DU PONT BALLISTIC GRANT STUDIES

THE UNIVERSITY OF MICHIGAN, ANN ARBOR, MICHIGAN

REPORT # 2

DEVELOPMENT OF BULLET JACKET FACTOR AND RIFLE BARREL FACTORS

by

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REPORT # 2 - THE DEVELOPMENT OF BULLET JACKET AND RIFLE BARREL FACTORS

I. RESUME'

For three years, studies in internal ballistics at The University of Michigan have investigated new ballistic methods. The first publication in Ordnance[1] described a new method for measuring simultaneously both the bullet travel in the barrel and the chamber pressure. The new technique involved the use of collimated beams of gamma radiation from the radioisotope iridium-192. Since then, two alternate methods have been developed and put into use as described in this report.

Technical Report # 1 on studies[2] conducted with aid of the DuPont Ballistic Grant described a procedure for measuring the true or absolute chamber pressure without the use of a special test barrel or crusher gages. A calibration curve to convert crusher values to true ones was included. A summary of this study was published in the third edition of the Handloader's Digest.[3]

In this Report # 2, a new procedure for predicting the maximum chamber pressure is presented. The bullet jacket factor (J) is a new characterization number which, with the sectional density of the bullet and the specific loading density of the powder, permits calculation of the maximum chamber pressure for a given powder, cartridge, bullet, and gun. Thus far the correlation has been limited to IMR No. 3031 powder but the correlation is of a fundamental type, being based upon the principle of dimensional similitude. Based on data for other powders, the correlation will be extended to include powder characterization factors.

II. BULLET RESISTANCE, A PROBLEM

For many years handloaders have been aware of the influence of bullet type on maximum chamber pressure (p). In the past, however, attempts to correlate the different values of (p) have proved unsuccessful. Waite[4] investigated such variables as bullet length, bearing length, core hardness, pressure, velocity, and the velocity-to-pressure ratio for bullets of identical weight and caliber but supplied by ten different manufacturers. In these tests, he used the identical seating depths, the same powder load (51 grains of IMR No. 3031 lot No. 157), Western cases, and No. 210 primers by Federal. For all tests, he used a Winchester pressure barrel, 24 inches long with four grooves and a twist of one turn in 10 inches. Ten firings of each bullet were made but no correlation was found to exist between the different bullet factors measured and the magnitude of the pressures and velocities. Waite stated that his most important observation was the presence of a pressure spread of more than 7,000 psi resulting from the use of different bullets. Waite[5] reported another series of tests entitled "Loads for the .30-'06." Here, the bullet weight was varied from 110 to 220 grains using over 40 combinations of bullet and powder loads. Again, using identical cases, primers and powder loads but different bullet types, there was a wide difference in pressures obtained. While Waite and other handloaders agree that bullet type does influence the maximum pressure, no serious attempt has been made to explain this phenomenon.

This study, based upon research conducted at The University of Michigan, introduces a correlation which successfully predicts the influence of the bullet "jacket factor" on the maximum pressure. The jacket factor (J) is a dimensionless characteristic of the bullet. It is measured in relation to a reference bullet, and is the relative ratio of maximum chamber pressures. We assume a theoretical frictionless reference bullet to have a (J) of 1.0; the bullets to be tested may then have a (J) of 1.10 to 1.40. Since the (J) is proportional to the ratio of maximum pressures, a bullet with a (J) of 1.30 will produce 30 per cent more pressure than a bullet with a (J) of 1.0, other loading conditions being equal. While such a range in pressures may seem surprising, ranges of this magnitude have been measured in test firings at Michigan.

The study of the jacket factor (J) developed from the continuation of studies on the maximum chamber pressure in the 30-06 cartridge described in Technical Report # 1. In the earlier studies, the bullet weight and type had been limited in most firings to the 220-grain Remington SPCL bullet. The studies of (J), however, utilized special bullets made from the 220-grain Remington SPCL bullet (see Fig. 1) but having various weights (from 110 to 476 grains), and which were test-fired with varying IMR No. 3031 powder loads. The equipment used to obtain the pressure measurements in the (J) studies (see Fig. 2) was the same as that described in "Absolute Chamber Pressure," Technical Report No. 1.[2] Data were obtained in the form of oscilloscope displays with 2ms time sweep (see Fig. 3).

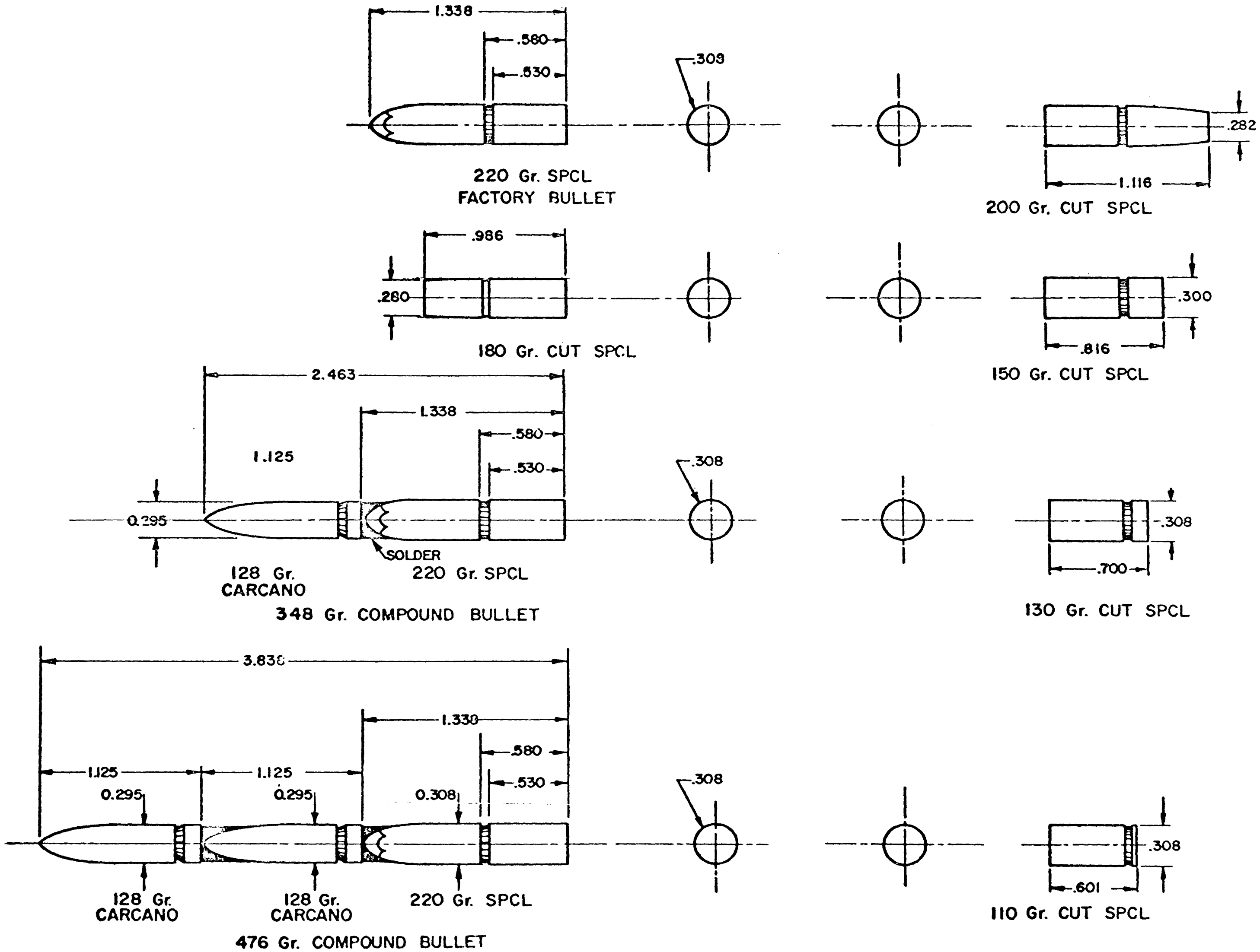


Fig. 1. Dimensions of Bullets of Various Mass's made from 220 gr. Rem. SPCL Bullet

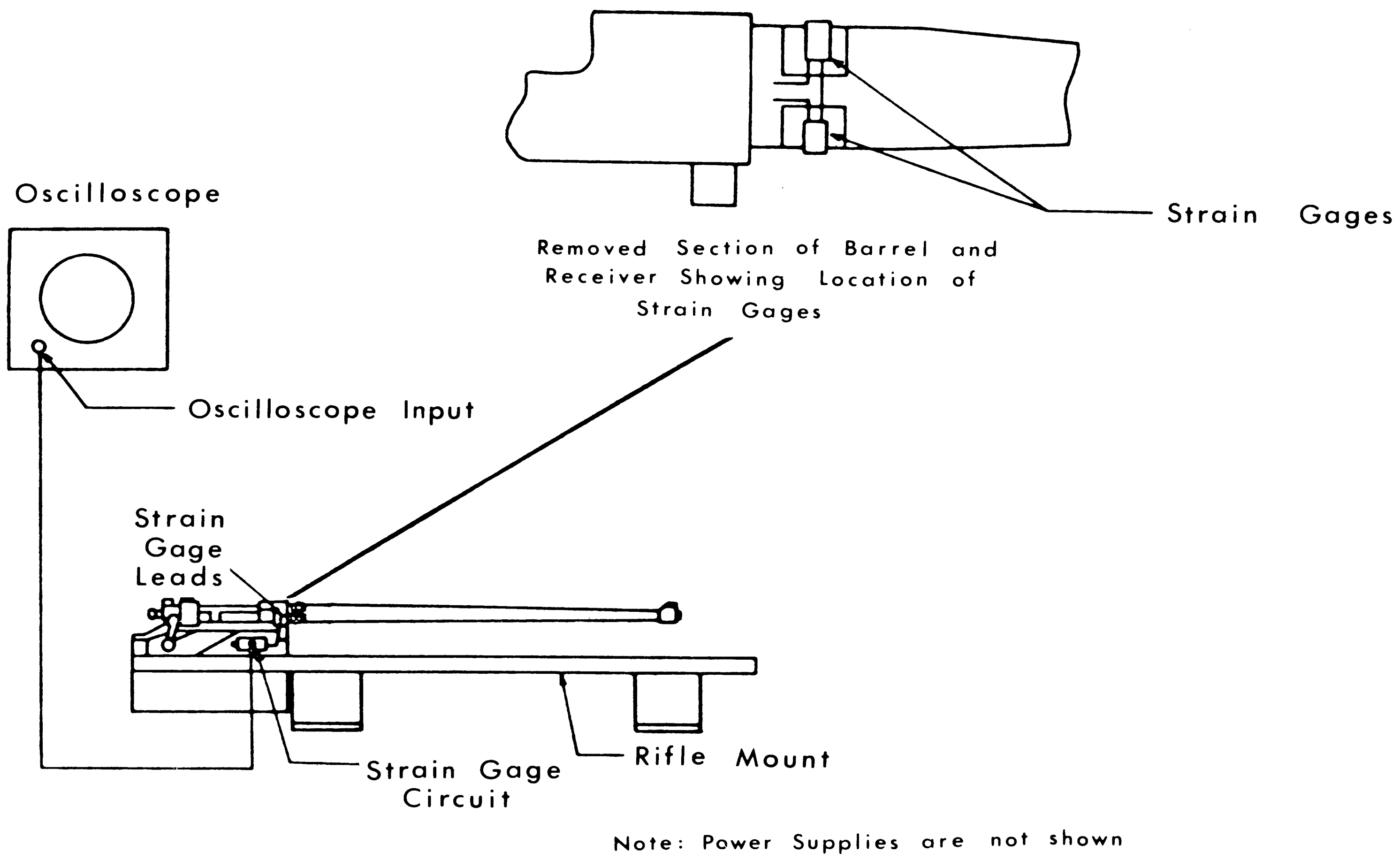
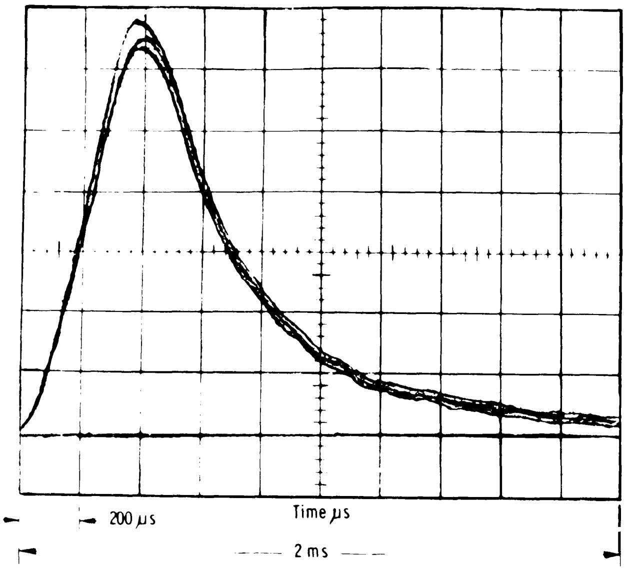
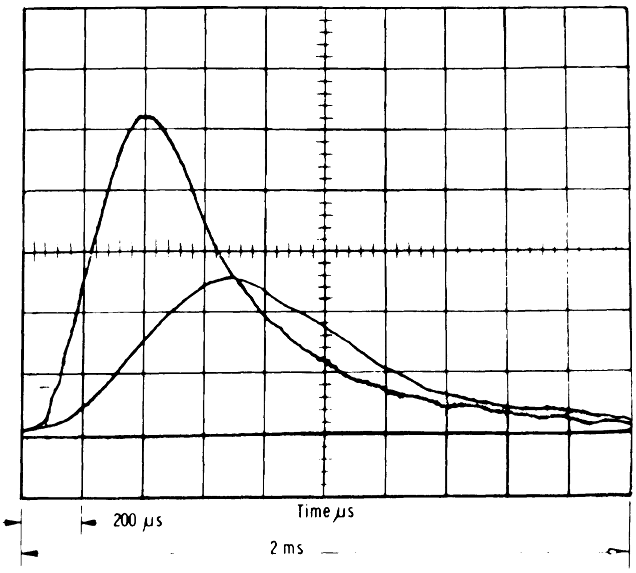


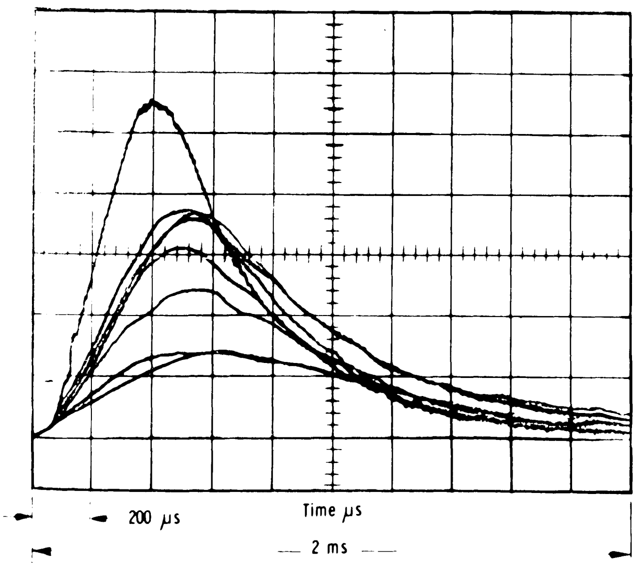
Fig. 2. Test Firing Facilities at The University of Michigan



a. Calibration Firing, 3 rds, 44 gr *3031, 220 gr SPCL
p=64,000 psi

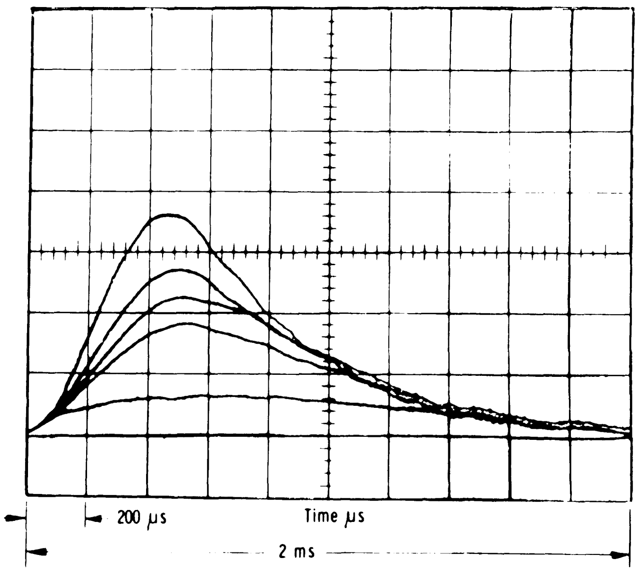


b. Herter 150 gr Banana Peel Bullet, 2 rds,
48 gr * 3031 - 51,200 psi
30 gr * 3031 - 25,600 psi



c. Remington 150 gr Bronze Pt, 8 rds,

48 gr	*3031,	54,200 psi
44 gr	"	36,000 "
42 "	"	34,500 "
38 "	"	30,500 "
36 "	"	23,600 "
30 "	"	13,800 "



d. Herters 150 gr Half Lead

44 gr	*3031,	35,500 psi
42 "	"	26,600 "
38 "	"	21,700 "
30 "	"	17,700 "
23 "	"	6,900 "

Fig. 3. Selected Oscilloscope Photo Negatives of 2 ms Display from Firings in 30-06 Test Rifle Using Barrel D and Lot 1229 IMR 3031 an 0.34" seat

III. METHODS OF CORRELATION

An understanding of the approach used to solve the problem of the effect of the jacket factor on the maximum pressure requires some discussion of the variables and theoretical concepts involved.

When considering possible solution procedures to an internal ballistics problem, one is faced with an extensive number of variables. Among these are: (1) maximum chamber pressure; (2) caliber of the gun; (3) weight of the bullet; (4) weight of the powder load; (5) case and barrel volume; (6) type of powder used; and (7) bore and bullet diameter. Still others are the powder position factor, powder temperature at ignition, case base diameter, primer hole diameter, primer ignition factor (manufacturer and type), bullet engraving factor (manufacturer and type), barrel rifling twist, the barrel-engraving factor, and the barrel-wear factor.

The "main" problem of internal ballistics is the solution of the relationships between chamber pressure (p), bullet velocity (v), bullet barrel-travel length (ℓ), and time (t). European ballisticians such as Charbonnier[7] of France have usually related these variables to "y," the fraction of powder burnt at any time, "t." American ballisticians for many years related "p," "v," and "t" to bullet travel length "ℓ" using the empirical methods developed by the French artillery officer Leduc.

Both of these procedures have certain disadvantages. The equations of Charbonnier are based on the energy balance. Various modifications of his equations have had worldwide use in solving problems in internal ballistics. A major difficulty in using Charbonnier's equations, however, is that the fraction "y" of powder burnt is not zero at bullet start where $v = 0$ but "p" is "p_0," a definite pressure required to "start" the bullet on its path. The methods of Leduc are not based on the equation of the conservation of energy (First Law of Thermodynamics) and so they lack generality. New equations and constants must be determined for each type of gun.

Most correlations in internal ballistics have been based on what is called energy balance. When the energy "stored" in the powder is known, it can be equated to the energy required to: (1) accelerate the bullet down the barrel; (2) accelerate a portion of the powder down the barrel; (3) work on atmosphere by muzzle blast; (4) work on shooter through recoil; (5) while at the same time accounting for heating the chamber, shell case and barrel; (6) frictional losses down the barrel; and (7) initial cold-working (grooving) the bullet into the rifling of the bore. But this method is also complex, and prediction of pressure, velocity or other variables difficult.

IV. DIMENSIONAL SIMILITUDE

Another approach which allows remarkably simple methods of solution for many problems in internal ballistics utilizes the principle of dimensional similitude. Dimensional similitude allows the correlation of the significant variables by means of compact equations without having to know the individual energy relationships involved.

Buckingham's Pi theorem of dimensional similitude[8] is a classic in its field. First presented in 1914, the theorem is based upon the Rayleigh[9] principle of dimensional similitude and logic. Stated in words the theorem might read as follows: Given any system, natural or artificial, and all of the variables affecting that system, one can, by appropriate grouping, explain the behavior of such a system no matter how complex. The phrase "by appropriate grouping" refers to the grouping of parameters in such a way that each group is dimensionless. These dimensionless forms raised to the appropriate power Buckingham called π groups.

Buckingham based his theorem on logic and made no attempt to prove it mathematically. It wasn't until 1951 that Langhaar[11] proved it in a rigorous manner. He defines the theorem as follows:

> If an equation is dimensionally homogenous, it can be reduced to a relationship among a complete set of dimensionless products. A set of dimensionless products is complete if each product in the set is independent of the others and every other product of the variables is a product of powers of dimensionless products in the set.

In a later paper,[12] Buckingham described a procedure for "dimensional reasoning," which is referred to today as "dimensional analysis.'' Buckingham gives as an example of dimensional reasoning for the case of the resistance (R) of still air to the motion of a smooth sphere and the influence on velocity, v. This relationship would apply to the external ballistics of spherical shot. Buckingham states that for this system some experience with fluid flow and the fundamentals of physics is necessary. One can proceed by listing the variables believed to influence the resistance (R). Obviously, the total resistance will increase with the diameter (D), one of the significant variables. The properties of the air affecting fluid flow are also important. These are: viscosity (μ), density (ρ), and compressability (C). Density and viscosity change with temperature. The energy of the sphere is partially dissipated by work done on the air and by frictional losses in the form of heat. The transfer of heat to the air will depend upon the specific heat (c_p), the thermal conductivity (λ), and the emmissivity (σ). Thus, a list of nine variables can be compiled:

$$R, V, D, \rho, \mu, C, c_p, \lambda, \sigma$$

By considering the order of magnitude of each of the nine variables, some may be eliminated. Since for practical problems, the temperature effect will be small, the variables c_p, λ, and σ will have an insignificant effect. If the velocity of the sphere is low, there will be virtually no compression of still air. Thus, compressibility (C) can also be eliminated. This leaves five significant variables which give the following equation:

$$F\,(R, V, D, \rho, \mu) = 0 \qquad (1)$$

Thus, by simple analysis and common-sense reasoning together with a general knowledge of the variables affecting the system, an equation including all the more important variables can be deduced. Whether the reasoning is sound and whether or not Eq. (1) is sufficiently complete can be determined by comparing experimental data with the theoretical equation.

Although Buckingham's publications are often referred to in discussions of dimensional similitude, the method did not originate with him. As early as 1892, Lord Rayleigh had used the principle in his own publications with great success.[9] However, he did not discuss the procedure until 1915,[10] nor did he consider it an original contribution on his part. In fact, he commented that Sir Isaac Newton, probably the greatest natural philosopher of all time, was well acquainted with this basic law of nature and used it extensively. In our review of Newton's Principia[13] we found ample evidence of its use. A very simple case is Newton's repeated use of the proportionality of similar sides of geometric figures of different size but similar shape. In a generalization, Newton states that "the ultimate principle is precisely expressible as one of similitude, exact or approximate, to be tested by the rule that mere changes in the magnitudes of the ordered scheme of units of measurement that is employed must not affect sensibly the forms of the equations that are the adequate expression of the underlying relations of the problems." Thus we can trace the origin of the name of similitude to Newton who refers to the principle as the "rule of similitude."

V. FOURIER'S EARLY WORK

An extensive search of the literature has shown that Jean Fourier, philosopher, mathematician, and friend of Napoleon, was the first to describe the procedure of dimensional analysis and the use of the principle of similitude as we know them today. He comments that the principle is "derived from basic concepts of 'quantities' used in geometry and mechanics and is the equivalent of fundamental lemmas which we inherited without proof from the Greeks." He makes this statement in his best known work, Theorie Analytique de la Chaleur, published in Paris in 1822.[14] In this remarkable treatise, he presents the development of the mathematical series and the theorem known by his name. However, his contribution to dimensional analysis in this publication has received little or no recognition, and authors of texts and articles on the subject of dimensional analysis omit him from their lists of references, probably because their literature surveys have been limited to more recent publications.

There is no question but that he was a master of the use of this versatile tool. In treating the "quantities" of thermal conductivity, heat capacity and the coefficient of heat transfer, he comments as follows:

> In order to measure these quantities and express them numerically, they must be compared using five different dimensions of: length, time, mass,

> temperature and, finally, the dimension which serves to measure the quantity of heat (or energy).....

One of the at first surprising facts shown by dimensional analysis is that all purely physical quantities, such as density, velocity, acceleration, work, area, volume, weight, resistance, conductivity, power, and the hundreds of other terms we use to describe physical phenomena, can be reduced to combinations of just five dimensions. Fourier continues to explain this in discussing the problem of heat transfer:

> In the analysis of the theory of heat any equation (E) which we use expresses a basic relation between the magnitudes: length (x), time (t), Temperature (T), and the coefficients c, h, and k (for heat capacity, heat transfer and thermal conductivity, respectively).
>
> The relation we obtain depends in no respect on the choice for the unit of length, because if we took a different unit to measure the linear dimension, the basic equation (E) would still be the same. For example, suppose the unit of length be changed, and its second value equal to the first divided by a number 'm.' Regardless of the value of 'x' in equation (E), it represents a certain distance 'ab' between two points 'a' and 'b.' This distance, ab, is some multiple of the unit length and thus in the new system of length units becomes 'mx.' Although we change the unit of length to mx the values of the time (t), and the Temperature (T) will not be changed. This is not the case with the quantities of h, k and c. The first, h, becomes h/m^2 because it expresses the quantity of heat which escapes during the unit of time from the unit of surface at temperature 1. Inspection of the thermal coefficient, k, shows that it becomes k/m because the flow of heat varies directly as the area of the surface, and inversely as the distance between. The heat capacity (c) also depends on the unit of length (as it expresses heat per unit volume) and becomes c/m^3.

Equation (E) must undergo no change when we write mx instead of x and, at the same time, k/m, h/m^2 and c/m^3 instead of k, h and c. Thus, the number m must disappear after these substitutions. The exponent on the dimension of length is 1, that of length in k is -1, that of h is -2, and that of c is -3. Now, if we attribute to each quantity its own exponent of dimension, the equation will be homogeneous, since every term will have the same total exponent number. Numbers such as S and V that represent surfaces and volumes have exponents of dimension of 2 and 3, respectively, on the linear measure. Angles, sines and other trigonometrical functions, logarithms or exponents of powers are, according to the principles of analysis, absolute numbers which do not change with the unit of length. Their exponent of dimension must therefore be taken as 0, which is the exponent of dimension of all abstract numbers.

If the unit of time is changed from 1.0, so that the new time is 1/n, the number t will become nt and the numbers x and T will not change. The coefficients k, h, and c will become k/n, h/n and c. Thus, the exponents of dimension of x, t and T with respect to the dimension of time are 0, 1, 0; and those of k, h, c are -1, -1, and 0.

If the unit of temperature be changed so that temperature 1 becomes a lower temperature in the ratio of 1 to the number p, T will become Tp, x and t will keep their values and the coefficients k, h and c will become k/p, h/p and c/p.

The following Table I indicates the exponents of dimension of the three undetermined quantities x, t and T and the three coefficients k, h and c with respect to each kind of dimension.

Table I.

Quantity of Constant	Length	Duration	Temperature
Exponent of dimension of x	1	0	0
Exponent of dimension of t	0	1	0
Exponent of dimension of T	0	0	1
The specific conducibility, K	-1	-1	-1
The surface conducibility, h	-2	-1	-1
The capacity for heat, c	-3	0	-1

To apply this rule to different equations and their transformations, the equation must be homogeneous to each kind of dimension, and every exponential quantity must cancel to nothing. If this is not the case, some error has been committed or an incomplete expression has been introduced. The following equation for heat transfer may be used as an example:

$$\frac{dT}{dt} = \frac{k}{c}\,\frac{d^2T}{dx^2} - \frac{h\ell}{cS}\,T \qquad (2)$$

Fourier's equation above (Eq. 2) which he uses as a demonstration states that the change in temperature (dT) with respect to change in time (dt) is equal to the constant k/c times the rate of change of temperature gradient (d^2T) with respect to change of distance per incremental length dx^2 minus the heat loss which is directly proportional to the heat transfer coefficient (h), thickness (ℓ) and temperature (T) and inversely proportional to the heat capacity (c) and the surface area (S). If we substitute the values of the exponents from Table I we have:

$$\frac{T^1}{t^1} = \frac{T^{-1}\,t^{-1}\,x^{-1}}{x^{-3}\,T^{-1}}\;\frac{T^1}{x^2} - \frac{x^{-2}\,t^{-1}\,T^{-1}}{x^{-3}\,T^{-1}}\;\frac{x^1}{x^2}\;T^1 \qquad (3)$$

Collecting exponents of dimensions we have

$$\frac{T^1}{t^1} = \frac{T^1}{t^1}\left[\frac{x^3}{x^3}\;\frac{T^1}{T^1}\right] - \frac{T^1}{t^1}\left[\frac{x^2}{x^2}\right] \tag{4}$$

The terms in the brackets cancel, leaving the dimensions of unit of temperature per unit of time or:

$$\frac{T^1}{t^1} = \frac{T^1}{t^1} - \frac{T^1}{t^1} \tag{5}$$

Equation (5) and therefore Eq. (2) are homogeneous. Dividing by $\frac{T^1}{t^1}$, the equation becomes dimensionless.

Returning to more recent times, Rayleigh in 1915 writes:

> I have often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of "laws" are put forward as novelties on the basis of elaborate experiments, which might have been predicted a priori after a few minutes' consideration. However useful verification may be, whether to solve doubts or to exercise students, this seems to be an inversion of the natural order. One reason for the neglect of the principle may be that, at least in its applications to particular cases, it does not much interest mathematicians. On the other hand, engineers, who might make much more use of it than they have done, employ a notation which tends to obscure it. I refer to the manner in which gravity is treated. When the question under consideration depends essentially upon gravity, the symbol of gravity (g) makes no appearance, but when gravity does not enter into the question at all, g obtrudes itself conspicuously.
>
> Lord Rayleigh

VI. RAYLEIGH'S SERIES

A simple statement of the principle is that all physical interrelationships may be expressed as functions and coefficients and are completely independent of any arbitrary system of units used in experimental measurements, provided the system is homogeneous. That is, there exists a general relationship for all physical phenomena: Regardless of the complexity of the functional relationships between the individual physical variables influencing the phenomenon independently or dependently, these variables need be expressed only in the form of a series of dimensionless ratios. For example, consider a system of "n" variables independent of dimension. We will call the variables $Q_1, Q_2, Q_3, \ldots Q_n$. If we wish to determine the relationship between Q_1 and the other variables we can let the power on Q_1 be unity and write the following equation:

$$Q_1 = K_1 Q_2^{a_2} Q_3^{a_3} Q_4^{a_4} \ldots Q_n^{a_n} \tag{6}$$

Where the greatest interest is in "Q_2" as a function of the other variables, a similar equation is written:

$$Q_2 = K_2 Q_1^{b_1} Q_3^{b_3} Q_4^{b_4} \ldots Q_n^{b_n} \tag{7}$$

In both Eqs. (6) and (7), "K_1" and "K_2" are dimensionless constants and the powers $a_2, a_3, a_4 \ldots a_n$; and $b_1, b_2, b_4 \ldots b_n$ are exponents determined by experimental measurements in which all variables but one are held constant. For example, the quantities Q_1, $Q_3, Q_4 \ldots Q_n$ may be held constant while "Q_2" is varied. Experimental data will then show how "Q_1" varies with "Q_2". If we take logarithms of Eq. (6) with quantities $Q_3, Q_4 \ldots Q_n$ held constant, we have

$$\ell n Q_1 = a_2 \ell n Q_2 + \ell n K_1 + \ell n [Q_3^{a_3} Q_4^{a_4} \ldots Q_n^{a_n}] \tag{8}$$

or and

$$\ell n Q_1 = a_2 \ell n Q_2 + \ell n C_2 \quad ; \quad Q_1 = C_2 Q_2^{a_2} \tag{9}$$

By holding quantities Q_3, Q_4 . . . Q_n constant, the product of "$Q_3^{a_3}$, $Q_4^{a_4}$ . . . $Q_n^{a_n}$" will also be constant so that Eq. (8) will reduce to Eq. (9) the equation of a straight line. Common practice in evaluating the constants is to plot the logarithm of "Q_1" versus the logarithm of "Q_2." The slope of the straight line obtained by plotting the experimental data will be equal to "a_2" and the intercept at $Q_2 = 1$ is equal to logarithm of C_2. The above procedure can be repeated for all the variables Q_3, Q_4 . . . Q_n to establish C_2, C_3, C_4 . . . C_n. Knowing these values for the constants, "C," we see that a substitution can be made for constants K_2, K_3, K_4 . . . K_n, and the exponents a_2, a_3, a_4 . . . a_n. Expressions for the other variables similar to Eq. (7) for "Q_2" can be derived from Eq. (6) by normalizing the exponent for the variable of interest. For example, let us assume that in solving Eq. (8) we find the slope of the line drawn through the data to be 0.50; this means that Q_1 is proportional to the square root of "Q_2", or:

$$Q = K_1 Q_2^{0.50} Q_3^{a_3} Q_4^{a_4} \ldots Q_n^{a_n} .$$

Now if we solve for "Q_2" we square all other quantities in the equation to obtain power of unity on Q_2, or:

$$(Q_2^{0.5})^2 = Q_2 = K_1^2 Q_1^2 Q_3^{2a_3} \ldots Q_n^{2a_n} \tag{10}$$

Comparison of Eq. (10) with Eq. (7) indicates that $K_2 = K_1^2$; similarly, $b_1 = 2a_1$, $b_3 = 2a_3$ . . . $2a_n$ where $a_2 = 0.50$. Thus, once the value of constant "K_1" has been established along with exponents a_2, a_3 . . . a_n, Eq. (6) may be rewritten for "Q_2" as in the case of Eq. (7) or for Q_3 or Q_4 by similar equations by the procedure described for normalizing the exponent to unity on the variable of interest.

Note that Rayleigh's method involves an infinite series from Q_1 to Q_n, whereas the methods of Fourier and Buckingham consolidate the dimensionless ratios into a smaller total number of groups. Both methods have their advantages. The variables influencing a particular system are most easily recognized as ratios. Then, by combining ratios according to the magnitude of the exponents, the total number of "Q" quantities may be reduced to shorten the equation. The greatest advantage to such combination is the great reduction in the number of experimental measurements required to determine the constants in the equation.

The effort required for experimental determination of a function can be very great. As Langhaar (11) comments, a function with a single variable may be plotted as one curve. If two variables are involved a family of curves may be used with one curve each for selected values of the second variable. With three variables, sets of curves are involved in a chart and so forth. Then we come to sets of charts. If we use five points to establish a curve, 25 points will be required for a chart, 125 for a set of charts, and 625 points for five sets. Thus the effort required increases exponentially with the number of variables. If we have several variables and determine more than the minimum number of values, the effort involved can become excessive. Thus the reduction of the number of variables by combination in dimensionless groupings can greatly reduce the labor of experimental investigations.

The proof that an equation of the form of Eq. 6 can be used to solve any physical relation is not self-evident. Let us first consider some forms the equation may take.

In its most reduced form, Eq. 6 for an infinite series can be limited to two quantities, Q_1 and Q_2, with the exponent of a_a reduced to unity, or:

$$Q_1 = kQ_2^{a2} = kQ_2^{1.0} = kQ_2 \tag{11}$$

Then as an example let us say that Q_1 is the dimensionless ratio of two units of length, x and y, and that Q_2 has a value of 1.0. This gives the equation of the simplest of curves, a straight line through the origin, or:

$$\left(\frac{x}{y}\right) = k\,(1.0),\ \text{or},\ y = kx \qquad (11a)$$

where k is the slope of the line. Then let us allow Q_2 to have a value other than 1.0 and be equal to the ratio of two velocities, v_1 and v_2, with exponent a_2 equal to 2.0. To explore the nature of Q_2 we will now let Q_1 have a value of unity:

$$1.0 = kQ_2^{\ 2} = k\left(\frac{v_1}{v_2}\right)^2 \qquad (12)$$

This could be the equation for the ratio of kinetic energies of two masses traveling at different velocities, v_1 and v_2, and k, the ratio of the masses. When plotted on plain coordinated paper this equation will give a curve that rises slowly at first and then more sharply.

We can continue in such a manner introducing $Q_3^{\ a3}$ with a_3 equal to a negative exponent, say -3. This also gives an exponential curve of a reciprocal form of $Q_2^{\ 2}$ and will curve downward. The curve of the total products of Q_1^{a1}, Q_2^{a2}, Q_3^{a3} can rise sharply or dip sharply or through various combinations first rise to a maximum, then dip downward, resembling a pressure time curve for a rifle. Inclusion of additional quantities Q_4^{a4}, Q_5^{a5} . . . Q_n^{an} permits great variation in the shape of the curve of the combined quantities. Herein lies part of the strength of the method.

In the experimental studies which are necessary for use of this method we hold all quantities constant except one. Then step by step we can determine the values of "K" and "a" for each "Q" by experimental measurement. When we complete our evaluation of these constants and substitute them into Eq. (6), we can use the combined equation to predict the result when several variables are changed simultaneously.

Although these examples demonstrate the power and versatility of the method, we still need proof that the equation must be homogeneous and that the homogeneous equation is adequate for any system.

VII. LANGHAAR'S PROOF (11)

We can prove quite easily that the equation must be homogeneous and either be free of dimensions or, in the case of the equation for a sum, all quantities must have the same dimension. For illustration take the case where "f" is the sum of the Q's, or:

$$Q_1 = f(Q_2, Q_3 \ldots Q_n) = Q_2 + Q_3 + \ldots Q_n \tag{13}$$

or,

$$K(Q_1 + Q_2 + \ldots Q_n) = K_1Q_1 + K_2Q_2 + \ldots K_nQ_n \tag{14}$$

This is an identity in Q's. Therefore:

$$K = K_1 = K_2 = K_n \tag{15}$$

This can be true only if $Q_1, Q_2 \ldots Q_n$ all have the same dimension. This requirement for homogeneity simply means that different dimensions such as inches and pounds are not directly additive.

Consider now the case of the product equation, a favorite of Rayleigh, as in Eq. (6) where $Q_1, Q_2 \ldots Q_n$ are dimensionless ratios:

$$Q_1 = K_1 Q_2^{a_2} Q_3^{a_3} \ldots Q_n^{a_n} \tag{6}$$

Taking logarithms as in Eq. 8 gives:

$$\ell n Q_1 = \ell n K_1 + a_2 \ell n Q_2 + a_3 \ell n Q_3 + \ldots a_n \ell n Q_n \tag{8a}$$

Equation (8a) reduces to Eq. 14 because the logarithms of numbers Q_1, Q_2, and Q_n are without dimension, as are the exponents $a_1, a_2 \ldots a_n$. Therefore, the equation is homogeneous. Equation 8a would not be homogeneous if $Q_1, Q_2 \ldots Q_n$ had different dimensions. The equation also is homogeneous if all quantities have the same dimensions as in Fourier's example. In this case, division of the equation by the common dimension leaves the equation dimensionless and demonstrates homogeneity.

In a similar manner but introducing the use of matrices, Langhaar (11) proves Buckingham's π theorem:

$$\pi_1^{h} \pi_2^{h_2} \ldots \pi_n^{h_n} = 1 \qquad (16)$$

where $\pi_1, \pi_2 \ldots \pi_n$ are not simple ratios but instead are groups of quantities arranged in dimensionless forms, such as:

$$\pi_1 = Q_1^{a'_1} Q_2^{a'_2} \ldots Q_n^{a'_n} \qquad (17)$$

$$\pi_2 = Q_1^{a''_1} Q_2^{a''_2} \ldots Q_n^{a''_n} \qquad (18)$$

$$\pi_n = Q_1^{a_1p} Q_2^{a_2p} \ldots Q_n^{a_np} \qquad (19)$$

Combining equations gives:

$$Q_1 (a'_1h_1 + \ldots a_1ph_p) \; Q_2 (a'_2h_1 + \ldots a_2ph_p) \ldots Q_n (a'_nh_1 + \ldots a_nph_p) \qquad (20)$$

This is an identity in Q's and the exponents vanish. Langhaar states that "this is contrary to the hypothesis that the rows in the matrix of exponents are linearly independent. Thus it is proved that when the rows in the matrix of exponents are linearly independent, the dimensionless products are independent." This is used to prove the theorem that "a necessary and sufficient condition that the products $\pi_1, \pi_2 \ldots \pi_p$ be independent is that the rows in the matrix of exponents be linearly independent."

The Buckingham π theorem basically constitutes a recognition that a truly "infinite" series, such as Rayleigh's Eq. (6), is not required for most problems and usually three to several dimensionless ratios may suffice. In the case of internal ballistics, our studies to date indicate that over 20 dimensionless ratios are required but that a number of these may be combined using the methods of Fourier and Buckingham. Buckingham's theorem also introduces an algebraic procedure for manipulation so that these quantities may be combined to produce the minimum number of combined groups required for a given number of dimensions and number of quantities. In our present stage of study we have been able to investigate only a portion of the known variables. We expect to discover new quantities in the course of future study that may have only minor influence but which will permit further refinement of the correlation. Thus, at present we are not in a position to make full use of Buckingham's method and will base the first correlations primarily on the use of the infinite series of Rayleigh.

VIII. DIMENSIONLESS GROUPS USED IN BALLISTICS

Let us now rationalize a bit about the factors that influence "p" in a rifle. First, there is the bullet, which can be characterized in external ballistics by the sectional density, "w/d^2", where "w" is the bullet weight and "d" is the bullet diameter. In external ballistics, a shape coefficient "C" is also used to calculate air friction on the bullet. Before reaching the muzzle, however, the bullet is in contact with the barrel rather than with air, and a different coefficient is necessary. This coefficient, called the jacket factor, (J) must be used for the barrel friction and resistance caused by cold working as the bullet enters the rifling and is "engraved" or grooved. As mentioned earlier, the jacket factor is defined as the ratio of the values of (p) produced by the test bullet to the value of (p) produced by a reference bullet; all other factors (barrel, powder type, powder load, primer, case, seating depth, etc.) are held constant. That is:

$$\frac{J\text{ (for bullet considered)}}{J\text{ (for reference bullet)}} = \frac{p\text{ (for bullet considered)}}{p\text{ (for reference bullet)}}$$

(all other factors constant) (21)

Other factors known to affect (p) are the load of powder (ℓ), the case volume (V_s) at a given bullet seating depth, the primer and the powder type. For the present, these last two, powder and primer type, will be held constant. The ratio of the powder load (ℓ) to case volume (V_S) is also known as the "loading density (ℓ/V_S)." In the past, this ratio has been used with success to correlate ballistic information, and, therefore, it will be used here. Loading density has the units, "lbs/cu in," in the English system and "gms/cc," in the metric system. These are the units of the absolute density (a) of the powder. IMR No. 3031 has an absolute density of 1.67 gm/cc in the metric system and 0.0604 lbs/cu in in the English system. The absolute density of the powder can be used to convert the loading density to the specific loading density (S) which is dimensionless and, therefore, independent of any system and of any units used.

$$S = \frac{\ell}{a\,V_S} \tag{22}$$

Other factors such as length of free bore, barrel wear and gas leakage also influence "p" but these too shall be considered constants for the present. It is now necessary to determine the number of π groups required to correlate these variables. This could be done using the mathematical procedure described by Buckingham, or by the intuitive method so often used by Rayleigh. We can see by inspection that one dimensionless π group is defined by "S," the specific loading density. The remaining factors, "w," "d^2," "p" and "gc" can be grouped into another dimensionless quantity as follows:

$$B' = \frac{\pi\,(p)\,(d)^2}{4(10^6)\,(w/gc)} \tag{23}$$

Or, for the 220-grain, 30 cal, Rem SPCL bullet:

$$B' = \frac{3.1416(32.17)\,(7000)\,(0.308)^2\,(p)}{4\,(10^6)\,\,(220)}$$

$$= 0.762\,(10^{-4})\,\,(p) \tag{24}$$

Now, for any bullet for 30-06

$$B' = 0.01675 \frac{(p)}{(w)} \text{, or rearranging:} \tag{25}$$

$$p = 59.7\,(w'\,B') \tag{26}$$

where: B' = Brownell number, megagees

p = maximum chamber pressure, psi

π = 3.1416

gc = 32.17 ft/sec^2

w = bullet weight, lb

w' = bullet weight, grains

d = bullet diameter, in.

The reason that the factors of $\pi/4(10^6)$ and "gc" were inserted was to give "B'" called the Brownell number for convenience, more significance. The numerator of Eq. (23) divided by 4 defines the maximum force on the bullet because force equals maximum pressure (p) times area ($\pi d^2/4$) of bullet base. Now, if the bullet mass is divided by the gravitational constant (gc) the result will be the gravitational acceleration, because, by Newton's Law of Motion, $F = wa/gc$, or $a = F/(w/gc)$. Thus, the numerator of Eq. 23 divided by "4 w/gc" is equal to the ratio of the maximum acceleration in the rifle barrel to the gravitational acceleration. This dimensionless ratio, also found in high velocity aircraft and in centrifugal machines such as the centrifuge, has been termed "gees." The "gee" value for bullet acceleration by the powder is very large (over 1 million). Therefore, the gee value can be divided by 10^6 (one million) resulting in the more convenient "megagee," equal to a million gees. According to the rules of dimensional similitude, our two groups can be related by:

$$B' = K \frac{\ell}{aV_s}^{b} = K\,(S)^b \tag{27}$$

Equation (27) is verified by determining the values of "K" and "b". If these are constants, Eq. (27) is valid.

IX. EVALUATION OF THE CONSTANTS "K" AND "b"

Experimental data for various firings are given in Table IIa through IIe, and data on "B" versus "S" are plotted in Figs. 4 and 5. Although none of the data produce identical curves, they do have a number of characteristics in common. All bullet masses and powder loads produce data that show a close grouping and a linear relationship for the logarithm of B′ versus the logarithm of S, when B′ is more than 3 megagees. The line is steep with respect to the logarithm of S scale and gives a slope of 4.0. An expanded scale is used for better analysis. Data for low-loading densities and for values of B′ less than 3 megagees show scatter indicating erratic pressures.

Bullets of one type and one "J" were produced from the 220-grain Remington SPCL (Soft Point Core Lokt) bullets. Lighter bullets were cut from the 220-grain ones, cut precisely on a lathe to a tolerance of ± 1 grain. The sizes cut were 200, 180, 150, 130, and 110 grains; the lengths are shown in Fig. 1. The objective in cutting bullets of different weights from the 220-grain bullet rather than using commercial bullets of various weights was to eliminate the variation in engraving resistance known to exist in commercial bullets of different weights. Bullets of identical weights but produced by different manufacturers have been shown to produce pressure variations of as much as 10,000 psi when fired in the same rifle with identical cases, primers and charges of the same powder.

For the purpose of analysis, data for different bullets but identical powder loads (40 to 52 grains of IMR No. 3031) are assembled in Table III. These powder loads give values for B′ of from 3.28 to 9.15. Data in Table III show good agreement in the values of B′ for 220, 200, 150, 130, and 110 grain bullets cut from the 220 grain Remington SPCL bullets. Data on these were more closely compared by determining the average values of B′ for 40, 42, 44, 49, 50, and 52 grains of load. None of the data for these

TABLE II (a, b, c, d) DATA FOR FIGURE 4

EXPERIMENTAL FIRING DATA FOR SPRINGFIELD 30-06 BARREL C

Load L,gr	S Δ/a	a 110g Cut (SPCL) p,psi	a B'	b 130g Cut (SPCL) p,psi	b B'	c 150g RN (SPCL) p,psi	c B'	d 150g Cut (SPCL) p,psi	d B'
20.2	0.201	---	---	---	---	---	---	6,000	0.67
23.0	0.232	---	---	7,000	0.90	---	---	11,000	1.23
25.0	0.252	---	---	9,000	1.16	---	---	---	---
26.0	0.262	---	---	---	---	---	---	11,000	1.23
27.0	0.272	---	---	11,000	1.42	---	---	---	---
28.0	0.282	---	---	---	---	---	---	13,000	1.45
29.0	0.292	---	---	---	---	18,000	2.01	13,500	1.51
30.0	0.302	---	---	12,000	1.54	---	---	---	---
33.0	0.332	10,500	1.60	13,000	1.67	20,000	2.23	19,000	2.12
36.0	0.363	14,000	2.13	17,400	2.24	---	---	21,000	2.35
37.0	0.373	---	---	---	---	21,000	2.34	---	---
38.0	0.383	---	---	22,500	2.90	---	---	---	---
39.0	0.393	---	---	---	---	---	---	29,000	3.24
40.0	0.403	21,500	3.28	26,000	3.35	25,000	2.79	31,000*	3.46
41.0	0.413	---	---	---	---	---	---	33,000	3.68
42.0	0.423	26,000	3.96	31,000	3.99	---	---	36,500*	4.08
43.0	0.433	---	---	---	---	34,000	3.80	40,300*	4.48
44.0	0.443	32,000	4.88	37,000	4.77	---	---	44,000	4.91
45.0	0.453	36,000	5.48	43,000	5.54	---	---	50,000	5.57
46.0	0.463	40,000	6.10	48,000	6.18	41,000	4.58	55,000	6.14
48.0	0.484	48,000	7.32	57,000	7.35	53,000	5.92	63,500	7.10
49.0	0.494	51,000*	7.77	59,800*	7.70	57,500	6.43	67,800*	7.58
50.0	0.504	54,000	8.24	62,500	8.05	62,000	6.93	---	---
52.0	0.524	60,000	9.15	---	---	---	---	---	---

*Interpolated or Extrapolated

POWDER DUPONT IMR 3031, LOT 1229; LOT 1229; BARREL "C", SPRINGFIELD 30-06, 03A3, 0.310" GROOVE DIAMETER AT BREECH (FOR 1/4"), BORE WEAR BY US ORDNANCE BREECHBORE GAGE 0.10; SEATING DEPTH 9/16" (TO CANNELURE); PRIMERS AND CASES REMINGTON COMMERCIAL: BULLETS CUT FROM 220 GRAIN REMINGTON SPCL EXCEPT FOR 150 g RN (30-30) SPCL.

TABLE II (e, f, g, h)

EXPERIMENTAL FIRING DATA FOR SPRINGFIELD BARREL C

		e 200g Cut (SPCL)		f 220gr Rem (SPCL)		g 348gr Compound (SPCL)		h 476gr Compound (SPCL)	
L,gr	S	p,psi	B'	p,psi	B'	p,psi	B'	p,psi	B'
20.0	0.201	11,000	0.86	---	---	---	---	---	---
23.0	0.231	13,200	1.11	---	---	---	---	---	---
25.0	0.251	15,400	1.28	---	---	---	---	---	---
26.0	0.262	---	---	---	---	---	---	27,000	0.95
27.0	0.272	17,000	1.43	19,200	1.47	25,000	1.21	---	---
29.0	0.292	---	---	24,000	1.84	29,000	1.40	35,500	1.25
30.0	0.302	20,000	1.68	---	---	34,000	1.64	---	---
32.0	0.322	23,200	1.95	---	---	---	---	47,000	1.65
34.0	0.342	27,500	2.32	---	---	---	---	---	---
35.0	0.353	---	---	30,000	2.30	53,500	2.59	61,000	2.14
36.0	0.363	29,700	2.50	---	---	---	---	---	---
37.0	0.373	---	---	35,000	2.68	56,000	2.70	71,000	2.50
38.0	0.383	36,300	3.06	37,500	2.88	---	---	---	---
39.0	0.393	---	---	41,000	3.14	64,000	3.09	83,000	2.92
40.0	0.403	41,800	3.51	44,000*	3.35	63,000	3.28	86,000	3.03
41.0	0.413	45,000	3.77	47,000	3.57	74,000	3.57	91,000	3.20
42.0	0.423	47,800*	3.99	52,000*	3.95	81,000	3.90	102,000	3.59
43.0	0.433	50,600	4.24	57,500	4.37	---	---	---	---
44.0	0.443	58,500	4.92	64,000	4.88	---	---	---	---
45.0	0.453	65,000	5.43	---	---	---	---	---	---
48.0	0.484	---	---	93,100	7.11	---	---	---	---
49.0	0.494	---	---	100,000	7.62	---	---	---	---

*Interpolated

POWDER IMR 3031, LOT 1229; BARREL "C", SPRINGFIELD 30-06, 03A3, 0.310" GROOVE DIAMETER AT BREECH (FOR 1/4"),

BORE WEAR BY US ORD BREECHBORE GAGE OF 0.10; SEATING DEPTH OF 9/16" (TO CANNELURE); REMINGTON CASES & PRIMERS.

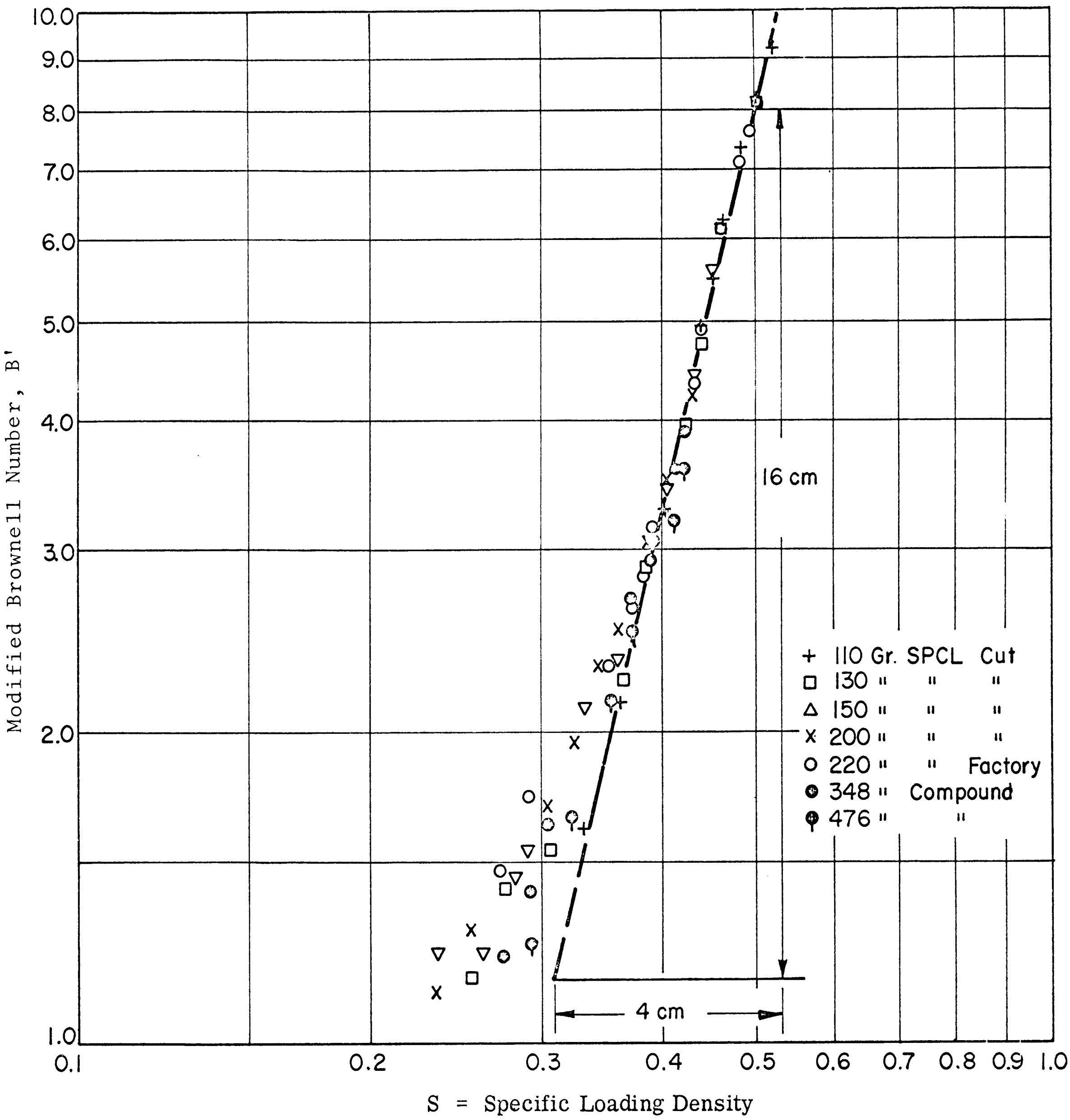


Fig. 4. Plot of B vs S For Bullets of 110 Grain to 476 Grain Mass
(Data from Table I)

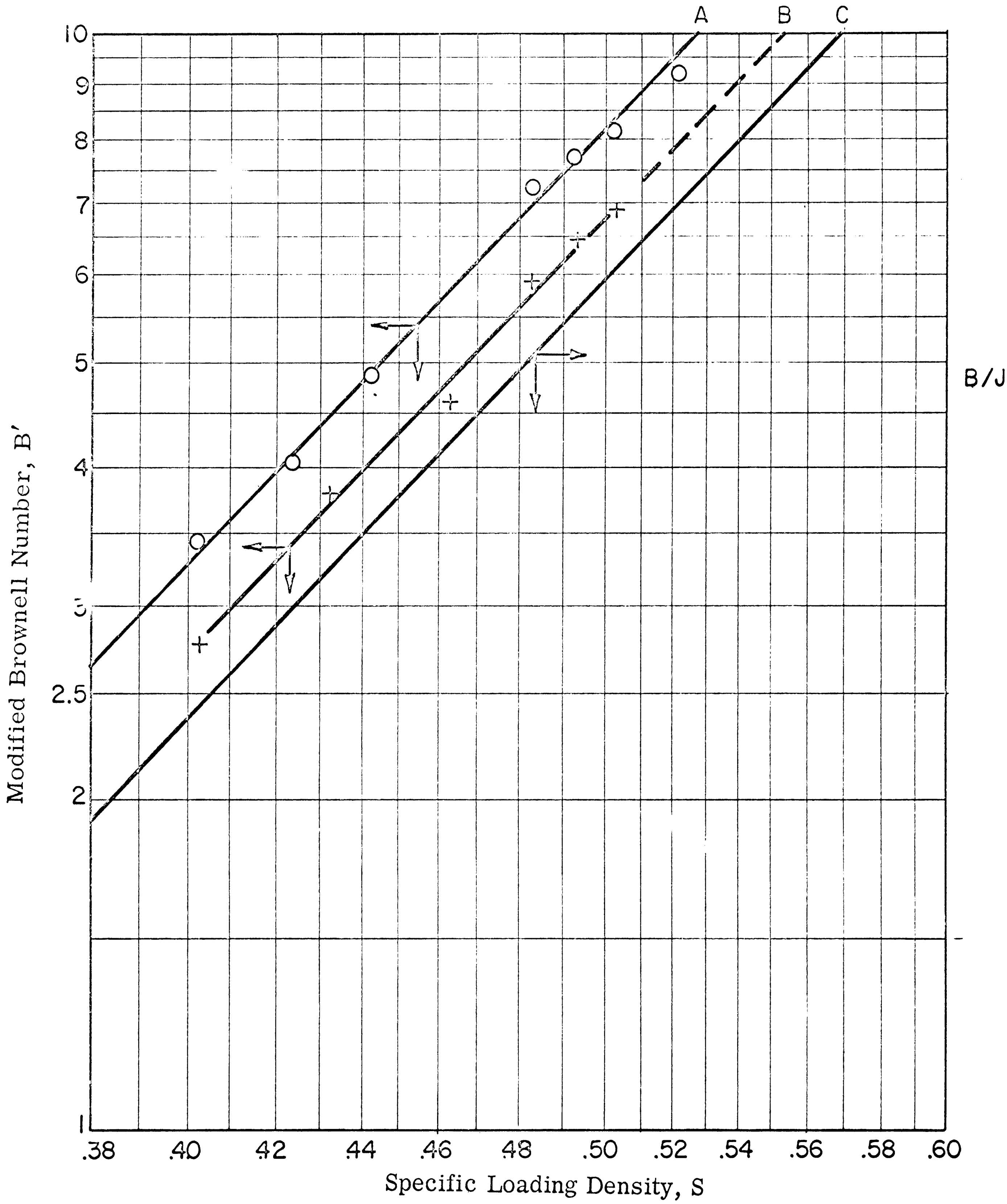


Fig. 5. Plots of Data of Table 2 and Equation 16

bullets and loadings deviated more than 3.5% from the respective average value. The data became more erratic at lower loadings of 30-38 grains. In the region of loads as low as 20-30 grains, "pressure excursions" giving over 100% variation were experienced (see "Pressure Horn" and "Pressure Excursions" in Technical Report No. 1);[2] these were not plotted.

In Fig. 5, the logarithm scale for S was expanded four times the logarithm scale of B′ permitting a more precise determination of the constants for the linear equation of the logarithm of B′ versus logarithm of S plot. The scale for the logarithm of S extends only from 0.38 to 0.60 because of the four-fold expansion. Note that five data points (indicated by circles ʘ) equaling S values of 0.403, 0.423, 0.443, 0.483, and 0.494 fall almost exactly along one straight line (labeled Curve A). These points correspond to powder loads of 40, 42, 44, 48, and 49 grains and represent average values of "B′" for the same values of "S" for three to five different bullet weights. The two points to the upper right are based on two bullet weights (130 and 110 grain) for a 50-grain load (S = 0.504) and one weight (110 grain) for a 52-grain load (S = 0.524). Since the other five points represent averages of five different bullet weights (110 to 220 grains), they were considered the most important in locating Curve A. The slope of the curve is 45° or, 4.3 in./4.3 in. or 1.0. If this is multiplied by the scale expansion of four to one, the product is 4.0. This value is the true slope of the curve and the value of the exponent, "b," in Eq. (27).

The next step is to determine the coefficient "K" by substitution of corresponding values of "B′" and "S" into Eq. (27). For convenience, we can let B′ = 3.00 and read from Curve A of Fig. 5 the corresponding value of "S." Inspection of Curve A shows that for B′ = 3.00, the value of "S" is 0.392. The only unknown in Eq. (27) now is "K." If we substitute the numerical values for "S," "b" and "B′" we can solve for "K":

$$B' = K\,(S)^b \tag{27}$$

$$3.00 = K\,(0.392)^4$$

$$K_{220} = \frac{3.00}{(0.392)^4} = \frac{3.00}{0.0236} = 127$$

Therefore: $B'_{220} = 127\ S^4$ (28)

To check Eq. (28), let S = 0.50 and solve for "B'," or:

$$B' = 127\ (0.50)^4 = 7.95$$

Reading from Curve A we obtain B = 8.00, for S = 0.50. The error is 5/800 or 0.6 per cent. Thus we have established the validity of Eq. (27) for the 220 grain Rem SPCL bullet. Note that Eqs. (27) and (28) are identical in form to the exponential version of Eq. (9).

X. JACKET FACTORS FOR OTHER BULLETS

In compiling Table II, only data for bullets of a similar type, fired in the identical rifle barrel were used. These data were for 220-grain, Rem SPCL bullets and for the special bullets made from them: 110, 130, 150, 200, 348, and 476 grains, respectively. The value of "J" for all of these bullets is the same, 1.38.

Table I lists one set of data for the Rem 150-grain, SPCL (RN 30-30) bullet which has a different value of J. The 220-grain Rem SPCL bullet has a thicker jacket than does the 150 grain Rem factory SPCL bullet. This is because the former is designed for use with large game where penetration is important. The heavier jacket keeps the bullet from mushrooming as quickly as lighter bullets do and allows more penetration before expending its energy. Therefore, bullets cut to 150 grains from the 220-grain Rem SPCL bullet will have thicker jackets than the factory made 150 grain bullet. The bullet with the tougher jacket will require more work from the powder gas to force it into the rifling of the barrel. The result is an increase in pressure for the more penetrating bullet, although the mass is the same for both.

In Fig. 5, Curve B (indicated by crosses +) representing 150-grain Rem SPCL bullets (low penetration) can be compared to Curve A representing the 220-grain Rem SPCL bullet (high penetration) and the specially cut 200, 150, 130 and 110 grain bullets. Curve B is parallel to

Curve A, but lower, corresponding to a lower pressure. The equation for Curve B as for Curve A can be determined using a slope of 4.0. Again, if we select a value of B′ = 3.00 for convenience, we read the value of S to be 0.410. Therefore:

$$B'_{150} = K_{150}\,(0.410)^4 = 3.00$$

$$K_{150} = \frac{3.00}{(0.410)^4} = \frac{3.00}{0.0282} = 106$$

$$B'_{150} = 106\ S^4 \qquad (29)$$

To show the significance of Eq. (29), we can take the ratio of Eq. (28) to Eq. (29) and obtain the ratio of "p" for 220 grain to that for 150 grain Rem SPCL bullets.

$$\frac{B'_{220}}{B'_{150}} = \frac{p_{220}}{p_{150}} = \frac{K_{220}}{K_{150}} = \frac{J_{220}}{J_{150}}$$

or:

$$J_{150} = J_{220}\,\frac{K_{150}}{K_{220}} = \frac{1.38(106)}{127} = .83(1.38) = 1.15$$

Thus, the "J" for 150-grain, Rem SPCL factory bullets (RN 30-30) is 83% of the values for the 220-grain Rem SPCL bullets. To demonstrate, consider a value of "S" of 0.433 which corresponds to a load of 43.0 grains. From Eq. (28) we have:

$$B' = 127\,(0.433)^4 = 4.48$$

and, from Eq. (26) for J = 1.38

$$p = 59.7\ w'B'$$

$$= 59.7(150)(4.48) = 40{,}200 \text{ psi}$$

(This checks with the value of 40,300 psi in Table Id for 150-grain, specially cut SPCL)

Multiply by 0.83 to correct for the lower jacket factor of the 150 grain factory bullet:

$$p = 0.83(40{,}200) = 33{,}400 \text{ psi (for 150 grain RN)}$$

(This checks with the value of 34,000 psi in Table Ic for 150-grain RN)

As demonstrated, we can switch from Curve A to Curve B by using the ratio of J values. Another procedure is to divide "B′" by "J" and plot the curve of "B′/J vs S." All bullets having values of "b = 4.0" would then be represented by the single line of Curve C (see Fig. 5).

The equation for Curve C may be written using Eqs. (28) or (29). Using the first:

$$B'/J = 127/1.38\,(S)^4 = 92\,S^4 = U \qquad (30)$$

or, the second:

$$B'/J = 106/115\,(S)^4 = 92\,S^4 = U \qquad (30)$$

Curve C of Fig. 5 will be useful for graphical solutions of Eq. (30) and, therefore, is not cluttered with data points. However, all of the data from Tables II and III in the range of S = 0.38 to S = 0.57 will fall on, or very close to the line of Curve C.

Handloaders interested in chamber pressures will have an interest in J values and the relative pressures for commercial bullets supplied by various manufacturers. More measurements need to be made to permit prediction of "J" from basic physical characteristics. Table III summarizes the data obtained as of June, 1966.

A few comments should be made about Table IV. In Part A, some data are included from Tables II and III, along with new data on additional firings using the same barrel, C, and a range of loads with each bullet type. These are presented with their corresponding J values. These

TABLE III

DATA USED FOR BASIC CORRELATION OF IMR #3031 POWDER IN 30-06 J VALUES FOR 220 GRAIN REMINGTON SPCL BULLETS IN SPRINGFIELD BARREL C

		Powder Load ℓ', Grains						
		40.0	42.0	44.0	48.0	49.0	50.0	52.0
		Specific Loading Density S, Dimensionless						
Bullet Weight, w' Grains		0.403	0.423	0.443	0.484	0.494	0.504	0.524
		B' = Brownell Number, Megagees						
220	B'	3.35	3.95	4.88	7.26	7.62	-----	-----
	% Dev	-1.2%	-1.2%	0.2%	0.0%	-0.6%		
200	B'	3.51	4.01	4.92	-----	-----	-----	-----
	% Dev	3.5%	0.3%	1.0%				
150	B'	3.46	4.08	4.91	7.10	7.58	-----	-----
	% Dev	2.1%	2.0%	0.8%	-2.0%	-1.2%		
130	B'	3.35	3.99	4.77	7.35	7.70	8.05	-----
	% Dev	-1.2%	-0.2%	-2.2%	1.2%	0.4%	-1.2%	
110	B'	3.28	3.96	4.88	7.32	7.77	8.24	9.15
	% Dev	-3.2%	-1.0%	0.2%	0.8%	1.3%	1.2%	0.0%
	Avg B'	3.39	4.00	4.87	7.26	7.67	8.15	9.15

POWDER DUPONT IMR 3031, LOT 1229; BARREL "C", SPRINGFIELD 30-06, 03A3, 0.310" GROOVE DIAMETER AT BREECH (FOR 1/4"), BORE WEAR BY US ORDNANCE BREECHBORE GAGE OF 0.10; SEATING DEPTH OF 9/16" (TO CANNELURE). CASES AND PRIMERS REMINGTON COMMERCIAL; BULLETS CUT FROM REMINGTON 220g SPCL.

TABLE IV A

J VALUES FOR VARIOUS BULLETS USED IN 30-06

(Determined by Range of Firings in Test Barrel C
with Groove Diameter of 0.310" at Breech)
IMR 3031, Lot 1229, Seating Depth 9/16", Rem. Cases and Primers

Bullet Weight w', gr.	Bullet Type	Supplier or Manufacturer	Jacket Factor J
220	SPCL	Remington	1.38
110,130 150,200	Cut from 220 g SPCL	Remington (Modified)	1.38
348	Compound 220 SPCL + 128 Carcano	Remington + FMJ (Italian) Military	1.38
476	Compound 220 SPCL + 2x128 Car.	Remington + FMJ (Italian)Military	1.38
150	SPCL RN(30-30)	Remington	1.20
150	Partition	Nosler	1.10
150	Spire Pt.	Hornady	1.18
150	Soft Pt. Spitzer	Speer	1.22

TABLE IV B

J VALUES FOR VARIOUS 150 GRAIN 30 CAL. BULLETS

Based on a Load of 51 Grains of IMR 3031 Powder (Lot 1229) and Triplicate Firings in 30-06 New Test Barrel D (Springfield) with Seating Depth of 0.34 Inches, Remington Cases and Primers

Bullet Type	Supplier or Manufacturer	Maximum Pressure pmax, psi	*B=Brownell Number, Megagees	Jacket Factor J**
Partition	Nosler	61,100	6.82	1.17
Sonic	Herter	61,100	6.82	1.17
Spire Pt	Herter	69,000	7.69	1.30
Half Lead	Herter	66,000	7.38	1.25
Banana Peel	Herter	63,000	7.04	1.20
FMJ-M2	US Army	63,000	7.04	1.20
Pt SPCL	Remington	63,000	7.04	1.20
S.P. Spitzer	Speer	64,000	7.16	1.21
Spire Pt	Hornady	65,000	7.26	1.23

* by Eq 12: $B_{150} = \frac{pmax}{59.7m} = \frac{pmax}{896}$

** $J_{150\,(9/16)} = \frac{B}{6.4}$

by Eq 17: $\frac{B}{J}_{(9/16)} = 92S^4 = U_{150} = 92(0.0695)^4 = 6.4$

$J_{150\,(0.34'')} = \frac{B}{5.9}$

$J_{150}(0.34'') = 1.08\ J_{150(9/16'')}$

Note: The quantity 1.08 is the "seating depth factor" and gives the pressure ratio for the same load of IMR 3031 when seating depth is decreased from 9/16" to 0.34" as determined from Fig 20 of Ref 2 (Technical Report No. 1).

data are plotted in Fig. 6. Basically, Fig. 6 is a graph of "J" times Eq. (30) with "B′" plotted versus 92 S^4. Because 92 S^4 is inconvenient we will let it equal U, University number (see Eq. 31). Note that in Fig. 6, "J" is not included with "B′." Therefore, only firings with a "J" of 1.0 will fall on the 45° line running from 1,1 to 10,10. Parallel lines of J = 1.10, 1.20, 1.30, 1.40, and 1.50 are drawn which permit interpolation for J. Equation (30) holds only for B′ values of 3 to 10 or higher because of erratic burning and pressure excursions at low loading

$$\frac{B'}{J} = U, \quad (\text{where } U = 92\ S^4) \tag{31}$$

densities (such as those giving pressures much under 30,000 psi).

Note the single Curve X at the upper left corner of Fig. 6. This line represents a cross plot of "J" versus "B′" for a specific value of U (or 92 S^4) equal to 6.40 (this is equal to 51.0 grains of IMR No. 3031 powder in a 30-06 case with bullet seated to 0.34 in.). Curve X was used to determine the J values for the firings in Table III, Part B, in which all of the loads were 51 grains of IMR 3031.

The data in Part B were taken from Figs. 3, 7, and 8 after the test barrel, C, had been replaced with a new one, D. Barrel C became slightly oversized (0.310 in. groove diameter for about the first 1/4 in.) as a result of earlier tests. Barrel D was a new Springfield 03-A3 which met government specifications when installed. For comparison with NRA data, several 150-grain, 30 caliber bullets from different suppliers were tested with 51 grains of IMR No. 3031 powder.

In order to use data in the literature for comparison with the data of Parts A and B, it is necessary to convert crusher values to absolute pressure. Curve Y in Fig. 6 is a plot which corresponds approximately to the relationship between crusher value and absolute pressure, psi (see Refs. 3 and 6). To use Fig. 6, enter with the crusher value, move to Curve Y and read absolute pressure. Note that the relation for Curve Y is not linear but exponential.

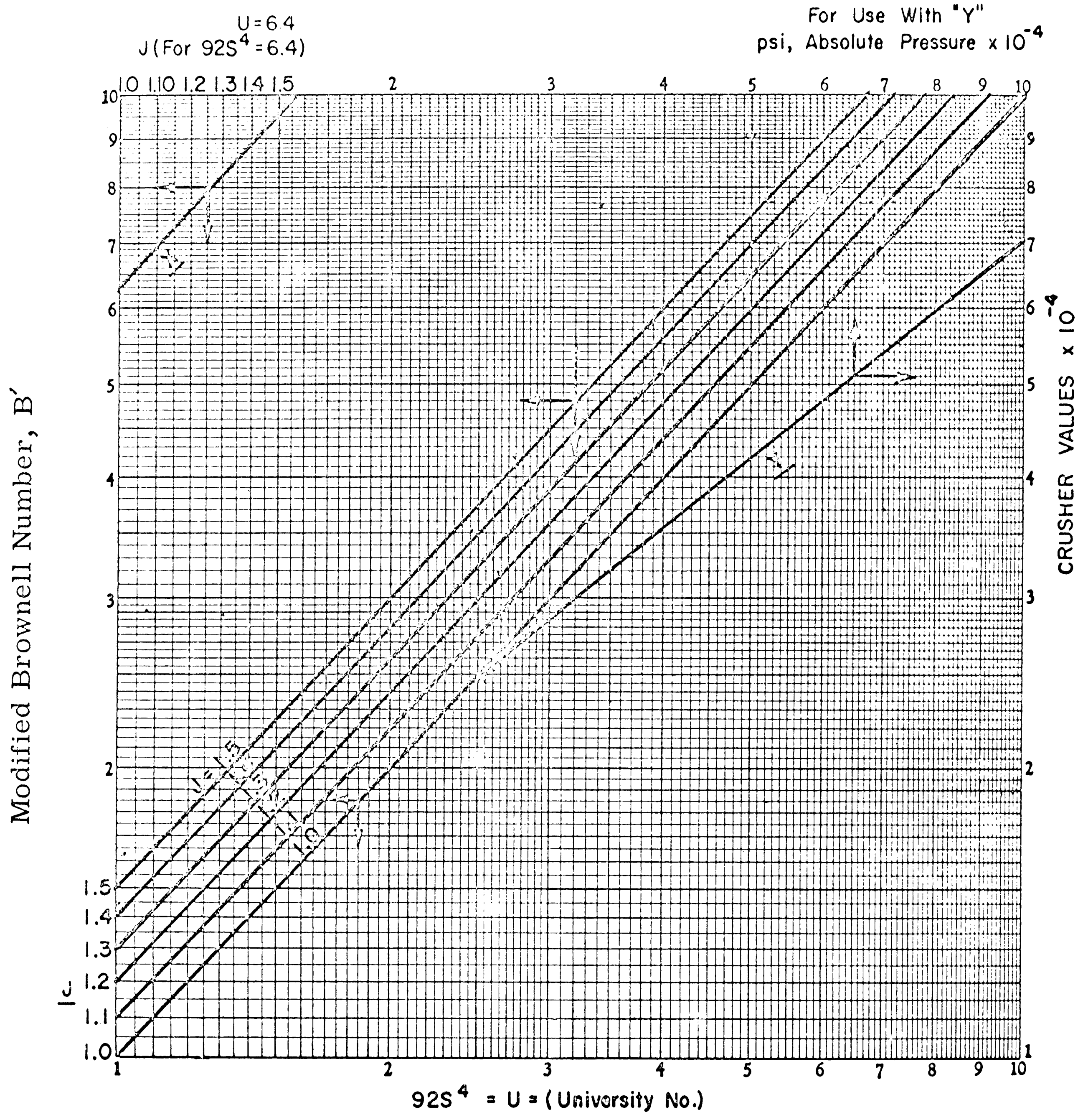
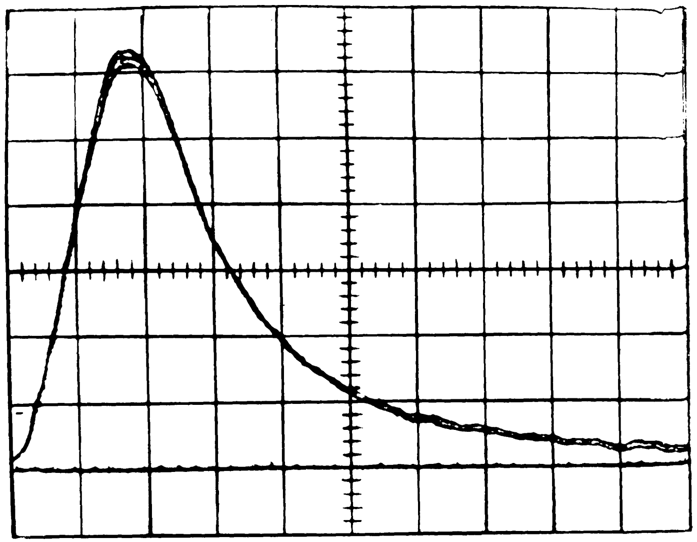
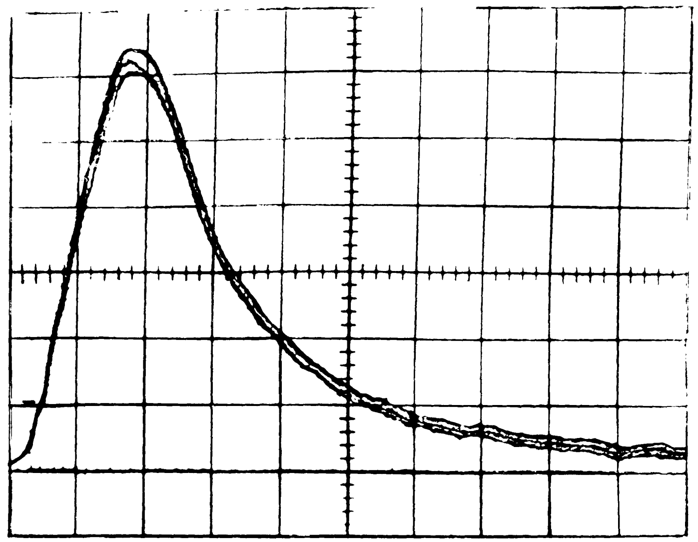


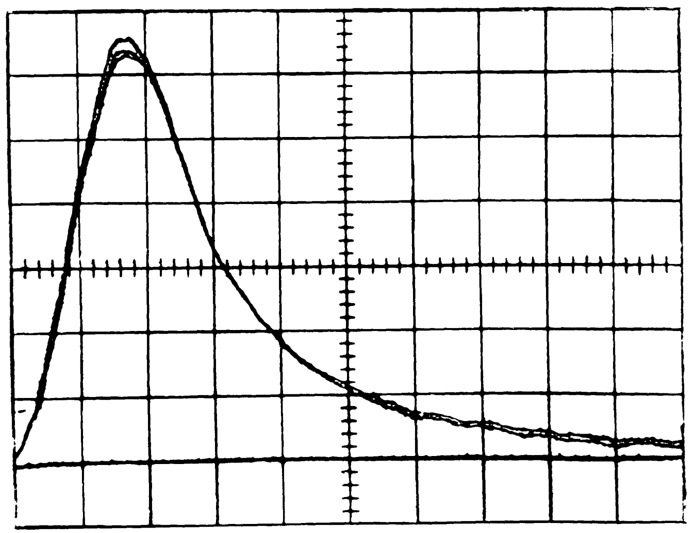
Fig. 6. Working Charts for Calculation of J



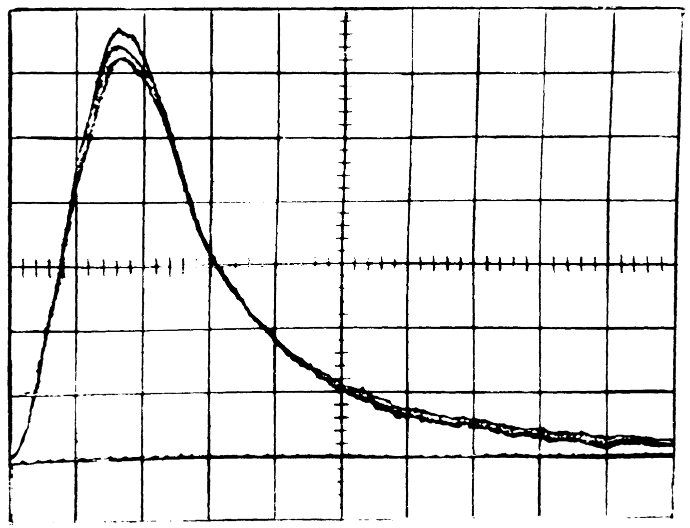
a. 150 gr Nosler Partition Bullet mean=61, 100 psi,
3 rds, 51 gr #3031, .34" seat



b. 150 gr Herter Sonic Bullet mean=61, 100 psi,
3 rds, 51 gr #3031, .34" seat



d. 150 gr US-M2 FMJ mean=63, 000,
3 rds, 51 gr #3031, .34" seat



c. 150 gr Rem. SPCL Ptd. mean=63, 000,
3 rds, 51 gr #3031, .34" seat

Fig. 7. Selected Oscilloscope Negatives with 2 ms Sweep of 3 Rd Firings of Various 150 gr. Bullets in 30-06 Test Rifle, Barrel D with 51 gr. IMR 3031 Powder Lot #1229.

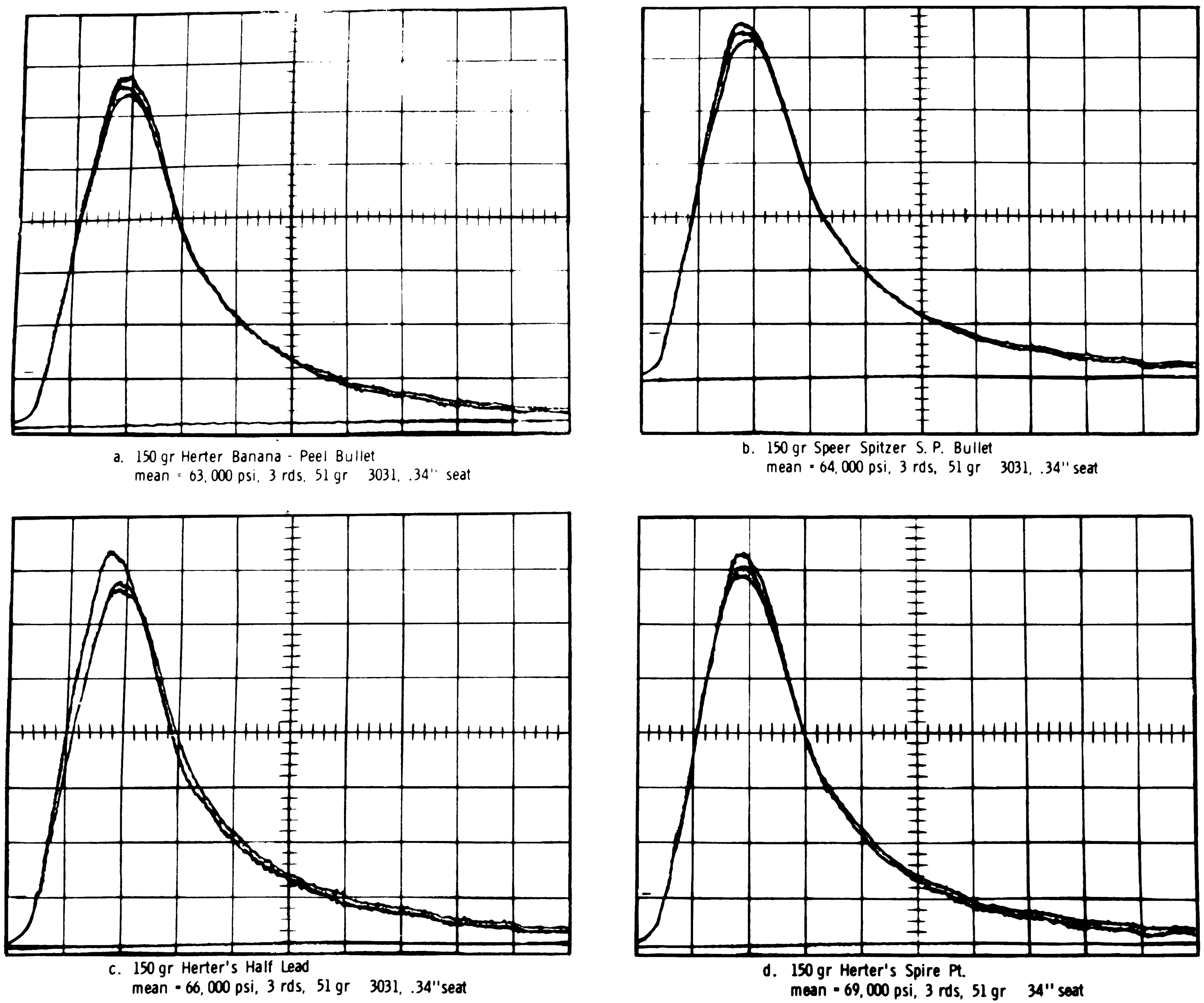


Fig. 8. Selected Oscilloscope Negatives with 2 ms Sweep of 3 Rd Firings of Various 150 gr. Bullets in 30-06 Test Rifle, Barrel D with 51 gr. IMR 3031 Powder Lot #1229.

XI. RIFLE BARREL FACTOR

A comparison of the data of Tables IIIA and IIIB shows that different barrels do not give the same values for J for certain bullets. This is particularly apparent with the 150 g Nosler bullets which had a J of 1.10 in barrel C and 1.17 in barrel D. This can also be observed with Hornady bullets and to a smaller degree with Speer bullets. Remington SPCL bullets gave about the same values for J in all barrels and continued to be used as a reference.

In the tests with barrel C the original calibration load of 44 g of IMR No. 3031 powder and the 220 g Rem SPCL bullets were used. It was found that barrel C, which had been fired in a large number of high pressure tests, had changed its barrel characteristics. It now gave lower pressures than measured previously. The groove diameter at the breech was determined by "slugging" with lead "00" Buck shot and was found to have increased its diameter from .308″ to .310″. The throat bore diameter was checked with a U. S. Ordnance breechbore gage and showed 0.10 units of wear. A new 30-06 military (Springfield) barrel identified as barrel "D" was installed in the test gun for the next series of firings, using bullets of various manufacture.

This brings us to the need for "R," the rifle barrel factor which is defined by Eq. (31) in a manner similar to J. The value of R is:

$$\frac{\text{R (for barrel considered)}}{\text{R (for reference barrel)}} = \frac{\text{p (for barrel considered)}}{\text{p (for reference barrel)}}$$

(with all other factors held constant) (31)

R is of primary importance to the handloader in finding whether his rifle is "typical", and if not, whether his rifle will give higher or lower pressures than those reported in the loading data that he is using as a guide. Many experienced riflemen testing a new rifle with standard ammunition may find that the rifle shoots "hot", possibly blowing primers and giving difficult case extraction, while other rifles apparently identical and of the same caliber and manufacture will give normal pressures with the same ammunition. These "hot" rifles have rifle barrel factors, R significantly greater than unity. On the other hand, either erosion as a result of firing many rounds or deliberate "freeboring" of a barrel will reduce the pressure and this reduces the rifle barrel factor to less than unity. The magnitude of the variation in R from rifle to rifle is rather surprising. We have tested a number of different barrels in 30-06 caliber in which R varied from about 0.6 for worn barrels to 1.3 for new "tight" barrels. This would mean that for R = 1.0 with p = 60,000 psi as a standard of reference, the barrel with R = 0.6 would give a p of only 36,000 psi, and the new tight barrel would give a p of 78,000 psi. To determine J values for commercial bullets and R values for different rifle barrels, a series of firings was conducted in which pressures were measured using the strain gage technique. Some of the test data are shown in Figs. 7 and 8 and are summarized in Table III B.

The next problem was the choice of the most suitable reference for R, the rifle barrel factor. The first procedure considered was to use Michigan military barrel "B" as the reference. This barrel was considered to be a "normal" barrel—neither new nor old, with some, but not excessive wear. However, on comparing our pressure data with those of the DuPont loading tables[15] it became obvious that our barrels were giving significantly lower pressure than the DuPont test barrels in 30-06 caliber. The choice of reference was changed in favor of using R = 1.0 for the loading data published by E. I. du Pont de Nemours and Co., Inc. (15). These data represent the major accumulation of recent ballistic information on rifle pressures available to the handloaders, other than the excellent data provided by NRA (16). Unfortunately, the duPont data tables give no description of the barrel characteristics. The NRA tables do provide data on barrel groove diameter, bore diameter and length. An inquiry to duPont provided the information that, in general, the data for each cartridge were the averge of results from numerous firings sometimes in several barrels. Therefore, as we were unable to characterize the duPont barrels, we concluded that the procedure for the best use of all data was to assign R = 1.0 to the rifle barrels used in the duPont tests.

Figures 9 and 10 show plots of data obtained using the Michigan military barrels B, C and D as well as duPont data and some of Waite's NRA data for the 30-06. Note that in the upper (high pressure) portion of the log-log plot of Fig. 9, the curves for all firings in all barrels are straight lines with the same slope but with different intercepts. Figure 10 shows this region on an expanded plain coordinate scale so that the intercepts can be read more easily.

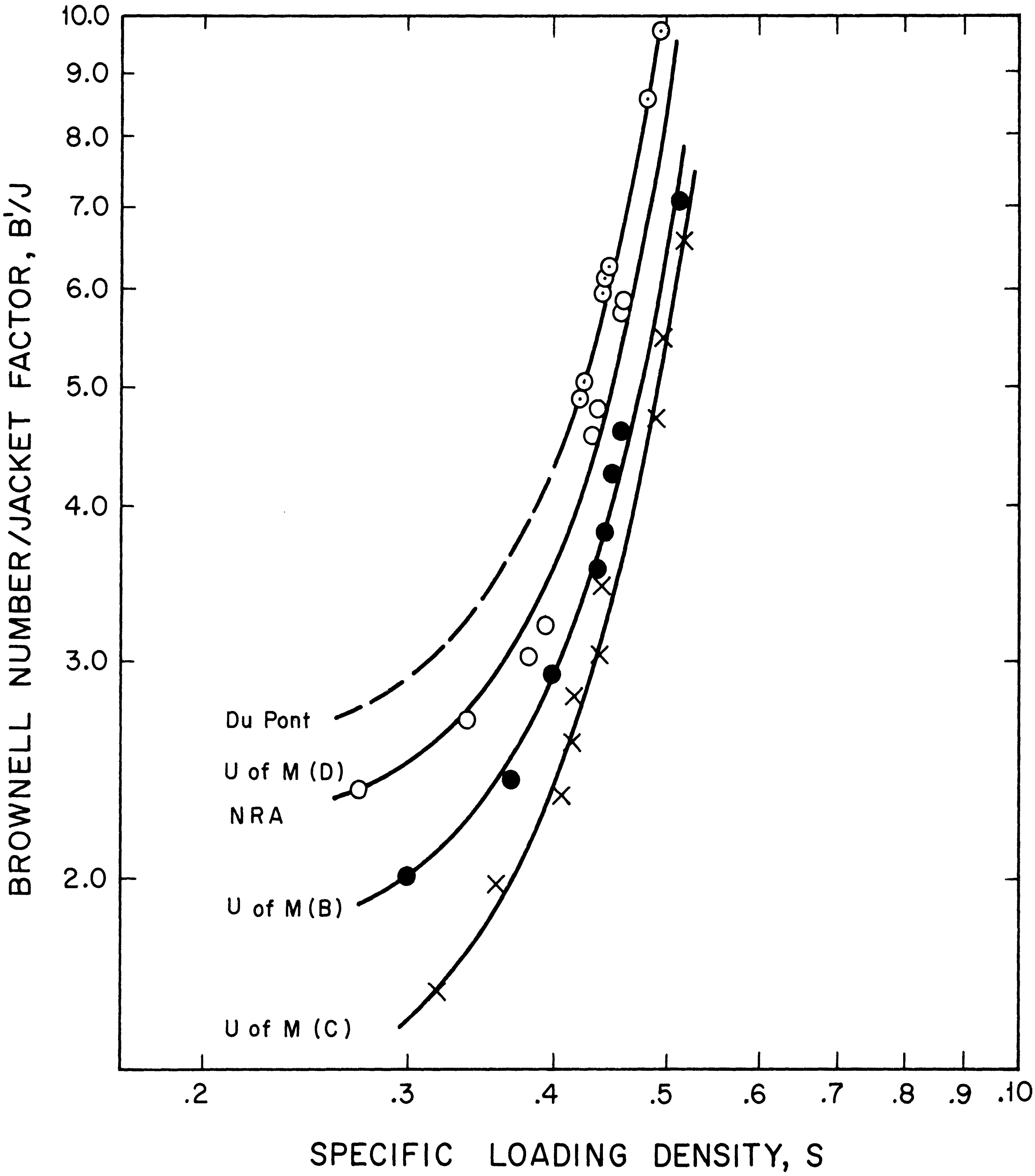


Fig. 9. B'/J versus S for five different barrels.

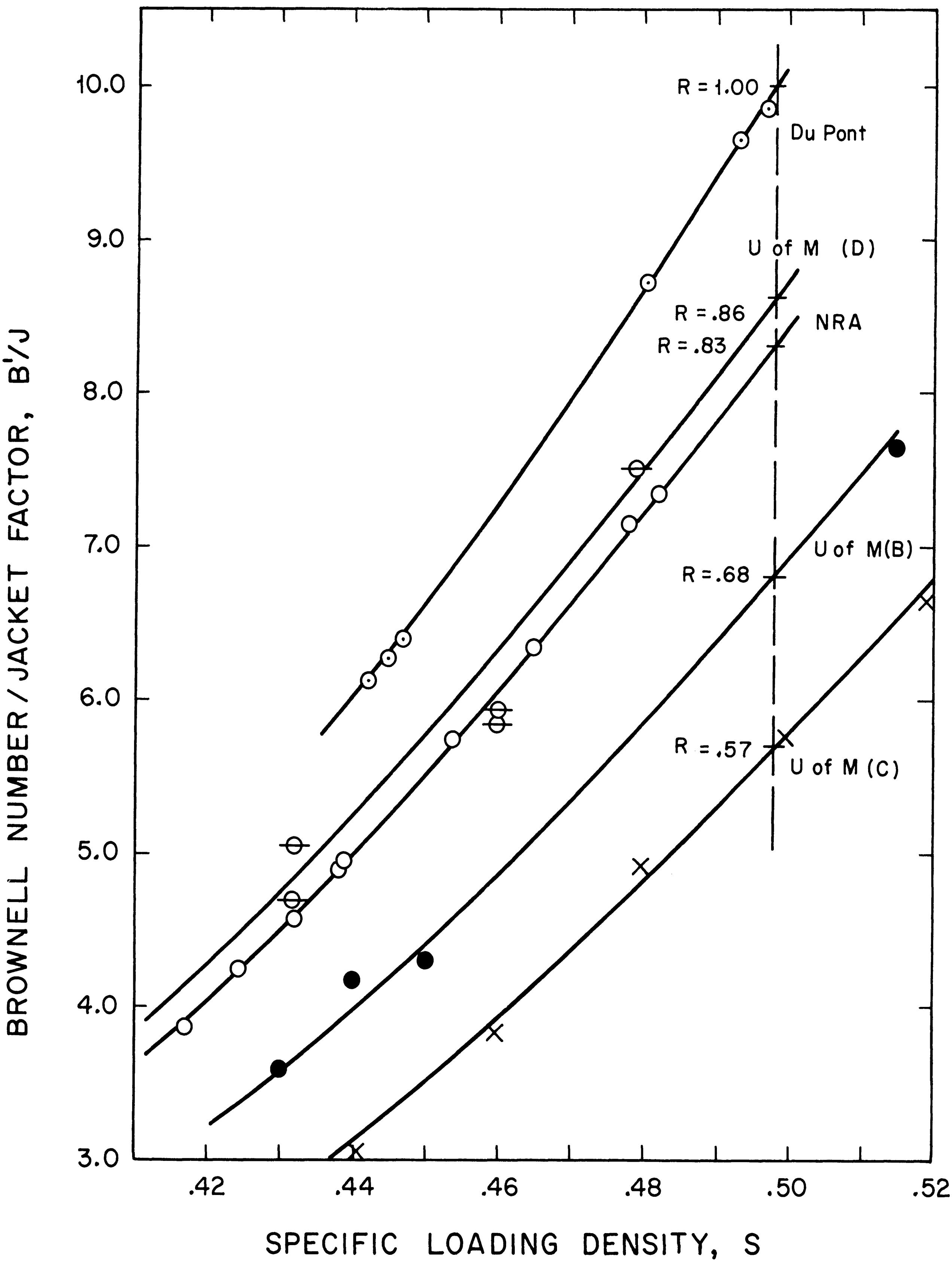


Fig. 10. B'/J versus S for five different barrels.

Table V

DATA USED FOR CORRELATION OF R VALUES FOR

FIVE DIFFERENT BARRELS USING IMR #3031 POWDER

S	B′/J	S	B′/J
	Dupont		
.442	6.12	.480	8.70
.445	6.25	.493	9.65
.447	6.40	.497	9.85
	N R A		
.417	3.85	.454	5.75
.424	4.25	.465	6.35
.432	4.60	.478	7.15
.438	4.90	.482	7.35
.439	4.95		
	Univ. of Mich. (B)		
.430	3.60	.450	4.30
.440	4.15	.515	7.65
	Univ. of Mich. (C)		
.441	3.05	.500	5.75
.460	3.85	.519	6.65
.480	4.95		
	Univ. of Mich. (D)		
.431	4.70	4.60	4.95
.431	5.10	4.79	7.50
.460	5.85		

For convenience in use of the duPont data as reference for R = 1 we can locate the intercepts at the value of S where B′/J is equal to 10.0. Figure 10 shows this to be at S = 0.498 for the duPont data. The other intercepts at the same value of S = to 0.498 are 8.60, 8.30, 6.50 and 5.70 for Michigan barrel D, NRA barrel for tests A, and Michigan barrels B and C respectively. Since these intercepts are directly proportional to p, and p to R, the corresponding values for R for the 5 barrels are obtained by dividing the values of B′/J by 10.0 to give 1.0, .86, .83, .68 and .57 respectively for the five barrels.

With the values of R determined for the different barrels involved, a further study of J the bullet jacket factor, now is possible.

If the bullets are of the same design, material, shape, hardness, yield point, etc., we can predict the value of J from the bullet diameter and sectional density. This relationship between J, sectional density and diameter exists because: (1) the basic characteristic of sectional density is that all bullets with a given sectional density require the same pressure for the same value of acceleration (or deceleration as by air resistance); and (2) diameter, d, is involved because as the size increases the ratio of engagement area to volume decreases. The work of bullet engraving by the rifling and the bullet friction in the barrel is a function of the surface area of engagement between the barrel and the bullet. Thus, as the bullet diameter increases and the ratio of surface area to bullet volume decreases the bullet becomes more efficient and J decreases. This relationship for J is shown in Fig. 11 which plots J versus bullet diameter d in inches with parameters of sectional density. As used here, the sectional density, w/d^2, is the weight of the bullet in pounds (grains/7000) divided by the bullet diameter squared (inches squared). The Speer Reloading Manual lists sectional density for each Speer bullet, and most other handloading books give tables of sectional density for bullets of various weights and caliber.

To determine J, simply enter Fig. 11 on the horizontal scale for diameter and move up to the estimated distance between the curved lines for constant sectional density. After locating the intersection of d and w/d^2, move across to the vertical scale and read J.

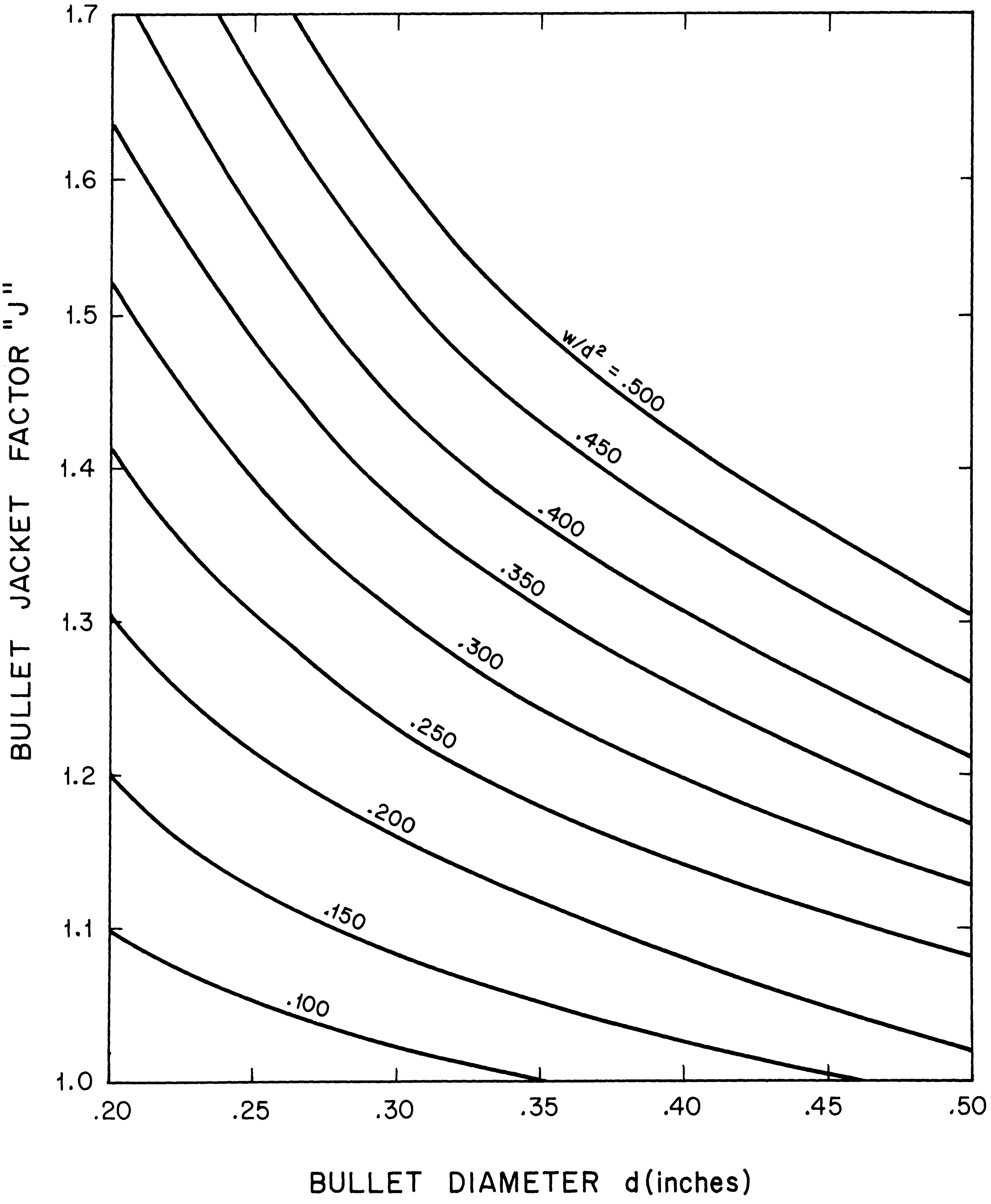


Fig. 11. "J" versus "D" with Parameters of "w/d^2" (pounds/square inch)

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THE APPENDICES

It is hoped that the Appendices of the University of Michigan Technical Reports in internal ballistics will serve as a source of information on ballistic research in the United States and abroad. When Michigan investigators looked for unclassified U.S. literature on ballistics they found it to be almost non-existent. The reasons for this seem to be that 1) the United States Army classifies nearly all of its ballistic research; and that 2) there is no journal in the United States which is suitable for printing technical articles on ballistic research. Therefore, the Appendices are an attempt to alleviate this problem in part by presenting technical articles from the University of Michigan (see Appendix I), pertinent articles not generally available (see Appendix II), and translations of significant articles published abroad (see Appendix III). Readers of the Technical Reports are encouraged to contact us about any articles and/or books they feel ought to be included in future appendices.

A second purpose behind the Appendices is to present data which will enable us to make comparisons between the conventional methods for internal ballistic calculations as used in Europe and the United States with those developed at the University of Michigan. The Michigan methods are based on the principle of dimensional similitude in contrast to the convention methods which involve the use of the energy balance, sometimes referred to as the First Law of Thermodynamics. By comparing the calculated results obtained from both conventional and the Michigan methods with experimental measurements, the Michigan correlation can be evaluated.

Now a word about the Appendices of Technical Report No. 2. Appendix I, ''Ballistic Breakthrough: A New Means of True Pressures,'' reprinted from the Third Anniversary Edition of the Handloader's Digest, was chosen because it includes the only known curves of pressure versus distance of bullet travel in which a curve for absolute pressure is compared with the corresponding curve for crusher pressure. The article also contains a very significant discussion of the two procedures for describing chamber pressure by Dr. Wendall F. Jackson, Associate Director of the Explosives Division, Du Pont Laboratories, Wilmington, Delaware.

Appendix II contains "Chamber Pressures in Small Arms" by Berwyn J. Andrus, a thesis describing his ballistic research and written as part of the Bachelor of Science Degree requirements at the University of Utah. The work by Andrus was done independently ten years before the studies at the University of Michigan began. Both Andrus and Michigan investigators have used the same type of devices for measuring pressure: the strain gage and the oscilloscope. The Michigan work was described in detail in Technical Report No. 1; "Absolute Chamber Pressure in Center-Fire Rifles." Andrus discusses points that were not mentioned in Technical Report 1. Andrus's thesis is well written and represents an important contribution to American internal ballistics literature. It is reprinted here with the special permission of Professor R. L. Sanks and the Library of the University of Utah, Salt Lake City, Utah.

Appendix III is a translation of Chapters 1 through 5 of an excellent and quite recent (1961) East German text on internal ballistics, Innere Ballistik by Professor Waldemor Wolff. Chapter 5 constitutes the major portion of the translation to date and covers the burning of smokeless powders at constant volume. This discussion provides the basis for the introduction of Abel's equation and the various powder constants. The translation of Chapter 6 and Chapter 7 is not quite complete and it is hoped it will be included in Technical Report No. 3.

APPENDIX I

BALLISTIC BREAKTHROUGH

A New Means of Determining True Pressures

by

L. E. Brownell and M. W. York

Department of Chemical and Metallurgical Engineering

The University of Michigan

Ann Arbor, Michigan

Reprinted by special permission from the Third Anniversary Edition of *The Handloader's Digest*, John T. Amber, Editor, 1966.

BALLISTIC BREAKTHROUGH

A New Means of Determining True Pressures

by L. E. Brownell and M. W. York

Absolute chamber pressures have been completely unknown to most reloaders until now! Here is the first popular account of a completely original method for recording true, accurate breech pressures, taken in unaltered rifle barrels—not in pressure barrels. As an added bonus, we can look forward to a reasonably priced, reliable instrument for recording chamber pressures in our own rifles

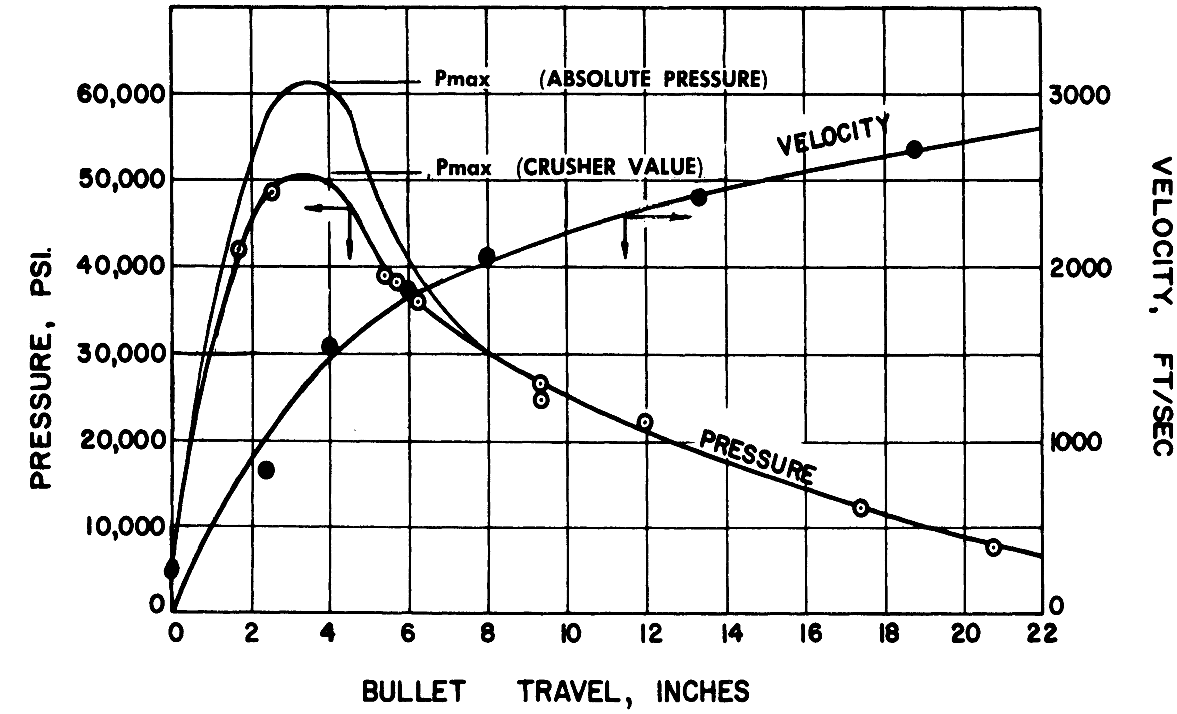


Fig. 1. Composite curves of chamber pressure and bullet velocity vs. bullet travel in inches. Data from several firings of 45.8 grs. Hi Vel 2, 173-gr. FJ bullet in 30-06 rifle.

FOR YEARS many reloaders have relied upon the powder companies and gun magazines for ballistic information. The more experienced handloaders work up new loads by slowly increasing powder charges while closely inspecting fired cases for signs of excessive pressures. Comparatively speaking, modern handloaders and shooters have a greater and more reliable body of ballistic information available than did their counterparts of 40 years ago. Yet a good look at current ballistic information, upon which so many must rely, indicates a need for more and better data if performance and safety factors are to be improved.

Such a conclusion should not be looked upon as a severe criticism of the limits of current ballistic information. Rather, it should be viewed as the logical outgrowth of the advanced technology which is permeating the entire fabric of our lives.

Fig. 4. Ballistic test apparatus used in Du Pont ballistic grant. Note oscilloscope sweep of pressure vs. time and strain gauge connections on barrel. M.W. York is reloading.

Pressure is of primary concern to the handloader and *absolute*, or true, pressure is the subject of this article.

Unfortunately, many handloaders do not have a proper understanding of the importance of rifle chamber pressures. This is not entirely their fault. Many loading handbooks give the reader only a smattering of pressure information. Let us look for a moment at how pressure is produced and review its importance in terms of safety and accuracy.

Production of Pressure

When the firing pin strikes the primer, the primer compound is detonated, producing a spurt of flame that enters the cartridge case through the flash hole in the primer pocket. This ignites the charge of smokeless powder

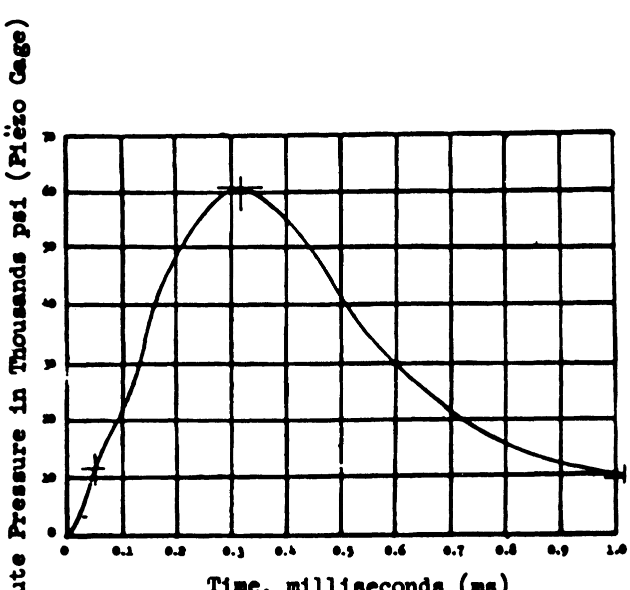
---

L. E. Brownell is Professor of Chemical and Nuclear Engineering and Supervisor, Du Pont Ballistic Grant, University of Michigan, Ann Arbor, Mich.

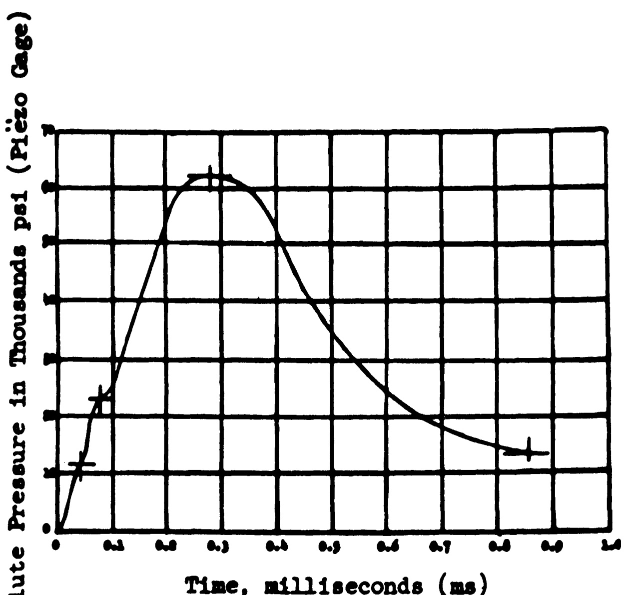
M. W. York is an engineering consultant, Ypsilanti, Mich., and research assistant, Du Pont Ballistics Grant, University of Michigan.

A grant in aid has been established at the University of Michigan by E. I. du Pont de Nemours & Co. to help continue these studies.

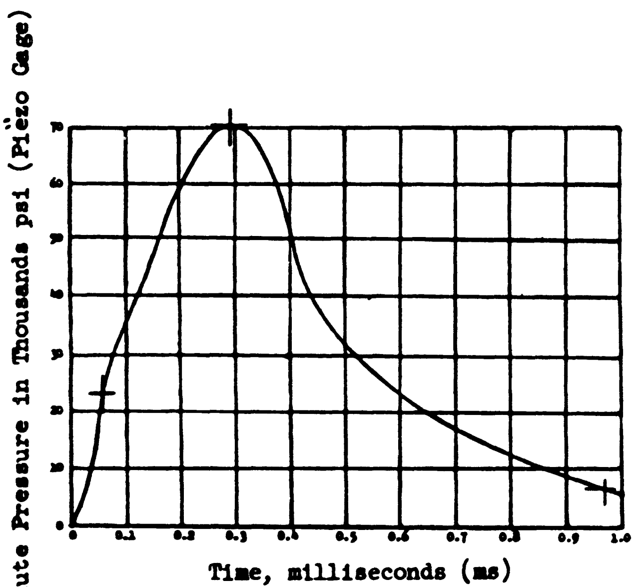
The research described in this article was supported in part by the Michigan Memorial Phoenix Project and the Department of Chemical Engineering at the University of Michigan, Ann Arbor, Michigan.



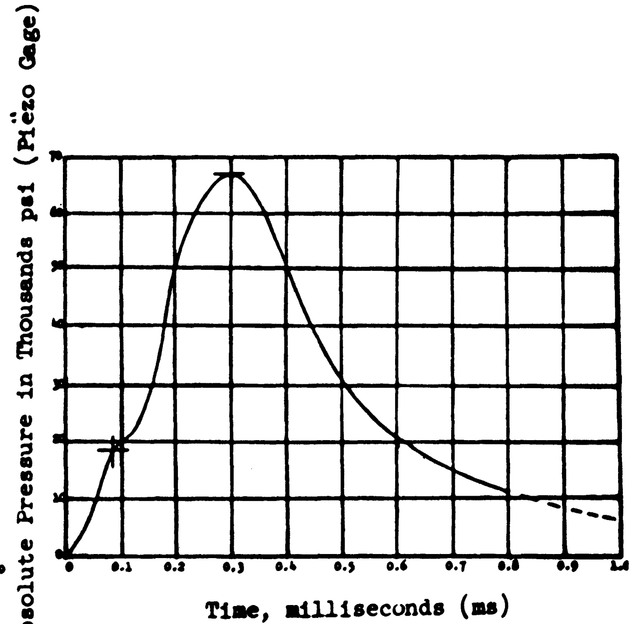
a. Piezo data: 180-gr. boattail bullet, 42 grains IMR 4895 Powder, Pmax = 60,500 psi. MV = 2680 fs. Copper crusher data: MV = 2620 fs, crusher value = 49,700 psi.



b. Piezo data: 110-gr. bullet, 51 grains IMR 4895 Powder, Pmax 63,000 psi, MV = 3370 fs. Copper crusher data: MV = 3320 fs. Crusher value = 50,700 psi.

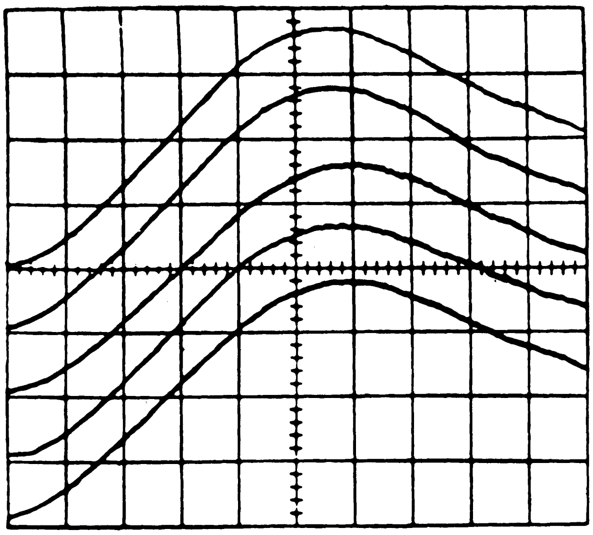


c. Piezo data: 180-gr. metal cased bullet, 35 grains IMR 4198 powder, Pmax = 70,500 psi, MV = 2620 fs. Copper crusher data: MV = 2540 fs, Crusher value = 60,300 psi.

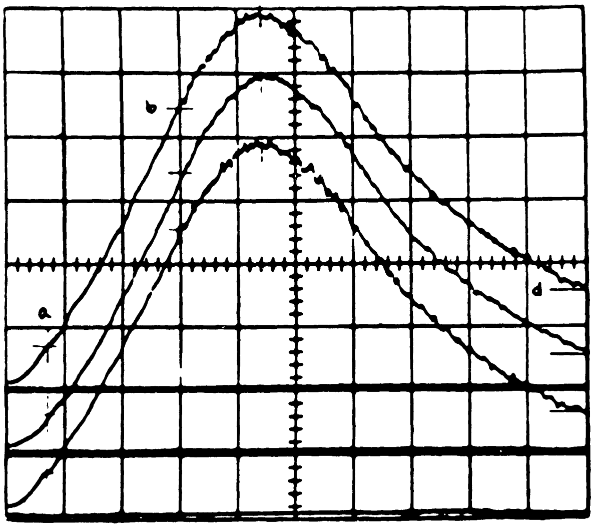


d. Piezo data: 110-gr. bullet, 41.5 grains IMR 4198 powder, Pmax = 67,000 psi, MV = .3310 fs. Copper crusher data: MV = 3300 fs, crusher value = 57,100 psi.

Fig. 2. Piezometric measurements of absolute pressure vs. time for 308 cartridge and 22" rifle barrel. (Reproduced by special permission of Du Pont Co.)



a. Five replicate firings of 43 grains Hi-Vel 2 powder, 180-gr. Sierra bullet with new Remington 30-06 cases and Remington primers.



b. Three replicate firings of 53 grains IMR 4064 powder, 150-gr. Remington SPCL bullet with new Remington cases and Remington primers.

Fig. 3. Negatives of displays on Model 564 oscilloscope showing excellent duplication of replicate firings of maximum charges.

in the cartridge case. The gases produced by the burning powder are contained in the case except for a small leakage after the case neck expands to free the bullet and before the bullet begins to move down the barrel. This confinement of the initial gas produced, plus the generation of heat from the burning powder, tends to produce a rapid rise in pressure before the bullet moves very far. However, the bullet gains velocity as it travels down the barrel and the total volume available to the confined powder gases (case capacity plus bore capacity) increases. This tends to decrease pressure. As a result of these opposing effects, chamber pressure usually reaches a maximum after a few inches of bullet travel and then begins to drop as the bullet advances. Fig. 1 illustrates the manner in which chamber pressure varies as the bullet moves within the bore. Fig. 1 shows curves obtained during tests at the University of Michigan for data on pressure and bullet velocity versus distance of bullet travel.[1] The curves are typical for a load of 45.8 grains of Hi-Vel 2 powder used with a 173-gr. GI bullet fired in a 1903-A3 Springfield 30-06 rifle. Chamber pressure is about 5000 pounds per square inch (psi) when the bullet starts to move and reaches a maximum after about 3 inches of bullet travel. Thereafter, pressure decreases even though only a portion of the powder has burned. This occurs because of the rapid increase in space behind the bullet which results in a reduction in chamber pressure even though some powder still is burning. In most cases of cor-

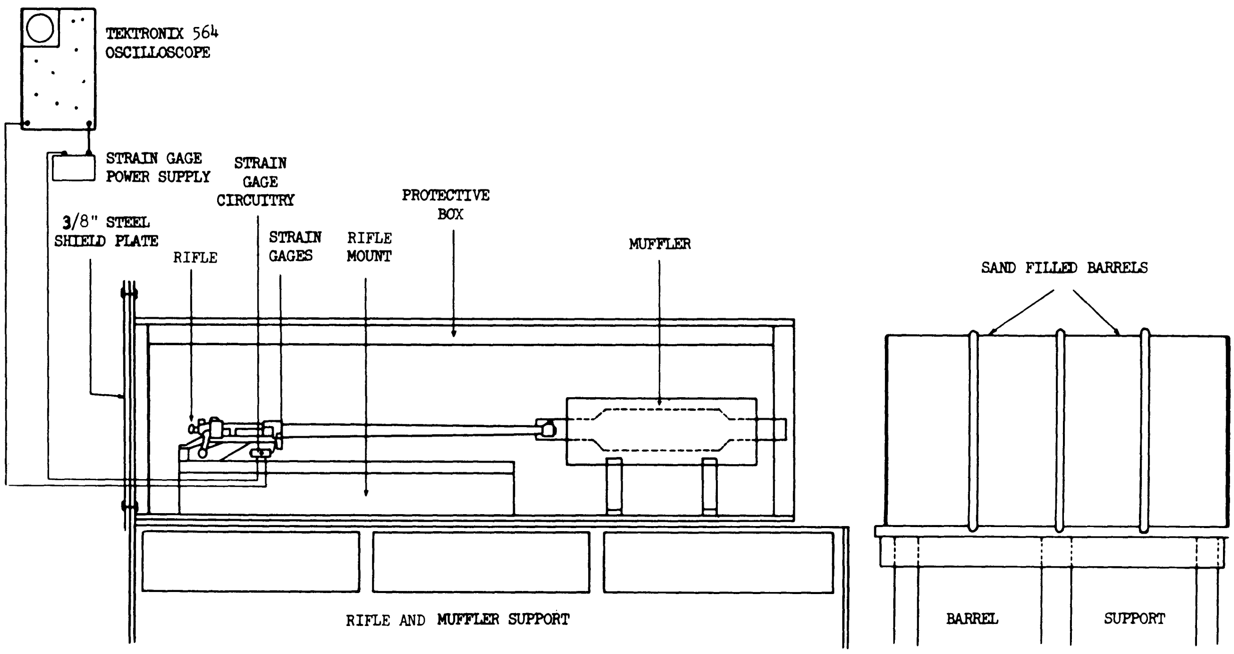


Fig. 5. Ballistic test apparatus as modified, August, 1964.

rect loading, all or nearly all of the powder is burned before the bullet reaches the muzzle. In the case of a fast-burning powder, all the powder is burned in the bore and the pressure decreases even more rapidly, because after "burn-up" no additional powder gases are produced.

The Importance of Pressure

When accidents occur at the breech end of a gun, the cause can often be traced to excessive chamber pressure in the cartridge and gun involved. Many modern high-powered bolt-action rifles operate with chamber pressures (crusher values) of approximately 50,000 psi. This represents a tremendous force which, if released in the breech outside the cartridge chamber, can disintegrate the receiver, magazine and surrounding stock. This not only wrecks the gun but can seriously injure or kill the shooter.

Accidents can also result from excessive head space of rimless cases which leaves the cartridge case without sufficient support for a chamber pressure that might be safe in a mechanically correct gun. Oil or grease in the chamber can reduce the friction between the case and chamber walls so that the pressure puts a greater part of its force on the bolt. This can cause a failure in some of the weaker types of actions. Plugged bores, excessive powder charges, or the use of the wrong type of powder can induce failures. Regardless of the conditions that result in failures, it is the chamber pressure that produces the force of destruction. Thus, whether we are designing the gun, testing it, or shooting it we must always be concerned with maximum chamber pressure. Safety depends upon keeping this maximum chamber pressure within safe limits.

In many instances adequate chamber pressure information is unavailable to the handloader. In such cases experienced handloaders feel their way through this darkness (created by the lack of pressure knowledge) by studying the appearance of primers and cases after firing. Another sign, difficult removal of cases, may indicate excessive pressure and a dangerous load. Although these signs are better than no information, they are no substitute for measured pressures. They give only *indications* of excessive pressure and cannot be relied upon in all cases to protect the shooter against dangerous loads.[2]

Muzzle velocities are, of course, influenced by chamber pressures. However, the influence is not as great as one might expect. For instance, we might ask how we can obtain accuracy with loads in which the maximum pressure can vary 10 per cent or more from shot to shot using the same gun and the same loads? Fortunately, the muzzle velocity of the bullet depends primarily upon the energy transferred from the powder gases to the bullet rather than upon the shape of the pressure-time curve. If the same powder charge is used and all the powder is burned in the barrel, the same amount of chemical energy will be released by the powder. The portion of this chemical energy that is transferred as kinetic energy to the bullet remains fairly constant for a given

Breech Pressure Device In the Works

In addition to ballistics research supported by E. I. du Pont de Nemours and Co., Inc., Mr. York and another engineer, Mr. Walter Hackler, are currently working on a portable rifle breech pressure measurement instrument that can be used without requiring any modification of the rifle. This measurement system is being specifically designed to bring the benefits of a modern ballistic laboratory to the serious shooter and handloader at a reasonable cost — perhaps as low as a few hundred dollars. Inquiries should be addressed to Mr. M.W. York, 547 Ivanhoe, Ypsilanti, Mich.

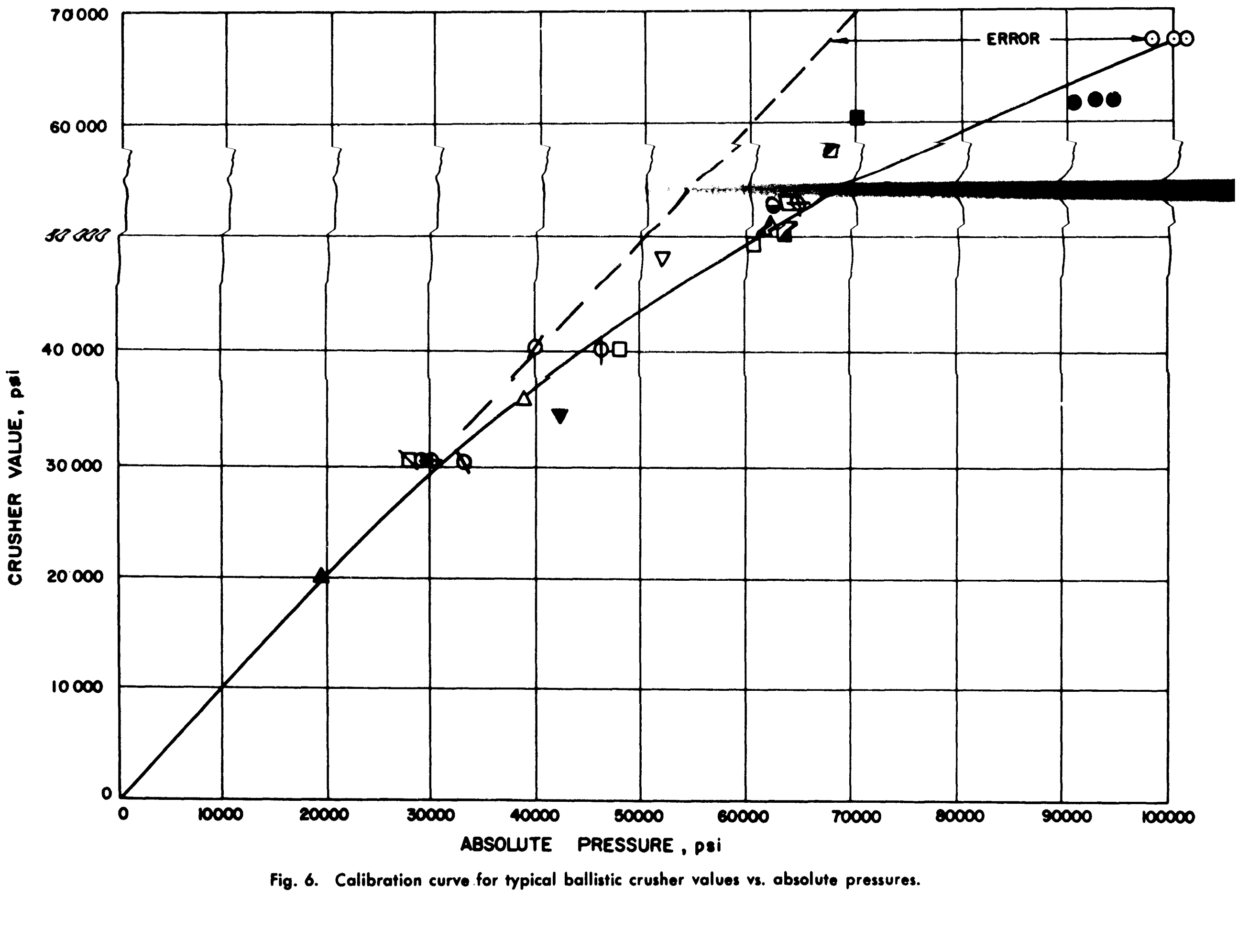


Fig. 6. Calibration curve for typical ballistic crusher values vs. absolute pressures.

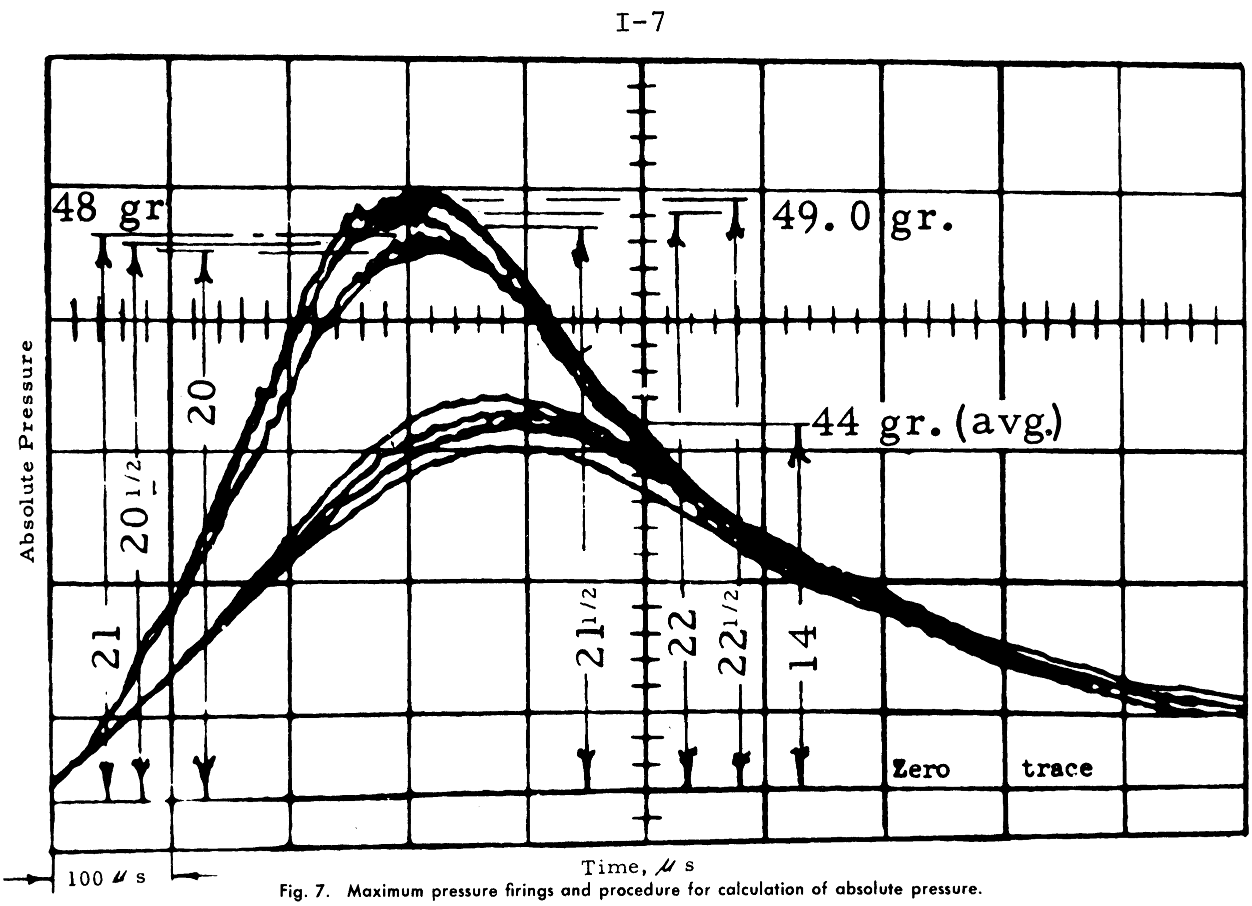


Fig. 7. Maximum pressure firings and procedure for calculation of absolute pressure.

length of barrel and chamber size. Therefore, the muzzle velocity for a given rifle bullet and powder type will depend primarily upon the weight of powder used.

The points mentioned above have been verified by test loads. Twelve loads using Hi-Vel 2 and additional tests with other powders in the 30-06 cartridge were reported in the *American Rifleman*.[3,4] In these tests bullets from various suppliers weighing 110, 125, 150, 180 and 220 grains were used. All were tested in the same barrel using cases and primers of one manufacturer. The mean 10-round dispersion was 66 feet per second or about 2 per cent in velocity, but 5000 psi or about 11 per cent in pressure.

Handloading Tables vs. Ballistic Correlations

Tables in handloading handbooks, with or without data on maximum pressures, have been used for half a century as a means of determining suitable powder charges. This procedure is simple to use but prevents correlation of the many factors involved. Differences in individual primers, in primer manufacturer, in diameter of the primer flash hole, in chamber volume, in bullet characteristics, in bullet seating depth, etc., can influence maximum pressure. If we express the maximum pressure for various cartridges by a single equation, we can apply correction factors for these variables. But as long as we continue to use handloading tables with hundreds of different loads we must use a separate correction factor for every variable with every load. We end up with the impossible situation of thousands of correction factors.

A major reason for the fluctuations in maximum pressure is the variation in the distance the bullet travels before the pressure peaks. For example, if the powder burns rapidly because of a "hot" primer or large primer hole, the pressure will peak before the bullet travels as far as it normally would. This produces a pressure higher than normal. With a "cold" primer or small primer hole, the powder will burn more slowly, the bullet will travel farther before the pressure peaks, producing a lower maximum pressure. Also, if the bore is worn or the bullet offers only slight resistance to engraving by the rifling, the resistance to bullet travel will be less and the travel will be greater before the pressure peaks.

The combined effects of the many variables influencing the maximum pressure can be taken into account by determining the influence of each variable on the distance of bullet travel to the pressure peak. From this the effect on the maximum pressure can be predicted. Thus, only by analysis and correlation of the data and by the use of ballistic equations rather than loading tables can we hope to make significant progress in the predictions of maximum pressures. The Ballistic Research Laboratories at Aberdeen have developed useful ballistic equations for small arms, but these rather complex equations are not suitable for use by the handloader.[5]

Although tables of maximum and moderate loads without pressures may serve the needs of most handloaders, these tables are a dead-end street so far as improvement in ballistic knowledge is concerned; the data cannot be used in any realistic evaluation of factors influencing maximum pressures. Only by knowing the *true* values of the maximum pressures can we be sure of our factor of safety.

Limitations of Ballistic Methods Based on Crusher Values

Ballistic information currently reported and available to the handloader is, in nearly all instances, based on data obtained by the use of the rifle "crusher" gauge. These data are obtained by firing cartridges in special test rifles designed to accommodate this gauge (See Fig. 8.) This type of measurement suffers from disadvantages and inherent inaccuracies*

The crusher gauge has a long history. It was developed and first used in black powder days to provide a means of indicating the magnitude of the maximum chamber pressure in firearms. For handloading purposes, pressures indicated by the crusher gauge procedure are sufficiently accurate to provide information on the

*See comment by Dr. W. F. Jackson, Associate Director, Explosives Division, Du Pont Laboratories, accompanying this article.

combinations of loading components that give safe loads. This is essentially the basis for the loading information in the *NRA Illustrated Reloading Handbook*.[4] However, data based on the crusher gauge technique are not considered sufficiently accurate for use in ballistic correlations.

The crusher gauge and its use are well-described in Norma's *Gunbug's Guide*.[6] The limitations of crusher measurements are mentioned briefly. The copper crusher is a rather crude device which is used to determine plastic deformation above the elastic limit of the copper used, and therefore is a measure of the work expended during that deformation. Copper crushers are not absolutely uniform in their physical properties, and some error is introduced by variations in yield point and cold working characteristics beyond the yield point. Physical properties such as yield point and modulus of elasticity also are temperature-dependent. For this reason measurement and control of temperature during firing is important in tests made with copper crushers. As the copper cylinder is compressed the copper becomes cold worked and the cross sectional area is increased. Thus, the measured deformation by compression is not directly proportional to the force applied by the chamber pressure. Frictional resistance of the steel piston used to transfer the force of the gas to the crusher absorbs energy. This is an added source of error. All of these factors decrease the accuracy of the crusher gauge.

The crusher gauge technique is based on the plastic compression of a cylinder of soft metal by the pressure in the chamber of the rifle. The change in the thickness of the crusher is then used as an indication of the pressure to which the gauge was exposed. The amount of deformation of the crusher for each pound of pressure is greater when the chamber pressure is relatively low than it is as the chamber pressures rise to higher values. Therefore, we find that almost universally *the reported crusher-gauge pressures for high pressure loads are lower than actual true pressures*. This is indeed unfortunate since the shooter should be more interested in obtaining true chamber pressures for high pressure loads than he should for the lower pressure loads.

Other Methods of Measuring Chamber Pressure

Two other techniques, the strain gauge and the pïezometric pressure gauge, have been used to measure chamber pressures with greater accuracy than with the crusher gauge. We have been informed that many powder manufacturers now are using "pïezo" gauges to obtain better pressure data. The use of strain gauges appears to be less extensive. Dr. E.L. Eichhorn, ballistician,[7] states that strain gauges are used by RWS Genschow, Nürnberg, Germany; Svenska P.F., Stock-

No.	Cartridge	Powder	Charge (gr 1)	Bullet	Crusher Reading psi	Source of Crusher Data	Pressure by Strain Gage, psi	Source of Strain Gage Data	Muzzle Vel. ft/sec	Pressure by Piezo Gage, psi	Source of Piezo Gage Data
1	308W	4895	42.0	180gr boat tail	49,700	E.I. DuPont*	-------	-------	2650	60,580	E.I. DuPont*(Fig. Z)
2	308W	4198	35.0	180gr boat tail	60,800	E.I. DuPont*	-------	-------	2580	70,000	E.I. DuPont*(Fig. Z)
3	308W	4895	51.0	110gr spitzer	50,700	E.I. DuPont*	-------	-------	3345	63,400	E.I. DuPont*(Fig. Z)
4	308W	4198	41.5	110gr spitzer	57,100	E.I. DuPont*	-------	-------	3305	66,860	E.I. DuPont*(Fig. Z)
5	30-06	3031 Lot 1224	44.0	220gr SPCL	52,700	E.I. DuPont**	62,000	U. of M.	2352	63,600	Du Pont
6	30-06	3031 Lot 1224	37.0	220gr SPCL	40,700	E.I. DuPont**	40,000	U. of M.	2078	48,000	Du Pont
7	30-06	3031 Lot 1224	29.0	220gr SPCL	30,400	E.I. DuPont**	33,000	U. of M.	1681	28,000	Du Pont
8	30-06	3031 Lot 1224	48.0	220gr SPCL	61,900	E.I. DuPont**	90,800 93,100 95,300			90,800	E.I. DuPont E.I. DuPont E.I. DuPont
9	30-06	3031 Lot 1224	49.0	220gr SPCL	67,400	E.I. DuPont**	97,700 100,000 102,300				E.I. DuPont E.I. DuPont E.I. DuPont
10	30-06	3031 Lot 1230	44.0	220gr SPCL	(52,700)	taken as for Lot 1224	64,000	U. of M.	----	(63,600)	------------
11	30-06	3031 Lot 1230	37.0	220gr SPCL	(40,700)	taken as for Lot 1224	40,000	U. of M.	----	------	------------
12	30-06	3031 Lot 1230	29.0	220gr SPCL	(30,700)	taken as for Lot 1224	29,000	U. of M.	----	(28,000)	------------
13	30-06	3031 Lot 194	37.0	220gr SPCL	(40,700)	taken as for Lot 1224	46,300	U. of M.	----	(48,000)	------------
14	30-06	3031 Lot 194	29.0	220gr SPCL	(30,700)	taken as for Lot 1224	30,000	U. of M.	----	------	------------
15	30-06	HiVel #2	23.0 (cor.primer)	220gr	20,000	P.Sharpe***	19,600	U. of M.	1400	------	------------
16	30-06	HiVel #2	38.0	220gr	36,000	P.Sharpe***	39,000	U. of M.	2140	------	------------
17	30-06	HiVel #2	43.6	220gr	51,000	P.Sharpe***	61,500	U. of M.	2315	------	------------
18	30-06	HiVel #2	37.0	180gr Sierra	34,150	N.R.A.+	42,600	U. of M.	2297	------	------------
19	30-06	HiVel #2	41.0	220gr SPCL	48,100	N.R.A.+	51,700	U. of M.	2357	------	------------

*Reproduced by special permission. (Data first brought to attention by Dr. Edgar L. Eichhorn, Ballistician)(9)
**Personal communication.
***Reproduced by special permission courtesy of Funk and Wagnalls Co., New York, N.Y.(11)
+The following credit line has been requested for special permission to reprint data on 30-06 loads "This material is reprinted from THE AMERICAN RIFLEMAN, official journal of the National Rifle Association of America, 1600 Rhode Island Avenue, N.W., Washington 6, D.C. THE AMERICAN RIFLEMAN goes monthly to the more than one-half of a million NRA members. Membership is available to citizens of good repute."

Table I — Calibration data for crusher readings vs. absolute chamber pressure.

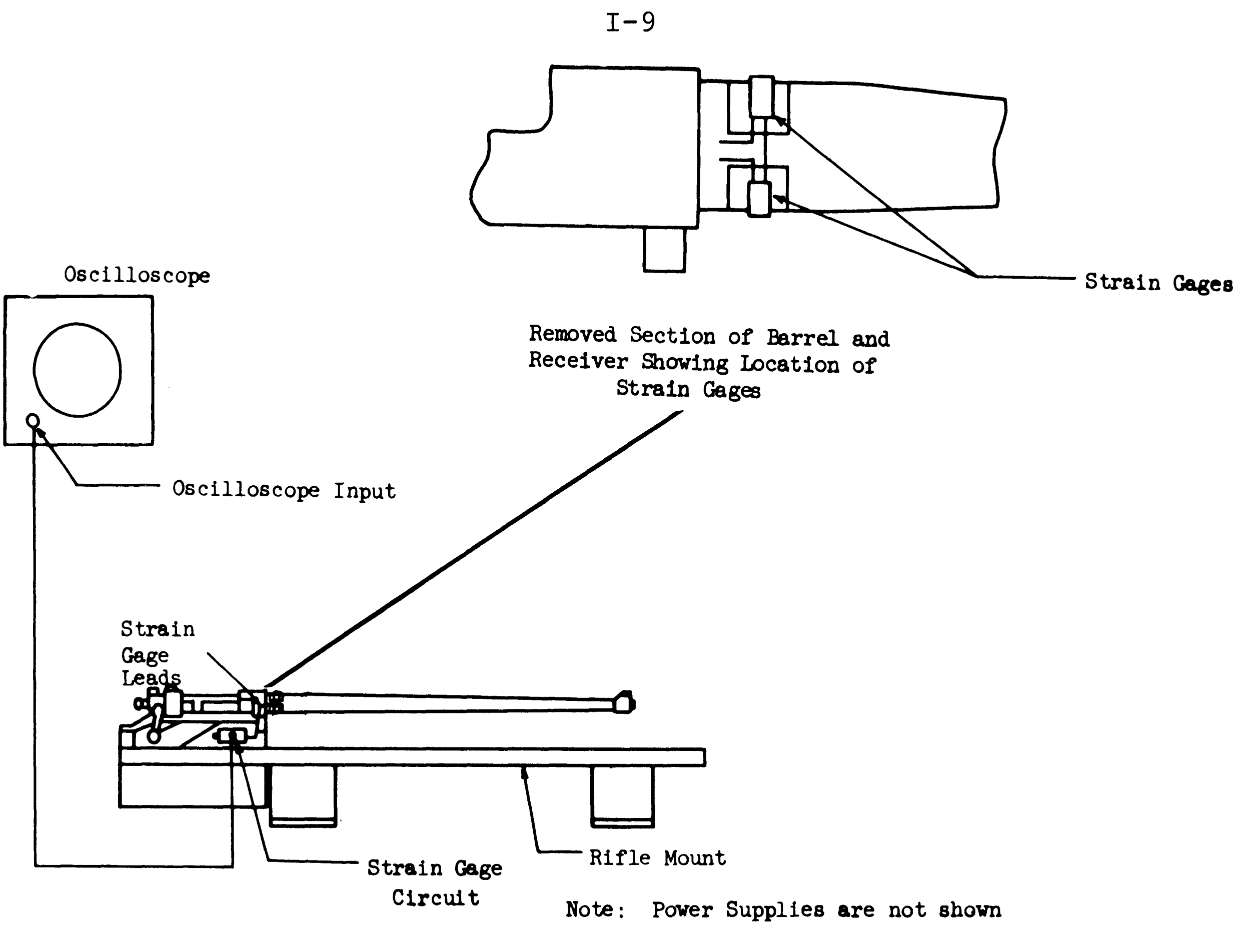


Diagram of pressure measuring system.

holm, Sweden; Norma, Amotfors, Sweden; Hembrug, Ijmuiden, Holland; and CCI, Lewiston, Idaho, and we know they are used by Detroit Testing Laboratory, Detroit, Mich.

Both the strain gauge and the piëzo gauge can be used in conjunction with an oscilloscope to give a pressure vs. time curve while the bullet is in the barrel. Using correct instrumentation, both systems can be made to produce a response that is linear with respect to pressure. In the case of the strain gauge the following relationship holds:

$$e = \frac{p}{E}\left[\frac{d_i^2(2-\mu)}{d_o^2-d_i^2}\right] = kp$$

The derivation of the equation above is given in Technical Report #1 of the DuPont Ballistic Grant, *Absolute Chamber Pressure in Center-Fire Rifles.*[8] It is based on Eq. 14.18 of Reference 9. For a given gun the value of Poisson's ratio μ, modulus of elasticity E, barrel outside diameter d_o, and chamber diameter d_i are constants. Therefore, the unit strain e will be directly proportional to the chamber pressure, p. If the strain gauge signal is directly proportional to unit strain e, the voltage change in the circuit will be linear with respect to pressure. We should point out that the modulus of elasticity, E, (sometimes called Young's modulus), is constant for a given material at a given temperature. Hooke's law states that stress *is directly proportional to unit strain* and definitely is applicable if the stress in the barrel does not exceed the elastic limit. If the elastic limit is exceeded, we will have a plastically and permanently deformed (bulged) barrel. Such conditions of plastic deformation exist only in destructive testing and are not applicable to the studies in the present tests.

At chamber pressures between 20,000 and 40,000 psi fair agreement is possible using the crusher technique and either the piëzo or strain gauge. At pressures of 50,000 psi and greater, the crusher does not give true absolute pressure. The magnitude of this difference is not known by most handloaders and gunsmiths. A handloader may be firing a cartridge with a charge of 42 grains of IMR 4895 powder and a 180-gr. bullet in a 308 Win. cartridge. This load gives a pressure of 49,700 psi as determined by crusher measurement. The true pressure for the identical load measured by piëzo gauges is 60,580 psi.[7,10] Some reloading handbooks list even higher charges than 42 grains for this powder and cartridge. Fig. 2 shows piëzo gauge data reproduced by special permission of Du Pont."

A New Approach to Ballistic Measurements

The concept that absolute pressures over 55,000 psi are unsafe should be dismissed. Commercial rimless cartridges for modern rifles are consist*ently loaded with charges that produce* actual absolute chamber pressures in excess of 60,000 psi. Belted cartridge cases for magnum loads are consistently loaded to produce actual absolute pressures in excess of 70,000 psi. *However, it is most important that the absolute chamber pressure and the values from crusher tests not be confused.* The possibility that crusher values might be confused with real pressures may have been one reason for continuation of the use of crusher values by the powder companies. In research at the University of Michigan the intent is to work with fundamentals and facts. References to pressure are to the true absolute pressure rather than the crusher values when the term pressure is used.

Because of the inherent inaccuracies associated with the crusher gauge technique of pressure measurement, a more accurate means of obtaining true chamber pressure measurements is being utilized for all ballistic research at the University of Michigan. The University method incorporates the use of strain gauges attached around the circumference of the chamber portion of the rifle barrel to obtain the chamber pressure in the rifle. The strain gauge and associated electric circuit provides a voltage signal that has a direct relationship to the pres*sure within the rifle chamber. Further*more, the change in voltage per pound of chamber pressure is essentially constant over a wide range of chamber pressures. Therefore, accurate chamber pressure measurements are possible at all pressure levels. This strain gauge and current are used with an oscilloscope and camera to record the voltage change from the strain gauge and hence the rifle chamber pressure. The oscilloscope records voltage as a function of time so that the use of the oscilloscope provides the added advantage of being able to determine the chamber pressure at any given time after the cartridge has been fired. Fig. 3 illustrates the type of picture obtained with this strain gauge technique with 2 different powders and replicate firing.

Another distinct advantage of the strain gauge is that it can be readily used in any sporting or military rifle without having to modify the rifle in any manner. This is a definite advantage compared to the crusher gauge technique, which requires a special test rifle or at least the permanent modification of the rifle that is to be tested. The strain gauge measurement technique is also relatively inexpensive in terms of the equipment involved. Cost of the strain gauges and associated circuit is less than $30. However, in the tests being conducted at the University of Michigan an oscilloscope is used to record the voltage output from the strain gauge circuit and this is a relatively expensive piece of equipment. Nevertheless, for the ordinary rifleman, use of a recording system and a strain gauge circuit can be constructed for less than the cost of a moderately priced chronograph currently being used by many shooters.

Fig. 4 shows a photograph of the test apparatus with Mike York, the first Du Pont fellow, loading a test rifle. A diagram of the test apparatus, as modified in August, 1964, is shown in Fig. 5. At the right side of Fig. 5, two sand barrels are indicated. They are used as a bullet trap and a shield for personnel working on the project. The plywood box surrounding the rifle and muffler effects some sound control and also acts as a personnel shield against minor fragments of bullets, etc. The 3/8" steel plate directly behind the bolt protects the immediate working personnel against a ruptured gun or ricochetting bullets. The rifle mount is constructed of steel. It receives the full recoil via the recoil lug and serves as a free floating stock in which no pressure is exerted on the barrel. For simplicity, the gun is triggered mechanically by a pull cord through the shield .A storage oscilloscope is used to record the strain gauge signals. This oscilloscope has the advantage of storing any trace so that data can be recorded or discarded without wasting camera film.

A simple strain gauge circuitry is used and involves a resistance in series with the strain gauge. After detonation of the primer, the pressure in the chamber begins to rise and the strain gauge voltage changes. A small voltage change triggers the scope and its trace displays the pressure in the chamber as a function of time.

Table 1 reports some of the absolute pressure measurements that have been obtained at the University of Michigan and data from various sources used to prepare a general relationship between absolute pressure and crusher values. The data in Table 1 are plotted in Fig. 6. It should be noted that different designs of crusher gauge apparatus give crusher values that vary somewhat for the same loads. Also, instruments for measuring absolute pressure, including piëzo and strain gauge, require calibration. Differences in calibration procedures will result in inconsistancies in evaluation of the same pressure. Thus, the curves shown in Fig. 6 have limited application and may not correlate data from some measurements. The calibration is based on the ratio of piëzo gauge pressures and crusher values determined in the Carney's Point Development Laboratory of the Explosives Department of E.I. Du Pont de Nemours and Co., Inc.,[10] and firings of the same loads at the University of Michigan.

Defense of the Crusher Gauge

"I must put in a word in defense of the crusher method — not because of its accuracy, which it does not have, but because it has been a simple indicator used in a reasonably standardized fashion for many years by both commercial and military ammunition testers and developers. We recognized at least 30 years ago that the crusher method would have been better served if "psi" had never been appended to the numbers on a tarage table. In the first place, the tables are prepared by calibrating compression as a function of load on a dead weight press. Even though the copper has been carefully treated during fabrication, and the deformation under static load determined with considerable precision, it is well-known that the same deformation is not encountered under dynamic loading.

"The results of crusher tests in guns might better have been reported as millimeters or mils from the beginning. The fact that increments of compression for a given increase in load are not constant as the load is raised is not surprising in view of both work hardening and increase in diameter as compression proceeds. The tarage table does take care of this. The real discrepancy lies in the difference between defection under dynamic vs. static loading.

"In spite of this situation, the community of gun, ammunition, and powder makers, aware of the fact that they should bite their tongue every time they attach "psi" to a crusher measurement, have found the crusher method a handy day-to-day procedure for powder and ammunition test, fairly reproducible at different stations, and by process of trial and error used it as a guide in gun and ammunition design.

"Conversion to a more scientific method of pressure measurement has met resistance in both industry and the military because of the tremendous backlog of crusher data available and the relative complexity of piezoelectric, magnetostriction or strain gauge equipment needed for the methods that operate within the elastic limits of the pressure sensing elements involved.

"I do not believe the powder companies have really been reluctant to bring this situation to the attention of the handloader. Rather, for many years, both the powder and ammunition companies refrained from discussing with the handloader pressures measured by any method. Only recently, with so many scientists and engineers involved in shooting as a hobby, have the powder companies found it desirable to share their information on pressures with the handloader. 'Pressure' I should put in quotes, because the data so far presented to the handloaders are crusher data.

"Your paper should go a long way toward educating the shooting fraternity in this complex area. I believe, however, that there is still more to be done before any method can be called the last word. Calibration of the strain gauge method by more direct means than those used so far would be desirable. Location of the measuring point along the chamber is another factor that deserves attention.

"All of this I am writing only to suggest that your readers may get too dismal a view of the crusher method from your presentation."

W. F. Jackson

The above letter was written to Mr. Brownell by Mr. W. F. Jackson, Assistant Director, Research and Development Division, E.I. du Pont de Nemours & Co., after reading this article. It is printed here to give a broader view on the subject. — Ed.

Proof Loads

To obtain data in the region of 90,000 to 100,000 psi for a calibration curve of crusher values vs. absolute pressure, two experimental loads were especially prepared and tested with copper crushers at Carney's Point Laboratory. The maximum pressure was selected to simulate the proof loads used to test U.S. Government rifles. In these tests a proof load giving a crusher value of 67,000 psi has been used for years. Because of the very high pressure, pre-deformed crushers are used in the government test. However, since we did not wish to introduce the additional factor of cold working of the crushers, simple copper crushers were used in all the Du Pont measurements.

Du Pont reported a crusher value of 61,900 psi using 48 grains of IMR 3031, Lot 1224, a 220-gr. Remington Core-Lokt bullet and Remington cases and primers. Another load, identical except that 49 grains of powder was used, gave a crusher value of 67,400 psi.

(**Warning:** *These are proof loads and should* never *be fired by an individual holding the gun. A proof-testing rack with suitable protection for the operators should* always *be used with tests of proof loads.)*

Du Pont did not measure pressures with piëzo apparatus because the upper limit for this equipment is about 70,000 psi absolute pressure. There is no such limit for strain gauges.

Strain gauge results for these loads gave, as an average of three shots with each, 93,066 psi with the 48-gr. load and 100,000 psi with the 49-gr. load. These results, obtained at the University of Michigan, are shown in Fig. 7. The data are listed in Table 1 and are plotted in Fig. 6. The four lowest traces of Fig. 7 show four calibration firings using 44 grains of IMR 3031 Lot 1224, and the 220-gr. SPCL bullet load. This load was found by Du Pont to produce a crusher value of 52,700 psi and a piëzo-gauge value of 63,600 psi absolute pressure. The lowest of the four curves for this loading was fired first in a comparatively cold rifle and produced an oscilloscope peak of only 13 vertical units. Then three loads of 48 grains of IMR 3031, Lot 1224, were fired to produce three nearly replicate curves with peaks of 20, 20½ and 21 oscilloscope units above the zero-voltage reference line. The headspacing was checked during firings and was found to be beyond the No-Go but less than the Field gauge limit and the tests were continued. Then three maximum loads of 49 grains of IMR 3031, Lot 1224, were fired to give three similar curves peaking at 21½, 22 and 22½ oscilloscope units respectively. Next, three additional calibration loads were fired and these firings were 1, 1¼ and 1¾ units higher than the original firings. A value of 14 units was taken as the average for the calibration firings.

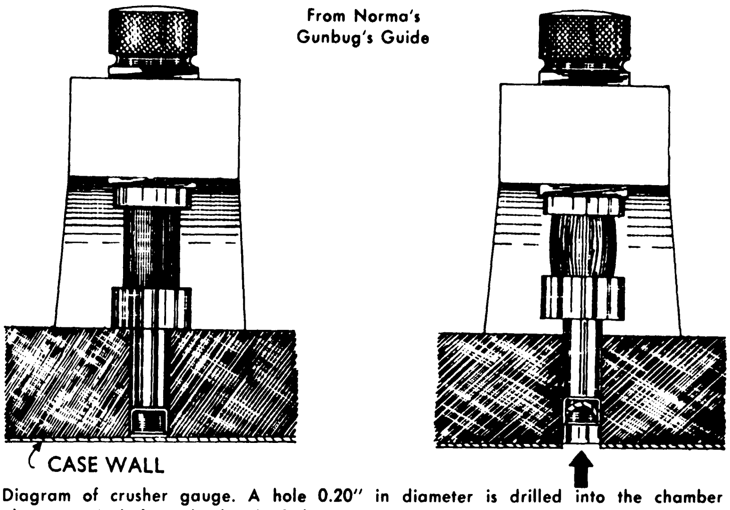


Diagram of crusher gauge. A hole 0.20" in diameter is drilled into the chamber about one inch from the head of the case at a right angle to the bore's axis. The hole is carefully lapped and a tight fitting steel piston is inserted above a gas check which protects the piston from hot powder gases and prevents gas leakage. A small cylindrical piece of copper of known dimensions and physical characteristics is placed on top of the piston and held by a heavy screw from the pressure gun frame. When the shot is fired, gas pressure punches out a disc of brass in the wall of the cartridge case and the pressure moves the piston upward, compressing the copper crusher pellet beyond its yield point. By comparing the copper pellet's length after firing with its original length, and use of a conversion table, pressures in (so-called) pounds per square inch are obtained. (Actually, as Dr. Brownell notes elsewhere in this article, true psi is not had via the crusher gauge system but the values still are used — and useful — as comparison means.)

Example Calculation of Pressure

Perhaps examples of the procedure for calculation of pressures would be appropriate here. If we divide 63,600 psi by 14 units we obtain 4540 psi absolute pressure per vertical unit on the oscilloscope sweep. For the 20, 20½ and 21 vertical units for the 48-gr. IMR 3031 loading we obtain absolute pressure of 90,800, 93,100 95,300 psi respectively (figures rounded off) by multiplying the scaling factor of 4540 psi per unit by the value of the vertical units at peak pressure. Similarly, for the load of 49 grains we obtain 97,700, 100,000 and 102,300 psi absolute pressure respectively. It is important to note that at the highest pressure of 102,300 psi absolute, the crusher value is only 67,400 psi — which gives a difference of 34,900 psi. This corresponds to a difference of over 50 per cent with respect to the crusher value.

We have observed that cartridges loaded to 0.1-gr. accuracy on powder scales give good reproducibility using full charges as shown in Fig. 3. Although the charges of 48 and 49 grains were weighed to 0.1 grains for the firings in Fig. 7, the 44-gr. loads were loaded with a powder measure. We believe this accounts in part for the spread in the 44-gr. firings.

Other Studies

One of the chief objectives of the research studies supported by the Du Pont Grant at the University of Michigan is to develop and explore new means of investigating the phenomena of internal ballistics in small arms, particularly internal ballistics for the centerfire, high powered rifle. The new techniques, among other tests, involve the use of gamma radiation and strain gauges for simultaneous determination of chamber pressure, of bullet location in the barrel and pressure at that point versus time!

New methods of correlation based on the use of dimensionless groups will be presented later. The results of these studies will be reported at suitable intervals and copies of these reports with original experimental data will be made available to the Du Pont Company and the general public without cost via the editorial offices of the various journals. The reports may be obtained by individuals for a small fee to cover the cost of reproduction (estimated for this report as $1.65 per copy).* ●

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*For *Absolute Pressure in Center-Fire Rifles*, write to Ulrich's Book Store, 549 E. University, Ann Arbor, Mich., or to Ann Arbor Arms and Sporting Goods, 1340 N. Main St., Ann Arbor, Mich.

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APPENDIX II

CHAMBER PRESSURES IN SMALL ARMS

by

Berwyn J. Andrus

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Requirements for the Degree

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University of Utah

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ABSTRACT

The purpose of this thesis is to attempt to determine the feasibility of using SR-4 strain gages to measure chamber pressures in a rifle.

The problem is approached as a structural strain gaging problem involving a thick-walled cylinder and the instrumental set-up is described in detail.

The pressures determined by this method were within the accepted values for the type of ammunition used. The overall accuracy of the method was not determined, but the accuracy of the strain gaging equipment was estimated.

The sources of error discussed include the inconsistencies between the ideal thick-walled cylinder and the chamber of the rifle. A static load test of the chamber pressure is recommended as a possible means of evaluating the errors in this approach.

From the findings of these experiments it is believed that further research will enable the expansion of the use of the SR-4 strain gage to regular use in the ballistics industry.

INTRODUCTION

There are three reasons for making chamber pressure measurements: It is necessary in cartridge manufacture to make routine control checks to insure uniformity of production; in ballistics research chamber pressures must be determined accurately in order to test theories of internal ballistics; and finally, it is often desirable to determine the safety of ammunition in a sporting rifle barrel without tests that destroy the barrel for sporting use. The first two reasons are solved at present by instruments that require a specially chambered rifle with a hole in the side so that a piston can be exposed to the chamber gases. This requires expensive machining and is good only for a limited number of shots. There are serious doubts as to the accuracy of this method because the chamber is altered which theoretically will alter the pressure developed. Furthermore this approach is of no value for testing sporting rifles because the rifle barrels are destroyed by machining the hole into the chamber.

PURPOSE

The primary purpose of this thesis is to determine whether SR-4 strain gages fastened to the outside of the chamber walls can be used to determine the internal chamber pressures. Basic theory, techniques, and electronic apparatus are described.

The problem is approached from the standpoint that the chamber of a rifle is a thick-walled pressure cylinder and that the circumferential strain on the outside surface of the chamber can be related to the internal pressure by utilizing thick-walled cylinder formulas.

SCOPE

The scope of these experiments is limited to the determination of the feasibility of this method and obtaining information as to whether or not the strain gaging equipment available is sensitive enough to give good results. The instrumental set-up is explored in an attempt to correct trouble spots or note for future work to correct.

The experiments are limited to one rifle and to one type of cartridge. No attempt is made to determine the overall accuracy of the method. Errors in the common thick-walled pressure cylinder theory are not considered.

The complex theories of interior ballistics are not considered except for a brief description of other means of pressure measurement in current use.

Because this was approached primarily as a strain gaging problem the history and operation of the SR-4 strain gage and recording equipment are presented in detail.

REVIEW OF LITERATURE

There are many textbooks and other publications which cover the fields of interior ballistics, strain gaging, and the theory of thick-walled pressure cylinders as individual subjects. The combination of these three is unique at present and no published literature was found on this approach to chamber pressure measurement. Mention was made in a publication of Baldwin Locomotive Works (3), manufacturer of the SR-4 strain gage, that the gage has been used in ballistic work with shotguns.

PRESENT METHODS OF PRESSURE MEASUREMENT

There are two methods in current use for determining chamber pressure. Both have the disadvantage of being destructive to the rifle used, as their principle of operation requires a drilled and lapped hole into the chamber to admit a piston that is acted upon by the powder gases.

Copper Crusher Gage

The copper crusher gage is used when only the maximum pressure is desired and details of the time-pressure curve are not important. It was invented by Noble in 1860 and is used successfully for routine checks in the production of cartridges (5).

Essentially the gage consists of a copper cylinder that is restrained at one end and compressed by a piston that is exposed to the powder gases on the other end. A table showing the reduction in length as compared to the pressure is prepared by calibrating the cylinder

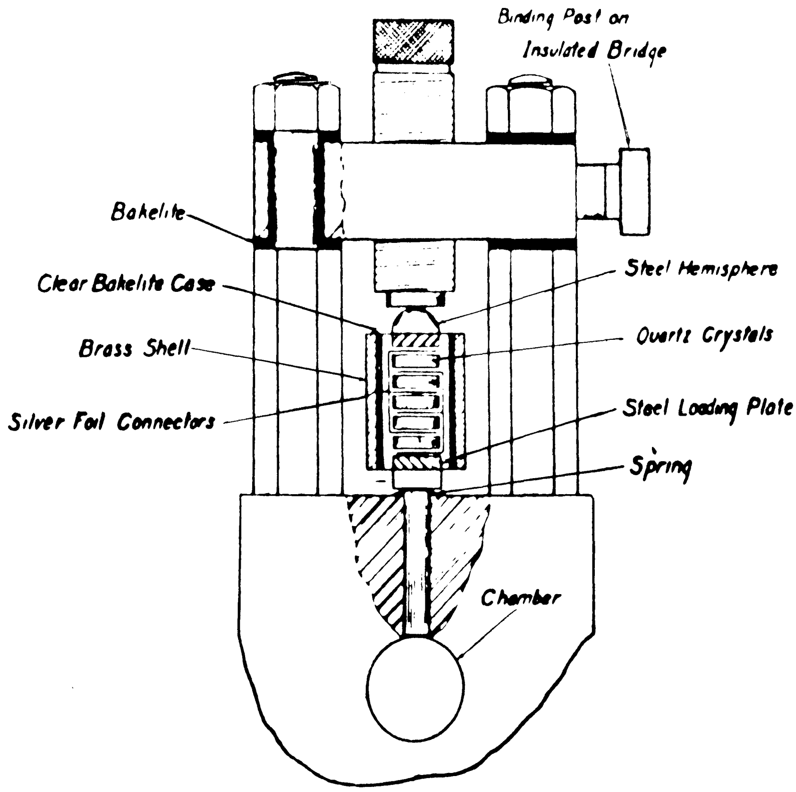

Fig. 1. Diagram of Piezoelectric gage. Copper Crusher gage is similar with copper cylinder in place of crystals. (7)

with dead weights. For pressures below five tons per square inch a lead cylinder is usually employed.

Piezoelectric Gage

This gage makes use of the fact that certain types of crystals have the property of developing an electrical charge when they are placed under load. The amount of charge is proportional to the amount of load placed on the crystals. The crystals are cut perpendicular to their electrical axes and secured with conducting cement to silver foil connecting the negative and positive sides of each crystal to the next, as shown in Fig. 1.

The complete apparatus for pressure measurement work consists of the following: a means for transmitting the chamber pressure to the piezo gage, the piezo gage proper, a resistance-coupled amplifier to convert the electrical impulse developed by the gage to a measurable current, and an oscillograph-camera combination to record the impulse(7).

This method can be used for time-pressure studies and is used more in ballistics research than routine checks.

BASIC ELEMENTS OF SR-4 STRAIN GAGES

By definition strain is the change in length of an article subjected to a load, divided by the original length from which the change was measured. The common unit of expression is inches per inch. This is what any strain gage measures.

The SR-4 strain gage is not a mechanical measurement of strain but is an indirect measure utilizing a principle of electrical resistance that was discovered by Lord Kelvin. He found that the electrical resistance of wires changed when they were stretched and that the change in resistance was proportional to the amount the wire was stretched. He also noted that different metals had different changes in resistance for the same change in length (6).

The SR-4 gage of present form is the result of work by E. E. Simmons at the California Institute of Technology and A. C. Ruge of the Massachusetts Institute of Technology. They concluded that if wires could be bonded to the material to be tested firmly enough that no slippage would occur they could measure the strain by measuring the change in resistance of the wires. The problem of bonding the wires to the material was solved by using wires 0.001 inch in diameter, because the ratio of circumferencial area to cross-sectional area increases as the diameter is reduced (3).

In order to use a long length of wire in a small space the wire is formed into a grid by looping it back and forth. The grid is then bonded in a matrix that can be applied to the material for test.

Each lot of gages has certain characteristics. These are gage factor and gage resistance. The gage factor is also known as the sensitivity factor and is the ratio of the change in resistance of the wire to the strain it undergoes. The change in resistance is expressed the same as strain—that is—the total change in resistance in ohms divided by the original resistance in ohms. Thus the gage factor or sensitivity is expressed in ohms per ohm over inches per inch. A higher gage factor indicates a greater sensitivity. The resistance of the gage is expressed in ohms and both characteristics are printed on each package of gages.

Two types of gages are sold in three types of matrices. One type of gage is made from a copper-nickel alloy and the other from a heavily cold-worked elinvar. The latter has a higher gage factor but is also highly sensitive to temperature changes. The matrices available are paper and Bakelite, the latter for use at higher emperatures. The newest type is an extra thin paper base used where speed of application is desirable or where clamping for long periods of time is impossible. Many different shapes of grids are available in all types and over 100 different gages are offered (3).

THEORY OF THICK-WALLED PRESSURE CYLINDERS

Cylinders subjected to pressures are divided into two general classes called "thick-walled" cylinders and "thin-walled" cylinders. The difference being that stress distribution through the thickness of the wall may be assumed constant without appreciable error when the cylinder wall is thin compared with the internal radius, but when the wall is thick errors in this assumption become too large to be acceptable. Just where the dividing line is depends on the accuracy required. In the case of this rifle chamber the ratio of thickness to radius is 1.65 and the error would be in the order of at least 120 per cent(8). This is too large for this type of work so the thick-walled theory must be applied in this case.

The solution of both cases makes use of several assumptions: A perfectly elastic, homogeneous material; and a prismatic, circular cylinder infinitely long. The solution of the thin-walled case is arrived at by simple statics because the stress across the section can be assumed constant.

The thick-walled case was solved orginally by Lame' and makes use of the fact that for any thin section of the thick-wall, the thin-wall formula will be true (9). By use of the properties of the material of the cylinder, Lame' derived a workable set of formulas for thick-walled cylinders. The formula he arrived at that relates circumferential strain to internal pressure is shown on the following page together with its application to this problem.

$$S_t = \frac{P_1R_2 - P_2R_2 + \frac{R_1^2R_2^2}{p^2}\ (P_1 - P_2)}{R_2^2 - R_1^2}$$

Where:

P_1 = Internal pressure

P_2 = External pressure

R_1 = Internal radius

R_2 = External radius

p = Radius of point being considered

S_t = Stress at point being considered

When $P_2 = 0$ and $p = R_2$, the equation becomes:

$$S_t = \frac{2\ P_1R_1^2}{R_2^2 - R_1^2}$$

Δ = Strain, inches per inch

E = Modulus of Elasticity = $\frac{S_t}{\Delta}$

Solving for Internal Pressure the equation becomes:

$$P_1 = \frac{(\Delta E)\ (R_2^2 - R_1^2)}{2\ R_1^2}$$

Substituting the values of E, R_2, R_1, it becomes:

P_1 = Δ microinches per inch X 89.395

EQUIPMENT AND APPARATUS

The equipment and apparatus used in this work are the property of the University of Utah with exception of the rifle barrel which was furnished by Mr. P. O. Ackley* for use in research. All of the preparatory work was done in the Structures Laboratory of the Civil Engineering Department and the test firing was done in the basement of the Civil Engineering Building. The individual pieces of equipment are described below.

TYPE OF GAGE

The duration of strain is so small in this work that temperature change affecting the gage during the recording period can be assumed to produce negligible effect, which allows the use of the higher sensitivity of the "C" type gage.

A convenient gage is the C-5-1, Fig. 2, because its 1/2 inch gage length makes it possible to apply two gages on the circumference and is of the special group of extra thin paper back gages for quick drying. This was desirable because of the difficulty encountered in clamping gages on the curved surface while they were drying. The C-5-1 required being held in place with the fingers a few

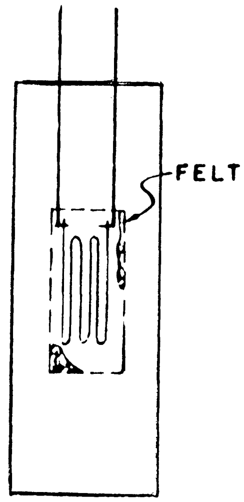

Fig. 2. SR-4 Strain Gage, C-5-1.

* Mr. P. O. Ackley, Barrel Maker, 2235 Arbor Lane, Salt Lake City, Utah

minutes and was ready for use almost immediately.

The gages used had a gage factor of 3.26± 2.0 per cent and a resistance of 350 ohms ± 5 ohms. The gages were mounted with their centers diametrically opposite and their length-wise axes on the same line. They were wired in series so that they acted as one gage with a resistance of 700 ohms and a gage factor of 3.26.

RECORDING EQUIPMENT

The recording equipment includes those instruments that were used to transcribe the changes in resistance of the wires in the gage into a form that is readable in terms of strain which could be converted to internal pressure. The equipment consisted of a Bridge Amplifier, Oscillograph, Trigger Circuit, Oscillograph Recording Camera, and Constant Voltage Transformer.

The Bridge Amplifier

This instrument is manufactured by Ellis Associates and is known as the BA-1. It contains the elements of a Wheatstone Bridge with variable resistors and a four step amplifier.

The Wheatstone Bridge is shown in simple form in Fig. 3. The gage is shown in one leg of the circuit and with no strain on the gage, or any other point desired to use as reference, the bridge is balanced so that the voltage E_0 is zero. When the gage is subjected to strain the resistance is changed and there is then an electrical potential at E_0. This small change in potential is impressed into the amplifiers which enlarge it depending upon the stage of amplification used. The output of the BA-1 is connected to the "Y" input of the oscillograph.

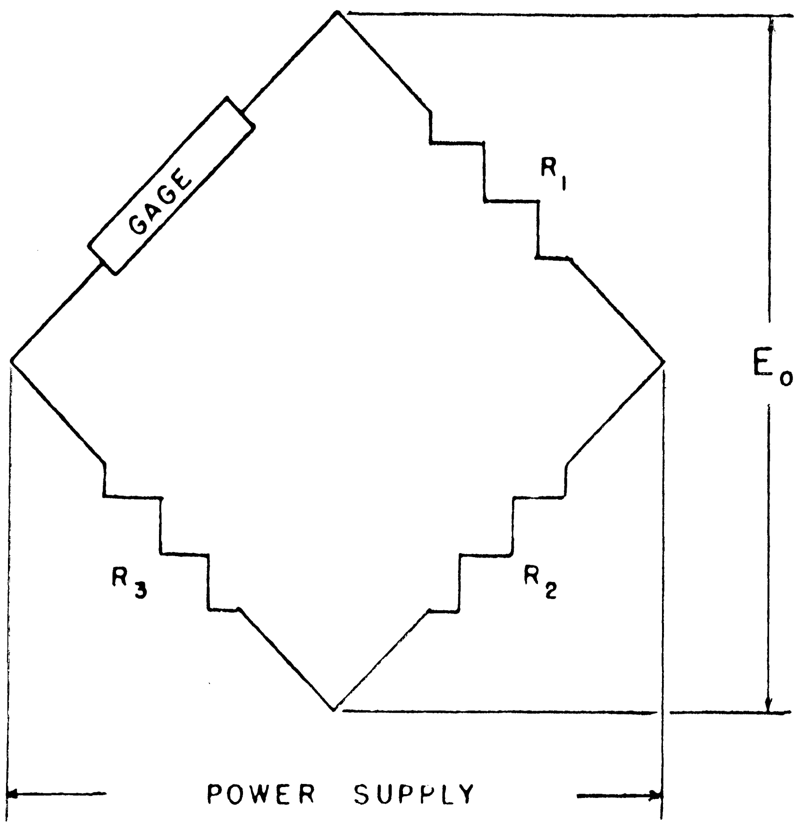

Fig. 3. Simple Wheatstone Bridge.

For purposes of calibration there are resistors of known value in the BA-1 that can be placed in the position of the gages by turning a switch. Thus calibration can be done at any convenient time during the test.

<u>The Oscillograph</u>

The oscillograph used in this experiment is a DuMont 304H. It is used for the purpose of putting into wave or graph form the voltage that is impressed into it. In this case the impressed voltage is from the BA-1 amplifier.

The oscillograph has two sets of parallel plates set perpendicular to each other to form a box through which electrons must pass. The impressed voltage from the amplifier is fed to one set of plates and the voltage on the other set of plates is controlled by the setting on the oscillograph. Depending on the voltage impressed on the plates, the electrons moving through are deflected a certain amount. The electrons are "aimed" at the sensitized screen on the front of the cathod ray tube, and with only the set of plates controlled by the oscillograph carrying voltage, the electrons make a luminous spot on the screen that moves straight across the screen. The speed of travel of thespot can be controlled from two crossings of the screen per second to 30,000, which appears to the eye to be a solid line.

When the voltage change that originates with the change in resistance of the strain gage is impressed on the other set of plates, the traveling spot is either deflected up or down depending on whether the strain is compression or tension. The amount of deflection of the spot is dependant on the amount of strain that the gage and material it is attached to undergo. During the time the spot is being deflected up or down by the voltage from the gage it is moving across the screen with a constant speed so it produces a wave form that can be used to time the change in strain on the gage.

Because in work of this nature the spot would be traveling so fast that even a camera would not record it, the screen has a certain amount of retentivity so that the trace of the spot remains after the beam of electrons has passed.

The Trigger Circuit

Since the traveling spot on the oscillograph keeps moving across and repeating instantaneously there is the need for allowing only one pass of the spot each time the gun is fired to keep the scope from being cluttered up with several traces of the spot because the camera had no shutter speed that could be timed to take only the trace at the time the rifle was fired.

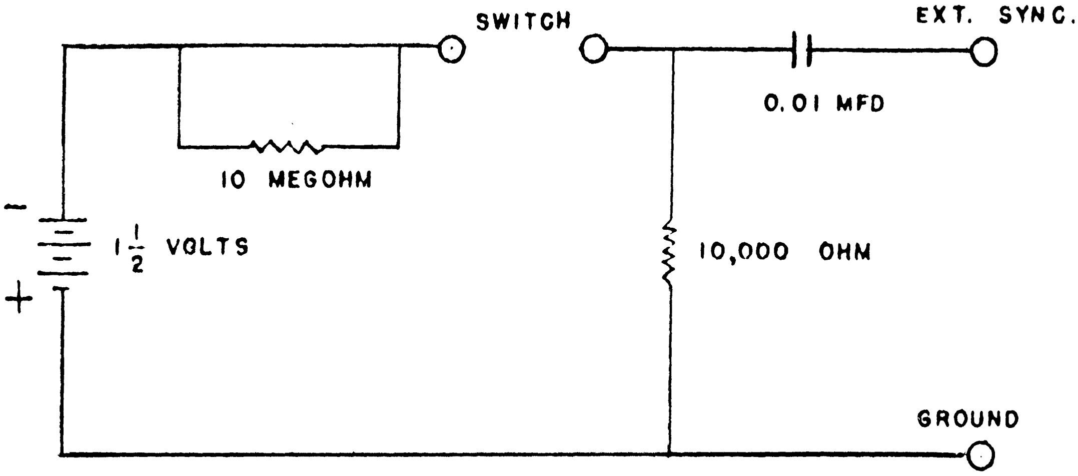

Fig. 4. The Trigger Circuit

The oscillograph can be set on "driven" which means that the spot must be driven across by an external voltage source. The trigger circuit was used for the purpose of developing sufficient voltage to drive the spot across the oscillograph. The control of the circuit was handled by a micro-switch mounted on the rifle so that it was tripped by the firing pin as it moved forward. By controlling the "Sync Amplitude" it was possible to get a single trace on the scope. The circuit used is shown in Fig. 4.

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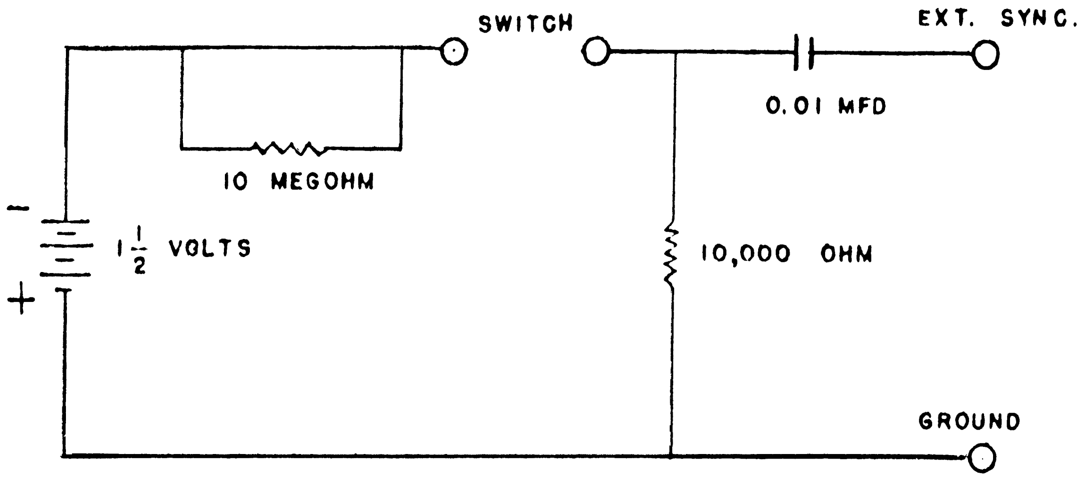

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The Recording Camera

The recording camera is a 35 mm camera that operates through a mirror and is mounted directly on the front of the oscillograph. With the trigger circuit in operation the shutter is opened before the rifle is fired and closed after firing because only one trace shows on the scope.

The camera is Model Number 35, manufactured by Allen B. DuMont Laboratories, Inc.

The film used was Super XX because it was desirable to have a fast film. Even with the small amount of retentivity and the diaphragm wide open it was necessary to use full intensity to get good pictures.

The Constant Voltage Transformer

A constant voltage transformer was used to insure a constant voltage source to the oscillograph. This eliminated the interference on the oscillograph due to slight variations in standard line voltage.

THE GUN

The gun was a standard 30-06 barrel mounted in a special jig that is used for test firing. The standard Army M-2 ball ammunition marked SL 45 was used in all test firing.

TESTING PROCEDURE

Since no published material was found to set forth definite procedure the work of this thesis was governed by standard procedures only so far as placement and operation of the SR-4 gages.

The technique that produced results is the one set forth below. Certainly many refinements could be made, but this procedure did yield good results without using any equipment that would not ordinarily be available to any laboratory equipped with strain gaging apparatus.

PREPARATION OF THE RIFLE

The modulus of elasticity was determined on the barrel blank after the barrel was bored but before the outside was turned to final dimensions. A twelve inch section was turned smooth and the strain measured by means of two Huggenberger Tensometers. This was performed by Professor R. L. Sanks and the modulus of elasticity was determined to be 29,600,000 psi.

The barrel blank was then chambered, turned to finish size and fitted to the test jig that was used for the firing.

The measurements of the barrel were taken by means of micrometers. The inside diameter of the chamber was determined by using a fitted dowel and checking against fired shells. The chamber is shown in cross-section in Fig. 5.

A base to mount the test firing jig on was made out of a four by four piece of wood. The jig was held in place by recessing into the wood and holding it in place with 1/4 inch U-bolts. Holes were provided in each end of the four by four for 1/4 inch rods to be driven into the

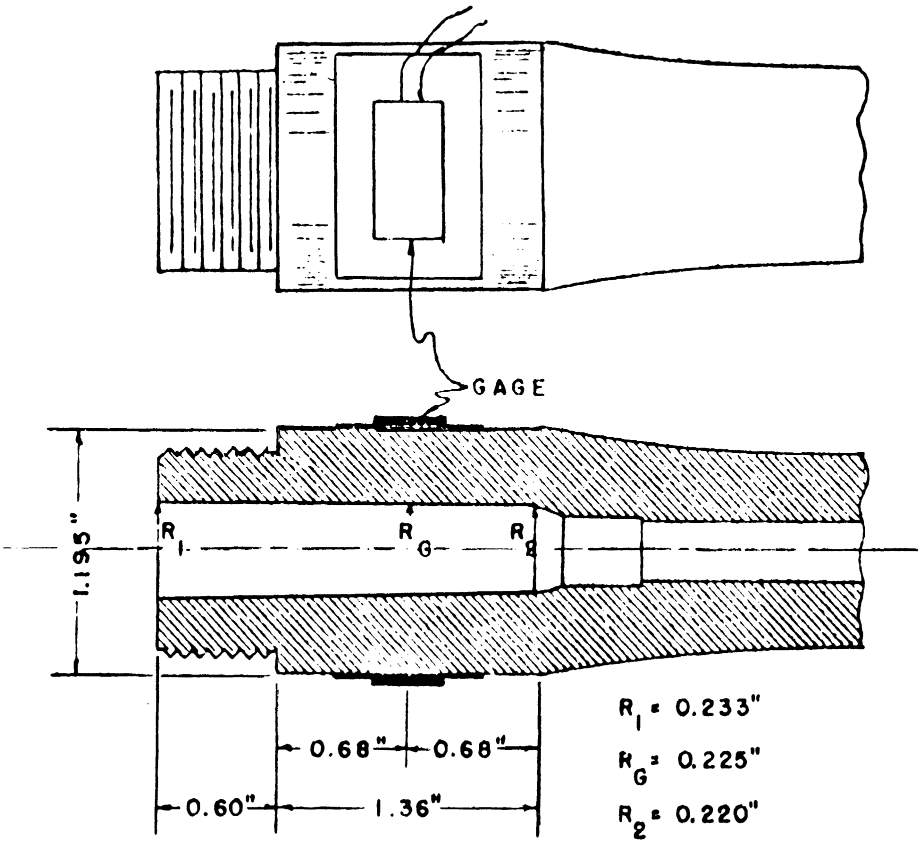


Fig. 5. Cross-section of chamber and location of gages.

ground to hold the block in place during firing.

The gages were applied to the rifle last in order to minimize any chances of damaging them. Two C-5-1 gages were mounted with their centers diametrically opposite and their lengthwise axes on the same plane. Both center lines were scratched on the outside of the chamber and then the surface was cleaned throughly with acetone. Standard SR-4 cement was applied to the surface and the gages were applied. They required holding in place with the fingers for about one minute. They were allowed to dry overnight before using.

After applying the gages and before and after firing the resistance of the gages and the resistance to ground were checked. The resistance of the gages was the same before and after, and a resistance of over 1,000 meg-ohms was present at all times to ground.

A ground wire of shield cable was attached to the barrel next to the gages by means of a hose clamp.

SETTING UP THE EQUIPMENT

Some preliminary firing on a different rifle was done at the Police Rifle Range but weather conditions, power difficulties and the effect of the shock from the firing upon the oscillograph made it obvious that it was not practical to do the firing in the open.

The tunnel in the Civil Engineering Building basement is an excellent location. The rifle is set up in the tunnel and all equipment is kept on the main floor where there are no vibrations to interfere with the equipment.

The rifle was mounted with the four by four against a concrete wall and the 1/4 inch rods driven into the ground. A four by four was grooved and placed under the barrel with a sandbag on top of the barrel to hold it firmly in place during firing.

The only way to load the gun involved unscrewing the barrel so the lead wires to the gages and the ground connection were brought to a plastic board with cap screw terminals. The bottom of the terminals were soldered to the wires leading up the outside of the building to the Soils Laboratory where the recording equipment was located.

All ground wires were connected in common to a water pipe by a hose clamp and lead sheet to insure complete bleed off of any ground current.

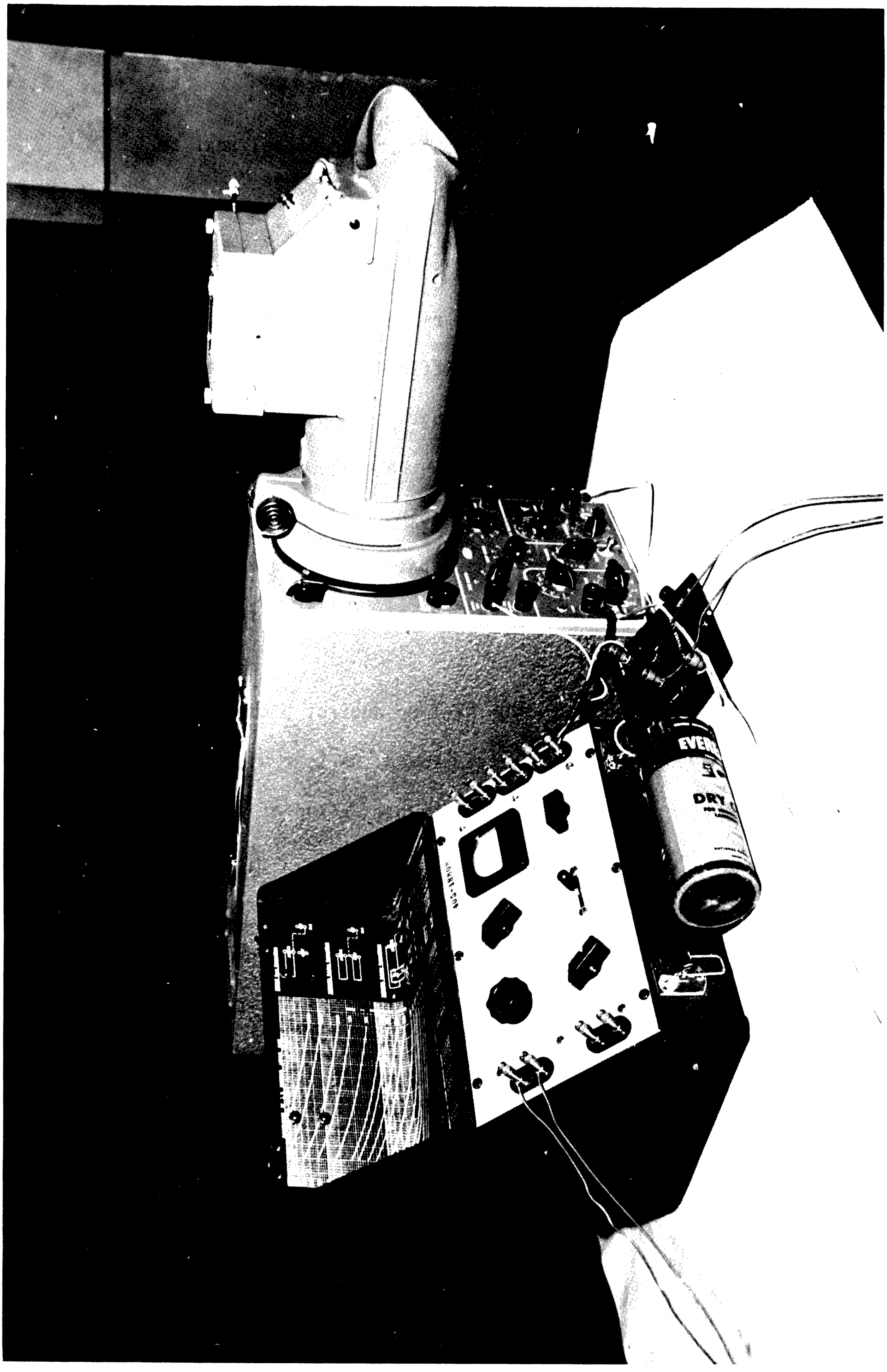


Fig. 6. Instrumental Set Up.

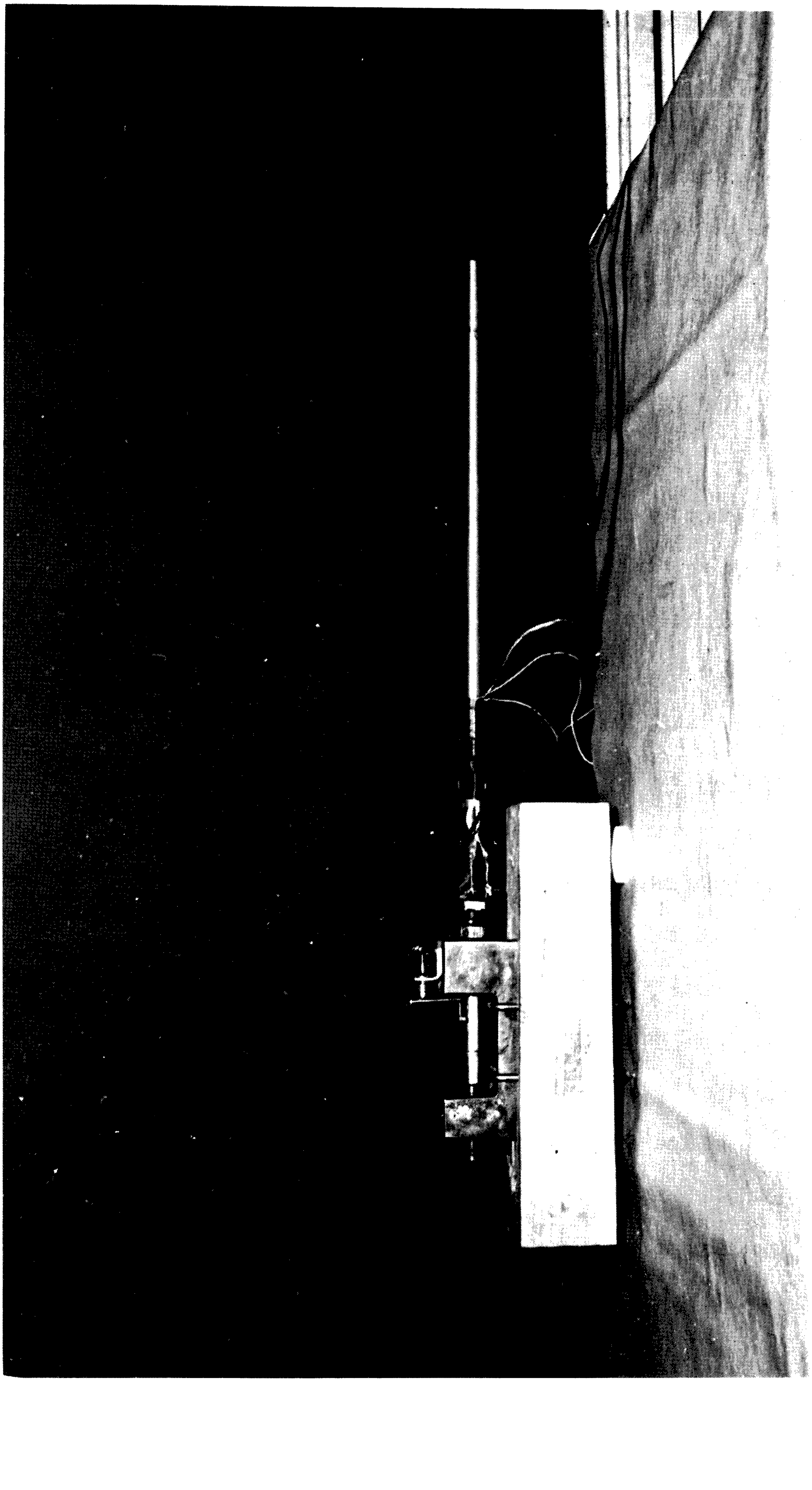


Fig. 7. Rifle Set Up.

Four bags of wet sand were placed three to four feet in front of the rifle to stop the bullet.

All of the recording equipment was set up in the Soils Laboratory on one table except the constant voltage transformer which was set up in another room because the operation of any type of coil near an oscillograph produces interference on the scope.

The oscillograph, BA-1, Trigger Circuit, and Camera were set up as shown in Fig. 6.

FIRING

Firing was accomplished by means of a wire running through guide loops to a lever attached to the firing pin. At first an attempt was made to use the trigger circuit to obtain but a single trace but since the firing pin would have to trip the micro-switch at the correct time in order to fire the rifle while the spot was traveling across nothing was shown on the screen but a straight line.

That meant that either the trigger circuit was out of time or else no results were coming from the gages. To determine which it was the trigger circuit was removed and the oscillograph left on "recur" which allowed the spot to continually travel across and repeat instantaneously. The next shot fired gave a good trace on the scope and by a rough calculation seemed to be of about the correct magnitude. This first firing was done with the oscillograph set for Sweep Speed of 20 and Sweep Range of 10-50.

This was much too slow to get the curve spread out across the scope. By trial and error a setting of 100 on the Sweep Speed and 50-250 on the Sweep Range was found to be the most desirable. This speeded up the trace so much that to get results on the film it was necessary to use a diaphragm opening of f1.5 and set the intensity of the trace up to "full."

The problem of adjusting the micro-switch so that it would trigger the scope off at the right time was solved by trial and error. It was fortunate that the first adjustment was close enough that by adjusting the controls on the oscillograph the trace was moved to the center of the scope.

The calibration signal with the 1 M resistor from the BA-1 was the most convenient one to use in this case. The decision is made by roughly approximating the strain anticipated and then using the resistor that produces a calibration signal near the same height as the expected pattern. After the correct resistor is used the "Y" amplitude control on the oscillograph can be used to make fine adjustments so that each division on the scope will be an even value of strain. In this case the adjustment was made so that each division on the scope grid represented 20 micro-inches of strain.

The calibration signal can be shown at any time during the test by throwing the chopper switch from "Dynamic" to "Calibration". It was taken several times during the firing to make sure that the calibration had not changed. The equipment with the setting of the dials for both firing and calibration are shown in Fig. 8 and Fig. 9.

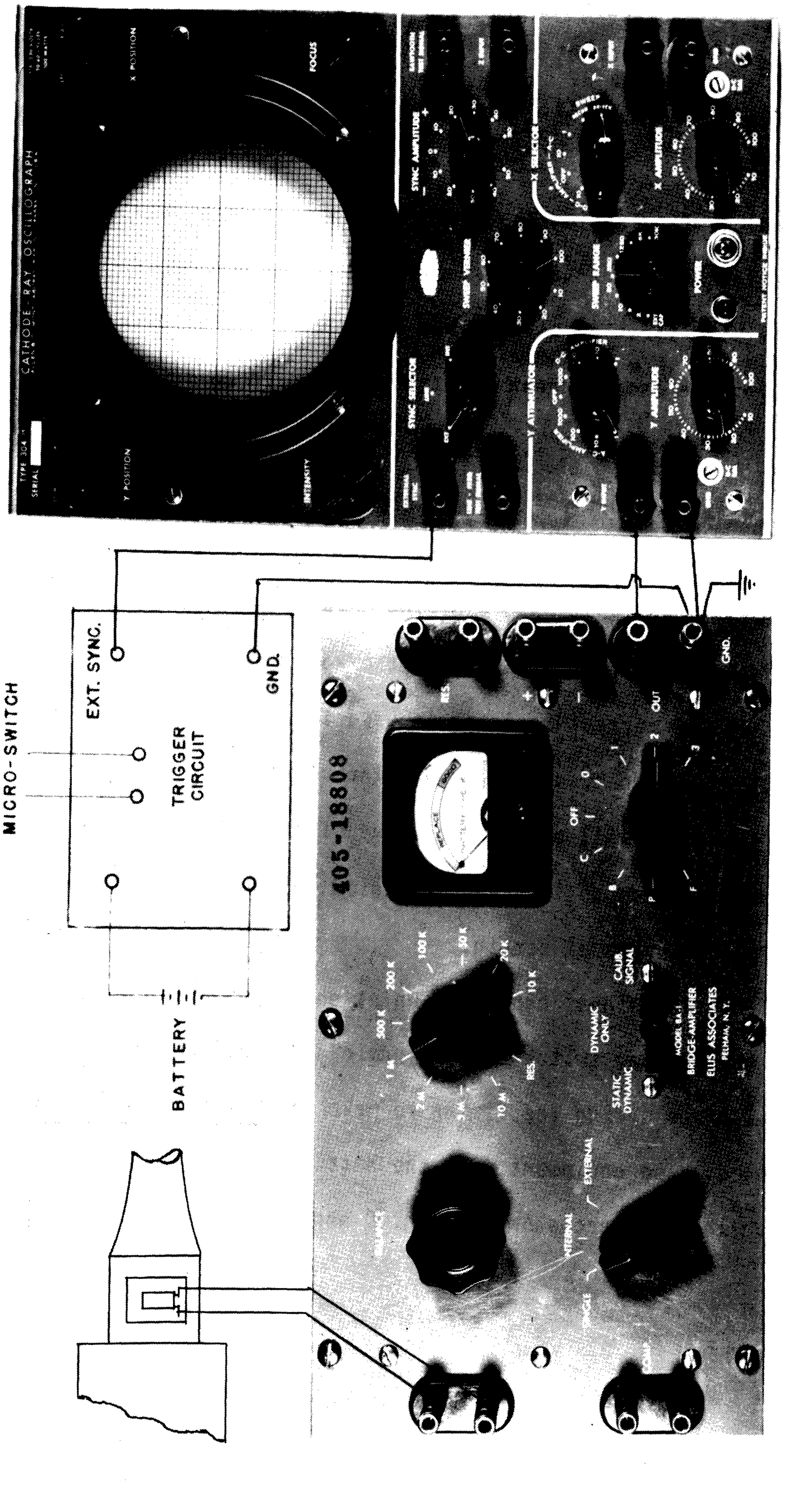


Fig. 8. Equipment with dials set for firing.

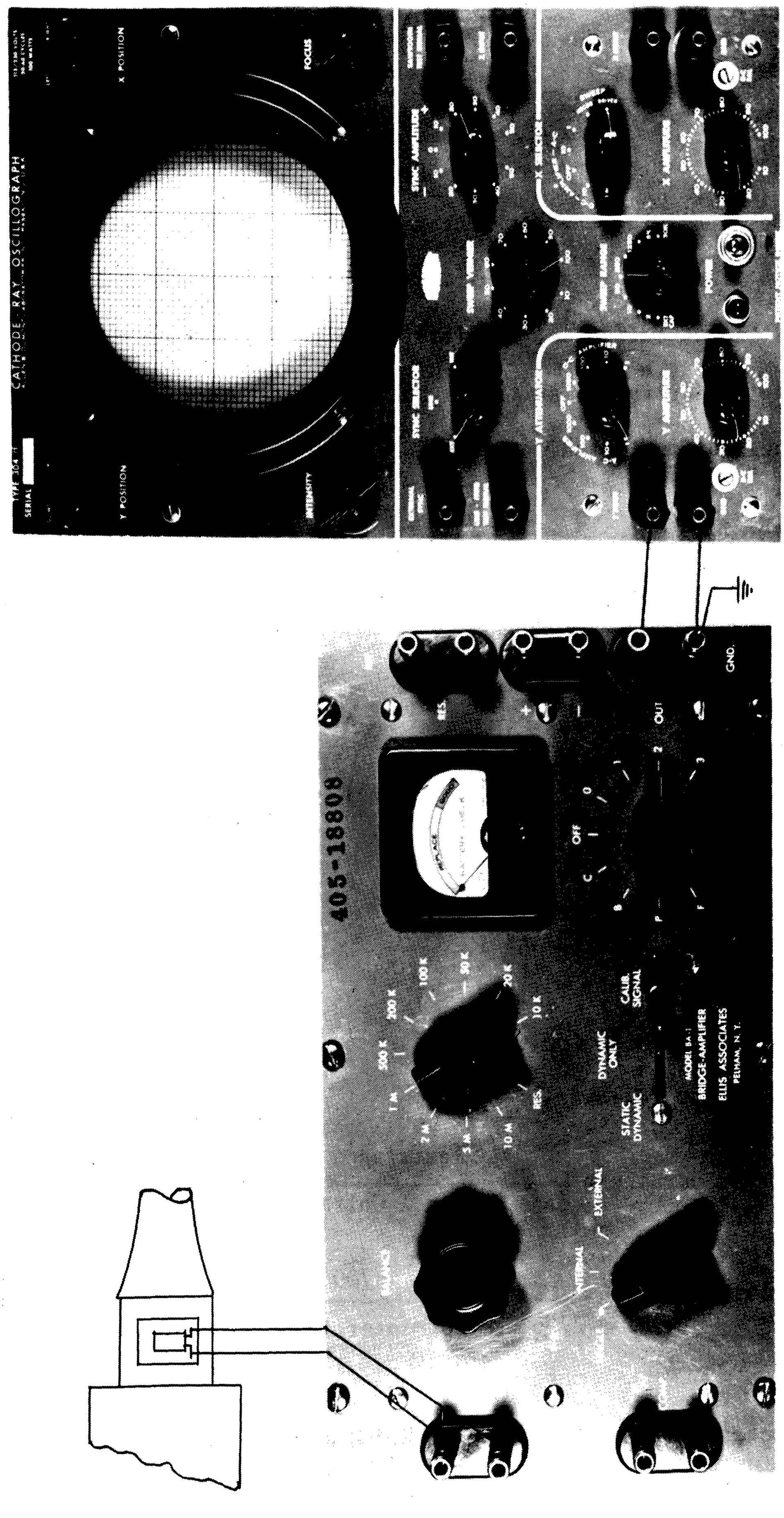


Fig. 9. Equipment with the dials set for calibration.

RESULTS

The results of these experiments indicate that this is a feasable approach to the problem of chamber pressure measurement when considered within the limits of these experiments. The equipment has demonstrated that it is of sufficient sensitivity to handle the strains involved without resorting to critically high amplification. The performance of the equipment used was very satisfactory.

CALIBRATION

The calibration signal was shown three times during the final firing, and was the same each time which results in only one calibration value for all pressure measurements. At the enlargement used for the pictures the calibration signal is 0.70 inches high. The amount of strain that it repsents is calculated as follows:

See Fig. 10, Page 25.

$$\text{Strain, inches per inch} = \frac{\text{Resistance of Gage}}{\text{Gage Factor X Calibration Resistor Used}}$$

$$\text{Strain, inches per inch} = \frac{\text{700 ohms}}{\text{3.26 ohms/ohm over inches/inch X 1 megohm}}$$

Strain, inches per inch= 0.0002148 = 214.8 micro inches per inch.

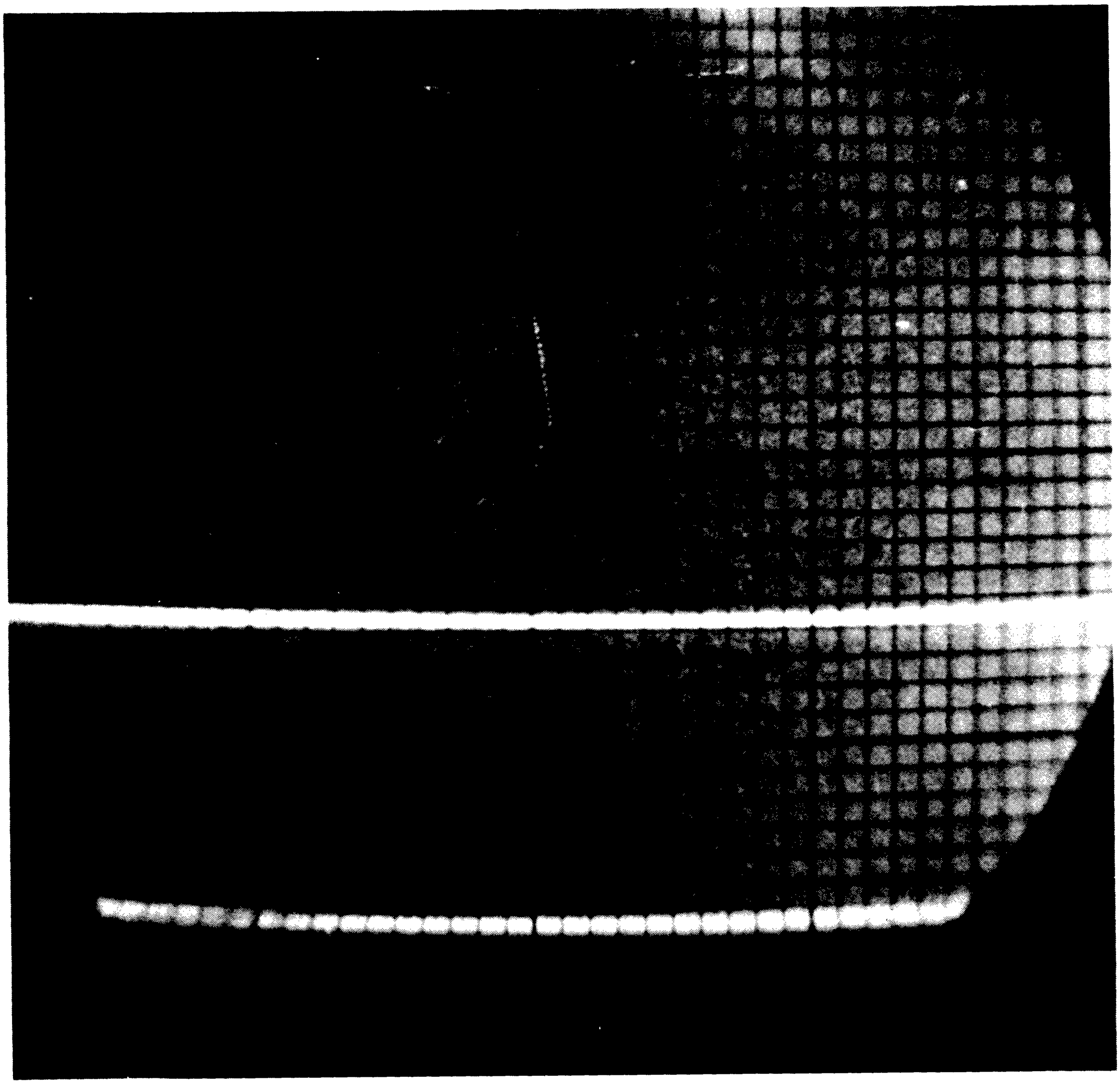


Fig. 10. Calibration Signal

The strain on the outside of the chamber during firing is equal to the height of the pressure curve divided by the height of the calibration curve times 214.8 micro inches per inch.

CALCULATION OF PRESSURES

The internal pressure is determined by substituting the strain as calculated from the above calibration into the formula derived from the basic thick-walled pressure cylinder theory on page 8. This calculation as performed for shot number one of Fig. 11 is as follows:

$$\text{Internal pressure} = \text{Strain, micro inches per inch} \times 89.395$$

$$\text{Internal pressure} = \frac{\text{Height of pressure curve}}{\text{Height of Calibration}} \times 214.8 \times 89.395$$

$$\text{Internal pressure} = 27{,}430 \times \text{Height of pressure curve.}$$

$$\text{Internal pressure} = 27{,}430 \times 1.58 = 43{,}300 \text{ psi.}$$

The photographs of all shots are shown in Fig 11 and Fig. 12. with the calculated pressure.

The range of pressure given for the ammunition at the time of purchase was 40,000 to 45,000 psi. All calculated pressures were in this range.

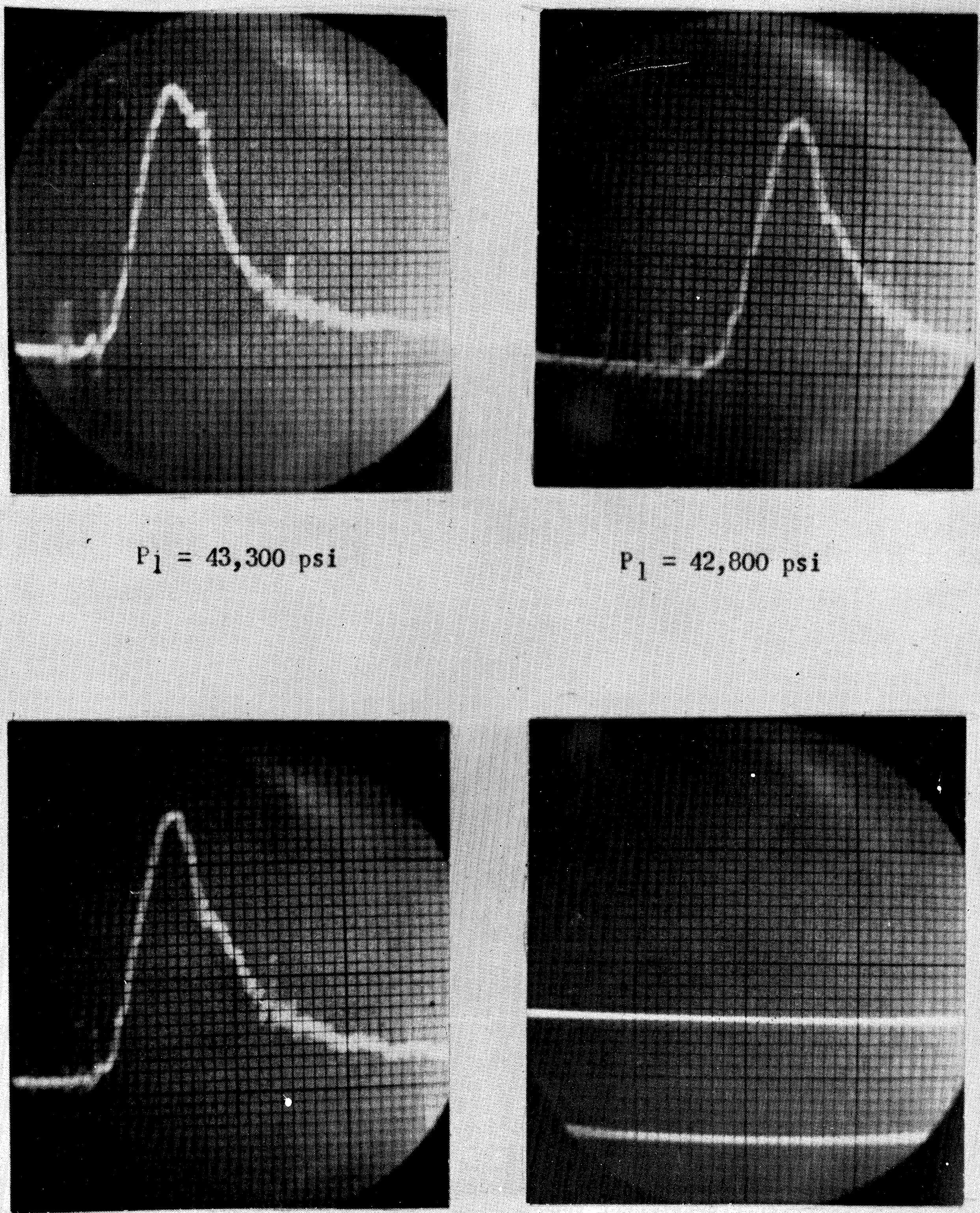
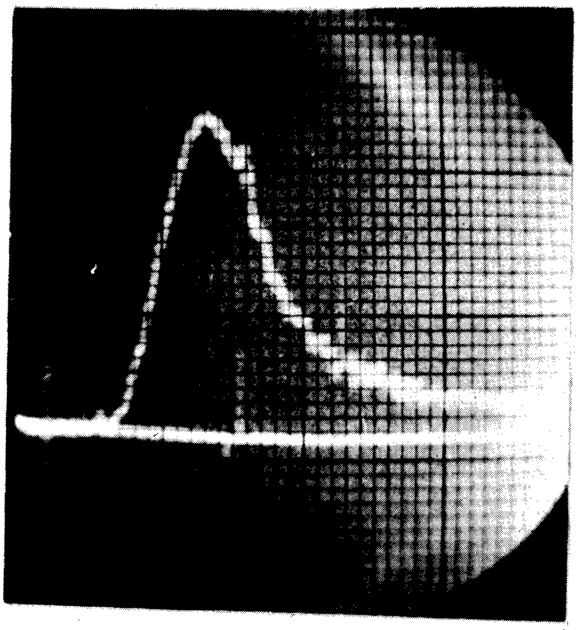
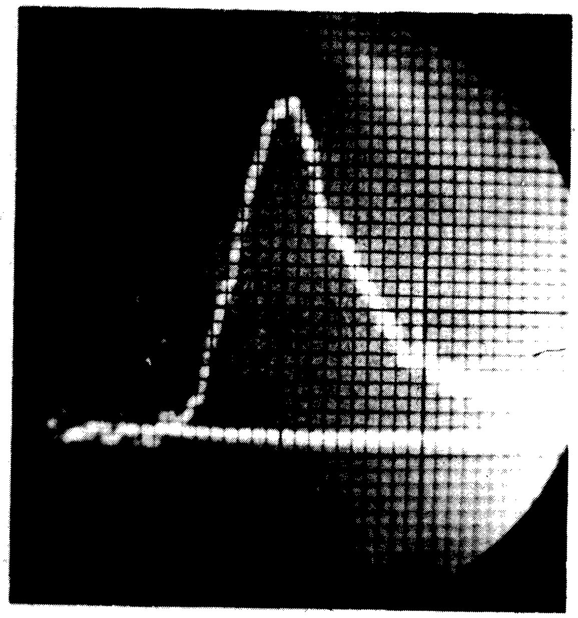


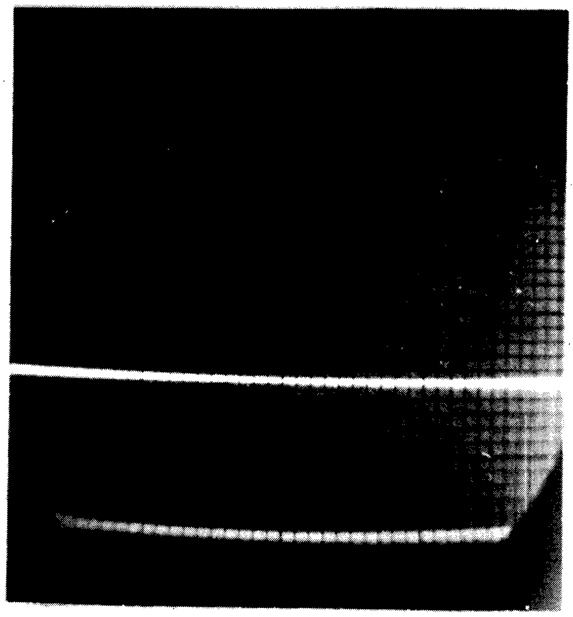
Fig. 11. Pressure Curves with Calibration Signal.



$P_1 = 41,200$ psi



$P_1 = 42,200$ psi



Signal Height = 0.70 inches.

Fig. 12. Pressure Curves with Calibration Signal.

SUMMARY AND CONCLUSIONS

These experiments have shown that the instrumentation is of sufficient sensitivity to record the magnitude of the strain on the outside of the rifle chamber. The accuracy of the strain measurements could be within the accuracy listed for the Gage Factor on the gage— two per cent— but errors in amplification, calibration and reading probably make the error in the strain measurements in the order of five per cent. The actual strain gaging technique used has been proved under so many different conditions that there is very little reason to suspect any large errors from this source.

A possible source of error is the measurements of the chamber, although they still should be within five percent. The measurements of the interior of the chamber could be made more accurately than those of these experiments by using a chamber casting metal.

The logical source of serious error is in the difference between the ideal conditions of the thick-walled pressure cylinder theory and the actual case of the gun chamber. The chamber is not of infinite length nor is it of uniform cross-section. Also, there are abrupt changes in cross-section near the strain gages— threads at one end and the shoulder at the other. In short, there is no formula at present that will give the relation of circumferencial strain to internal pressure precisely.

There is also the question of whether or not the cartridge case should be included in the thickness of the wall of the cylinder.

The fact that the calculated pressures were in the range of anticipated pressures indicates that the errors are not of great magnitude.

The simplest procedure for making a check of the results of these experiments would be to make a static pressure load test on the chamber by using some sort of fluid under pressure and recording the strain indicated. Comparison of these two along with research to determine the effect of the end areas and the short length of the cylinder should enable the determination of the possibilities of using this method in both industry and research.

The writer feels that the surface has just been touched in the possibilities of using the SR-4 strain gage in ballistics industry and research. At present further research has been undertaken at the University of Utah to enlarge upon these experiments, to determine the backthrust on the bolt and the possibilities of using a static load test on the chamber as a means of verifying the results obtained during firing.

Time calibration of the pressure curves and location of the bullet within the barrel with respect to the pressure curve are all possible with the use of SR-4 strain gages.

The possible applications of the SR-4 gage in the field of ballistics are almost unlimited and the writer feels that eventually this approach will be used in extensive ballistics research. There is a possibility that this method may replace the copper crusher gage for production checks.

Then, too, there are advantages offered the sportsman who uses non-standard or "wildcat" calibers. Several of our present outstanding calibers are the result of "backyard" experimenters. Few or none of these experimenters have the means to afford standard pressure tests, and, therefore the safety of their experiments is doubtful. The SR-4 gage may make the testing of any "wildcat" quite practical and inexpensive.

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APPENDIX III

Elements of Internal Ballistics

I. Russo-German Methods

by

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1. RUSSO-GERMAN METHODS PROCEDURES FOR THE SOLUTION OF THE MAIN PROBLEM OF INTERNAL BALLISTICS

by L. E. Brownell

The "main" problem of internal ballistics is the solution of the relationships between chamber pressure (p), bullet velocity (v), bullet barrel-travel length (ℓ), and time (t). European ballisticians such as Charbonnier[1] of France have usually related these variables to "y," the fraction of powder burnt at any time, "t." American ballisticians for many years related "p," "v," and "t" to bullet travel length "ℓ" using the empirical methods developed by the French artillery officer Leduc.[2]

Both of these procedures have certain disadvantages. The equations of Charbonnier are based on the energy balance. Various modifications of his equations have had worldwide use in solving problems in internal ballistics. A major difficulty in using Charbonnier's equations, however, is that the fraction "y" of powder burnt is not zero at bullet start where v = 0 but "p" is "p_0," a definite pressure required to "start" the bullet on its path. The methods of Leduc are not based on the equation of the conservation of energy (First Law of Thermodynamics) and so they lack generality. New equations and constants must be determined for each type of gun.

In 1903, the Russian ballistician Drosdow first proposed that "x," the fraction of powder thickness burnt after bullet start, be used as the independent variable rather than the total weight fraction "y." He let "e_1" be half the initial wall thickness of tubular powder and "e" be the thickness at any time after burning starts and, let "z" be the fraction, e/e_1, and "z_0" the fraction at bullet start. Then he defined $x = z - z_0$. The value of "x" is zero at bullet start, and will be $1 - z_0$ when the powder is all burnt. Thus, the value of "x" increases from 0 to a maximum as does the bullet velocity. This simplification reduces the definition of velocity

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[1] Charbonnier, P., "Ballistique interieure," Droin, Paris, 1908.

[2] Challeat, J., "Theorie des affute a deformation," Rev. d'art, LXV, p.184, 1905.

[3] Drosdow, N. F., (in "Innere Ballistik" by Serebrjakow,) (Moscow, 1949,) Moscow, 1903.

to a constant times x. This in turn simplifies the integration of velocity to give bullet-travel length (ℓ). Serebrjakow[4] developed the method of Drosdow further and explored the expression of burning rate of the powder as directly proportional to chamber pressure.

In 1961, Wolff, a professor in East Germany, reviewed the Russian methods in Innere Ballistik,[5] prepared for use in Officer Schools of East Germany and Russia and his book was translated by L. E. Brownell[6] in 1966.

---

[4] Serebrjakow, M. E., Innere Ballistik, Oborongis (Russ.), Moscow, 1949.
[5] Wolff, W., Innere Ballistik, Deutscher Militarverlag, Dresden, 1961.
[6] Wolf, W., Innere Ballistik, translated by Brownell, L. E., (Ulrich's Book Store, East University Street) Ann Arbor, Michigan, 1966.

PREFACE

The international MKS (Mass) System of units was officially accepted in the (East) German Democratic Republic on August 14, 1958. This change is of special importance to the study of internal ballistics, because the general use of new units necessitates a change from the ''engineering'' system previously used in ballistics. Some of the benefits associated with this new system can be seen in the following example. The impulse, a basic unit in the rocket field, in the engineering system has the dimensions: ''kg s/kg;'' it is written in this manner to avoid writing ''s'' only. In the new system the impulse has the dimensions: ''m/s,'' which is sufficient without additional description. The advantage of the new system become more obvious throughout this text whenever calculations with combined units are given, and becomes especially significant in the solution of the ''main'' problem of internal ballistics.

Chapter 2, ''The MKS Mass System and Its Units,'' is included to assist the reader in becoming familiar with the new system. All formulae and equations are written as unit-equations; and, conversions of non-consistent dimensions are clearly indicated.

This book is not intended for the ballistic specialist, but rather for anyone connected somehow with weapons and ammunition who is interested in the processes inside a weapon, e.g., for officers and weapon-technicians. The field of internal ballistics may also be of importance to students in the field of thermodynamics since many problems related to thermodynamics still exist in internal ballistics. Only an elementary mathematical background is required to understand this book; seldom are basic concepts of differential and integral calculus necessary. The Appendix, however, contains two sections which require a somewhat greater knowledge of mathematics.

The solution of the ''main'' problem of internal ballistics is certainly the heart of internal ballistics. Because of the limited scope of this book, only one ballistics method was selected from the many developed throughout the years, and this is a simplified and approximate one. The original plan was to demonstrate the ballistics solution of the French ballistician, Charbonnier. However, a detailed examination showed that Charbonnier's assumption that the covolume is equal to the reciprocal of the density, led to errors which increase during burning of the powder and which became excessively large at the end of the burning process. The same examination has shown that the method of the Russian ballistician, Drosdow, is more accurate. Therefore, the latter has been chosen; the more elaborate mathematical calculations involved in Drosdow's method are more than justified by the greater accuracy obtained. The illustration of the Drosdow method used here is closely related to the method described in the more recent book by the Russian ballistician, Serebrjakow, Inner Ballistics, Moscow, 1949. Unfortunately, this book is not available at the present and no German (or English) translation exists.

W. Wolff

Dresden, März, 1961

1. INTRODUCTION

Ballistics is the study of projectile motion; internal ballistics is involved with the motion of the projectile in the barrel under the influence of the powder-gas pressure, whereas external ballistics treats the motion of the projectile under the influence of gravity and air resistance after leaving the gun barrel. This separation is made necessary by the differences in the forces acting on the projectile. For the intermediate case in which the projectile has just left the gun barrel, the projectile will be passed by part of the escaping gas and thereby influenced to a limited extent. The study of this transition state is not treated as a new branch, "transition" ballistics, because the effects on projectile velocity are not sufficient to warrant such a consideration.

Precise knowledge of the forces acting on an object permits precision in the calculation of its motion. In the case of internal ballistics, the most important force to be considered is the pressure of the powder gas. Examination of the pressure process as well as the other processes in weapons is made difficult because of the high pressures (2000-3000 kp/cm^2, or 30,000-45,000 psi) and high temperature (2500-3000°C, or 4500-5400°F) involved; also, the processes exist only a very short time. For example, the time which an artillery projectile spends in the gun barrel is between 0.001 second and 0.060 second. This time is determined by the burning process which in turn depends upon the gas pressure in the weapon. Several "laws" of combustion have been defined of which two will be examined more closely. Recent experiments conducted in Russia show that these "laws" are valid only under certain conditions (see Section 5.34 and 5.35), i.e., values which were considered constant are only constant under certain assumed conditions. The burning "laws" must be treated as approximations because the combustion process is far more complicated than they take into account.

Internal ballistics can be divided into two main stages: the pyrostatic and the pyrodynamic periods. The pyrostatic period is involved with combustion at constant volume, such as in the manometric or pressure bomb, or in the chamber of the weapon before the projectile begins to move. The pyrodynamic period, however, deals with the events concurrent with the motion of the projectile. Experimental measurements with the powder in a pressure bomb give the data characteristics of the powder and provide basic information necessary for ballistic calculations. The examination of the pressure process and the motion of the projectile in the gun barrel lead to a system of equations, the solution of which is considered the "main" problem of internal ballistics. An exact solution requires extensive calculations. For convenience, other methods may be used which are based upon simplifying assumptions and require fewer calculations. However, such methods must be recognized as not being absolutely rigorous; some of the simplified assumptions can lead to large errors if the loading density is high. Internal ballistics for mortars and recoilless guns is appreciably different from that for cannons, howitzers and rifles and, thus, requires special treatment.

An important problem of internal ballistics is the determination of the magnitude of the maximum chamber pressure and the muzzle velocity which depend in large part upon the weight of the charge of powder (load), projectile mass, combustion space, and the dimensions of the powder grains. In the sixteenth century, mathematicians Daniel Bernoulli and Leonhard Euler treated problems in internal ballistics. Euler established the theory of the elasticity of gasses. And, the invention of the ballistic pendulum, by Robins in 1740, was of prime importance as it permitted measurement of projectile velocity. Only recently has the magnitude of the forces acting on the projectile been appreciated.

An extensive development of modern internal ballistics began at the end of the last century when smokeless powders, composed of nitroglycerine and nitrocellulose, were discovered. Since these powders can be formed in definite shapes and sizes, it became possible to control the rate of burning as well as the quantity of gas produced. The development of propellants for solid-fuel rockets is still in its early stages, but propellants for artillery pieces and small arms have reached a degree of perfection.

2. THE MKS MASS SYSTEM AND ITS UNITS

In the past, the engineering (Force) system of units has been used in the field of ballistics. The international (Mass) system used here is that made public in Report No. 149 of the German Bureau of Standards of the (East) German Democratic Republic. At the tenth general conference of the Bureau of Standards in 1954, six basic units and their abbreviations were defined. Those of interest in internal ballistics are as follows:

Dimension	Unit	Abbreviation
Lenth	the meter	m
Mass	the kilogram	kg
Time	the second	s
Temperature	the degree Kelvin	^oK

The adoption of this system provides a common set of units for both Physics and Engineering, and thus eliminates the simultaneous usage of the engineering system and the physics or absolute system. The use of two systems has caused extensive confusion in the past. The change described above is important in the field of ballistics and results in a number of advantages.

Some comments are in order here to demonstrate the difference between the MKS Mass System and the MKS Force System. First the letters M-K-S are the abbreviations for "meter," "Kilogram," and "second." Note that in the MKS Mass System the kilogram is taken as a unit of mass whereas in the MKS Force System (called engineering system by Wolff) the kilogram is a unit of force. In the MKS Mass system the unit of force is the newton and is that force which will produce an acceleration of 1 m/sec^2 with a one kilogram mass. A newton by definition is also equal to 10^5 dyns or about 0.225 lbs. of force. Work is equal to force time distance. Thus, in the MKS Mass System the newton meter also called the "joule" is the unit of work. One joule also is equal to 16^6 ergs and one joule per second is equal tothe "watt," the unit of power. This simple system of units has been most widely used by electrical engineers but should be used by all engineers to eliminate existing confusion about the units of force and mass.

The essential difference between the engineering and the physics systems is in the use of the unit of mass, the kilogram (kg) as a basic unit. Non-homogeneous derived units exist from simple product and quotient formation. If no other factor than unity (1) appears in the result, the derived units are called homogeneous or consistent with each other.

In connection with the above considerations, the concepts of mass and force may be given a brief review. The "mass" of a projectile may be determined by placing the projectile on one side of a weighing scale and balance weights on the other. If the projectile is equal to the weights, the scale will remain in equilibrium, i.e., the scale shows equality of masses. This simple method determines the "mass," for example, of a 100 mm projectile to be 13.2 kg, and the "mass" of a 150 mm projectile to be 43.8 kg. The term "weight" would be wrong here because it signifies a force. Furthermore, the "mass" determination, unlike "weight," is independent of location; the same result would be obtained if the measurement were done at the North pole, at the equator or even on the moon. Thus, "mass" is an intrinsic property of a particular body.

As indicated, the above discussion does not apply equally to the term "weight," and in this book, the term will be avoided. A 100 mm projectile, placed on a table, exerts a force which becomes apparent by the bending of the table-top. This bending is rarely visible, but can be detected easily with sensitive instruments. Obviously, a 150 mm projectile exerts a larger force on the table. From Sir Isaac Newton we have the equation:

$$F = \int ma$$

where "F" is the force, "m" the mass and "a" the acceleration. On the basis of this relationship, a consistent unit for the force is defined. This unit, called a newton (N), is defined as follows: One newton is the force which gives a mass of 1 kg the acceleration of 1 m/s^2. Thus, the unit of force becomes

$$1\ N = 1\ m\ kg\ s^{-2}$$

Units from other systems may be converted to the international system. Such derived units are not necessarily consistent. An example is the kilopond (kp) which is defined as equal to 9.80665 newtons. Thus, we have:

$$\begin{aligned} 1\ kp &= 9.80665\ N \\ &= 9.80665\ kg^{-2} \end{aligned}$$

In earlier days the unit of force in the engineering system was one kg. In order to distinguish between the units of mass and force the term one kilopond (kp) was introduced. One pond (p) is 1/1000 kilopond. Hence:

$$\begin{aligned} 1\ p &= 10^{-3}\ kp \\ &= 9.80665\ (10)^{-3}\ kg\ s^{2} \end{aligned}$$

The determination of the unit of the force, kp (formerly kg), is arbitrary, whereas the determination of the former unit, the kg, was based on the normal gravitational acceleration at 45° geographical latitude. Both projectiles mentioned earlier would cause a different deflection of the table at the pole than at the equator, because gravitational acceleration is larger at the pole than at the equator. This reveals one weakness of the engineering system of units.

Units are also required for pressure, work, energy, heat and power. The unit for pressure is newton per square meter, i.e., it is the uniformly distributed force of one N exerted on one m^2. Thus:

$$1\ N/m^{2} = 1\ m^{-1}\ kg\ s^{-2}$$

Often the engineering term, atmosphere (at), is used for pressure and is equal to 10,000 kp/m^2. Hence:

$$1\ at = 10^{4}\ kp/m^{2} = 98065.5\ m^{-1}\ kg\ s^{-2}$$

For the ballistician not familiar with the kp as a unit of force, it is convenient to remember that 1 kp/cm^2 is equal to one atmosphere (at) or 14.7 psi. The expression of pressure in atmosphere of force per unit area also avoids confusing mass and force units.

The product of force times distance is equal to "work." Work is equivalent to energy and heat. The Joule (J), the wattsecond (Ws) or the newtonmeter (Nm) each define the amount of work done by moving a fixed point the distance of one meter in the direction of applied force of one newton.

Hence:

$$1\ \text{J} = 1\ \text{Ws} = 1\ \text{Nm} = 1\ \text{m}^2\ \text{kg}\ \text{s}^{-2}$$

The unit of heat used is one calorie (cal). This is equal to:

$$1\ \text{cal} = 4.1868\ \text{J} = 4.1868\ \text{m}^2\ \text{kg}\ \text{s}^{-2}$$

One kilocalorie (kcal) is equal to 1000 cal; hence:

$$1\ \text{kcal} = 4168.8\ \text{m}^2\ \text{kg}\ \text{s}^{-2}$$

"Power" is the amount of work done per unit time; its unit is the watt (W) and it is equal to:

$$1\ \text{W} = 1\ \text{J/s} = 1\ \text{m}^2\ \text{kg}\ \text{s}^{-3}$$

The unit of one horse-power (PS) is also used and is equal to 75 kp m/s, or 735.5 W. In addition to the three basic units: length, time and mass, a unit for temperature, the degree Kelvin (°K), is required. The well-known centigrade degree is equal to the Kelvin degree. However, 0°C corresponds to 273 °K, i.e., 15°C corresponds to 288°K. The term "degree" to indicate temperature differences may be used instead of "Kelvin degree." Thus, °K is replaced by the notation: degree.

The following table illustrates the most important derived units:

TABLE 2.1

Important Derived Units

Derived Unit	Units Involved	Abbreviation	Relation to Basic Unit
Velocity	meters/second	m/s	1 m/s = 1 m s^{-1}
Acceleration	meters/second sq.	m/s^2	1 m/s2 = 1 m s^{-2}
Density	kilogram/cubic m.	kg/m^3	1 kg/m^3 = 1 m^{-3} kg
Force	newton	N	1 N = 1 m kg s^{-2}
	kilopond	kp	1 kp = 9.80665 N = - 9.80665 m kg s^{-2} = 1 at cm^2 = 10^{-4} at m^2
	pond	p	1 p = 10^{-3} kp = 9.80665 $(10)^{-3}$ m kg s^{-2}
Pressure	newton/sq. meter	N/m^2	1 n/m^2 = 1 m^{-1} kg s^{-2}
	atmosphere	at	1 at = 10^4 kp/m^2 = 1 kp/cm^2 = 98066.5 m^{-1} kg s^{-2}
Work, energy and heat	Joule	J	1 J = 1 Ws = 1 Nm = 1 m^2 kg s^{-2}
	watt second	Ws	
	newton meter	Nm	
	calorie	cal	1 cal = 4.1868 J = 4.1868 m^2 kg s^{-2}
Power	watt	W	1 W = 1 J/s = 1 m^2 kg s^{-3}
	horsepower	PS	1 PS = 75 kp m s^{-1} = 735.49875 W

Conversion factors are needed for non-homogeneous units The calculations shown in this book generally use the rounded values of conversion factors, i.e., 9.81 is used instead of 9.80665, and 735.5 is used instead of 735.49875; this is within slide rule accuracy.

In all calculations, formulae and equations are written as dimensional equations, i.e., each physical value is given as a product of the actual numerical value and the corresponding unit.

For the velocity "v" we have:

$$v = \ell t$$

where "ℓ" is the length of the completed path and "t" is the time in which the distance was covered. For example: $\ell = 16$ km and $t = 4$ min; then:

$$v = \frac{16\ \text{km}}{4\ \text{min}} = 4\,\frac{\text{km}}{\text{min}}$$

Note that 1 km = 1000 m and 1 min = 60 s; thus:

$$v = 4\,\frac{1000\ \text{m}}{60\ \text{s}} = 4(16.66)\ \text{m/s} = 66.66\ \text{ms}^{-1}$$

For a weapon producing a maximum gas pressure of 2200 atm, the pressure can be re-expressed as follows:

$$2200\ \text{at} = 2200\,\frac{10^4}{\text{m}^2} = 2200\,\frac{10^4\ (9.81)\ \text{N}}{\text{m}^2}$$

$$= 2200\,\frac{10^4\ (9.81)\ \text{m kg}}{\text{S}^2\ \text{m}^2} = 2152\ (10^8)\ \text{m}^{-1}\ \text{kg S}^{-2}$$

$$= 2200\,\frac{(1)\ \text{kp}}{\text{cm}^2} = 2200\,\frac{(14.7)\ \text{lbs}}{\text{in}^2} = 32{,}340\ \text{psi}$$

3. THE MUZZLE VELOCITY AND SOME ADDITIONAL BASIC CONCEPTS

Firearms are heat engines which convert the chemical energy of the powder into the kinetic energy of the projectile. Although only a fraction of the available chemical energy is converted into kinetic energy, it is desirable to obtain the highest conversion percentage possible. The energy which the projectile possesses when leaving the muzzle, the "muzzle energy," is represented by:

$$E_k = \frac{1}{2} m v_o^2 \qquad (1)$$

where "m" is the projectile mass and "v_o" the projectile velocity, or "muzzle velocity," at the moment when the projectile leaves the muzzle. At zero travel from the muzzle, this "muzzle velocity" is identical to the "initial velocity" in external ballistics.

Equation (1) gives the kinetic energy of the projectile. A spin-stabilized projectile rotates about its cylindrical axis and possesses rotational energy, but such energy can generally be neglected as it is often less than 0.5 per cent of the translational energy.

The following examples will illustrate the calculation of the muzzle-energy and other closely associated parameters. The examples also illustrate the application of the mass system and the units mentioned in Chapter 2.

Example 1. What is the value of the muzzle energy of a 150 m projectile with a mass of m = 44.2 kg and a muzzle velocity v_o = 680 m/s^2?

$$E_k = \frac{1}{2} (44.2\ \text{kg})\ 680^2\ \text{m}^2/\text{s}^2$$

$$E_k = \frac{1}{2} (44.2)\ (4.624)\ (10^5)\ \text{m}^2\ \text{kg s}^{-2}$$

$$E_k = 1.022\ (10)^7\ \text{J}$$

Example 2. What is the value of the muzzle energy for a carbine, if m = 9 grams (139 grains) and v_o = 840 m/s (2760 ft/sec.)?

$$E_k = \frac{1}{2} (0.009) \; (70.65) \, (10)^4 \; m^2 \; kg \; s^{-2}$$

$$E_k = 3175 \; J$$

Only about 1/3 of the energy contained in the powder is transferred as muzzle energy to the projectile. The remainder of the energy escapes as heat content and kinetic energy of the escaping powder gases. Based on Example 1 and a conversion efficiency of 1/3, determine the energy content (E_p) of the charge:

$$E_p = 3(1.022) \, (10)^7 \; J = 0.7323 \; (10)^7 \; cal$$

$$E_p = 7323 \; kcal$$

The heat content of nitroglycerine powder is 1150 kcal/kg. In order to obtain the required muzzle energy, the following loading mass is necessary:

$$\frac{7323 \; kcal}{1150 \; kcal/kg} = 6.37 \; kg$$

The corresponding values for Example 2 are as follows:

$$E_p = 3(3175 \; J) = 9525 \; J = 2275 \; cal = 2.275 \; kcal$$

Using nitrocellulose powder with a heat content of 834 kcal/kg, the required loading mass is:

$$\frac{2.275 \; kcal}{834 \; kcal/kg} = 0.00273 \; kg = 2.73 \; gm$$

$$\left(2.73 \times 5.4 \frac{grain}{gram} = 42.0 \; grains\right)$$

The powder gases produced by burning the load exert a pressure on the base of the projectile. These gases accelerate the projectile until it emerges from the muzzle. In order to obtain an approximate average value of the magnitude of the pressure, we can assume a mean or equivalent pressure of the powder gases that remains constant. Note that this is not correct, but is a convenience since the gas pressure increases to a maximum and decreases thereafter.

This assumption permits calculation of an integrated mean gas pressure which will be of interest. A pressure of "$\bar{p}$" acts on the base of the projectile. If "$\bar{p}$" indicates the mean pressure of the powder gases, "q" the projectile cross section, and "ℓ_o" the length of the rifle barrel, the work done by the powder gases is "$\bar{p}q\ell_o$". Neglecting friction losses (small for large guns), this work may be set equal to the muzzle energy:

$$\bar{p}q = \frac{1}{2} m v_o^2 \qquad (2)$$

and the mean pressure becomes:

$$\bar{p} = \frac{m v_o^2}{2\, q \ell_o} = \frac{E_k}{q \ell_o} \qquad (3)$$

Example 3. Using the same data as given in Ex. 1, what is the mean gas pressure if the rifle barrel has a length of 3.75 m?

$$\bar{p} = \frac{E_k}{q\ell_o} = \frac{1.022\ (10)^7\ J}{0.0177\ m^2\ (3.75) m} = 15.4\ (10)^7 \frac{J}{m^3} = 15.4\ (10)^7 \frac{kg}{ms^2}$$

Now:

$$1\ at = 9.81\ (10^4) \frac{kg}{ms^2}$$

Thus:

$$\bar{p} = 1570\ at$$

(Note: The maximum gas pressure is significantly higher than the mean pressure $\bar{p}$.)

Firearms use comparatively large amounts of energy. To show this, we shall calculate the kinetic energy of an automobile with a mass of 1800 kg, (3960 lbs.) moving with a velocity of 80 km/h. (The above velocity is v = 22.2 m/s, or about 50 m.p.h.) Hence:

$$E_k = \frac{1}{2} 1800 \text{ kg } (492.84) \text{ m}^2/\text{s}^2 = 4.44 \, (10)^5 \text{ m}^2 \text{ kg s}^{-2}$$

The muzzle energy of the 150 mm projectile of Ex. 1, was twenty-three times as large.

These large amounts of energy must be liberated in a very short period of time. The approximate time elapsed between the instant the projectile starts its motion and the instant it leaves the muzzle may be determined easily. The projectile motion starts with velocity = 0 and ends with velocity = v_0. Thus, the arithmetic average projectile velocity in the barrel is:

$$\bar{v} = \frac{0 + v_0}{2} = \frac{v_0}{2} \tag{4}$$

If the length of the rifle barrel is ℓ , the approximate time for the transit becomes:

$$t = \frac{\ell}{v} \frac{\text{m}}{\text{m/s}}$$

$$= \frac{2}{v_0} \text{ (s)} \tag{5}$$

In reality the true transit times are somewhat longer. Equation (6) is an approximate formula which gives more realistic values than Eq. (5):

$$t = \frac{2.4}{v_0} \tag{6}$$

For Exs. 1 and 3 in which ℓ = 3.75 m; and v_0 = 680 m/s, Eq. (6) gives:

$$t = \frac{2.4 \text{ (3.75m)}}{680 \text{ m/s}} = 0.0132 \text{ s}$$

and for Ex. 2 with a length of ℓ = 0.48 m for the carbine barrel:

$$t = \frac{2.4\ (0.48\ \text{m})}{840\ \text{m/s}} = 0.00137\ \text{s}$$

The examples above show that a significant difference in transit-time exists between a cannon and a small arm. Here, this difference amounts to a ratio of 10:1. The transit-time is of major importance in determining the lifetime of the weapon, i.e., the number of firings possible with one barrel before it is rendered useless. This is true because for the same gas temperature, the longer the transit-time, the shorter will be the barrel lifetime. The combustion of the powder in the barrel develops temperatures from 2000 to 3500°C which exceed significantly the melting temperature of the steel (1300 - 1500°C). Thus, the longer the exposure of the interior of the barrel to the hot powder gases during a firing process, the shorter the lifetime of the barrel.

Firearms have the characteristic of producing a large amount of work in a very short time. The work done by a force in a unit time of one second (s) is a unit of power such as the watt (W). Hence:

$$1\ \text{W} = 1\frac{\text{J}}{\text{s}} = 1\ \text{m}^2\ \text{kg s}^{-3}$$

The power may be determined by dividing the work of kinetic energy by the time, or for the power, N:

$$N = \frac{E_k}{t}$$

Referring to Ex. 1, we have:

$$N = \frac{1.022\ (10)^7}{0.0.32\ \text{s}}\ \text{J} = 0.774\ (10)^9\ \text{W} = 774000\ \text{kW}$$

(kW = Kilowatt = 1000 watt)

This is a very significant amount of power as shown by comparison with the output of a modern powerplant. Excluding some of the huge powerplants of the USSR (and the USA), the average power of a typical powerplant lies between 100,000 and 500,000 kW. Converting to horsepower (PS) for the case of Ex. 1, where 735.5 W = 1 PS, gives:

$$N = \frac{0.744\ (10)^9\ \text{W}}{735.5\ \text{W}} - 1.052\ (10)^6\ \text{PS}$$

(or about the same as the design power level of McNary Dam on the Columbia River, USA)

i.e., approximately one million PS. The carbine, used in Ex. 2 also produces considerable power:

$$N = \frac{3175\ \text{J}}{0.00137\ \text{s}} = 2.32\ (10)^6\ \text{W} = 2320\ \text{kW}$$

or

$$N = \frac{2.32\ (10)^6\ \text{W}}{735.5\ \text{W}} = 3154\ \text{PS}$$

(or about ten times the horsepower of the larger 1966 automobiles of the USA)

4. SMOKELESS POWDER PROPELLANTS

We have seen that firearms are heat engines of extraordinary power output, i.e., large amounts of energy are converted in a very short time. The combustion process is the responsible event, and, therefore, must occur very quickly. In order to obtain the high power required, a propellant called a "powder" is used. The term has nothing to do with the characteristic implied in "powderlike," but is an inheritance from the days of black "powder."

Before discussing modern propellants, we will consider briefly the precursor black powder which has been used for the past 500 years and continues in limited use today. The action of black powder can be demonstrated by the following simple experiment. A small amount (5g) of saltpeter (potassium nitrate, KNO_3) is heated in a pyrex-glass beaker until it becomes liquid. A pea-sized lump of sulfur now introduced into the molten KNO_3 will burn, developing an intense flame and giving off so large an amount of heat that the beaker will melt. Such rapid combustion is possible because KNO_3 contains a large amount of oxygen which is given up almost instantly. A similar but slower process occurs if charcoal is used instead of sulfur.

Black powder consists of a mixture of saltpeter, charcoal, and sulfur. The proportions of the mixture are selected for the intended purpose. One widely used formula for black powder consists of 75 per cent saltpeter (KNO_3), 13 per cent coal (C), 12 per cent sulfur (S). The combustible materials are the coal and the sulfur, whereas, the saltpeter furnishes the oxygen. Through the addition of sulfur, the mixture becomes more easily ignitable. Thus, the burning of black powder is an ordinary combustion process with the exception that it occurs much more quickly than say that of coal in a furnace; an additional difference is that black-powder combustion takes place in a closed space at a constant or confined volume.

Combustion occurs according to the following equation:

$$2\ KNO_3 + 3\ C + S \rightarrow K_2S + 3\ CO_2 + N_2$$

The right side of the equation shows that the products of black-powder combustion are potassium sulfide (K_2S), carbon dioxide (CO_2) and nitrogen (N_2). Carbon dioxide and nitrogen are gases; potassium sulfide is a solid compound part of which remains as a layer on the walls of the barrel, the rest escaping as smoke. Here, the solid combustion products amount to nearly 41 per cent and influence the overall combustion process in an undesirable manner.

In the firing of modern powders, the propellant grains are converted completely into gas and are, in contrast to the black powder, "smokeless" (or better, "less smoke") powders. The basic ingredient of all modern smokeless powders is nitrocellulose (pyroxilin) which is a combination of cellulose and nitrate groups from nitric acid. Cotton is nearly pure cellulose and scrap-cotton can be used for the production of nitrocellulose. But, it is not necessary to rely on cotton alone, since pure cellulose may also be obtained from wood and straw. In the production of nitrocellulose, very small pieces of cellulose fibers are added to a mixture of nitric and sulfuric acids. The cellulose combines with the nitric acid and the end result is an explosive product, "guncotton."

Explosives are not suitable as propellants because of their high conversion velocity; rapid conversion of explosives is called detonation. To avoid the almost instantaneous conversion of nitrocellulose into gas (which renders it unusable as a propellant) the fibers of nitrocellulose are gelatinized, i.e., the fibers of "gun cotton" are transformed into a uniform mass. This is accomplished with the help of a mixture of ether and alcohol in which nitrocellulose is soluble. The plastic mass thus obtained can be shaped and then dried to evaporate the solvent. However, one disadvantage of this method is that in the evaporation process the nitrocellulose powder becomes porous and hydroscopic. A second disadvantage is that of using a solvent which must be removed later.

Therefore, years ago, attempts were made to find a solvent which could remain in the mass-such a solvent is nitroglycerine. Nitroglycerine is also an explosive, which, when mixed with diatomaceou earth, is known as "dynamite." The mixing process reduces the extrem sensitivity of nitroglycerine so as to lessen its explosiveness; diatomaceous earth is very porous and consists of the shells of dead microscopic organisms, "diatoms." Nitroglycerine is obtained through a process, similar to that used for nitrocellulose, using the combined effect of nitric acid and sulfuric acid on glycerine. In the reaction, the nitric acid combines with the glycerine. "Nitroglycerine powder" consists of a mixture of nitroglycerine and dissolved nitrocellulose; formulations for the powder vary in the percentage of each of these components as well as in the addition of a third component. This is "centralite," an organic nitrogen compound related to the urea, which improves the gelatinizing process, and increases the storage life and reduces the heat content of the powder. A controlled and lower-heat content is important since this allows a better adjustment of the powder to various purposes. Also, propellants with a high-heat content lead to a reduced barrel lifetime.

As mentioned earlier, nitrocellulose and nitroglycerine powders burn completely to gases, and, in this respect, are superior to black powder. A further advantage lies in the possibility of forming the powder into definite shapes. Distinctions are made between the shapes of individual powder particles: leaf-powder, strip or band powder, ring powder, tubular powder, and multichannel powder. Multichannel powder, often called just "channel" powder, consists of powder cylinders, each containing seven channels which run parallel to the cylinder axis. The length of tubular powder grains in general varies with the length of the combustion space. The wall thickness is decisive for the length of the combustion period; hence, the ratio of the diameter of the empty space in the tubes to the total diameter should be about 1:2.5. Instead of strip or band powder common in Germany, some countries such as France and England use powder in the form of long, thin, full-cylinders. Since this powder has the form of a rope or string, it is also referred to as cordit (French corde=rope, string) or cordite (England).

Since the powder grains burn in successive layers, the combustion velocity of the total charge can be regulated to a large degree through the proper selection of the shape. The mathematical treatment of the combustion processes is made possible simply through defining the shape of the burning surface. The internal ballistician may not only calculate the required powder charge for a weapon using a certain powder composition, but may also determine the necessary shape and size of the powder grains. Before the development of the "smokeless" powder, attempts were made to regulate the combustion process of black powder through selection of the proper powder shape. However, the individual grains pressed from black powder tended to fall apart during the combustion process.

After World War I, Germany developed new powder mixtures such as diglycol powder and gudol powder. The incentive for this was the catastrophic need for fat. Glycerine as part of a fat can be obtained essentially only from fats, and, during World War I, the total fat supplies in Germany were used for the production of powder. In the new powder mixtures, nitroglycerine was replaced by diglycol, i.e., diglycol and nitroguandin, respectively. The basic substances required to produce diglycol are coal and chalk. It has been shown that these new powder mixtures possess some favorable properties which are derived from their relatively low heat content. This results in better protection of the barrels giving them a longer lifetime, and in a reduction in muzzle flash.

Diglycol powder also can be adapted to rockets. Inorganic propellants (their major component is ammonia) are also used since such propellants are significantly cheaper than nitrocellulose or nitroglycerine powder.

5. BURNING OF SMOKELESS POWDER AT CONSTANT VOLUME

5.1 THE EQUATION OF STATE AND ABEL'S EQUATION:

The conversion of the propellant from the powder load to powder gases takes place in the gun during the definite, but very brief, interval of time called the combustion period. Factors influencing the length of the combustion period present the most important problems in internal ballistics.

Analysis of the propellant combustion in the chamber and in the barrel of the gun is complicated because the acceleration of the projectile during the combustion period increases the space available for the powder gases, and this prevents the direct use of the simpler relationships of combustion at a constant volume. However, the problem can be overcome by using approximate procedures which permit the use of constant-volume data.

In the determination of constant-volume data, powder is burned in a combustion space of an apparatus known as a pressure, or manometric, bomb. The pressure bomb in Fig. 1 consists of a hollow cylinder (1) made of high-strength alloy steel. It is closed at one end by a movable piston (3). The piston presses against a small copper cylinder (crusher gage) (4) which is held in place by a threaded closure (5). The powder is placed inside the bomb and ignited by an electrically heated glow-wire. The pressure created by the burning powder is transferred by the piston (3) to the copper cylinder (4) which it compresses. The dimensional change in the axial direction is used as a measure of the pressure exerted on the crusher body.

This method measures only the maximum pressure. Pressure as a function of time can be determined by replacing the copper crusher with a piezo-electric quartz crystal or a strain gage used with an oscilloscope. Such methods will be discussed in more detail later.

Usually a series of test firings are conducted in the manometric bomb over a range of maximum pressures to determine the powder constants. In addition to pressure, other important variables include the identity of propellant, amount of load, bomb volume, and maximum gas temperature. A classic relationship known as the ideal gas law exists for ideal or perfect gases. Real gases, such as the powder gas at high pressure, do not precisely satisfy the ideal relationship. In general, the lower the gas density, the more closely the gas follows the ideal equation of state. As an example, the equation of state can be used to predict the behavior of air. The relationship between pressure, volume, weight of gas, and temperature is:

$$pV = LRT \tag{1}$$

where "p" is the pressure, "V" the volume of the bomb, "L" the weight of the gases (taken as equal to the weight of the propellant load), "T" the absolute temperature, and "R" the gas constant which is a characteristic of the gas. Dividing Eq. (1) by L gives:

$$pv = RT \tag{3}$$

where

$$\frac{V}{L} = v \tag{2}$$

Here, V/L is the specific volume of the gas:

$$\frac{1}{v} = \frac{L}{V} = \rho$$

The quantity "ρ" is the density of the gas. From Eq. (3) it follows that:

$$\frac{pv}{T} = R \tag{4}$$

Thus, the gas constant (R) which is characteristic of a particular gas can be determined if the values of p, v, and T are known.

Example 1. Let us determine the gas constant for air. For a pressure of

$$p = 760 \text{ mmHg} = 10332 \text{ kp/m}^2 \times 9.807\text{N/kp} = 101{,}325 \text{ N/m}^2$$

and a temperature of

$$T = 273°K = 0°C$$

and an air density of

$$\rho = \frac{1}{v} = 1290 \text{ kg/m}^3$$

the gas constant for air is:

$$R = \frac{pv}{T} \frac{101325 \text{ N/m}^2}{1290 \text{ kg/m}^3 \ (273°K)}$$

$$R = 286.9 \frac{m^2}{s^2 K} = 286.9 \frac{T}{kg°K}$$

If the given weight (L) of a gas is heated at constant pressure (p) from T_1 to T_2, and the volume increases from V_1 to V_2, the following relationships hold:

$$pV_1 = LRT_1$$

and

$$pV_2 = LRT_2$$

Subtracting the first equation from the second gives:

$$p\,(V_2 - V_1) = LR\,(T_2 - T_1)$$

The product on the left side has the dimensions of N/m^2 and m^3.

Writing only the units:

$$\frac{N}{m^2} m^3 = Nm = J$$

Thus, the left side represents work which is performed by the expansion of a gas having a weight of L kg if the gas is heated from T_1 to T_2.

The gas constant can now be considered as equal to the work done by 1 kg gas when the temperature of the latter is increased 1°K under constant pressure. The gas constant of air is:

$$R = 286.9 \frac{J}{kg°K}$$

The dimensions J/kg °K are consistent with the definition above.

In the engineering system of units, gas constants with other dimensions and other numerical values are used. The dimension of work in this system is written as kpm. Thus:

$$1 \text{ kpm} = 9.807 \text{ Nm} = 9.807 \text{ J}$$

Hence:

$$R = 286.9 \frac{J}{kg\ °K} = \frac{286.9 \text{ kpm}}{9.807 \text{ kg } °K}$$

$$R = 29.27 \frac{kpm}{kg\ °K}$$

The above value of R is found in the literature of ballistics and thermodynamics.

The equations of state, Eqs. (1) and (3), are accurate for ideal gases of low density. Therefore, since the processes in firearms involve high pressures and powder gases with large densities, the equations of state must be corrected. This correction is based on the assumption that the gas consists of molecules which are small with respect to their separation distance. Substituting

T = 0 into Eq. (3) will eliminate the term pv. But this implies that gas has no volume at some finite pressure, contradicting fact. If the gas has a large density, the volume of gas molecules becomes significant. Subtracting the volume of gas molecules from the total volume gives the value of the free volume available between molecules. Thus, the free volume (V) in Eq. (1) may be replaced by the term $V-\alpha L$. Here, α is the volume of 1 kg of powder molecules. It is called the "covolume" and has the dimension, m^3/kg. The product αL can be considered as the characteristic volume of the powder gas molecules. Eq. (1) when corrected for this volume becomes:

$$p\,(V-\alpha L) = LRT \tag{5}$$

If the powder is burned in a pressure bomb, a certain temperature, T_{ex} (the explosion temperature), and a certain gas pressure are reached. T_{ex} and R are characteristic constants for a particular powder. Combining both constants gives a new constant "f," sometimes called the powder force and equal to RT_{ex} or:

$$p = f\,\frac{L}{V-\alpha L} \tag{6}$$

In order to use the values of f given in the existing literature, the dimensions of kpm/kg are used as shown in Table 5.1.

Equation (6) is called Abel's equation after the ballistician, F. Abel who proved its validity for pressures up to several thousand atmospheres. If L is small compared to V, the product αL can be neglected and Eq. (6) becomes the equation of state for perfect gases.

TABLE 5.1

Values of "f" and "α" for Different Powders

Powder Types	$f = \left[\frac{kpm}{kg}\right]$	$\alpha = \left[\frac{m^3}{kg}\right]$
Single Base (Nitrocellulose)	84,000 --- 105,000	$0.90 --- 1.1\times10^{-3}$
Double Base (Nitroglycerine)	90,000 --- 120,000	$0.75 --- 0.85\times10^{-3}$
Diglycol	100,500	1.00×10^{-3}
Black Powder	28,000 --- 30,000	$\approx 0.5\times10^{-3}$

Dividing the numerator and denominator of Eq. (6) by V gives:

$$p = f \frac{L/V}{1-\alpha L/V}$$

The ratio L/V is called the loading density and is indicated by Δ, or:

$$\Delta = L/V \quad kg/m^3$$

and Eq. (6) becomes:

$$p = f \frac{\Delta}{1-\alpha\Delta} \tag{7}$$

The term "loading density" is applied not only in the study of powder constants determined using the pressure bomb, but also in the general study of firearms. This density is equal to the ratio of the powder load to the combustion space volume (V_o) in the chamber. The unit of dimension for V_o is the liter which can be set equal to one cubic decimeter dm^3, or 1000 cm^3. Equation (7) shows that the pressure is a function of the loading density (Δ) and that the pressure increases with increasing loading density.

Figure 2 shows a plot of the final pressure p (atmospheres), i.e., after the powder load is burnt, as a function of the loading densities (kg/dm^3) of single-base (nitrocellulose) and double-base (nitrocellulose and nitroglycerine) powders. The upper curves of Fig. 2 indicate that the rate of pressure rise is not constant but increases with the loading density. If the covolume is neglected ($\alpha = 0$), the lower linear curves indicated by dotted lines are obtained. The two sets of curves deviate strongly, particularly at high loading densities. Thus, covolume can be neglected only up to pressures of approximately 500 atm. Firearms (except mortars) have loading densities which are significantly larger than those used in the pressure bomb. Typical values for loading densities of various weapons are given in Table 5.2.

TABLE 5.2

Typical Loading Densities of Various Weapons

Weapon	Δ, kg/dm^3
Small Arms	0.80 -- 0.85
Long Range Weapons	0.65 -- 0.78
Cannons	0.55 -- 0.70
Howitzers, Full Load	0.45 -- 0.60
Howitzers, Reduced Load	0.10 -- 0.35
Mortars	0.03 -- 0.12

In spite of some of the high loading densities listed in Table 5.2, the high pressures shown in Fig. 2 are not actually reached in the weapons. This is because the projectile moves during the combustion period thereby increasing the volume of containment. Thus, ordinarily the maximum gas pressure does not exceed 3000 atmospheres; the danger of producing excessive pressure if the projectile were hindered in its motion, however, should be obvious.

Although the validity of Abel's equation has been proved for several thousand atmospheres, it has not been proved for extremely high pressures which would occur in pressure bombs with loading densities of 0.6 and 0.7 kg/dm^3.

Example 2. A charge of 25 g of single-base (nitrocellulose) powder is ignited in a manometric bomb having a volume of 200 cm^3. The powder has the constants; $f = 95{,}000$ kpm/kg, and $\alpha = 1.1 \times 10^{-3}\ m^3/kg$. What is the pressure in the bomb at the end of combustion?

$$\Delta = \frac{0.025kg}{0.2\ dm^3} = \frac{0.025\ kg}{0.0002\ m^3} = 125\ kg/m^3$$

$$p = \frac{95{,}000\ kpm/kg\ (125\ kg/m^3)}{1-1.1\times10^{-3}\ m^3/kg\ (125\ kg/m^3)}$$

$$p = \frac{11.875\times10^6\ kp/m^2}{0.8625} = 13.77\times10^6\ kp/m^2$$

$$p = 1377\ at \approx 1380\ atmospheres$$

The powder force (f) and the covolume (α) can be found from the pressure measurements in the manometric bomb. Two equations are required for the simultaneous solution of the two unknowns. These equations can be obtained from pressure measurements using at least two different loading densities. The solution can be performed either analytically or graphically.

Using the graphical-solution method, Eq. (7) may be rewritten as the equation for a straight line, or:

$$p/\Delta = \alpha p + f \tag{8}$$

Values of p/Δ on the y-axis versus p on the x-axis may be plotted as shown in Fig. 3. A series of firings with different loading densities produces a range of pressures which lie on a straight line (g) corresponding to the points obtained from

Eq. (8) (see Fig. 3). The coefficient α is equal to the tangent of the angle β which is the slope of the straight line; hence:

$$\beta = \text{arc tan } \alpha$$

The intercept of the straight line (g) with the ordinate axis gives (f).

Figure 4 shows three characteristic straight lines, 1, 2, and 3, for three different types of powder:

1. Double-Base (Nitrocellulose plus Nitroglycerine) Powder $f = 120{,}000$ kpm/kg; $\alpha = 0.85 \times 10^{-3}$ m^3/kg

2. Single-Base (Nitrocellulose) Powder $f = 95{,}000$ kpm/kg; $\alpha = 1.1 \times 10^{-3}$ m^3/kg

3. Black Powder $f = 30{,}000$ kpm/kg; $\alpha = 0.5 \times 10^{-3}$ m^3/kg

Since two points are required to define a line, pressure measurement at two different loading densities must be made to determine powder constants f and α. If the loading densities are Δ_1 and Δ_2 and the corresponding pressures are p_1 and p_2, then the points A and B are obtained with the following coordinates:

A $(p_1/\Delta_1, p_1)$

B $(p_2/\Delta_2, p_2)$

These points connected by a straight line are shown in Fig. 5. The point of intersection of the straight line AB with the ordinate axis is the powder force f; the slope (angle β) gives the covolume according to the relationship:

$$\alpha = \tan\beta$$

In order to obtain accurate values, very small loading densities should be avoided; also, A and B should be separated so that $\Delta_2 - \Delta_1$ is approximately 0.1 kg/dm^3 or more. Loading densities of $\Delta_1 = 0.15$ and $\Delta_2 = 0.25$ kg/dm^3 are recommended for single-base (nitrocellulose) powder, while values of $\Delta_1 = 0.12$ and $\Delta_2 = 0.20$ to 0.22 kg/dm^3 suffice for the more potent double-base (nitroglycerine) powders.

Better accuracy can be obtained by making more than two test firings using the same kind of powder but different loading densities. If all of the points obtained do not lie on one straight line, an average straight line between all the points may be drawn such that the mean deviation of the individual points from the line is a minimum (method of least squares).

The analytical procedure for determining the two unknowns α and f also requires a minimum of two equations which can be obtained by two firings using two loading densities, Δ_1 and Δ_2. This gives the following two equations (9a) and (9b):

$$p_1/\Delta_1 = f + \alpha p_1 \tag{9a}$$

and

$$p_2/\Delta_2 = f + \alpha p_2 \tag{9b}$$

Subtracting the first equation from the second gives:

$$(p_2/\Delta_2)-(p_1/\Delta_1) = \alpha(p_2-p_1)$$

It follows that:

$$\alpha = \frac{(p_2/\Delta_2)-(p_1/\Delta_1)}{p_2 - p_1} \tag{10}$$

and

$$f = \frac{(p_1/\Delta_1)\ (p_2/\Delta_2)\ (\Delta_2-\Delta_1)}{p_2 - p_1} \tag{11}$$

The analytical method may also be used with more than two test firings by utilizing the method of least squares.

In order to obtain an accurate determination of f and α, the loading densities used must be sufficiently large--a lower limit of $\Delta = 0.1$ kg/dm^3 is recommended. Systematic experiments by A. I. Kochanow have shown that the straight line predicted by Eq. (8) is not obtained when very small loading densities are used (0.015-0.020 kg/dm^3). Instead, the results (A, B, and C) lie on a hyperbolic curve which approaches asymptotically to the straight line g predicted by Abel's equation, (see Fig. 6). To explore this phenomenon, Serebrjakow[1] experimented with powder grains of different thicknesses using equal loading densities. His work showed that the maximum pressure was lower with the thicker powder than with the thinner powder. In Fig. 6, point 1 indicates the point of maximum pressure for the thicker powder, and point 2, that for the thinner powder. The reason for the deviation from Abel's equation was found to be related to heat transfer to the bomb wall. Thick powder has a longer burning time than thin powder and so it transfers a larger amount of heat to the bomb wall before reaching maximum pressure. Thus, the powder force (f) which equals RT_{ex}, and the measured pressures, are low. On the basis of such experiments it is possible to determine correction values for heat losses in the bomb.

5.2 THE POWDER CONSTANTS

In addition to the powder force (f) and the covolume (α) discussed in Section 5.1, other characteristic constants are required to define a particular powder. The heat of combustion (Q) is one of special importance. This quantity is the amount of heat which is created during the burning of 1 kg of powder. The unit used for the heat of combustion is usually the kcal (1000 calories), where 1 cal = 4.1868 m^{2}kg s^{-2} = 1 kcal/1000. Thus, the dimension for the heat from a kilo of powder is kcal/kg. The heat of combustion for double-base (nitrocellulose plus nitroglycerine) powder is about 1100-1200 kcal/kg, whereas diglycol powder has a heat of combustion of only about 740 kcal/kg. Therefore, to obtain the same power with diglycol as with double-base powder, a significantly larger amount of diglycol powder is needed.

The value of Q is determined experimentally by burning a known amount, L kg, of the powder in a calorimetric bomb. The calorimetric bomb is used in many fields besides ballistics to measure the heat of combustion of various compounds. Essentially it is a pressure vessel with a screw top through which an insulated wire is introduced for powder ignition. It has a thinner wall than the manometric bomb and cannot be used at very high pressures. The combined assembly of bomb plus a water bath is known as the calorimeter. To measure the heat of combustion, the bomb is immersed in the bath. The heat given off during combustion causes an increase in the temperature of the calorimeter, i.e., the water and the vessel, and this temperature rise is used to calculate the heat of combustion. Additional details concerning the apparatus and experimental techniques of bomb calorimetry are described in most Physics and Chemistry texts and laboratory books.

Reliable procedures for measuring the explosion temperature (T_{ex}) have not as yet been developed. One difficulty lies in the fact that most temperature recording devices do not follow the temperature with sufficient rapidity, nor have optical methods given satisfactory results. However, the explosion temperature can be calculated with considerable accuracy; the following relationship can be used for this calculation:

$$RT_{ex} = f$$

If the gas constant R is known and the powder force f measured, the temperature T_{ex} may be determined by:

$$T_{ex} = f/R \tag{1}$$

The gas constant R may be determined from Eq. (4) of Section 5.1:

$$pv/T = R$$

The specific gas volume (v_0) expressed in units of m^3/kg is equal to the volume of a unit mass. It may now be introduced to give:

$$\frac{p_o v_o}{T_o} = R$$

For standard conditions:

$T_o = 273°K = 0°C$

$p_o = 760$ mm Hg $= 1$ atm

Thus, if v_o is known, then R is known. However, v_o must be calculated from the reaction equation, which requires a knowledge of the equilibrium conditions for high temperatures, and these values are usually known only approximately. As an alternative method, the value of v_o may be measured, but this procedure includes the same uncertainties.

TABLE 5.3

Values of Q, T_{ex}, and v_o for Three Different Powders

Powder	Heat of Combustion, Q kcal/kg	Explosion Temperature, T_{ex} °K	Specific Gas Volume, v_o m^3/kg
Single-Base (Nitrocellulose)	800 -- 900	2500 -- 3000	0.900 – 0.970
Double-Base (Nitroglycerine)	1100 -- 1200	3000 -- 3800	0.800 – 0.860
Diglycol	720 -- 800	2500 -- 2700	≈ 1.00

Table 5.3 lists general values for typical powders. However, since the heat of combustion and the explosion temperature vary with the formula of the powder, it is possible for values to lie outside the limits indicated if special powder formulae are used.

Example 3. Consider a double-base (nitroglycerine) powder for which $f = 120{,}000$ kpm/kg, and the specific volume (v_o) = 0.850 m^3/kg, and calculate:

a) the gas constant, R;

b) the explosion temperature, T_{ex}.

$$R = p_o v_o / T_o = \frac{10{,}332 \text{ kp/m}^2 \text{ } (0.850 \text{ m}^3\text{/kg})}{273.16 \text{ °K}} = 32.2 \frac{\text{kpm}}{\text{kg °K}}$$

$$T_{ex} = \frac{f}{R} = \frac{120{,}000 \text{ kpm/kg}}{32.2 \text{ kpm/kg °K}} = 3727 \text{ °K}$$

5.3 THE BURNING PROCESS

5.31 The Linear Burning Velocity

The linear burning velocity (u_1) is another characteristic constant of a particular powder;this constant is more complex;thus, the process of combustion must now be considered in some detail. If a long cylindrical grain of smokeless powder in the shape of a pencil such as a piece of cordite is ignited at one end in free air, it will burn with a constant velocity. In a certain time period t, a thickness e of the cylindrical body is burnt:

$$u = \frac{e}{t}$$

This quantity e/t is the linear burning velocity. If the burning velocity is not constant, then e/t is replaced by the differential rate de/dt:

$$u = \frac{de}{dt}$$

Propellant powder differs from most other combustible materials because it can burn in an enclosed space without requiring the oxygen present in the air. It is able to do this because its chemical composition contains the oxygen necessary for its complete combustion. If a powder grain is ignited in an enclosed space, it will start burning only at the point where it has been ignited. However, hot combustion gases develop which soon cover the entire surface of the powder grain with flame, and, at this point, all of the powder grain burns in successive layers perpendicular to its surface. The "combustion velocity" concept of linear burning perpendicular to the surface and in the direction normal to the surface is still applicable. However, the differential linear burning velocity de/dt is not a constant. As burning progresses, the powder gases in the enclosed volume increase, causing the pressure to increase. This increase in pressure in turn causes an increase in the rate of reaction and, concomitantly, a further increase in pressure. The over-all process is increased somewhat like an avalanche with an accelerated chain-reaction type of combustion having a velocity that accelerates, and it eventually terminates in what is commonly called an "explosion."

Thus, an "explosion" is a combustion process in which the reaction velocity continuously increases, in contrast to the burning of propellants in a weapon where the reaction velocity increases to a maximum and then decreases. The reaction velocity and its influence on the system of the gun and bullet can only be determined by experimentally burning the powder in the gun.

In order to represent the combustion process of smokeless powder by an equation, the French ballistician, Vieille, (who produced the first smokeless powder truly successful as a propellant), introduced the following relationship for the linear burning velocity:

$$u = de/dt = A\,p^{\nu} \qquad (1)$$

where A and ν depend on the chemical formulation of the powder, and both are constant for a given powder. For French smokeless powder, Vieille correlated his results using $\nu = 2/3$. However, detailed examinations in 1913 by

Schmitz, a German ballistician at the Krupp laboratories, showed that the value v = 1.0 may be used with many powders. For v = 1.0 Eq. (1) reduces to:

$$u = de/dt = Ap \qquad (2)$$

In Eq. (2) A is the combustion velocity at p = 1 atm (10^4 kg/m^2). The equation for the linear burning velocity (u_1) can be written so that the combustion law now assumes the form:

$$u = u_1 \, p \qquad (3)$$

Here, u_1 is an additional constant which is a characteristic of the powder.

The following values have been established for the two most common types of propellant:

a. Single-Base (nitrocellulose) powder $\qquad u_1 = 0.6 \text{ -- } 0.9 \times 10^{-4} \frac{m/s}{at}$

b. Double-Base (nitrocellulose plus nitroglycerine) powder $\qquad u_1 = 0.7 \text{ -- } 1.5 \times 10^{-4} \frac{m/s}{at}$

These may be used as guide values for initial calculations.

As mentioned previously, the powder grains burn over their entire surface after sufficient pressure and temperature have been reached in the combustion space, and this burning occurs in layers. The combustion velocity (u) is equal to the thickness burned per unit time, and must be defined at some value of the powder-layer thickness. It is usually taken at u_1, which is the initial rate and which corresponds to e_1, the initial thickness.

If the combustion velocity were constant, a layer "a" would burn in one unit time period, etc., as indicated in Fig. 7. But, ordinarily, the linear burning velocity is not constant, and the layer thicknesses indicated in Fig. 7 are different. The values of u_1 (mentioned above) are theoretical ones since they have been calculated for a combustion process at high temperatures using Eq. (3). Actually u_1 depends on several factors such as the chemical composition of the powder, its moisture content, and its temperature.

The value of u_1 is generally larger for double-base (nitroglycerine) than for single-base (nitrocellulose) powders. The percentages of solid nitrocellulose and of liquid nitroglycerine both influence the linear burning velocity: the larger the percentage of these two explosives, the larger the value of u_1. On the other hand, the addition of camphor or vaseline as plasticizers which are, in addition, "desensitizers," will result in the reduction of u_1. Thus, one method of regulating the combustion velocity involves controlling the formula of the powder.

Moisture content has a marked influence on u_1 because water also acts as a desensitizer. Powder with a high moisture content burns more slowly than powder with a normal moisture content. This desensitization can be quite serious. For example if the powder is too damp the burning may be incomplete when the projectile leaves the barrel, causing a sharp reduction in the muzzle velocity of the projectile and erratic grouping at the target. The single-base (nitrocellulose) powders are especially sensitive to moisture because they are quite porous. The average moisture content in powder is about 1 per cent, but can increase to 2 per cent.

The temperature of the powder load before firing also plays an important role. Firing tables for artillery weapons are calculated for a selected powder temperature, usually an ambient temperature of 15°C. At higher temperatures, however, the entire combustion process occurs more quickly, the maximum pressure is larger, and, ordinarily, the muzzle velocity of the projectile is slightly increased. Because of these changes, the influence of the temperature must be determined before a set of firing tables can be prepared. For this purpose a series of "cool," "normal" and "warm" loads are fired, and the muzzle velocity and maximum gas pressure are measured.

Figure 8 shows typical results of such measurements for a range of temperatures. It indicates that the muzzle velocity (v_o) varies with the temperature of the load in a nearly linear fashion for the temperature range measured. On the basis of similar measurements for other powders, the influence of temperature and a correction table for its influence may be established for use with artillery tables. Although the peak of the flame temperature during combustion is limited to a very short time interval, its influence on u_1 should not be overlooked. And, as briefly discussed previously, the thickness of the powder grains has a marked influence on the value of u_1.

Experimental measurements of the influence of temperature on u_1 can also be used to determine whether the powder burns uniformly at both low and high temperatures and if gas pressure excursions might occur at high temperatures which would be unsafe.

An important phenomenon of internal ballistics, sometimes referred to as the Krupp-Schmitz law, is related to the behavior of gas pressure as a function of time. Schmitz studied this behavior using a bomb with a relatively large volume of 3.35 liters. By this technique he was able to contain a volume of powder gas comparable to the chamber volume used in the smaller artillery weapons. A diagram of a cross section of the Schmitz bomb is shown in Fig. 9.

The Schmitz bomb is a high-pressure vessel consisting of a thick-walled, closed steel cylinder. A movable piston (1) is introduced through the front surface and makes contact with a strong spring (2) attached to the outside. This spring carries a mirror (3) which reflects a beam of light onto a rotating drum (4). The drum surface contains a light-sensitive paper. The angle of the light beam is varied with pressure. The time is determined by the angular velocity of the drum.

The record of the combustion process obtained from the drum is similar to the curve shown in Fig. 10. The subscript "e" indicates the end of the combustion process; thus, p_e and t_e are the pressure and the time, respectively, at the end of the combustion. The pressure impulse (I_e) is the pressure impulse of the powder gases at the end of the combustion and is given by the integral,which may be calculated from the area under the curve p(t). Thus:

$$I_e = \int_0^{t_e} p \, dt \tag{4}$$

The pressure impulse (I) for any time t indicates the time dependent area under curve p(t). It is given by:

$$I = \int_0^{t} p \, dt \tag{5}$$

Schmitz performed numerous experiments with tubular powders using loading densities Δ between 0.12 and 0.26 kg/dm^3, and discovered that for the <u>same</u> powder I_e is constant and independent of the loading density. The term <u>same</u> powder implies that the powder has not only the same chemical composition, but also the same shape, and the same dimensions.

Figure 11 gives typical results of three experimental firings using the <u>same</u> powder, but different loading densities. The value shown for p_{1e} is the end pressure for the lowest loading density. With larger loading densities, higher end pressures p_{e2} and p_{e3} are obtained,while the times, t_{e2} and t_{e3}, required for burning become shorter. This occurs because a larger weight of powder produces a more rapid rise in the pressure and thus increases the burning velocity. For the areas under the three curves are proportional to the integrals of the relationship of p(t) and as these areas are equal:

$$I_{e1} = I_{e2} = I_{e3}$$

From this Schmitz concluded the validity of the law:

$$u = u_1 \, p$$

In one of the following chapters, the reasons supporting the Schmitz law as well as some exceptions will be analyzed more closely.

The Schmitz law states that the linear combustion velocity is proportional to the pressure p and to a characteristic constant of the powder u_1 giving the relationship:

$$u = u_1 p$$

The pressure impulse I_e of the powder gases for the same powder at the end of the combustion is constant and independent of the loading density.

This sentence may be reversed and stated as follows: If for a given powder, the pressure impulse I_e of the powder gases at the end of the combustion is found to be independent of the loading density, the linear burning velocity of the powder may be expressed by the following equation:

$$u = u_1 p$$

Another important relationship can be derived from Schmitz's law. This relationship will be illustrated with tubular powder but can be applied to many powders of different shapes. Tubular powder may be assumed to burn uniformly at the same rate from the inside and outside. Thus, the wall thickness may be considered as $2e_1$ (see Fig. 12). After a certain time period, a layer of thickness ''e'' is burnt on the inside and outside. The wall thickness, then, becomes $2e_1-2e$.

$$\frac{de}{dt} = u_1 p$$

gives:

$$de = u_1 p \, dt$$

and thus:

$$e_1 = u_1 \int_0^{te} p\,dt = u_1 I_e \qquad (6)$$

Rearranging gives:

$$I_e = \frac{e_1}{u_1} \qquad (7)$$

or:

$$u_1 = \frac{e_1}{I_e} \qquad (8)$$

Equation (8) permits the determination of the powder constant u_1 because the half-wall thickness e_1 is known and I_e can be measured in the bomb. For all powder shapes, the dimension $2e_1$ controls the burning limit, i.e., when the burnt thickness is $2e_1$ all of the powder is burned.

5.3.2 Dependence of Burning Rate on Size and Shape of the Powder Particles

The powder charge consists of a large number of powder grains whose burning rate depends not only on the chemical composition of the propellant, but also on individual grain shape. By the proper selection of these two factors, the burning rate characteristic of the powder can be adjusted to give the required rate by the system of a given weapon and projectile.

Figure 13 shows six different simple shapes used with powders for small arms. The powder burns perpendicularly to the exposed surface, and for long cylinder (a) it burns both in the direction of its longitudinal axis and in the direction perpendicular to the axis, i.e., radially inward. During combustion, the cylinder becomes both shorter and thinner, so that the ratio of the length to the diameter increases. For the total burning period of long cylinder (a), the radius e_1 (for a diameter of $2e_1$) is the factor controlling the burning rather than the height of the cylinder. On the other hand, for the short cylinder (b), the burning period depends on the height of $2e_1$ rather than on the diameter. A similar relationship (burning controlled by $2e_1$) is valid for the strip powder (e). Similarly, for tubular powder (d) the burning time also depends on the thickness $2e_1$. In German ballistic literature the quantity $2e_1$ is usually called the "wall thickness" and corresponds to what the American-British literature refers to as "web size." Tubular and leaf shapes are used in most German powders. Figure 13-f shows how the dimension $2e_1$ may be used in irregularly shaped powder grains. The Soviet Union and the United States use multi-channel powder in addition to simple powder shapes, but even for these more complex shapes the dimension $2e_1$ is useful.

Figure 14 shows the section of a "seven-channel" powder; diagram (a) shows the cross section at the beginning of combustion and diagram (b) at the moment of disintegration. In Fig. 14a, (D) is the outer diameter of the powder "grain" or body, and (d) is the inner diameter of one channel before the start of combustion. The condition at the end of combustion is shown in Fig. 14b. The powder body disintegrates into 12 three-edged columns. In order to make these columns as small as possible, the powder may be given the cross-section shown in Fig. 15. Grains of this shape are called rosette-powder.

The combustion space in the gun available to the propellant changes during the combustion process because the volume becomes larger with projectile travel. The question can be raised, of what is the value of regulating powder shape? In answer, note that this increase in volume does not occur uniformly, because the projectile velocity steadily increases. Thus, it is desirable--and here is the basic importance of regulating powder shape--that the powder charge burn slowly at first, with the rate increasing as the projectile travels through the barrel at an increasing rate of speed.

The volume of the powder before combustion is indicated by Λ_1. This volume becomes smaller because of combustion during time t; the volume remaining at any time is called Λ. Thus, the relative fraction of the charge which is burnt at a certain time t is indicated by "y," and can be expressed as follows:

$$y = \frac{\Lambda 1 - \Lambda}{\Lambda_1} \tag{1}$$

Consider a spherical powder shape (Fig. 13-c) with a diameter $2e_1$. The initial volume is:

$$\Lambda_1 = \frac{4}{3} \pi e_1^3$$

A layer of thickness e is burnt in time t. The remaining radius is $(e_1 - e)$. Thus, the remaining volume Λ is:

$$\Lambda = \frac{4}{3} \pi (e_1 - e)^3$$

Hence:

$$y = \frac{4/3 \pi e_1^3 - 4/3 \pi (e_1 - e)^3}{4/3 \pi e_1^3} = \frac{e_1^3 - (e_1 - e)^3}{e_1^3}$$

$$y = 3 \frac{e}{e_1} - 3 \left(\frac{e}{e_1}\right)^2 + \left(\frac{e}{e_1}\right)^3 \tag{2}$$

In Eq. (2) e/e_1 is the relative thickness of the burnt layer. For simplicity it is given the symbol "z," or:

$$z = \frac{e}{e_1} \tag{3}$$

Thus:

$$y = 3z - 3z^2 + z^3 \tag{4}$$

Equation (4) is one form of a general relationship that can be shown to hold for all powder shapes, or:

$$y = \ell z + mz^2 + nz^3 \tag{5}$$

Equation (5) is valid for any powder as y increases from 0 to 1.0. At the beginning of burning z = 0, and, according to Eq. (5), y = 0. At the end of the combustion z = 1 and Eq. (5) gives:

$$1 = \ell + m + n$$

Thus, only two coefficients are independent of each other and Eq. (5) may be written as:

$$y = \Psi z\ (1 + \lambda z + \mu z^2) \tag{6}$$

where "Ψ", "λ", and "μ" are called shape factors. Using this notation for the sphere gives:

$$y = 3z\ (1 - z + \frac{1}{3} z^2) \tag{7}$$

and

$$\Psi = 3,\ \lambda = -1,\ \mu = \frac{1}{3}$$

Table 5.3 gives the relationship between shape and the independent values of α and β of some common powder shapes. The values α and β are defined as follows:

Strip powder-

$$\alpha = \frac{2e_1}{2b}\ ,\ \beta = \frac{2e_1}{2c} \qquad \text{(see Fig. 16)}$$

Tubular powder-

β = ratio of wall thickness to length ℓ, or $\frac{2e_1}{\ell}$

Quadratic leaf powder -

β = ratio of thickness to edge length

Solid cylinder powder-

β = ratio of diameter to length

TABLE 5.3 Table of Powder Shape Factors

Powder Shape	Ψ	λ	μ
Strip	$1 + \alpha + \beta$	$-\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}$	$\frac{\alpha \beta}{1 + \alpha + \beta}$
Tubular	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0
Quadratic leaf	$1 + 2 \beta$	$-\frac{2 \beta + \beta^2}{1 + 2 \beta}$	$\frac{\beta^2}{1 + 2\beta}$
Solid cylinder	$2 + \beta$	$-\frac{1 + 2 \beta}{2 + \beta}$	$\frac{\beta}{2 + \beta}$

Example 4: Determine the expression for the value of y as a function of z for a quadratic leaf powder which has a thickness equal to 0.1 of the edge length. By definition, β = ratio of thickness to length, or β = 0.1, and:

$$\Psi = 1 + 2 \beta = 1 + 2(0.1) = 1.2$$

$$\lambda = -\frac{2 \beta + \beta^2}{1 + 2 \beta} = -\frac{2(0.1) + 0.01}{1 + 2(0.1)} = -0.175$$

$$\mu = \frac{\beta^2}{1 + 2 \beta} = \frac{0.01}{1.2} = 0.0083$$

Hence:

$$y = 1.2 z(1 - 0.175z + 0.008 z^2) \qquad \text{(Curve 1)}$$

Figure 17, curve 1, shows this function. The curve 2 of the same Fig. also shows the function y for a sphere:

$$y = 3z\left(1 - z + \frac{1}{3}z^2\right) \qquad \text{(Curve 2)}$$

Figure 17 shows that the fraction (y) of load burnt increases more uniformly for the leaf powder than for the spherical powder. Thus, to understand the burning process better, one must know the velocity at which powder is transformed into gas, i.e., the change of y per unit time or, dy/dt. The quantity dy/dt is called the spatial combustion velocity, and it is discussed in the following section.

The fraction (y) of load burnt increases a differential amount dy in a differential increment, dt. In this time period, the total powder load of weight "L" has been reduced by "dL." Thus, the following relationship exists:

$$dL = Ldy \qquad (8)$$

If "S" is the total surface of the powder charge at time t, the following equation is valid:

$$dL = \rho\, Sde \qquad (9)$$

where ρ is the density of the load.

Combining both equations yields:

$$dy = \rho \frac{S}{L} de \qquad (10)$$

According to Eq. (3), Section 5.31, de can be written:

$$de = u_1\, pdt$$

Substituting this into Eq. (10) gives:

$$\frac{dy}{dt} = \rho \frac{S}{L} u_1\, p \qquad (11)$$

Using S_1 to indicate the initial surface of the total load and expanding the right side of Eq. (11) yields:

$$\frac{dy}{dt} = \rho \frac{S_1}{L} u_1 \frac{S}{S_1} p \qquad (12)$$

This equation gives a relationship between the linear burning velocity and the spatial combustion velocity (dy/dt). The right side can be considered a product of three factors. The first factor consists only of constants and is itself a constant indicated by A:

$$A = \rho \frac{S_1}{L} u_1 \qquad (13)$$

The term A gives the "brisance" or the "quickness" constant of the powder. The ratio L/ρ is the initial volume (Λ_1) of the powder load. Hence:

$$A = \frac{S_1}{\Lambda_1} u_1 \qquad (14)$$

The size of the powder grain influences its characteristics. For example, the ratio S_1/Λ_1 becomes larger as the size of individual powder grains is decreased. This is because the surface area S_1 varies as the square of the size, whereas the volume Λ_1 varies as the cube of the size. Thus the spatial combustion velocity becomes smaller as the powder burns.

The ratios S/S_1 and S_1/Λ_1 are the same whether they refer to a single powder grain or to all the grains in the load. In the following discussion it is often useful to consider individual powder grains. For a cube with an edge length "a," we have:

$$\frac{S_1}{\Lambda_1} = \frac{6a^2}{a^3} = \frac{6}{a} \qquad (15)$$

and, for a cylinder with radius "r" and height "h" we have:

$$\frac{S_1}{\Lambda_1} = \frac{2\pi r^2 + 2\pi rh}{\pi r^2 h} = \frac{2}{h} + \frac{2}{r} \tag{16}$$

Cylindrical powder is usually in the form of long strings, e.g., cordite. Therefore, since h is much larger than r, in most cases 2/h is so small with respect to 2/r that it can be neglected. For the case under consideration $r = e_1$. Hence, the expression for cordite powder may be written as:

$$\frac{S_1}{\Lambda_1} \approx \frac{2}{e_1} \tag{17}$$

For a hollow cylinder (tubular powder) with the outside radius "R," the inner radius "r," and the height "h," the expression becomes:

$$\frac{S_1}{\Lambda_1} = \frac{2\pi (R^2 - r^2) + 2\pi h (R + r)}{\pi h (R^2 - r^2)}$$

$$\frac{S_1}{\Lambda_1} = \frac{2}{h} + \frac{2 (R + r)}{R^2 - r^2} \tag{18}$$

$$\frac{S_1}{\Lambda_1} = \frac{2}{h} + \frac{2}{R - r}$$

where $(R-r) = 2e_1$. Ordinarily h is significantly larger than R-r; so, once again, 2/h may be neglected as compared to 2/(R-r). The expression then reduces to:

$$\frac{S_1}{\Lambda_1} \approx \frac{1}{e_1} \tag{19}$$

These considerations apply equally well to other powder shapes, further demonstrating that e_1 is a constant of basic importance to the spatial combustion velocity (dy/dt). The larger e_1, the smaller the spatial combustion velocity.

A comparison of Eqs. (17) and (19) also shows that the shape, too, plays an important role. In Eq. (17) a 2 appears in the numerator and in Eq. (19) a 1 is present. However, this consideration is valid only for the beginning of combustion.

Besides A, other factors such as S/S_1 and p must be considered. For the case of an exponent of unity on the pressure term, the spatial combustion velocity is directly proportional to the pressure. The factor S/S_1 is the ratio of the instantaneous (burning) surface to the initial surface. For tubular powder this ratio is equal to 1 if end effects are neglected, i.e., if the length of the grain as compared to its diameter is so large that the burnt-off sections at both ends are not significant. For all other powder forms, the ratio S/S_1 is variable.

The surface changes of other powder shapes will now be examined more closely, i.e., the behavior of:

$$\sigma = \frac{S}{S_1} \tag{20}$$

where "σ" is the relative surface of the powder body. As a simple example we may consider spherically-shaped powder. According to Fig. 13-c we may use e_1 as the radius. After a certain time a layer of thickness e is burnt. Hence:

$$\sigma = \frac{S}{S_1} = \frac{4\pi (e_1 - e)^2}{4\pi e_1^2} = 1 - 2\frac{e}{e_1} + \left(\frac{e}{e_1}\right)^2 \tag{21}$$

where e/e_1 is the relative thickness of the burnt layer. We indicate this layer by z whose value changes from 0 to 1. Equation (21) may now be written

$$\sigma = 1 - 2z + z^2$$

Figure 18 shows σ as a function of z for spherically-shaped powder (curve 1). Notice that during the combustion process σ decreases rapidly with increasing z. This means that we are dealing with a "degressive" powder shape.

As a second case, consider a very long solid cylinder such as a "grain" of cordite. In order to simplify the calculation, the burnt-off sections at both ends will be neglected, as will the end surfaces (because of their smallness). Again, the radius is equal to e_1. Indicating the height of the cylinder by h, we have:

$$\sigma = \frac{S}{S_o} = \frac{2\,(e_1 - e)\,\pi\,h}{2\,\pi\,e_1\,h} = \frac{e_1 - e}{e_1} = 1 - z \tag{22}$$

which gives a descending straight line (see Fig. 19, curve 1).

An exact calculation, i.e., including the surface reduction and end surface which were neglected above, for a solid cylinder whose length is 20 times its diameter gives (Fig. 19, curve 2)

$$\sigma = 1 - 1.073\,z + 0.073\,z^2 \qquad \text{(curve 2)} \tag{22a}$$

and, for a solid cylinder whose length is 100 times its diameter:

$$\sigma = 1 - 1.015z + 0.015\,z^2 \tag{22b}$$

Curve 1, Fig. 19 corresponds nearly to the straight line, $\sigma = 1 - z$; curve 2 is also nearly a straight line. This shows that the approximations described previously are valid.

This approximation, however, is only valid if the length of the grain is essentially larger than the diameter. A solid cylinder whose length is equal to its diameter gives the following expression:

$$\sigma = 1 - 2z + z^2 \qquad \text{(curve 3)} \tag{22c}$$

The corresponding curve 3 in Fig. 19 has a definite deviation from a straight line.

In addition to the curves for spherically shaped powder (curve 1) and for solid cylinder powder where its length is 20 times its diameter (curve 2), Fig. 18 presents curves for quadratic leaf powder with a thickness equal to 1/10 of the edge length (curve 3):

$$\sigma = 1 - 0.350\, z + 0.025\, z^2 \qquad (23)$$

and for strip powder with the ratio of the edge lengths 1:10:100 (curve 4):

$$\sigma = 1 - 0.2\, z + 0.003\, z^2 \qquad (24)$$

During the burning of tubular powder, the surface remains practically constant. For long tubular powder where the burnt-off sections at the ends and the end surfaces are negligible, the following expression applies:

$$\sigma = \frac{S}{S_1} = \frac{2\,(r + e)\,\pi\, h + 2\,(R - e)\,\pi\, h}{2\, r\, \pi\, h + 2R\, \pi\, h} = 1 \qquad (25)$$

where "R" is the outer radius, "r" the inner radius, and "h" the height. An exact calculation for tubular powder where the length is 50 times the wall thickness (including the end surfaces and burnt off sections) gives:

$$\sigma = 1 - 0.039\, z \qquad \text{(curve 5)} \qquad (25a)$$

Thus Eq. (25a) demonstrates that during the burning of tubular powder, the surface decreases very slowly and is only 4 per cent smaller at the end of the combustion.

The equation for the spatial combustion velocity, Eq. (12), can also be written without applying the previous relationships; thus:

$$\frac{dy}{dt} = A\, \sigma\, p \qquad (26)$$

As pointed out earlier, a continuous increase in the combustion velocity is desirable. At the beginning of combustion in the gun, this is indeed the case, and p increases rapidly. However, the maximum chamber pressure is soon reached and p decreases thereafter. One means of increasing the combustion velocity is by increasing the relative surface of the powder body. However, Fig. 18 shows that with nearly all simple powder shapes, the relative surface of the powder body decreases, and that most shapes are, therefore, "degressive." The only exception to this is tubular powder where $\sigma = 1$ [see Eq. 25].

Powder shapes in which σ does increase are called "progressive," and these have been the subject of extensive research. One progressive shape is the seven-channel powder (see Figs. 14 and 15). Here, the central channel serves to compensate for the decrease of the outer surface just as in the case of simple tubular powder. The remaining channels contribute to the enlargement of the burning surface. Figure 20 shows the behavior of σ as a function of z for a seven-channel powder with channel diameters $d = e_1$, making a total diameter of 11d and a height of 25d. Note that σ increases to 1.37 at $z = 1$. The powder body then disintegrates into columnar slivers; z increases further but σ decreases quickly, making the powder degressive after the disintegration of the seven channels. Thus, for seven-channel powder, 85 per cent of the powder body burns progressively, and 15 per cent burns degressively.

Test firings, however, show that multichannel powder strongly deviates from theoretical behavior; this has been discussed in Section 5.35.

Another type of definitely progressive powder is "coated powder." Such powders are produced by coating the outer surface of tubular powder with a substance called a deterrent. This coating causes the powder to burn more slowly at the outer surface and, as a result, to burn primarily at the inner surface. Deterrents can, however, create new problems. Some perform well except that the coated powder may then develop a large amount of smoke. Dinitrotoluene is used successfully as a deterrent coating in American rifle powders.

We have seen that $\sigma = S/S_1$ can be represented as a function of:

$$z = \frac{e}{e_1}$$

The same is true for the fraction of the powder burnt (y). Thus, an inter-relationship exists between both equations which permits y to be expressed in terms of z and the powder shape constants.

$$\frac{dy}{dz} = \Psi \ (1 + z \lambda z + 3 \mu z^2) \tag{27}$$

By definition we can write:

$$\frac{dy}{dz} = \frac{dy}{dt} \cdot \frac{dt}{de} \cdot \frac{de}{dz}$$

By substitution we have:

$$\frac{dy}{dt} = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} u_1 p$$

$$\frac{de}{dt} = u_1 p$$

$$\frac{de}{dz} = e_1$$

By combination of terms:

$$\frac{dy}{dz} = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} e_1 \tag{28}$$

Considering Eq. (27) and substitution of terms yields:

$$\frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} e_1 = \Psi(1 + 2 \lambda z + 3 \mu z^2) \tag{29}$$

For $z = 0$, $S = S_1$. It follows from Eq. (29):

$$\frac{S e_1}{\Lambda_1} = \Psi \tag{30}$$

Dividing Eq. (29) by Eq. (30) gives:

$$\sigma = \frac{S}{S_1} = 1 + 2\,\lambda\, z + 3\,\mu\, z^2 \tag{31}$$

The function σ thus depends on the coefficients λ and μ and the variable z whereas Ψ is defined by Eq. (30).

So far we have considered y and σ as functions of the relative burning thickness, z. In some cases, however, it is more convenient to give σ as a function of y, or $\sigma(y)$. This approach is found in both the French and German internal ballistic literature. Its origin is the pioneer work of the French ballistic expert Charbonnier, who considered this relationship as a shape function. His method has been used extensively and permits the expression of all calculations in terms of the independent variable y.

The curves $\sigma(y)$ can be obtained by calculating the values of y and σ for a series of values of z using Eqs. (6) and (31) and cross-plotting with respect to y. Figure 21 shows the curves $\sigma(y)$ for the powder shapes used in Fig. 18. In Fig. 21, curve 1 corresponds to spherical powder, curve 2 to solid cylinder powder, curve 3 to quadratic leaf powder, curve 4 to strip powder, and curve 5 to tubular powder with the same dimension ratios as given for Fig. 18.

5.3.3 The Basic Equation of Pyrostatics

Data obtained from test measurements in the manometric bomb can be combined under the heading "Pyrostatics," so named because combustion is studied under static conditions rather than being associated with expanding gases.

The relationship between pressure and volume is given by Abel's equation which was derived in Section 5.1. This equation applies to the end conditions when the load is completely burnt.

In determining the pressure behavior during the combustion process, it should be recognized that the total combustion space of the bomb is not available, since part of this space is still occupied by the volume of unburnt powder. In addition, the covolume of the powder gases already formed must be considered.

Figure 22 shows three different stages in the burning process. Before the start of combustion, $y = 0$. The free space is shown in Fig. 22 to be equal to the combustion space (V_o) of the bomb minus the space occupied by the load (L), i.e., L converted to volume as L/δ where "δ" is the density of the powder. In the second stage, a fraction y of the load is burnt. The free space (V_y) is reduced by the powder gases, or:

$$V_y = V_o - \frac{L}{\delta}(1-y) - \alpha Ly$$

where $L(1-y)$ is the unburnt load and αLy is the covolume of the powder gases already created. The third stage begins at $y = 1$, the point at which the total load is burnt. At this point:

$$V_y = V_o - \alpha L$$

To use the equation of state Eq. (1) Section 5.1, the pressure and volume of the gases must be related at the moment when the fraction y of the load is burnt. Thus:

$$p_y V_y = RTLy = fLy \qquad (1)$$

As derived above, Eq. (2) may be written to solve for V_y:

$$V_y = V_o - \frac{L}{\delta}(1 - y) - \alpha Ly \qquad (2)$$

By substituting for V_y Eq. (3) is obtained:

$$p_y = \frac{fLy}{V_o - \frac{L}{\delta}(1-y) - \alpha Ly} \qquad (3)$$

or:

$$p_y = \frac{fLy}{V_\circ - \frac{L}{\delta} - L\,(\alpha - \frac{1}{\delta})\,y} \qquad (3a)$$

This gives the dependence of the pressure on y. Introducing the loading density $\Delta = L/V_\circ$, Eq. (3a) may be rewritten as:

$$p_y = \frac{f\,\Delta\,y}{1 - \frac{\Delta}{\delta} - \Delta\,(\alpha - \frac{1}{\delta})y} \qquad (4)$$

or

$$p_y = \frac{fy}{\frac{1}{\Delta} - \frac{1}{\delta} - (\alpha - \frac{1}{\delta})y} \qquad (4a)$$

Equation (4) becomes Abel's equation when $y = 1$.

Figure 23 shows the pressure behavior (p_y) for nitrocellulose powder where $f = 95{,}000$ kpm/kg, $\alpha = 1.1\,(10)^{-3}\ m^3/kg$, and, $\delta = 1650\ kg/m^3$ at a loading density of $180\ kg/m^3$.

This figure shows that the pressure as a function of y deviates only slightly from a straight line which goes through the origin (0,0). This is more precisely so if $\alpha \approx 1/\delta$, as is the case for pure nitroglycerine.

Values of y as a function of p can be found from Eq. (4) using Eq. (4a). Thus:

$$y = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_y} + \alpha - \frac{1}{\delta}} \qquad (5)$$

Equation (5) permits the calculation of the fraction py of the load burnt y that corresponds to a particular pressure.

Since the quantity $\alpha - (1/\delta)$ can usually be neglected as compared to f, the following simplified equation results:

$$y = \frac{p_y}{f} \left(\frac{1}{\Delta} - \frac{1}{\delta}\right) \qquad (6)$$

This is sufficiently accurate for most cases. Equation (6) is, then, a simplified version of Abel's equation.

Examples: A load of 30 gm of single-base nitrocellulose powder is burnt in a manometric bomb which has a combustion space of $V_o = 200$ cm^3. The powder has the constants $f = 101{,}000$ kpm/kg, $\alpha = 0.984$ $(10)^{-3}$ m^3/kg, and, $\alpha = 1620$ kg/m^3.

1. What pressure exists at the end of the combustion?

2. What fraction of the total charge is burnt when the pressure was equal to 1/2 of the final pressure?

   With regard to (1) the loading density is:

$$\Delta = \frac{L}{V_o} = \frac{0.030 \text{ kg}}{0.2 \text{ dm}^3} = 150 \text{ kg/m}^3$$

Using the simplified version of Abel's equation, the end pressure (p_e) may be calculated from Eq. (6) rewritten as:

$$p_e = f \frac{\Delta}{1 - \alpha \Delta}$$

$$p_e = \frac{101{,}000 \text{ kpm/kg } (150 \text{ kg/m}^3)}{1 - 0.984\ (10)^{-3} \text{ m}^3\text{/kg } (150 \text{ kg/m}^3)}$$

$$p_e = \frac{1.515\ (10)^7 \text{ kp/m}^2}{1 - 0.1476}$$

$$= 1.777\ (10)^7 \text{ kp/m}^2 = 1.777\ (10)^3 \text{ kp/cm}^2$$

With regard to (2), Eq. (5) is used to calculate the fraction y of the total load which is burnt to reach one half of the end pressure is:

$$1/2\ p_e = 0.8885\ (10)^7 \text{ kp/m}^2$$

$$\frac{1}{\Delta} - \frac{1}{\delta} = 6.050\ (10)^{-3} \text{ m}^3\text{/kg}$$

$$\frac{f}{p_y} = \frac{1.01\ (10)^5 \text{ kpm/kg}}{0.8885\ (10)^7 \text{ kp/m}^2} = 1.1368\ (10)^{-2} \text{ m}^3\text{/kg}$$

$$\frac{1}{p_y} + \alpha - \frac{1}{\delta} = 11.735\ (10)^{-3} \text{ m}^3\text{/kg}$$

$$y = \frac{6.050\ (10)^{-3} \text{ m}^3\text{/kg}}{11.735\ (10)^{-3} \text{ m}^3\text{/kg}} = 0.516$$

Thus, for the case of a pressure equal to 1/2 final pressure, it turns out that only slightly more than one-half of the charge was burnt.

5.3.4 The Linear Burning Law of Muraour and Aunis*

The representations given so far have been mostly theoretical. In deriving the spatial combustion velocity, geometrical considerations were used as the basis. Also, it was assumed that at any given pressure the powder grains burn uniformly over their total surface with the same linear velocity, and that the powder grains do not influence each other.

The linear burning law, $u = u_1 p$, was verified experimentally by Schmitz for loading densities of 0.12...0.26 kg/dm^3. This showed that the pressure impulse for the same powder has the constant value:

$$\int_0^{t_e} p\,dt = I_e = \frac{e_1}{u_1} \tag{1}$$

Muraour and Aunis have established a different equation for the linear burning velocity based on theoretical considerations. This law is given by:

$$\frac{de}{dt} = a + bp \tag{2}$$

where "a" and "b" are constants. In theory, the quantity "a" involves loss in pressure because of the transfer of heat by conduction, and "b" involves the increase in pressure because of the transfer of energy by molecular motion.

Calculating the impulse integral for this new law gives:

$$e_1 = at_e + b\int_0^{te} p\,dt \tag{3}$$

or

*Muraour-Aunis, "Verbrennungsgesetz Rolloidaler Pulver," Explosivstoffe 2, 1954, S. 154 ff.

$$I_e = \int_0^{t_e} p\,dt = \frac{e_1}{b} - \frac{a}{b} t_e \qquad (4)$$

This equation does not consider the impulse integral (I_e) a constant, but instead it is considered to be a function of the burning period t as well as of the loading density (Δ). The result of the subtraction is that I_e becomes smaller; and u_1 in Eq. (1) is no longer a constant for the entire burning. Now u_1 depends on the length of the burning period, becoming larger as the burning period becomes longer.

In experiments conducted in Russia by M. E. Serebrjakow and A. I. Kochanow, changes in I_e were observed only in rather thick types of powder having a considerable burning thickness (e_1) which had longer burning periods. On the other hand, fast burning powders did not show any influence of I_e on the loading density. This implies that the length of the combustion process plays an important role in slow-burning powders. This might be expected, because in slower combustion processes the heat of the hot combustion gases has more time for transfer and to influence the temperature of the total powder mass. Consequently, the temperature of the interior of the powder grains is increased, which in turn increases u_1. The net result is to reduce the dependence of I_e on time. Experiments with loading densities significantly smaller than those used by Schmitz ($\Delta < 0.10$ kg/dm^3) have verified the statements above regarding the increase of u_1.

In summary, for rifles with full loads of fast or medium burning powders the following law is valid:

$$u = u_1 p$$

For longer burning periods such as for cannons and slow burning powder, however, u_1 is not constant. It becomes larger because of heating during the combustion process, and this increase in turn causes a reduction in the total impulse (I_e). This can be expressed by the following burning relationship:

$$u = \frac{de}{dt} = a + bp$$

or by an equation of the form:

$$u = \frac{de}{dt} = Ap^{\nu} \tag{5}$$

with $\nu < 1$. The relationship expressed by Eq. (5) may be written:

$$de = Ap^{\nu}\,dt = Ap^{\nu}\,\frac{p^{1-\nu}}{p^{1-\nu}}\,dt = \frac{A}{p^{1-\nu}}\,pdt$$

A constant average value, $(p^{1-\nu})m$, obtained through integration may be substituted for $p^{1-\nu}$. Thus:

$$e_1 = \frac{A}{(p^{1-\nu})m} \int_0^{te} pdt$$

or

$$I_e = \int_0^{te} pdt = \frac{e_1}{A}\,(p^{1-\nu})m \tag{6}$$

Increasing the loading density increases $(p^{1-\nu})m$, since increasing the largest value also increases the average value, where $\nu < 1$, and I_e is no longer constant. The total impulse becomes larger with increasing loading density and vice versa.

5.3.5 Experimental Examination of Progressivity

Degressive and progressive powder shapes were defined in Section 5.3.2. For degressive forms:

$$\sigma = \frac{S}{S_1}$$

The value of σ decreases for degressive forms and increases for progressive forms. At constant pressure p:

$$\frac{dy}{dt} = \frac{S_1}{\Lambda_1} u_1 \frac{S}{S_1} \quad p$$

The value of dy/dt decreases or increases with degressive or progressive powders, respectively. In order to obtain a measure of progressivity, the relationship dy/dt must be obtained at constant pressure. Of course, dy/dt is the rate of the conversion of powder grains to powder gas and, as indicated previously, this rate is greatly influenced by pressure. If u_1 is directly proportional to pressure, however, we can "normalize" dy/dt by dividing by p to give:

$$\frac{1}{p} \frac{dy}{dt}$$

For convenience, this relation will be given the symbol Γ and called the intensity of the gas formation or:

$$\Gamma = \frac{1}{p} \left(\frac{dy}{dt}\right) \tag{1}$$

or:

$$\Gamma = \frac{1}{p} \left(\frac{dy}{dt}\right) = \frac{S_1}{\Lambda_1} u_1 \frac{S}{S_1} = \frac{S_1}{\Lambda_1} u_1 \sigma \tag{2}$$

The value of Γ can be determined experimentally on the basis of pressure measurements in the manometric bomb.

Figure 24 shows the results of test firings of tubular powder and gives Γ as a function of y. An experiment with strip powder gave nearly the same results. In Fig. 24 Γ_T is the theoretical curve based upon the following equation:

$$\Gamma_T = \frac{S_1}{\Lambda_1} u_1 \sigma$$

The experimental curve of Γ sometimes deviates considerably from the theoretical curve. As shown in Fig. 24, the curve can be divided into four sections for the purpose of analysis. These sections will now be considered in more detail.

Section I. The curve does not start with the theoretical value. The reason for this is that the total powder load does not ignite instantaneously as assumed for the theoretical condition. The value of Γ reaches a maximum at $y = 0.05...0.08$. (The little excursion above the theoretical value may be related to deviation from the linear relation between pressure and burning rate.)

Section II. The value of Γ decreases and slowly approaches the theoretical curve. This section includes the values of $y = 0.05...0.08$ to $y = 0.30$, and covers an accelerated combustion process.

Section III. This section indicates normal combustion which corresponds to the geometrical law and occurs in the region $y = 0.3...0.85$ to 0.90.

Section IV. In this last section, the experimental curve deviates strongly from the theoretical curve. The experimental curve decreases quickly and approaches a value of zero.

Figure 25 shows $\Gamma(y)$ for a seven-channel powder. Whereas for tubular and strip powders Γ still lies in the approximate vicinity of the theoretical curve over a comparatively large section, a completely different behavior is now experienced. In the case shown, the predicted progressivity does not occur. The reason for this is that some stoppage occurs in the narrow channels, preventing the powder gases from escaping uniformly. Due to the pressure increase resulting from this confinement, the powder grains are prematurely disintegrated.

The location of the peak in the curve of Γ for tubular and strip powders with respect to the magnitude of Γ depends largely on the composition of the powder. Powder with solid solvent (Trotyl and Pyroxillin) has a significantly lower peak than powder with a liquid solvent. It is believed that the number and the size of the pores have a great influence on the height of the peak.

The behavior of Γ can be modified significantly if the outer layers of the powder are desensitized. This can be accomplished by coating the powder with a deterrent (see Section 5.3.2). Following this, the more favorable behavior of curve 2 instead of curve 1 of Fig. 20 occurs.

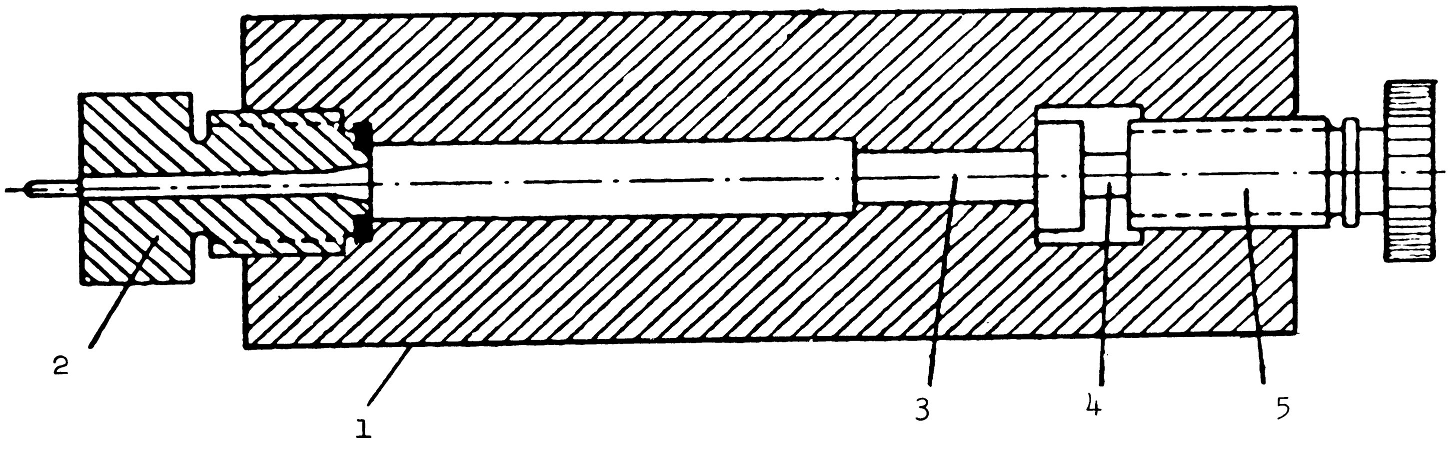


Fig. 1. Sectional diagram of a pressure bomb for measuring properties of propellants.

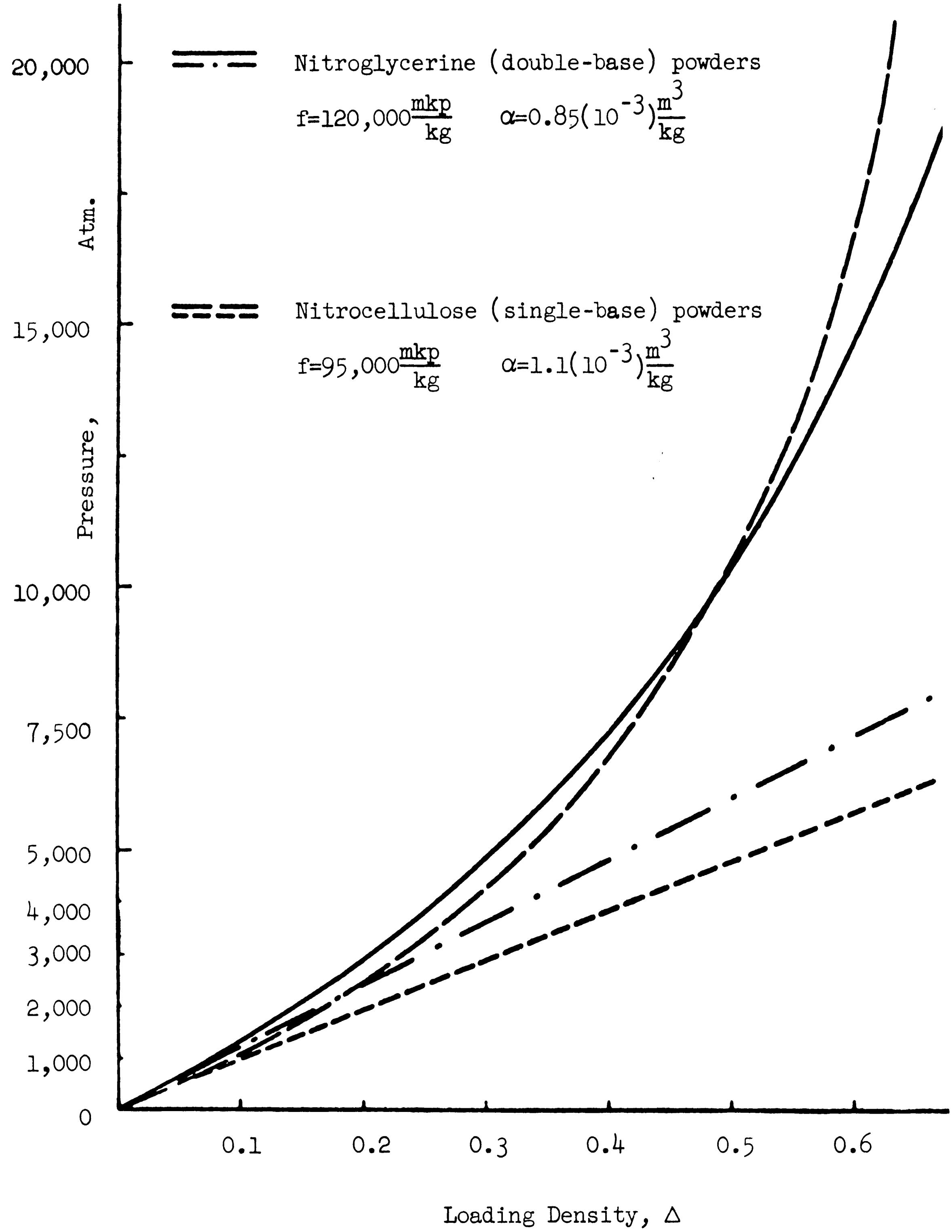


Fig. 2. Combustion bomb pressures plotted as a function of loading density for single-base nitrocellulose and double-base nitroglycerine powders.

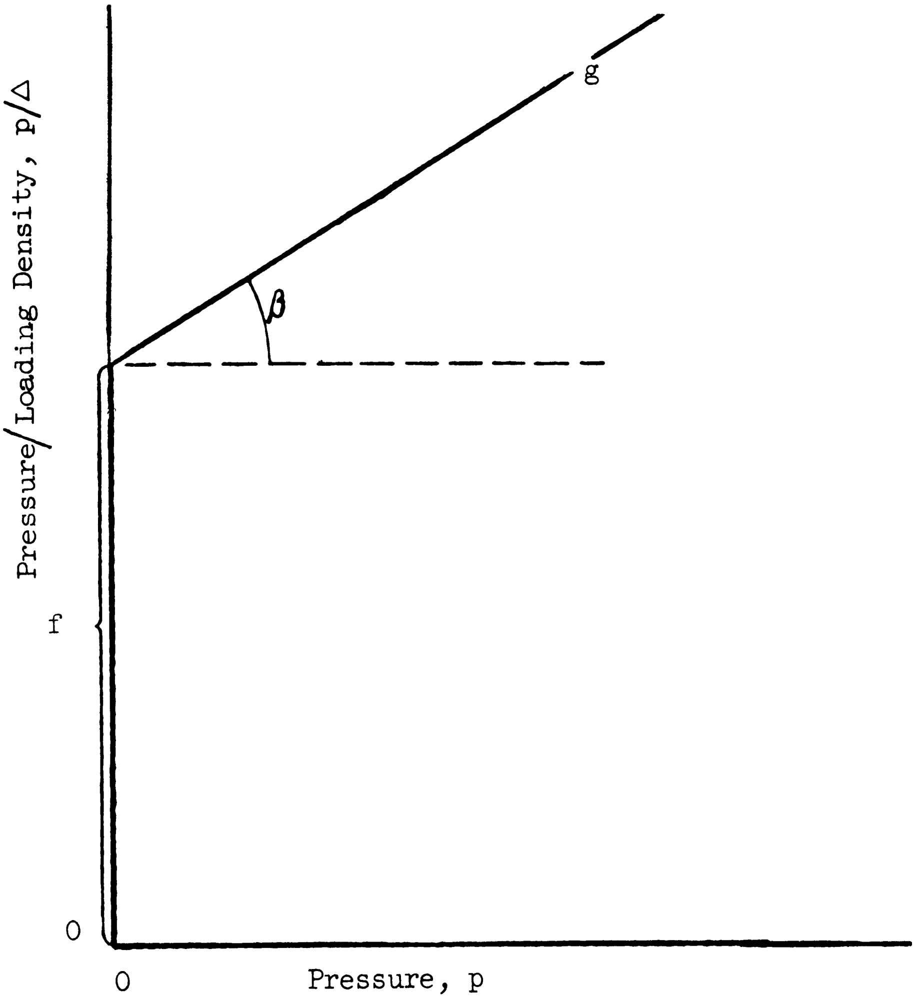


Fig. 3. Pressure as a function of $(f+p/\Delta)$ from pressure bomb firings of one powder using different loading densities.

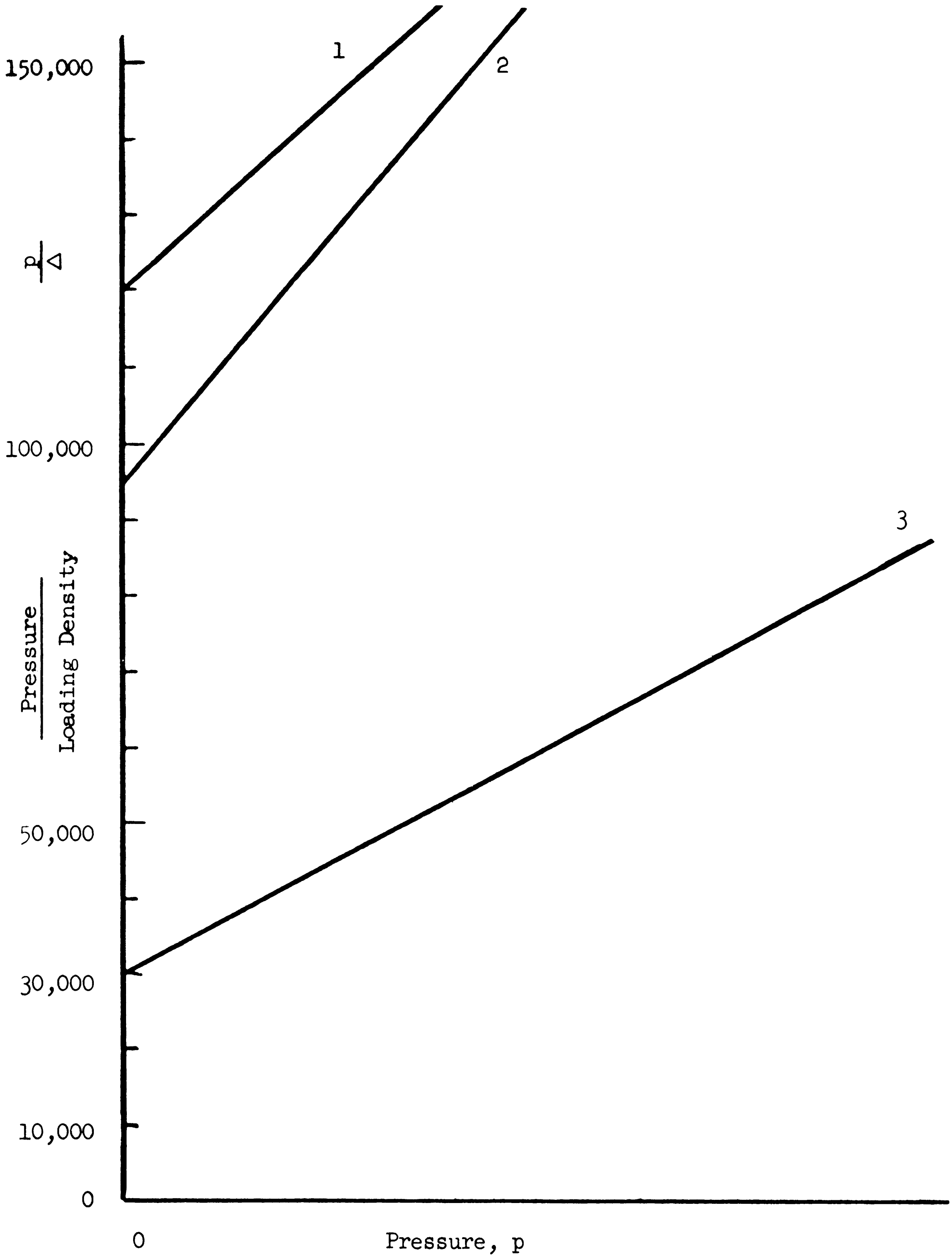


Fig. 4. Comparison of data from pressure bomb firings of three types of powder.

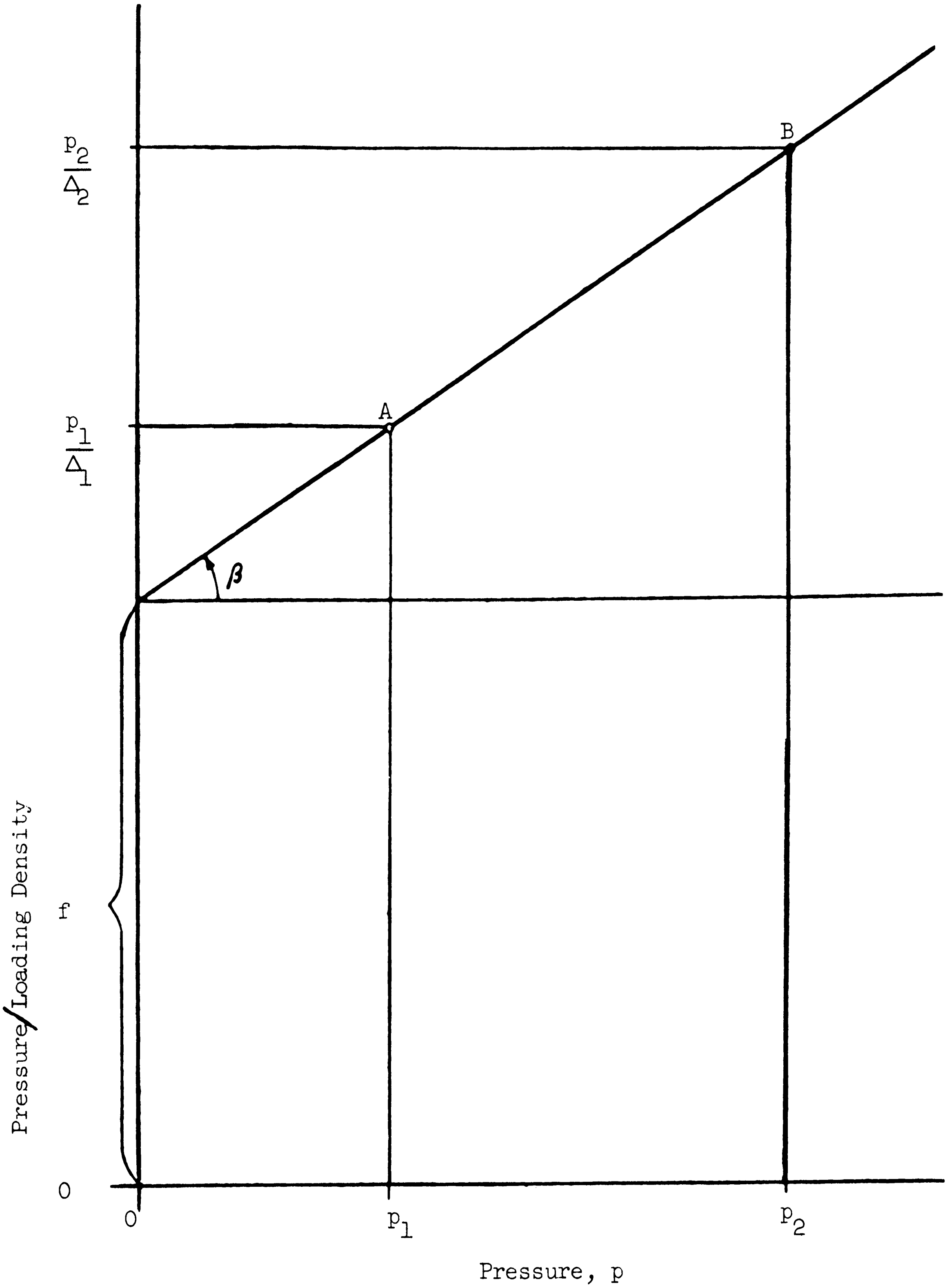


Fig. 5. Graphical procedure for determining f and α from two firings of one powder.

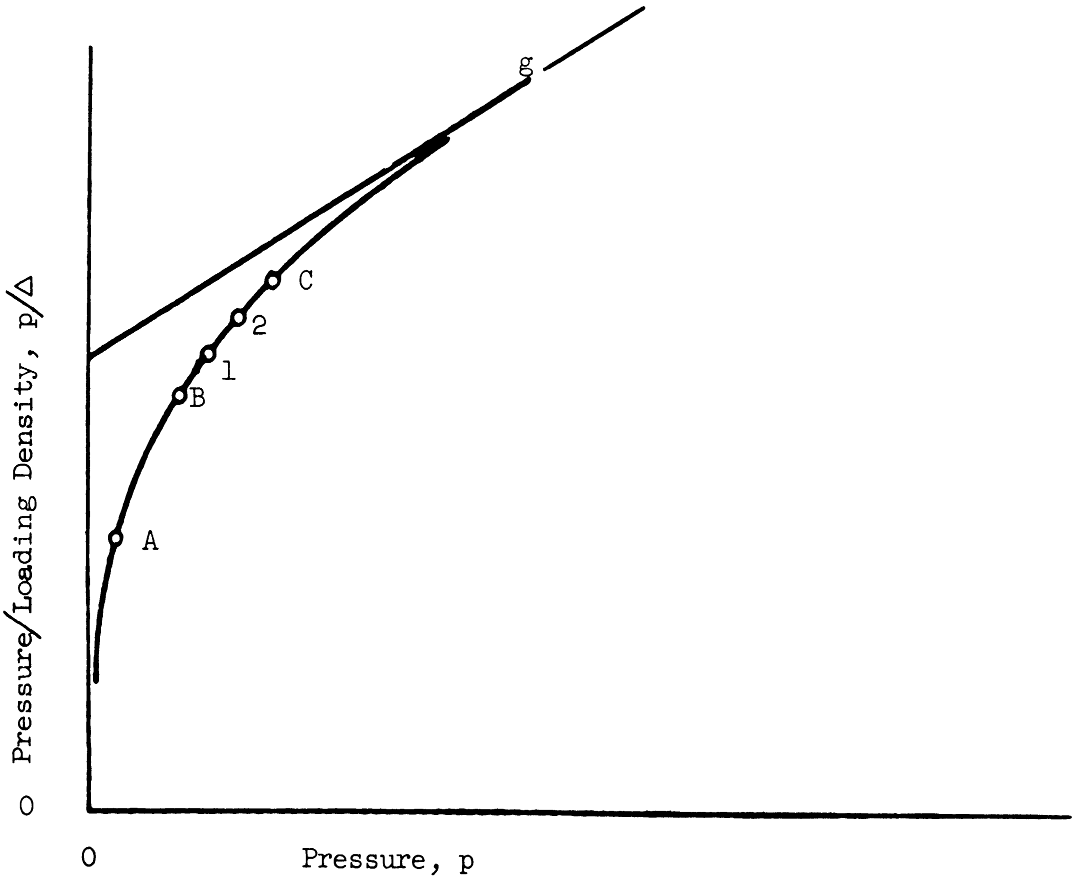


Fig. 6. Serebrjakow's data for pressure bomb firings with low loading densities.

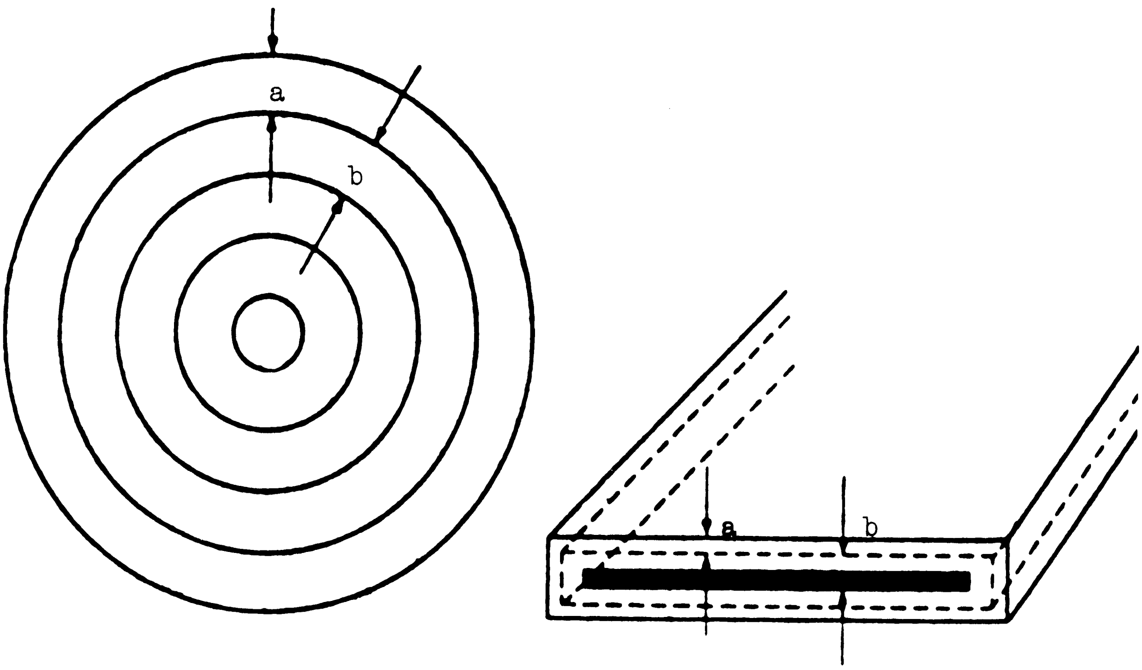


Fig. 7. Diagram of powder layers as they burn successively from outside to inside.

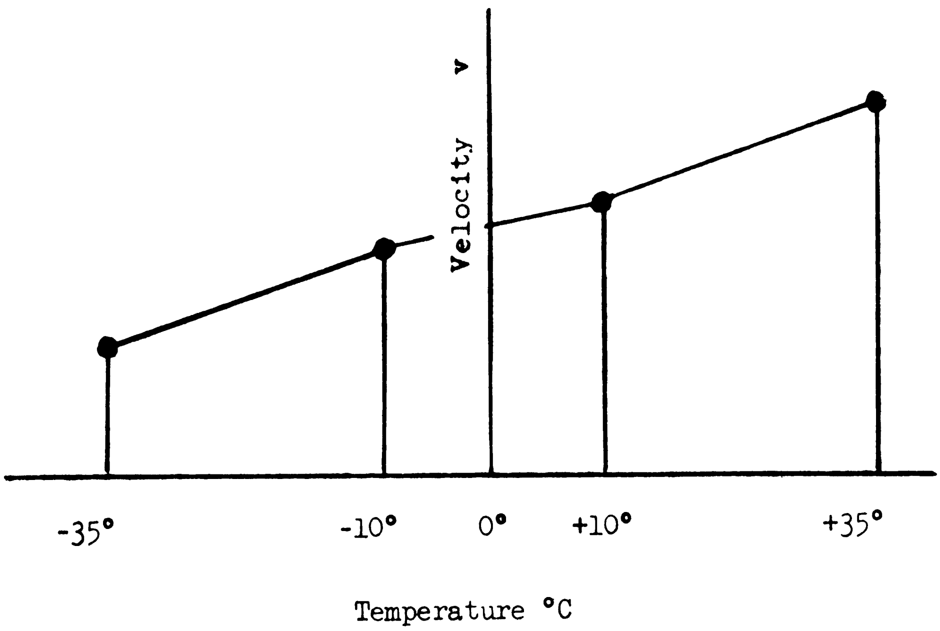


Fig. 8. The typical influence of powder temperature on muzzle velocity.

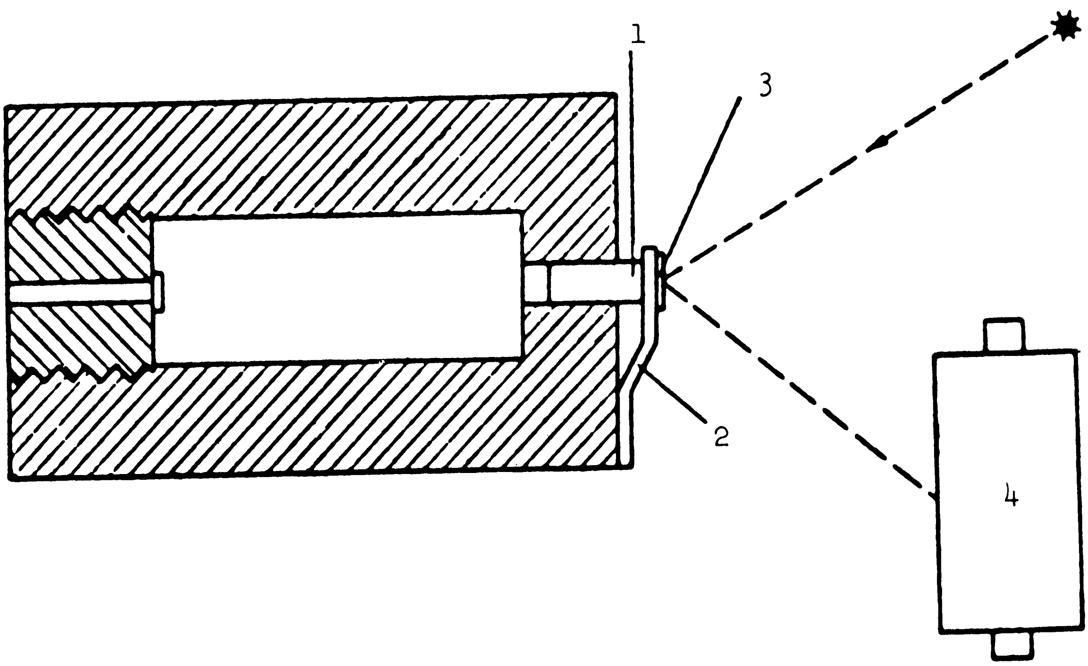


Fig. 9. Diagram showing construction of the Schmitz Bomb.

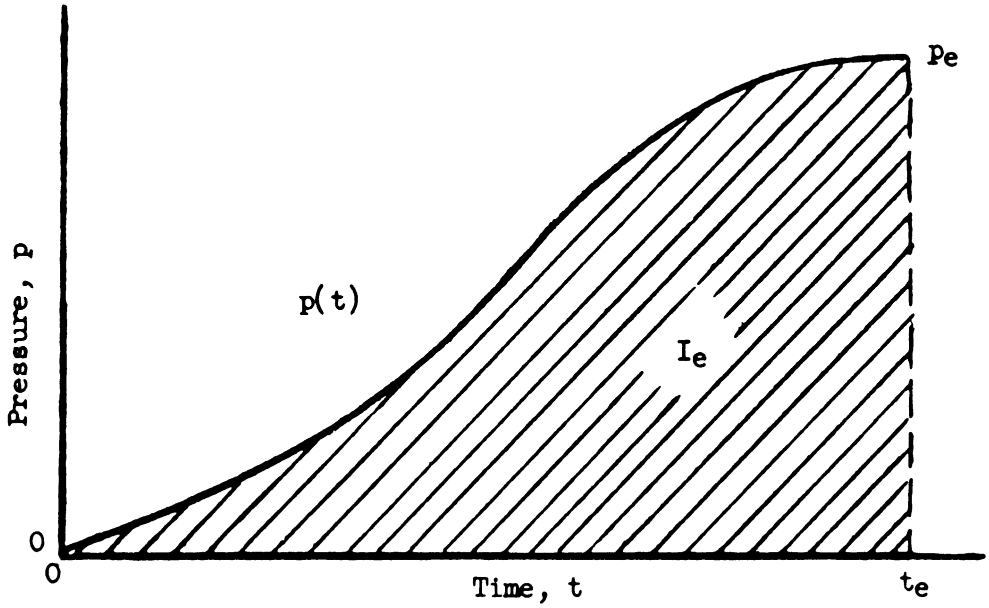


Fig. 10. Curve of pressure versus time as obtained from the Schmitz Bomb.

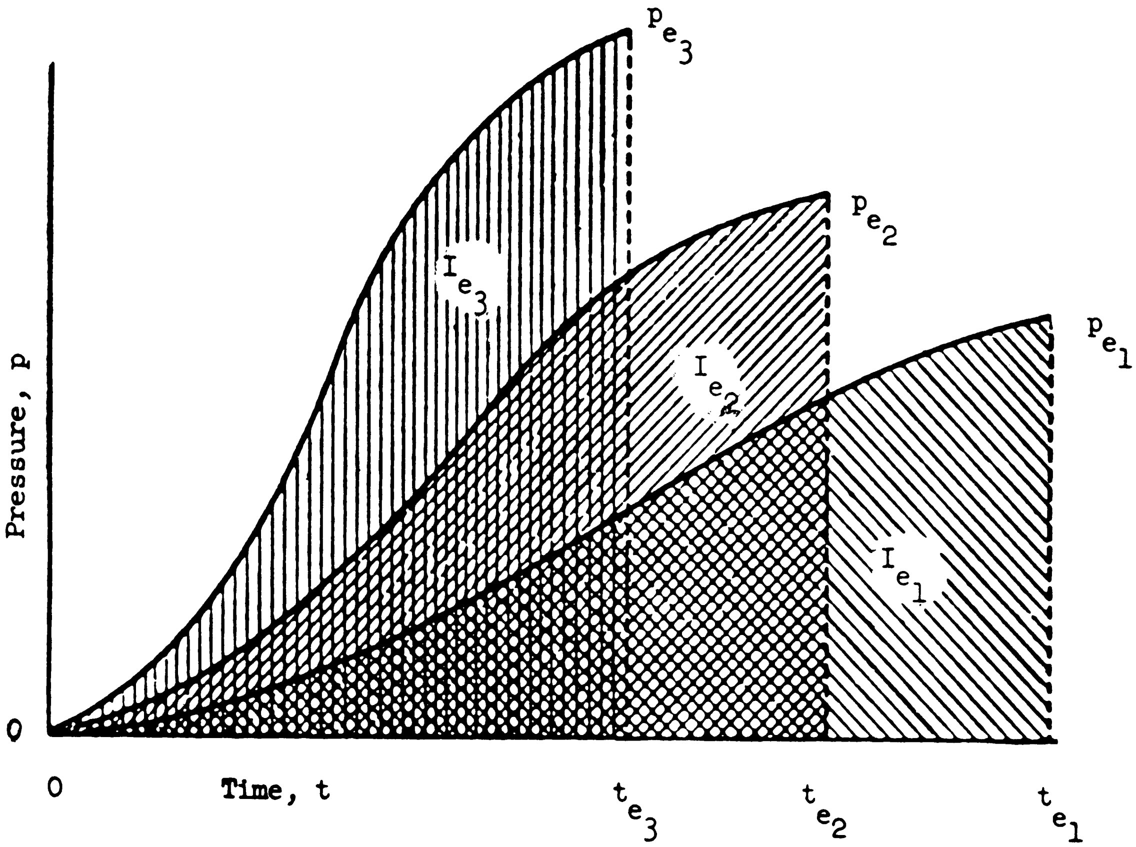


Fig. 11. Schmitz curves for three loading densities and one powder which give constant values of pressure impulse, I_e.

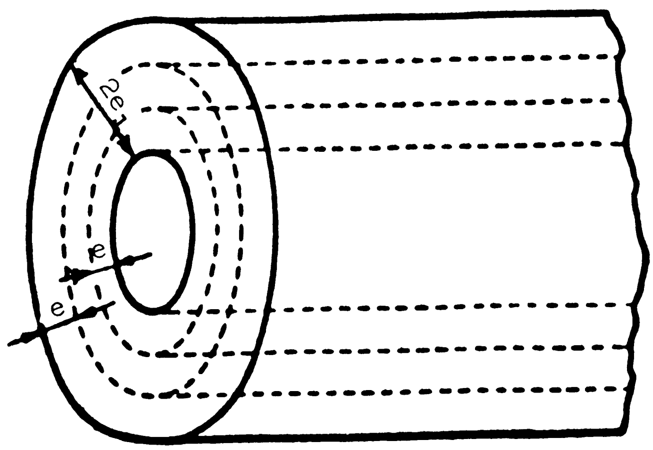


Fig. 12. Diagram of burning layers in tubular powder.

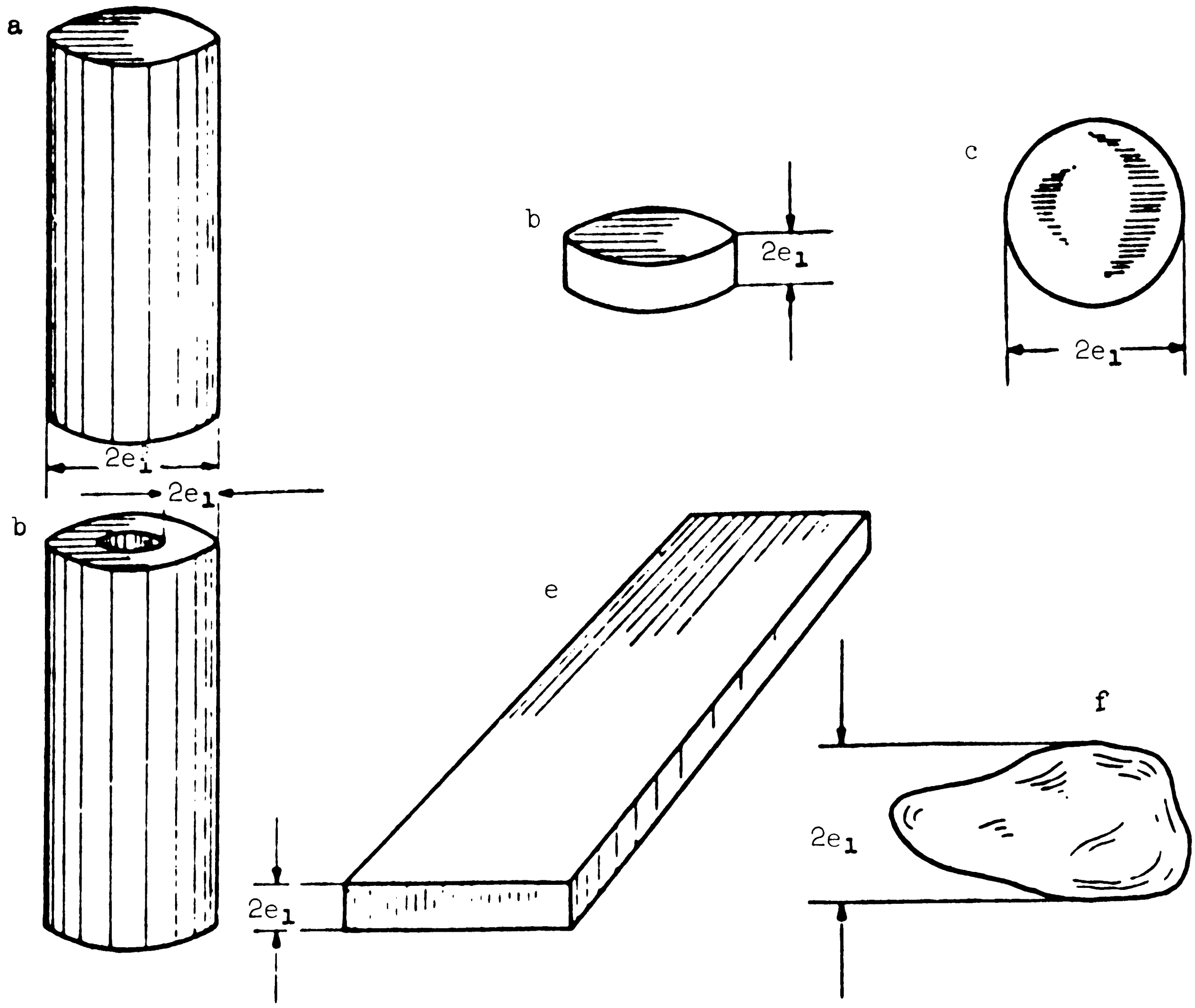
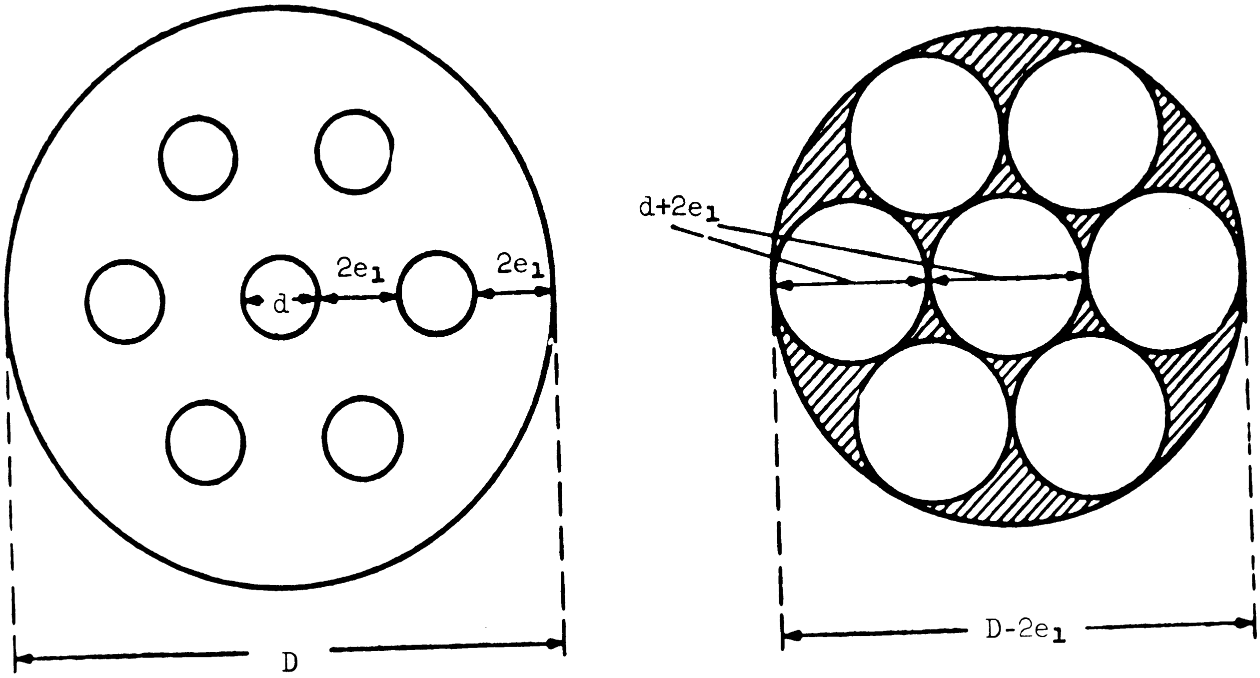
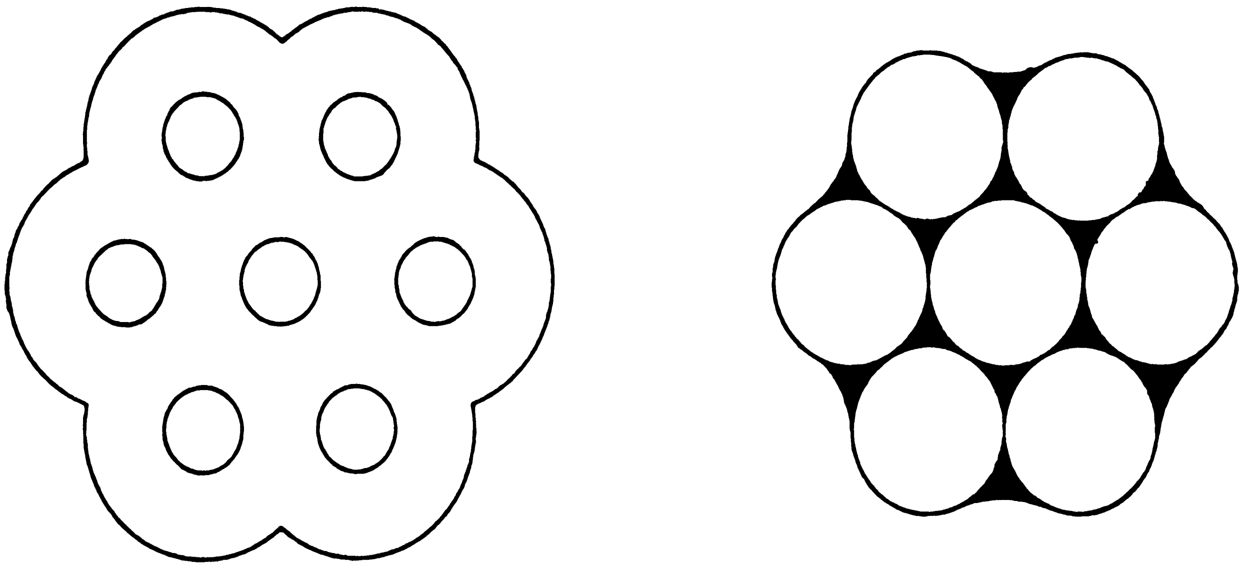


Fig. 13. Six different shapes commonly used for rifle powders.



a) b)

Fig. 14. Seven-channel artillery powder: (a) before burning, and (b) at the instant of splintering.



b)

a)

Fig. 15. Rosette artillery powder: (a) before burning, and (b) at the instant of splintering.

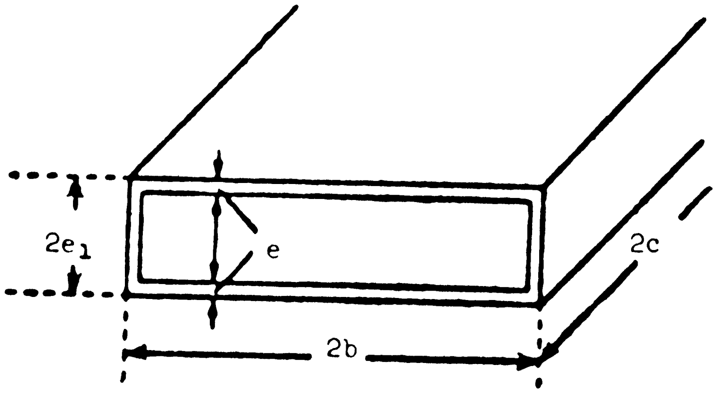


Fig. 16. Significant dimensions for burning of strip powder.

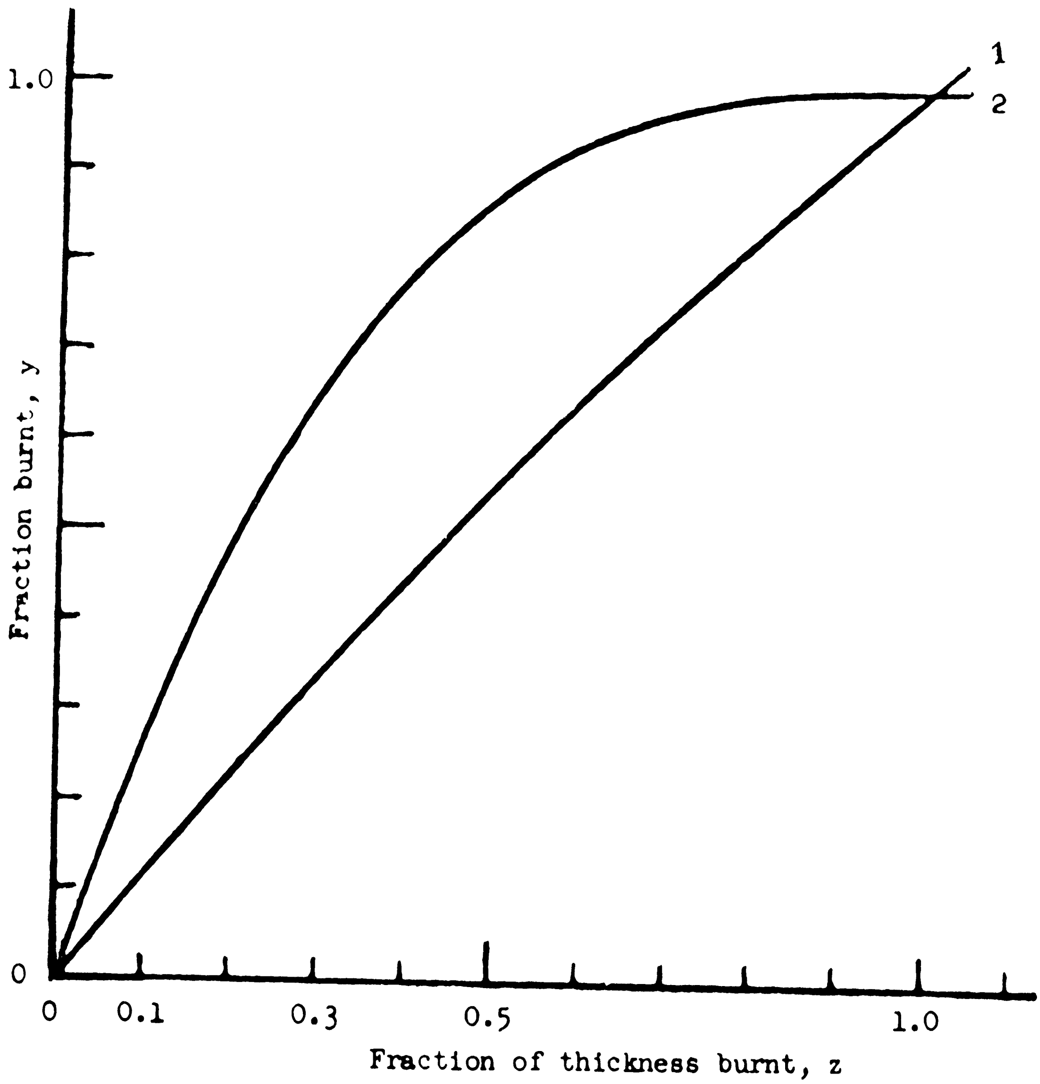


Fig. 17. Fraction of thickness burnt (z) versus fraction burnt (y) for quadratic leaf powder (Curve 1) and spherical powder (Curve 2).

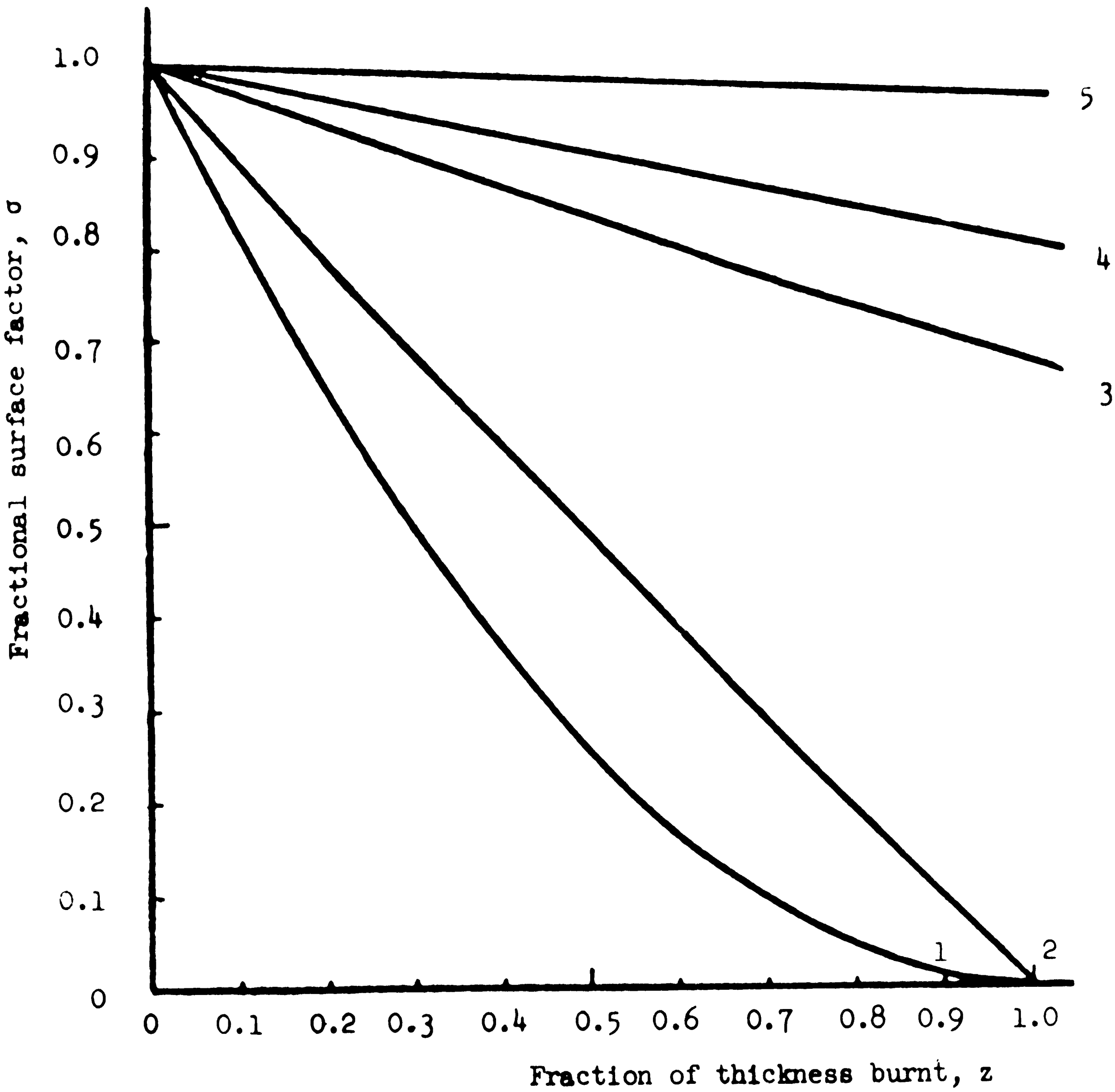


Fig. 18. Influence of the fractional surface factor (σ) of the powder grain on the fraction of thickness burnt (z) for different powder shapes.

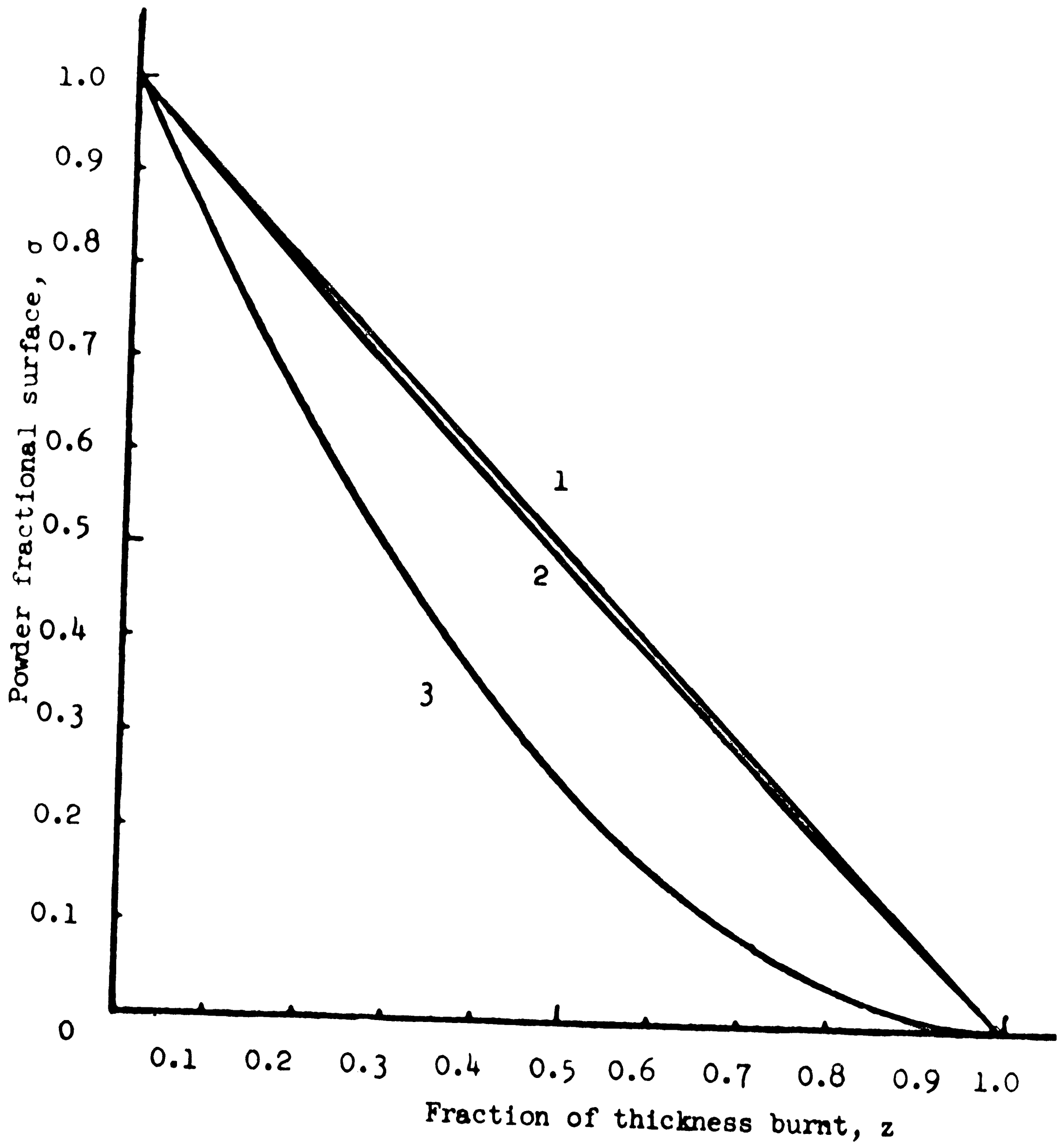


Fig. 19. Powder fractional surface (σ) for three cylindrical powder lengths: long (Curve 1), intermediate (Curve 2), and short (Curve 3).

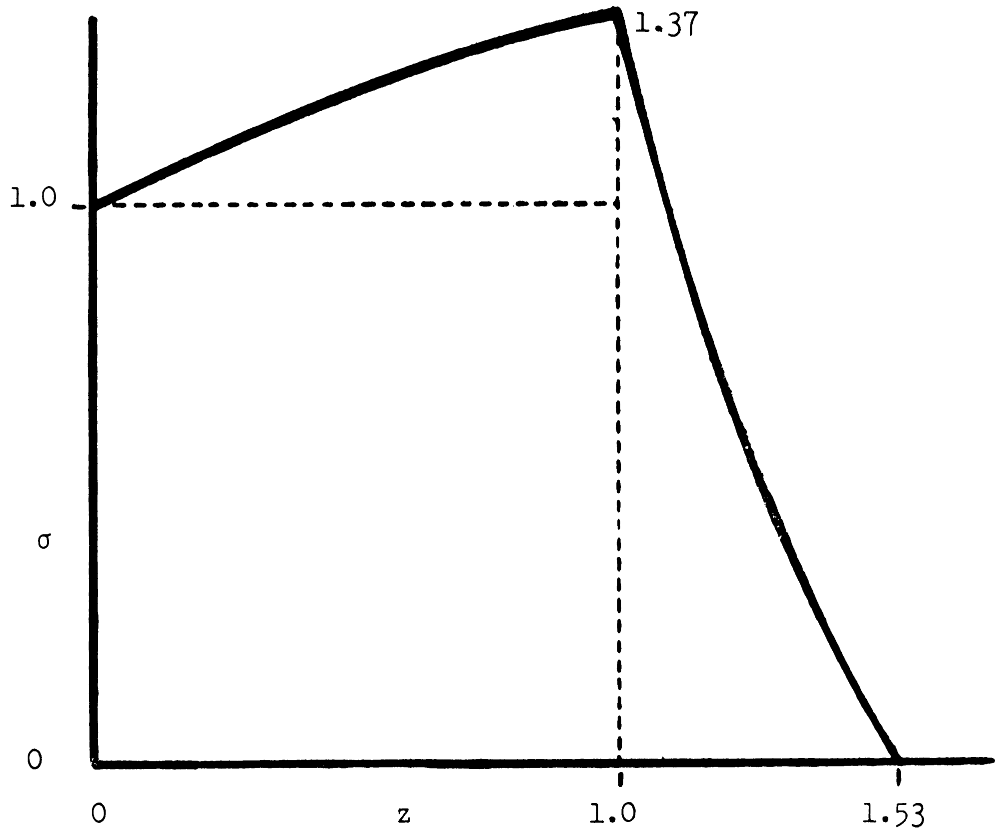


Fig. 20. Increase in the powder fractional surface (σ) versus z for seven-channel progressive powder (σ = 1.37 max.).

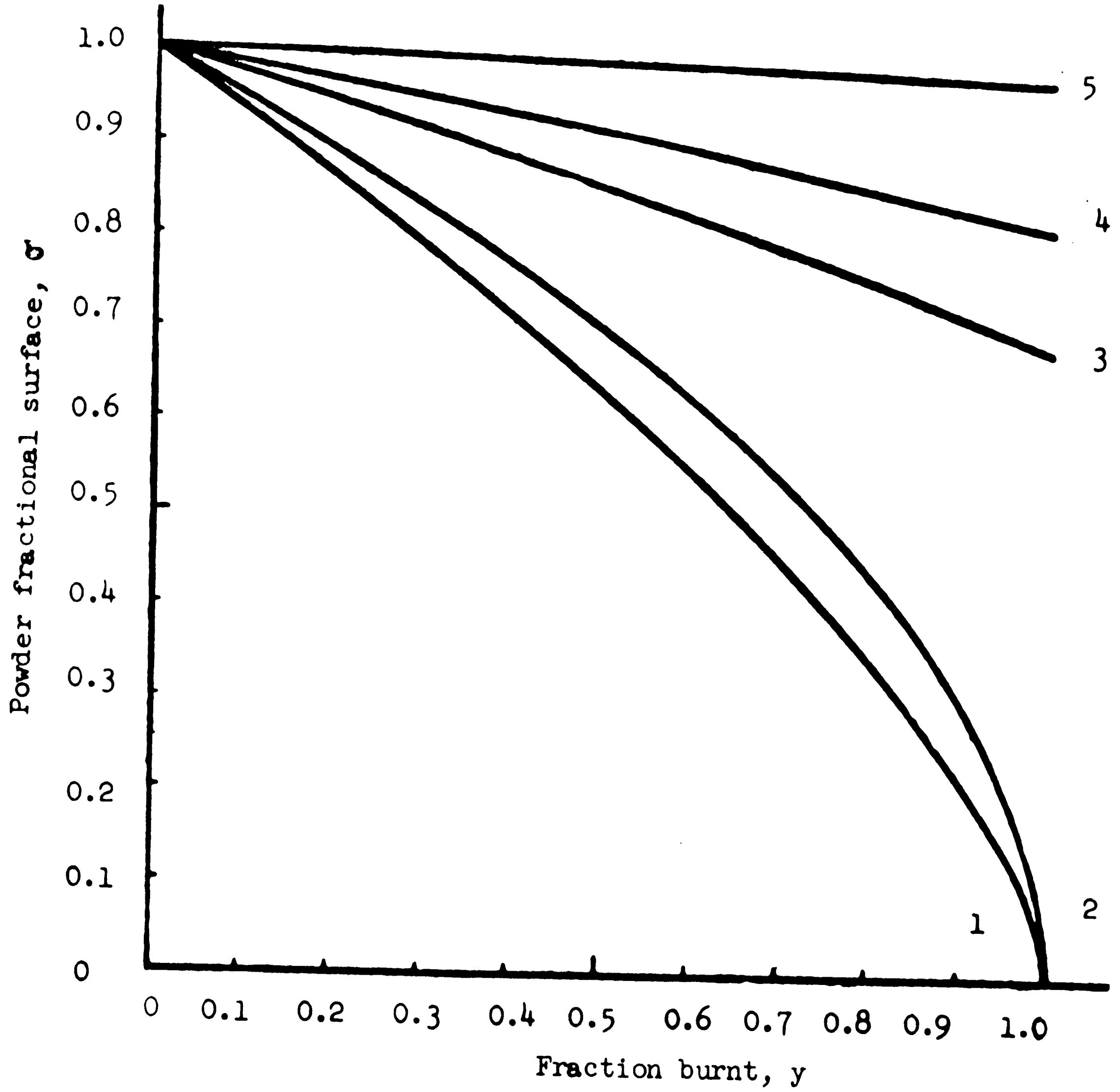


Fig. 21. Powder fractional surface (σ) as a function of fraction burnt (y).

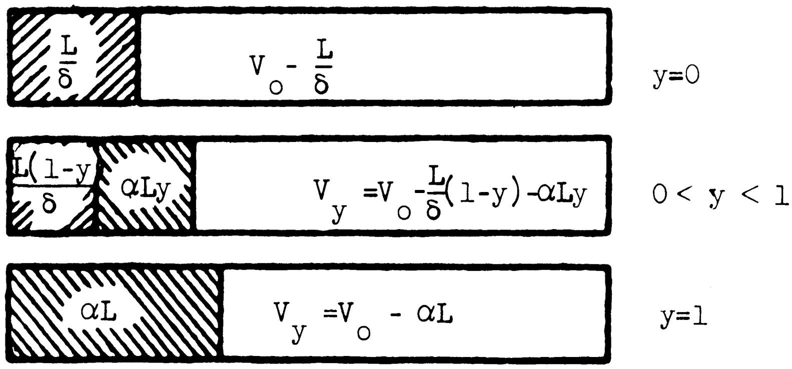


Fig. 22. Three states in the burning of powder at constant volume.

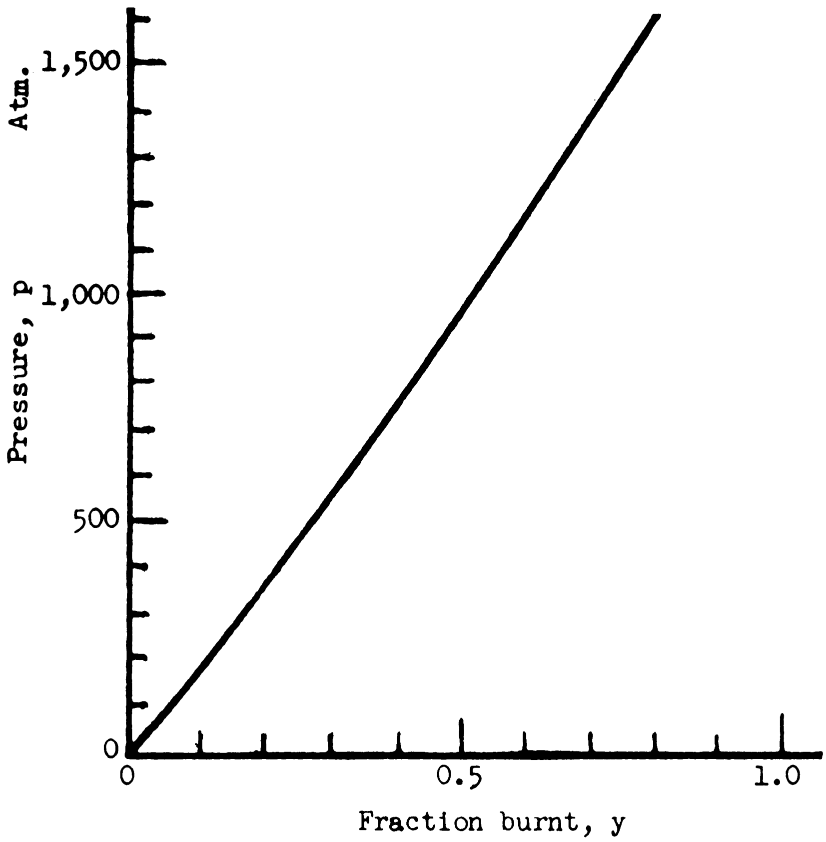


Fig. 23. Pressure (p) versus fraction burnt (y) for the combustion of nitrocellulose-based powders burnt at constant volume.

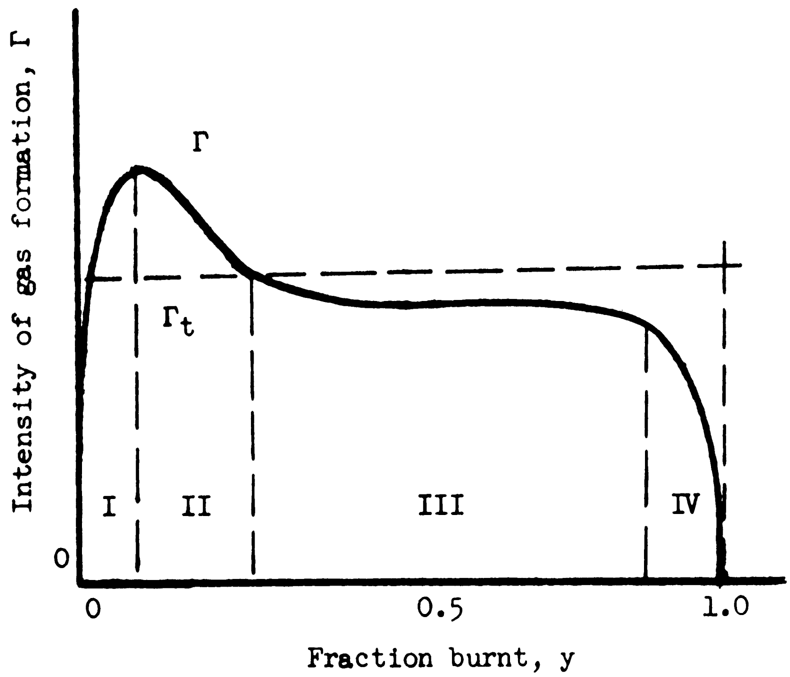


Fig. 24. Intensity of gas formation (Γ) versus fraction burnt (y) for tubular and strip powders.

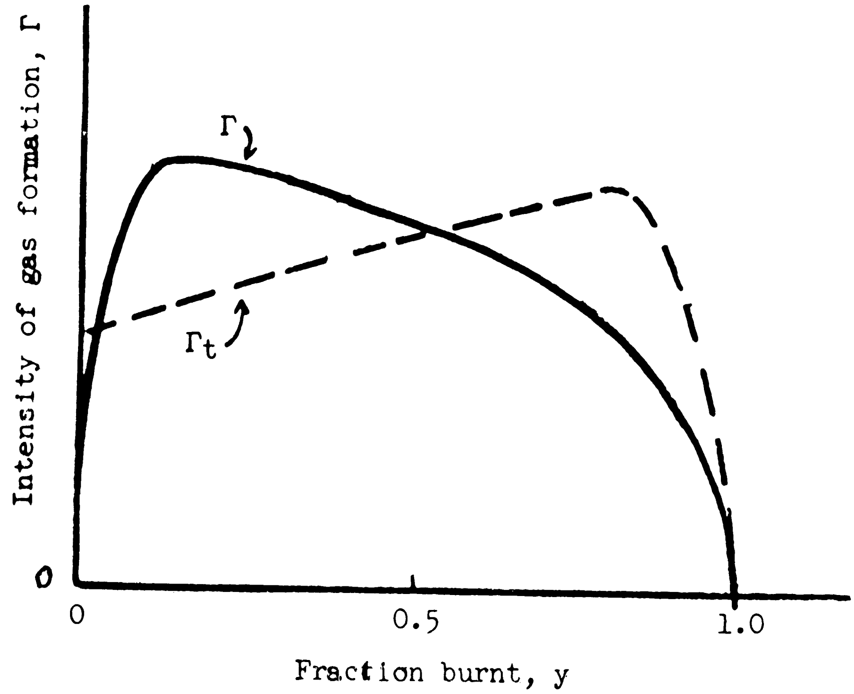


Fig. 25. Intensity of gas formation (Γ) versus fraction burnt (y) for seven-channel powder.

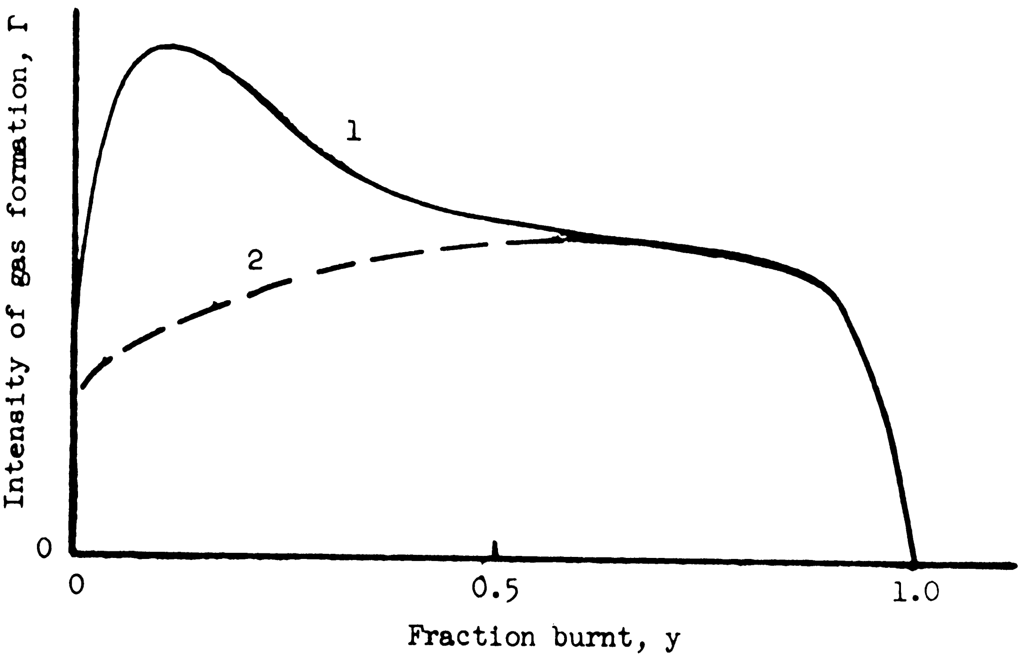


Fig. 26. Intensity of gas formation (Γ) versus fraction burnt (y) for tubular powders having untreated (Curve 1) and deterrent-treated (Curve 2) surfaces.