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FINAL REPORT

# DESIGN AND DEVELOPMENT OF AN UNDERWATER SOUND VELOCITY METER

By

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ABSTRACT

A new type of underwater sound velocity meter is described which makes continuous recordings of the velocity of sound in the ocean as a function of depth. The meter improves the accuracy of present methods of velocity determination by its ability to measure sound velocity directly rather than merely the temperature of the water. It employs a phase-comparison system of measurement, transmitting a 510-kilocycle acoustic wave through a 1-foot sample of sea water. Two quartz crystals are used as transducer elements. The voltage across the transmitting crystal is compared in phase with the voltage across the receiving crystal, and the phase difference between these two voltages is shown to be a nearly linear function of the velocity of propagation of the acoustic wave.

As a calibration for the velocity meter, a comparison system for measuring the wavelength and velocity of a sound wave in the water of a laboratory tank has been set up. Wavelength is determined by measuring the distance between equiphase planes. The results of this series of tests indicates sound velocities higher than those predicted from tables in current use.

Velocity recordings and measurements of temperature, salinity, and pressure are reported for several test locations in the Atlantic Ocean. The measured difference in the sound velocity between ocean layers has been compared with the predicted velocity difference as determined from tables, and the agreement has been found to be good.

ACKNOWLEDGEMENTS

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# INTRODUCTION

The velocity of sound in the ocean depends upon the temperature, the pressure, and the amount of dissolved salt in the water. These three velocity-influencing factors are not constant. All three may change with depth, and temperature and salinity may change from point to point in a horizontal plane. The purpose of this report is to describe an entirely new underwater sound velocity measuring and recording instrument and a series of experiments in which the velocity of sound in the ocean was measured with high accuracy as a continuous function of depth.

The velocity of sound in the ocean acquired considerable practical importance when sonic depth finders were first developed and began to be used as reliable navigational instruments. Before that time the study of bottom topography had been limited to the extremely laborious and somewhat inaccurate method of taking depth soundings by means of rope or wire cable. In the 19th century few soundings were obtained from depths of greater than a few hundred meters, and it was not until the 1920's, when the determination of ocean depth by sonic echo methods became a practical reality, that the ocean bottom was found to be as rugged as the topography of any mountain landscape. A knowledge of ocean depth has been a great aid to navigation, since the passage over irregularities on the sea floor can be used to check a ship's position.

The development of echo-ranging equipment, spurred by the tremendous effectiveness of the submarine in the first and second world wars, has increased greatly the importance of knowing the velocity of underwater sound accurately. Sound waves in general follow the well-known laws of optics. When a sound wave passes from one region into another which has a different velocity, the wave is refracted in the direction of the water having the lower velocity. Thus, with a velocity structure unfavorable for echo ranging, the presence of an approaching submarine may not be detected until it has come much closer than the maximum range of the sonic detection equipment under good sound conditions. A knowledge of the velocity of sound at various depths will permit the detecting ship to predict its maximum range of detection, or conversely, will tell a submarine how best to operate so as to avoid being detected.

## The Range of Sound Velocity Encountered in the Ocean

Sound velocity increases with temperature, pressure, and salinity.* Differences in temperature with depth, which may range from about 85°F to 30°F in extreme cases, account for the greatest variation in the velocity of sound. The basic thermal structure of the deep ocean at mid-latitudes is sketched in Fig. 1. The surface layer is generally characterized by great variability and may differ considerably from the isothermal condition indicated in Fig. 1. It is influenced, for example, by such factors as wind and sea state, seasonal changes in temperature, and geographical variations in

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*Salinity is defined as the number of grams of dissolved salt in 1000 grams of sea water. It is generally indicated in parts per thousand by the symbol 0/00.

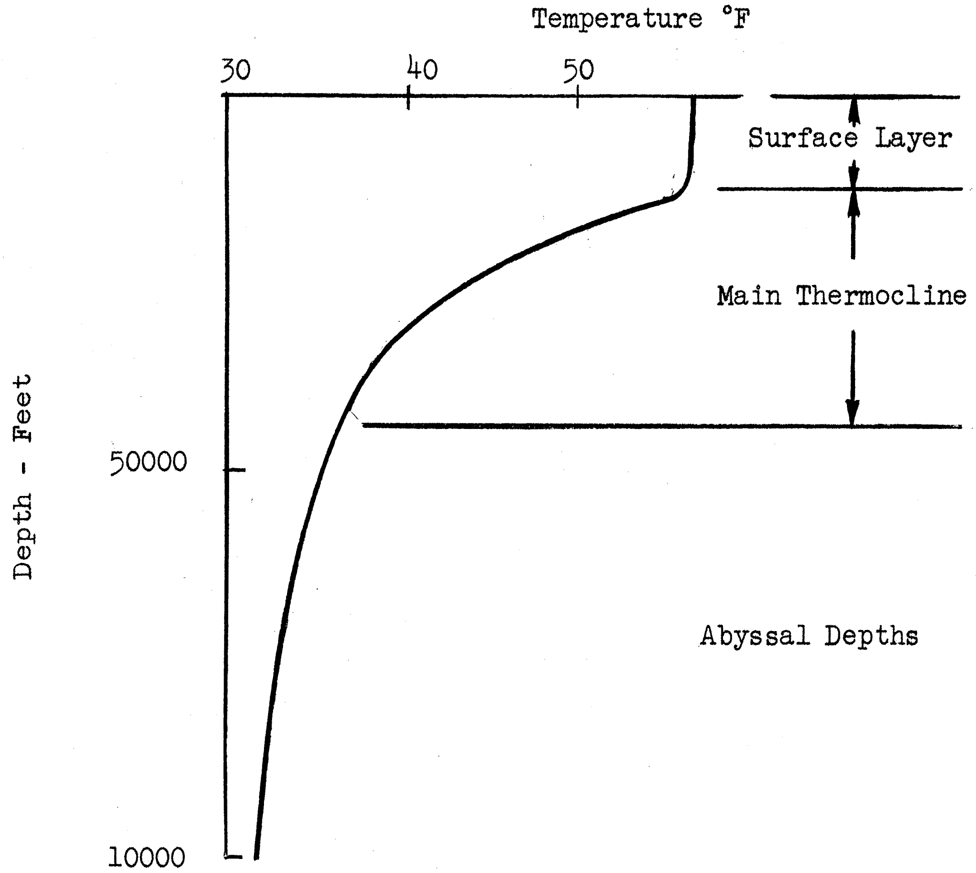



Fig. 1. **Basic thermal structure of the deep** ocean

climate; in tropical latitudes the isothermal surface layer may become relatively thin or even disappear. The main thermocline has the property of rapidly decreasing temperature with depth. Below the main thermocline the temperature falls much more gradually, and in this deep ocean conditions are relatively static. It is interesting to note that the temperature structure of many Michigan lakes in summer is exactly similar to Fig. 1, although of course the depth scale is quite different. An increase of 40° in the temperature of the water, from 40°F to 80°F, for example, increases the sound velocity by about 5 per cent.

The underwater sound velocity meter has been constructed to operate from the ocean surface down to depths of 400 feet. In this region the change in sound velocity due to an increase in pressure is small but not negligible. Increasing the depth by 100 feet increases sound velocity by about 1.8 feet per second, an increase of approximately 0.036 per cent. Similarly, the change in sound velocity due to a change in the salinity of the ocean is usually considerably less than that associated with a temperature difference. An increase of one part per thousand in salinity causes an increase of about 4.3 feet per second in velocity, or approximately 0.086 per cent. Salinity in the ocean usually ranges by only a few parts per thousand from a value of 35 0/00, but there are cases, for example in the Grand Banks of Newfoundland, where salinity changes from 32 to 36 0/00 in a small region.

From a consideration of the expected range of temperature, pressure, and salinity it was concluded that the measurements of sound velocity would fall within the limits of about 4750 and 5050 feet per second. The velocity figures mentioned here are based upon tables of sound velocity in the ocean and are described more fully in a later section of this report.

## Methods for Measuring the Velocity of Sound in the Laboratory

A number of instruments are in existence for the measurement of the velocity of sound. In general, however, these instruments are laboratory devices which require careful adjustment and the constant attention of an operator. Furthermore, they are devices not easily adapted to the continuous recording of sound velocity or suitable for operation in the field.

Perhaps the most common velocity-measuring instrument is the interferometer first described by Pierce in 1925.[1] It employs an x-cut quartz crystal, vibrating in a thickness mode, and a movable reflecting plate which is always maintained carefully parallel to the face of the crystal. An acoustic wave starts from the crystal, travels a short distance in the fluid being measured and is returned to the crystal after reflection from the plate. When the path length between the crystal and the plate is an odd multiple of one-half the wavelength, a resonance condition is set up in the fluid column between these two elements. This condition is indicated by a sharp change in the radio-frequency current in the electric circuit driving the crystal. The interferometer is actually a self-calibrating instrument for measuring wavelength. Resonance occurs every time the reflecting plate is moved through a distance of one-half the wavelength. It is interesting to note that somewhat similar principles are used in a completely different field, microwave engineering, to determine wavelength by means of a coaxial wavemeter.

An entirely different method of measuring the velocity of ultrasonic waves in a liquid is based upon the fact that plane sound waves of ultrasonic frequency passing through a liquid cause alternate regions of higher and lower density. These regions act in much the same manner as the rulings on a diffraction grating, producing diffraction of a beam of light traversing the liquid in a direction normal to the direction of propagation of the acoustic wave. This phenomenon has been described by Debye and Sears.[2] Wavelength may be determined from photographs of the diffraction pattern.

A third method, which has been used to measure sound velocities in recent times, makes use of pulse techniques which are similar to those employed in radar. Essentially the measurement consists of determining the time required for the transmission of a pulse, usually containing ten or more cycles of the acoustic wave, over a known distance. Either the time of direct transmission between two spaced transducers or the time of the return of an echo to a single transducer is measured. This latter system has also been used for the location of flaws in metals when the sound velocity is known.[3]

The above methods and modifications of these methods were considered for use in the measurement of underwater sound velocity. However, it appeared desirable to construct an entirely new type of velocity meter, a continuous recording instrument that could be made to operate without the use of movable reflecting plates, photographic equipment or even elaborate time-measuring circuits. Such a device has been built and is described in the next section.

## THE UNDERWATER SOUND VELOCITY METER; GENERAL DESCRIPTION

A system of velocity measurement involving the phase comparison of two acoustic wavefronts seemed to be promising from the standpoint of both simplicity and accuracy. A plane, progressive acoustic wave can be sampled at two points a fixed distance apart in the direction of propagation. The phase difference between the two points is a function of the phase velocity of the acoustic wave. This is more easily understood in terms of the sketch, shown in Fig. 2. In the figure the quantity varying

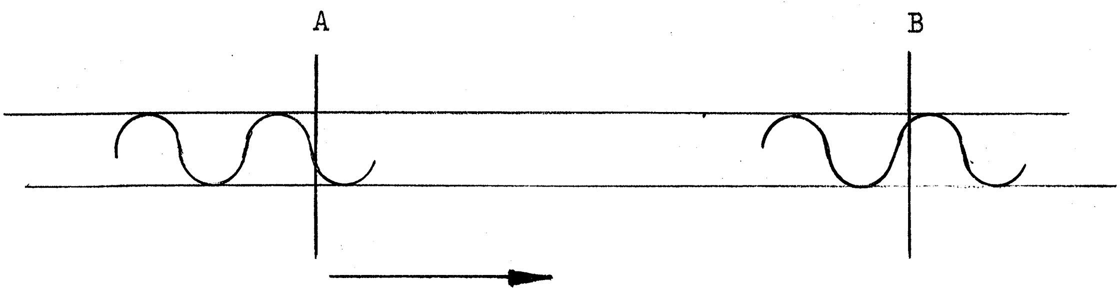



Fig. 2. Wave motion of a plane progressive wave.

sinusoidally with time and distance may be, for example, the acoustic pressure; and the wave propagation is assumed to be in a direction from A to B. The wavefront at A, a plane in which all particles have the same phase, advances and crosses plane B at some later time. The time for a particular wavefront to advance from A to B is, of course, related to the velocity of the sound wave and to the separation distance of the two planes; that is, $t = d/c$, where $t$ = time in seconds, $d$ = separation in feet, and $c$ = velocity in feet per second.

This time delay may be expressed as a phase difference between the two pressure wavefronts, one at A and the other at B. Expressing the time delay indicated above as a fraction of a period, T, and multiplying this fraction by 360° yields the phase delay by which B lags A in degrees. Symbolically this may be written as

$$\theta = 360\ t/T = 360(d/c)f, \text{ where } f = \text{frequency} = 1/T.$$

Thus it is evident that if the separation and the frequency are known and are held constant, then the velocity of the sound wave can be determined from a knowledge of the phase delay. Furthermore, if the product of d and f is large, then small changes in the velocity will produce sizeable changes in the phase angle. The sensitivity of the system is increased by increasing distance d or increasing frequency f. It should be pointed out that an increase in the sound velocity produces a decrease in the phase angle. The selection of a good operating frequency for a velocity-measuring instrument employing this phase-comparison system involves a considerable amount of engineering judgment. It is convenient to build an instrument of small dimensions; on the other hand, the reduction of the crystal spacing distance will reduce the sensitivity of the meter. Operating at higher frequencies will restore the sensitivity lost in going to smaller dimensions. Interferometers and other precision laboratory instruments for measuring velocity operate most successfully at ultrasonic frequencies of a megacycle or higher. However, the frequency region of primary interest at present for the measurement of velocity is from 15 to 30 kc, somewhat above the upper limit of hearing.

At the present time there is little evidence for assuming the existence of frequency dispersion in water. In fact, a fairly large amount of experimental evidence tends to point to the absence of dispersion. Records of the arrival of explosive sound waves which have traveled over considerable distances in deep sound channels show the almost simultaneous time of arrival of sounds of various frequencies. For this reason it was decided that velocity information obtained at a moderately high ultrasonic frequency would be useful, and that a frequency of 510 kc would represent a satisfactory compromise between physical dimensions and sensitivity.

The underwater sound velocity meter makes use of the phase-comparison system described above to measure small changes in the velocity of sound in the ocean. The present meter, designed to operate from a surface ship, has been built in two parts. The submerged portion of the meter, capable of operating down to a depth of 400 feet, is connected by means of a cable to the recording equipment, which remains in the surface ship. The submerged portion includes two quartz crystals as the transducer elements and also the electronic circuits associated with the generation of the sound wave and the measurement of phase. The portion of the meter remaining on the surface consists of a recorder and a servo-mechanism to drive the recorder chart through a distance proportional to the depth of the submerged instrument. With this recording arrangement the coordinates of the chart are sound velocity vs depth, and the velocity is continuously recorded as the instrument is lowered from the ship. Fig. 3 below illustrates the method of measurement from a surface ship. Fig. 4 shows the various components of the velocity meter. Fig. 5 is a photograph of the assembled meter.

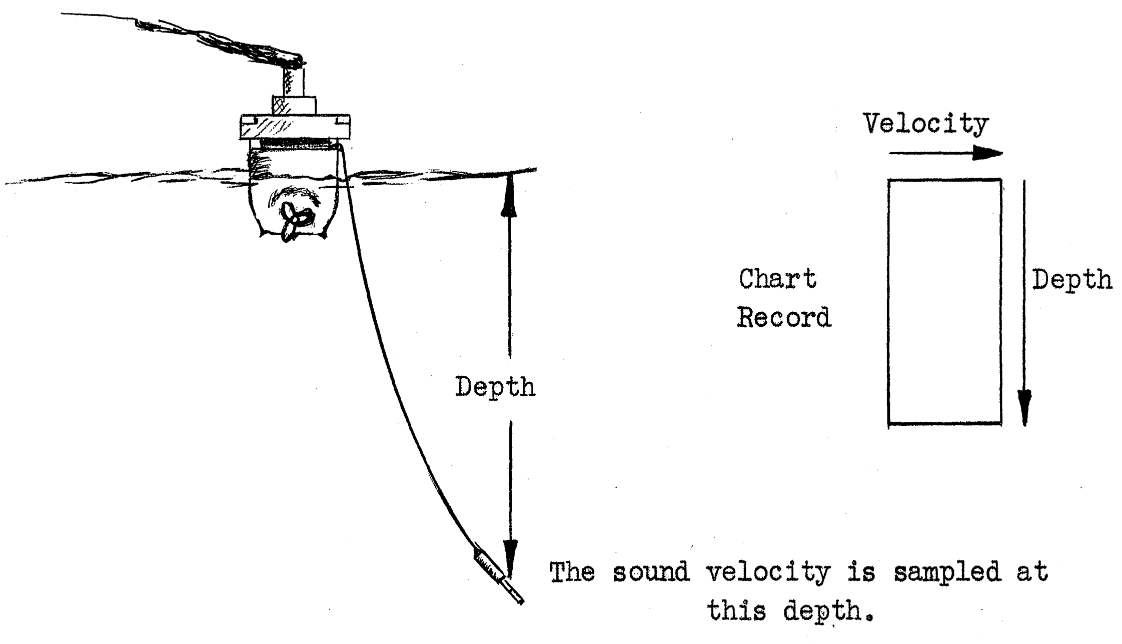



Fig. 3. Lowering meter from a ship; the coordinate system.

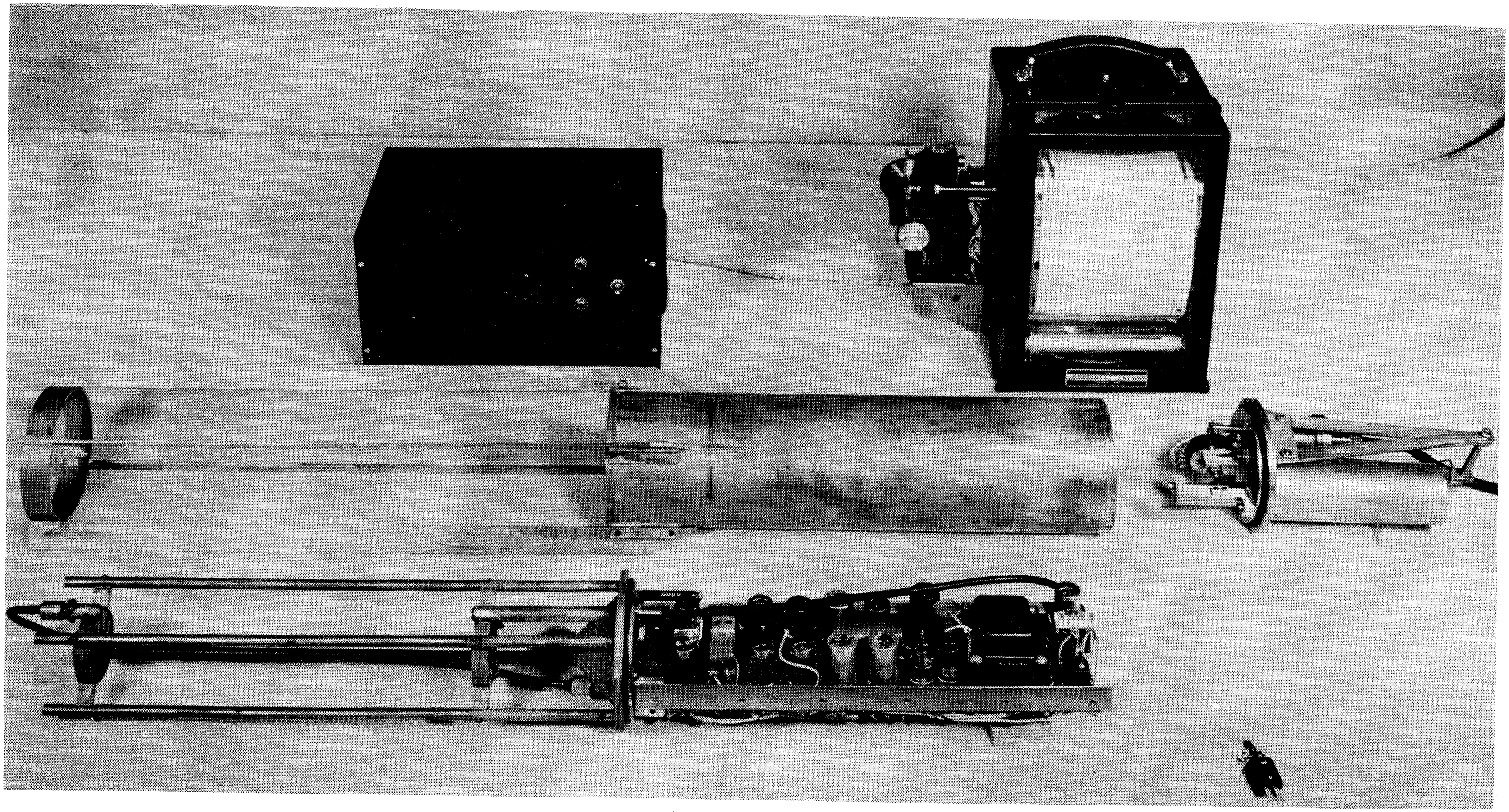


Fig. 4. An exploded view of the meter, with the electronic chassis removed from the watertight chamber. The recorder, which remains on the surface ship, is shown being driven by a small servo-motor. The black cabinet at the left of the recorder contains circuits used in the servo-system.

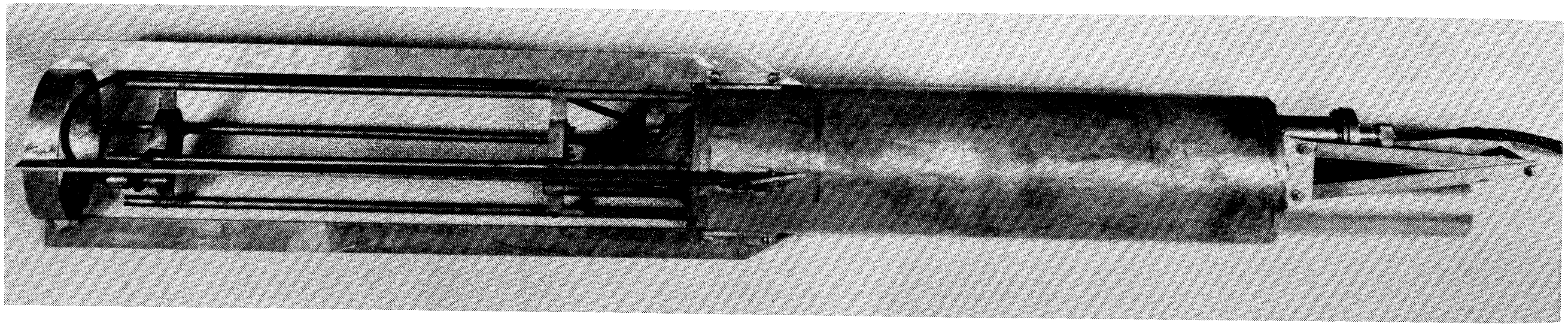


Fig. 5. This is a portion of the meter which is dropped from the surface ship. Crystals and protecting fins are shown at the left and the four-conductor connecting cable is attached to the right-hand end of this unit

# ACOUSTIC PRINCIPLES RELATED TO THE VELOCITY METER

The measuring process used in the underwater sound velocity meter depends upon the transmission of a sound wave from a quartz transmitting crystal to a quartz receiving crystal. In between these two crystals is the sample of water whose velocity is measured. In the design of the present meter the crystal separation distance has been made 1 foot, a distance which corresponds to approximately 100 wavelengths of the ultrasonic wave transmitted through the water at a frequency of 510 kc. Greater crystal separation has been shown to increase the sensitivity of the meter, but it also increases the size of the sample measured. Since it is desired to study the velocity microstructure, that is, small changes in velocity which occur from point to point, crystal separation distances greater than 1 foot were excluded.

In order to understand the mechanism of sound propagation between the two crystals it is appropriate to review the wave equation for longitudinal acoustic waves and to examine carefully the harmonic solutions of this equation. A number of testbooks[4] on acoustics derive a general wave equation in terms of a scalar velocity potential $\phi$. The presence of a velocity potential is indicated by the fact that the particle velocity vector (not to be confused with the velocity of propagation of the wave) is an irrotational vector. That is, the particle velocity vector field has no curl, and it can therefore be represented as the gradient of a scalar potential function $\phi$, the velocity potential. The wave equation in terms of vector notation is

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \Delta^2 \phi \quad , \tag{1}$$

where c is the velocity of propagation of the sound wave and $\Delta^2$ is the Laplacian operator. The Laplacian operator $\Delta^2$ depends in form upon the type of coordinate system used. For example, in the case of a three-dimensional rectangular-coordinate system the wave equation may be rewritten as follows:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) . \tag{2}$$

In the development of this equation it has been assumed that the fluid medium transmitting the acoustic wave is homogeneous, isotropic, and perfectly elastic. This last condition, indicating the absence of dissipative forces, should be examined critically to establish whether the above wave equations may be applied to the underwater sound velocity meter or if a more complicated equation whose solution includes attenuation should be considered. The measured attenuation of ultrasonic 510-kc signals in water will be investigated to establish the magnitude of attenuation over a distance of 1 foot.

The attenuation of an acoustic wave is described in terms of the following equation:

$$I = I_0 \, e^{-2\alpha r} \quad . \tag{3}$$

Here I and $I_0$ are intensities at points on plane wavefronts r cm apart and $\alpha$ is the absorption coefficient. A considerable body of literature has been accumulated on the absorption of sound in liquids.[5] In most liquids, absorption is found to be proportional to the square of the frequency. This frequency dependence of absorption appears to be true of fresh water, but is not strictly correct for salt water. More than a century ago Stokes developed a classical theory to account for the absorption of sound waves. This theory predicts sound absorption to be proportional to the square of the frequency, but the magnitude of the absorption coefficient as predicted from the classical theory is rarely in agreement with measurements. Lieberman describes a series of experiments in fresh water at frequencies of 120, 480, and 940 kc to determine absorption coefficients, and reports a value of $2\alpha = 8.3 \times 10^{-5}$ $cm^{-1}$ at 480 kc.[6] This is in good agreement with predictions based upon laboratory measurements at higher frequencies. In sea water the experimentally determined absorption coefficient reported by Lieberman was found to be considerably greater, $2\alpha = 40 \times 10^{-5}$ $cm^{-1}$ at a frequency of 480 kc.

Assuming a value of $2\alpha = 45 \times 10^{-5}$ $cm^{-1}$ for the sound velocity meter operating in sea water at 510 kc, the intensity ratio for a plane wave and a distance of 1 foot is readily calculated. It is

$$I/I_o = e^{-45 \times 10^{-5} \times 2.54 \times 12} = .985 \quad .$$

The ratio of pressures is equal to the square root of the intensity ratio. In this case it is

$$p/p_o = \sqrt{.985} = .992 \quad .$$

These ratios of intensity and pressure for ultrasonic transmission over a distance of 1 foot, such as is encountered in the underwater sound velcoity meter, indicate that absorption has a relatively unimportant influence upon the operation of the meter. Harmonic plane-wave solutions of the wave equation (2) will now be considered, and these solutions will neglect attenuation.

It is possible to derive wave equations in terms of acoustic variables other than velocity potential. However, the various quantities are interrelated, and a solution for the velocity potential of a plane wave is easily transformed into a solution for acoustic pressure p, for x-directed particle velocity u, or for any of the other acoustic variables, by the use of appropriate multiplying factors. In particular, the following table gives the definition of various commonly encountered acoustical quantities and indicates their connection with the velocity potential for a plane wave.

Table I

Relationship between the Velocity Potential and Other Acoustical Quantities

| Symbol | Definition | Relating Equation |
|---|---|---|
| u | particle velocity in the direction of the x-axis | $u = \frac{d\phi}{dx}$ |
| p | excess pressure, that is, the difference between the instantaneous pressure and the constant mean pressure | $p = -\rho \frac{d\phi}{dt}$, where $\rho$ is the constant mean density |
| $\xi$ | particle displacement in the direction of the x-axis | $\xi = \int u \, dt$ |

A plane acoustic wave traveling in the positive x-direction will have all particle motions in the x-direction. Therefore $\partial\phi/\partial y$, the y-directed particle velocity, and $\partial\phi/\partial z$, the z-directed particle velocity, are both zero. Eq 2 is thus reduced to

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \frac{\partial^2 \phi}{\partial x^2} \quad . \tag{3}$$

for x-directed plane waves. Complex harmonic solutions in the form of $\phi_+ = Ae^{j(\omega t-kx)}$ and $\phi_- = Be^{j(\omega t+kx)}$ can be shown to satisfy Eq 3 by differentiation and substitution, provided $k = \omega/c$. The solution indicated by $\phi_+$ is a wave traveling in the positive x-direction having a complex amplituce A, and the solution $\phi_-$ is a wave traveling in the negative x-direction with a complex amplitude B. The sum $\phi_+ + \phi_-$ is also a solution of the wave equation. By using solutions for the velocity potential and the relationships given in Table I, it is not difficult to obtain expressions for the other acoustic variables. For example,

$$p = -\rho\, \partial\phi/\partial t = -j\omega\rho \left[Ae^{j(\omega t-kx)} + Be^{j(\omega t+kx)}\right]$$
$$u = \partial\phi/\partial x = -jk \left[Ae^{j(\omega t-kx)} - Be^{j(\omega t+kx)}\right] \quad .$$

The term in these equations involving the complex exponent $+j(\omega t-kx)$ refers to a wave traveling in the positive x-direction may be identified as p+ or u+, respectively. That is,

$$p_+ = -j\omega\rho\ Ae^{j(\omega t-kx)} = -j\omega\rho\ \phi_+$$
$$u_+ = -jk\ Ae^{j(\omega t-kx)} = -jk\ \phi_+ \quad .$$

Similarly, the term involving the complex exponent $+j(\omega t+kx)$ refers to a wave traveling in the negative x-direction and may be designated as $p_-$ or $u_-$, where

$$p_- = -j\omega\rho\, Be^{j(\omega t+kx)} = -j\omega\rho\, \phi_-$$

$$u_- = +jk\, Be^{j(\omega t+kx)} = +jk\, \phi_- \quad .$$

It is interesting to note that the ratio $p_+/u_+ = \omega\rho/k = +\rho c$ and the ratio $p_-/u_- = -\omega\rho/k = -\rho c$, where $\rho c$ is a real number. These relationships are exactly analogous to the ratio of the positive moving voltage to the positive moving current wave and to the ratio of the negative moving voltage to the negative moving current wave on a lossless electric transmission line. In that case $E+/I+ = +Z_o$ and $E-/I- = -Z_o$, where $Z_o$ is the characteristic impedance, a real number for a lossless line. The quantity $\rho c$ is frequently called the characteristic impedance of an acoustic medium.

The Influence of Standing Waves

The transmission of an acoustic wave from one medium having a characteristic impedance $\rho_1 c_1$ to a second medium having a characteristic impedance $\rho_2 c_2$ will now be considered. It is assumed that transmission through the boundary occurs at normal incidence. In this type of wave propagation two boundary conditions must be satisfied:

(a) The acoustic pressures on each side of the boundary must be equal

(b) Particle velocities normal to the boundary on one side must equal particle velocities normal to the boundary on the other side.

The first of these boundary conditions arises from the fact that pressure is a scalar quantity, continuous throughout the media, and the second condition from the necessity of maintaining actual physical contact of the two materials at the boundary layer. Reflection from a boundary of the sort described here is shown in Fig. 6.

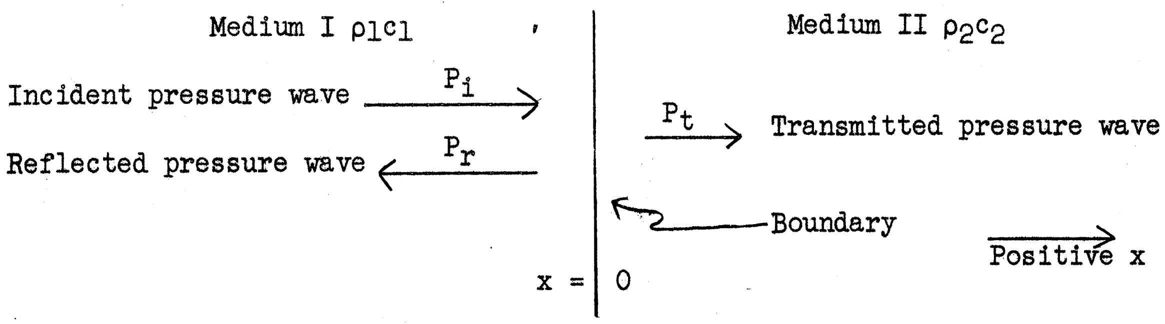



Fig. 6. Acoustic transmission at normal incidence through a boundary.

It is convenient to assume complex pressure solutions satisfying the wave length equation and to apply boundary conditions to these solutions which are appropriate to the transmission phenomena considered here. The following pressure waves will be considered:

Incident pressure wave $p_i = A_1 e^{j(\omega t - k_1 x)}$

Reflected pressure wave $p_r = B_1 e^{j(\omega t + k_1 x)}$

Transmitted pressure wave $p_t = A_2 e^{j(\omega t - k_2 x)}$ ,

where $k_1 = \omega / c_1$ and $k_2 = \omega / c_2$, and $A_1$, $B_1$, and $A_2$ are complex pressure amplitudes. At the boundary, where x is taken equal to zero, the equality of pressures requires that

$$A_1 e^{j\omega t} + B_1 e^{j\omega t} = A_2 e^{j\omega t} \tag{4}$$

or

$$A_1 + B_1 = A_2 .$$

The equality of normal particle velocities yields

$$\frac{A_1}{\rho_1 c_1} + \frac{B_1}{-\rho_1 c_1} = \frac{A_2}{\rho_2 c_2}$$

or

$$\rho_2 c_2 (A_1 - B_1) = \rho_1 c_1 A_2 . \tag{5}$$

Eqs 4 and 5 may be combined to obtain an expression for the ratio of pressure amplitude of the incident wave to pressure amplitude of the reflected wave:

$$\frac{B_1}{A_1} = \frac{\rho_2 c_2/\rho_1 c_1 - 1}{\rho_2 c_2/\rho_1 c_1 + 1} .$$

This ratio is a real number. Likewise, an expression may be obtained from Eqs 4 and 5 to show the relationship between the transmitted pressure amplitude and the incident pressure amplitude. It is also a real number and has the form

$$\frac{A_2}{A_1} = \frac{2\rho_2 c_2/\rho_1 c_1}{\rho_2 c_2/\rho_1 c_1 + 1} .$$

Two separate pressure wavefronts will now be considered, one located a distance d cm to the left of the boundary and the other located exactly at the boundary. The phase difference between these two wavefronts will be examined to determine if this phase difference will yield information from which the velocity of propagation $c_1$ of the acoustic pressure wave in medium I can be determined easily. To simplify the arithmetic involved, the characteristic impedance of medium II, $\rho_2 c_2$, will be taken equal to three times the characteristic impedance $\rho_1 c_1$ of medium I. In this example medium I may represent the sea water being measured and medium II the receiving cyrstal of the velocity meter. The 3:1 ratio makes the example similar to an experimentally observed reflection which will be described later. With

$\rho_2 c_2/\rho_1 c_1 = 3$

$$\frac{B_1}{A_1} = \frac{3 - 1}{3 + 1} = 0.5 \quad \text{and} \quad \frac{A_2}{A_1} = \frac{2 \times 3}{3 + 1} = 1.5 \quad ,$$

The acoustic pressure at any point to the left of the boundary is the sum of the two pressure waves moving in opposite directions. At a distance d cm to the left of the boundary

$$p_{(x=-d)} = A_1 e^{j[\omega t - k_1(-d)]} + 0.5\ A_1 e^{j[\omega t + k_1(-d)]}$$

or

$$p_{(x=-d)} = A_1 e^{j\omega t}\left[e^{jk_1 d} + 0.5\ e^{-jk_1 d}\right] \quad .$$

Directly at the boundary layer the effect of the transmitted pressure is

$$p_{(x=o)} = 1.5\ A_1 e^{j\omega t} \quad .$$

In order to compare the phases of these two pressure waves it is convenient to consider the ratio $\frac{p(x=-d)}{p(x=o)}$ . This is done in the following steps:

$$\frac{p(x=-d)}{p(x=o)} = 1/3\ (e^{-jk_1 d} + 2e^{+jk_1 d})$$

$$= 1/3\ (3 \cos k_1 d + j \sin k_1 d)$$

$$= De^{j\Theta}$$

where

$$\tan \Theta = 1/3 \tan k_1 d = 1/3 \tan \omega/c_1 d \quad . \qquad (6)$$

Angle $\Theta$ is the phase difference between the two pressure wavefronts, one at $x = -d$ and the other at $x = o$, and D is the amplitude ratio. The underwater sound velocity meter is not sensitive to changes in amplitude. For this reason, only the phase difference $\Theta$ will be investigated. In the actual meter the distance of sound transmission d is 1 foot, the frequency is approximately 500 kc, and both distance and frequency are carefully maintained constant.

Fig. 7 shows the relationship between the phase angle $\Theta$ and the velocity of propagation $c_1$ for such a normal incidence boundary problem where the amplitude of the reflected wave is equal to one-half the amplitude of the incident pressure wave; that is, where the characteristic impedances of the two media are in the ratio of 3:1. Numerical values of this calculation are given in the appendix p. . The phase angle is actually 100 times 360 degrees plus the number of degrees indicated on the phase-angle scale of Fig. 7. It is at once evident that the relationship between the phase angle and the sound velocity for this condition of standing waves is neither simple nor linear. The phase angle decreases by 360 degrees whenever the number of wavelengths between the two wavefronts, one at $x = -d$ and the other at $x = o$, is decreased by one. In the particular application considered here this decrease in the number of wavelengths is of course associated with an increase in the velocity of sound propagation. That is,

$$c_1 = \lambda f, \quad (\lambda \text{ is the wavelength})$$

and an increase in the velocity increases the wavelength so that fewer wavelengths appear between the two planes. Fig. 7 shows that the phase angle $\Theta$ goes through a cyclical variation whenever the the sound velocity is changed by approximately 25 feet per second. This cyclical variation is related to the number of half wavelengths between the two planes and is a direct consequence of standing waves.

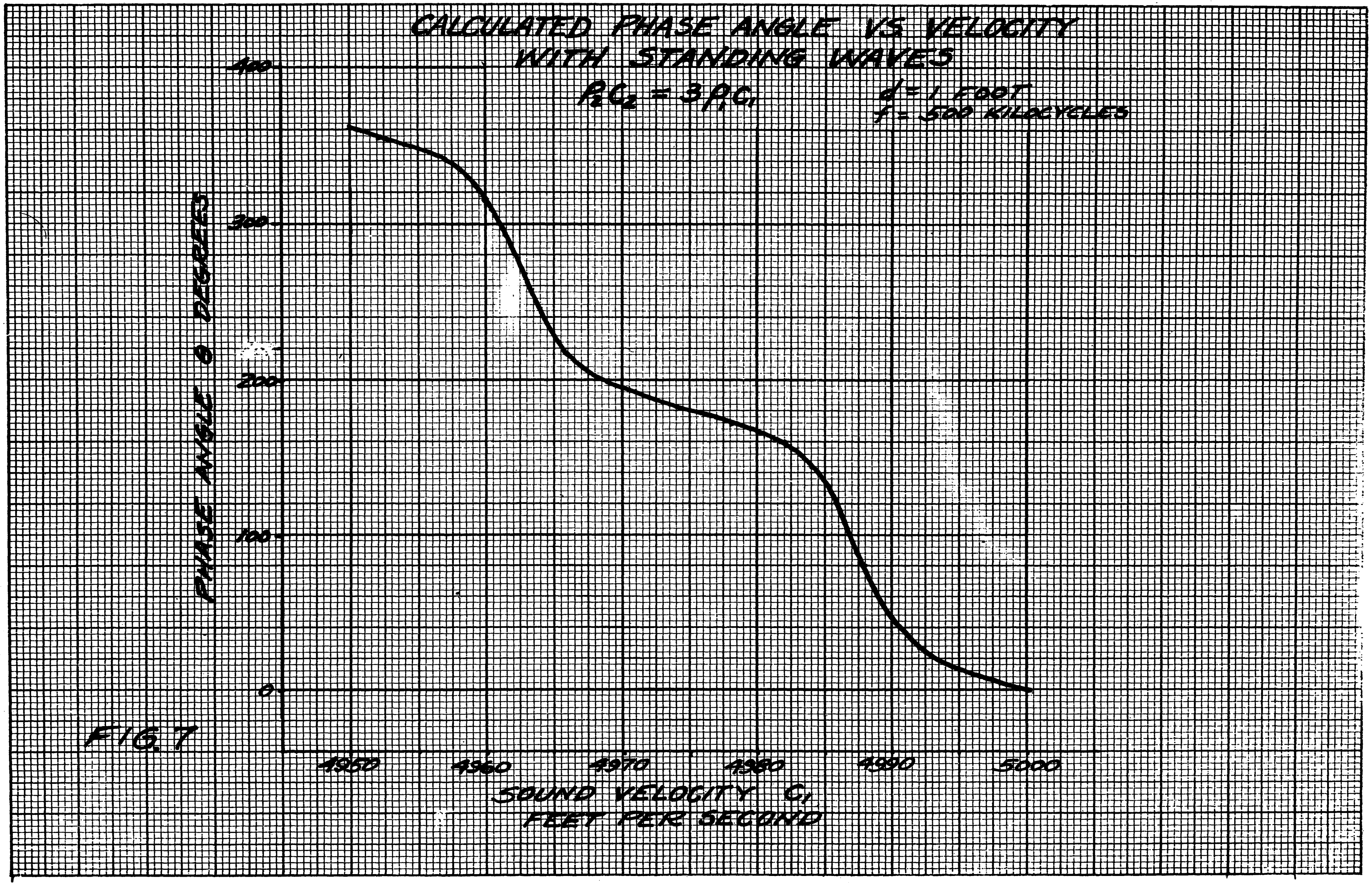

CALCULATED PHASE ANGLE VS VELOCITY
WITH STANDING WAVES
$\rho_2 C_2 = 3\rho_1 C_1$
d = 1 FOOT
f = 500 KILOCYCLES
PHASE ANGLE θ DEGREES
400
300
200
100
0
4950
4960
4970
4980
4990
5000
SOUND VELOCITY $C_1$
FEET PER SECOND

FIG. 7

The phase angle vs sound velocity relationship shown in Fig. 7 is highly undersirable in an underwater sound velocity meter for several reasons. Calibration of the meter would be difficult under such conditions, and the use of a variable coordinate scale would make the reading of the meter extremely awkward. Scale accuracy would vary from one velocity to another. Fortunately it is possible, by means of suitable crystal-backing material, spacing, and crystal dimensions, to reduce greatly this disturbing effect of standing waves.

Experimental verification of the presence of standing waves has been obtained for a number of different crystal arrangements. It is somewhat more convenient experimentally to observe the effect of standing waves by changing crystal-spacing distance d rather than by changing the velocity of propagation c. The complex expressions previously given for velocity potential, acoustic pressure, instantaneous particle velocity, etc., include a term $e^{j(\omega t-kx)}$ for plane waves propagated in the positive x-direction and and a term $e^{j(\omega t+kx)}$ for plane waves propagated in the negative x-direction. In these terms k is the wavelength constant and is equal to $\omega/c$ or $2\pi/\lambda$. It is evident that the exponent $kx = \omega x/c$ is a function of angular frequency $\omega$, distance x, and velocity of propagation c, and that an increase in distance x is essentially equivalent to a decrease in the velocity c. To show the effect of standing waves experimentally, the phase difference between two wavefronts, one at $x = o$ and the other at $x = -d$, was measured and plotted as a function of separation distance d. Quartz crystals were used as transducer elements and the ultrasonic signals were heterodyned down to a lower frequency before the phase measurement was made. A more detailed description of the equipment used in making phase measurements will be given in a later section of this report.

Fig. 8 presents the results of a series of phase measurements plotted as a function of crystal separation distance when the standing waves were rather prominent. In this particular case the separating medium was fresh water, the crystals were 1-inch-square x-cut quartz vibrating at their fundamental frequency of 1.5 mc, and the crystal-backing material was air. The crystals had copper-plated electrodes, one electrode in direct contact with the water. The presence of acoustic standing waves is immediately apparent from Fig. 8. A change of distance d by exactly one wavelength brings about a phase change of 360 degrees, but the phase also goes through a cyclical variation every time the crystal-spacing distance changes by one-half a wavelength. This latter variation is a direct consequence of standing waves. The similarity to the calculated data presented in Fig. 7, where velocity rather than crystal-separation distance was given, is evident. In the absence of standing waves it can be shown that the curve plotted in Fig. 8, would be a perfectly straight line. The presence of standing waves can also be noted experimentally by the change in the amplitude of the signal received from the quartz receiving crystal as either spacing or velocity is changed. This amplitude change, amounting to an approximate ratio of 2:1 for the experimental case described above, is removed in the phase-measuring electronic circuits and is therefore of little consequence in the measurements.

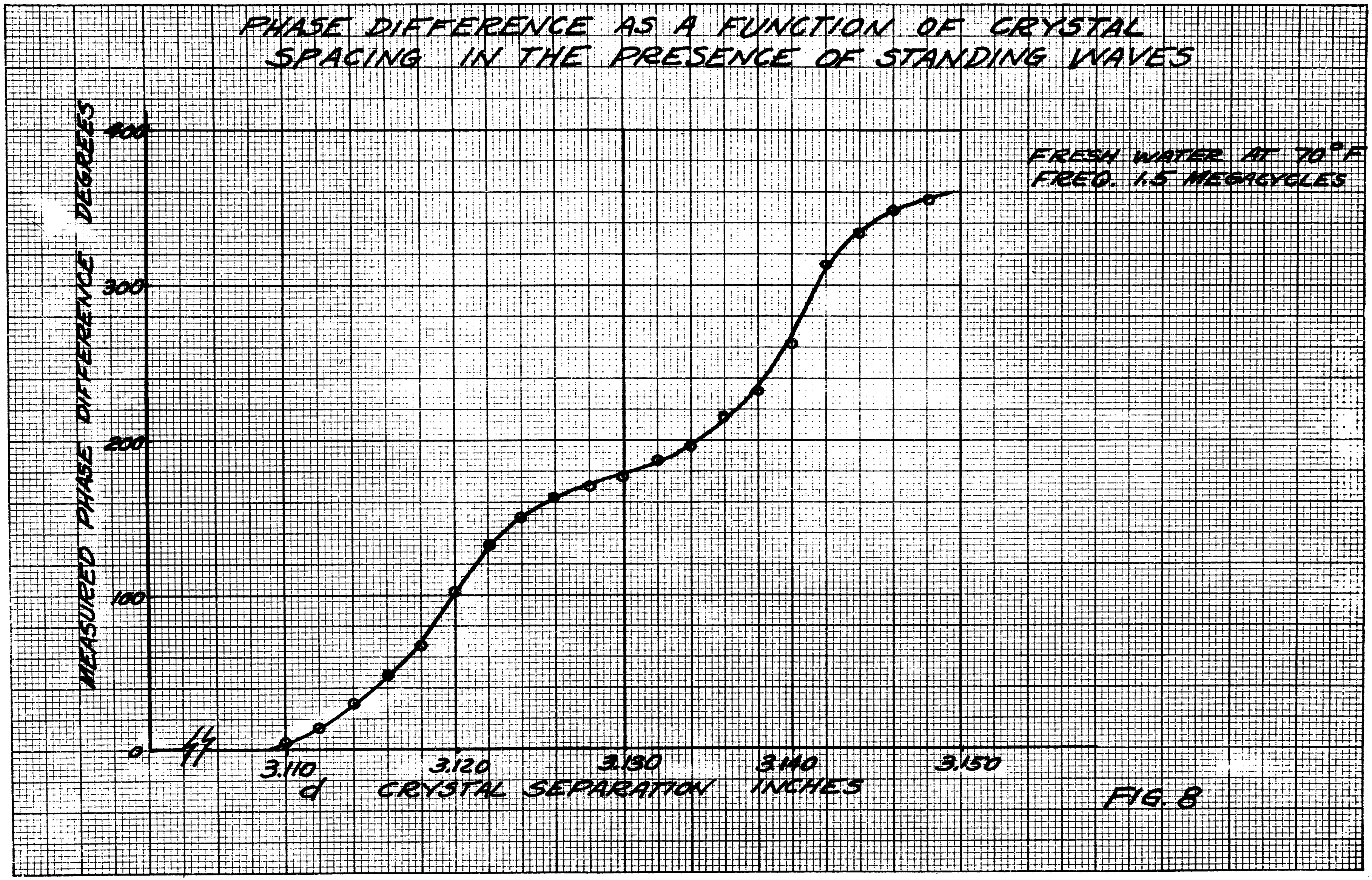

PHASE DIFFERENCE AS A FUNCTION OF CRYSTAL
SPACING IN THE PRESENCE OF STANDING WAVES
FRESH WATER AT 70°F
FREQ. 1.5 MEGACYCLES
MEASURED PHASE DIFFERENCE DEGREES
400
300
200
100
0
3.110
3.120
3.130
3.140
3.150
d CRYSTAL SEPARATION INCHES

FIG. 8

Relationship between the Velocity of Propagation and Phase Angle in the Absence of Standing Waves

The relationship between the velocity of propagation and the phase angle of an acoustic wave when standing waves are not present will now be considered. Plane waves of velocity potential $\phi$ and acoustic pressure p traveling in the positive x-direction may be described in the complex form by the following equation:

$$\phi = Ae^{j(\omega t - kx)} \tag{7}$$

$$p = -j\omega\rho Ae^{j(\omega t - kx)} = A_1 e^{j(\omega t - kx)} \; . \tag{8}$$

The complex pressure amplitude $A_1$ is related to the velocity potential amplitude A. That is,

$$A_1 = -j\omega\rho A \; .$$

In these expressions $\rho$ is the constant mean density of the medium transmitting the acoustic wave, t is time, x is distance from the origin, and $\omega$ is the angular frequency. Eq 7 can be shown to satisfy the velocity-potential wave equation if the wavelength constant k is defined as $k = \omega/c$, where c is the velocity of propagation. The actual velocity potential and acoustic pressure are found by taking the real parts of Eqs 7 and 8, respectively.

The phase difference between two plane wavefronts, one at x = d and the other at x = o, may be determined easily from the equation of acoustic pressure at these two locations. Absorption will be neglected.

$$\frac{p_{x=d}}{p_{x=o}} = \frac{A_1 e^{j(\omega t - kd)}}{A_1 e^{j\omega t}} = e^{-jkd} = e^{-j\theta} \; .$$

The pressure wavefront at x = d lags the pressure wavefront at x = o by an angle $\theta$, where $\theta = kd = \omega d/c$ radians. It is immediately apparent that this phase angle $\theta$ is directly proportional to the angular frequency and to the separation distance of the two planes and is inversely proportional to the velocity of **propagation** of the acoustic wave between the two planes. In the underwater sound velocity meter the frequency and the separation distance are carefully maintained constant.

The expression obtained above for the phase angle in radians,

$$\theta = \omega d/c,$$

or in degrees,

$$\theta = \frac{360\ d}{fc} \tag{9}$$

is exactly the same as that obtained earlier in this report from a slightly different approach. Eq 9, with a constant spacing d and frequency f, is actually a hyperbola with the velocity and phase coordinates as asymptotes. Assuming that the sound velocity could vary over a much greater range than that actually found in the ocean and taking values of d and f approximately equal to those of the velocity meter, Fig. 9 has been drawn to indicate the shape of this hyperbola. Since the actual range of sound velocity in the ocean is only from about 4750 to 5050 feet per second under the most extreme conditions, the operation of the velocity meter is restricted to a very small section of the hyperbola, a segment which is very nearly a straight line. This approximate straight-line velocity-phase relationship has been verified experimentally.

Eq 9 may be solved for velocity

$$c = 360\ df/\theta\ . \tag{10}$$

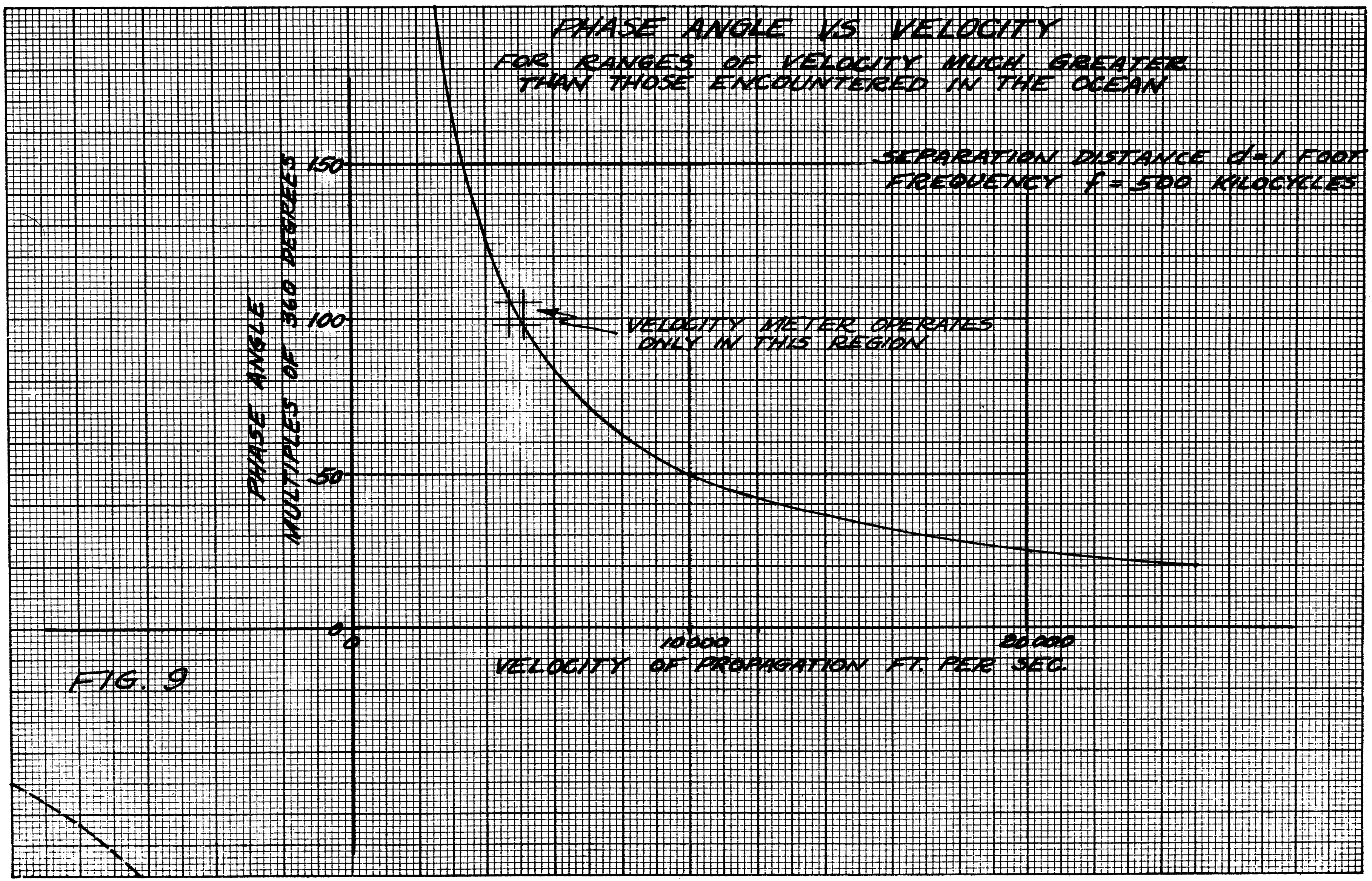

PHASE ANGLE VS VELOCITY
FOR RANGES OF VELOCITY MUCH GREATER THAN THOSE ENCOUNTERED IN THE OCEAN
SEPARATION DISTANCE d = 1 FOOT
FREQUENCY f = 500 KILOCYCLES
PHASE ANGLE
MULTIPLES OF 360 DEGREES
150
100
50
0
0
10000
20000
VELOCITY OF PROPAGATION FT. PER SEC.
VELOCITY METER OPERATES ONLY IN THIS REGION

FIG. 9

When the velocity c, as indicated by Eq 10, changes to some other value $c_1$, spacing and frequency remaining constant, the phase angle will also change to some new value $\theta_1$. It is useful to describe the fractional velocity change $(c - c_1)/c$ in terms of the change in phase angle. This is indicated in Eq 11 below:

$$\frac{c - c_1}{c} = \frac{\theta_1 - \theta}{\theta_1} \quad . \tag{11}$$

If the change in velocity is small, as it is in practice, then the denominator $\theta_1$ of the second member of Eq 11 may be replaced by the initial phase angle $\theta$ without appreciable error. This modified equation then shows that the fractional change in sound velocity is directly proportional to the change in phase angle $\theta - \theta_1$.

To demonstrate the magnitudes involved in these equations it is interesting to calculate the change in phase produced by a change in velocity from 5000 to 5001 feet per second. In this case

$$\theta_1 - \theta = - \frac{360 \times 100}{5000} = - 7.2 \text{ degrees} \quad .$$

Although the velocity change here is only 0.01 per cent, the phase-angle change of 7.2 degrees is readily measurable.

In the underwater sound velocity meter there are additional phase delays introduced in the system besides those encountered in the simple acoustic transmission between the two quartz crystals. In particular, one amplifier stage (which will be described later) introduces a phase shift of 180 degrees when its tuned circuits are exactly at resonance. These nonacoustic phase delays will, however, be considerably smaller in all cases than the phase delay involved in the actual sound transmission, less than 1 per cent in fact. They remain constant during the operation of the meter.

## DESCRIPTION OF ELECTRONIC CIRCUITS

The general operation of the electronic circuits involved in the underwater sound velocity meter will be described in connection with the block diagram, Fig. 10, and the schematic diagrams, Figs. 11 and 12. These electronic circuits, with the exception of the servo-amplifier, are designed to fit inside a watertight cylinder 5 inches in diameter and 18 inches long, and are lowered in the ocean with the instrument. Power, consisting of approximately 80 watts at 110 volts 60 cycles, is supplied to the electronic circuits by means of a connecting cable to the surface ship, and additional wires in this same cable transmit the velocity-of-sound information and also a signal proportional to the depth of the instrument back to the surface ship.

Essentially the electronic circuits of the underwater sound velocity meter are designed to measure the phase difference of two voltages of ultrasonic frequency. One of these voltages is the voltage applied to the electrodes of an x-cut quartz transmitting crystal, a crystal which vibrates in a fundamental thickness mode with displacements in the x-direction, parallel to the applied electric field. The electronic circuits

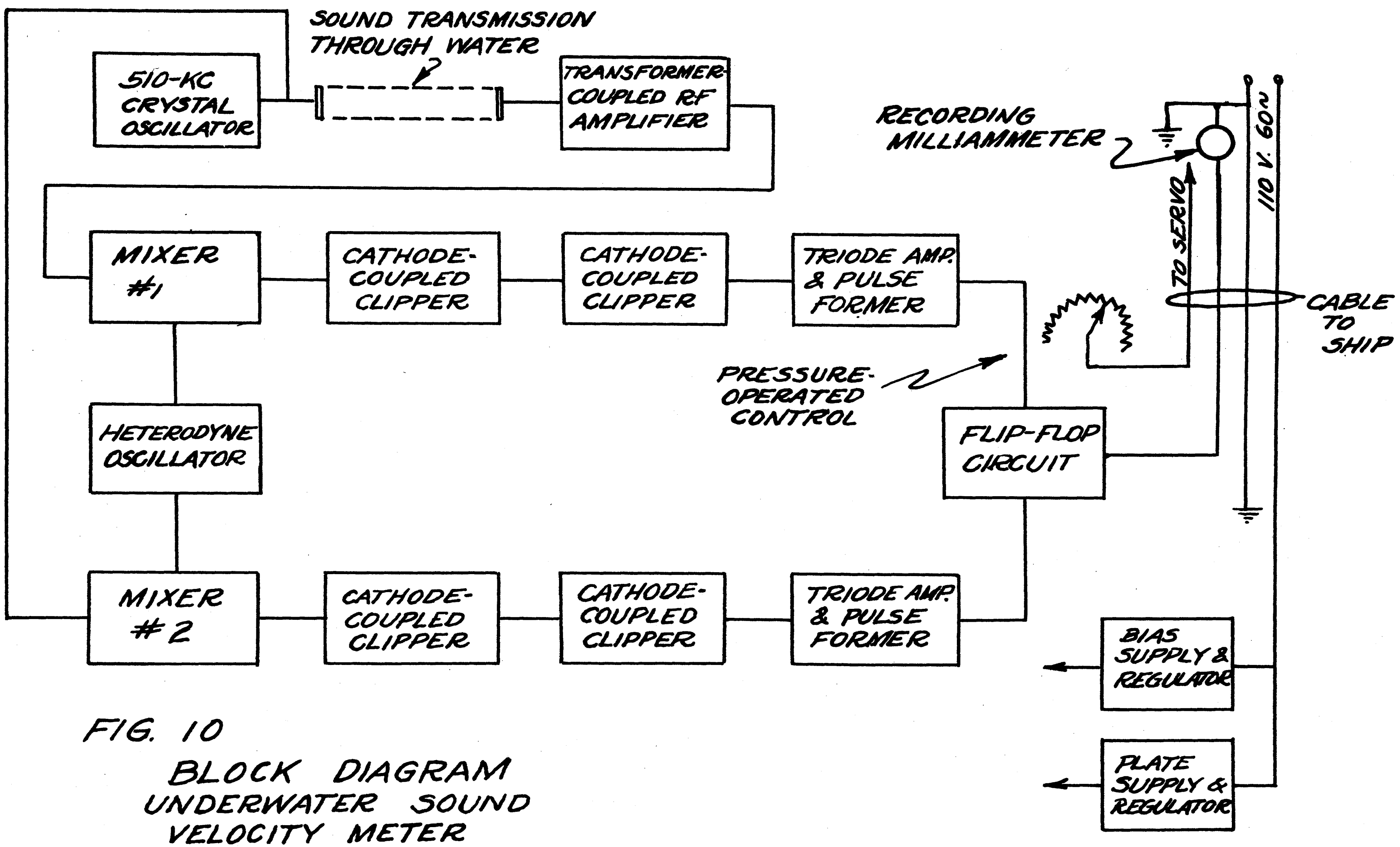



FIG. 10
BLOCK DIAGRAM
UNDERWATER SOUND
VELOCITY METER

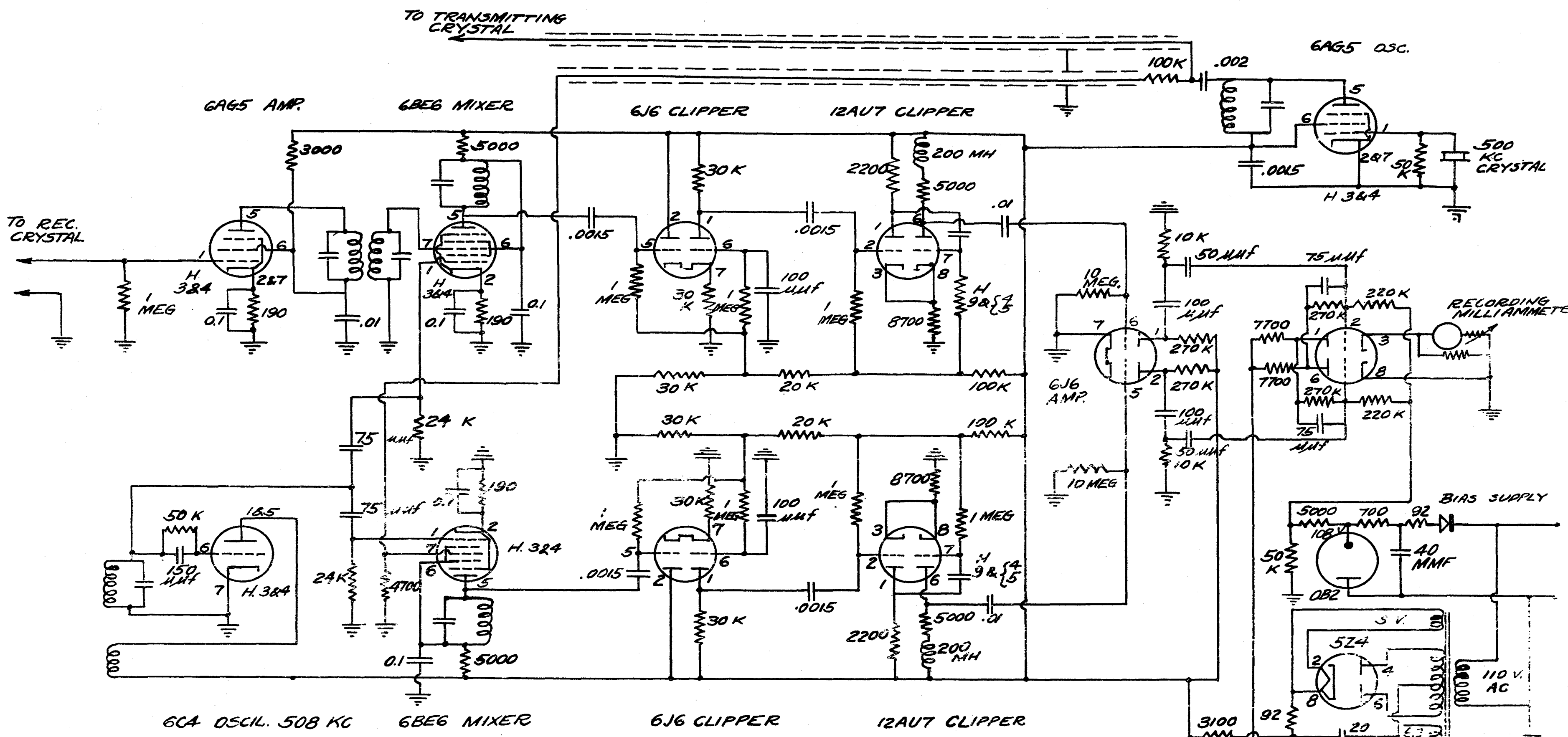



FIG 11
SOUND VELOCITY METER
RKB
10 JULY 1951

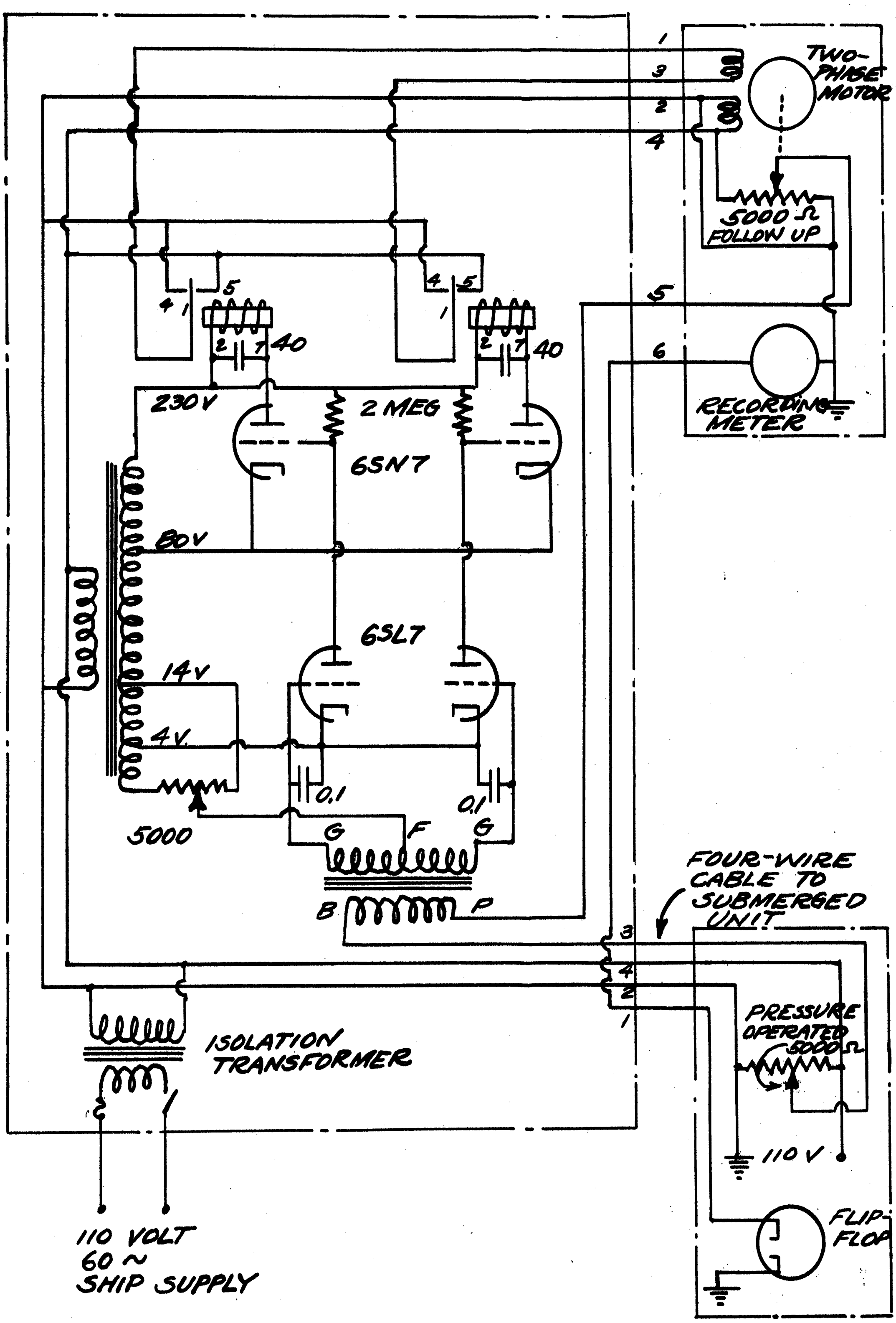



FIG12 SERVO SYSTEM AND SUPPLY CIRCUITS SOUND VELOCITY METER

BKB 2 AUG. 1951

associated with this voltage will be described as the comparison channel, or channel B. The second voltage is that across the electrodes of a second x-cut quartz crystal, a crystal which receives the acoustic wave through the water from the transmitting crystal and transforms the variations of acoustic pressure into a voltage of ultrasonic frequency. The electronic circuits associated with this second voltage will be referred to as the acoustic channel, or channel A. The acoustic transmission distance between the faces of the two crystals is 1 foot. The crystals and their mountings will be described in a separate section of this report.

In order to facilitate the electronic measurement of phase, the two voltages mentioned above are each heterodyned down into the audio-frequency region before the phase comparison is made. This procedure considerably simplifies the operation of the pulse-forming circuits and the flip-flop circuit which are involved in the phase-measurement system. The block diagram. Fig. 10, indicates how the two audio-frequency signals, one from each mixer, are sent through two cathode-coupled clipping stages which form square-voltage waves from the original sinusoidal voltages. These square voltages are then amplified and differentiated, and the resulting sharp negative pulses are applied to the two grids of a flip-flop circuit of the Eccles-Jordan type. The flip-flop circuit, employing a twin-triode tube, is of such a nature that only one triode section conducts at a time. The presence of a negative voltage pulse on the grid of the conducting tube causes that tube to become nonconducting abruptly and the other triode to start to conduct suddenly. A d-c recording milliammeter in the cathode of one of the triodes in the flip-flop circuit indicates the phase angle between the two original voltages. The electronic circuits outlined here will now be described in greater detail.

Frequency-Controlling Circuits

The phase-comparison system employed in the underwater sound velocity meter is essentially a continuous-wave system as contrasted with pulse-transmitting-and-receiving systems, such as, for example, are encountered in most radars. It is desirable to hold the operating frequency as nearly constant as possible. This is apparent from the fact that the phase difference between two wavefronts on a plane acoustic wave is directly proportional to frequency as well as inversely proportional to the velocity of propagation of the acoustic wave. It is shown on page 29 of this report that a change in the velocity of propagation from 5000 to 5001 feet per second, with a constant frequency of 500 kc and a separation distance of 1 foot, produces a decrease of 7.2 degrees in phase. It is not difficult to show that if the velocity of propagation had remained constant at 5000 foot per second and the frequency of the system varied, then the same decrease of 7.2 degrees in phase angle could have also been produced by a decrease of 100 cps in frequency. In the example this would mean a drop in frequency from 500,000 cps to 499,900. In view of this frequency dependence of phase it was decided to employ a crystal-controlled oscillator to supply radio-frequency voltage to the transmitting crystal and to depend upon this third crystal in the system to maintain constant frequency. The frequency-controlling element of the circuit is a quartz crystal, type PM-3 (TR), obtained from the Clark Crystal Company, designed to have a low temperature coefficient of frequency in the region 0 to 100°C.

Because of the limitation on space in the electronic equipment the use of a constant-temperature oven for the frequency-controlling crystal was not considered. The PM-3 (TR) crystal operates in an air gap holder, and the tube employed in the crystal oscillator is a miniature type 6AG5 with the screen grid operating at the same d-c potential as the plate. The amplitude of the sinusoidal 510-kc voltage delivered by this oscillator depends somewhat upon the tuning of the oscillator, but it is approximately 50 volts. This voltage is applied through a 0.002-microfarad condenser to the quartz transmitting crystal and by means of a resistive voltage-dividing network to the signal grid of the mixer in the comparison channel.

<u>Transformer-Coupled Radio-Frequency Amplifier</u>

The signal received by the x-cut quartz receiving crystal is amplified by a single-stage 6AG5 transformer-coupled amplifier and is then applied to the signal grid (grid No. 3) of a pentagrid mixer. Voltage amplification at this point of the circuit was considered desirable in order to obtain signals in the acoustic channel approximately equal in amplitude to those in the comparison channel; that is, to apply signals of almost equal amplitude to the two mixer stages. The coupling transformer of the radio-frequency amplifier is a midget intermediate-frequency transformer of the type employed in small broadcast superheterodyne receivers operating with 455-kc intermediate-frequency amplifiers. The primary and secondary of this transformer are tuned to resonance at 510 kc. Voltage amplification of the stage is approximately 31. It should be noted here that the phase shift across this 6AG5 amplifier depends upon the tuning of the coupling transformer. This phase shift is equal to 180 degrees only at the resonance frequency of the transformer. For this reason, calibration of the sound velocity meter should be made only after the final adjustment of the amplifier coupling transformer.

The Double Mixer and Common Heterodyne Oscillator

Accurate phase measurement of two voltages becomes increasingly difficult at higher and higher frequencies because of the increased accuracy required of the electronic circuits, especially if the phase measurement is made by the use of shaping circuits and trigger pulses. A number of phase-measuring systems are available at audio frequencies. Some of these work at frequencies up to approximately 100 kc. For example, a number of such circuits were described by E. R. Kretzmer[7] in 1949, and a meter, the Technology Instrument Corporation type 320-A phasemeter, is commercially available. In order to make use of such phase-measuring circuits, which are reliable in the audio-frequency region, the two ultrasonic-frequency voltages of 510 kc, whose phase is measured in the underwater sound velocity meter, are both heterodyned down to an audio frequency of 8 kc. By the use of two mixers, operating with a common heterodyne frequency oscillator, the phase difference between the two 510-kc voltages is preserved in the phase difference between the two 8-kc voltages, and the actual phase measurement is made at the lower frequency. Heterodyne techniques of this type have been employed in high-frequency direction-finding equipment; however, the technique as applied to the measurement of sound velocity appears to be entirely new.

A double input mixer, that is, a mixer circuit where oscillator and signal voltages appear on different grids, may be described as a device in which the mutual conductance $g_m$ of the tube is made to vary in accord with the instantaneous value of the oscillator voltage. Stockman describes mixer operation of this type in terms of a more or less straight-line section of the extended characteristic between plate current $i_b$ of the mixer and the signal-grid voltage $e_s$.[8] The slope of the characteristic is determined by the oscillator voltage $e_o$. Fig. 13 below illustrates the relationships described.

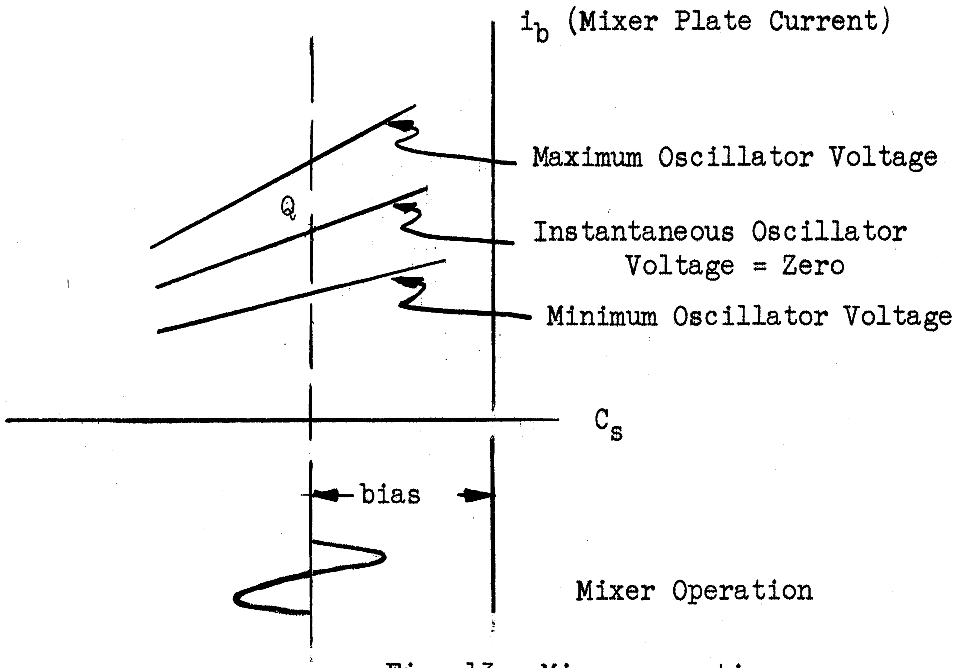



Fig. 13. Mixer operation.

The instantaneous oscillator voltage causes the point of operation Q to move up and down on the diagram, and the slope of the characteristic through the various Q points determines the mutual conductance $g_m$. The value of $g_m$ changes because of the fact that the various characteristics have different slopes.

To demonstrate the manner in which phase difference is preserved by the use of two mixers with a common oscillator, it will be assumed that the mutual conductance is directly proportional to the instantaneous voltage. This assumption is not a necessary condition, and in fact any non-linear current-voltage device could be employed as a mixing element. The plate voltage of the mixer stage remains almost constant, for there is essentially no a-c signal or oscillator voltage across the tuned plate load, and the difference-frequency voltage is small. If $g_{mo}$ is the mutual conductance with zero oscillator voltage and $kE_o$ is the amplitude of the variation of mutual conductance with instantaneous oscillator voltage, then the instantaneous mutual conductance of the mixer may be written as

$$g_m = g_{mo} + kE_o \sin \omega_o t \quad .$$

The oscillator angular frequency is $\omega_o$. The plate current flowing in the mixer will consist of a number of frequencies. Writing current as the product of $g_m$ and signal voltag $e_s = E_s \sin (\omega_1 t + \theta_1)$ and substituting the above expression for $g_m$

$$i = (g_{mo} + kE_o \sin \omega_o t) \left[E_s \sin (\omega_1 t + \theta_1)\right] \quad .$$

Also

$$i = g_{mo} E_s \sin (\omega_1 t + \theta_1) + \frac{kE_oE_s}{2} \cos \left[(\omega_o - \omega_1)t - \theta_1\right]$$

$$- \frac{kE_oE_s}{2} \cos \left[(\omega_o + \omega_1)t + \theta_1\right] \quad .$$

The term of interest in the application considered here is the low or difference frequency $(\omega_o = \omega_1)/2\pi$.

If an exactly similar mixer stage is employed, using the same oscillator voltage but having a voltage input to the signal grid

$$e_{s_1} = E_{s_1} \sin (\omega_1 t + \theta_2) \quad ,$$

then it can be shown by exactly similar reasoning that the plate current of the second mixer is

$$i = g_m E_{s_1} \sin (\omega_1 t + \theta_2) + \frac{kE_o E_{s_1}}{2} \cos \left[(\omega_o - \omega_1) t - \theta_2\right]$$
$$+ \frac{kE_o E_{s_1}}{2} \cos \left[(\omega_o + \omega_1) t + \theta_2\right] \quad .$$

The plate currents of the mixer stages flow through tuned circuits which offer high impedance only to the difference frequency. The a-c plate voltage of the mixer is a small but sinusoidal voltage at this low difference frequency. Comparison of the two low-frequency terms

$$\frac{kE_o E_s}{2} \cos \left[(\omega_o - \omega_1) t - \theta_1\right] \quad \text{and} \quad \frac{kE_o E_{s_1}}{2} \cos \left[(\omega_o - \omega_1) t - \theta_2\right] ,$$

shows that they differ in phase by exactly the same phase angle $(\theta_1 - \theta_2)$ which characterizes the phase difference of the original high-frequency terms. Thus, the heterodyning process described above is a method of reducing the frequency of two sinusoidal voltages without disturbing the phase difference between the two voltages.

Initial attempts to construct a double mixer with a common oscillator were not highly successful because of the cross-interference between the two channels. Even when the acoustic signal was reduced to zero, for example, by blocking the water path between the two x-cut quartz crystals by a completely reflecting piece of wood, a voltage still appeared across the load of the mixer in the acoustic channel. These initial circuits employed circuit coupling with 6AG7 mixer tubes as is shown in Fig. 14. This cross-interference is believed to have originated through the oscillator coupling condensers. Fig. 14 indicates how the common oscillator signal was applied in this circuit to the signal grids of the mixer tubes through 5-micromicrofarad condensers, and it also shows how any signal voltage appearing on the signal grid of one of the mixer tubes could also appear at the signal grid of the other mixer tube by way of the oscillator coupling condensers. Cross-interference of the type described here makes the circuit behavior somewhat similar to the operation of the meter when pronounced standing waves are present, and this type of interference can easily be confused with standing waves.

Double input mixers in which the signal and oscillator voltages are applied to different grids of the mixer tube are nearly free from this cross-interference. Therefore, circuits of the double-input type, rather than mixing circuits having only a single input, are employed in the underwater sound velocity meter. The actual schematic diagram, Fig. 11, gives values of the various components used with the two 6BE6 miniature-type mixing tubes. As a further step in isolating the two channels the wiring is kept well separated on the chassis and the amplifier in the acoustic channel is decoupled from the plate power supply by an r-c decoupling network. When the sound transmission path in the present meter is blocked but the comparison channel is operating normally, the signal appearing at the plate of the acoustic channel mixer is reduced more than 35 decibels from its normal value. This isolation of the two channels is adequate for the operation of the meter.

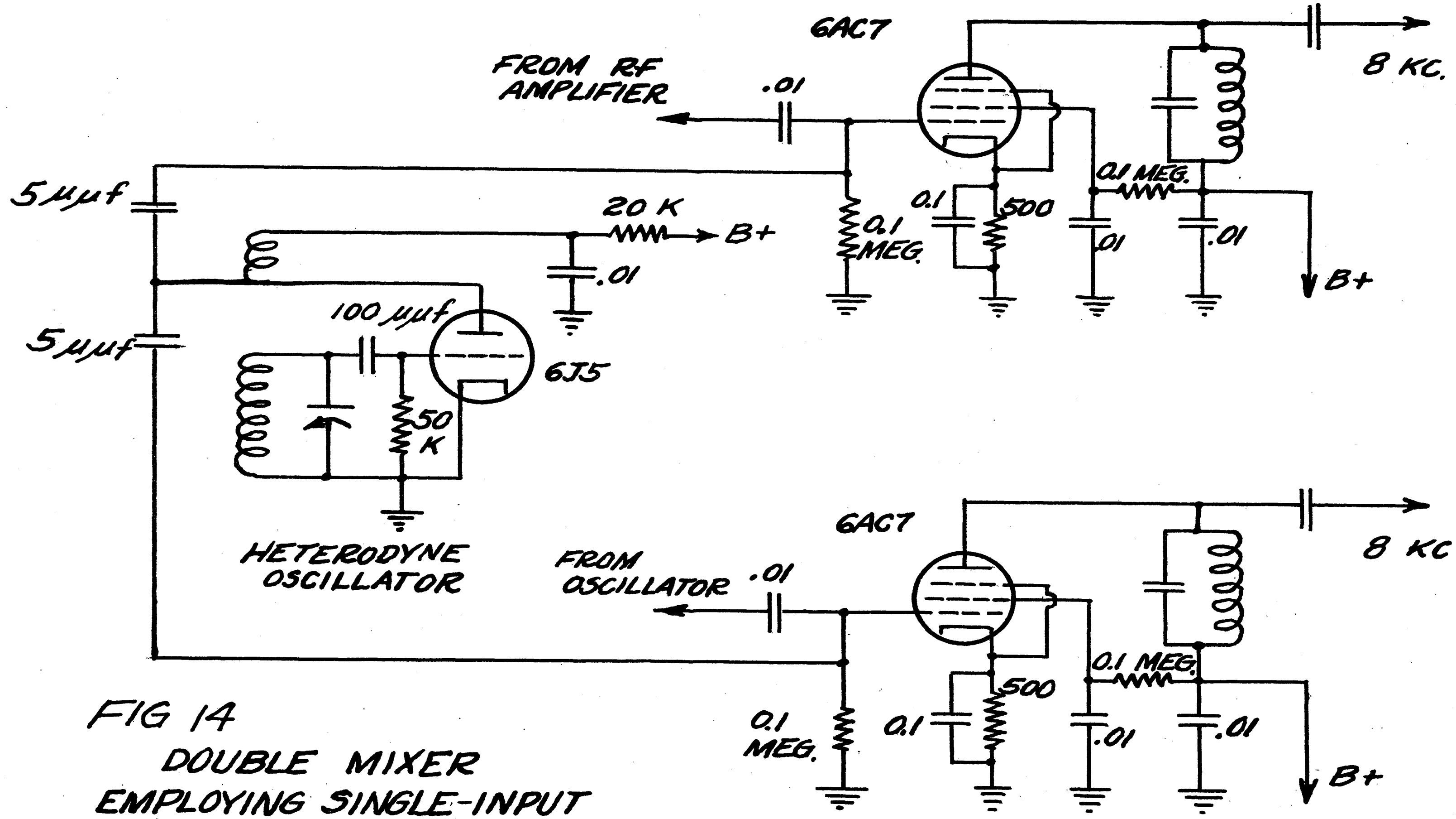



FIG 14
DOUBLE MIXER
EMPLOYING SINGLE-INPUT
CIRCUIT COUPLING

Under normal operating conditions the amplitudes of the 510-kc voltages applied to the mixer signal grids are approximately 2 volts in both channels. The voltage received from the x-cut quartz receiving crystal before amplification is about 6.4 mv.

The heterodyne oscillator, common to both mixers, employs a 6C4 miniature triode with a tuned circuit in the grid and inductive coupling to the plate. The frequency of this oscillator is set 8 kc higher than the frequency of the crystal-controlled oscillator which drives the quartz transmitting crystal.

The frequency-stability requirement of the heterodyne oscillator is considerably less severe than that of the oscillator controlling the frequency of the acoustic channel. The phase-measuring circuits after the mixer stages are not sensitive to frequency, so that a frequency drift in the heterodyne oscillator merely means that the mixer tubes work into somewhat reactive loads rather than into purely resistive loads. Furthermore, the two channels are identical beyond the input of the mixer stages. Therefore a shift in heterodyne-oscillator frequency which introduces a phase delay in one channel will introduce an identical phase delay in the other channel. The phase difference will remain unchanged. This ideal situation, of course, occurs only when the two channels are perfectly balanced.

In order to test the behavior of the instrument with a shift in the heterodyne-oscillator frequency, the velocity meter was mounted in a tank of isothermal water and the heterodyne oscillator was tuned to various frequencies on either side of 518 kc. The difference frequency and the velocity meter milliammeter readings were recorded. Table II summarizes the results of this test:

Table II

The Effect of the Heterodyne-Oscillator Frequency upon Phase

| Difference frequency, cps | Velocity meter, Milliamperes | Phase Angle, Degrees |
|---|---|---|
| 6600 | 6.1 | 220 |
| 7050 | 6.0 | 216 |
| 7480 | 5.9 | 213 |
| 8100 | 5.6 | 202 |
| 8170 | 5.7 | 205 |
| 8700 | 5.9 | 213 |
| 9150 | 6.0 | 216 |
| 10,200 | 6.05 | 218 |

Fresh water was used in the tank; its temperature was 54.5°F. A change of almost 2 kc in the heterodyne-oscillator frequency caused a change in phase of less than 20 degrees.

In the operation of the velocity meter in the field, it was standard procedure to tune the heterodyne oscillator for the maximum signal at the mixer plate. This signal was observed by means of a cathode-ray oscilloscope, and the condition of maximum signal occurred when the difference frequency was approximately 8 kc.

## The Cathode-Coupled Clipping Stages

Two cathode-coupled clipping stages are employed in each channel of the underwater sound velocity meter to form square voltage waves with steep leading and trailing edges. Circuits of this type are described by Goldmuntz and Krauss.[9] The schematic diagram, Fig. 11, gives details of the circuit components. For small signal inputs the cathode-coupled clipper acts as a linear amplifier. However, larger grid signals drive the two triodes alternately to cutoff, and the output is a clipped or square wave. The second of the cathode-coupled clippers in each channel employs regenerative feedback to increase the sharpness of the wavefront, and this second clipper also has a shunt-peaking inductance in series with its plate load resistance. This inductance, larger than that used in most shunt-peaking video-frequency amplifiers, gives considerable overcompensation which results in a large overshoot of the leading and the trailing edge of the square wave. This overcompensation has the effect of greatly reducing the rise time and sharpening the pulses used to trigger the flip-flop circuit.

Voltage-wave shapes in their proper time relationship were observed by means of a DuMont Type 279 dual-beam cathode-ray oscillograph. These wave shapes for both channels are sketched in Fig. 15 for the phase condition where the acoustic signal (channel A) lags the signal across the transmitting crystal (channel B) by 90 degrees. The sketches show clearly that the rise time of the voltage-wave output from the second clipper is much less than the rise time of the voltage output from the first clipper.

## The Triode Amplifier and Differentiating Network

A single triode amplifier is used in each channel after the second clipper stage to produce a sharp negative-going voltage form the square wave. The grid of this triode amplifier is driven positive during the short time of overshoot of the leading edge of the square wave, and the tube is blocked for the major portion of the cycle. As a result of this blocked condition, the plate voltage of the triode amplifier is not influenced by the negative traveling edge of the square wave applied to its grid. The triode is cutoff at this time.

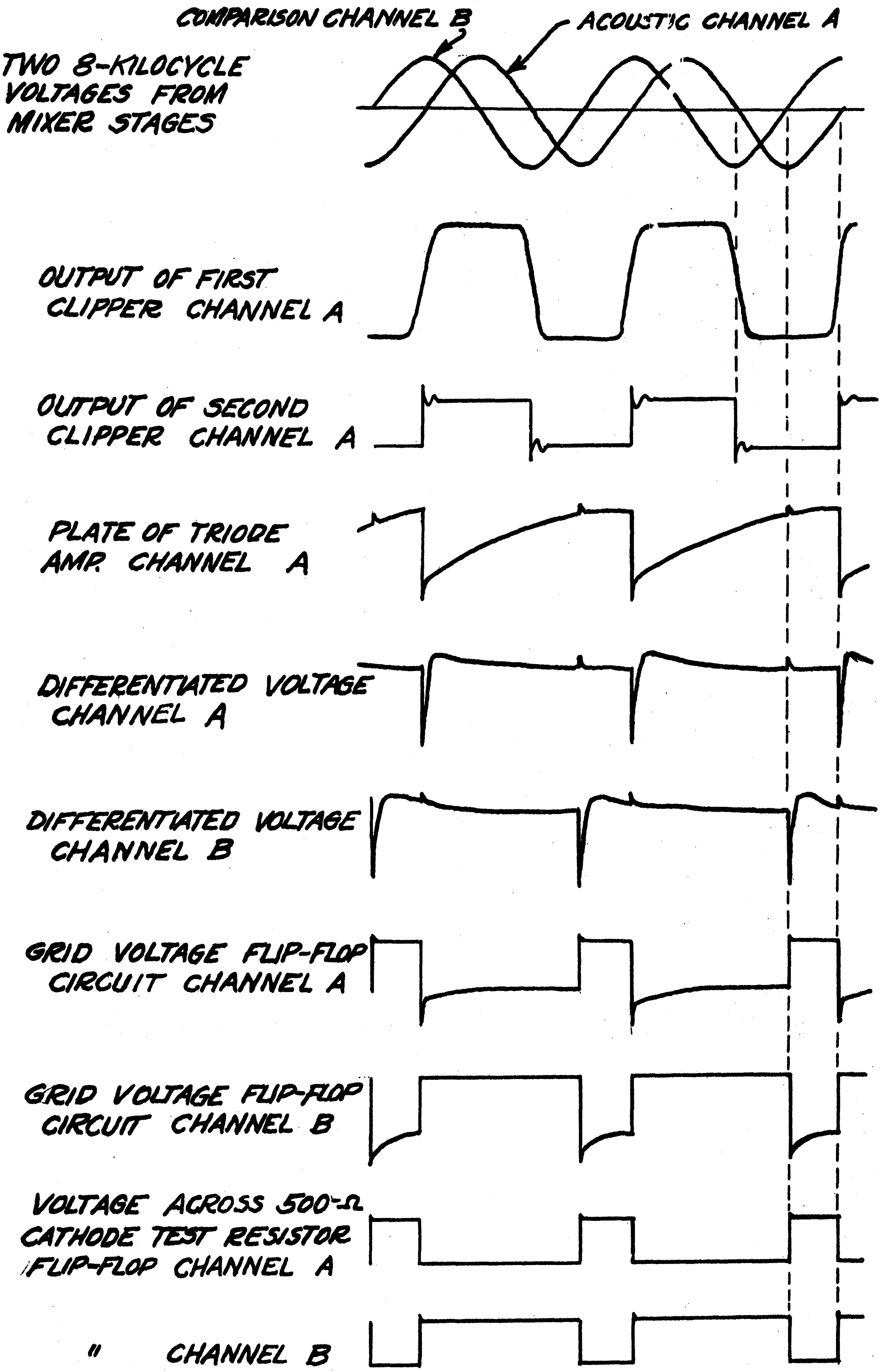



FIG. 15

SCALE - 20 VOLTS PER CENTIMETER

The triode-amplifier plate voltage, sketched in Fig. 15, is next applied to the terminals of a differentiating network consisting of a 100-micromicrofarad condenser in series with a 10,000-ohm resistor. The time constant of the r-c differentiating circuit is 1 microsecond, considerably less than the 125-microsecond period of the 8-kc voltages. This results in sharp negative output pulses of voltage which are used to trigger the flip-flop circuit at the exact time of the axis crossing (from negative to positive) of the original sinusoidal 8-kc voltages from the mixers. These triggering pulses have amplitudes of approximately 26 volts in the underwater sound velocity meter.

## The Flip-Flop Circuit and Recording Milliammeter

The flip-flop circuit employed in the underwater sound velocity meter is essentially a switching device in which the timing is controlled exactly. Two sections of the miniature twin-triode 12AU7 tube are used for this purpose. Triggering or switching circuits were first described by Eccles and Jordan in 1919, and descriptions of their operation are found in most electronic circuit books.[10]

Two stable conditions of equilibrium exist in the circuit. One of the tubes may be conducting strongly and the other tube completely cut-off or the exactly opposite condition may exist; that is, the first tube may be cutoff and the second conducting strongly. The switchover from the one equilibrium to the other is a violent and a very rapid transient. It may be brought about by applying to the conducting-tube grid a negative voltage pulse of sufficient amplitude to carry the grid over a certain voltage barrier as the grid voltage decreases, at which point the switch is initiated. Switching of this type is ideal for the measurement of phase angles, providing the frequency is not too high. A d-c permanent-magnet moving-coil type of current-indicating meter, when connected in the cathode lead of one of the flip-flop tubes, will read the fraction of a period when that particular tube is in its conducting condition. Triggering pulses, which occur at the time the two sinusoidal voltages cross the axis, are applied to the two grids of the flip-flop circuit causing the d-c meter to indicate the phase of one voltage with respect to the other. The d-c indicating meter of the velocity meter reads the phase angle by which the voltage in the acoustic channel lags the voltage in the comparison channel.

The calibration of the flip-flop for phase measurement is extremely simple. The tube having the d-c meter in its cathode can be made to conduct continuously merely by removing the negative voltage pulse (which would normally switch that tube off for a fraction of every cycle) from its grid. This is accomplished by grounding point A of the circuit shown in Fig. 16. The resistance in shunt with the meter is then adjusted until the meter reads full-scale, a reading corresponding to a 360-degree phase angle. The ground is then removed from the grid. An intermediate phase condition between 0 and 360 degrees is now indicated linearily on the meter. As a check on proper circuit operation, the other grid of the flip-flop may be grounded temporarily (point B, Fig. 16); the meter should read zero with this grid grounded.

As a test of the linearity of the phase-measuring circuits in the underwater sound velocity meter, two sinusoidal voltages of 8-kc

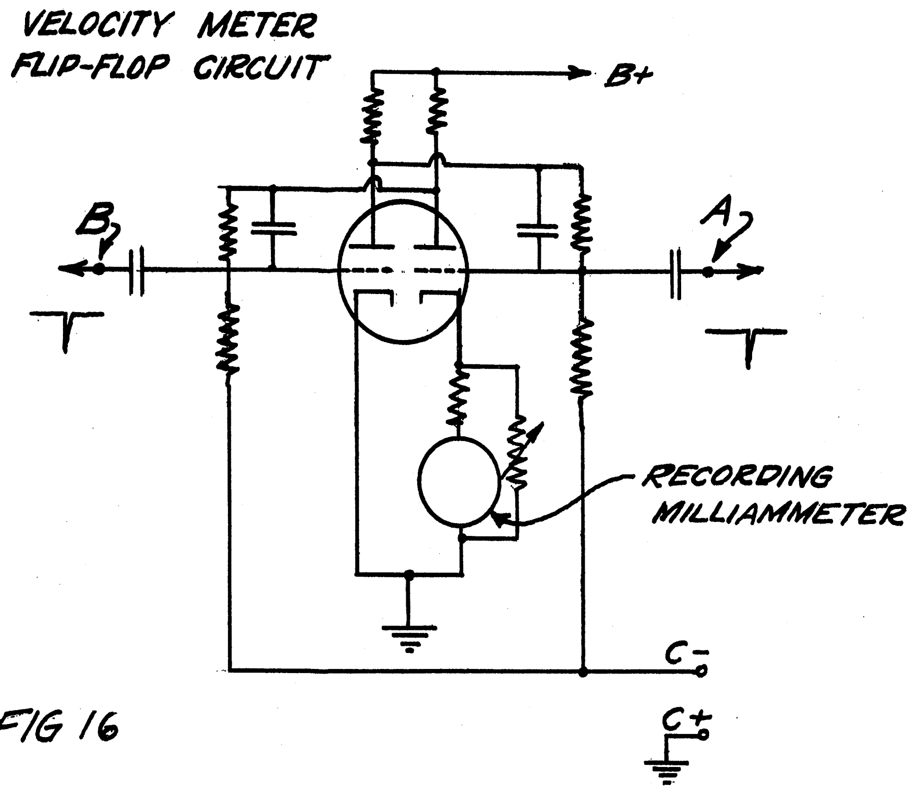

VELOCITY METER
FLIP-FLOP CIRCUIT
B+
B
A
RECORDING
MILLIAMMETER
C−
C+

FIG 16

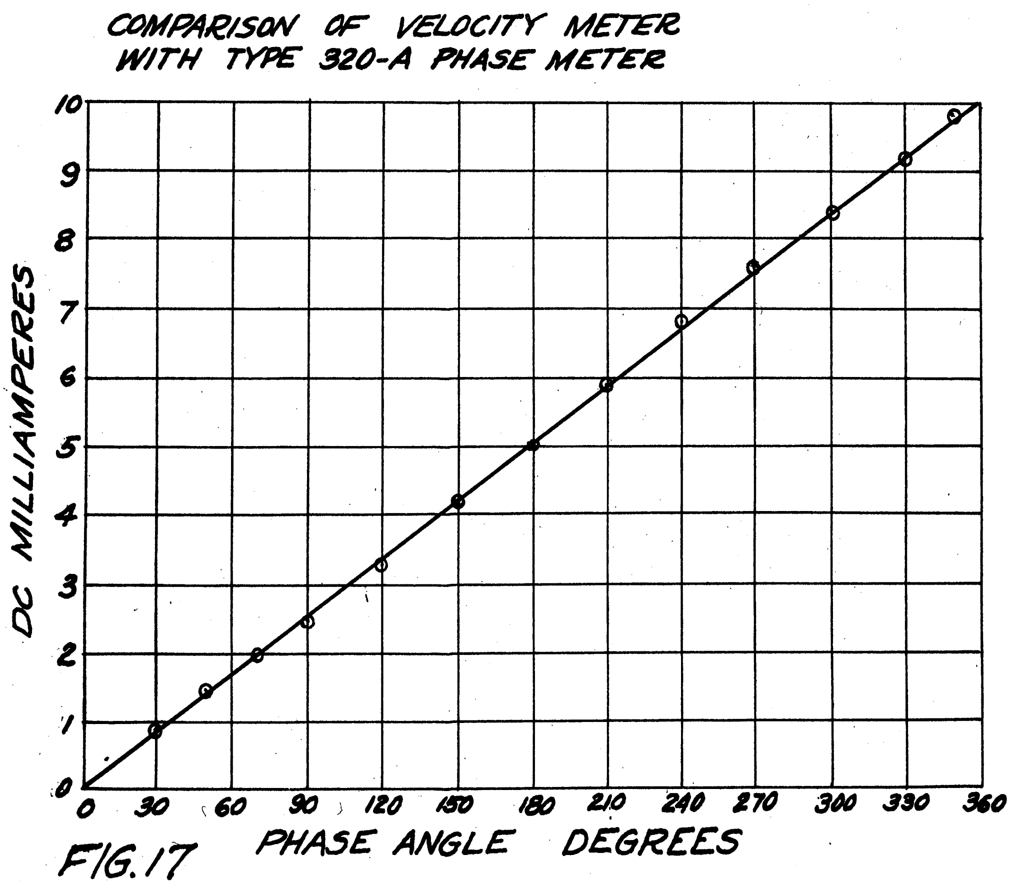

COMPARISON OF VELOCITY METER
WITH TYPE 320-A PHASE METER
DC MILLIAMPERES
0
1
2
3
4
5
6
7
8
9
10
0
30
60
90
120
150
180
210
240
270
300
330
360
PHASE ANGLE DEGREES

FIG. 17

frequency and variable phase were connected to the inputs of the acoustic and the comparison channels. The Technology Instrument Corporation Type 320-A phasemeter was also connected to indicate the phase angle of these two voltages. The phase angle indicated by the Type 320-A phasemeter has been plotted as a function of the d-c cathode current of the velocity meter in Fig. 17. The linearity of the comparison is at once apparent.

The circuit constants of the flip-flop were not found to be critical. However, the circuit operates properly only within a certain range of bias and plate voltages. Bias voltage may be obtained by means of a resistor connected between a junction point of both cathodes and ground, in which case the d-c meter in the cathode of one of the tubes would have both its terminals at a positive potential with respect to ground. In the velocity meter this biasing arrangement would have necessitated the use of two separate metering wires in the cable connecting the submerged portion of the instrument with the surface ship. Such an uneconomical use of cable wires would have been undesirable. A second biasing arrangement, the one actually used in the present meter, obtains a negative voltage from a small bias power supply and applies this voltage to the flip-flop grids. Selinium rectifiers are used and the source of a-c voltage is the 110-volt 60-cycle line. It was found that the use of a plate power supply voltage of 150 volts called for a bias voltage of about 84 volts. Any deviation of the bias voltage by as much as $\pm$ 8 volts from the 84-volt value made the circuit trigger spuriously or not at all, and furthermore the value of bias voltage influenced the full-scale reading of the d-c cathode-circuit meter. For these reasons the bias power supply for the flip-flop circuit was regulated by the use of a OB2 voltage-regulator tube.

When the two triggering pulses to the flip-flop circuit arrive at exactly the same time, as they will when the phase angle is exactly 360 degrees, there is usually an uncertainty of triggering, and occasionally the circuit responds only to alternate pulses. In this case the d-c meter indicates half- rather than full-scale. As the phase angle of the two voltages being compared is changed slowly through 360 degrees, the meter climbs to full scale, then suddenly jumps to exactly half-scale, and finally drops abruptly to zero. The time of the intermediate step is greatly reduced by the use of sharper triggering pulses. With the present meter this region of uncertainty is of the order of only two degrees in phase. It is not noticeable in field operation of the meter.

The d-c meter is a type AW 0-5-ma recording milliammeter manufactured by the Esterline-Angus Company. This recording milliammeter remains on the surface ship. It operates with a multiplying resistor and shunt to make full-scale deflection occur at ten milliamperes.

## Plate Power Supply

The plate power supply of the underwater sound velocity meter is typical of supplies designed to deliver 80 ma d-c from a full-wave rectifier. The 150 volts delivered to the plate circuit of the flip-flop is regulated by means of a OA2 voltage-regulator tube.

## The Servo-Mechanism

A servo-mechanism is incorporated in the velocity meter to move the chart of the recording milliammeter through a distance proportional to the depth of the meter in the ocean. The chart coordinates with this system become velocity vs depth, a system sometimes referred to as an x-y system which is independent of time. The electronic circuits associated with the servo are contained largely in a cabinet which remains on the surface ship.

The servo-mechanism of this type is commonly described as a follow-up control. A potentiometer located in the submerged portion of the velocity meter is made to operate from static ocean pressure by the compression of a spring inside a sylphon bellows. A rack-and-pinion drives the potentiometer (see Fig. 22, p. 65). A second potentiometer, the follow-up, is connected through gears to the chart of the recording milliammeter, which in turn is driven by a small two-phase 60-cycle a-c motor. The difference in the voltage from the two potentiometers is an error voltage which actuates the servo to drive the chart either in the forward or in the reverse direction, the actual direction depending upon the polarity of the error voltage. The system is in balance when the error voltages becomes zero. The name "follow-up" is highly descriptive in this system, since in effect the servo-motor drives the follow-up control to make it follow the motion of the remote pressure-actuated potentiometer. Servos of this type are commonly employed in the remote control of motors, and the circuit is similar to one suggested by Chute.[11] Two relays connected in the plates of a 6SN7 triode switch the motor from forward to reverse. The complete schematic diagram of the servo-system is shown in Fig. 12.

Interconnecting Cables

The velocity meter is designed to operate with only four wires in the cable connecting the meter with the surface ship. This cable carries:

(1) velocity-of-sound information;

(2) depth-of-the-instrument information; and

(3) power for the electronic circuits.

One of the wires is a ground, common to all of the above functions. The velocity-of-sound indication is a d-c signal, and the depth of the instrument is indicated by a 60-cycle a-c signal from the depth potentiometer. With so many circuits depending upon a common ground, there is a possibility of a cross-ground connection with the ship's 60-cycle power supply. To prevent this, an isolation transformer was connected between the ship's supply of power and the velocity meter. These cable circuits are also shown in the schematic diagram, Fig. 12. The submerged cable consisted of 500 feet of rubber-jacketed Belden type 8454 four-conductor cable which proved satisfactory for test purposes but lacked the strength and general ruggedness for continuous use at sea.

Auxiliary Electronic Equipment

For the testing and development of the phase-measuring circuits of the underwater sound velocity meter, an auxiliary circuit was needed to produce two voltages having a variable phase difference and also an adjustment of the amplitude of each individual voltage. Such a circuit, to operate from the 8-kc voltage of an audio oscillator, was constructed for test purposes. Its schematic diagram is shown in Fig. 18. This circuit

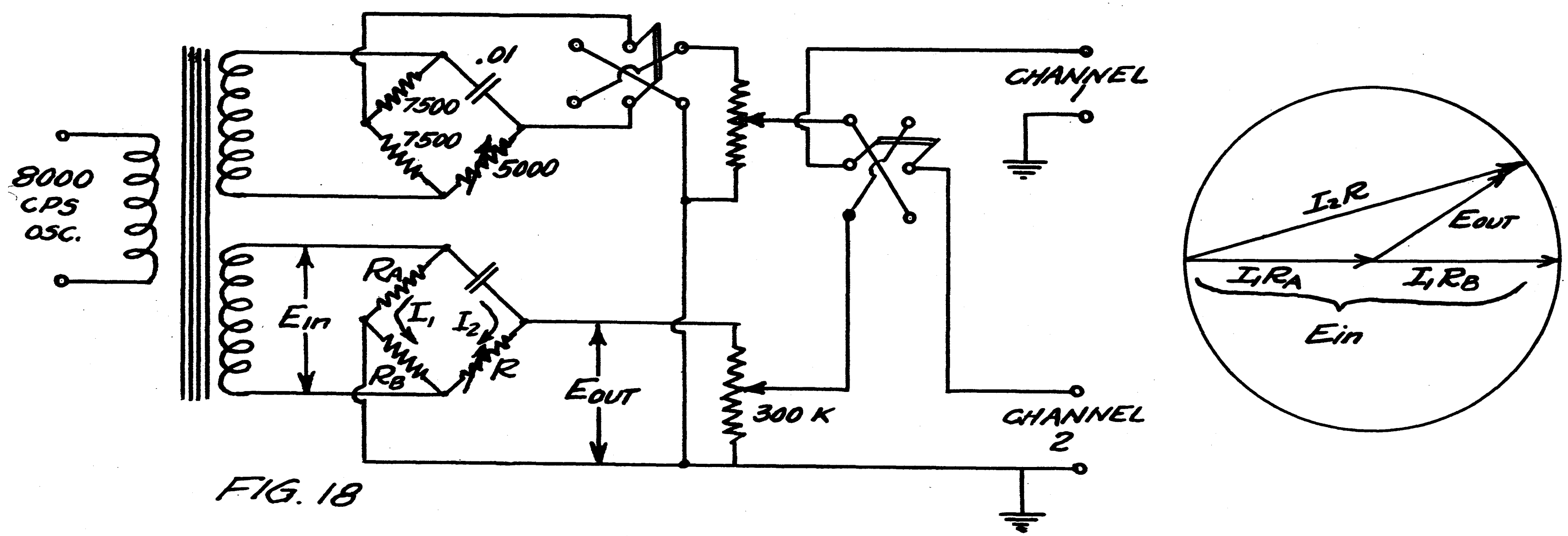



FIG. 18

CIRCUIT FOR PRODUCING
TWO VOLTAGES HAVING
VARIOUS PHASE DIFFERENCES

can be made to produce continuous shifts in phase up to 360 degrees. Operation is indicated by the vector diagram included with Fig. 18.

## TRANSDUCERS OF THE UNDERWATER SOUND VELOCITY METER

Two transducers are required for the operation of the underwater sound velocity meter. Both employ resonant x-cut quartz crystals vibrating in the fundamental thickness mode, and each crystal is mounted in the same type of holder. The transmitting crystal, with a 510-kc voltage applied to its electrodes, starts a plane acoustic pressure wave in the water by its direct piezoelectric effect. This ultrasonic plane wave is radiated from both faces of the quartz crystal; however, the back radiation is largely scattered by means of a 45-degree hollow cone which has been machined on one end of the watertight chamber containing the electronic equipment. The forward wave is transmitted through the 1-foot sample of sea water whose velocity is being measured. The receiving crystal, because of its inverse piezoelectric effect, produces an electric signal from the action of forward transmitted acoustic pressure wave, and this electric signal is the input to the acoustic channel of the electronic circuits described earlier in this report. Fig. 19, sketched below, shows the general arrangement of the transducer elements.

The design of crystal vibrating systems is covered rather extensively in the literature;[12] therefore, only a brief summary of the design theory as it applies to the transducer elements of the underwater sound

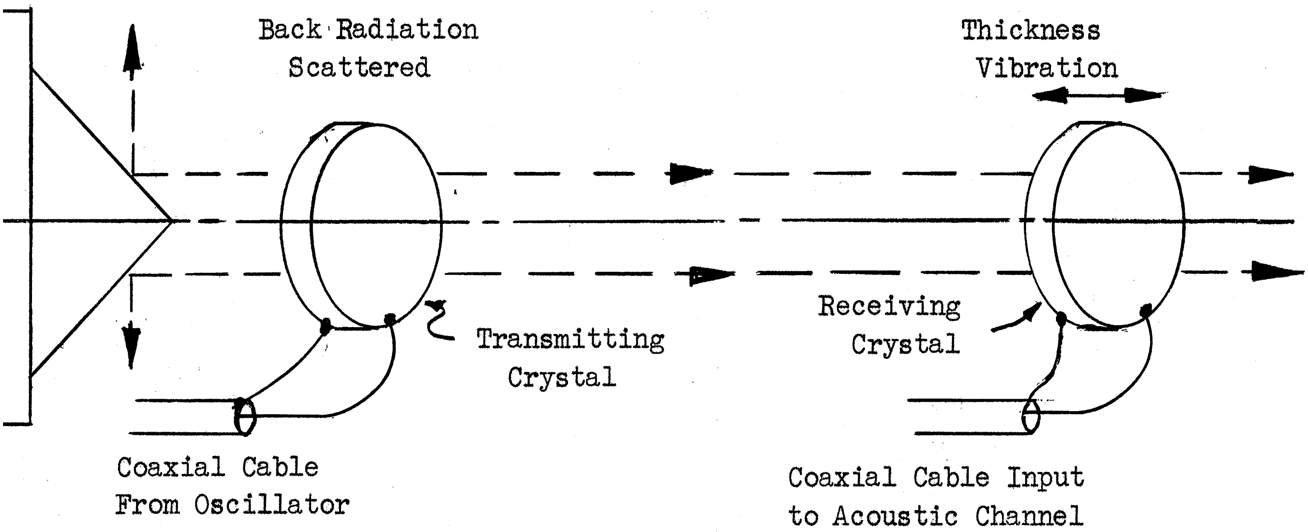



Fig. 19

velocity meter will be given here. The vibration of the x-cut quartz crystals used in the transducers is heavily damped in the water by radiation resistance and viscous forces of the crystal mounting; therefore the resonance is quite broad. It has been verified experimentally that the resonance frequency of the transducers is not changed appreciably by immersion of the crystals in water. This indicates that the radiation impedance of the radiating face of the crystal (an impedance having the units of mechanical impedance, force/velocity) is entirely resistive. This is expected from theoretical studies of circular piston radiators having a diameter of about ten wavelengths. An attempt has been made in the crystal holders to make the specific acoustic impedance (pressure/velocity) of the back plate of the holder, where it joins the crystal face, equal to the specific acoustic impedance of sea water. In the receiving crystal this has a tendency to reduce standing waves, which will be explained later.

The Transmitting Crystal

Fig. 20 is a sketch of one of the crystal holders. Detailed drawings of the various components will be found in the appendix to this report. In studying the design of the transducers it was considered highly desirable to have as little material in the actual path of sound transmission as possible, besides the actual sea water whose velocity is being measured. For this reason crystals were obtained with very thin copper electrodes plated directly on the crystal faces, and one of these electrodes, operating at ground potential, is directly in contact with water. The back electrode is maintained in good acoustical contact with a bakelite plate one-half a wavelength thick, and the back side of this plate is also in contact with the water. Thus, the transducers actually consist of two half-wavelength plates, namely, the quartz crystal itself and the bakelite backplate.

A number of different expressions are used for describing impedance in the field of acoustics; it is convenient to use certain of these in discussing the crystal holder of the velocity meter. The specific acoustic impedance $z$ is defined as the acoustic pressure p divided by the particle velocity $\mu$. It has been mentioned earlier in this report that for a plane progressive acoustic wave the specific acoustic impedance is $\rho c$, where $\rho$ is the constant mean density and c is the velocity of propagation of the wave. The mechanical impedance $Z_M$ is defined as force divided by velocity. It is related to the specific acoustic impedance; that is $z = Z_M S$, where S is the area. Still another impedance, the acoustic impedance $Z$, is defined as the acoustic pressure p divided by the volume velocity

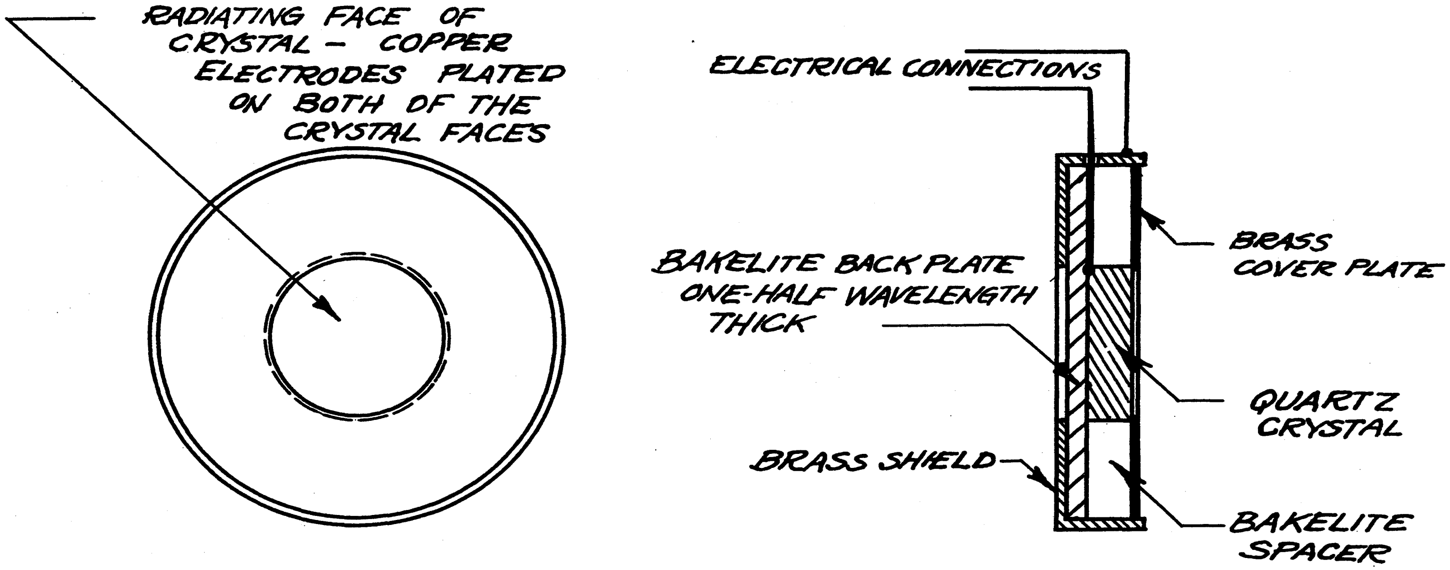



FIG 20

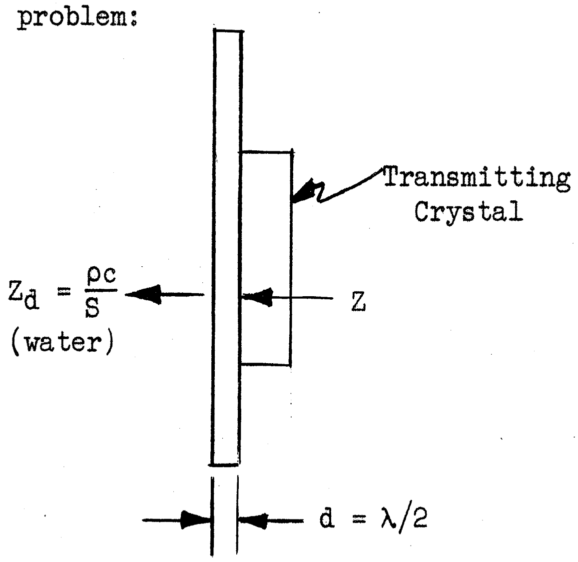
SKETCH OF CRYSTAL HOLDER

dX/dt. The volume displacement X of a fluid through a surface is the integral of the product of the displacement $\xi$ and the surface element,

$$X = \iint \xi \cdot dS \quad ,$$

or merely $\xi S$ when the particle displacements are normal to the surface. Acoustic impedance is equal to specific acoustic impedance divided by the area; that is, $Z = z/S$.

The backplate of the crystal holder is a distributed acoustic element. The acoustic impedance relationships involved are exactly analagous to the electric impedances of a transmission line half a wavelength long. Neglecting losses in the backplate, we can write for the distributed acoustic problem:




$$Z = \frac{\rho c_b}{S} \; \frac{Z_d + j\frac{\rho c_b}{S} \tan kd}{\frac{\rho c_b}{S} + j\, Z_d \tan kd}$$

$$kd = \omega/c_b \; d = \pi \text{ when } d = \lambda/2$$

$$\tan kd = 0$$

$$Z = Z_d = \rho c/S \quad ,$$

where the area S is the area of the crystal face, the acoustic impedance Z is the impedance presented to the back side of the quartz crystal by the backplate; $c_b$ represents the velocity of propagation of an acoustic wave in bakelite (plate velocity); and c represents the velocity of propagation in sea water. The acoustic impedance presented to the front face of the transmitting crystal by the water to the right in the above sketch is also $\rho c/S$, provided reflections from the receiving crystal are not present. This is very nearly true in the underwater sound velocity meter.

The particle velocity at the front face of the transmitting crystal is a function of the applied voltage, the impedances described above, and certain properties of the x-cut quartz crystal. It is interesting to calculate this particle velocity and also the acoustic pressure at the front face of the transmitting crystal for the case of 50 volts applied to the electrodes. The equations for this calculation are outlined on page 156 of Design of Crystal Vibrating Systems,[12] and the solution is given in the appendix to this report. With sea water, the calculated pressure is $1.54 \times 10^6$ dynes per square centimeter.

The electrical input impedance of the transmitting crystal can also be calculated at resonance. It consists almost entirely of the reactance associated with the capacitance of the crystal electrodes.

The transmission of acoustic waves in solids is in general a highly complicated problem because of the fact that shear as well as longitudinal stresses and strains are involved. Experimental investigation of crystals has demonstrated that the vibration of a crystal face is usually not that of a simple piston. With the type of crystal holder developed for the underwater sound velocity meter, the use of the edge clamping (by the brass coverplate, see Fig. 20) further complicates the theoretical study of the crystal vibration. For these reasons the relationships described above are only approximately correct, and an experimental calibration has therefore been made of the meter response.

In order to make the crystal holder watertight, the entire unit is assembled with Permatex No. 2, a nonhardening gasket-sealing compound. This material, leakproof to salt water, does exhibit some electrical resistance, but the d-c resistance of the transducers and coaxial cable is over one megaohm. Good acoustical contact between the back of the quartz crystals and the bakelite backplate is made by means of a very thin layer of Permatex.

## The Receiving Crystal

The receiving crystal has been mounted in a holder of almost exactly the same design as that of the transmitting crystal. The half-wavelength backplate is used in this case in an attempt to reduce standing waves which are caused by the reflection of the acoustic wave from the receiving crystal. It has already been demonstrated that the half-wavelength layer of bakelite with one side in contact with water presents a specific acoustic impedance to the back face of the crystal equal to that of sea water. In a similar fashion, the crystal itself is a half-wavelength plate. Therefore, to a first approximation, the specific acoustic impedance looking into the front face of the receiving crystal is also equal to that of sea water. If this were exactly true, reflections would not occur from the crystal face. Actually some reflection does occur from the crystal, indicating that the impedance relationships are not entirely as simple as outlined here, perhaps because the crystal does not vibrate as a simple piston, the backplate may introduce some loss in the system, and the electrical load on the crystal is not an infinite impedance.

Experiments were made with different backing materials and also with bakelite backing plates of different thicknesses to determine the effect of the backing plate upon the standing waves in the velocity meter. To accentuate the presence of standing waves, these tests were made with a crystal spacing of approximately 1 inch rather than 1 foot as it is in the actual meter. The receiving crystal is moved through a distance of about 1 wavelength, always with its face parallel to the face of the transmitting crystal. The ratio of the maximum receiving crystal voltage to the minimum receiving crystal voltage is then recorded. The following table presents the results of this series of measurements:

| Backing material | Voltage ratio |
|---|---|
| Air | 2.86 |
| 0.058-inch polystyrene | 2.5 |
| 0.063-inch bakelite | 2.0 |
| 0.090-inch bakelite | 1.43 |
| 0.100-inch bakelite | 1.67 |
| 0.125-inch bakelite | 3.33 |
| 0.239-inch bakelite | 3.33 |

One-half a wavelength in bakelite at a frequency of 510 kc is slightly less than 0.1 inch. Therefore these experimental results are approximately as expected, although standing waves are not eliminated entirely. With a crystal separation distance of 1 foot instead of the 1 inch used in the measurements above, the slight divergence of the radiation would help to reduce standing waves further.

The voltage generated by the crystal is a function of the pressure applied to the face, the electrical load, and the mechanical impedance of the holder. Equations are developed in Design of Crystal Vibrating Systems[12] and the solutions of these for the velocity meter are given in the appendix.

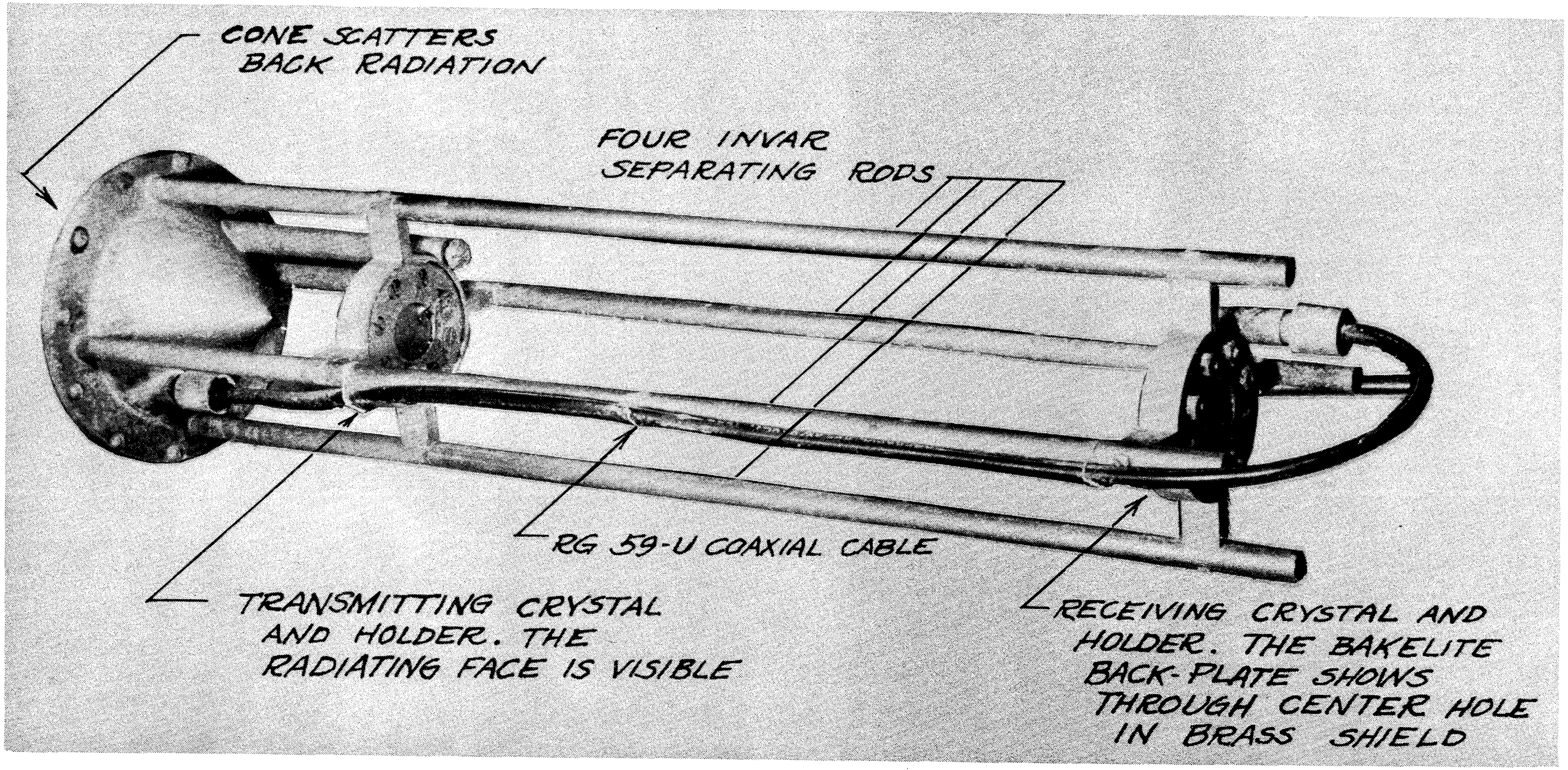



FIG. 21
PHOTOGRAPH OF TRANSDUCERS

Fig. 21 is a photograph of the two transducers as they are mounted in the underwater sound velocity meter.

## MECHANICAL CONSTRUCTION OF THE VELOCITY METER

Only brief mention will be made of the mechanical construction of the various components of the velocity meter. The transducer units have already been described in detail in this report, and construction and assembly drawings of these units are given in the appendix.

The watertight brass chamber containing the electronic components of the meter is 5 inches in diameter and 18 inches long. The 1/8-inch walls of this hollow cylinder have been calculated to withstand crushing pressures up to 390 pounds per square inch, about twice the pressure the instrument will be subjected to at its maximum depth of operation, 400 feet. The bulkheads at each end of the cylinder are sealed by means of standard 1/8-inch (cross-sectional diameter) "O" rings. These have proved absolutely leakproof and are very convenient to assemble.

The pressure-operated potentiometer unit and the main cable seal are located on one of the cylinder bulkheads. These are shown in Fig. 22. The sylphon bellows, the internal spring, and the end pistons of the potentiometer unit were purchased from the Wallace and Tiernal Products, Inc. They are actually the pressure-bellows assembly for a 400-foot bathythermograph. This unit as received from the manufacturer was modified slightly by connecting a shaft to the inside of the piston head. The longitudinal

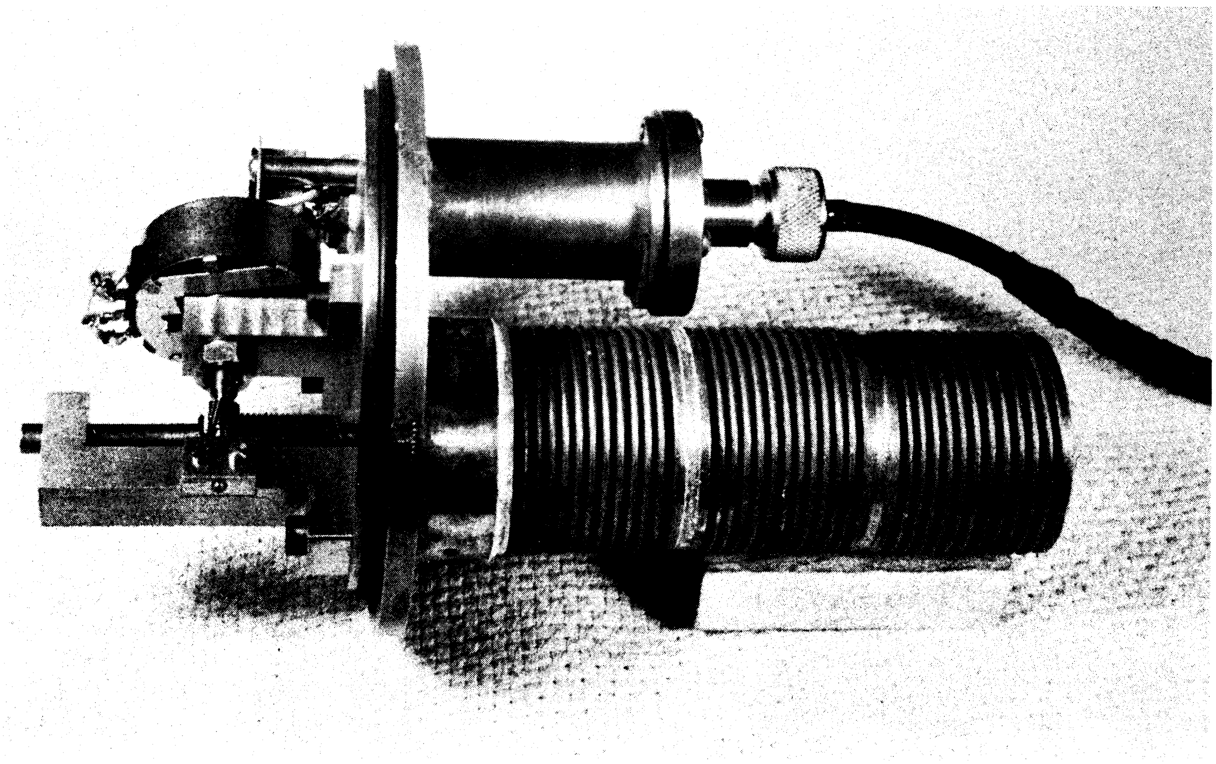


Fig. 22. The pressure-actuated potentiometer and cable seal.

movement of this shaft operates a rack-and-pinion which in turn rotates the arm of the 5000-ohm depth potentiometer. The portion of this shaft coming through the bulkhead is shown in Fig. 22.

All the cables entering the watertight chamber of the velocity meter were sealed twice, once around the outer covering of the cable and a second time at each individual wire. This procedure removes the possibility of having a leak in the sheath of the cable send water into the electronic circuits.

It was recognized in the design of the velocity meter that the presence of a number of electron tubes and electronic equipment in a small enclosed chamber would cause a considerable increase in temperature. This was overcome to a certain extent by constructing the chassis of the electronic unit of copper and mounting it in the brass cylinder so that heat could flow easily to the outside. Two copper slide tracks, soldered to the brass cylinder, hold the chassis in place.

The transducer units and their cables are protected by four brass fins and a connecting ring. This arrangement and other mechanical details are shown in Figs. 4 and 5.

## CALIBRATION OF THE VELOCITY METER IN THE LABORATORY TANK

The underwater sound velocity meter has been calibrated in the laboratory tank by changing the water temperature and recording the accompanying change in phase angle. By making use of sound-velocity tables or by making direct experimental measurements of the velocity of sound (the tables and measurements are described in the following section of this report) the velocity can be determined for various water temperatures. This calibration has been carried out in fresh water at atmospheric pressure.

The laboratory tank, constructed of 1/16-inch steel, is 41 inches long, 23 inches wide, and 19 inches deep. One end of the tank has a hole cut in the wall so that the velocity meter can be mounted through this wall and the opening sealed by means of the watertight gasket. The quartz crystals and their mountings are submerged inside the tank, but the electronic portion of the meter remains outside the tank and is positioned conveniently for electrical tests. A coil of steel tubing inside the tank is connected to a hot-water outlet and drain. By running hot water through this coil, the temperature of the water in the tank can be raised slowly. This method of changing water temperature has been found especially convenient when tests are made with salt water, where the salinity is not to be altered by dilution. To maintain uniformity of temperature the water is stirred constantly, and the temperature is measured with a high-quality mercury thermometer (calibrated in steps of 0.1°C). Fig. 23 is a photograph of the transducer elements of the velocity meter as they are mounted in the laboratory tank. It also shows one of the transducers used in a comparison velocity-measuring system. In order to reduce standing waves in the tank, a reflecting surface has been placed at 45 degrees to the surface of the water at the far end so that the ultrasonic waves arriving at the far end of the tank are largely scattered. As an additional step in the reduction of standing waves in the tank, and 45-degree reflector has also been covered with a 1/2-inch plate of bakelite, which tends to increase the amount of absorption.

The results of this calibration test are summarized in Fig. 24, where velocity-meter milliamperes, a quantity directly related to phase angle, is plotted as a function of water temperature. Numerical data for this test are given on page 127 of the Appendix. A smooth curve has been drawn closely through the experimental points. The slight scattering of the points will be shown later to arise from a small residual standing wave between the velocity-meter crystals. The phase angle changes by a little more than 4 x 360 degrees as the salt water (salinity 35°/00)

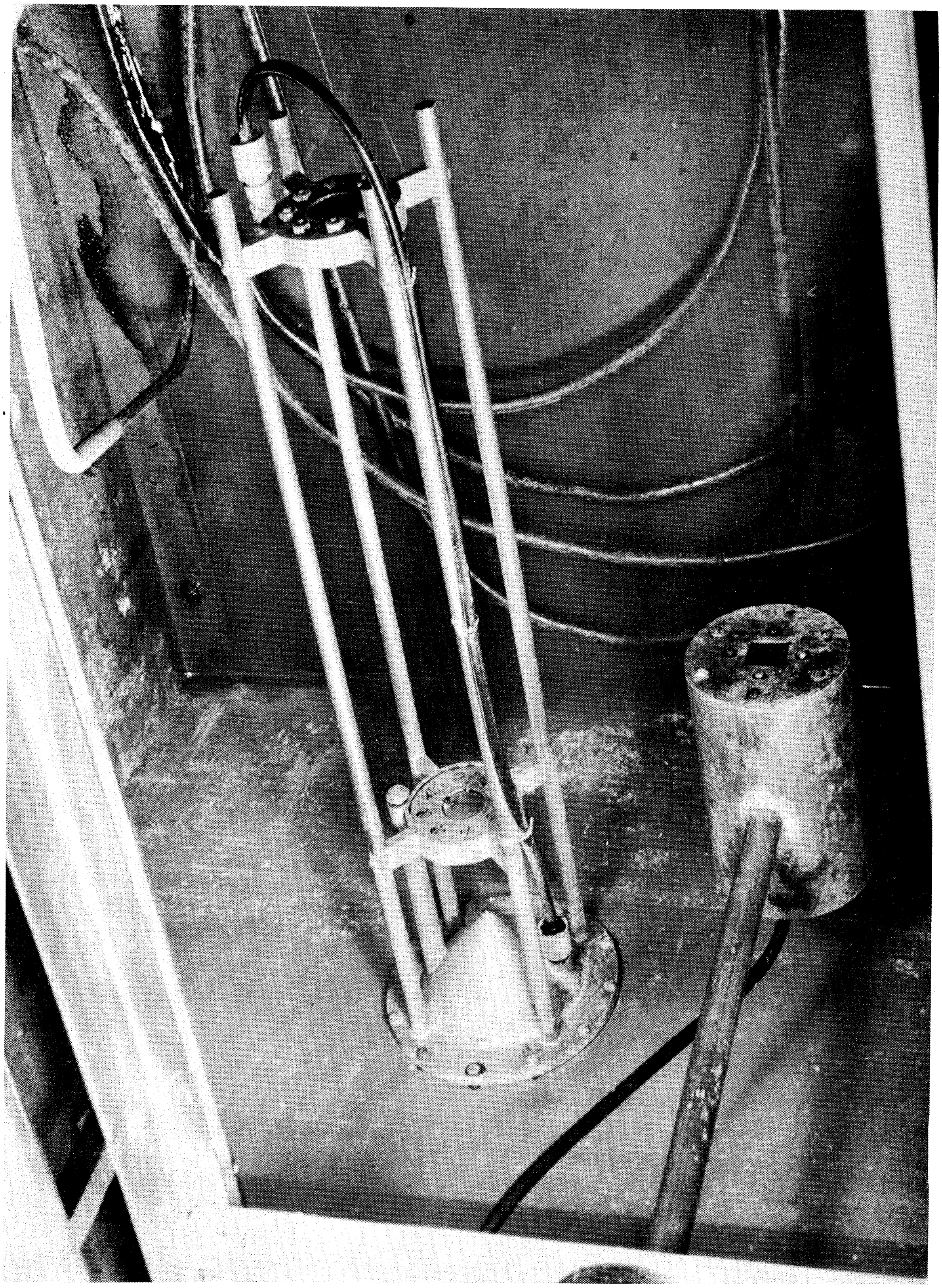


Fig. 23. Velocity meter transducer elements mounted in the laboratory tank for calibration. A transducer element, used in a comparison velocity-measuring system, is also shown.

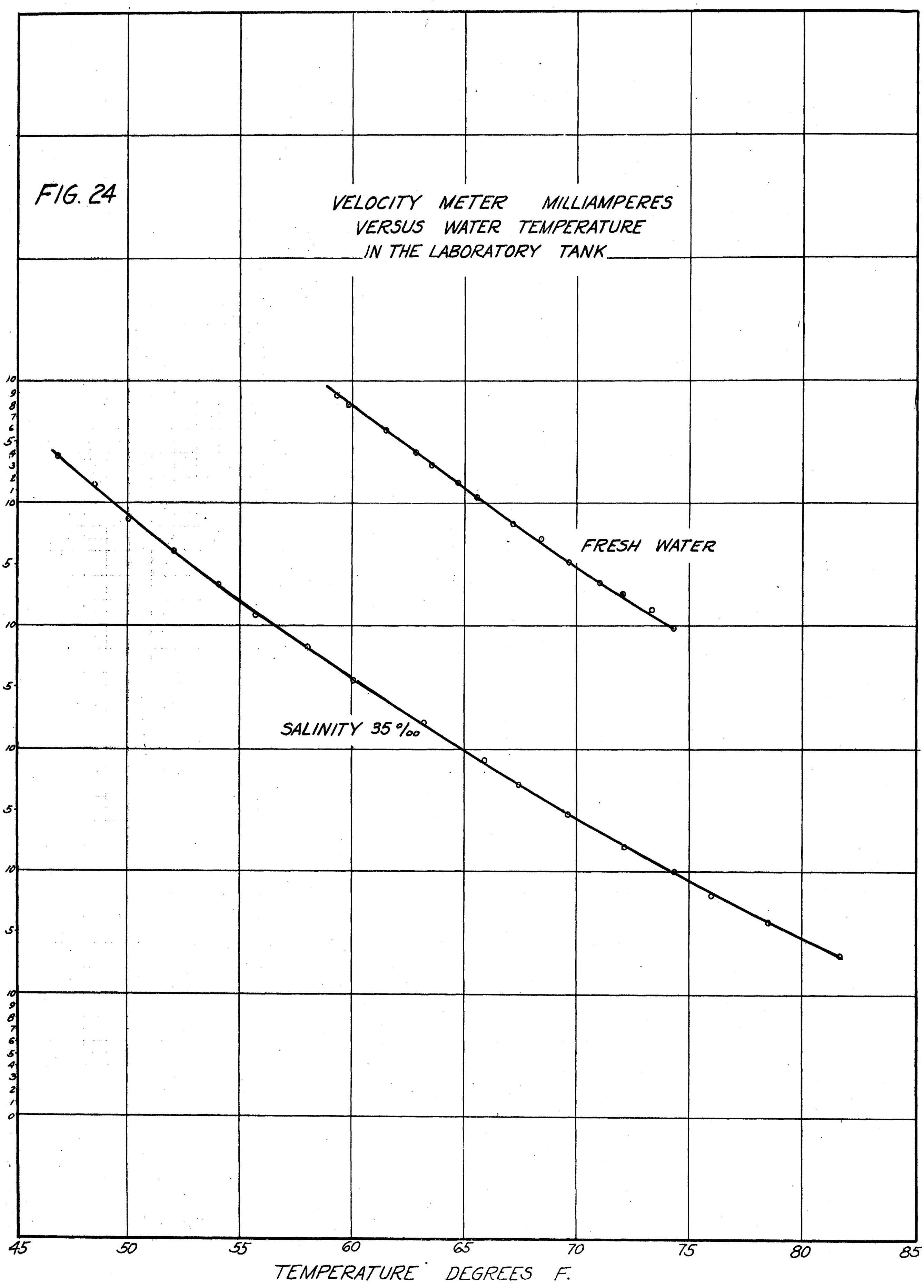


FIG. 24
VELOCITY METER MILLIAMPERES
VERSUS WATER TEMPERATURE
IN THE LABORATORY TANK
FRESH WATER
SALINITY 35 ‰
TEMPERATURE DEGREES F.
45
50
55
60
65
70
75
80
85

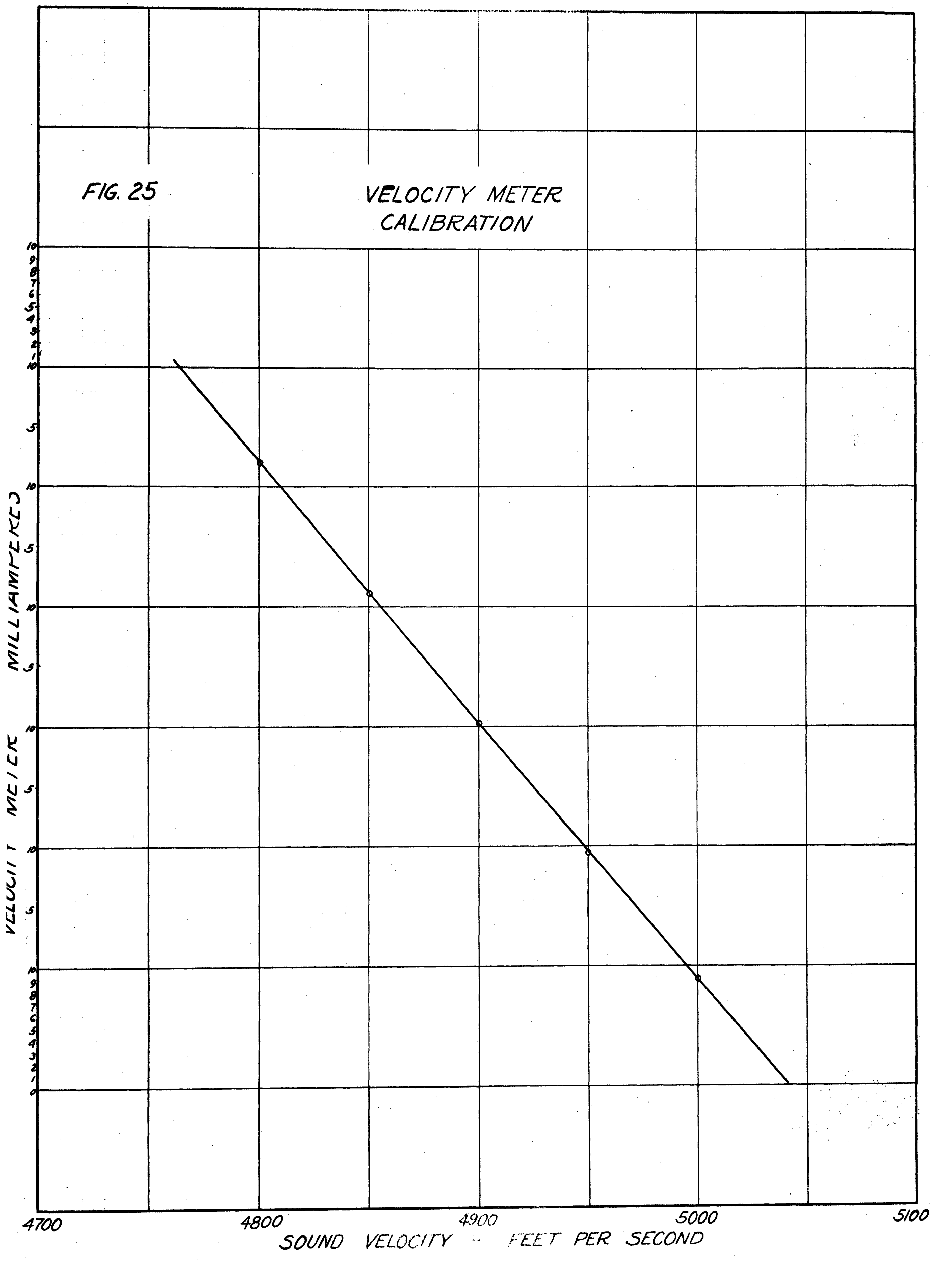
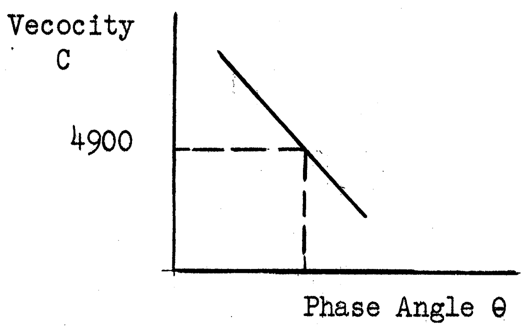


FIG. 25
VELOCITY METER
CALIBRATION
VELOCITY METER MILLIAMPERES
4700
4800
4900
5000
5100
SOUND VELOCITY - FEET PER SECOND

temperature drops from 80 to 45°F. A reading of 0 milliamperes corresponds to 0 degree phase angle (or more exactly to some integral multiple of 360 degrees), 5 milliamperes to 180 degrees, 10 to 360, and so on. As the phase increases slowly through 360 degrees the milliammeter climbs to full scale, suddenly jumps to zero, and then rises again slowly. Phase angles such as 355 degrees will therefore be indicated by slightly less than 10 milliamperes and angles such as 365 degrees as a current slightly more than 0. The scale of the velocity meter repeats in this way every time the phase angle changes by 360 degrees.

Fig. 25 gives the data of Fig. 24 in terms of sound velocity rather than water temperature. The points indicated in Fig. 25 by small circles were taken from the smooth curve in Fig. 24 rather than from the actual experimental points. It is immediately apparent that the relationship between phase angle and sound velocity is strikingly linear in this calibration.

A convincing check on the validity of the calibration curve is to calculate its midpoint slope from the simple basic equations of velocity and phase change when the frequency and the separation distance of the crystals are known. To do this it will be assumed that the meter operates over such a small segment of the hyperbola, (Fig. 9, page 28) that the curve is essentially a straight line.



It has been demonstrated that phase angle $\theta = 360\ f\ d/c$ degrees, or $c = 360\ f\ d/\theta$, where c is velocity, $\omega$ is angular frequency, d is crystal-separation distance, and $\theta$ is the phase angle. The slope $dc/d\theta$ can be evaluated at the approximate midpoint of the curve, where c = 4900 feet per second.

$$\text{slope} = dc/d\theta = 360\ f\ d\ (-\ 1/\theta^2)$$
$$= -\ c^2/(360\ f\ d) \quad .$$

Using values of frequency = $509.3 \times 10^3$ cps and distance d = 1 foot, consistent with the velocity meter, and substituting in the above equation

$$\text{slope} = -\ \frac{\overline{4900}^2}{360 \times 509.3 \times 10^3 \times 1} = -\ 0.131 \text{ ft per sec per degree.}$$

This is in good agreement with the slope - 0.129 of the calibration curve. The predicted equation of the calibration curve is easily shown to be $c = -0.131\ \theta + 9800$ feet per second when $\theta$ is in degrees.

When the experimental results of the tank calibration are studied closely the presence of a residual standing wave between the two transducers can still be detected. Fig. 26 is an enlargement of one section of the experimental data where the effect of standing waves is clearly evident. The experimental curve differs from the straight line by the velocity values given on the following page. For accurate results and when the calibration curve, Fig. 25, is used to convert velocity-meter milliamperes into sound velocity, the corrections listed on the following page should be added to the velocities obtained from the curve. It is to be noted that zero correction is needed at four distinct points on the scale.

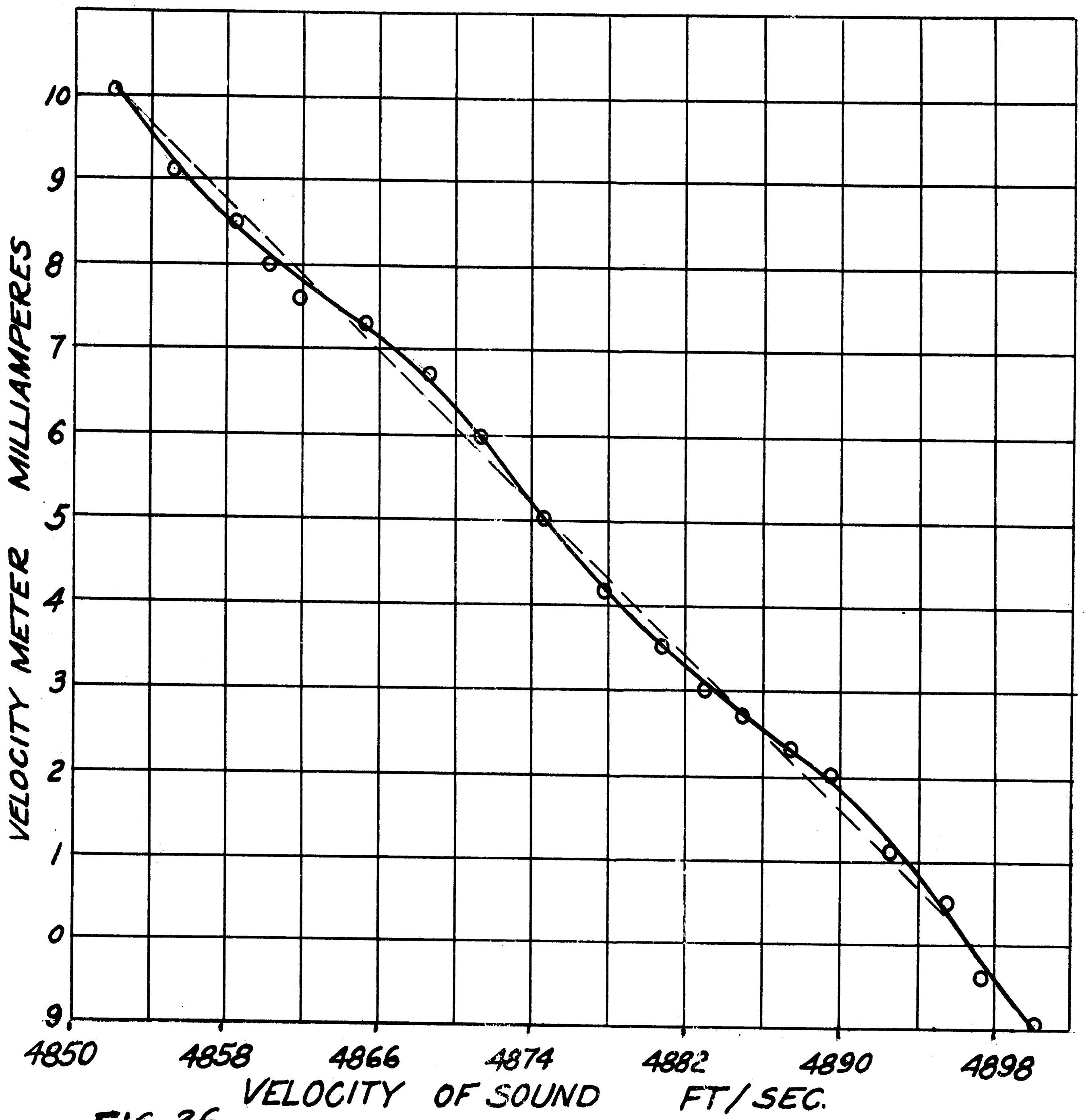



FIG. 26

EFFECT OF STANDING WAVES UPON THE VELOCITY-METER SCALE

| Milliamperes | Velocity, ft/sec |
|---|---|
| 0 | 0 |
| 0.5 | 0.6 |
| 1.0 | 0.9 |
| 1.5 | 0.9 |
| 2.0 | 0.6 |
| 2.5 | 0 |
| 3.0 | -0.6 |
| 3.5 | -0.9 |
| 4.0 | -0.9 |
| 4.5 | -0.6 |
| 5.0 | 0 |
| 5.5 | 0.6 |
| 6.0 | 0.9 |
| 6.5 | 0.9 |
| 7.0 | 0.6 |
| 7.5 | 0 |
| 8.0 | -0.6 |
| 8.5 | -0.9 |
| 9.0 | -0.9 |
| 9.5 | -0.6 |

While the corrections needed for the standing waves are small, the presence of even a slight standing wave can alter the slope of the phase-velocity characteristic considerably in a limited region and lead to errors in interpreting small velocity changes if this change in slope is not considered. This will be demonstrated in a later section of the report when experiments to measure the pressure effect upon velocity are described.

## Tables of the Velocity of Sound in Water

The velocity of sound in water can be determined either by direct experimental measurement or indirectly by calculations based upon the laboratory measurement of certain physical properties. Because of the absence of velocity-measuring instruments suitable for field use, almost all the existing reliable knowledge of sound velocity in the ocean is based upon calculations which are summarized in sound-velocity tables. These tables give velocity as a function of temperature, depth, and salinity. Probably the most accurate tables in existence today are those of Kuwahara,[13] prepared for the Japanese Hydrographic Department, and those of Mathews,[14] prepared for the British Admiralty. In only a very limited number of cases, samples of sea water have been brought into the laboratory for the measurement of sound velocity.[15]

The indirect determination of sound velocity, the approach used in the preparation of sound-velocity tables, is based upon calculation using the formula

$$c = \sqrt{\gamma B_T/\rho} \ ,$$

or its equivalent

$$c = \sqrt{v\gamma/K} \ ,$$

where c is the phase velocity of propagation of the sound wave, $\gamma$ is the ratio of the specific heat at constant pressure and constant volume, $B_T$ is the isothermal bulk modulus of elasticity, and $\rho$ is the density, and K is the compressibility of water. Values of the physical constants $\gamma$, v, and K are known for water at different temperatures, pressures, and salinities. The ratio of specific heats, $\gamma$, appears in the above equations to allow for the fact that a sound wave, a wave of compression, heats the water as it passes through. Water is a poor conductor of heat, and the wave has passed away from each heated portion before that portion has had time to cool down to the temperature of the surroundings. Thus the sound wave travels in a region warmer than that for which the velocity is calculated.

It is interesting to note that an experimentally determined velocity of sound has often been used to yield information on the ratio of specific heats and the compressibility of a fluid.

A summary of sound-velocity tables, converted to English units of feet per second and °F, is given in Table V and Fig. 40, pages 125 and 126 of the appendix. The data given there for a salinity of 35 0/00 is from Kuwahara's tables, and the data for fresh water is from the British Admiralty Tables.

Sound velocity in water increases with pressure. The relationship is nearly a direct proportionality, and the increase is also approximately independent of salinity. According to the velocity tables the pressure effect of velocity can be calculated with sufficient accuracy by the relation

$$\Delta c = .00182\ D,$$

where $\Delta c$ is the increase in sound velocity in water (feet per second) due to the pressure associated with a water depth D feet. Here the proportionality constant is calculated on the basis of a salinity of 35 0/00 and would be slightly different for fresh water. There have been few experimental instruments to measure sound velocities in the laboratory at various pressures, and surprisingly little has been published on the actual results of experiments to measure the change in velocity as a function of pressure.

Sound velocity also increases with salinity. In the range of salinities normally encountered in the ocean, the velocity change is very nearly directly proportional to the salinity; that is,

$$\Delta c = 4.27 \text{ (S - 35.00 0/00)} \quad ,$$

where $\Delta c$ is the velocity to be added to the velocity occurring at a salinity of 35 0/00 in order to determine the velocity at a salinity S 0/00. The British Admiralty tables give values of sound velocity for fresh water as well as for sea water.

The equations above, giving the pressure and salinity corrections to the velocity of sound in water at various temperatures are good approximations, but they are not exact. When both pressure and salinity effects occur simultaneously, second-order corrections should be applied. However, these in general affect only the fifth decimal place in the table values of the velocity of sound, and they are therefore not of great importance.

Measurements have been made in the laboratory tank, by means of a phase-comparison system somewhat similar to that employed in the underwater sound velocity meter, to determine the experimental velocity and to permit comparison with the table velocity. Both the method of measurement and the numerical results are outlined on pages 77 to 84 of this report.

## The Direct Measurement of Wavelength

The phase-comparison system is easily adapted to the direct measurement of wavelength. It may be used in a modified form as a self-calibrating device for the measurement of sound velocity whenever the frequency is known. Instead of maintaining crystal-separation distance constant, as it is in the underwater sound velocity meter, the receiving crystal can be moved along the axis of sound transmission. The electronic circuits are identical with those of the velocity meter. Whenever the receiving crystal is moved through a distance of one wavelength the phase changes by 360 degrees. By measuring distance carefully, for example by counting the angular rotation of a precision thread, the wavelength may be determined accurately; and with a knowledge of the frequency, the velocity of propagation is readily calculated. The accuracy of the system is increased by moving the crystal through a number of wavelengths, preferably a whole number rather than a fractional number, so that standing wave effects are eliminated. Systems of this sort do not appear to have been used previously for the direct measurement of ultrasonic wavelength, probably because of the absence, until rather recent times, of adequate electronic phase-measuring instruments. Interferometers, which are extensively used for the measurement of one-half a wavelength, depend upon the reaction of a reflected wave upon a transmitting crystal. They are therefore basically different from the system described here.

Fig. 27 is a photograph of the original equipment for measuring the wavelength of sound in the laboratory tank by means of the phase-comparison system. The two vertical rods form part of the crystal-mounting system. The transmitting quartz crystal and its holder are located in the tank at the lower end of the first rod. They are fixed in position. The receiving crystal and its holder are held in position by the second rod, which in turn is mounted on a movable carriage. This carriage travels on the two tracks over the tank as is shown in the photograph, and the distance of travel can be determined by means of a calibrated micrometer-thread arrangement. The underwater sound velocity meter is also shown in the photograph, mounted through one end of the tank so that its readings

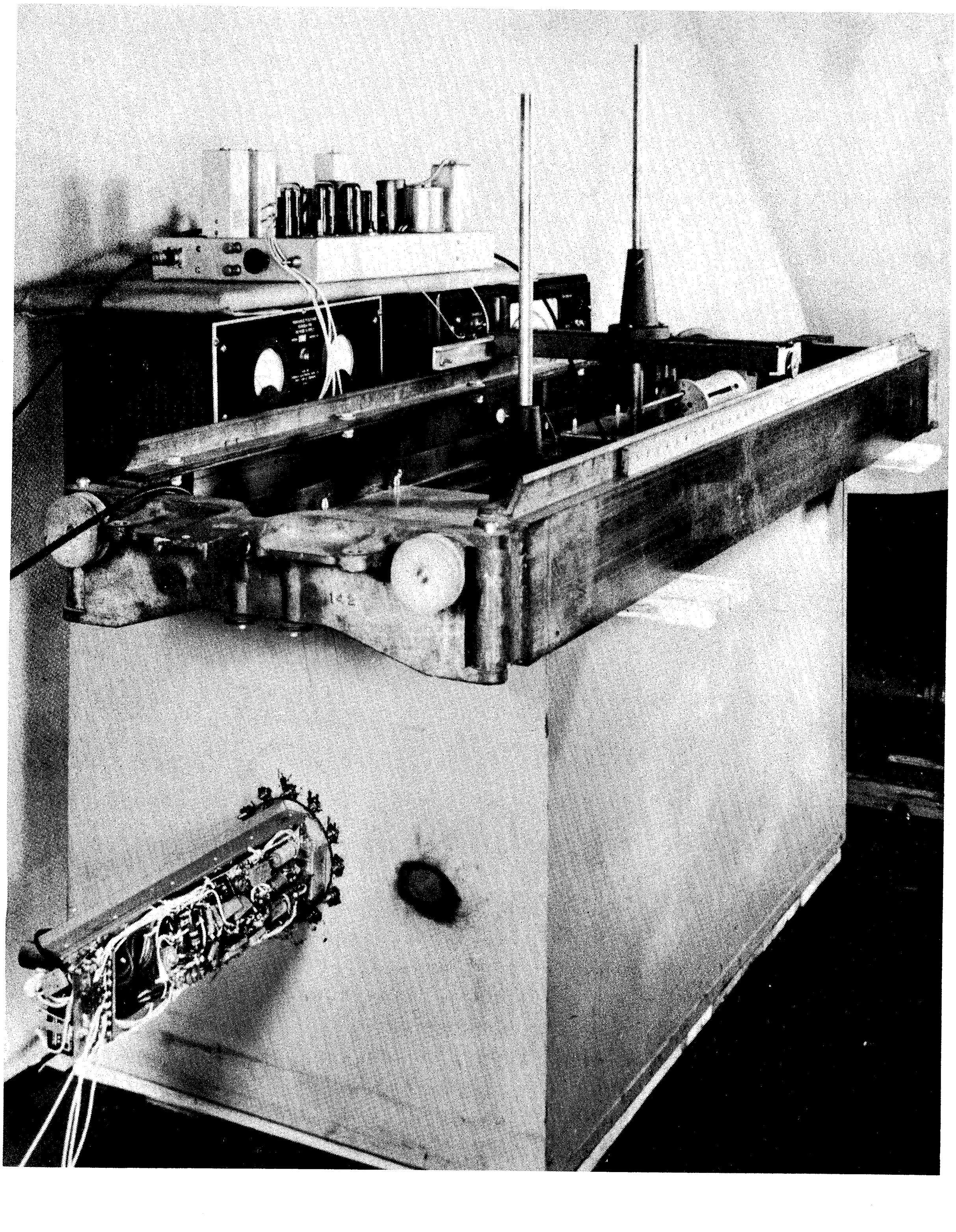


Fig. 27. The electronic portion of the velocity meter is visible at the end of the laboratory tank. Electronic equipment and a carriage for moving two crystals in a comparison velocity-measuring system appear above the tank.

may be checked with the comparison velocity-measuring system. The electronic equipment shown above the tank is a prototype velocity meter of which the radio-frequency amplifier, the two mixers, and the heterodyne oscillator are used. A power supply and Technology Instrument Corporation Phasemeter also appear above the tank.

The mechanical system described above was modified at a later date into a system having the crystal mounting rods run through the walls of the tank in watertight sliding gaskets. This latter arrangement removed errors in the system caused by slight displacements of the rods from a parallel position, which had introduced corresponding errors in the measurement of displacement. A sketch of this modified mechanical arrangement and also the crystals is shown in Fig. 28. The x-cut quartz crystals were edge-clamped and air-backed.

Original attempts to measure wavelength were made with the equipment described above, using test oscillators such as the Hewlett-Packard type 650-A wide-range oscillator or the General Radio type 700-A beat-frequency oscillator as the frequency-controlling element. It was immediately evident that the frequency stability of these oscillators was not adequate for the accuracy needed in the measurements. A crystal-controlled oscillator, the same as used in the underwater sound velocity meter, improved system stability to a point where individual measurements of wavelength were considered reliable to better than $\pm 0.08$ per cent. All measurements were made at the one frequency, 509.3 kc with the crystals moved through a distance of 25 wavelengths.

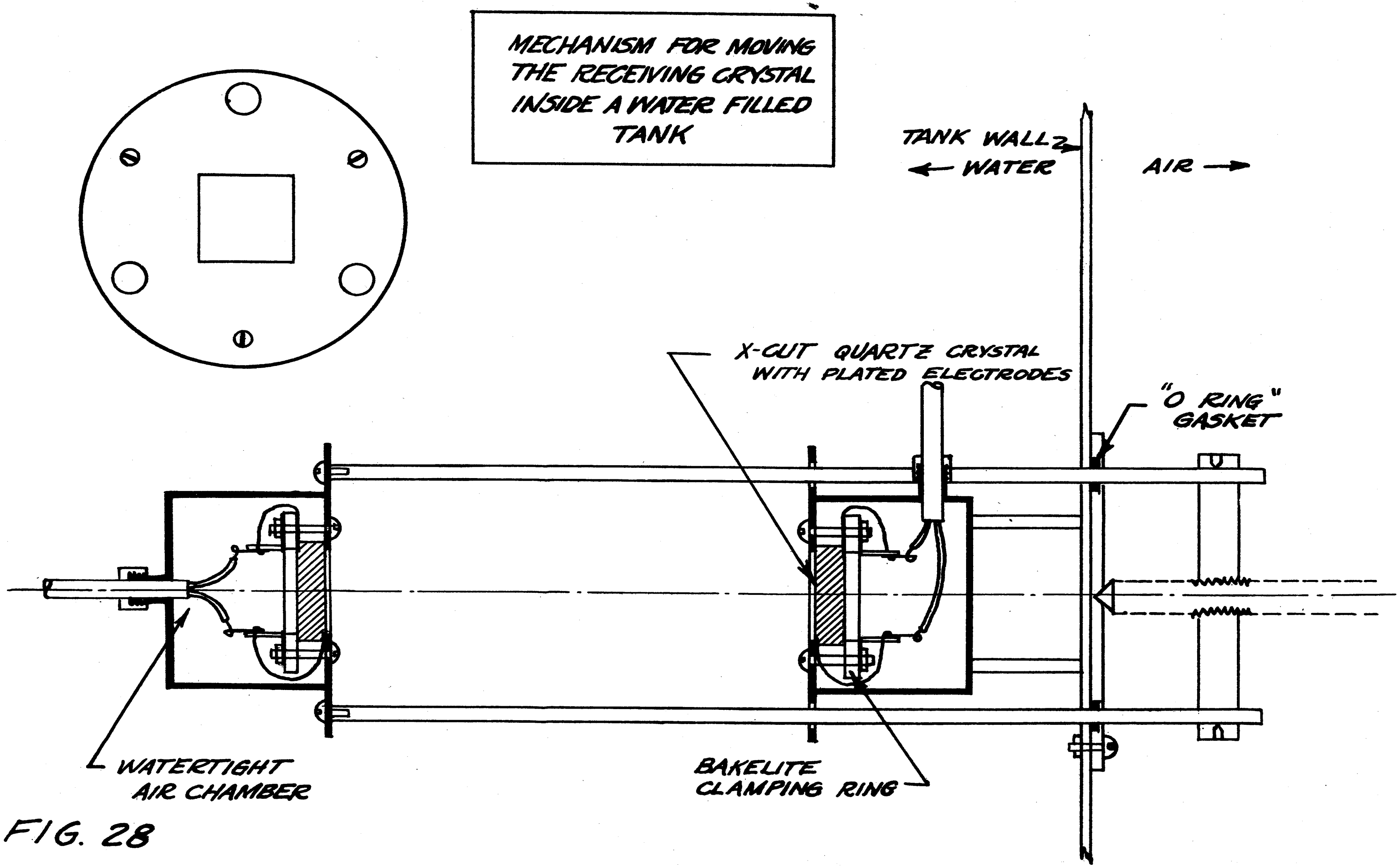

MECHANISM FOR MOVING THE RECEIVING CRYSTAL INSIDE A WATER FILLED TANK
TANK WALL
← WATER
AIR →
X-CUT QUARTZ CRYSTAL WITH PLATED ELECTRODES
"O RING" GASKET
WATERTIGHT AIR CHAMBER
BAKELITE CLAMPING RING

FIG. 28

Tables III and IV give the results of a series of measurements of the velocity of sound in the laboratory tank at various temperatures and compare these with the Kuwahara and the British Admiralty Table values of the velocity. Table III is for fresh water (water from the City of Ann Arbor water system), and the comparison is made with the British Admiralty Tables for fresh water at atmospheric pressure. Table IV is for water with sodium chloride added to produce a salinity of 35 0/00, and the comparison is made with the Kuwahara Tables.

Table III

Measured Velocity of Sound in Water of Salinity 35 0/00
Compared with Table Value of the Velocity of Sound

| Temperature °F | Wavelength, ft | Measured Velocity ft/sec | Table Velocity ft/sec | Difference, ft/sec |
|---|---|---|---|---|
| 53.0 | .009658 | 4918 | 4898 | 20 |
| 56.0 | .009697 | 4939 | 4923 | 16 |
| 59.6 | .009732 | 4956 | 4937 | 19 |
| 63.7 | .009767 | 4974 | 4959.5 | 14.5 |
| 63.8 | .009778 | 4980 | 4960 | 18 |
| 68.3 | .009820 | 5001 | 4984 | 17 |
| 71.4 | .009857 | 5020 | 5000 | 20 |
| 75.5 | .009895 | 5040 | 5019 | 21 |
| | | | | Average 18.2 |

Calculation of weight of salt added to the tank:

Tank dimensions: 41 x 23 x 16 inches

Density of sea water: 63.99 pounds per cubic foot at 15° C

Weight of salt for salinity of 35 0/00 =

$$\frac{41 \times 23 \times 16}{1728} \times 63.99 \times .035 = 19.53 \text{ pounds.}$$

Table IV

Measured Velocity of Sound in Fresh Water Compared with Table Values of the Velocity of Sound

| Temperature, °F | Wavelength, ft | Measured Velocity, ft/sec | Table Velocity ft/sec | Difference ft/sec |
|---|---|---|---|---|
| 53.9 | .009403 | 4789 | 4767 | 22 |
| 56.5 | .009430 | 4803 | 4785 | 18 |
| 59.2 | .009475 | 4826 | 4804 | 22 |
| 61.7 | .009502 | 4839 | 4818 | 21 |
| 64.3 | .009528 | 4853 | 4833 | 20 |
| 66.5 | .009570 | 4872 | 4847 | 25 |
| 68.8 | .009587 | 4883 | 4858 | 25 |
| 71.4 | .009613 | 4896 | 4874 | 22 |
| 74.3 | .009640 | 4910 | 4887 | 23 |
| | | | Average | 22 |

Frequency: 509.3 kilocycles

With fresh water the measurements appear to average about 22 feet per second higher than the predicted values; for salt water, about 18.2 feet per second higher. These differences do not seem unreasonable. Weissler and Del Grosso[15] report measured values of the velocity of sound in samples of sea water approximately 3 meters per second higher than the predicted values from the Kuwahara Tables. They used a laboratory ultrasonic interferometer operating at a frequency of 3 mc. A difference in the chemical composition of the water measured in Ann Arbor may possibly account for some of the higher velocity values measured. However, the experimental evidence seems to be consistent in indicating that the tables in current use give velocities that are too low. With so much velocity information at the present time depending upon the use of sound-velocity tables, it is surprising that more attention has not been directed to their experimental verification.

The underwater sound velocity meter has been calibrated in the laboratory tank in salt water having a calculated salinity of 35 0/00. The calibration process consists of measuring water temperature and then determining velocity from the sound-velocity tables. To make the tables agree with experimental results a correction of 18 feet per second is added to these tabulated velocities.

## Pressure-Tank Tests of the Velocity Meter

In order to calibrate the movement of the chart in terms of ocean depth, the velocity meter was placed inside a laboratory pressure tank at the Woods Hole Oceanographic Institute and the chart displacement measured as a function of pressure. The average displacement was found to be 1 inch for every 41.6 feet of depth. This makes the adjacent arcs on the recording milliammeter paper (arcs spaced 3/4 inch apart along the axis of the chart paper) corresponds to a depth change of 31.2 feet. A pressure calibration of this type is perhaps even more accurate than measuring cable length on the meter in the actual ocean, because of the uncertainty of the angle the cable makes with the vertical when it is connected with a drifting ship.

Fortunately, a velocity-meter record was taken simultaneously with the test described above. This was actually a carefully controlled measurement of the influence of pressure upon sound velocity. Fig. 29 is one of the experimental records; it shows a linear shift in phase with pressure up to the maximum pressure of the test, that corresponding to 282 feet of ocean depth. The small, sharp notches of this record are present because the pen was pushed by hand to mark certain pressure intervals, so

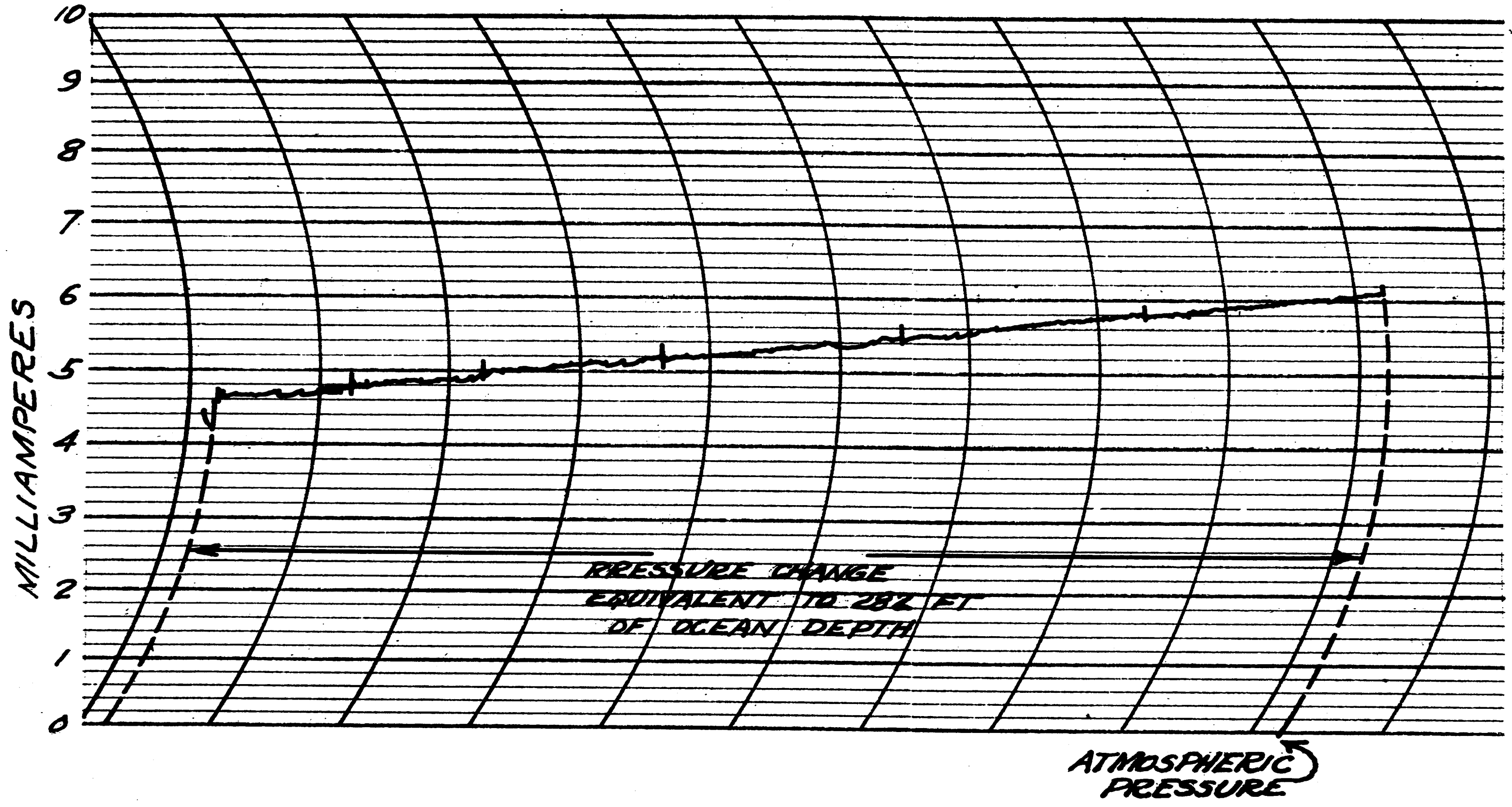



FIG 29 INDICATED VELOCITY CHANGE WITH PRESSURE
VELOCITY METER INSIDE THE WOODS HOLE OCEANOGRAPHIC
INSTITUTE PRESSURE TANK - FRESH-WATER TEST

they are not actually a part of the velocity record. The experiment was performed in fresh water at a water temperature of 71°F. The increase in the temperature of the water in the tank from adiabatic compression is negligible for the pressure changes used in this test; therefore, the water temperature remained essentially at 71°F.

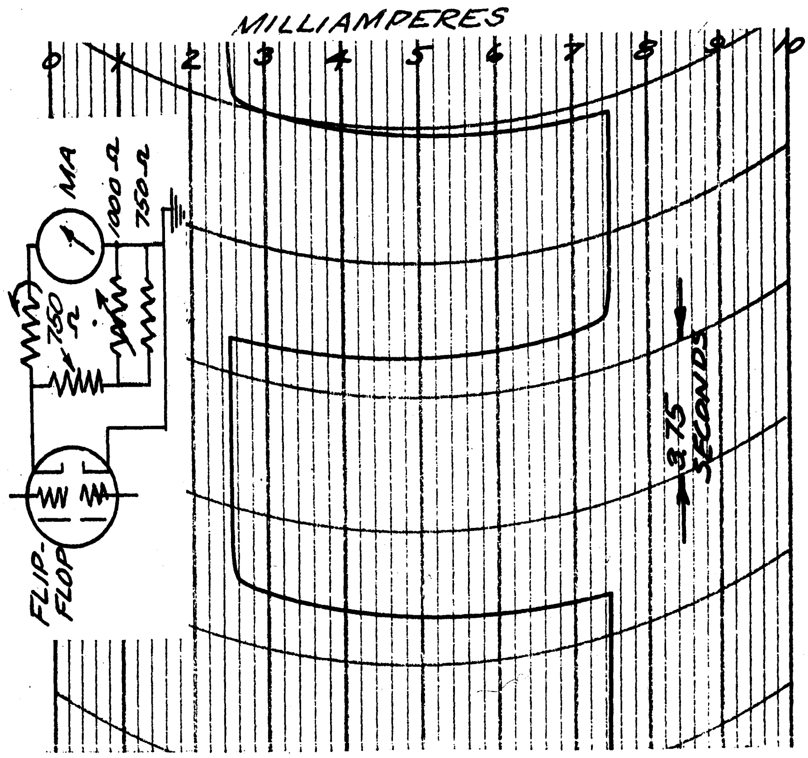
The expected change in sound velocity for a change of depth of 272 feet is $\Delta c$ = 0.00182 x 272 = 5.13 feet per second. This is actually for sea water, but it will not differ greatly for fresh water. The observed change in phase angle is $\frac{(6.1 - 4.6)}{(10 \times 360)} = 54°$. As seen by reference to the slope of the calibration curve, Fig. 25, page 70, this indicates a velocity change of 7.06 feet per second, or 38 per cent greater than predicted. However, when the effect of the standing wave, which is shown on page 73, Fig. 26, is included, the agreement between the predicted and the measured result is very much closer. Most of the pressure-phase record in this tank experiment occurs in the neighborhood of 5 ma on the velocity-meter milliammeter. Fig. 26 indicates a decreased slope in this region, a slope of -0.0972 rather than the -0.131 foot per second per degree called for by the average calibration curve. With this correction, due to the presence of a slight residual standing wave, the measured velocity becomes $\Delta c$ = 0.0972 x 54 = 5.25 feet per second, which is in fair agreement with the table velocity change of 5.13 feet per second. The difference is, of course, extremely small when compared with the total velocity of sound in the water at this temperature.

Transient Response of the Velocity Meter

The transient response of a velocity-measuring instrument is important when the measurement of abrupt changes in velocity is considered and the instrument itself travels rapidly through the water. Sudden changes in velocity are likely to occur between ocean layers of different temperature and salinity.

The speed of response of the present velocity meter is essentially determined by the response of the recording milliammeter, which in turn is influenced greatly by the arrangement of shunt and series resistance. The recording meter is a 0-5-ma permanent-magnet, moving-coil element which is connected by means of a series resistance and shunt to the flip-flop circuit. Full-scale deflection, at very nearly 10 ma, is adjusted by means of a variable shunt resistance and a test described in the electronic-circuit section of this report.

Fig. 30 shows the response of the phase-measuring sections of the velocity meter, that is, from the outputs of the two mixers to the recording milliammeter, to a sudden reversal of phase. The lower record, with a simple shunt connected across the meter, shows a considerably slower response than the upper record, which is for the actual circuit used in the velocity meter. The velocity meter attains 90 per cent of its final response in 0.4 second.




VELOCITY - METER CIRCUIT

SIMPLE SHUNT

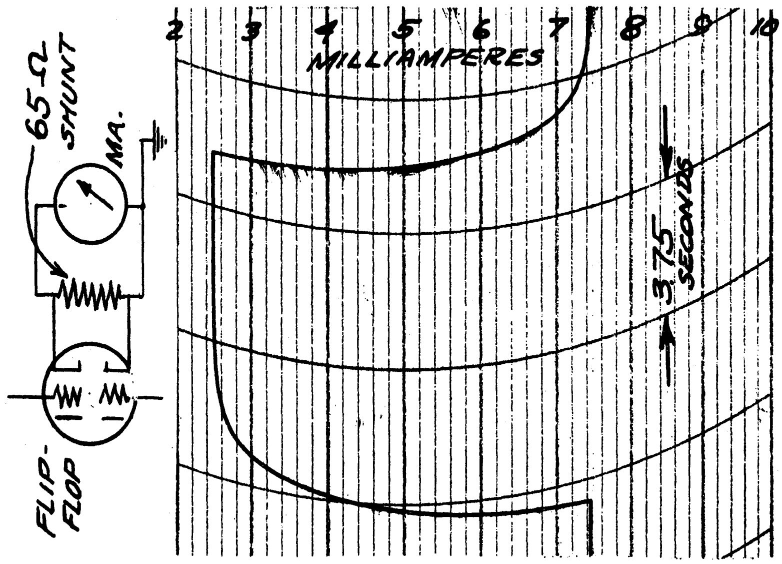



FIG. 30

RESPONSE OF PHASE-MEASURING CIRCUITS TO SUDDEN REVERSAL OF PHASE

## SOURCES OF ERROR

The sources of error in the underwater sound velocity meter can in general be grouped under the following headings:

(a) Frequency drift
(b) Changes in the phase response of the electronic components and the transducers'
(c) Changes in the dimensions of the mechanical components
(d) Disturbances from the water flow around the transducers

These will now be examined in detail.

### Frequency Drift

The errors introduced in the phase-comparison system of velocity measurement due to changes in frequency have already been described in the electronic-circuit section of this report (page 32). It has been pointed out that a change of 100 cps in the frequency of the acoustic signal transmitted, for example from 500,000 to 499,900 cps, produces a change in phase of 7.2 degrees or the equivalent of about 1 foot per second change in sound velocity. Also, it has been experimentally observed that a change of 1 kc in the frequency of the heterodyne oscillator introduced an error of about twice this amount in the indicated sound velocity. The frequency drift of the oscillators in the velocity meter is believed to be associated with the changes in the temperature of the various electronic components, and for this reason is a slow variation. The readings of the meter for a single drop from a surface ship, when the readings are taken in a period of a minute or less, appear to be essentially free of errors introduced by frequency changes of the oscillators.

When the present meter is operated for long periods of time there is a tendency for the electronic circuits, which are completely enclosed in the watertight cylinder, to overheat. This condition could be overcome by a different physical arrangement of the various components. Rather than attempt to house all the electronic circuits in the submerged portion of the meter, the circuits could be modified so that all but the radio-frequency portions of the meter are located on the surface ship. This would simplify the problems of heat dissipation and tend to improve the stability of the two oscillators over longer time intervals. An additional improvement in the design of the meter would be the use of crystal control for the heterodyne oscillator. This would greatly stabilize the frequency of that oscillator.

Changes in Phase Response

Changes in the phase response of one channel of the velocity meter without compensating changes in the other channel can introduce errors in the readings of the meter. The most likely place for such errors to be introduced is in the transducer elements or in the radio-frequency amplifier.

In order to investigate the influence of temperature change upon the phase response of the transducers, a special experimental transducer arrangement was made up so that the water path between the two crystals was eliminated entirely. The two crystals in this test unit had their front faces in direct contact with each other and each crystal was backed by a half-wavelength backplate as in the velocity meter. This experimental unit was submerged in the laboratory tank and the crystals connected to the electronic circuits as they are in the actual meter. It was found that an increase in the temperature of the water in the tank from 40 to 80°F produced no observable change in the phase angle. This experiment tends to support the belief that the shift in phase measured by the velocity meter is entirely attributable to the changes in the velocity of sound in the water sample and is not caused by changing properties of the crystals with temperature.

It has been mentioned earlier in this report that the radio-frequency amplifier in the acoustic channel of the velocity meter has a tuned transformer between its output and the mixer grid. Both the primary and the secondary of this transformer are tuned to resonance by small adjustable mica condensers. If these condensers are changed after the velocity meter is once calibrated, the calibration curve could be shifted by as much as 15 feet per second under extreme conditions. This would correspond to a phase change of approximately 90 degrees introduced after calibration of the meter. In spite of this phase change introduced in the electrical system, the slope of the calibration curve would be changed very little, and the velocity meter would still give correct velocity differences.

## Changes in Dimensions

The two crystals acting as transducers in the underwater sound velocity meter are separated by four invar rods approximately 1 foot in length. The meter operates in various temperatures of sea water. These various temperatures bring about a thermal expansion or contraction of the separating rods, and it is worth while to calculate the error occasioned in the velocity measurement because of these changes in the lengths of the rods. The cases of both invar and brass rods will be considered.

The thermal coefficient of expansion $\epsilon$ of a metal is the increase in its length per unit length per °C when the temperature is increased. For brass the coefficient is in the neighborhood of $19 \times 10^{-6}$ and for invar, a nickel-steel alloy containing 36 per cent nickel, it is only $0.9 \times 10^{-6}$. Consider the case where the temperature of the sea increases from 10 to 20°C and the instrument is immersed long enough in these two distinct temperature regions for thermal equilibrium to be reached in the separating rods. A change in phase will occur as the instrument is moved from the one temperature region to the other. This change in phase may be considered to be caused by both an increase in the length of the rods and an increase in the velocity of propagation of the sound wave in the water region between the transducers. It is not difficult to separate these two effects and calculate the magnitude of each one as if it were acting alone. The phase shift due to the thermal expansion of the rods will be examined first.

The increase in rod length $\Delta d$ is related to the coefficient of expansion $\epsilon$, the original length d, and the two temperatures as follows:

$$\Delta d = \epsilon d \, (t - t_o) \quad .$$

In the example considered here $t_o$ is 10°C and t is 20°C. For the two metals the expansion in a rod 1 foot long is,

for brass, $$\Delta d = 19 \times 10^{-6} \times 1 \times (20 - 10) \quad ;$$
$$= 190 \times 10^{-6} \text{ feet}$$

for invar, $\Delta d = 0.9 \times 10^{-6} \times 1 \times (20 - 10)$

$= 9 \times 10^{-6}$ feet.

The phase difference $\Theta$ between two plane acoustic wave fronts separated a distance d has been demonstrated to be (see page 29)

$$\Theta = \omega d/c \quad ,$$

where $\omega$ is the angular frequency and c is the velocity of propagation of the sound wave. It will be assumed in the following calculation that the velocity c remains constant at 4930 feet per second, a value corresponding to the average of the two sound velocities for temperatures of 10 and 20°C in an ocean having a salinity of 35 0/00. Calling the two phase angles $\Theta_{10}$ and $\Theta_{20}$ for the two temperatures, then the change in the phase angle due to the increase in rod length from $d_{10}$ to $d_{20}$ is

$$\Theta_{20} - \Theta_{10} = \omega/c\ (d_{20} - d_{10}) \quad \text{radians or}$$

$$\Theta_{20} - \Theta_{10} = 360f/c\ (d_{20} - d_{10}) \quad \text{degrees.}$$

The frequency is f, approximately 500 kc in the velocity meter. The second equation may be rewritten in terms of the increase in length $\Delta d$:

$$\Theta_{20} - \Theta_{10} = 360f/c\ (d_{10} + \Delta d - d_{10})$$

or

$$\Theta_{20} - \Theta_{10} = 360f/c\ \Delta d \quad .$$

This difference in phase angle for brass and invar is calculated below.

for brass, $\Theta_{20} - \Theta_{10} = \dfrac{360 \times 500 \times 10^3}{4930} \times 190 \times 10^{-6} = 6.83$ degrees for invar, $$\theta_{20} - \theta_{10} = \frac{360 \times 500 \times 10^3}{4930} \times 9 \times 10^{-6} = 0.328 \text{ degrees.}$$

These changes in phase angle, due only to the increase in the length of the separating rods, will next be compared with the phase change caused by the different sound velocities in the water at the two temperatures. The separation distanced will be assumed constant in the calculation of the angles associated with velocity differences. In an ocean of salinity 35 0/00 the sound velocity at 10°C is 4879 feet per second and at 20°C, it is 4982 feet per second. The difference in phase angle caused by these two different velocities is calculated below.

$$\theta_{20} - \theta_{10} = 360 \text{ f d } (1/c_{20} - 1/c_{10}) \text{ degrees}$$

$$\theta_{20} - \theta_{10} = 360 \times 500 \times 10^3 \times 1 \times (1/4982 - 1/4879)$$

$$= 763 \text{ degrees.}$$

This change in phase angle is considerably greater than that produced by thermal expansion of the rods; in fact the thermal-expansion angle difference is 0.9 per cent for brass, and for invar only 0.043 per cent, of the angle difference-of-velocity change. The figures of the example above show the relative **unimportance** of the errors introduced by thermal expansion of the crystal separating rods in the readings of the underwater sound velocity meter. The use of invar rods makes the error negligible.

Disturbances from the Flow of Water around the Transducers

The readings of the velocity meter are influenced to a certain extent by the flow of water around the transducer units. Some of the velocity records which will be given later in this report show sharp spikes of the order of 1 foot per second which were observed to be related to the roll of the ship in time. They are apparently not connected with actual changes of sound velocity in the water.

In order to investigate in the laboratory the influence of towing the velocity meter in the water, several tests were made in the University of Michigan Engineering Mechanics Department fluid flow laboratory. The velocity meter transducer units were mounted in the flow tank with the axis of sound transmission roughly parallel to the direction of water flow. The entire transducer assembly could be turned end for end in the tank to compare the shift in phase between the condition (1) of water flowing away from the transmitting crystal and (2) the condition of water flowing toward the transmitting crystal. Turbulence obscured these tests to a certain extent, but a definite shift in phase was observed. With water velocities of the order of 1 foot per second the reversal of the transducer assembly produced a phase change of about 15 degrees. This would correspond to an apparent change in sound velocity of approximately 2 feet per second. Tests at sea and also in the flow tank tend to indicate that this disturbing shift in phase due to the flow of water is reduced or eliminated if the flow of water is at right angles to the direction of sound transmission.

## VELOCITY METER FIELD TESTS

A series of field tests will be described on the following pages in which the velocity of sound was measured in a fresh-water lake and also in the Atlantic Ocean. It is believed that these experiments represent the first successful attempt to measure sound velocities as a continuous function of depth under field conditions. All of the records shown have a vertical scale giving the depth of the instrument and a horizontal scale indicating the d-c current from the output of the phasemeter. This latter scale is easily converted into sound velocity by means of the calibration curve for the velocity meter given in Fig. 25, page 70. On certain of the records this velocity scale has been indicated, while other records show only the current scale.

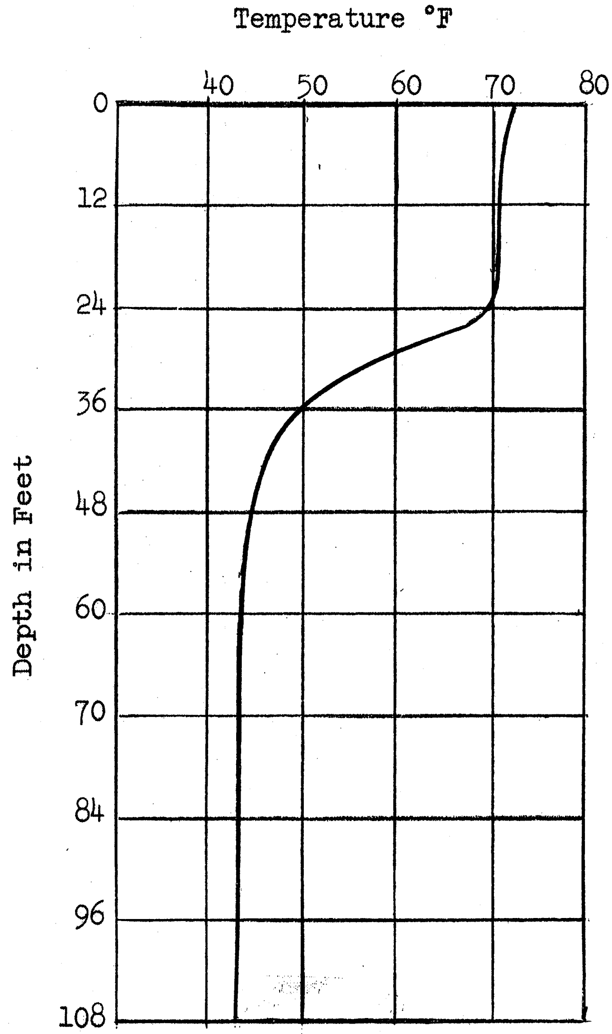
An attempt has been made to compare the velocity records with what would be expected from the sound-velocity tables which have been described in earlier sections of this report.

Preliminary Field Tests

Preliminary field tests of the velocity meter were made in early August, 1951, in fresh water at Orchard Lake, Michigan, where water depths of over 100 feet were available. Power was obtained from a small vibrator running from storage batteries, so that the equipment was highly portable and could be operated from a small boat. This initial test served primarily as a shakedown to find troubles which might develop in the field and under conditions where the water pressure was no longer negligible. This was a fortunate precaution, because the depth-indicating potentiometer, which is driven by the comparession of a spring under static water pressure, was found to bind and drive the servo in small jumps rather than continuously. This trouble was remedied by reducing the mechanical friction in the depth-potentiometer system, and the servo functioned more satisfactorily on tests at a later date in the ocean.

At this time calibrated oceanographic temperature-measuring equipment was not available for the experiment, so a rather awkward but effective system of measuring lake-water temperature at various depths was used. Two thermocouples were lowered into the lake. One of these was exposed to the lake water, and the other, a comparison junction, was kept inside a small thermos flask and maintained at 32°F in the presence of melting ice. A portable potentiometer was employed to measure thermocouple voltage. The following lake-water temperatures were recorded:

| Depth, ft | Temperature, °F |
|---|---|
| surface | 72 |
| 6 | 71 |
| 12 | 70.5 |
| 18 | 70.5 |
| 24 | 69.5 |
| 30 | 57.5 |
| 36 | 50 |
| 42 | 46 |
| 48 | 44.5 |
| 54 | 44 |
| 60 to 96 | 43.5 |
| 108 | 43 |




Because of the friction in the servo system, about the only information obtainable from the Orchard Lake velocity-meter test record was the total shift in phase from the surface of the lake to a depth of

100 feet. The recording milliammeter went through slightly more than four complete cycles, starting at 3.4 ma and ending at 3.8 ma at the 100-foot depth. This corresponds to an increase in phase angle of 4 x 360 + 7.2 = 1447.2 degrees. If the calibration curve of the meter is extended to cover the fresh-water velocities of this test, such an indicated phase shift corresponds to a velocity change of 188 feet per second.

The British Admiralty Tables for fresh water give sound velocities of 4875 and 4685 feet per second for temperatures of 72 and 43°F, respectively, a decrease in velocity of 190 feet per second from the higher to the lower temperature. The pressure correction for 100 feet is approximately 1.8 feet per second, an increase in velocity with the higher pressure. Therefore, the total predicted change in sound velocity is 190 - 1.8 = 188.2 feet per second, in excellent agreement with measured results.

Tests of the Underwater Sound Velocity Meter in the Atlantic Ocean

From August 22 to August 28 the underwater sound velocity meter was given a series of field tests in the Atlantic Ocean with the cooperation of the Woods Hole Oceanographic Institute at Woods Hole, Massachusetts. These tests were designed to observe the operation of the meter under actual field conditions and to compare the values of sound velocity obtained from the meter with those obtained from sound-velocity tables on the basis of water temperature, pressure, and salinity. Coincident with the lowering of the velocity meter in these tests, oceanographers of the Institute made recordings of water temperature as a function of depth and took samples of water at various depths which were later analyzed for salinity. A careful check can therefore be made in most cases between measured and predicted results.

On the following pages a series of velocity-meter records will be compared with bathythermograms made simultaneously. A bathythermogram is a water temperature-depth record made by means of a bathythermograph. The records reproduced in this report are photographs of small, smoked slides from the instrument. The horizontal scale of the slides is temperature in °F and the vertical scale is depth in feet. Each of the slides have the following identification information: (a) the number of the slide; (b) the time; (c) the date; (d) the number of the bathythermograph; and (e) the name of the ship. The tests at Woods Hole were made from the Bear. Fig. 31 is a photograph of the velocity meter about to be lowered in the ocean and Fig. 32 shows the bathythermograph coming out of the water after a trip to the bottom and back.

The first ocean test was made in about 100 feet of water off Tarpaulin Cove in Vinyard Sound (lat. 41° 26.4', long. 70° 46.1'), where a rather strong tidal current produces thorough mixing of the water. The bathythermogram indicates isothermal water, and the analysis of water samples from several depths shows a nearly constant salinity of 31.79 0/00. The velocity-meter records for this location, such as the sample shown in Fig. 33 on page 101, indicate in general a slight increase in velocity with depth (a decrease in phase). Comparison of the velocity-meter records with those of the pressure test in the laboratory tank tends to support the belief that the average increase in the velocity with depth as recorded by the meter is due to the increase in pressure. The record shows a

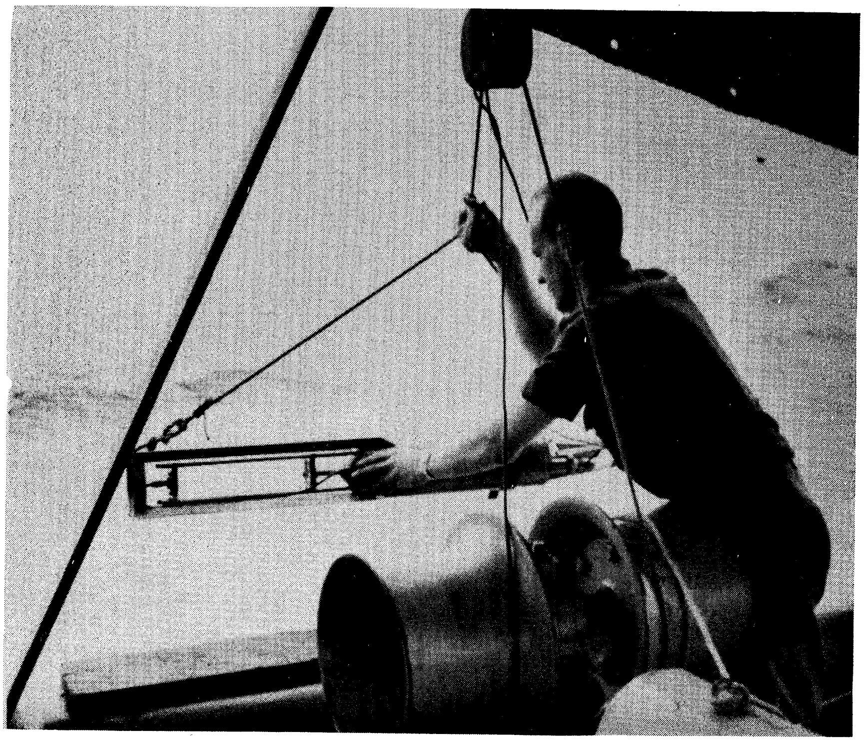


Fig. 31. The velocity meter about to be lowered from the surface ship. The meter is horizontal. Earlier experiments were made with the axis of the meter vertical.

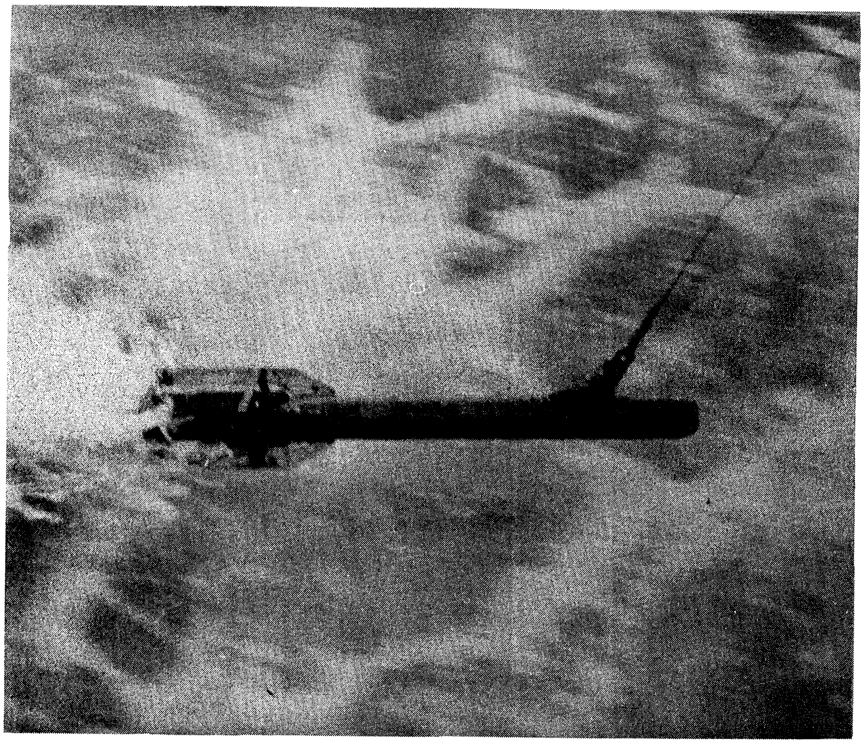


Fig. 32. The BATHYTHERMOGRAPH after a measurement of water temperature. This instrument is lowered by means of a small steel cable.

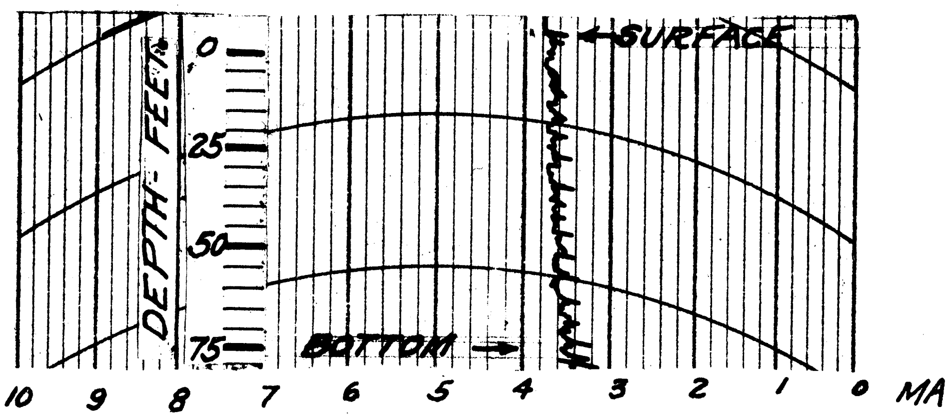



FIG. 33(a) VELOCITY METER TRACE

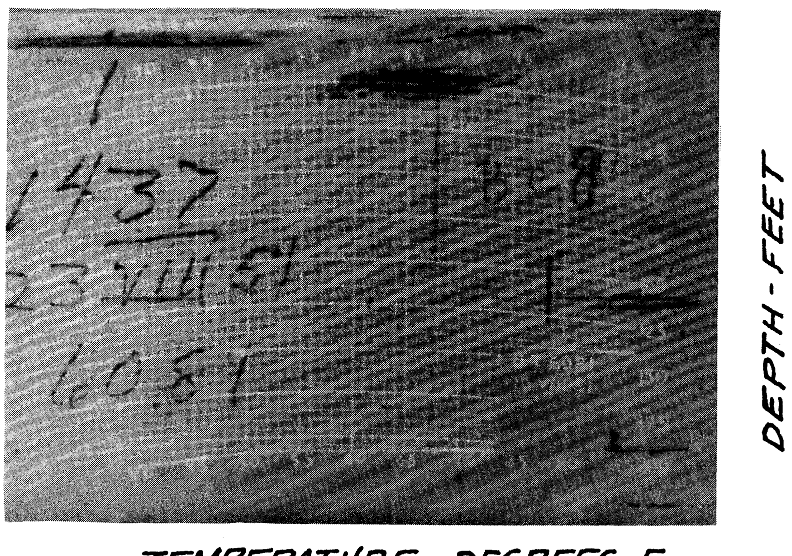
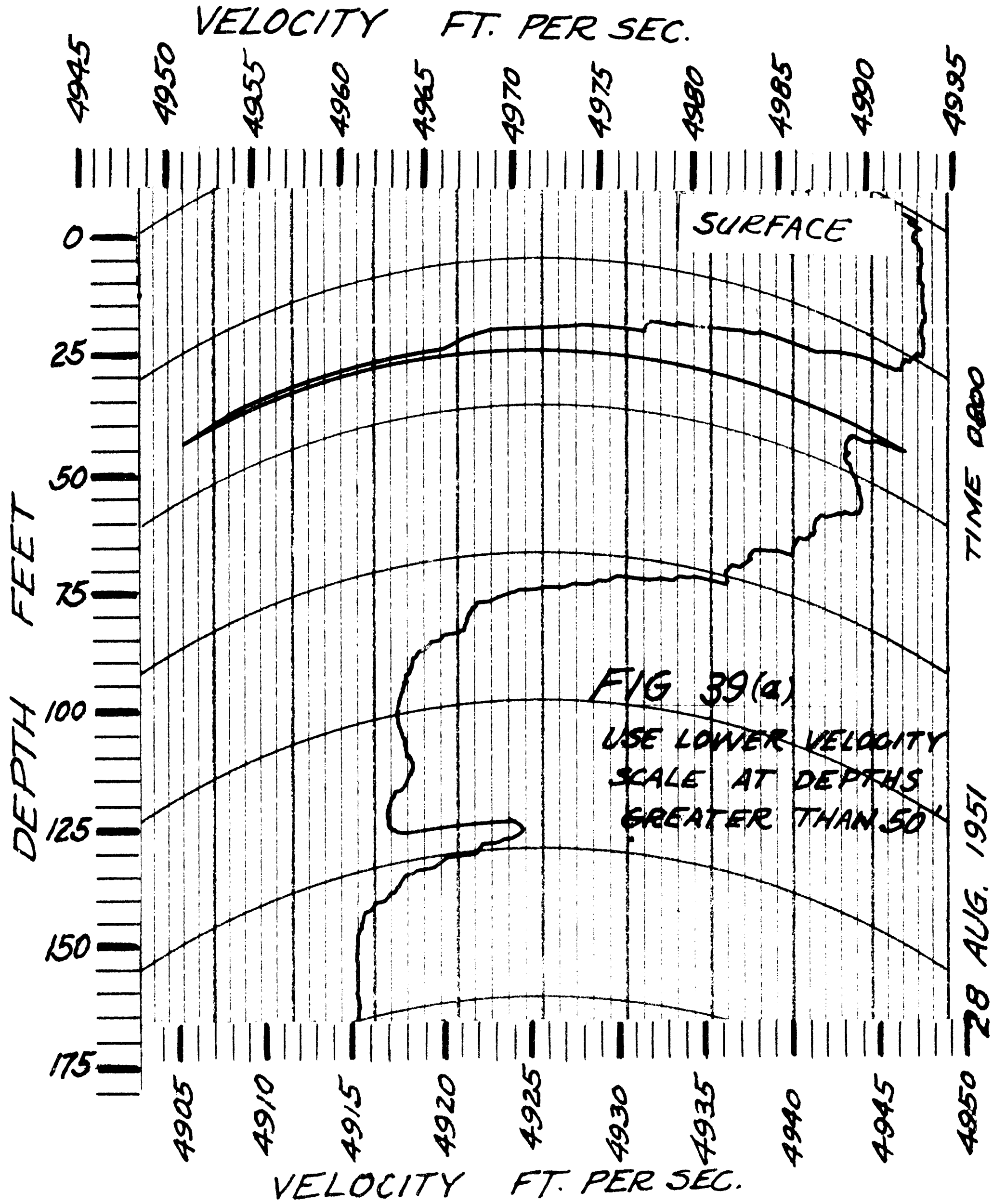



FIG. 33(b) BATHYTHERMOGRAM

FIRST OCEAN TEST IN VINYARD SOUND WHERE THE WATER IS NEARLY ISOTHERMAL AND OF UNIFORM SALINITY series of sharp spikes, having a peak value of roughly 1 foot per second. These spikes appear to be directly associated with the roll of the ship or with jerks on the cable used to lower the velocity meter. The meter itself was lowered in a more or less vertical position in this test, so that sound transmission is along the direction of movement of the meter in the water. Later drops of the velocity meter were made with the axis of the meter horizontal, and these later records do not show the spikes described above. Tests made in the model towing tank at the University of Michigan to determine the effect upon the velocity record of towing the meter have been described earlier in this report.

The first ocean test where prominent thermal gradients were present was made near the southern entrance to Vinyard Sound, 1-1/2 miles west of the sea buoy (lat. 41° 17.3', long. 71° 02.0'). Fig. 34 compares the velocity-meter records with the bathythermogram for this test site. Two velocity-meter records are shown, one for the lowering of the meter and the other for the raising. The chart was displaced several inches by hand from one record to the other so that the two velocity traces would not overlap and obscure each other. The salinity near the ocean surface was 31.89 0/00 and at the bottom 32.58 0/00.

The thermal structure of the ocean at this location, determined from the bathythermogram, is presented on page 104. In general, the velocity meter record follows a variation which is in agreement with the temperature structure. Increasing current from about 4 to 5 ma represents the decreasing velocity in the layer of water from 5 to 35 feet, where there is a very slight decrease in temperature with depth. When the



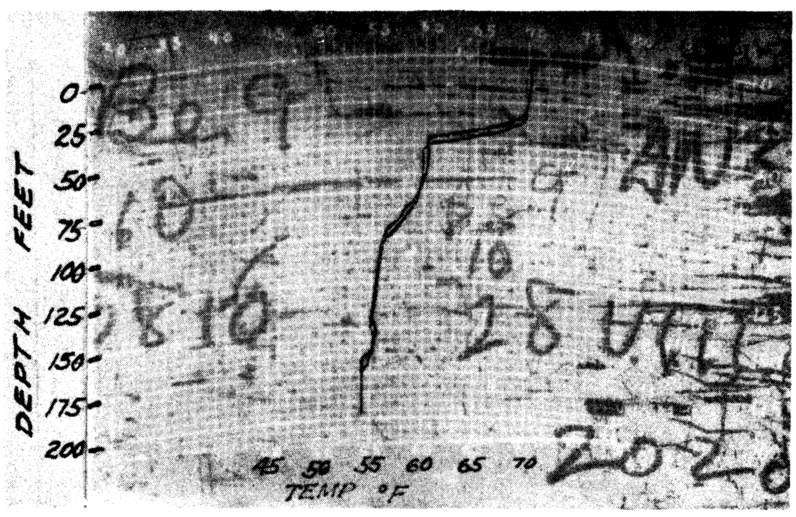




FIG 39(b) BATHYTHERMOGRAM

| Depth | Temperature |
|---|---|
| Surface | 67.9°F |
| 0-5 ft | Surface warming; drop of about 1°F |
| 5-35 ft | Slight drop to 67.0-66.9°F |
| 35-75 ft | Abrupt drop to 59°F |
| 75-120 ft | Gradual decrease to 58°F |
| 120-135 ft | Bottom layer at 57.5°F |

temperature begins to fall abruptly at 35 feet, the velocity-meter current jumps suddenly, and the meter passes through one of its changes in scale which correspond to changes in phase through a multiple of 360 degrees. The recorder, because of the inertia of the pen and moving coil, does not swing quite from full-scale to zero, but rather from a value of current slightly less than full scale to a value somewhat greater than zero. If the temperature gradient had been less steep, the recorder pen would have swung closer to the extreme limits of the velocity-meter scale. Changes of scale of this sort on the records are simple to identify and follow, since the arc drawn in the process of changing scale is exactly parallel to the printed arc on the chart paper. From 2 to 3.5 ma the record indicates a more gradual decrease of velocity with depth, and finally it shows a layer near the bottom where the velocity becomes abruptly less than it is in the region directly above. The smallest division on the velocity scale, a difference of 0.2 milliampere in current or 7.2 degrees in phase, corresponds closely to a velocity difference of 0.93 feet per second. The apparent surface velocity of 4976 feet per second indicated by the velocity meter compares with the table velocity (corrected by 18 ft per second as explained on page 83) of 4986 feet per second.

A comparison can be made of the difference in velocity between the surface and the bottom layer. When the velocity-meter readings for these two regions are referred to the calibration curve, the indicated velocity difference is 47 feet per second. The velocity difference predicted from tables for these two regions, taking into account differences in pressure and salinity, is 50.7 feet per second. The velocity meter was lowered in a vertical position during this test, so that when the top of the meter was at the surface of the ocean the transducers and the sample of water being measured were actually about three feet below the surface and at a slightly lower temperature. For this reason, there is very little indication of surface warming on the velocity-meter trace, although surface warming is indicated on the bathythermogram. This could account for the disagreement in the two velocity differences mentioned above, a disagreement which is equivalent to an error in temperature of about 0.4°F.

The velocity records demonstrate the presence of some backlash in the rack-and-pinion used to drive the pressure-operated potentiometer. For example, the velocity-meter downtrace has a good record of the bottom layer, while the uptrace does not appear to move in depth until this layer, about ten feet thick, has been almost completely passed. The use of anti-backlash gears might have corrected this situation.

Figs. 35(a) and 35(b) show the velocity-meter record and bathythermogram for a slightly different location from the one just described.

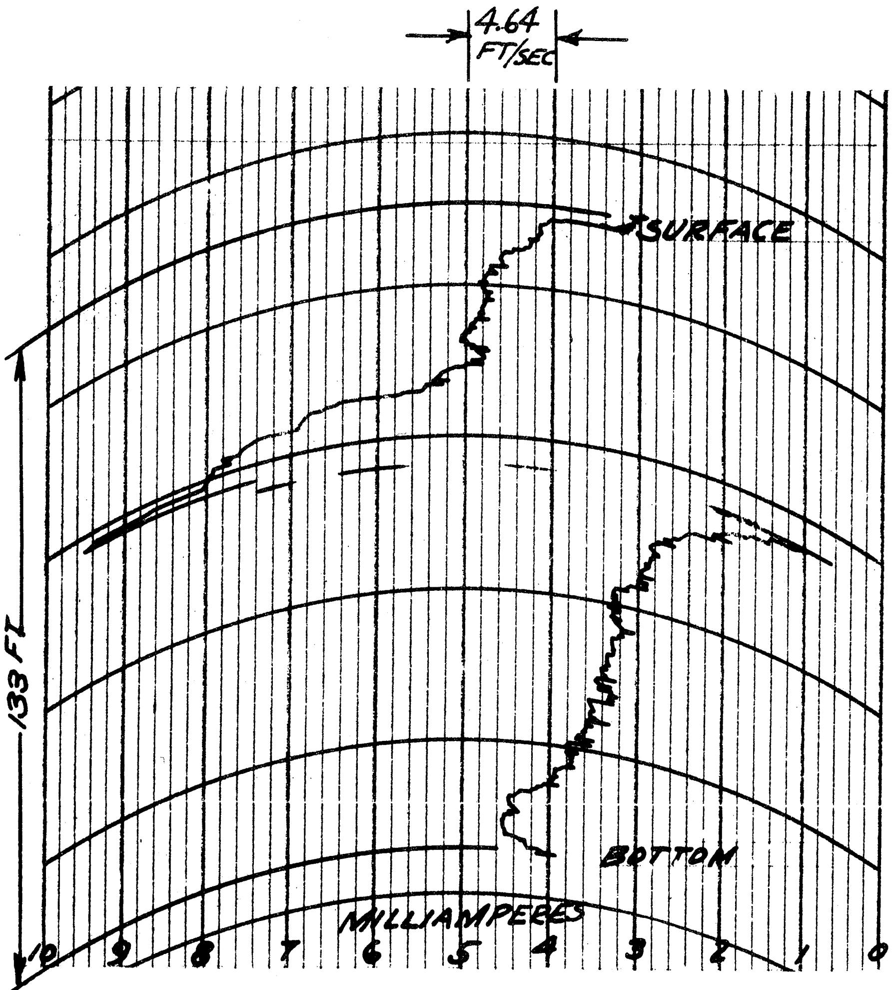



FIG 35(a) VELOCITY METER DOWNTRACE

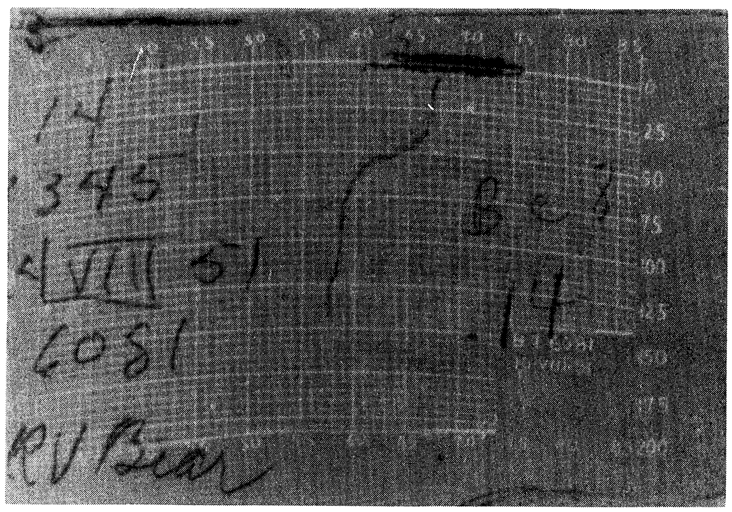


FIG 35(b) BATHYTHERMOGRAM

A rather pronounced surface-warming condition is shown, although otherwise the records are almost the same as before.

A velocity-meter record and a corresponding bathythermogram are shown in Figs. 36(a) and 36(b) for a test site where the thermal gradients are not as steep as those of the previous two records. This location (lat. 41° 19.2', long. 70° 55.4') is about three miles in the direction of Vinyard Sound from the test location of Fig. 35. Due to a different pulley and winch arrangement, the records of the velocity meter were made with a smoother descent and ascent than the previous drops, and they do not show the sharp velocity spikes characteristic of earlier records. Again definite temperature layers may be identified on both the bathythermogram and the velocity-meter traces. The analysis of water samples at this location indicates a change in salinity from 31.91 0/00 at the surface to 32.41 0/00 at the bottom.

At the top of the velocity-meter records a scale has been prepared in terms of both ma and feet per second. The measured velocity difference between the surface and the bottom is 37 feet per second, which compares favorably with the value of 38 feet per second determined from sound-velocity tables. This latter figure includes corrections for both pressure difference and the difference in salinity.

Both the velocity-meter traces and the bathythermograph traces appear to have some hysteresis between ascent and descent. The reason for this behavior is not known, but it is likely that the two instruments show hysteresis for different reasons; the bathythermogram is shifted slightly in depth, while the velocity record is shifted slightly in velocity.

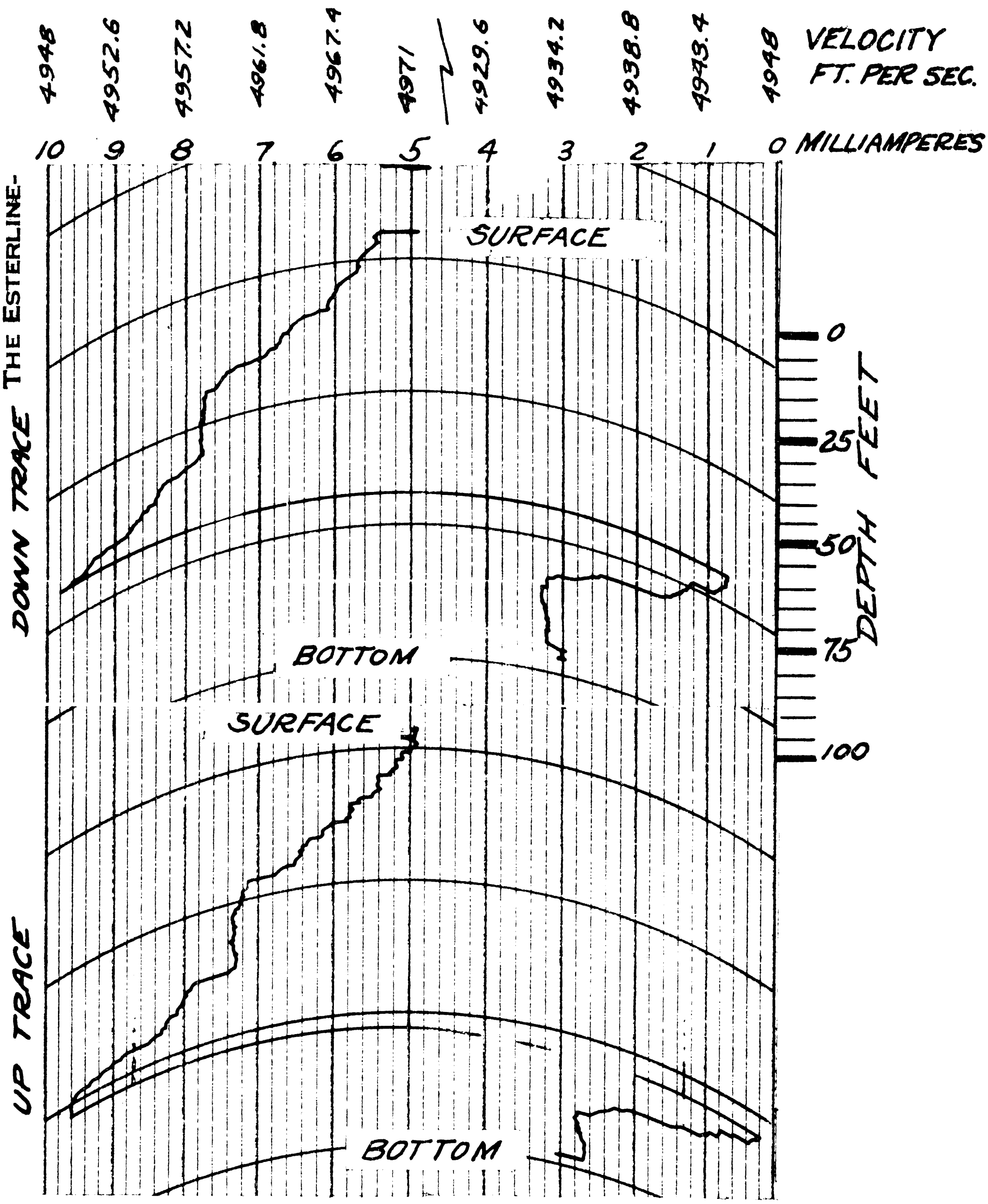



FIG 36(a) VELOCITY METER TRACES

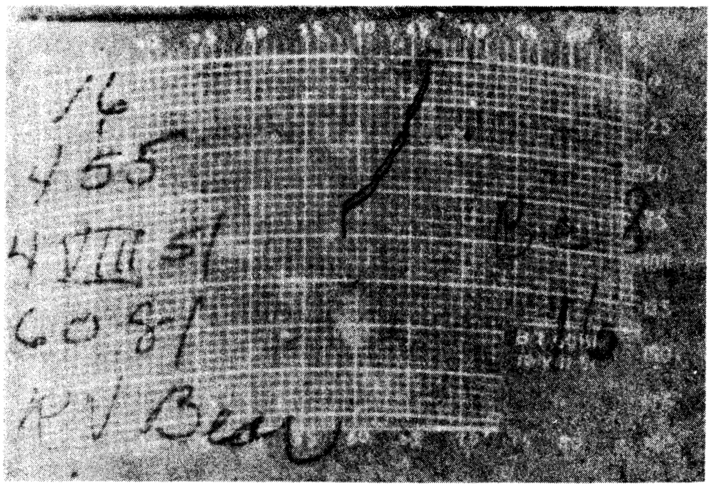


FIG 36(b) BATHYTHERMOGRAM

The records of Fig. 37 were made about 65 miles from land, but still on the continental shelf (lat. 40° 20', long. 71° 09'). Salinity measurements were not taken at this location; however, a bathythermogram was made of the temperature structure. Two distinct, nearly isothermal layers are evident, one extending from 10 feet to 70 feet of depth and the other from 145 feet to the ocean bottom. Between these two layers the temperature decreases rather sharply from approximately 74 to 54°F. The recorded sound velocities of the surface and the bottom layer are 4971 and 4824 feet per second, respectively. The lower layer is especially interesting in that the average velocity is very nearly constant for 100 feet of depth. This must mean that the pressure effect is balanced by either a slight change in the temperature or a gradual change in salinity, neither of which appears on the bathythermogram.

The records were made late in the afternoon of a warm August day. There is a measured increase of velocity of 3 feet per second near the ocean surface, which is probably due to surface warming.

Fig. 38 is a record of sound velocity made after the ship had drifted about three miles from the location for Fig. 37. The velocity structure of the ocean is similar to that of Fig. 37; however, the position of the layers in depth is quite different. For example, the bottom layer starts at 115 feet rather than 145 feet, and the lower edge of the surface layer is at 55 feet rather than 70. Again surface warming is prominent in the velocity record.

The records of Fig. 39 were made at a test site some 50 miles south of Vinyard Sound (lat. 40° 44', long. 71° 12'). Surface temperature

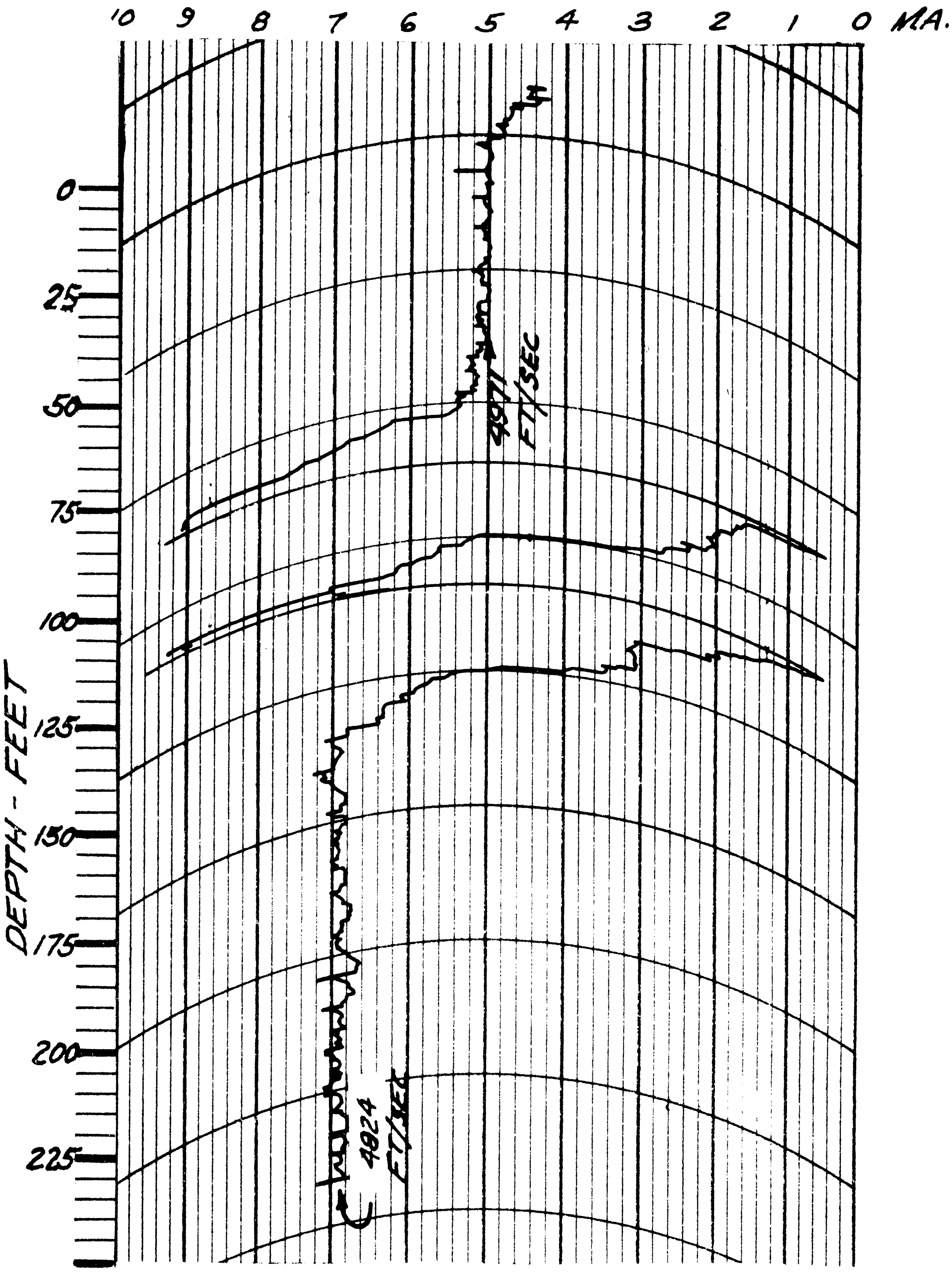



FIG. 37(a) VELOCITY RECORD

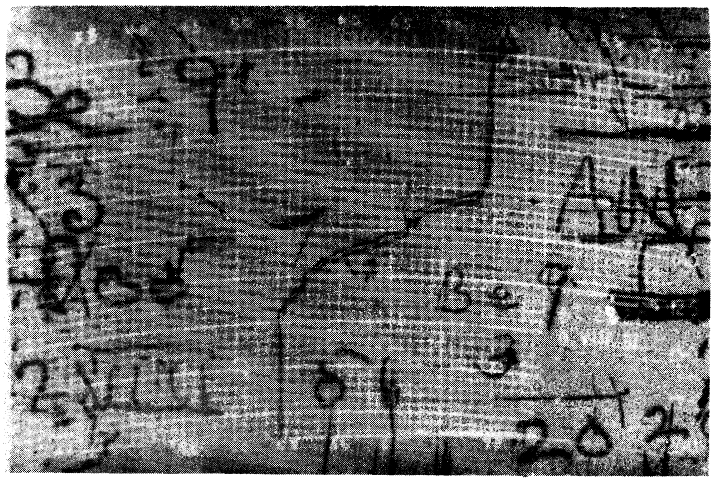


FIG. 37(b) BATHYTHERMOGRAM

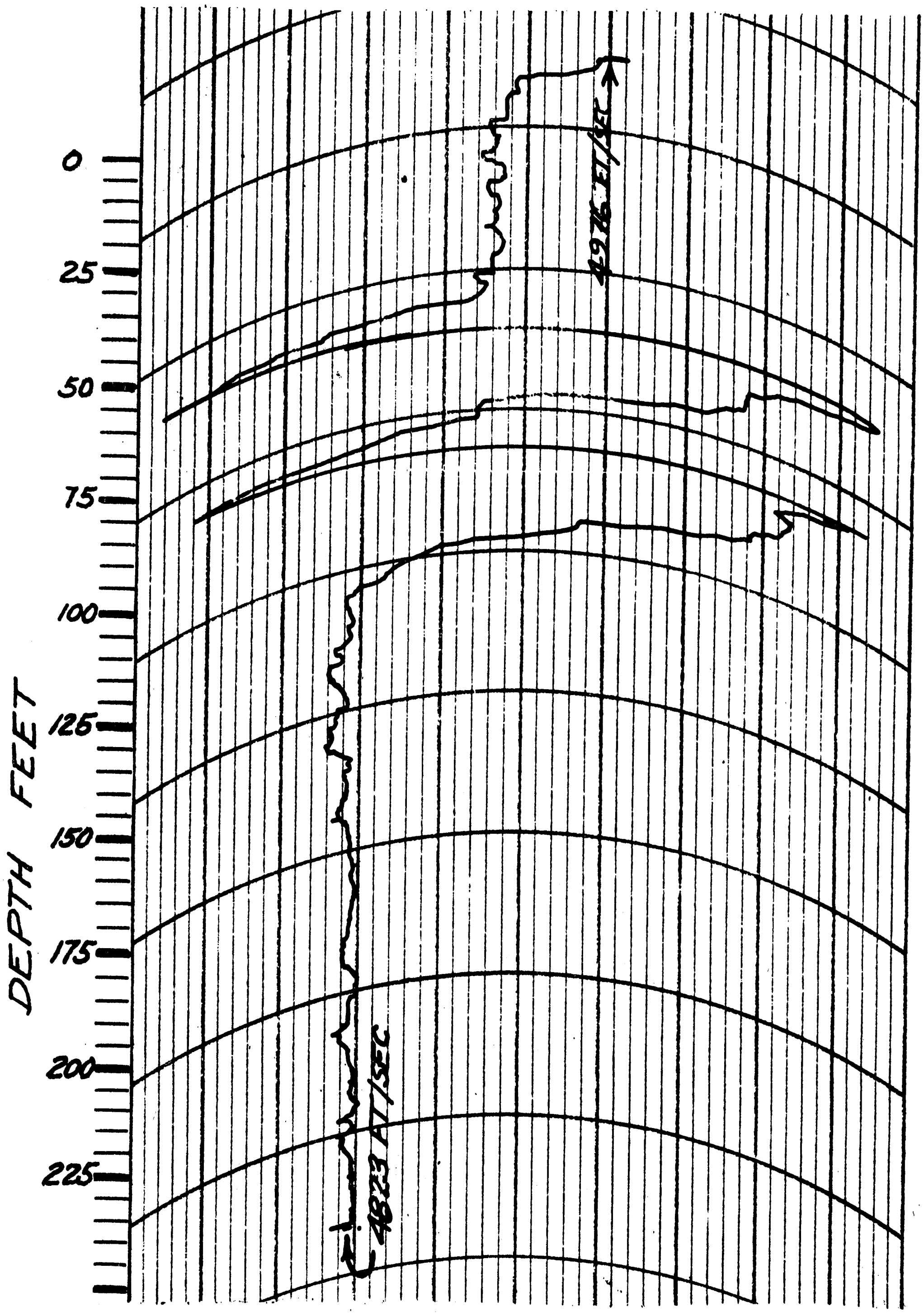
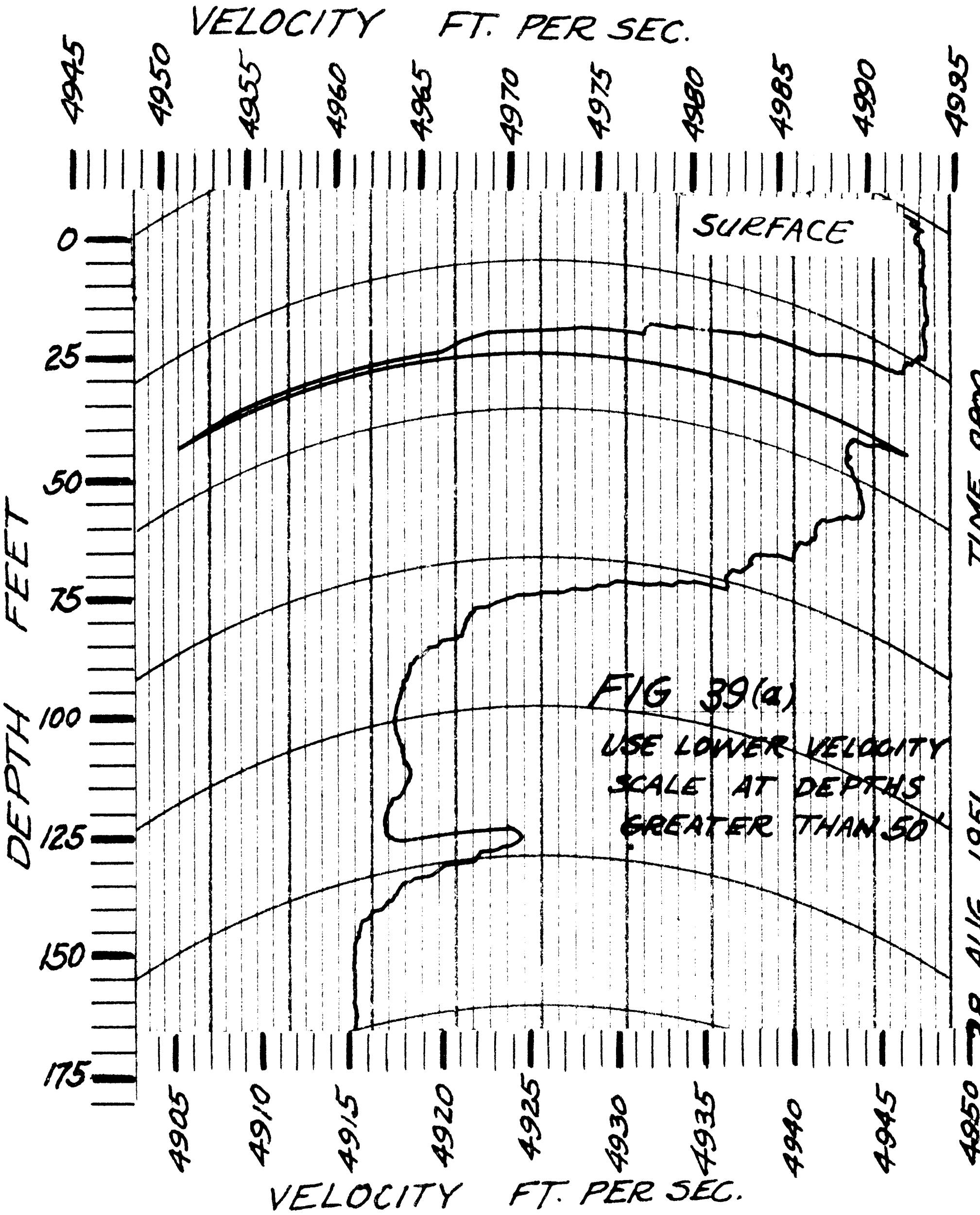



FIG 38 VELOCITY RECORD

TAKEN ABOUT THREE MILES FROM THE LOCATION OF FIG. 37



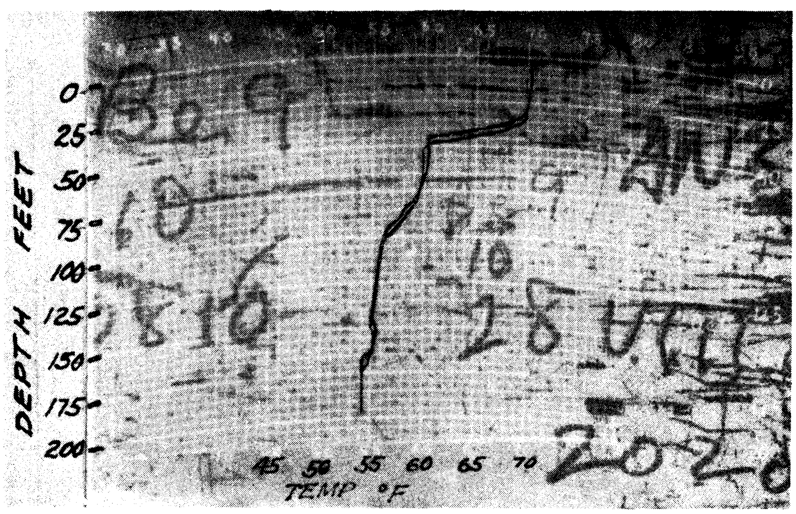




FIG 39(b) BATHYTHERMOGRAM

was 70.0°F, and the bathythermogram shows the general thermal structure of the ocean at this location. Salinity measurements were not made.

Again the velocity record follows the thermal structure closely. A nearly uniform velocity region exists down to a depth of 30 feet, although there is a slight decrease in the velocity at the ocean surface which is probably due to surface cooling at this early hour in the morning. The record was made at 8:00 a.m. At 30 feet the velocity decreases rapidly, falling 38 feet per second in the next 15 feet of depth. At 160 feet the velocity record indicates the apparent beginning of a uniform velocity region near the ocean bottom. It is especially interesting to notice the abrupt increase in velocity of about 10 feet per second occurring at a depth of 150 feet. The steepness of this jump is not well indicated on the bathythermogram, and the velocity change appears to be somewhat greater than would be expected from the increase in temperature alone.

The change in velocity between the surface and the bottom layer as measured by the velocity meter is 77 feet per second. On the basis of uniform salinity the sound-velocity tables record a velocity change of 85.8 feet per second. The difference, 8.8 feet per second, may possibly be due to a change in salinity of 2 0/00 between these two regions.

In this test the velocity meter was lowered in a horizontal position, with the axis of sound transmission at right angles to the motion of the meter in the water. The record appears much smoother than most of the earlier records. The sharp spikes, which are indicated in most of the early records, are probably connected with the motion of the velocity meter in the water.

## IMPROVEMENTS IN THE VELOCITY METER

As a result of the field tests of the velocity meter, certain suggestions for the improvement of the meter were made. All of these are related to its accuracy or to its use under field conditions. The suggestions are as follows:

(1) Improve the velocity-meter frequency stability, especially that of the heterodyne oscillator.

(2) Rearrange the electronic circuits to reduce the heat-dissipation problem and the size of the submerged meter.

(3) Develop transducers to eliminate standing waves completely.

(4) Redesign the amplifier for greater stability of phase response.

(5) Improve the voltage regulation of the power supply.

(6) Mount the transducers so that the direction of water flow is mostly at right angles to the direction of sound transmission.

(7) Redesign the servo-system and the recorder to increase their accuracy and reduce the time of response.

With the above suggestions built into an improved velocity meter, it is believed that a large amount of experimental information could be gathered on the velocity structure of the ocean.

## CONCLUSIONS

During the course of the research described in this report an entirely new method has been developed to measure sound velocities in the ocean. An instrument using this new principle has been constructed and thoroughly tested, and for the first time accurate velocity measurements have been made as a continuous function of depth in the ocean. Certain general conclusions have been reached. These are summarized below:

(a) Measurements made in the laboratory on fresh water and salt water tend to support the belief that the sound-velocity tables in current use give velocities which are too low.

(b) Velocity records made with the underwater sound velocity meter confirm the agreement between the measured difference in the velocity of sound between various ocean layers and the predicted velocity difference as determined from sound-velocity tables.

(c) It has been demonstrated that it is possible to measure, under actual field conditions, small changes in the velocity of sound in the ocean caused by such factors as the difference in pressure or salinity as well as velocity changes associated only with the difference in temperature. In effect, this means that an improvement in accuracy has been made over present velocity-measuring techniques.

Conclusion (a) is certainly important because of the fact that almost all the present knowledge of the velocity of sound in the ocean is based on velocity tables. It is fortunate that the refraction of sound is related to the relative change in the velocity rather than to the absolute value of the velocity. Changes in velocity, obtained by subtracting two table values of velocity for different temperature, pressure, or salinity conditions, appear to be reliable. However, it would seem that a carefully controlled series of experiments should be made to check the tables. Actual measurements of the effect of pressure and salinity upon sound velocity in water should be made with high precision, preferably by several methods and at various frequencies. There appears to be a shortage of experimental verification in this important field.

The chief contribution of the velocity meter to the field has been in its ability to supply second-order corrections which have not heretofore been measured. A temperature-recording instrument, such as the bathythermograph, is both convenient and rugged. It will supply velocity information by way of sound-velocity tables to a good approximation, and it certainly has the great advantage of simplicity. However, the velocity meter has greatly increased the accuracy of the measurements by it ability to respond directly to sound velocity rather than to some other physical property of the water which merely exerts an influence upon the velocity. The velocity meter described in this report, with the improvements suggested on page 114, should therefore be a highly useful research tool.

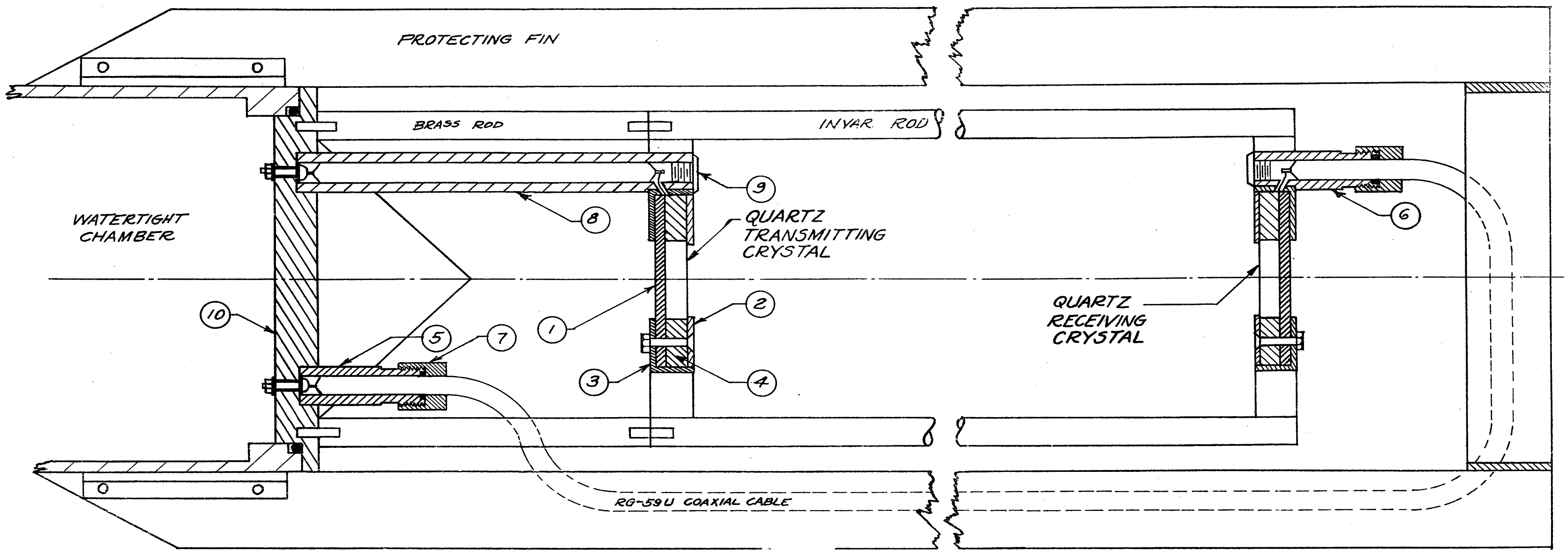
# APPENDIX

CALCULATION OF THE EFFECT OF STANDING WAVES UPON PHASE ANGLE

**Fig. 7, page 22, is based upon the following numerical calculations:**

$$\tan \theta = 1/3 \tan \omega d/c_1$$

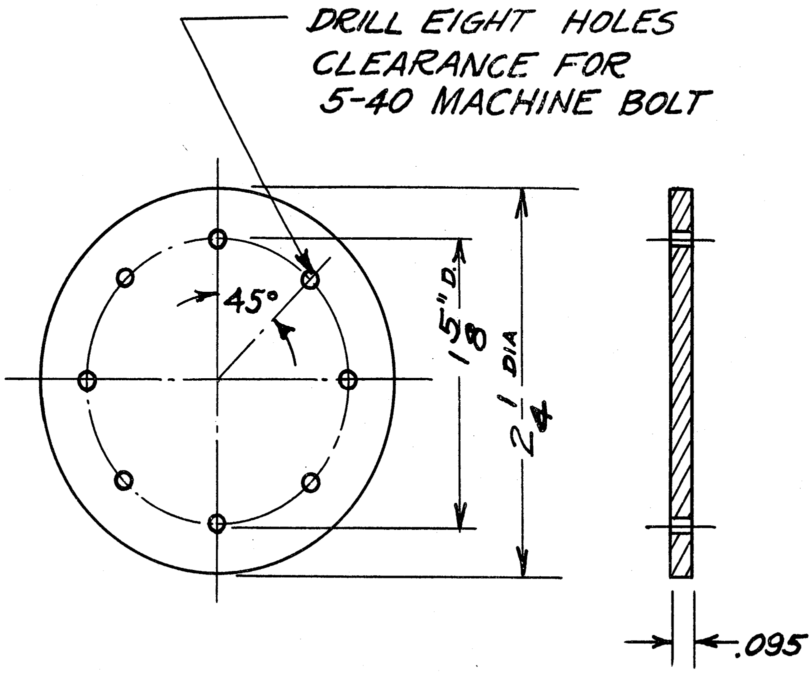
| $c_1$ ft/sec | $\omega d/c_1$ degrees | tan $\omega d/c_1$ | tan θ | θ Degrees |
|---|---|---|---|---|
| 5000 | 100x360+0 | 0 | 0 | 100x360+0 |
| 4998 | 100x360+14.4 | .2568 | .085 | 100x360+4.87 |
| 4996 | 100x360+28.2 | .5498 | .1832 | 100x360+10.38 |
| 4994 | 100x360+43.2 | .9391 | .313 | 17.38 |
| 4992 | 57.6 | 1.5757 | .5252 | 27.71 |
| 4990 | 72.14 | 3.102 | 1.034 | 45.95 |
| 4988 | 86.6 | 16.83 | 5.61 | 79.9 |
| 4986 | 101.1 | - 5.097 | -1.699 | 120.6 |
| 4984 | 115.6 | - 2.087 | - .696 | 145.2 |
| 4982 | 130.1 | - 1.188 | - .363 | 160.05 |
| 4980 | 144.6 | - .7107 | - .2366 | 166.68 |
| 4978 | 159.1 | - .3819 | - .1273 | 172.75 |
| 4976 | 173.6 | - .1122 | - .0374 | 177.87 |
| 4974 | 188.2 | + .1441 | + .0480 | 182.75 |
| 4972 | 202.7 | + .4183 | + .139 | 187.92 |
| 4970 | 217.3 | + .7618 | .254 | 194.27 |
| 4968 | 231.9 | + 1.275 | .415 | 202.55 |
| 4966 | 246.5 | + 2.300 | .767 | 217.50 |
| 4964 | 261.1 | + 6.386 | 2.13 | 244.87 |
| 4962 | 275.7 | - 9.816 | 3.272 | 287.0 |
| 4960 | 290.3 | - 2.703 | - .901 | 297.63 |
| 4958 | 305.0 | - 1.428 | - .475 | 317.97 |
| 4956 | 319.6 | - 0.851 | - .284 | 344.15 |
| 4954 | 334.3 | - .4813 | - .1605 | 350.88 |
| 4952 | 348.9 | - .1962 | - .0653 | 356.27 |
| 4950 | 363.6 | + .0629 | - .021 | 361.2 |




TRANSDUCER ASSEMBLY
SOUND VELOCITY METER

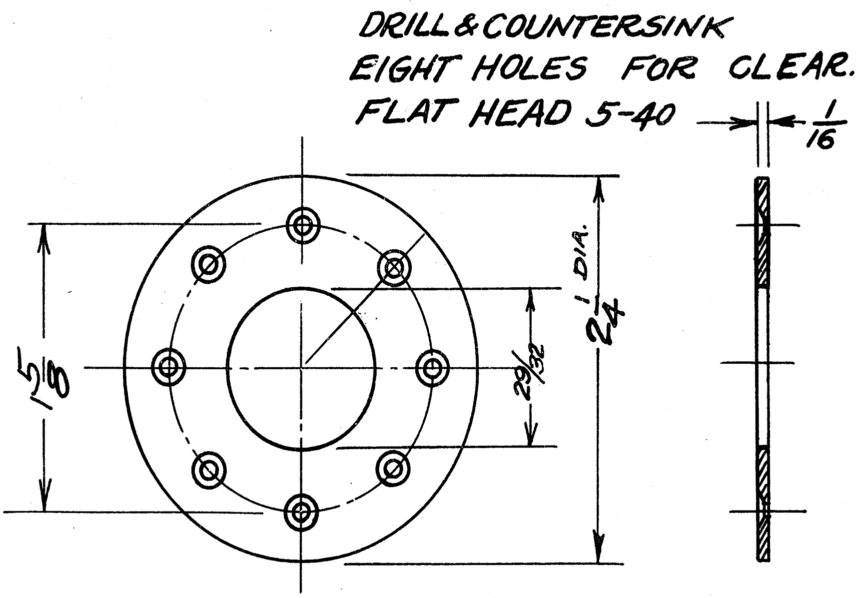
FULL SCALE
R.K.B. 29 MARCH 1951

1




MATERIAL BAKELITE
MAKE TWO

2

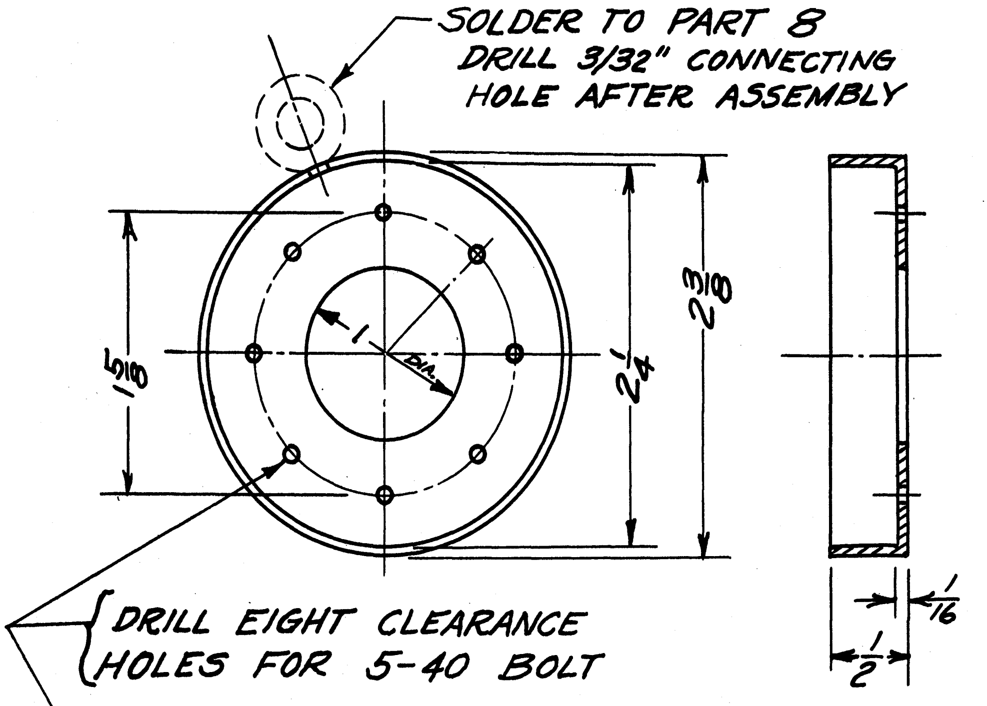



MATERIAL BRASS
MAKE TWO

SOUND VELOCTY METER

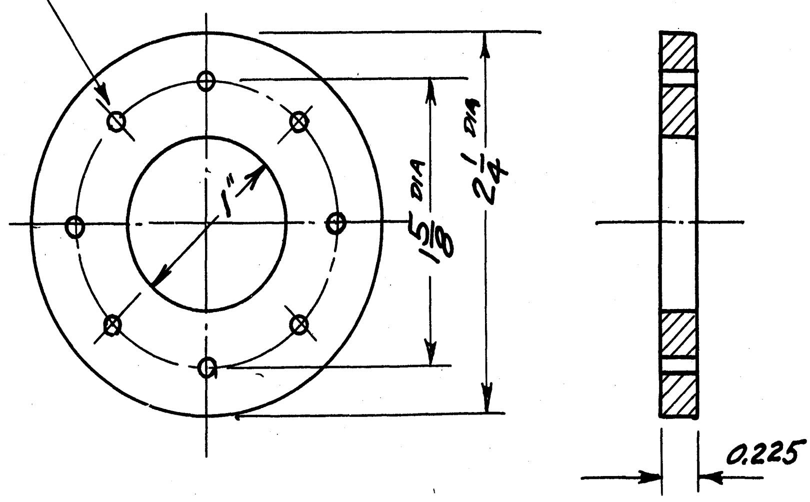
31 MARCH 51 R.K.B.

3




MATERIAL BRASS MAKE TWO

4

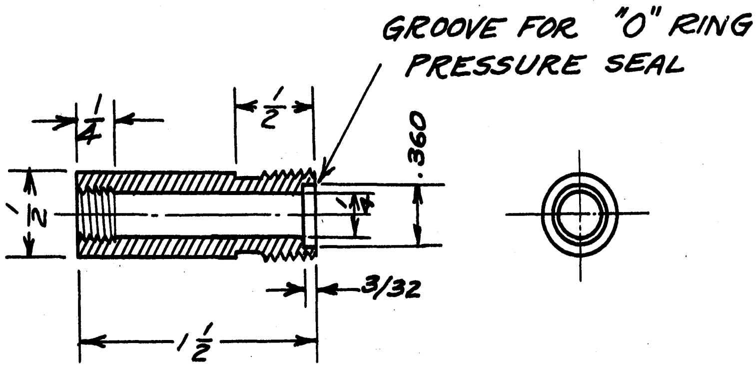



MATERIAL BAKELITE
MAKE TWO

SOUND VELOCITY METER

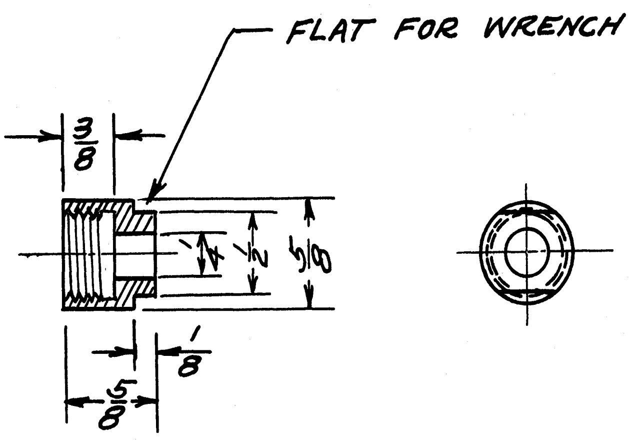
31 MARCH 51 R.K.B

5 & 6




MATERIAL BRASS
MAKE TWO (ONE WITHOUT INSIDE THREAD)

7



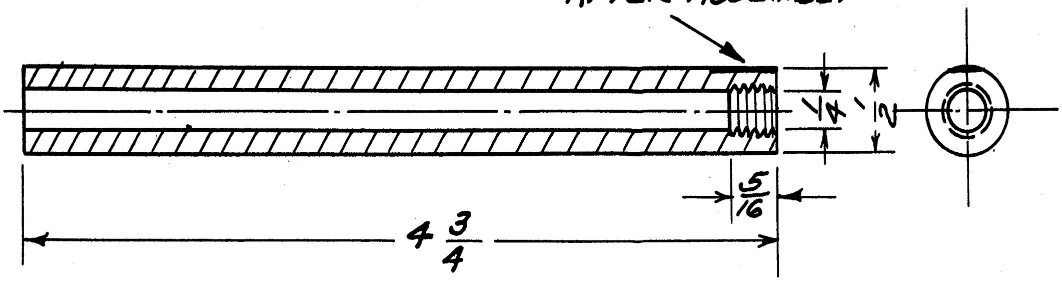

MATERIAL BRASS
MAKE TWO

SOUND VELOCITY METER

31 MARCH 51 R.K.B.

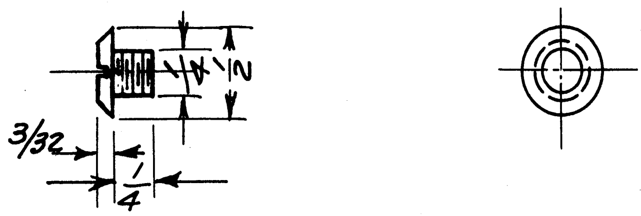
8

FILE FLAT & SOLDER TO PART 3
DRILL CONNECTING HOLE
AFTER ASSEMBLY




MATERIAL BRASS
MAKE ONE

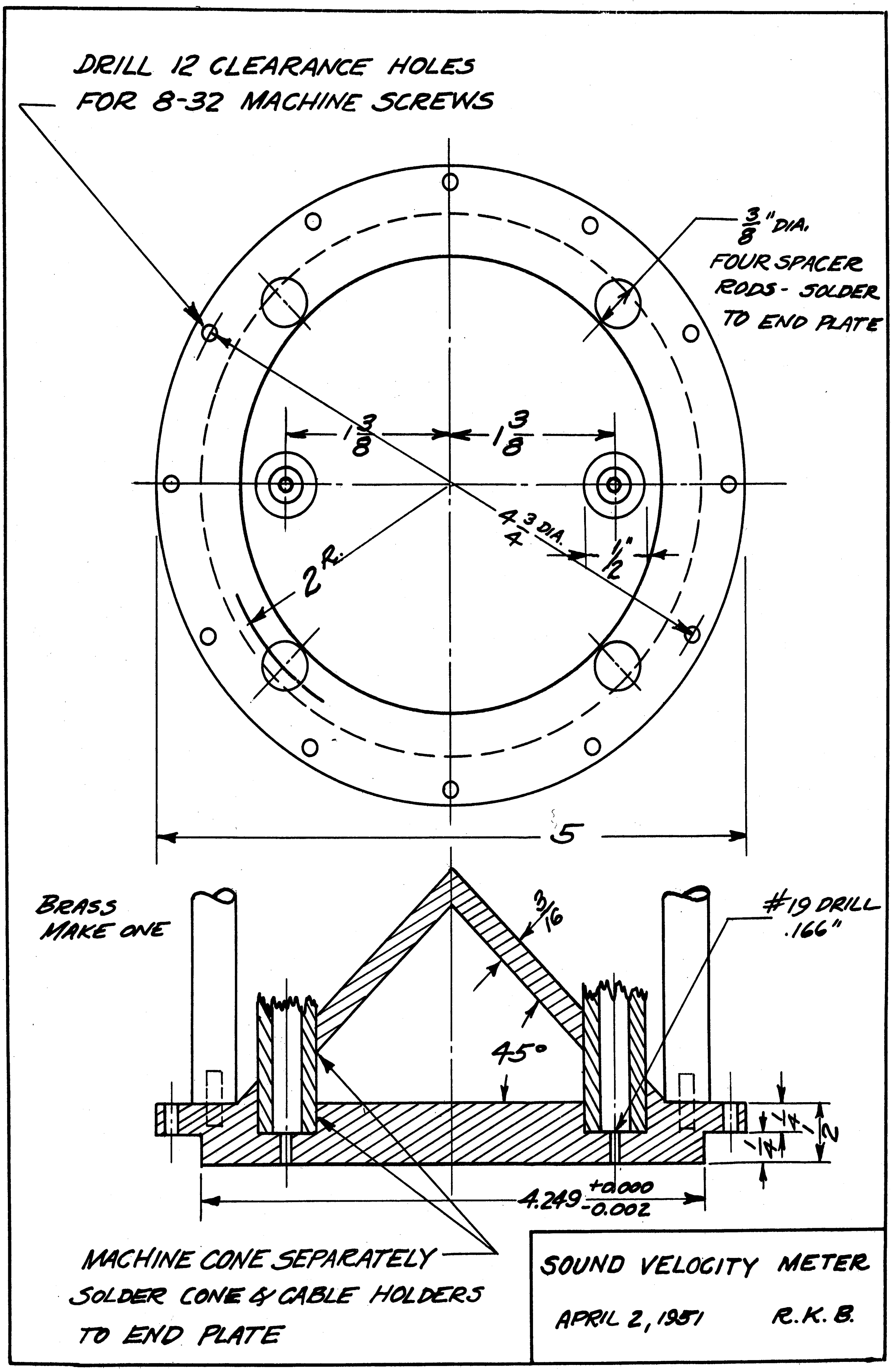
9




MATERIAL BRASS
MAKE TWO

SOUND VELOCITY METER

31 MARCH 51 R.K.B.


DRILL 12 CLEARANCE HOLES
FOR 8-32 MACHINE SCREWS
3/8" DIA.
FOUR SPACER
RODS - SOLDER
TO END PLATE
1 3/8
1 3/8
4 3/4 DIA.
2 R.
1/2"
5
BRASS
MAKE ONE
3/16
#19 DRILL
.166"
45°
1/4
1/4
1/2
4.249 +0.000 -0.002
MACHINE CONE SEPARATELY
SOLDER CONE & CABLE HOLDERS
TO END PLATE
SOUND VELOCITY METER
APRIL 2, 1951
R.K.B.

VALUES OF THE VELOCITY OF SOUND IN WATER FROM SOUND-VELOCITY TABLES

| Temperature °F | Sound Velocity, ft/sec (salinity, 35 0/00) | Sound Velocity, ft/sec (pure water) |
|---|---|---|
| 35 | 4768.2 | 4617.3 |
| 40 | 4806.2 | 4659.7 |
| 45 | 4843.4 | 4700.3 |
| 50 | 4877.9 | 4738.5 |
| 55 | 4910.1 | 4774.2 |
| 60 | 4939.5 | 4807.1 |
| 65 | 4966.2 | 4837.3 |
| 70 | 4992.5 | 4864.6 |
| 75 | 5016.3 | 4890.0 |
| 80 | 5038.1 | 4911.8 |
| 85 | 5058.8 | 4933.1 |

Salt-water velocities from Kuwahara's Tables and fresh-water velocities from the British Admiralty Tables.

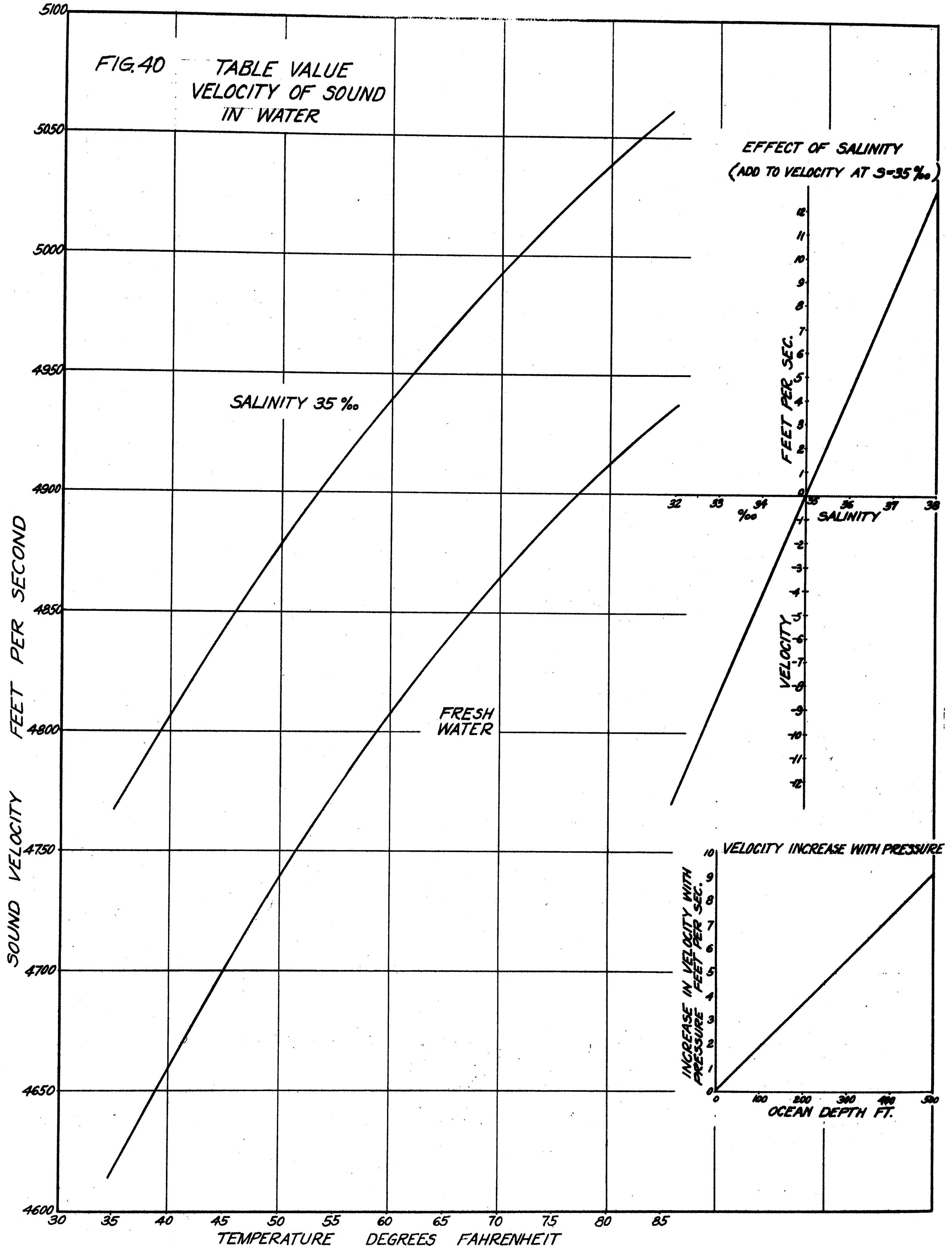


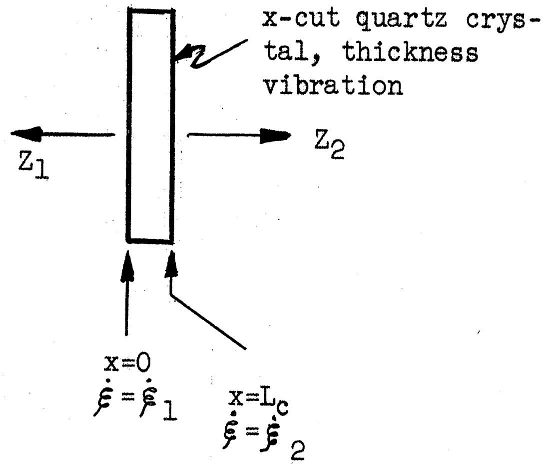
FIG. 40
TABLE VALUE
VELOCITY OF SOUND
IN WATER
SALINITY 35 ‰
FRESH WATER
SOUND VELOCITY FEET PER SECOND
TEMPERATURE DEGREES FAHRENHEIT
EFFECT OF SALINITY
(ADD TO VELOCITY AT S=35 ‰)
FEET PER SEC.
VELOCITY
SALINITY
‰
VELOCITY INCREASE WITH PRESSURE
INCREASE IN VELOCITY WITH PRESSURE FEET PER SEC.
OCEAN DEPTH FT.

RESULTS OF TEMPERATURE TEST IN LABORATORY TANK

| Water Temperature, °F | Table Velocity ft/sec | Corrected Velocity,* ft/sec | Velocity Meter Reading, ma |
|---|---|---|---|
| NaCl Added to 35-0/00 Salinity | | | |
| 46.8 | 4838 | 4856 | 3.8 |
| 48.4 | 4849 | 4867 | 1.5 |
| 50.0 | 4860 | 4878 | 8.6 |
| 52.0 | 4874 | 4892 | 6.0 |
| 54.0 | 4886 | 4904 | 3.2 |
| 55.7 | 4897 | 4915 | 0.8 |
| 58.0 | 4910 | 4928 | 8.2 |
| 60.1 | 4923 | 4941 | 5.4 |
| 63.2 | 4939 | 4957 | 2.0 |
| 65.9 | 4953 | 4971 | 9.0 |
| 67.4 | 4960 | 4978 | 7.0 |
| 69.7 | 4973 | 4991 | 4.6 |
| 72.1 | 4985 | 5003 | 2.0 |
| 74.3 | 4995 | 5013 | 9.8 |
| 76.0 | 5003 | 5021 | 8.0 |
| 78.5 | 5014 | 5032 | 5.8 |
| 81.6 | 5027 | 5045 | 3.0 |
| Fresh Water | | | |
| 59.3 | 4803 | 4825 | 8.8 |
| 59.8 | 4806 | 4828 | 8.0 |
| 61.5 | 4817 | 4839 | 5.95 |
| 62.8 | 4825 | 4847 | 4.0 |
| 67.1 | 4850 | 4872 | 8.2 |
| 71.0 | 4870 | 4892 | 3.4 |
| 74.2 | 4886 | 4908 | 9.8 |

*Correction to table values are +18 feet per second for salt water and + 22 feet per second for fresh water.

## CALCULATION OF THE PARTICLE VELOCITY AT THE FACE OF THE TRANSMITTING CRYSTAL




$Z_1$ = mechanical impedance of load
$= \rho c \, A_c$

$Z_2$ = mechanical impedance of crystal holder
$= \rho c \, A_c$

$A_c$ = area of the crystal face ($5.06 \times 10^{-4}$ square meters)

$L_c$ = thickness of crystal ($0.572 \times 10^{-2}$m)

d/s = piezoelectric interaction constant
= 0.176 coulombs/square meter for x-cut quartz

$$\phi = (d/s)\,(A_c/L_c) = 0.176\,\frac{5.06 \times 10^{-4}}{0.572 \times 10^{-2}}$$
= .0156 coulomb/meter

$$a_{11} = \cos\frac{\omega L_c}{V_c} + j\,\frac{Z_1}{Z_o}\sin\frac{\omega L_c}{V_c}$$

$Z_o = A_c \rho_c V_c$

$$a_{21} = Z_1 \cos\frac{\omega L_c}{V_c} + jZ_o \sin\frac{\omega L_c}{V_c}$$

$V_c$ = velocity of propagation in crystal

$\rho_c$ = density of crystal

$$a_{12} = -1$$

$\rho$ = density of sea water, $1.0234 \times 10^3$ kg/cu meter

$$a_{22} = Z_2$$

c = velocity of propagation in sea water, $1.5 \times 10^3$ meters/second

$$b_1 = \frac{j}{Z_o}\sin\frac{\omega L_c}{V_c}$$

$\dot{\xi}_2$ = particle velocity at the right face of the crystal

$$b_2 = \left(1 - \cos\frac{\omega L_c}{V_c}\right)$$

E = applied voltage

From <u>The Design of Crystal Vibrating Systems</u>:

$$\dot{\xi}_2 = E\,\phi\,\beta_2, \text{ where } \beta_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & -b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

For a half-wavelength crystal, $\frac{\omega L_c}{V_c} = \pi$ and,

$$\dot{\xi}_2 = E\,\phi\,\frac{-2}{Z_1 + Z_2}$$

$$= -\,E \times 1.56 \times 10^{-2} \times \frac{2}{2(1.5\times10^3\times1.0234\times10^3\times5.06\times10^{-4})}$$

$$= -\,.0201\,E \text{ meters/sec or } 2.01 \text{ cm/sec} \;.$$

The acoustic pressure in the water adjacent to the crystal face is

$$p = \rho c \times \text{particle velocity}$$

$$= 2.01 \times 1.535 \times 10^5 \times E \quad \text{or}$$

$$= 2.01 \times 1.535 \times 10^5 \times 50 = 1.54 \times 10^6 \text{ dynes/sq cm with 50 volts applied}$$

## VOLTAGE GENERATED BY THE RECEIVING CRYSTAL, $E_R$

$$E_R = -\,1/2\,\delta P_b\,\frac{Z_L}{Z_e + Z_L}$$

$$\delta = \frac{L_c}{d/s} = \frac{0.572 \times 10^{-2}}{0.176} = 3.25 \times 10^{-2}$$

$$Z_L = -\,\frac{j}{\omega(C_o + C_L)}\,,$$ where $C_o$ = clamped capacity of crystal

$C_L$ = capacity of electrical load across crystal

This assumes that the electrical load across the crystal is entirely capacitive. This is approximately true, although it actually consists of a capacitance in parallel with a 1-megohm resistance. The sum $C_o + C_L$ is approximately $150 \times 10^{-12}$ farads.

$$Z_e = \frac{Z_o}{\phi^2} \cdot \frac{B}{4} \text{ ohms}, \quad \text{where } B = \frac{\rho c}{\rho c V_c} = \frac{1.535 \times 10^5}{2.65 \times 5.44 \times 10^5} = 0.1064$$

$$Z_o = A_c V_c = 5.06 \times 10^{-4} \times 2.65 \times 10^3 \times 5.44 \times 10^3$$

$$= 7.3 \times 10^3 \text{ kg/sec}$$

$$Z_e = \frac{7.3 \times 10^3 \times 0.1064}{(0.156 \times 10^{-2})^2 \times 4} = 7.98 \times 10^6 \text{ megohms} \quad .$$

The other terms are defined in the appendix section dealing with the transmitting crystal.

$$E_R = -1/2 \times 3.25 \times 10^{-2} \times P_b \times \frac{-j2080}{7.98 \times 10^6 - j\,2080}$$

$$E_R \doteq -1.63 \times 10^{-2}\, P_b \quad ,$$

where $P_b$ is the pressure on the front face of the receiving crystal.

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