satisfactorily over the range of $(v_G + v_L)$ investigated at an inlet column pressure of 10 lb./sq. in. gauge. A least-squares analysis of the data

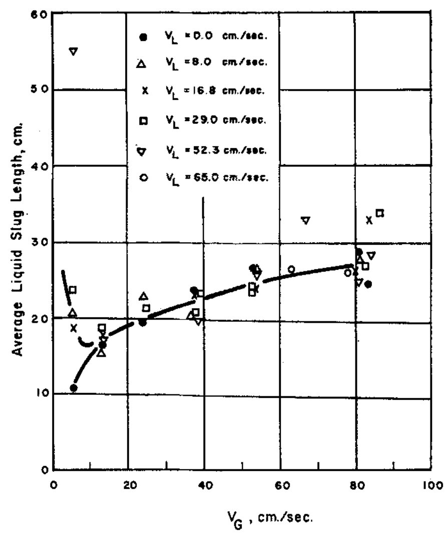

Fig. 6. Average liquid slug length vs. superficial gas velocity with superficial liquid velocity as parameter in a 1⅛-in. pipe at a 10 lb./sq.in. gauge inlet pressure.

gave a value of 0.18 for the slope of the w vs. $(v_G + v_L)$ line. The standard deviation of the experimental values from those predicted by the correlation $w = 0.35\sqrt{gD} + 0.2\ (v_G + v_L)$ is 10.3% with a bias of 0.7%. However, it appears that the velocities measured when the column pressure is near atmospheric are higher than the velocities encountered in a column whose inlet pressure is 10 lb./sq. in. gauge.

Figure 5 contains a correlation for w of the form

$$w = b(v_G + v_L) + 0.35\sqrt{gD} \qquad (14)$$

where the value of b was determined to be 0.35 by a least-squares analysis of the data. Here the five points for $v_L = 0$ which lie well below the bulk of the data were omitted because of the observed breakdown of the slug flow regime over this range of flow rates. The standard deviation of the experimental values from those predicted by this correlation is 15.8% with a bias of 3.4%.

The data taken at atmospheric pressure in this pipe do not show the marked rise in bubble velocity as do the data for the 1⅛-in. pipe. Either the rise in bubble velocity does not exist in this pipe, or the rise is masked by the scatter in the data.

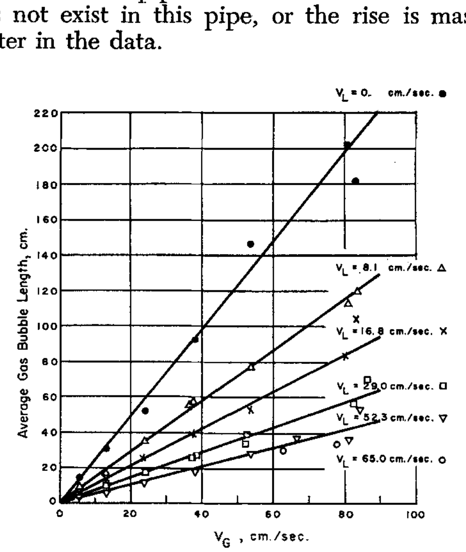

Fig. 7. Average gas bubble length vs. superficial gas velocity with superficial liquid velocity as parameter in a 1⅛-in. pipe at a 10 lb./sq.in. gauge inlet pressure.

Table 3. Summary of the Specific Cases Used to Compare Experimental and Predicted Pressure Drops

Figure	Pipe diam., in.	v_G, cm./sec.	v_L, cm./sec.	w, cm./sec.	α	Inlet pressure, lb./sq. in. abs.	T, °F.	Experimental $\overline{\Delta P_D}$	Predicted $\overline{\Delta P_D}$
8	2¼	87.8	29.0	78.8	0.27	24.7	80	0.37	0.41
9	1⅛	83.6	8.1		0.01	24.7	78	0.23	0.21

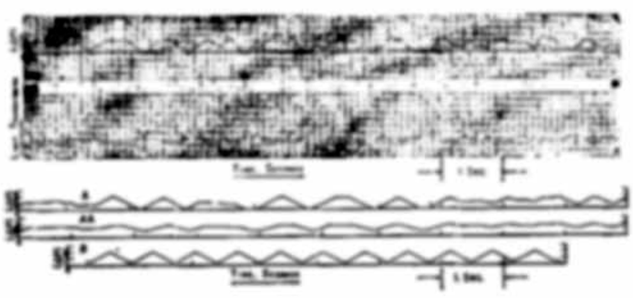

Fig. 8. Experimental and predicted pressure drops as functions of time for the specific case presented in Table 3.

Now consider an alternate method of correlation for w as a function of D and $(v_G + v_L)$. If the following equation is proposed

$$w = \sqrt{gD}\,[a + b\,(v_G + v_L)] \tag{15}$$

the data of this study and the data of Nicklin et al. (9) yield the following values for a and b:

D	a	b, cm.$^{-1}$	Reference
1 in.–1⅛ in.	0.35	0.0038	This work, Nicklin et al. (9)
2¼ in.	0.35	0.0047	This work

These results indicate that (15) may be a method of correlating the bubble-rise data in such a way that a and b are only functions of the physical properties of the two phases, that is, the particular two-phase system under consideration. Obviously the paucity of the data does not allow a final judgment to be made. A wide range of diameters and gas-liquid systems would have to be investigated to verify this hypothesis. However, it does appear that a generalized correlation such as (15) is possible.

In the theoretical analysis of gas-liquid slug flow the assumption has been made that the liquid slugs travel at a superficial velocity $(v_G + v_L)$. This leads to the interpretation of w as the velocity with which the gas bubbles rise through the liquid slugs. This assumption can be substantiated by measuring the pressure drop-time relation over a segment of pipe which is only partly filled with liquid ($v_L = 0$). If a gas bubble is not breaking through the liquid surface, the liquid level should rise at a velocity v_G. Therefore, when one neglects frictional effects, the slope of the pressure drop-time trace when $(-\Delta P)$ is increasing should equal v_G, where $(-\Delta P)$ is measured in centimeters of water. When a gas bubble enters the segment L, $(-\Delta P)$ should decrease. The rate of decrease (the slope of the decreasing pressure drop-time trace) equals the difference between the bubble-rise velocity and the velocity of rise of the liquid level, that is, the decreasing pressure drop-time trace should have a slope equal to w.

Several pressure drop-time traces have been taken over a segment of the 1⅛-in. pipe. A tabulation of the results of seven traces is presented in Table 2. The results show, within the experimental error of measuring the slopes, that the liquid slugs do travel at a superficial velocity v_G. Secondly the slopes measured when $(-\Delta P)$ is decreasing agree with the values of w predicted by Nicklin et al. (9) within the experimental error.

Therefore it may be concluded that the liquid slugs do move at a superficial velocity v_G for zero liquid flow, and the gas bubbles do move through the liquid at a velocity w. It is only required now that this result extrapolate to the case of continuous liquid flow.

The data on liquid slug and gas-bubble lengths taken in the 1⅛-in. pipe, when used in (4), indicate α to be small (of the order 0.05). The data in Figure 6 indicate that the liquid slug lengths are nearly independent of v_L and are a weak, nearly linear function of v_G over the range of 10 cm./sec. $\leqq v_G \leqq$ 90 cm./sec. Equation (12) indicates therefore that the following proportionality should hold:

$$\overline{l_G} \propto K \frac{v_G}{v_L + w/2} \tag{16}$$

The data in Figure 7 confirm this proportionality over the range of flow rates studied.

The theoretical predictions of pressure drop as a function of time have been compared with experimental data. These comparisons have been carried out for a number of cases which are presented in (12). Two examples are presented here in Figures 8 and 9. The data for these examples are given in Table 3 which also contains the time-averaged pressure drops. The pressure-drop prediction labelled "A" in Figure 8 utilized experimental bubble and slug lengths measured by the light source-photocell system (the light-transmission data in the figure). The pressure-drop prediction labelled "B" utilized idealized bubble-slug data (time-averaged lengths). Figure 9 contains a prediction calculated from the theory and utilizes only the correlations in Figures 4, 6, and 7.

The A prediction indicates that the theoretical model satisfactorily predicts the pressure drop as a function of time for slugging gas-liquid systems. The discrepancies between the experimental trace and the theoretical prediction are thought to be due to turbulent fluctuations in the fluids and occasional unstable bubble motion which gives rise to coalescense and acceleration effects.

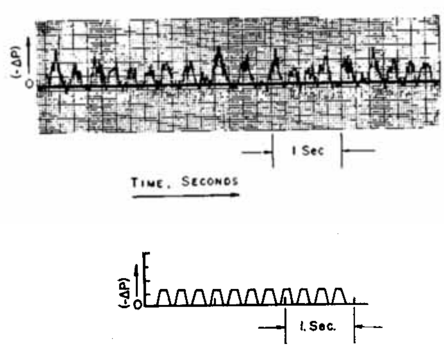

Fig. 9. Experimental and predicted pressure drops as functions of time for the specific case presented in Table 3.

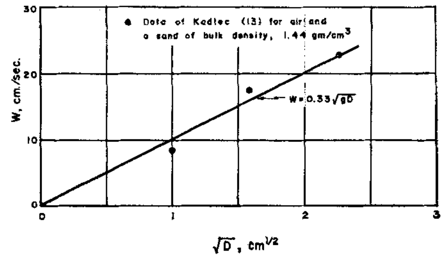

Fig. 10. Gas bubble velocity as a function of pipe diameter for a gas-fluidized solids system.

It will be recalled at this point that the initial assumption was made that the gas bubbles were regions of constant pressure. Suppose for the moment that this result is not applied, that the effects of acceleration of the liquid around the bubble are neglected, and therefore that a pressure drop exists across the gas bubble equal to the holdup of liquid less the wall shear acting on the liquid film. When one uses the theoretical model of a rising gas bubble developed by Street and Tek (*11*), this pressure drop can be calculated for any specification of bubble-rise velocity, liquid properties, and gas and liquid flow rates.

Figure 8 contains a prediction of the pressure drop-time function incorporating this effect of a pressure drop across the bubble (labelled "*AA*"). The poor agreement between the predicted and experimental pressure drops is to be noted. During certain time intervals the pressure drop experimentally goes to zero. This is not predicted by curve *AA* but is predicted by curve *A* for which the assumption of zero pressure drop across the gas bubble was made. Secondly the time-average pressure drop $\overline{\Delta P_D}$ predicted from curve *AA* equals 0.55 which is significantly different from the experimental value 0.37 and the value predicted by curve *A*, 0.41.

The "*B*" prediction in Figure 8 indicates that the use of the idealized boundary conditions may lead to an underestimation of the magnitude of the pressure-drop fluctuations. This can be attributed to the existence of turbulent fluctuations in the fluids and more significantly irregular bubble generation. However, the results in Figure 9 indicate that the idealized boundary conditions can yield pressure drop-time functions which realistically predict the experimental data.

GENERALIZED CONCEPTS OF SLUGGING IN TWO-PHASE SYSTEMS

The phenomenon of slugging has been observed and investigated in gas-fluidized solids and immiscible liquid-liquid systems as well as in gas-liquid systems. These phenomena naturally raise the question of the existence of quantitative similarities and analogies between the three types of observed slugging.

Kadlec (*13*) has investigated the behavior of slugging gas-fluidized solid systems. His experimental work and theoretical model indicate that the pressure drop is periodic in time. In addition his measurements of bubble velocities indicate that the quantity w is proportional to the square root of the pipe diameter. Figure 10 contains Kadlec's data and the straight line drawn through it:

$$w = 0.33\sqrt{gD} \tag{17}$$

The value of the slope 0.33 corresponds surprisingly to the value of 0.35 observed in air-water systems. In addition Kadlec has shown that the bubble velocity w is independent of the superficial gas velocity v_G. In terms of the correlation (15) the constant b equals zero for gas-solid systems.

Harmathy (*14*) has studied the simultaneous flow of immiscible liquids and has observed the existence of slugs similar to those occurring in gas-liquid flow. He has measured the velocity with which the liquid bubbles rise through a stagnant continuous liquid. The results may be predicted by applying a density correction to Dumitrescu's theory:

$$w = 0.35\sqrt{\frac{\Delta\rho}{\rho_c}}\sqrt{gD} \tag{18}$$

In order to extend the theoretical model in this work to immiscible liquid-liquid slug flow, account would have to be taken of the existence of a pressure drop across the liquid bubbles.

The results of this work and the results of other investigators (*9, 13, 14*) therefore indicate the existence of generalized concepts of slugging in two-phase systems in that:

1. Alternate regions of dense phase and light phase material exist in the pipe in which the light phase moves at a well-defined velocity with respect to the dense phase.
2. The velocity with which the light-phase bubbles move through the dense-phase slugs is possibly predicted by an equation of the form

$$w = \sqrt{gD}\,(a + b(v_G + v_L))$$

where the subscripts G and L refer to the light and dense phases, respectively, and where the constants a and b are possibly dependent only upon the particular two-phase system under consideration.

3. The pressure drops occurring are periodic in nature because of the periodic nature of the fluid density at any pipe location and are predictable from macroscopic mass and momentum balances of the type presented in this work and elsewhere (*13*).

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NOTATION

- A_G = cross-sectional area of pipe occupied by gas bubble
- A_P = cross-sectional area of pipe
- D = pipe diameter
- g = acceleration of gravity
- l_G = gas bubble length
- l_L = liquid slug length
- l = length of gas bubble, liquid slug, etc.
- L = length of pipe, length of gas bubble
- L_g = length of gas bubbles
- L_l = length of liquid slugs
- L_{GS} = total input length of gas bubbles
- P = pressure
- Q = volumetric flow rate
- t = time
- v = velocity measured with respect to pipe wall
- v_B = bubble rise velocity measured with respect to pipe wall
- v_G = superficial gas velocity measured at pipe inlet, Q_G/A_P
- v_L = superficial liquid velocity, Q_L/A_P
- w = velocity of gas bubbles with respect to liquid slugs
- z = axial distance

Greek Letters

- α = volumetric gas content of liquid slug, vol. gas/vol. pipe
- Δ = denotes a difference
- $\Delta\rho$ = density difference between the liquids
- ρ, ρ_L = liquid density
- ρ_c = density of the continuous phase
- ρ_G = gas density
- τ_w = wall shear acting on liquid slugs
- μ = liquid viscosity
- μ_G = gas viscosity

Subscripts

- i = specific gas bubble in macroscopic model (Figure 1)
- D = dimensionless quantity
- G = gas phase
- L = liquid phase
- N = number of gas bubbles in macroscopic model

Superscripts

- f = liquid film around gas bubble
- — = time-averaged quantity

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Unsteady Heat Transfer to Slug Flows: Effect of Axial Conduction

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Solutions are obtained for three illustrative cases of unsteady heat transfer to slug flows, taking axial conduction into account. The magnitude of the correction to the solutions neglecting this effect is shown to be quite appreciable near the leading edge. In agreement with previous steady state estimates, the axial conduction correction becomes negligible for Peclet numbers in excess of 100.

Slug flows comprise the class of incompressible flows in which the velocity field is everywhere uniform. Despite their relative simplicity, solutions of the equations for unsteady transport of heat or mass to slug flows have not been extensively investigated, possibly because there are at least three independent variables. Such problems arise in such diverse applications as the continuous annealing of bars and the heating of oil or gas in underground strata. For definiteness it will be assumed herein that the diffusant is heat; the authors shall also be concerned only with problems in which two space variables, as well as time, are involved, and in which the fluid velocity remains constant with time. Such problems are conveniently solved by multiple transform methods, with the Laplace transform to remove the time variable, and a finite Sturm-Liouville transform or Laplace transform to remove one of the space variables. In some cases it is permissible to neglect axial conduction, and for the interested reader, a number of problems of this type are solved in reference *1*. Siegel (*2*) has shown that these problems can also be solved by considering separately the fluid particles which were before and beyond the tube leading edge at the instant of imposition of the new boundary conditions. No heat exchange can occur between these two regions on account of the neglect of axial conduction. Hence, it is possible to solve for the temperature in each region sep-

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