UNIVERSITY OF MICHIGAN
ENGINEERING RESEARCH INSTITUTE

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Willow Run Laboratories
Willow Run Airport
Ypsilanti, Michigan

Signal Property Officer
Building No. 1150
Fort Monmouth
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Subject: Contract DA-36-039-SC-64748, Phase I-A

Notice is hereby given that all technical aspects of the subject phase, of the subject contract have been completed.

This letter is incorporated as part of the final report of Phase I-A, of the contract.

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DMB:em

UNIVERSITY OF MICHIGAN

COMPUTATION OF SPATIAL RADIATION PATTERNS OF RHOMBIC ANTENNAE

Final Report

on

Contract DA36-039-SC-64748-Phase I-A

To

Signal Corps Radio Propagation Agency
Fort Monmouth, N. J.

Period 15 July 1955 to 31 March 1956

James H. Brown

ERI Project 2411

Report 2411-1-F

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ABSTRACT

A method of numerical analysis to compute the antenna gain for determination of the spatial radiation patterns for the various types of rhombic antennae is formulated. A program for solving the problem on the Michigan Digital Automatic Computer (MIDAC) is listed in external MIDAC language. Parameters for computing fourteen different types of antennae are listed (Table I), and an example of antenna gain for one type is given (App. B).

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1. Statement of Problem.

Phase Ia of contract DA-36-039 SC-64748 requires the computation of the antenna gain for 14 different types of rhombic antennas. This gain is to be calculated for the parameter variations listed in section 1.2.

1.1 Equations for Antenna Gain.*

The antenna gain, A, in decibels, is given by the equation

$$A = 10 \log_{10} \frac{/E_H/^2 + /E_V/^2}{(186.3)^2} . \qquad (1.1)$$

The horizontal and vertical polarization components, E_H and E_V respectively, are expressed in millivolts per meter at one mile by the equation

$$\left. \begin{aligned} E_H &= F' \; G'_R \\ E_V &= F'' \; G''_R \end{aligned} \right), \qquad (1.2)$$

where $F' = K \cos \phi \; U_1 U_2 (\cos \beta - \sin \phi \cos \Delta)$, (1.3)

$F'' = K \cos \phi \; U_1 U_2 \sin \beta \sin \Delta .$ (1.4)

* The derivation of the following equations is given in reference 1.

$$\left.\begin{aligned} K &= \text{constant} \\ U_1 &= \frac{\sin \frac{\pi L K_1}{\lambda}}{K_1} \\ U_2 &= \frac{\sin \frac{\pi L K_2}{\lambda}}{K_2} \end{aligned}\right) \qquad (1.5)$$

$$\left.\begin{aligned} K_1 &= 1 - \cos \Delta \sin (\phi + \beta) \\ K_2 &= 1 - \cos \Delta \sin (\phi - \beta) \end{aligned}\right) \qquad (1.6)$$

Figure 1.1 defines the geometric quantities of the above equations.

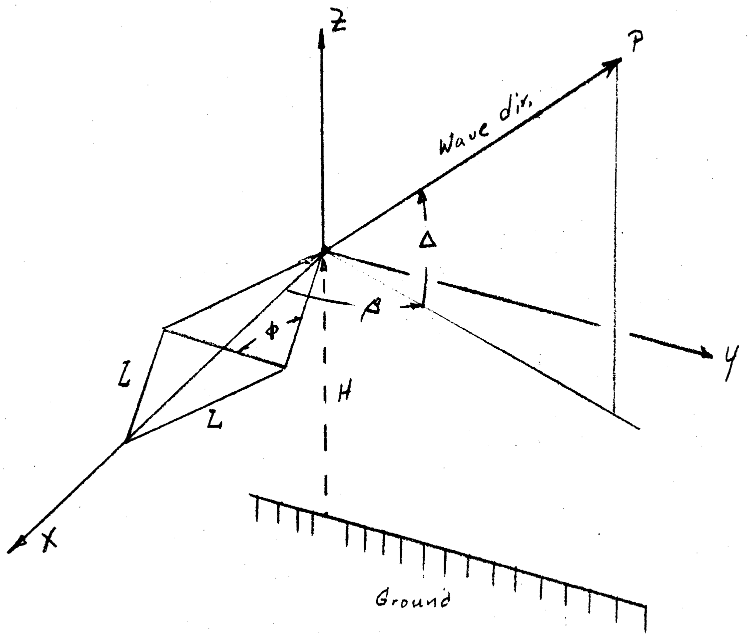

Fig. 1.1

Geometry of Rhombic Antenna

$$G_R' = 1 + R' e^{-j[\rho' + (4\pi H/\lambda)\sin\Delta]} \tag{1.7}$$

$$G_R'' = 1 - R'' e^{-j[\rho'' + (4\pi H/\lambda)\sin\Delta]} \tag{1.8}$$

where
$$R' e^{-j\rho'} = \frac{\sin\Delta - (\epsilon' - \cos^2\Delta)^{\frac{1}{2}}}{\sin\Delta + (\epsilon' - \cos^2\Delta)^{\frac{1}{2}}} \tag{1.9}$$

$$R'' e^{-j\rho''} = \frac{\epsilon' \sin\Delta - (\epsilon' - \cos^2\Delta)^{\frac{1}{2}}}{\epsilon' \sin\Delta + (\epsilon' - \cos^2\Delta)^{\frac{1}{2}}} \tag{1.10}$$

$$\epsilon' = \epsilon - j\, 6\, \sigma\lambda \cdot 10^{12} \tag{1.11}$$

ϵ = constant, ratio of earth-air dielectric constants.

σ = ground conductivity, emu.

The quantities G_R' and G_R'' are the ground reflection factors for horizontal and vertical polarization, respectively.

1.2 Parameter Variation.

The parameters for the calculation of the antenna gain are:

L = leg length, in feet.

H = height above ground, in feet.

ϕ = one-half the included angle, in deg.

Δ = vertical angle of wave arrival or departure, in degrees.

β = azimuthal angle of wave arrival or departure, in degrees.

λ = 300/f = wave length in meters, where f = frequency in mc.

The first three parameters are functions of the antenna type. There are 14 of these types, which are listed in Table I. The other three parameters have the ranges

$$\left.\begin{array}{l} 2 \text{ mc} \leqq f \leqq 30 \text{ mc} \\ 2 \text{ deg.} \leqq \Delta \leqq 85 \text{ deg.} \\ 0 \text{ deg.} \leqq \beta \leqq 180 \text{ deg.} \end{array}\right\} \quad (1.12)$$

The calculation of the antenna gains is made for each type of antenna for:

29 frequencies, 2 through 30 mc., inclusive.

30 values of Δ, each two degress from $\Delta = 2^\circ$ to 42°, and each five degrees from $\Delta = 45^\circ$ to 85°.

46 values of β, each two degrees from $\beta = 0^\circ$ to 30°, and each five degrees from $\beta = 35^\circ$ to 180°.

This gives a total of 560,280 values of the antenna gain required.

TABLE I

Antenna Types

Type	L, ft.	H, ft.	ϕ, deg.
A	375	65	70
B	350	60	70
C	315	57	70
D	290	55	67.5
E	270	53	65
F	245	51	62.5
G	225	50	60

(cont.)

TABLE I (cont.)

Type	L, ft.	H, ft.	ϕ, deg.
RD-1	400	130	67
RD-2	400	130	68
RD-3	400	130	69
RD-4	400	130	71
RD-5	450	130	62
RD-6	450	130	64
RD-7	450	130	70

2. Solution.

2.1 Ground reflection coefficients.

The ground reflection coefficients, G_R' and G_R'' of equations (1.7) and (1.8) are complex quantities; therefore, the magnitudes of the polarization components used in equation (1.1) may be expressed as

$$\left.\begin{array}{l} /E_H/^2 = F'^2 (G' \, G'^*) \\ /E_V/^2 = F''^2 (G'' \, G''^*) \end{array}\right\} \qquad (2.1)$$

Let

$$\left.\begin{array}{l} G_R' = G_1' + j\, G_2' \\ G_R'' = G_1'' + j\, G_2'' \end{array}\right\} \qquad (2.2)$$

$$\left.\begin{array}{l} R' e^{-j\rho'} = R_1' + j\, R_2' \\ R'' e^{-j\rho''} = R_1'' + j\, R_2'' \end{array}\right\} \qquad (2.3)$$

$$e^{-j\,4\pi\,H/\lambda\,\sin\Delta} = \cos\Psi + j\sin\Psi. \qquad (2.4)$$

Then,

$$\left.\begin{aligned} G'_R &= 1 + Q'_1 + j\,Q'_2 \\ G''_R &= 1 - Q''_1 - j\,Q''_2 \end{aligned}\right) \qquad (2.5)$$

where

$$\left.\begin{aligned} Q'_1 &= R'_1\cos\Psi - R'_2\sin\Psi \\ Q'_2 &= R'_1\sin\Psi + R'_2\cos\Psi \end{aligned}\right) \qquad (2.6)$$

$$\left.\begin{aligned} Q''_1 &= R''_1\cos\Psi - R''_2\sin\Psi \\ Q''_2 &= R''_1\sin\Psi + R''_2\cos\Psi \end{aligned}\right) \qquad (2.7)$$

Therefore,

$$\left.\begin{aligned} G'_R\,G'^{*}_R &= (1 + Q'_1)^2 + Q'^2_2 \\ G''_R\,G''^{*}_R &= (1 - Q''_1)^2 + Q''^2_2 \end{aligned}\right) \qquad (2.8)$$

Equations (1.9) and (1.10) contain the term $(\epsilon' - \cos^2\Delta)^{\frac{1}{2}}$. Since ϵ' is a complex quantity, it is necessary to determine the sign to be used upon taking the square root.

Let $\epsilon' - \cos^2\Delta = c + j\,n = B\,e^{j\theta}$ (2.9)

The factor n is always negative, and the factor c is always positive; therefore, the angle θ is in the fourth quadrant. The square root requires $\theta/2$. The sign was chosen so that the half-angle was also in the fourth quadrant. Subsequent checks with known values proved that this was the correct assumption.

2.2 Antenna Gain.

The factors F' and F" used in determining the antenna gain involve straight-forward algebraic and trigonometric computations. For purposes of digital computation, equations (1.6) were expanded in terms containing the sines and cosines of ϕ and β.

3. Mechanization.

The computation of the antenna gain by the MIDAC involves not only the solution of the given equations, but also an organization of the program so that as little time is spent in the computation of each value of antenna gain as possible. This is necessary because of the large number of values to be computed.

The complete problem was divided into four parts. Part I controls the parameter variation, and some sines and cosines are computed in this part. Part II computes the ground reflection coefficients; Part III completes the computation of the antenna gain. Part IV is the print-out program.

3.1 Part I, Parameter Variation.

The parameter variation was chosen so as to minimize the amount of calculation required. Each of the 14 antenna types requires the calculation of 40,020 values of the antenna gain. This constitutes a run. Within a run, the angle β is varied the most; the frequency, f, and the angle Δ follow in decreasing order of variation. Thus, the β-variation is considered to be in the "inner loop", with the f-variation in the middle loop; and the Δ-variation being the outer loop. Part I controls the Δ- and f-variation. Associated with these two parameters are two tallies, i and j, respectively. By setting these tallies to the proper integral value, as shown in Table II, any desired combination of the parameter variation for Δ and f for this problem may be obtained. The β-variation is controlled by Part III.

Part I also computes the quantities: sin Δ, cos Δ, sin Ψ, and cos Ψ. This is done at this point to help minimize the amount of computation. These values are stored and used later on in the other parts.

3.2 Part II, Ground Reflection Coefficients.

The computation of the ground reflection coefficients requires the square root of a complex quantity (equation 2.9); thus

$$(\epsilon' - \cos^2\Delta)^{\frac{1}{2}} = B^{\frac{1}{2}}(\cos \tfrac{1}{2}\theta + j \sin \tfrac{1}{2}\theta) \qquad (3.1)$$

where

$$B = [(\epsilon - \cos^2\Delta)^2 + (-6\,\sigma\lambda\, 10^{12})^2]^{\frac{1}{2}} \qquad (3.2)$$

$$\theta = \tan^{-1}\left[\frac{-6\,\sigma\lambda\, 10^{12}}{\epsilon - \cos^2\Delta}\right] \qquad (3.3)$$

In order to minimize computation, sin ½θ and cos ½θ are found from the relations:

$$\left.\begin{aligned} \sin \tfrac{1}{2}\theta &= -(\tfrac{1}{2} - \tfrac{1}{2}\cos\theta)^{\frac{1}{2}} \\ \cos \tfrac{1}{2}\theta &= (\tfrac{1}{2} + \tfrac{1}{2}\cos\theta)^{\frac{1}{2}} \end{aligned}\right\} \qquad (3.4)$$

The real and imaginary parts of equations (2.3) are obtained from equations (1.9) and (1.10); thus,

$$R_1' = \frac{\sin^2\Delta - B}{D'} \qquad (3.5)$$

$$R_2' = \frac{-2\, B^{\frac{1}{2}} \sin\Delta \sin\tfrac{1}{2}\theta}{D'} \qquad (3.6)$$

where

$$D' = \sin^2\Delta + B + 2\, B^{\frac{1}{2}} \sin\Delta \cos\tfrac{1}{2}\theta \qquad (3.7)$$

$$R_1'' = \frac{\underline{H}^2 \sin^2\Delta - B}{D''} \qquad (3.8)$$

$$R_2'' = \frac{2(\underline{\underline{n\, B^{\frac{1}{2}} \sin\Delta \cos\tfrac{1}{2}\theta}} - \underline{\epsilon}\, B^{\frac{1}{2}} \sin\Delta \sin\tfrac{1}{2}\theta)}{D''} \qquad (3.9)$$

where

$$D'' = \underline{H} \sin\Delta + B + 2(\underline{\underline{n\, B \sin\Delta \sin\tfrac{1}{2}\theta}} + \underline{\epsilon}\, B \sin\Delta \cos\tfrac{1}{2}\theta) \qquad (3.10)$$

$$H = \epsilon^2 + n^2 \qquad (3.11)$$

Note that if the single underscored terms in equations (3.8) to (3.10) are replaced by 1, and the double underscored terms are replaced by 0, these equations reduce to the corresponding equations (3.5) to (3.7). This fact is used in programming the equations. Table III lists the equations for the computation of the ground reflection coefficients which were actually programmed. Equations (211) through

(220) are common to both the horizontal and vertical coefficients. Equations (221) through (234) are repeated twice, with the modifications as indicated in equations (221) through (224) for the horizontal and for the vertical coefficients. The final results are the magnitudes, squared, of the ground reflection coefficients.

3.3 Part III, Antenna Gain.

Part III completes the calculation of the antenna gain. Table IV lists the equations used in the program for this calculation. The values of the sine and cosine of β are stored in a table, rather than computing the sine and cosine of each β as it is used. Part III is the inner loop, and is computed 46 times for each time Parts I and II are computed. The table look-up of the sines and cosines of β is faster than the computation of the function would be each time it is needed.

The 46 values of the antenna gain are stored, to be used by the print-out routine in Part IV.

3.4 Part IV, Print-out.

The print-out program for this problem was designed around the fact of the large amount of output required. The antenna gain is required to one decimal digit accuracy. The limit on the magnitude of the gain is less than 100; therefore, only three numbers need be printed out for each value of antenna gain. By using a fixed-point print-out (always printing out three decimal digits, even though the first are zero), the decimal point need not be printed. This reduces the number of characters to be printed out. If the value of the antenna gain is negative, the negative sign is printed after the number. **Two print-out formats are available; single column and page. The running variable for the column is β. A page consists of 10 columns, with the column variable being Δ; three pages are used for one value of f.**

Before the gain is printed out, it is converted from binary (the internal language of the MIDAC) to decimal. Also the number is rounded in the second decimal place.

In case the antenna gain becomes equal or larger than 100 in magnitude, the print-out program indicates this within the list of values. An identity word is printed out preceding the group of 46 values of gain for each Δ-f variation.

4. Organization.

4.1 Operation.

The program for the calculation of the antenna gain on the MIDAC is given in Appendix A. The program was divided into two sections to best fit into the high-speed storage space of the MIDAC. The first section contains parts I and II, and the second section contains parts III and IV. These two sections are stored on the drum, and are called into the acoustic memory when needed.

The program starts by reading in the parameter tape specifying the antenna type. The first section (parts I and II) is called in to the acoustic memory, and the ground reflection coefficients for a particular Δ-f combination are computed and stored. The second section is then called in. The antenna gain for the 46 values of β are computed and stored, the computer cycling through Part III 46 times. The print-out routine is then entered, and the Δ- and f-tallies and the 46 values of the antenna gain are printed out.

Section 1 is recalled, a new value of f is chosen and the process repeated. When the program has cycled through all the values of f, a new Δ is chosen. This continues until a run is completed. A run consists of 40,020 values of antenna gain.

The program is designed so that it may be halted at any point, and the computation continued from that point at a later date. This is accomplished by setting the Δ- and f-tallies to the desired value (cf. Table II) after reading in the antenna-type parameters. Table V is the flow diagram for the computation of the antenna gain.

The program is written to be used with the MAGIC I system of automatic programming, developed for the MIDAC (cf. ref. 2).

4.2 Error Halts.

During the period of continuous operation of the MIDAC, random machine malfunctions will occur. These occur at such a frequency, that for runs over an hour in length, it is advisable to provide recovery procedures. The recovery procedures should be as automatic as possible, and this may be done by programmed recoveries.

The main method of recovery after a machine malfunction is the use of a "rollback". At specified intervals during the computation, the contents of the acoustic memory is stored on the drum. Then, if a recovery is required from a computer malfunction, this stored contents is brought back into the acoustic memory, and computation may be "rolled back" to the point at which the acoustic memory was stored. Thus, the entire computation need not be repeated.

The programming of a rollback routine requires the consideration of some factors. The first factor is the assurance that the contents of the acoustic memory is correct before it is stored, i.e., the assurance that registers in the acoustic memory have not picked up or lost information. A built-in checking circuit in the computer automatically indicates such an error as each register is used. Thus, prior to storing for rollback, a cycling routine is used which refers to every register in the acoustic memory. If everything is correct, the acoustic memory is stored and the program continues. If there is an error, the computer halts automatically. The faulty component must be corrected, and the program may be continued by calling in the previous rollback.

When the rollback is used, the computer must be started in the same state as at the time of storage for rollback. This requires that the instruction counter and the base counter be stored as part of the rollback. The first instructions of a rollback use these stored values to reset the counters.

A rollback routine has been incorporated in the program for calculation of the antenna gain. It consists of two parts: checking the acoustic memory, storing the counters, and storing the acoustic memory; and recalling the stored memory, resetting the counters, and continuing the program.

The checking of the acoustic memory is accomplished by adding each register to a sum-register. This uses every register in the acoustic memory and will cause an error halt if information has been altered by a malfunction. This sum is also stored, to be used by the second part. The values of the instruction and base counters are also stored.

The second section performs the actual rollback. The stored memory is recalled to the acoustic memory, summed and checked against the sum obtained in the first section. This checks the call-in from the drum. If there is a discrepency, the stored memory is called in again. This is repeated three times, after which the computer halts. Such a halt usually indicates major trouble, so the run is discontinued at this point.

Sometimes the information called in from the drum can be called in incorrectly. A drum read-in check program has been incorporated into the program from calculation of the antenna gain. As each of the two sections are called in from the drum, each word of the section is automatically added to a sum register. Upon completion of the call-in, this sum is compared with the known sum for that section.

If the sums agree, the program continues. If the sums do not agree, the call-in is repeated as many as three times. If, after the third time, the sums still do not agree, the computer prints out the call-in order and the two sums being compared and halts. This gives information which helps locate the malfunction. Upon running again, the program automatically proceeds to the rollback routine and rolls back to the last previous memory store.

It was mentioned in describing the rollback routine, that the sum of the entire acoustic memory was taken prior to rollback. The negative of this sum is stored as the last word of the acoustic memory. Upon calling in this stored memory for rollback, the complete sum should then be zero.

The program may be halted at any point and the computation resumed at a later time with very little loss in computation time. This results from the use of tallies to cycle the parameters. The identity word which is printed out before each group of antenna gains, indicates the i and j tallies, as listed in Table II. Before beginning computation again, these tallies are set to their next value. By this method, the program will continue computation from where it left off.

5. Subroutines.

Standard MIDAC subroutines are used in the program for the calculation of antenna gain. Part I uses the sin-cos subroutine; Part II uses the square root subroutine. Part III uses the sine subroutine, and the log subroutine. The sines and cosines of the 46 values of β were computed separately, and were incorporated into the main program as a table of values.

6. Results.

The 560,280 values of the antenna gain were punched out on paper tape, via the high-speed teletype punch. Column format output was used with types D, E, F, G. Editing with this form of output was difficult because of the physical handling of the tapes and paper required. As a result, the program was revised so that the output was in page form. The output for the remaining types used this page format output.

The following statistics resulted from this problem:

Output -- 14 runs.		
	87 pages/run	1218 pages.
	3290 characters/page	4,007,220 characters.
	10 char./in. of tape	400,722 in. tape
		6.3 mi. tape.
Production time on the MIDAC.		
	Computation of antenna gain	60.9 hours
	Print-out	18.5 hours
	Total	79.4 hours
Editing time -- 10 minutes/page		203 hours

Appendix B is an example of the edited output. The sample used is for Type C, with a frequency of 2 megacycles. The column variable is Δ; the row variable is β. The first word of the column below the column heading is the identity word.

One complete copy of the edited output (1218 pages) has been supplied to the contracting officer.

Table II

PARAMETER VARIATION

$\Delta = x_0$

$x_0^i = x_0^{0'} + i\,\Delta_1 x_0$ for $0 \leq i \leq 20$

$= x_0^{0''} + i\,\Delta_2 x_0$ $21 \leq i \leq 29$

where: $x_0^{0'} = 2$ deg. $\Delta_1 x_0 = 2$ deg.

$x_0^{0''} = -60$ deg. $\Delta_2 x_0 = 5$ deg.

i hex	i dec	x_0^i deg.
00	0	2
01	1	4
02	2	6
03	3	8
04	4	10
05	5	12
06	6	14
07	7	16
08	8	18
09	9	20
0a	10	22
0b	11	24
0c	12	26
0d	13	28
0e	14	30
0f	15	32
10	16	34
11	17	36
12	18	38
13	19	40
14	20	42

i hex	i dec	x_0^i deg.
15	21	45
16	22	50
17	23	55
18	24	60
19	25	65
1a	26	70
1b	27	75
1c	28	80
1d	29	85

$f = x_3$ mc

$x_3^j = x_3^0 + j\,\Delta x_3$ where: $x_3^0 = 2$ mc

$\Delta x_3 = 1$ mc

j hex	j dec	x_3^j mc
00	0	2
01	1	3
02	2	4
03	3	5
04	4	6
05	5	7
06	6	8
07	7	9
08	8	10
09	9	11
0a	10	12
0b	11	13
0c	12	14
0d	13	15
0e	14	16
0f	15	17
10	16	18
11	17	19
12	18	20
13	19	21
14	20	22
15	21	23
16	22	24
17	23	25
18	24	26
19	25	27
1a	26	28
1b	27	29
1c	28	30

$\beta = x_6$

$x_6^k = x_6^{0'} + k\,\Delta_1 x_6$ for $0 \leqq k \leqq 15$ where $x_6^{0'} = 0$; $\Delta_1 x_6 = 2$ deg.

$= x_6^{0''} + k\,\Delta_2 x_6$ $16 \leqq k \leqq 45$ $x_6^{0''} = -45$ deg; $\Delta_2 x_6 = 5$ deg.

k hex	k dec	x_6^k deg
00	0	0
01	1	2
02	2	4
03	3	6
04	4	8
05	5	10
06	6	12
07	7	14
08	8	16
09	9	18
0a	10	20
0b	11	22
0c	12	24
0d	13	26
0e	14	28
0f	15	30
10	16	35
11	17	40
12	18	45
13	19	50
14	20	55
15	21	60
16	22	65
17	23	70
18	24	75
19	25	80
1a	26	85
1b	27	90
1c	28	95
1d	29	100
1e	30	105
1f	31	110
20	32	115
21	33	120
22	34	125
23	35	130
24	36	135
25	37	140
26	38	145
27	39	150
28	40	155
29	41	160
2a	42	165
2b	43	170
2c	44	175
2d	45	180

REFERENCES

1. Radio Propagation Unit Technical Report 7: Linear Communication Antennas. May 1949.

2. "Digital Computers and Data Processors", Notes for the University of Michigan Summer Session, 1955. Section II.3.1: "Programming for the MAGIC I System".

Appendix A

Program for Calculation of Antenna Gain. **-- Column print-out.**

0016ff001
2G4 M4 12-7-55 Antenna Gain Brown
1↑

EXPLANATION

ad1025m

fdd39

ac008

cd

faa00	b00	001	b03	fi	call in part III	S
	343	d37	a00	ri	call in part III	S
	f00	-0	a08	cm	forced ---> 101	
faa01	e00	e00	e00	su	0 ---> i	I
	1e00	1e00	1e00	su	0 -> j	
	2e00	2e00	2e00	su	0 -> k	
faa02	7e10	c00	511	mr	x_5(rad) .2^{-3}	
	a03	001	1a03	fi	file -> sin-cos	
	f00	510	4e11	ad	sin $x_5 \cdot 2^{-1}$	
	f00	511	5e11	ad-	cos $x_5 \cdot 2^{-1}$	
faa04	e00	1c00	a05	cn	Is i ≥ 21?	100
	e00	2c00	a06	cn	Is i ≥ 30?	
	0	0	0	ri	halt	
	f00	-0	a01	cm	forced -> I	
faa05	e00	5c00	e01	ml	i $\Delta_1 x_0^0 \cdot 2^{-8}$	
	3c00	e01	e10	ad-	$x_0^{0'} + i\, \Delta_2\, x_0^0 \to x_0^i \cdot 2^{-8}$	
	f00	-0	a07	cm	forced	
faa06	e00	6c00	e01	ml	i $\Delta_2\, x_0^0 \cdot 2^{-8}$	
	4c00	e01	e10	ad-	$x_0^{0''} + i\, \Delta_2\, x_0^0 \to x_0^i \cdot 2^{-8}$	
faa07	e10	c00	511	mr	$x_0^i \cdot 2^{-3}$	
	a03	001	1a03	fi	file -> sin-cos	
	f00	510	2e10	ad	sin $x_0^i \cdot 2^{-1}$	

	f00	511	1e10	ad-	cos $x_0^i \cdot 2^{-1}$	
faa08	b15	001	1b15	fi	store rollback	101
	1e00	7c00	a09	cn	Is $j \geq 30$?	
	8c00	e00	e00	ad	$1 + i \to i$	
	1e00	1e00	1e00	su	$0 \to j$	
	f00	-0	a04	cm	forced -> 100	
faa09	1e00	9c00	e01	ml	$j \Delta x_3 \cdot 2^{-8}$	
	10c00	e01	e01	ad	$(f^i = x_3^0 + j \Delta x_3) \cdot 2^{-8}$ mc	
	e01	11c00	3e10	dv-	$(\lambda = x_3^i = 984.3/f^i) \cdot 2^{-12}$ ft	
	12c00	6e10	e01	mr	$2 x_4 \cdot 2^{-19}$	
	e01	2e10	e01	mr	$2 x_4 x_2 \cdot 2^{-20}$	
	3e10	e01	e01	dv	$(2x_4x_2/x_3) \cdot 2^{-8}$ (rev.)	
	e01	14c00	e01	sn	θ (rev. 12^{-0})	
	13c00	e01	511	mr-	$2\pi\theta \cdot 2^{-3}$ rad.	
	a03	001	1a03	fi	file -> sin-cos	
	f00	510	e11	su	$1/7 = -\sin(4\pi x_4 x_2/x_3) \cdot 2^{-1}$	
	f00	511	1e11	ad-	$1/6 = \cos$ (") $\cdot 2^{-1}$	
	f00	-0	a24	cm	forced	

faa03

da061

(sin-cos subroutines)

```
02902a
038038038e4
027028038e5
0261ff002ac
02d1ff1ff85
1ff0290066d
1ff0290036d
1ff02a1ff44
021022003ed
0271ff1ff84
01f030030e4
01e1ff004ad
1fe1fe1fe04
01f01c1ffc5
01b01c017ed
1ff01f0056d
01c0191fec5
1ff0181fe41
```

(sin-cos subroutine continued)

```
1ff1ff1ff04
016017feded
01a1ff027ab
026026027e9
013011003e5
01201b1ffc5
0241ff1ff89
1ff01a1ff45
011fffffe5
ffe00dffded
00e01d01de5
01e1ff1ff89
00c01b007ed
00b00a01ce1
00900d01ce5
00801a01be1
00801a019e5
0061ff1fe85
005006ff1ed
015004ffc1
003004fdaed
1ff01a1ff45
1ff0201ff45
-00000000000
00000000001
00000100000
80000000000
ffffffffff
3243f6a8886
96cbe3f9991
6487ed5110b
c90fdaa2217
00000074f6a
-00001e28998
000541dad70
-00996964021
0a335e3337c
-52aef3988ae
c90fdaa2211
0
0
0
0
```

fac00

da003	8efa351294f			c00	$\pi/180 \cdot 2^{5}$	
	00000000015			1c00	$21 \cdot 2^{-44}$	
	0000000001e			2c00	30.2^{-44}	
def	.2	1d	-8b	3c00	$x_0^{0'} \cdot 2^{-8}$	init Δ'
	-.60	2d	-8b	4c00	$x_0^{0''} \cdot 2^{-8}$	init Δ"
	.2	1d	-8b	5c00	$\Delta_1 x_0^0 \, 2^{-8}$	
	.5	1d	-8b	6c00	$\Delta_2 x_0^0 \, 2^{-8}$	

da002	0000000001d				7c00 $29 \cdot 2^{-44}$
	00000000001				8c00 $1 \cdot 2^{-44}$
def	.1	1d	-8b		9c00 $\Delta x_3 \cdot 2^{-8}$
	.2	1d	-8b		10c00 $x_3^0 \cdot 2^{-8}$
	.9843	3d	-20b		11c00 $(984.3) \cdot 2^{-20}$
da003	04				12c00 $2 \cdot 2^{-7}$
	c90fdaa2216				13c00 $2\pi \cdot 2^{-3}$
	00000000008				14c00 $8 \cdot 2^{-44}$
fae01					e01 t_0
cd					
faa24	2c10	3e10	3e37	mr	$(y_1 = k_2 x_3) \cdot 2^{-12}$
	3e37	17c10	3e37	sn	$y_1 \cdot 2^{-8}$
	3e37	3e37	4e37	mr	$y_1^2 \cdot 2^{-16}$
	1e10	1e10	e38	mr	$x_1^2 \cdot 2^{-2}$
	e38	4c10	e38	sn	$x_1^2 \cdot 2^{-8}$
	1c10	e38	8e37	su	$(y_{2.1} = k_1 - x_1^2) \cdot 2^{-8}$
	8e37	8e37	e38	mr	$y_{2.1}^2 \cdot 2^{-16}$
	e38	4e37	511	ad	$(y_2^2 = y_{2.1}^2 + y_1^2) \cdot 2^{-16}$
	a22	001	1a22	fi	file -> sq. rt.
faa18	f00	511	e37	ad-	$y_2 \cdot 2^{-8}$
	a22	001	1a22	fi	file sq. rt.
	f00	511	5e37	ad	$(y_3 = y_2^{\frac{1}{2}}) \cdot 2^{-4}$
	8e37	5c10	e38	sn	$1/2\ y_{2.1} \cdot 2^{-9}$
	e37	e38	e38	dv	$1/2\ y_{2.1} / y_2 \cdot 2^{-1}$
	6c10	e38	511	ad	$(y_4^2 = \frac{1}{2} + y_{2.1}/2y_2) \cdot 2^{-1}$
	511	7c10	511	sn	$y_4 \cdot 2^{-2}$
	a22	001	1a22	fi	file -> sq. rt.
	f00	511	6e37	ad	$y_4 \cdot 2^{-1}$
	6c10	e38	511	su	$(y_5^2 = \frac{1}{2} - y_{2.1}/2y_2) \cdot 2^{-1}$
	511	7c10	511	sn	$y_5^2 \cdot 2^{-2}$

	a22	001	1a22	fi	file -> sq. rt.	
faa21	f00	511	7e37	su-	$y_5 \cdot 2^{-1}$	
	5e37	2e10	e38	mr	$y_3 \cdot x_2 \cdot 2^{-5}$	
	e38	6e37	1e37	mr	$(y_8 = y_3\, y_4\, x_2) \cdot 2^{-6}$	
	e38	7e37	2e37	mr	$(y_9 = y_3\, y_5\, x_2) \cdot 2^{-6}$	
faa25	f00	8c10	a26	ad	2a -> 2	1A
	f00	10c10	7e37	ad	$1 \cdot 2^{-16} \rightarrow y_{10}$	
	f00	11c10	9e37	ad	$1 \cdot 2^{-8} \rightarrow y_{11}$	
	10e37	10e37	10e37	su	$0 \rightarrow y_{12}$	
	5e37	5e37	5e37	su	$0 \rightarrow y_{13}$	
	f00	-0	a29	cm	forced	

faa30	f00	9c10	a26	ad	2B -> 2	1B
	12c10	4e37	7e37	ad	$(y_{10} = k_1^2 + y_1^2) \cdot 2^{-16}$	
	f00	1c10	9e37	ad	$(y_{11} = k_1) \cdot 2^{-8}$	
	3e37	1e37	10e37	mr	$(y_{12} = y_1\, y_8) \cdot 2^{-14}$	
	3e37	2e37	5e37	mr	$(y_{13} = y_1\, y_9) \cdot 2^{-14}$	
faa29	2e10	2e10	e38	mr	$x_2^2 \cdot 2^{-2}$	
	7e37	e38	6e37	mr-	$(y_{14} = y_{10}\, x_2^2) \cdot 2^{-18}$	
faa31	9e37	1e37	e38	mr	$y_{11} \cdot y_8 \cdot 2^{-14}$	
	5e37	e38	5e37	ad	$2(y_{13} + y_{11} \cdot y_8) \cdot 2^{-15}$	
	5e37	13c10	5e37	sn	(") $\cdot 2^{-12}$	
	e37	14c10	e38	sn	$y_2 \cdot 2^{-12}$	
	e38	5e37	5e37	ad	$[y_2 + 2(y_{13} + y_{11}\, y_8)] \cdot 2^{-12}$	
	6e37	15c10	e38	sn	$y_{14} \cdot 2^{-12}$	
	e38	5e37	5e37	ad-	$(y_{15} = y_{14} + y_2 + 2y_{13} + 2y_{11}y_8)2^{-12}$	
faa32	9e37	2e37	e38	mr	$(y_{11} \cdot y_9) \cdot 2^{-14}$	
	10e37	e38	10e37	su	$[y_{16} = 2^0(y_{12} - y_{11}y_9)]2^{-15}$	
	6e37	13c10	9e37	sn	$y_{14} \cdot 2^{-15}$	
	e37	16c10	e38	sn	$y_2 \cdot 2^{-15}$	
	9e37	e38	9e37	su	$(y_{17} = y_{14} - y_2) \cdot 2^{-15}$	

faa33	10e37	e11	6e37	mr	$(y_{18} = y_{16}\, y_7) \cdot 2^{-16}$	
	9e37	1e11	7e37	mr	$(y_{19} = y_{17}\, y_6) \cdot 2^{-16}$	
	10e37	1e11	10e37	mr	$(y_{21} = y_{16}\, y_6) \cdot 2^{-16}$	
	9e37	e11	9e37	mr	$(y_{22} = y_{17}\, y_7) \cdot 2^{-16}$	
	7e37	6e37	e38	su	$(y_{19} - y_{18}) \cdot 2^{-16}$	
	5e37	e38	6e37	dv	$[y_{20} = (y_{19} - y_{18})/y_{15}] \cdot 2^{-4}$	
	9e37	10e37	e38	ad	$(y_{22} + y_{21}) \cdot 2^{-16}$	
	5e37	e38	7e37	dv	$[y_{23} = (y_{22} + y_{21})/y_{15}] \cdot 2^{-4}$	
	7e37	7e37	7e37	mr	$(y_{23}^{\ 2}) \cdot 2^{-5}$	
faa26	f00	-0	a27	cm		2
faa27	3c10	6e37	2e11	ad	$1 + y_{20}$	2A
	2e11	2e11	2e11	mr	$(1 + y_{20})^2$	
	2e11	7e37	2e11	ad-	$[y_{25} = (1+y_{20})^2 + y_{23}^{\ 2}] \cdot 2^{-8}$	
	f00	-0	a30	cm	Forced -> 1B	
faa28	3c10	6e37	3e11	su	$(1 - y_{20}) \cdot 2^{-4}$	2B
	3e11	3e11	3e11	mr	$(1 - y_{20})^2 \cdot 2^{-8}$	
	3e11	7e37	3e11	ad-	$[y_{26} = (1-y_{20})^2 + y_{23}^{\ 2}] \cdot 2^{-8}$	
	f00	-0	a00	cm	forced	

def					
fac10	.0	0d	0b		c10 -0
	.10	2d	-8b		1c10 $k_1 = \Theta \cdot 2^{-8}$
	-.36756	-1d	0b		2c10 $(k_2 = -6\,\sigma \cdot 10^{12}) \cdot 2^0$
	.1	1d	-4b		3c10 $1 \cdot 2^{-4}$
	-.6	1d	-44b		4c10 $-6 \cdot 2^{-44}$
	-.2	1d	-44b		5c10 $1 \cdot 2^{-44}$
	.5	0d	-1b		6c10 $.5 \cdot 2^{-1}$
	-.1	1d	-44b		7c10 $-1 \cdot 2^{-44}$
cd	f00	-0	a27	cm	8c10 2A
	f00	-0	a28	cm	9c10 2B
def	.1	1d	-16b		10c10 $1 \cdot 2^{-16}$

.1	1d	-8b	11c10	$1 \cdot 2^{-8}$
.100	3d	-16b	12c10	$k_1^2 \cdot 2^{-16}$
.3	1d	-44b	13c10	$3 \cdot 2^{-44}$
-.4	1d	-44b	14c10	$-4 \cdot 2^{-44}$
.6	1d	-44b	15c10	$6 \cdot 2^{-44}$
-.7	1d	-44b	16c10	$-7 \cdot 2^{-44}$
.4	1d	-44b	17c10	$4 \cdot 2^{-44}$

faa22

da027 sq. rt.

```
 0130140000cd
 0121ff002ad
 011012ffeed
 0101ff002ac
-0011ff0000f
 1ff00001426
 01300f014e7
 013013012e5
 1ff01101167
 01000c010e7
 00900a00de5
 00800c1ffc5
 1ff00900b67
 1ff00c00b6b
 00900a009e5
 00000000cad
 00300a00ae4
 1ff0091ff47
 001002feeed
-00000000000
 ffffffffff
-00000000001
 0
 0
 0
 0
 0
```

fae38 e38 t_0

fae37 e37

ac429 10e37

cd

fab00	f00	-0	0	cm	exit
fab01	f00	b11	1b12	ad	xfer Σ_I
	f00	-0	b05	cm	forced

1

fab02	f00	1b11	1b12	ad	xfer Σ_{II}
	f00	-0	b05	cm	forced
fab03	f00	2b11	1b12	ad	xfer Σ_{III}
	f00	-0	b05	cm	forced
fab04	f00	3b11	1b12	ad	xfer Σ_{IV}
fab05	b07	-001	-001	-fi	file c_b and clear
	b00	5b11	4b11	ex	γ of exit -> t_0'
	b00	1b11	b00	ad	modify exit
	4b11	6b11	4b11	sn	shift t_0'+12 bits -> t_0'
	4b11	7b11	b13	ex	set up xfer order
fab13	f00	0	b06	ad	xfer r ∘ i order
fab06	0	0	0	ri	r∘i order
	f00	f37	b12	ad	xfer Σ
	b12	1b12	b10	cn	is $\Sigma \geq \Sigma_i$?
	1b12	b12	b10	cn	Is $\Sigma_i \geq \Sigma$?
fab07	0	4095m	-001	ba	reset c_b
	f00	-0	b00	cm	forced -> exit
fab10	-001	003	b06	ba	repeat ri 3 times
	001	b06	0	ro	print ri order
	002	b12	0	ro-	print Σ, Σ_i and halt
	f00	-0	b14	cm	forced -> rollback
fab11					
da008	0				b11 Σ_I
	000000001				1b11 $1 . 2^{-36}$
	0				2b11 Σ_{III}
	0				3b11 Σ_{IV}
	0				4b11 t_0'
	000000fff				5b11 γ exit
	0000000000c				6b11 $12 . 2^{-44}$
	000fff				7b11 b exit
fab12					b12 Σ

```
ac2b12                                          1b12 Σ i
cd
fab15    0       4095m   0      ba     set c_i and c_b
         b15     -001    -001   -fi    file c_b and clear
         f00     0       6b17   ad     save overflow
         f37     f37     f37    su     0 -> Σ
         f00     7b17    0      ad     normal overflow
fab16    f37     -0      f37    -ad  }
         -001    511     b16    ba   } mem Σ
         f00     f37     511    su     - Σ -> 511
         512     0       d35    ro     store mem.
         f00     6b17    0      ad     reset overflow
         f00     -0      b15    cm     forced -> exit
--------------------------------------------------------------
fab14    0       0       0      ba     clear c_b
fab09    -002    b17     0      ro     print rb
         -001    5b17    0      ro     print rb
         512     d35     0      ri     call in rollback
         f00     f37     b18    cm     is Σ = 0?
         f00     6b17    0      ad     reset overflow
         f00     -0      b15    cm     forced -> exit
--------------------------------------------------------------
fab18    -001    003     b09    ba     try 3 times
         -006    b17     0      ro     print "rb ng"
         003     e00     0      ro-    print i, j, k tallies and stop
         f00     -0      b14    cm     forced
--------------------------------------------------------------
fab17
da008    c8                            b17     r
         2c                            1b17    b
         40                            2b17    sp
         4c                            3b17    n
         88                            4b17    g
```

	d4				5b17 c.r.
	0				6b17 store overflow
	00000000001				7b17 normal overflow
ad1408m					
fdd35					d35 r.b. store
ac8b17					
fae00					e00 i . 2^{-44} Δ
ac3e00					1e00 j . 2^{-44} λ
					2e00 k . 2^{-44} β
fae10	e10 $x_0^i = \Delta \cdot 2^{-8}$ deg				
ac8e10	1e10 $x_1^j = \cos\Delta \cdot 2^{-1}$				
	2e10 $x_2^i = \sin\Delta \cdot 2^{-1}$				
fae11	3e10 $x_3^i = \lambda \cdot 2^{-12}$ ft.				e11 $y_7 \cdot 2^{-1}$
ac008	4e10 $x_6^k = \beta \cdot 2^{-8}$ deg				1e11 $y_6 \cdot 2^{-1}$
ad1088m	5e10 $x_7 = 1 \cdot 2^{-12}$ ft.				2e11 $y_{25} \cdot 2^{-8}$
	6e10 $x_4 = h \cdot 2^{-12}$ ft.				3e11 $y_{26} \cdot 2^{-8}$
fdd37	7e10 $x_5 = \phi \cdot 2^{-8}$ deg				4e11 $y_{27} \cdot 2^{-1}$
cd					5e11 $y_{28} \cdot 2^{-1}$
faa34	b00	001	b01	fi	call in part I
	231	d39	a00	ri	
fab33	0	0	0	ba	0 -> c_b
faa35	2e00	c02	a36	cn	is k $\geq$ 16?
	2e00	1c02	a37	cn	is k $\geq$ 46?
	2e00	2e00	2e00	su	0 -> k
	f00	-001	a11	fi	forced clear c_b
faa36	2e00	2c02	e02	ml	$k\ \Delta_1 x_6 \cdot 2^{-8}$
	3c02	e02	4e10	ad	$(x_6^k = x_6^{0'} + k\ \Delta_1 x_6) \cdot 2^{-8}$
	f00	-0	a38	cm	forced
faa37	2e00	4c02	e02	ml	$k\ \Delta_2 x_6 \cdot 2^{-8}$
	5c02	e02	4e10	ad	$(x_6^k = x_6^{0''} + k\Delta_2 x_6) \cdot 2^{-8}$
faa38	2e00	8c02	e02	sn	$(2k) \cdot 2^{-24}$
	b34	-001	-001	-fi	file c_b and clear
	e02	7c02	e02	ad	set up xfer order
	e02	9c02	a39	ex	
faa39	f00	-e04	-e03	-ad	sin-cos $x_6 \cdot 2^{-1}$

	-001	002	a39	ba	~~sin~~-cos $x_6 \cdot 2^{-1}$
fab34	0	4095m	-001	ba	reset c_b
	4e11	1e03	1e02	mr	$y_{27}\, y_{30} \cdot 2^{-2}$
	5e11	e03	4e02	mr	$y_{28}\, y_{29} \cdot 2^{-2}$
	1e02	4e02	e02	ad	$(y_{31} = y_{27}\, y_{30} + y_{28}\, y_{29}) \cdot 2^{-2}$
	1e02	4e02	1e02	su-	$(y_{32} = y_{27}\, y_{30} - y_{28}\, y_{29}) \circ 2^{-2}$
fab24	12c02	5e10	4e02	mr	$(\pi \cdot x_7) \circ 2^{-18}$
	3e10	4e02	2e03	dv	$(y_{35} = \pi\, x_7/x_3) \cdot 2^{-6}$
	2e03	2e03	3e03	mr	$(y_{35.1} = y_{35}) \cdot 2^{-12}$
	1e10	e02	e02	mr	$(x_1 y_{31}) \cdot 2^{-3}$
	10c02	e02	e02	su	$(y_{33} = 1 - x_1 y_{31}) \cdot 2^{-3}$
	1e10	1e02	1e02	mr	$(x_1 y_{32}) \cdot 2^{-3}$
	10c02	1e02	1e02	su-	$(y_{34} = 1 - x_1 y_{32}) \cdot 2^{-3}$
fab25	e02	11c02	b20	cn	is $y_{33} \geq 0$?
	2e03	e02	510	mr	$(y_{36} = y_{35}\, y_{33}) \cdot 2^{-9}$
	b28	001	b27	fi	file -> $\sin\theta/\theta$
	f00	511	2e02	ad-	$(y_{38} = \sin y_{36}/y_{36}) \cdot 2^{-1}$
	f00	-0	b21	cm	forced
fab20	f00	13c02	2e02	ad	$1 \circ 2^{-1} \rightarrow y_{38}$
fab21	1e02	11c02	b22	cn	is $y_{34} \geq 0$?
	2e03	1e02	510	mr	$(y_{37} = y_{35}\, y_{34}) \circ 2^{-9}$
	b28	001	b27	fi	file -> $\sin\theta/\theta$
	f00	511	3e02	ad-	$(y_{39} = \sin y_{37}/y_{37}) \cdot 2^{-1}$
	f00	-0	b23	cm	forced
fab22	f00	13c02	3e02	ad	$1 \cdot 2^{-1} \rightarrow y_{39}$
fab23	18c02	5e11	e02	mr	$(k_3 y_{28}) \cdot 2^{-9}$
	e02	3e03	e02	mr	$[(\;\text{"}\;)\, y_{35.1}] \cdot 2^{-21}$
	e02	2e02	e02	mr	$(\;\text{"}\;)\, y_{38} \cdot 2^{-22}$
	e02	3e02	e02	mr-	$(y_{40} = k_3 y_{28} y_{35.1} y_{38} y_{39}) \cdot 2^{-23}$
	4e11	1e10	4e02	mr	$(y_{27} x_1) \cdot 2^{-2}$
	1e03	19c02	1e02	sn	$y_{30}\; 2^{-2}$

	1e02	4e02	1e02	su	$(y_{41} = y_{30} - y_{27}x_1)\ 2^{-2}$
	1e02	1e02	1e02	mr	$(y_{42} = y_{41}^2)\ 2^{-4}$
	1e02	2e11	2e03	mr	$(y_{43} = y_{42}y_{25})\ 2^{-12}$
	e03	2e10	1e02	mr	$(y_{44} = y_{29}x_2)\ 2^{-2}$
	1e02	1e02	1e02	mr	$(y_{45} = y_{44}^2)\ 2^{-4}$
	1e02	3e11	3e03	mr	$(y_{46} = y_{45}y_{26})\ 2^{-12}$
	2e03	3e03	3e02	ad-	$(y_{47} = y_{43} + y_{46})\ 2^{-12}$
fab29	e02	0	4e02	px	$Px\ (y_{40})\ 2^{-44}$
	e02	4e02	e02	sn	$y_{40}\ 2^{-(23-px)}$
	20c02	4e02	4e02	su	$(23 - px)\ 2^{-44}$
	4e02	6c02	4e02	sn	$2(23 - px)\ 2^{-44}$
	e02	e02	e02	mr	$y_{40}^2\ 2^{-2(23-px)}$
	e02	0	2e02	px	$P'x\ (y_{40}^2)$
	e02	2e02	e02	sn	$y_{40}^2\ \ 2^{-2(23-px)+px'}$
	4e02	2e02	4e02	su-	$[2(23 - px) - px']\ 2^{-44}$
fab30	3e02	21c02	3e02	mr	$y_{47}(1/186.3)\ \ 2^{-12}$
	3e02	0	2e02	px	$Px''\ (y_{47})$
	3e02	2e02	3e02	sn	$y_{47}\ \ 2^{-(12-px'')}$
	22c02	2e02	2e02	su	$(12 - px'')\ 2^{-44}$
	3e02	e02	511	mr	$(y_{47}y_{40}^2)\ 2^{-58+px+px'+px''}$
	511	0	1e02	px	$Px'''(y_{47}y_{40}^2)$
	511	1e02	511	sn-	$(y_{47}y_{40}^2)\ \ 2^{-58+px+px'+px''+px'''}$
	2e02	1e02	2e02	su	$(12 - px'' - px''')\ 2^{-44}$
	4e02	2e02	2e02	ad-	K = exponent of $(y_{47}y_{40}^2)$
	b31	001	1b31	fi	file -> $\log_2$ routine
fab32	511	25c02	3e02	sn	$\log_2\ \ 2^{-8}$
	2e02	26c02	2e02	sn	$K\ 2^{-8}$
	2e02	3e02	3e02	ad	$\log_2(y_{47}y_{40}^2)=(K+\log_2)\ \ 2^{-8}$
	3e02 -e12	23c02 24c02	-e12 -e12	- mr- -ad	$[y_{48} = 10\log_{10}y_{47}y_{40}^2 - 10\log_{10}186.3]2^{-12}$
	6c02	2e00	2e00	ad	1 + k -> k
	-001	047	a35	ba	Repeat 46 times

	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	f00	-0	b14	om	forced -> rollback
fab27	14c02	510	4e02	mr	$(\theta/\pi) \cdot 2^{-9}$
	4e02	15c02	4e02	sn	$(\theta/2\pi) \cdot 2^{0}$ (rev) frac. part
	16c02	4e02	511	mr	$\theta \cdot 2^{-3}$ (rad)
	b26	001	1b26	fi	file -> sin rout.
	511	17c02	4e02	sn	$(\sin\theta) \cdot 2^{-10}$
	510	4e02	511	dv	$\sin\theta/\theta \cdot 2^{-1}$
fab28	f00	-0	0	om	exit
fab26					
da041					b26 sin rout

```
 01901a000cd
 0181ff002ac
 01c1ff1ff85
 1ff01900a6d
 1ff0190036d
 1ff0191ff44
 013014002ed
 0181ff1ff84
 1ff0140056d
 0100120lee5
 1ff00f01d61
 00e01c1ffc5
 00d00eff4ed
 00f1ff01aab
 00b009004e5
 018018019e9
 0100091ffc5
 0171ff1ff89
 1ff00f1ff45
 fff007fffe5
 ffe004ffded
 0121ff1ff89
 003004feaed
 1ff00f1ff45
 1ff0151ff45
-00000000000
 00000100000
 80000000000
 3243f6a8886
 96cbe3f9991
 c90fdaa2217
 6487ed5110c
 00000074f6a
-00001e28998
 000541dad70
-00996964021
 0a335e3337c
-52aef3988ae
```

```
c90fdaa2211
0
0

fab31

da032                                               b31 log2 rout

40000100006
1ff01c0126c
1ff01b1ff48
1ff01101b64
1ff01001b65
01a019019eb
00f018018eb
017017018e9
40000900365
40000d1ff45
1ff0151ff49
1ff00c1ff45
fff010fffe5
ffe005ffded
0101ff1ff89
1ff00e1ff44
400001ff026
1ff00c1ff45
1ff0111ff45
-0011ff0000f
5a827999fcf
2bec3330189
00000004a32
000000b0f06
00001e44962
00059f73f4f
013e5a65819
7ebbe72c5fe
000001
8
0
0

def
```

fac02	.16	2d	-44b		c02	16.2^{-44}
	.46	2d	-44b		1c02	46.2^{-44}
	.2	1d	-8b		2102	$\Delta_1 x_6 = 2.2^{-8}$
	.0	0d	0b		3c02	$x_6^{0'} = 0$
	.5	1d	-8b		4c02	$\Delta_2 x_6 = 5.2^{-8}$
	-.45	2d	-8b		5c02	$x_6^{0''} = -45.2^{-8}$
	.1	1d	-44b		6c02	1.2^{-44}
cd	0	e04	0	ri	7c02	initial sinβ add.
def	.21	2d	-44b		8c02	21.2^{-44}

da001	000fff			9c02 β ext.
def	.1	1d	-3b	10c02 1.2^{-3}
	.1	1d	-40b	11c02 $1.2^{-40} \approx 0$
da001	0c90fdaa221			12c02 $\pi.2^{-6}$
def	.1	1d	-1b	13c02 1.2^{-1}
da001	517cc1b7272			14c02 $(1/\pi).2^{0}$
def	.8	1d	-44b	15c02 8.2^{-44}
da001	c90fdaa2217			16c02 $2\pi.2^{-3}$
def	-.9	1d	-44b	17c02 -9.2^{-44}
	.128988	3d	-8b	18c02 $k_3.2^{-8}$
	-.1	1d	-44b	19c02 -1.2^{-44}
	.23	2d	-44b	20c02 23.2^{-44}
	.005367686	0d	0b	21c02 (1/186.3)
	.12	2d	-44b	22c02 12.2^{-44}
	.3010299957	1d	-4b	23c02 $10(\log_{10}2).2^{0}$
	.22702128546	2d	-12 b	24c02 $(10\log_{10}186.3$ 2^{-12}
	-.8	1d	-44b	25c02 -8.2^{-44}
	.36	2d	-44b	26c02 36.2^{-44}

fae04

da092

e04 sin-cos $x_6.2^{-1}$

00000000000
80000000000
047796327e0
7fec09e2fc1
08edc7b6b51
7fb02dc5c3f
0d61304d9e2
7f4c7e53e19
11d06c968d7
7ec11aa4c25
163a1a7e0b6
7e0e2e32047
1a9cd9ac426
7d33f0c9e80
1ef74bf2e4a
7c32a67de6e
234815ba651
7b0a9f8d79e

91e04

278dde6e5fe
79bc384d114
2bc750e9140
7847d90948b
2ff31bddcb3
76adf5e65f2
340ff2420c1
74ef0ebbfe0
381c8bb57c0
730baeed58e
3c17a4e848b
71046d3db49
3ffffffffff
6ed9eba1614
496af3e1f13
68d9f964561
5246dd48f03
620dbe8b3d5
5a827999fcf
5a827999fcf
620dbe8b3d2
5246dd48f07
68d9f964561
496af3e1f13
6ed9eba1614
3ffffffffff
7401e4c0b75
36185aee6f5
7847d90948b
2bc750e9141
7ba3751d22e
2120fb83262
7e0e2e32046
163a1a7e0b6
7f834ed0605
0b27eb5c616
80000000000
-00000000005
7f834ed0605
-0b27eb5c616
7e0e2e32045
-163a1a7e0ba
7ba3751d22e
-2120fb83262
7847d90948a
-2bc750e9144
7401e4c0b75
-36185aee6f5
6ed9eba1612
-40000000003
68d9f96455f
-496af3e1f17
620dbe8b3d2
-5246dd48f07
5a827999fcb
-5a827999fd2
5246dd48f03
-620dbe8b3d5
496af3e1f0f

```
-68d9f964564
 3ffffffffff
-6ed9eba1614
 36185aee6f0
-7401e4c0b77
 2bc750e9140
-7847d90948b
 2120fb8325e
-7ba3751d22f
 163a1a7e0b6
-7e0e2e32047
 0b27eb5c611
-7f834ed0605
-00000000004
-80000000000
 cd
 faa11    3e15    3e15    3e15    su    0 -> p.o.
          f00     7e15    6e15    ad    c.r. -> p.o.
          e00     3c01    511     sn    set up i for p.o.
 faa13    a12     001     1a12    fi    file -> digit store
          f00     510     -1e15   -ad   store i  for p.o.
          -001    002     a13     ba    ba 2 times
          1e00    3c01    511     sn    set up j for p.o.
 faa14    a12     001     1a12    fi    file -> digit store
          f00     510     -4e15   -ad   store j for p.o.
          -001    002     a14     ba    ba 2 times
          -007    e15     002     ro    print i and j (punch)
 faa16    -e12    11c01   -e12    -ad   round a_k  to 2 dec.        200
          -e12    5c01    a17     -cm   is a_k ≥ 100?
          f00     6c01    3e15    ad  }
          f00     6c01    4e15    ad  } x -> t_1  t_2  t_3
          f00     6c01    5e15    ad  }
          f00     -0      1a20    cm    forced
-----------------------------------------------------------------
 faa17    -e12    7c01    511     -mr   a_k . 2^12/10^4
          511     001     511     bd    conv. 511
          511     8c01    511     sn    conv. no. 2^-6
          a20     -001    -001    -fi   file c_b and clear
 faa19    a12     001     1a12    fi    file -> digit store
          f00     510     -3e15   -ad   d_10(d_11)-> t_1(t_2)
```

	-001	002	a19	ba	ba 2 times
faa20	0	4095m	-001	ba	k -> c_b
	-e12	c01	b35	-cn	is $a_k \geq 0$?
	f00	7e15	6e15	ad	c.r. -> p.o.
	-004	3e15	002	ro	print + a_k
	f00	-0	b36	cm	forced
fab35	f00	9c01	6e15	ad	㊀ -> p.o.
	-005	3e15	002	ro	print $-a_k$
fab36	-001	046	a16	ba	ba 46 times
	-002	7e15	002	ro	print c.r. and stop code
	2e00	2e00	2e00	su	0 -> k
	12c01	1e00	1e00	ad	1 + j -> j
	f00	-0	a00	cm	forced
cd					
faa12	f00	-0	0	cm	exit
	511	1c01	510	ex	b_1-b_6 -> 510
	c01	1c01	511	ex	0 -> b_1-b_6
	511	2c01	511	sn	shift 511 (+4) bite
	f00	-0	a12	cm	forced -> exit
fac01					
da007	-0				c01 -0
	fc				1c01 b_1-b_6 ext
	00000000004				2c01 4.2^{-44}
	00000000022				3c01 34.2^{-44}
	fff				4c01 α ext.
	064				5c01 100.2^{-12}
	5c				6c01 x
def	.4096	0d	0b		7c01 $2^{12}/10^4 \cdot 2^0$
da004	00000000006				8c01 6.2^{-44}
	60000000000				9c01 ㊀
	01				10c01 $1 \cdot 2^{-8}$

	0000d		A19	11c01 $d.2^{-20}$ $5x10^{-2}$
def	.1	1d	-44b	12c01 1.2^{-44}
fae15				
da009	d4			e15 c.r.
	0			1e15
	0			2e15
	0			3e15 t_1
	0			4e15 t_2
	0			5e15 t_3
	0			6e15 c.r. or $\ominus$
	d4			7e15 c.r.
	8e			8e15 stop code
fae02				e02 t_0
ac5e02				1e02 t_1
fae03				2e02 t_2
ac4e03				3e02 t_3
fae12				4e02 t_4
bca01				

e03 $y_{29} \cdot 2^{-1}$

1e03 $y_{30} \cdot 2^{-1}$

2e03 $y_{35} \cdot 2^{-6}$

3e03 $y_{35.1} \cdot 2^{-12}$

e12
⋮
45e12 } $a_k \cdot 2^{-12}$

APPENDIX B

The following are the values of the antenna gain for the types of antennas as listed in the headings.

The constants used are:

$K = 128.988$

$\epsilon = 10$

$\sigma = 2 \cdot 10^{-14}$ e.m.u.

Each table gives the value of the antenna gain for the 46 values of β as listed in the table of parameter variations. The values of Δ and f are listed in the heading.

The first word of each table is an identity word, indicating the value of Δ and f as listed in the table of parameter variations. (Table II).

TYPE C f = 2 mc

Δ=2	4	6	8	10	12	14	16	18	20
00000	01000	02000	03000	04000	05000	06000	07000	08000	09000
491-	429-	390-	361-	336-	315-	295-	276-	259-	243-
479-	419-	383-	356-	332-	312-	293-	275-	258-	242-
455-	400-	367-	343-	322-	304-	287-	270-	254-	239-
432-	379-	349-	327-	309-	293-	278-	263-	249-	235-
412-	361-	332-	312-	296-	282-	268-	256-	243-	230-
396-	345-	317-	298-	283-	271-	259-	247-	236-	224-
382-	331-	304-	286-	272-	260-	249-	239-	228-	218-
370-	319-	293-	275-	261-	250-	240-	231-	221-	212-
359-	309-	283-	265-	252-	241-	232-	223-	214-	206-
350-	300-	274-	256-	243-	233-	224-	215-	207-	199-
342-	292-	265-	248-	236-	225-	217-	209-	201-	194-
335-	285-	258-	241-	228-	218-	210-	202-	195-	188-
328-	278-	252-	235-	222-	212-	204-	196-	189-	183-
322-	272-	246-	229-	216-	206-	198-	191-	184-	178-
317-	267-	241-	223-	211-	201-	193-	186-	179-	173-
312-	262-	236-	219-	206-	196-	188-	181-	174-	168-
303-	253-	226-	209-	196-	186-	178-	171-	165-	159-
296-	246-	219-	201-	188-	178-	170-	163-	157-	151-
292-	241-	214-	196-	183-	173-	165-	158-	151-	146-
291-	240-	212-	194-	181-	170-	162-	154-	148-	142-
292-	240-	213-	194-	181-	170-	161-	154-	147-	141-
296-	244-	216-	197-	183-	172-	163-	155-	149-	143-
303-	251-	222-	203-	189-	177-	168-	160-	153-	147-
314-	261-	232-	212-	198-	186-	176-	168-	161-	155-
328-	275-	245-	225-	210-	198-	188-	180-	172-	166-
347-	293-	263-	243-	227-	215-	205-	196-	188-	181-
372-	318-	287-	267-	251-	238-	227-	217-	209-	202-
405-	351-	320-	298-	282-	268-	256-	246-	237-	229-
453-	397-	365-	343-	325-	311-	298-	287-	277-	267-
532-	476-	442-	418-	398-	381-	365-	351-	337-	325-
697-	650-	635-	645-	699-	739-	600-	535-	489-	454-
539-	483-	453-	434-	421-	413-	408-	407-	409-	416-
505-	449-	418-	397-	382-	371-	363-	357-	354-	352-
497-	440-	408-	387-	371-	358-	349-	342-	336-	333-
503-	445-	413-	391-	374-	361-	351-	342-	336-	331-
521-	463-	430-	407-	390-	376-	365-	355-	348-	341-
555-	496-	463-	439-	420-	405-	393-	382-	373-	365-
622-	562-	527-	501-	480-	463-	447-	434-	421-	410-
773-	724-	707-	715-	769-	809-	669-	605-	561-	526-
600-	543-	512-	491-	478-	469-	465-	464-	467-	475-
551-	493-	461-	439-	424-	413-	405-	400-	398-	398-
525-	466-	433-	411-	395-	383-	374-	368-	364-	362-
508-	450-	416-	394-	377-	365-	356-	349-	344-	341-
498-	439-	406-	383-	367-	354-	345-	337-	332-	329-
493-	434-	400-	378-	361-	348-	338-	331-	326-	322-
491-	432-	399-	376-	359-	346-	336-	329-	324-	320-

TYPE C f = 2 mc

Δ=22	24	26	28	30	32	34	36	38	40
0a000	0b000	0c000	0d000	0e000	0f000	10000	11000	12000	13000
228-	214-	200-	188-	176-	165-	155-	145-	136-	128-
227-	213-	200-	188-	176-	165-	155-	145-	136-	128-
225-	212-	199-	187-	175-	164-	154-	145-	136-	128-
222-	209-	197-	185-	174-	163-	153-	144-	135-	127-
218-	206-	194-	183-	172-	162-	152-	143-	135-	127-
213-	202-	191-	180-	170-	160-	151-	142-	134-	126-
208-	198-	187-	177-	167-	158-	149-	141-	132-	125-
202-	193-	183-	174-	165-	156-	147-	139-	131-	124-
197-	188-	179-	171-	162-	153-	145-	137-	130-	123-
191-	183-	175-	167-	159-	151-	143-	135-	128-	121-
186-	179-	171-	163-	156-	148-	141-	133-	127-	120-
181-	174-	167-	160-	152-	145-	138-	131-	125-	119-
176-	169-	163-	156-	149-	142-	136-	129-	123-	117-
171-	165-	159-	152-	146-	140-	133-	127-	121-	116-
167-	161-	155-	149-	143-	137-	131-	125-	120-	114-
163-	157-	151-	146-	140-	134-	129-	123-	118-	113-
153-	148-	143-	138-	133-	128-	123-	119-	114-	110-
146-	141-	136-	132-	127-	123-	119-	115-	111-	107-
141-	136-	131-	127-	123-	119-	115-	112-	108-	105-
137-	133-	128-	124-	121-	117-	113-	110-	107-	104-
136-	132-	127-	124-	120-	117-	113-	110-	108-	105-
138-	133-	129-	125-	122-	118-	115-	112-	110-	107-
142-	137-	133-	129-	126-	122-	119-	117-	114-	112-
149-	144-	140-	136-	132-	129-	126-	123-	121-	118-
160-	155-	150-	146-	142-	139-	136-	133-	130-	127-
175-	170-	165-	160-	156-	152-	148-	145-	142-	139-
195-	189-	184-	178-	174-	169-	165-	161-	158-	154-
222-	215-	209-	203-	197-	192-	187-	182-	177-	173-
258-	250-	242-	235-	228-	221-	215-	208-	203-	197-
313-	301-	291-	280-	270-	261-	252-	243-	235-	227-
424-	398-	376-	356-	338-	321-	306-	292-	279-	267-
427-	448-	484-	573-	560-	460-	410-	375-	347-	325-
353-	356-	362-	371-	385-	405-	440-	514-	565-	441-
330-	330-	331-	334-	339-	347-	358-	374-	398-	439-
327-	324-	323-	323-	325-	328-	334-	342-	353-	369-
336-	332-	329-	327-	327-	327-	330-	334-	340-	349-
358-	352-	347-	343-	341-	339-	339-	340-	342-	347-
400-	390-	382-	375-	369-	364-	360-	358-	357-	359-
498-	475-	455-	438-	423-	411-	400-	392-	386-	383-
489-	511-	550-	642-	633-	536-	491-	462-	441-	427-
401-	406-	414-	426-	444-	469-	508-	588-	646-	530-
362-	364-	368-	375-	384-	396-	412-	435-	466-	516-
340-	341-	343-	348-	354-	362-	374-	389-	408-	433-
327-	327-	329-	332-	337-	344-	353-	365-	380-	400-
320-	320-	321-	323-	328-	334-	342-	353-	367-	384-
318-	317-	318-	321-	325-	331-	339-	350-	363-	379-

TYPE C f = 2 mc

Δ=42	45	50	55	60	65	70	75	80	85
14000	15000	16000	17000	18000	19000	1a000	1b000	1c000	1d000
120-	110-	096-	086-	079-	076-	076-	081-	089-	102-
120-	110-	096-	086-	079-	076-	076-	081-	089-	102-
120-	110-	096-	085-	079-	076-	076-	081-	089-	102-
120-	110-	096-	085-	079-	076-	076-	081-	090-	103-
119-	109-	096-	085-	079-	076-	077-	081-	090-	103-
119-	109-	095-	085-	079-	076-	077-	081-	090-	103-
118-	108-	095-	085-	079-	076-	077-	081-	090-	103-
117-	108-	095-	085-	079-	076-	077-	082-	090-	103-
116-	107-	094-	085-	079-	076-	077-	082-	090-	103-
115-	106-	094-	085-	079-	077-	078-	082-	091-	103-
114-	105-	094-	085-	079-	077-	078-	083-	091-	103-
113-	105-	093-	085-	079-	077-	078-	083-	091-	104-
112-	104-	093-	085-	079-	077-	079-	083-	092-	104-
110-	103-	092-	085-	080-	078-	079-	084-	092-	104-
109-	102-	092-	085-	080-	078-	080-	084-	092-	104-
108-	101-	092-	085-	080-	079-	080-	085-	093-	105-
105-	099-	091-	085-	081-	080-	082-	086-	094-	105-
103-	098-	091-	086-	082-	082-	084-	088-	096-	106-
102-	097-	091-	087-	084-	084-	086-	090-	097-	107-
101-	098-	092-	089-	087-	087-	089-	093-	099-	109-
103-	099-	095-	091-	090-	090-	092-	096-	102-	110-
105-	102-	098-	095-	094-	094-	095-	099-	104-	111-
110-	107-	103-	100-	099-	098-	100-	102-	107-	113-
116-	113-	109-	106-	104-	104-	104-	106-	109-	114-
125-	122-	117-	114-	111-	110-	110-	110-	112-	116-
136-	133-	127-	123-	120-	117-	116-	115-	116-	117-
151-	146-	140-	134-	129-	125-	122-	120-	119-	119-
169-	163-	154-	147-	140-	134-	129-	125-	123-	121-
192-	184-	172-	161-	152-	144-	137-	131-	126-	123-
220-	209-	193-	178-	166-	155-	145-	137-	130-	125-
256-	240-	217-	198-	181-	166-	154-	143-	134-	126-
305-	281-	247-	220-	198-	179-	163-	149-	138-	128-
388-	338-	284-	246-	216-	192-	173-	156-	142-	130-
546-	445-	331-	275-	236-	207-	182-	162-	146-	132-
394-	476-	401-	309-	258-	221-	192-	169-	150-	134-
363-	397-	573-	350-	281-	236-	202-	175-	153-	135-
355-	376-	478-	400-	305-	251-	212-	182-	157-	137-
362-	374-	432-	465-	330-	266-	222-	187-	160-	138-
382-	386-	423-	569-	354-	280-	230-	193-	163-	139-
417-	411-	430-	692-	377-	293-	239-	198-	166-	141-
486-	454-	451-	598-	397-	304-	246-	202-	168-	142-
632-	550-	486-	601-	413-	314-	252-	206-	170-	142-
469-	570-	548-	645-	425-	322-	257-	209-	172-	143-
425-	482-	716-	787-	432-	327-	260-	211-	173-	144-
406-	452-	623-	720-	436-	331-	262-	212-	174-	144-
400-	444-	590-	682-	437-	332-	263-	213-	174-	144-