

***Search-Money-And-Barter Models of Financial
Stabilization***

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Non-technical summary: A macroeconomic model based on search-theoretical foundations is built to show that in an economy with structural deficiencies of the Russian Virtual Economy, money substitutes appear as a result of optimizing behavior of agents. Moreover, the volume of money substitutes is typically large, and it is impossible to reduce their volume significantly by using standard instruments as an increase of the money supply or decreasing the tax level. The result obtains for an economy, where there are large natural monopolies and widespread informal networks. Many interesting properties of the economy are derived; in particular, it is shown that money substitutes serve as a transmission belt of value from restructured effective firms to old ineffective ones thereby decreasing an incentive to restructure. In addition, the presence of large natural monopolies and widespread informal networks increase the incentives for the capital flight.

SEARCH-MONEY-AND-BARTER MODELS OF FINANCIAL STABILIZATION

S.I.Boyarchenko and S.Z.Levendorskii

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Abstract One of the key features of the Russian "virtual economy" is the wide-spread usage of barter and money substitutes. We apply the search-theoretical approach to Monetary Theory to construct a macroeconomic model, which allows us to analyze the appearance of money substitutes which are issued by some agents of the economy, and the interaction of these substitutes with genuine money.

We model Russian economy as consisting of two sectors: a sector of old, mainly ineffective agents (firms) (1-sector), who can collude so that promissory notes one of them issues are redeemed by others, while a sector of new, effective firms (0-sector) cannot. The sizes of the sectors, the money supply, the utility function and marginal production costs of agents are given exogenously, and trading strategies, prices, the amount of notes in circulation and the distribution of the money between sectors are found in an equilibrium as the result of optimizing behavior of agents of the economy and the colluding sector.

The main finding is that in an economy with such structure, the money substitutes appear and circulate, and it is impossible to make them disappear by changing the money supply unless the economy splits into two disjoint ones. By issuing notes, the ineffective sector preserves its stability, and ensures the transfer of value from the effective sector.

From the point of view of our model, the most effective way to restrict the usage of money substitutes is the simultaneous

- increase of the money supply;
- demonopolization of note-issuing large firms, like Gazprom and UES – notes of smaller firms are less acceptable, and
- heavy taxation of intermediaries involved in organizing of barter chains.

We show that there exists an optimal level of the money supply in the economy from the point of view of the colluding 1-sector, so that there is a strong incentive for 1-sector to transfer new money out of the economy, should the new money arrive. This observation may provide an additional explanation for the capital flight. The more effective 1-sector, the lower an optimal amount of the money in the economy for this sector, hence the larger the incentive for the capital flight. It means, in particular, that the investment in the energy sector (a core of the sector of colluding agents) at the expense of restructured enterprises of 0-sector increases the incentive for the capital flight.

At the same time, at low levels of the money supply, its increase is good for effective agents. Similarly, the welfare of agents of different types can move in opposite directions as other exogenous parameters of an economy change. This means that the total welfare for

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such an economy is an ill-defined concept, and one has to analyze the welfare of different types of agents separately.

We introduce such characteristics of the trading process, as *the trading friction* into the search-theoretical approach to monetary economics, and show that the type of an equilibrium crucially depends on the value of friction.

We analyze how such factors, as the trading friction, the level of ineffectiveness, the size of the note-issuing sector and the money supply influence the state of the economy, and show that the stability of an economy with interacting sectors is preserved if and only if *the instability index* = the product of the level of ineffectiveness, the defect of the money supply and the inverse of trading friction does not cross a certain *instability threshold*. Thus, if the ineffectiveness is large, large trading friction and/or money supply are needed to prevent an economy from splitting.

We also show that if both the trading friction and ineffectiveness of agents are small, then there may exist three types of equilibria:

- without trade between the sectors, with money circulating in 0-sector;
- when 1-sector uses only notes in trades with 0-sector, and
- when both money and notes circulate between sectors.

In the last two cases, both money and notes circulate inside 0-sector.

Depending on parameters' values, either of these equilibria can be optimal from the point of view of one sector or both. If the trading friction is very small then the second equilibrium is superior from the point of view of the welfare of 1-sector, so that it does not use money, but if the trading friction is not very small, the money supply is fairly large, and 1-sector is rather effective, then the usage of money becomes optimal for 1-sector. A type of an equilibrium optimal for 1-sector may be optimal for the effective 0-sector, and may be non-optimal. In particular, it is possible that an equilibrium without interaction between sectors is optimal for type-0 agents, but they will continue to trade with 1-sector only due to inability of agents of 0-sector to collude and change the type of an equilibrium by a joint action.

The equilibrium with all types of exchange is more fragile than the equilibrium with only notes circulating between sectors in the sense that the instability threshold for the former is much lower than that for the latter.

Numerical examples show that the economy can change the type of equilibrium due to a small change of the trading friction, the level of ineffectiveness, the money supply or size of note-issuing sector, and this may lead to the steep decline of the welfare of one of the sectors or both, and the same happens in a variant of the model with taxation, due to a small change of the level of taxes.

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0. INTRODUCTION

0.1. **”Virtual Economy”, Barter and Money Substitutes.** Starting with Karpov report (1997) (see Kuznetz (1998)) and papers Gaddy and Ickes (1998a, 1998b), it has become a widely spread perception of the Russian Economy as the *virtual economy* with a number of special features different from both a normal market economy and a command economy (see Guriev and Pospelov (1998), Ericson (1998), Gaddy and Ickes (1999), Guriev and Ickes (1999), Ericson and Ickes (1999) and the bibliography there). In principle, any transition economy differs from both a normal market economy and a command economy, but a word ”transition” presupposes a passing stage of a transformation from a command economy into a normal market one whereas the Russian Economy enjoys several stable abnormal properties which do not exhibit a tendency to fade out. This means that the Russian Economy may be in a new steady state, which rises the question of constructing appropriate macroeconomic models.

One of the key features of the Russian Economy is the wide-spread usage of barter and money substitutes (see e.g. empirical studies Aukutsionek (1994, 1998), Commander and Mumssen (1998), Dolgopiatova (1998), Guriev and Ickes (1999)), which increases rather than decreases as time goes by and is much more prominent than in other Transition Economies (Carlin, Schaffer and Seabright (1999)).

Many explanations for these phenomena have been suggested. Ericson and Ickes (1999) give the following list:

- insufficient liquidity due to misplaced ‘monetarism’ [Commander-Mumssen (1998), etc.],
- irrationally high (controlled) monetary prices, especially for energy, inducing barter as a means to effect price cuts [Woodruff (1998)];
- tax evasion [Hendley *et al.* (1998), Yakovlev (1999)];
- inefficient monetary and credit systems [Poser (1998)];
- rent-seeking in monetized transactions by commercial and monetary intermediaries, raising transactions costs above those of barter and quasi-monies [Guriev-Pospelov (1998)];
- a lack of serious industrial restructuring, implying an inability to reproduce value with the inherited configuration of technologies, production facilities, social obligations, etc. [Commander-Mumssen (1998), Gaddy-Ickes (1998b), Ericson (1998)].

They also observe that the common feature of all these explanations is that the prevalence of barter and money substitutes is the result of either bad policy or bad structure. Bad policy means (among other things) a very high level of taxation and the lack of liquidity, and the usage of money substitutes and barter facilitate tax evasion and provide necessary liquidity; bad structure means structural distortions in the economy which make many enterprises unviable in the pure monetary economy, so that they have to resort to non-monetary means of payment in order to survive. Ericson and Ickes (1999) construct a partial equilibrium model which describes the interaction among the government, the value-adding energy sector and a sector of loss-making enterprises. (For other approaches to structural deficiencies of the Russian economy, see e.g. Polterovich (1998).)

We look at the structural distortions in the Russian Economy from a different perspective. Certainly, all the factors mentioned contribute to the wide-spread usage of barter and money substitutes but a model we construct clearly demonstrates that even if we assume that

- the agents in the economy are sufficiently effective so that they can survive in the pure market economy;
- there is no taxation;
- there is no price control, and all the prices in the economy are determined endogenously, as a result of optimizing behavior of individual agents;
- the money supply is sufficient by any reasonable standards,

money substitutes will appear in the economy simply because there exist agents in the economy like Gazprom, UES who can issue universally accepted notes (IOU's).

As Gaddy and Ickes (1998a) points out, these IOU's typically circulate among chains of enterprises short of cash and are eventually redeemed for gas, electricity etc. by some of them. The usage of money substitutes is also facilitated by wide-spread stable business networks and relations which help to organize barter² chains which can use IOU's of smaller agents. For empirical study of transactions conducted using non-monetary methods and instruments, see Guriev and Ickes (1999), Carlin, Schaffer and Seabright (1999) and the bibliography there.

Our model also shows that if a part of an economy is very ineffective then it cannot interact with an effective part. All the money circulate in the latter, and the former can survive only by using non-monetary transactions.

The endogenous appearance of money substitutes in an economy is an old issue in Monetary Economics; we are interested in money and money substitutes as media of exchange. In the next two Subsections, we discuss approaches to Monetary Economics, which we are going to use.

0.2. Wallace's Dictum for Monetary Theory and a Random Matching Model (Wallace (1998)). Walrasian equilibrium models implicitly assume an auctioneer, who observing the goods suggested for trade chooses market clearing prices.

Hence, Walrasian equilibrium (WE) models have no role for a valued fiat medium of exchange, and since they always assume complete markets, all assets can be traded at given prices in any circumstances. Therefore, all monetary theories have to depart from WE. However, some monetary models (like money-in-the-utility/production-function, or transaction costs models) depart only from Walrasian physical environment (agents, preferences, resources, technology, information structure), while others (like trading-post models or cash-in-advance models) depart from the equilibrium concept (rules governing interactions among agents). Finally, there is a class of models which depart both from physical environment and equilibrium concepts of WE. One fraction of this class consists of models with an absence of double coincidence of wants.

Monetary theories should not contain money as a primitive – this is Wallace's dictum for Monetary Theory. Models which assume real balances being arguments of utility or production functions or impose cash-in-advance constraints do not satisfy this dictum.

²Here the word "barter" is used in the Russian sense: non-monetary transaction

The main reason is that they do not permit the assets' role in exchange to be endogenous. This role is given to the assets in a model with no double coincidence of wants. Absence-of-double-coincidence notion goes naturally with pairwise meetings of agents, therefore one theory which satisfies the dictum is a random matching model. Random meetings imply that agents cannot choose whom to meet with, therefore they have to search. Monetary search models show how fiat currency can be valued, how endogenous commodity money can arise; they can also discuss international monetary issues and address a variety of other questions in monetary economics.

Random matching monetary models with indivisible goods and money study exchange processes where, once agents meet, they exchange and part company. However, this framework is not interesting enough because prices are given exogenously. One of the ways to generalize these models is to make goods divisible, then the rate at which agents exchange can be determined by bilateral bargaining. A strategic bargaining model is due to Rubinstein and Wolinsky. This is an essentially dynamic model, however it is possible to show that the equilibrium outcome of the strategic bargaining game can be approximated by the generalized Nash bargaining solution which is inherently static, and therefore tractable.

0.3. Inside and Outside Monies. One of the important questions addressed by monetary economics is whether the private sector should be allowed to create money. One concern, which holds for any economy, is what mechanisms could prevent a private monetary system from printing too much money. Monetary models which use random matching to represent a trading process, were introduced in Kiyotaki and Wright (1989; 1991; 1993); for subsequent developments, see Trejos and Wright (1995), Aiyagari, Wallace and Wright (1995), Shi (1997), Wallace (1997) and review Wallace (1998). Models of this type decentralize the trading frictions, abandon the Walrasian fiction and naturally generate transaction demand for money.

Recently, there appeared several models, incorporating "inside money", where the label "inside" stands for "inside the private sector". The word "money" indicates the object which is used as a tangible medium of exchange among the agents who recognize it as an asset. In Cavalcanti and Wallace (1998; 1999) and Cavalcanti, Erosa, Temzelides (1998), Williamson (1999) inside money is given a role both for credit and tangible medium of exchange.

There is also a paper of Burdett, Trejos and Wright (1998), which introduces endogenous money as a commodity (general good), which can be either consumed or stored and used as a medium of exchange.

While the endogenous (commodity) money does not require any kind of pre commitment, since the exchange, if it takes place, is always *quid pro quo*; models with inside money need incentives of the agents to be taken into consideration. Though, originally, credit was completely ruled out of the random matching framework, it is possible to introduce some form of credit into monetary models by assuming that either people can commit to future actions, or (complete or partial) public information about trading histories is available, or both.

The first paper to put partial public knowledge into the random matching setting, was the one of Kocherlakota and Wallace (1998). In Cavalcanti and Wallace (1998; 1999) and Cavalcanti, Erosa and Temzelides (1998), it is assumed that a fraction of population - a banking sector - has access to a private note-issuing technology, while the rest of the economy - a non-banking sector - uses inside money as a medium of exchange. There is a record keeping technology (clearing house) in the first sector and privacy-of-trading histories in the second sector. There is also a note redemption technology inside the banking sector which allows to discipline the amount of notes issued by the banking sector.

Williamson (1999) explores a model with claims on banks as private money. Agents can choose between investing into low or high-return projects, so there may exist welfare dominated equilibria where banks hold low-return assets. Also it is shown that in case of private information, private money may be subject to lemons problems.

In addition to the media of exchange produced in the private sector, all the above models with inside and endogenous money, incorporate the exogenous provision by a public sector of fiat currency usually referred to as "outside or exogenous money". In Burdett, Trejos and Wright (1998) and Cavalcanti and Wallace (1998; 1999), it is shown that an equilibrium can be achieved in an economy with only endogenous or inside money (respectively) in circulation.

The former paper also shows that if the supply of exogenous money is sufficiently small, both types of money coexist. The work of Cavalcanti, Erosa and Temzelides (1998) deals with the environment where inside and outside moneys circulate. Cavalcanti and Wallace (1998; 1999) consider separately economies with inside and outside money.

Our model considers the case when inside and outside money coexist. We visualize the note-issuing sector not as a banking sector, but as a coalition of large producers, like Gas, Oil, Electricity companies, therefore we rule out the possibility of bankruptcy in a sense that all resources are assumed to be unlimited. We assume both complete public information and pre commitment in the banking sector, which makes it unavoidable to face certain incentive compatibility constraints in the model. The punishment for deviation is not the shutdown of the whole economy, like in Cavalcanti and Wallace (1998; 1999), but the denunciation of a deviator, after which nobody accepts her notes.

Our agents are placed in a standard money-search environment of Kiyotaki and Wright, in which different people have different preferences over a large number of differentiated goods.

Our model combines more essential features than models in op.cit. The closest to our paper are papers Cavalcanti and Wallace (1998, 1999), Burdett et al (1998) and Cavalcanti et al (1998).

Cavalcanti and Wallace (1998, 1999) consider implementable allocations which arise with inside and outside money separately, they do not examine coexistence of both kinds of monies.

Burdett et al (1998) consider only commodity money as inside money, so they do not have to take into account incentive compatibility constraints etc, since the trade if it takes place is always quid pro quo.

In Cavalcanti et al (1998), there is a finite number of consumption goods and individuals. Agents of type i can consume good i and produce good $i + 1$. A banking sector is a real banking sector with clearing house, reserve keeping and possibility of being dissolved if notes redeemed exceeds reserve balance. Coexistence of private and government money is studied only for the case of discount factor close to 1, but neither analytical results for endogenous variables are obtained nor the case of agents of different levels of ineffectiveness is studied (the last remark concerns other papers as well).

0.4. A Monetary Model for the Virtual Economy: A General Set-Up. We are going to construct a tractable benchmark model, so that an economy enjoys only basic features of the Virtual Economy, which are necessary and sufficient for a reproduction of an effect of wide-spread usage of money substitutes. The reader may regard some of our assumptions as over-simplifications, but as she will see below this over-simplified model is rather complicated already, and exhibit a number of interesting features even in this simple set-up. Further, the model is constructed in such a way that it can be modified in many directions in order to incorporate additional features of the reality, and when convenient, we will indicate how it can be done.

The economy consists of two sectors:

- a sector of effective new or restructured and privatized firms (0-sector), and
- a sector of old, mainly non-restructured and ineffective firms (1-sector), who can collude and issue universally accepted notes.

A note may circulate among type-0 agents (agents of 0-sector), and eventually one of them redeems it for a good an agent of 1-sector produces. Since agents of 1-sector collude, they agree on conditions for note-issuing and redemption and on the amount they produce to each other in meetings inside 1-sector. These conditions and amount are chosen to maximize the welfare of 1-sector, given optimizing behavior of agents of 0-sector. The usage of money is decided by each agent, and the interaction of all these optimizing actions results in endogenously determined trading strategies, prices, the amount of notes in circulation and the distribution of the money between sectors.

We assume that agents are heterogeneous, and we specify their preferences and production opportunities so that in any meeting, there is no double coincidence of wants. This naturally generates the demand for media of exchange.

In the real counterpart of our model economy, notes of such large firms as Gazprom and UES are accepted because these firms are large and omnipresent, and notes of smaller firms can circulate due to the existence of well-established connections and barter chains, which play the role of the clearing house in search-theoretical models with the banking sector; the redemption of IOU's for goods is wide-spread in the Russian Economy, as was mentioned above. So far, our assumptions agree with the reality, except for the fact that the real old sector consists of two subsectors with essentially different production properties – a value adding energy subsector (Gazprom, UES and Oil companies), and low/negative value adding (loss making) old manufacturing sector. Thus, a more realistic model should describe an economy with three sectors:

- 0-sector (the same as in our model);

- 1-sector (a note-issuing sector of relatively efficient colluding agents);
- 2-sector (non-restructured ineffective old manufacturing enterprises).

By analogy with our results for a two-sector economy, we expect that if 2-sector is sufficiently ineffective then in an extended version of our model with 3 sectors, in an equilibrium, it is non-optimal for 1-and 2-sectors to use money in mutual trades. If we make an assumption that the effective 0-sector does not consume inferior goods produced by 2-sector or 2-sector is so ineffective that it is non-optimal for 0-sector to trade with 2-sector on conditions acceptable for 2-sector, then 0-sector and 2-sector do not trade. Finally, if we assume that type-1 agents are sufficiently effective then type-0 agents and type-1 agents have an incentive to trade, and if type-1 agents can trade with type-0 agents independently of trades with 2-sector (so that if type-1 agents are production constrained, this constraint affects only trades with 2-sector, not 0-sector), then our model completely determines optimal trading strategies, equilibrium prices, volume of notes in circulation and distribution of money between 0-sector and 1-sector in an equilibrium.

After that we can take these prices, etc., as exogenous for a model describing relations between 1-sector and 2-sector. Notice that Ericson and Ickes (1999) construct a partial equilibrium model describing interaction between a value-adding sector and loss-making one, with the exogenously given the government policy, which facilitate the transfer of value from 1-sector to 2-sector and explain the appearance of barter and money substitutes in trades between 1-sector and 2-sector.

Thus, our model and Ericson and Ickes (1999) model taken together can give the whole picture. Notice, that for some parameters' values, our model demonstrates the additional transfer of value: from 0-sector to 1-sector.

The agreed rules for note-issuing, redemption and production inside 1-sector are absolutely natural if we assume that agents of 1-sector are branches of only one large firm as Gazprom. More realistic model should include several sectors with properties of 1-sector; in principle, our model can be tractable in this more general set-up.

In the framework of a consistent (from the point of view of Monetary Theory) approach, the next step to the reality can be taken if, in addition, we allow the appearance of notes issued by ineffective and unreliable agents and intermediaries, who enforce the redemption of these notes in separate "islands" of the ineffective sector – at a cost paid by other ineffective agents.

We plan to construct the corresponding models in the future.

0.5. Pairwise Meetings, Participation Constraints and Incentive Compatibility Constraints. Agents meet pairwise and at random, and in each meeting, an individual decision: to trade or not to trade, and how much good to produce for a unit of money or a note, is the result of the optimizing behavior of individual agents, with the exceptions of note-issuing rules imposed on type-1 agents by the "social planner" of 1-sector.

The optimizing behavior of individual agents is described by *participation constraints*. The rules imposed by the "social planner" may be non-optimal on an individual level (for instance, it is not individually optimal to redeem a note issued by another agent), and in order that the deviation be non-optimal on the individual level, it is necessary that the gain from a deviation be less than the gain from obeying the rules. After formalizing,

we obtain a group of *incentive compatibility constraints*. Both groups of constraints are derived in Section 1.

0.6. Characteristic Parameters and Trading Friction. Agents discount the future at the rate $r > 0$, and an agent meets other agents at random according to the Poisson process with the parameter $\alpha > 0$. Since one is free to choose a unit of time, one may assume that $\alpha = 1$ and simplify some formulas in the model. The same simplification obtains when one introduces the normalized discount rate $\rho = r/\alpha$, as one routinely does in random-matching models.

We consider agents with idiosyncratic tastes, so that for a given agent, only a fraction x_* of agents can produce goods which she consumes. This means that only a fraction x_* of all meetings can provide an opportunity to trade. In effect, this is the same as if the Poisson rate of arrival is multiplied by x_* , and the normalized discount rate is divided by x_* . We obtain a fraction $h = \rho/x_*$, which we call *the trading friction*. If we choose a unit of time so that $r = 1$, then $1/h$ is the Poisson rate of arrival of partners with whom an agents may wish to trade.

The smaller h , the more effective is the trading process.

Notice that for unstructured firms, both α and x_* are small: such firms can neither afford to actively look for trading partners, since this is costly, nor diversify their production good and make it desirable for a large fraction of agents in an economy. Hence, for unstructured firms, h is larger than for non-structured ones.

Notice the importance of the value of h for a pattern of exchange observed in an equilibrium. If h is small, an agent can be patient, so that when a trading partner offers rather unfavorable conditions, she can walk away and wait for a better trading opportunity to arrive, but if h is large, then in effect, agents have to heavily discount the future and accept small amounts of good for a unit of money, and produce for a unit of an inferior mean of exchange; note-issuing agents can issue more notes since the moment of redemption is in the distant future.

The interaction of these factors leads to an economy with very bad properties – many notes in circulation and small amount of good produced for both a note and a unit of money, and according to the argument above, this situation is expected in an economy with a large fraction of non-structured firms.

The well-known effect in the Game Theory is the difficulty in supporting cooperation when the discount factor is small and agents heavily discount the future. The similar situation in our model arises when h is large: the incentives to deviate become large and some of incentive compatibility constraints become binding (we constructed such equilibria in Boyarchenko and Levendorskiĭ (1999) for an economy with only notes in circulation, and similar equilibria can be constructed when both notes and money circulate). In the present paper, we consider mainly the case of small h , and prove that in this case, all the incentive compatibility constraints are non-binding.

0.7. Exogenous and Endogenous Parameters. The list of exogenous parameters of our model consists of r, α, x_* , the amount of the money in the economy, M , the size of the note-issuing sector, and parameters characterizing utility functions and marginal production cost of agents.

In order that agents of 0- and 1-sector differ, we have to assume that they have consumption technologies with different levels of effectiveness and/or different marginal production costs. We choose them to have different marginal production costs; the case of consumption technologies with different levels of effectiveness can be reduced to the case of different marginal production costs by a change of variables.

The trading strategies of agents, amounts of good produced for a unit of money and a note in meetings of different types, note-issuing rules, the amount of notes in circulation and the distribution of money between sectors are determined endogenously as the result of optimal choices of agents of an economy and a "social planner" of a colluding sector.

0.8. Stationary Equilibria, Balance Equations and Bellman's Equations. We consider only stationary equilibria, hence the volume of notes in circulation, the distribution of money between sectors, the flows of notes and money between sectors must be independent of time. This requirement leads to a group of *the balance equations*. Similarly, the value function of an agent in a given state must be time-independent, and we obtain a group of *the stationary Bellman's equations*.

0.9. Types of Equilibria. We consider an economy with non-zero level of money supply, and we mainly concentrate on cases when there are notes in circulation as well. The case of only notes in circulation was treated in Boyarchenko and Levendorskii (1999), and pure monetary equilibria in Boyarchenko (1999) (for more general utility functions and types of heterogeneity of agents than here).

0.9.1. Splitted Economies. Possible pure monetary equilibria in our case are equilibria when there is no trade between sectors, the economy splits into two disjoint economies, and all the money circulate inside 0-sector. We call them type 0_M equilibria, with 0 indicating the number of types of exchange between sectors, and M stresses the fact that there is money in circulation.

A splitted economy may arise due to our assumptions that 1) there are only two types of agents in an economy; and 2) there is no constraint on consumption tuples so that a type-0 agent can substitute goods type-0 agents produce for goods type-1 agents do. If they cannot, certain amount of trade between sectors remains in any case; on the formal level, we must introduce an additional constraint into our model, and this constraint is binding when the present more simple model yield a splitted economy. If there are many levels of effectiveness, there may be as groups of sectors with no exchange among them (petty manufacturers do not buy brand-name computers, and Compaq does not need goods these manufacturers produce) as groups of sectors who trade among themselves.

The effect of splitting is demonstrated in Boyarchenko (1999) for the case of pure monetary economy with agents of many levels of effectiveness; since in her model there are neither inside money nor collusion, starting with certain level of ineffectiveness, ineffective agents are unable to trade at all.

Notice that an economy which splits from the point of view of non-existence of stationary steady states with trade between sectors may have non-stationary steady states – cycles or more complex forms – with trade between sectors. The study of such non-stationary equilibria goes beyond the scope of the present paper.

0.9.2. *Economies with Trades between Sectors.* Properties of equilibria and equilibria specification depend on a note-issuing rule, which the note-issuing sector uses: the sector may find it optimal to

- restrict issue of notes, so that an agent has no right to issue notes in some cases when it is optimal on an individual level;
- allow for agents to print notes if and only if this is optimal on the individual level;
- print too many notes, more than it is optimal on an individual level, in order that the stability be preserved.

It is interesting that depending on values of parameters, characterizing the economy (such as the trading friction, the degree of specialization in consumption, and the level of ineffectiveness), all possible types of equilibria can be realized (we have shown it in Boyarchenko and Levendorskiĭ (1999) for the case of an economy with only notes in circulation), but when the friction vanishes, many equilibria disappear as well. In particular, if agents are not very picky, an ineffective sector rather ineffective, and the friction is small, then only type- 0_M equilibria are possible.

On the other hand, if the trading friction is sufficiently large then there may exist an equilibrium where effective type-0 agents continue to trade with very ineffective type-1 agents.

Thus, a possible way for ineffective sector to preserve itself is to increase the trading friction, i.e. make the infrastructure worse (by using political connections, say), but if a policy maker willing to improve the economy invests in the trading infrastructure at the expense of the effectiveness of the production sector, the economy loses stability and splits into two disjoint economies, which may cause huge welfare losses³

If the trading friction is small, 1-sector is sufficiently effective and agents are not very picky ($x_* \gg h$), there may exist two types of equilibria:

- type- I_M equilibria (only notes circulate between sectors), and
- type- II_M equilibria (both money and notes circulate between sectors).

In both cases, agents of type 0 use both money and notes in trades inside 0-sector.

Thus, 0, $I(= 1)$ and $II(= 2)$ indicate the number of different types of trades between sectors in an equilibrium of a given type.

Type I_M equilibrium can be perceived as too unrealistic, but it provides a good approximation to more realistic situations when 1-sector needs some inflow of money to pay taxes or acquire goods produced by a fraction of agents who do not accept notes at any circumstances, – if the tax level is not very high and the fraction of such agents is not very large. One can also generalize our model and treat this situation explicitly.

When the trading friction vanishes, the welfare of the note-issuing sector becomes larger in a type- I_M equilibrium if I_M and II_M equilibria can coexist. Thus, 1-sector would always choose not to use money at all if the former does not need the latter for some exogenous purposes (to pay taxes, for instance). If the trading friction is not very small, the money supply is rather large, and 1-sector is fairly effective, then the usage of money becomes optimal for 1-sector.

³cf. Castanheira and Roland (1995) for general equilibrium analysis of the optimal speed of Transition

At the same time, at low levels of the money supply, its increase is good for effective agents. Similarly, the welfare of agents of different types can move in opposite directions as other exogenous parameters of an economy change. This means that the total welfare for such an economy is an ill-defined concept, and one has to analyze the welfare of different types of agents separately.

A type of an equilibrium optimal for 1-sector may be optimal for the effective 0-sector, and may be non-optimal. In the last case, the trade between sectors can be preserved only due to inability of agents of 0-sector to collude. Numerical examples illustrate how it can happen: when one of the parameters of the model, e.g. the money supply or the size of note-issuing sector, changes, an economy can move from a region where a type- II_M equilibrium, say, was optimal for both sectors, into a region where a type- II_M equilibrium continues to be optimal for 1-sector (so the latter does preserve the former) but for 0-sector, it becomes optimal not to trade with 1-sector at all. Still, type-0 agents cannot collude and change the type of the equilibrium, and if a policy maker wishes to stop circulation of notes, she has to introduce some additional obstacles which would prevent an economy from using them; notes will continue to circulate in an economy, if it is left alone.

From the point of view of our model, the most natural thing to do is the simultaneous

- increase of the money supply;
- demonopolization of note-issuing large firms, like Gazprom and UES – notes of smaller firms are less acceptable, and
- heavy taxation of intermediaries involved in organizing of barter chains.

Of course, political feasibility of this program is questionable, to say the least.

0.10. Optimal Level of Money Supply and Capital Flight. For type- I_M equilibria, we show that for an effective 1-sector, an optimal money supply in the economy is zero, and for larger levels of the ineffectiveness, there is an optimal positive level of the money supply. The first fact has a clear economic interpretation: a note-issuing sector can issue notes on more favorable condition if notes is the only mean of exchange, but an explanation of the second fact – a positive optimal level of the money supply – is more subtle. Even in a non-monetary economy, a symmetric steady state does not exist if the ineffectiveness is large: an effective type-0 agent can obtain a large amount of good for a note when she meets another type-0 agent, and if the trading friction is small, she can be patient and not accept essentially smaller amount of good from type-1 agents, but if the latter are very ineffective, they cannot produce that much, and prefer not to trade with 0-sector at all (we proved it in Boyarchenko and Levendorskiĭ (1999)), and the same effect is observed when money are scarce. Thus, if 1-sector is rather ineffective, then a non-zero level of the money supply is needed to prevent the economy from splitting. An increase of the money supply plays a stabilizing role because it makes a unit of money and a note less valuable, and at the same time, the relative value of a note w.r.t. a unit of money increases. At some level of the money supply, the note issuing becomes optimal for 1-sector, on conditions acceptable for agents of 0-sector.

Notice that if 1-sector is not very ineffective, an optimal money supply from the its point of view is less than that from the point of view of 0-sector.

If the existing money supply is at or above an optimal level (from the point of view of 1-sector), there is a strong incentive for 1-sector to transfer new money out of the economy, should the new money arrive. This observation may provide a new explanation for the capital flight. The more effective 1-sector, the lower is the optimal amount of the money in the economy for this sector, hence the larger is the incentive for capital flight. It means, in particular, that

the investment in the energy sector (a core of the sector of colluding agents) at the expense of restructured enterprises of 0-sector facilitates the capital flight.

0.11. Instability Index and Instability Threshold. We show that the economy does not split if and only if *the instability index* = the product of the level of ineffectiveness, the defect of the money supply and the inverse of trading friction does not cross a certain *instability threshold*. Thus, if the ineffectiveness is large, large trading friction and/or money supply are needed to preserve the trade between sectors.

Similarly, if a policy maker invests in the trading infrastructure (means of communications, etc.), which increases the Poisson rate of arrival of trading partners and decreases the trading friction, without investment in the ineffective 1-sector, or decreases the money supply, than the economy loses stability and splits, but if 1-sector is as effective as 0-sector, then the sectors continue to trade for all values of the trading friction.

This observation leads to a conjecture on 3-sector variant of our model made in Subsection 0.4: an ineffective 2-sector cannot trade with an effective 0-sector, but if 1-sector is sufficiently effective, it continues to trade with 0-sector.

The final remark on stability issues: numerical examples show that the economy can lose stability due to a small change of the money supply or size of note-issuing sector; similarly, this can happen in an analogous model with taxation, due to not large increase of taxes. This may lead to a steep decline of the welfare of one of the sectors or both of them.

1. MODEL SPECIFICATION

1.1. Fit the First (Preferences, Production and Meeting technology). Consider an economy with a continuum of infinitely lived agents, the size of population being normalized to one. Time is continuous. The agents discount the future at the rate $r > 0$. Agents are indexed by points on a circle of circumference two. There is also a continuum of goods/services indexed by points on the same circle. The agents have idiosyncratic tastes for goods: there exists $x_* \in (0, 1/2)$ such that if the distance between an agent and her favorite good is greater than x_* then the former enjoys 0 utility from consuming any amount of the latter, and if the distance is less than or equal to x_* then the agent enjoys utility $u(q)$ from consuming q units of the good. This implies, in particular that x_* is the probability that an agent will like a certain good.

We assume that u enjoys standard properties:

u is smooth, increasing and concave, and satisfies the Inada conditions.

To acquire goods, agents search in a productive sector. Suppose that search is costless and production is instantaneous. Each production opportunity yields to an agent i some amount of a perfectly divisible good j at a fixed distance z , $2x_* < z < 2 - 2x_*$, clockwise from i .

Due to properties of the preferences and production opportunities, agents do not consume their own output, so they have to trade it in the exchange sector. Trading partners arrive according to a Poisson process with the constant rate α . In other words, we restrict our consideration to a CRS meeting technology. We consider the case when the distance between partners in a meeting is drawn randomly from a uniform distribution $U[0, 1]$. It is clear that the specification of preferences and production opportunities rules out the double coincidence of wants. Thus, we have either single or no coincidence of wants, and the probability of single coincidence of wants is x_* .

Goods are perishable, so if an agent decides to acquire a good, she has to consume it immediately.

1.2. Fit the Second (Means of exchange and Types of agents). There is an exogenous money supply $M \in (0, 1)$. Money is indivisible, and an agent can carry either 1 unit of money or none. These assumptions essentially simplify the study of the model, and still allow us to endogenize prices as inverses to amounts of good produced for a unit of money.

We rule out $M = 0$, since the economy is non-monetary then, and $M = 1$ means that the money cannot be used for transaction purposes, since everybody has a unit of money and cannot find a seller.

Agents may issue indivisible and perfectly storable promissory notes (IOU's). A fraction of agents - we call them type-1 agents (and the others - type-0 agents) - issue notes, which are distinguishable in a sense that no counterfeiting is possible, and collude in the following sense:

(i) they agree on a rule for note-issuing and an amount of good, q_N , to be redeemed for any type-1 note (i.e. a note issued by any type-1 agent);

- (ii) they agree not to use means of exchange when they trade with each other, and fix q_1 , the level of production for a trade between two type-1 agents in the case of single coincidence of wants;
- (iii) each type-1 agent signs a note she issues and writes a date of issue on it;
- (iv) they record information about every type-1 agent and spread this information among themselves;
- (v) if a type-1 agent deviates from any of the agreed rules, the other type-1 agents make publicly known that they regard notes issued by a deviator after this moment as notes issued by any type-0 agent and do not redeem them;
- (vi) they choose the rule and quantities q_N and q_1 in (i) and (ii) so that to maximize the welfare of type-1 agents subject to the incentive compatibility constraints (which ensure that a deviation is not optimal).

The type of the rule deserves a special comment. On an individual level, a type-1 agent would not issue a note unless she enjoys a positive utility from consumption of a given good, i.e. an individual agent issues a note with probability x_* . However, a choice of the rule affects all endogenous parameters of the economy and the welfare of type-1 agents. Thus, it may be optimal to issue a note with probability $x_1 > x_*$ in order to increase the amount of liquidity in the economy and make the outside money less valuable; in this case, "the social planner" of 1-sector (a sector of type-1 agents) obliges a type-1 agent to issue a note even if the agent does not like the good. It may also be optimal to issue a note with probability $x_1 < x_*$ in order to make notes more scarce, increase the transaction value of a note, and receive more goods in exchange for a note. In this case, "the social planner" of 1-sector forbids to issue a note in some cases even if an agent likes a good.

So a rule for note-printing, as far as the mathematical structure of the model is concerned, is as follows:

- (for the case $x_1 \leq x_*$) "never issue a note if the distance of the good from your favorite one $z > x_*$, and if $z \leq x_*$, issue a note with a probability x_1/x_* ";
- (for the case $x_1 > x_*$) "if $z > x_*$, issue a note with a probability $(x_1 - x_*)/(1 - x_*)$ ".

When a buyer is of type 1 and has to issue a note even when she does not like the good, we also use the label "single coincidence meeting".

Notes of type-0 agents are distinguishable one from another so that neither agent except for an agent i of type 0 is in any sense under obligation to redeem a note issued by the latter. Hence, each note will be redeemed with 0 probability, and in any symmetric equilibria, which we are going to consider, it cannot be optimal for anyone to accept a type-0 note. Thus, we may assume that type-0 notes do not circulate at all, and hereafter, we call type-1 notes simply "notes".

We assume that any type-0 agent can carry either 1 unit of money or a note or none of these; a type-1 agent can carry 1 or 0 units of money.

Types 1 and 0 agents also differ in their production: type-0 agents may be endowed with better production opportunities, so that they suffer lower disutility (cost) of production per unit of good. Namely, we set the marginal cost 1 for a type-0 agent, and $k \geq 1$ for a type-1 agent.

The motivation is that in the Russian economy, it is a sector of old, mainly unstructured enterprises which manages to survive due to collusion, informal networks and widespread usage of IOU's.

1.3. Fit the Third (States of agents, Single-coincidence-of-wants-meetings and Participation Constraints). Each type-0 agent can be in three states:

- a note holder (a buyer carrying a note);
- a type-0 money holder (a buyer with a unit of money);
- a type-0 seller.

We denote by V_N^0, V_m^0, V_s^0 the value functions of an agent in these states.

A type-1 agent can be in two states:

- a type-1 money holder (a type-1 agent with a unit of money);
- a type-1 agent without money.

The corresponding value functions are denoted by V_m^1 and V^1 , respectively.

We assume that a buyer has a bargaining power which enables her to extract all seller's surplus from trade. More precisely, we determine a quantity produced in each round of trade as a generalized Nash bargaining solution, which satisfies

$$q = \operatorname{argmax}[u(q) + V_s - V_b]^\theta [V_b - kq - V_s]^{1-\theta},$$

where $V_s \in \{V_s^0, V^1\}$, $V_b \in \{V_N^0, V_m^0, V_m^1\}$ are value functions of a seller and a buyer respectively; and θ is the bargaining power of a buyer. Due to the assumption above, buyers have absolute bargaining power in this model, i.e. $\theta = 1$. This also means that if a trade occurs, a seller produces her "reservation quantity", i.e. the quantity which makes her indifferent between an alternative: produce or not produce.

If a seller and/or buyer find it optimal not to trade, both leave the meeting remaining in the same state as before the meeting.

When *two agents of type-0 meet*, they cannot trade unless the buyer has either a note or a unit of money, the other has neither, and the buyer wants to consume the production good of the seller.

Suppose, the buyer carries a note. Evidently, the seller does not produce if she is worse off after the trade, therefore in exchange for a note, a type-0 seller produces an amount q_0 given by

$$V_N^0 - V_s^0 - q_0 = 0. \tag{1.1}$$

The note holder decides whether to spend her note or not given the amount of good the seller agrees to produce. We denote by x_0 the probability with which a type-0 buyer spends her note in a meeting with a type-0 seller. We assume that the buyer trades if she is not worse off after the trade, therefore x_0 is either 0 or x_* and satisfies

$$x_0 = x_* \iff u(q_0) \geq V_N^0 - V_s^0;$$

on the strength of (1.1), this is equivalent to

$$x_0 = x_* \iff u(q_0) \geq q_0. \tag{1.2}$$

Now, let the buyer be a type-0 money holder. The seller does not produce if she is worse off after the trade, therefore in exchange for a unit of money, a type-0 seller produces an amount q_{m0} given by

$$V_m^0 - V_s^0 - q_{m0} = 0. \quad (1.3)$$

The money holder decides whether to spend her unit of money or not given the amount of good the seller agrees to produce. We denote by x_{m0} the probability with which a type-0 money holder spends her money in a meeting with a type-0 seller. We assume that the buyer trades if she is not worse off after the trade, therefore x_{m0} is either 0 or x_* and satisfies

$$x_{m0} = x_* \iff u(q_{m0}) \geq V_m^0 - V_s^0;$$

on the strength of (1.3), this is equivalent to

$$x_{m0} = x_* \iff u(q_{m0}) \geq q_{m0}. \quad (1.4)$$

Now suppose that *two agents of different types meet*. We have to consider two cases:

- (i) type-0 agent is a buyer, type-1 agent is a seller;
- (ii) type-0 agent is a seller, type-1 agent is a buyer.

In the first case, if the type-0 agent has a unit of money, and the type-1 agent has no money, the former decides whether to spend her unit of money or not, given the maximal amount, q_{m1} , the latter agrees to produce:

$$V_m^1 - V^1 - kq_{m1} = 0. \quad (1.5)$$

We denote the probability of acquiring a good for money by x_m ; assuming that the buyer spends her money if she is not worse off after the trade, we conclude that x_m is either 0 or x_* , and

$$x_m = x_* \iff u(q_{m1}) \geq V_m^0 - V_s^0.$$

By taking into account (1.3), we obtain

$$x_m = x_* \iff u(q_{m1}) \geq q_{m0}. \quad (1.6)$$

If the type-0 agent has a note, she takes into account the amount q_N , which type-1 agents have agreed to produce while redeeming a note, and decides, whether to spend her note or not. We denote the probability of returning a note for redemption by x_N ; it is equal x_* , if the buyer decides to spend her note ever, and 0 otherwise.

Thus, we must have (1.1) and

$$x_N = x_* \iff u(q_N) \geq q_0. \quad (1.7)$$

In the second case, a trade cannot take place unless the type-0 agent carries neither notes nor money. Suppose, the buyer has no money. The maximum amount, q_0 , which the seller agrees to produce for a note, is given by (1.1), and the buyer issues a note in exchange with probability x_1 , following the rule assigned by the "social planner".

If the type-1 agent has a unit of money, her decision rule depends on the note-issuing rule x_1 .

If the assigned probability of note-issuing $x_1 \leq x_*$, and $z > x_*$, the buyer walks away. If $z \leq x_*$, she first decides whether to spend her money or not, and the decision depends on z . If $z \leq x_1$, then the buyer spends her money with probability x_{m1} . It is equal to 0

provided the agent decides not to pay with the money and issues a note instead (given she gets q_0 in exchange), and x_1 otherwise (given she gets q_{m0} in exchange). Thus,

$$x_{m1} = x_1 \iff u(q_{m0}) - (V_m^1 - V^1) \geq u(q_0)$$

(if the agent spends her money, she is not worse off than when she pays with a note), which is equivalent to

$$x_{m1} = x_1 \iff u(q_{m0}) - kq_{m1} \geq u(q_0), \quad (1.8)$$

on the strength of (1.5); and

$$x_{m1} = 0 \ \& \ z \leq x_1 \Rightarrow \text{the agent issues a note.} \quad (1.9)$$

If $x_1 < z \leq x_*$, then the probability of spending money is denoted by x_{m1}^+ . It is equal to 0 if the buyer decides to walk away, and $x_* - x_1$ otherwise. Thus,

$$x_{m1}^+ = x_* - x_1 \iff u(q_{m0}) - (V_m^1 - V^1) \geq 0$$

(if the agent spends her money, she is not worse off when she walks away). By using (1.5), we obtain

$$x_{m1}^+ = x_* - x_1 \iff u(q_{m0}) - kq_{m1} \geq 0. \quad (1.10)$$

Now consider the case when the socially optimal (for 1-sector) probability of note-issuing $x_1 > x_*$. If $z \leq x_*$, the buyer first decides whether to spend her money or not. The probability of spending money is denoted by x_{m1} ; it is equal to 0 if the agent decides not to pay with the money and issues a note instead, and x_* otherwise:

$$x_{m1} = x_* \iff u(q_{m0}) - (V_m^1 - V^1) \geq u(q_0),$$

which is equivalent to

$$x_{m1} = x_1 \iff u(q_{m0}) - kq_{m1} \geq u(q_0), \quad (1.11)$$

on the strength of (1.5); and

$$x_{m1} = 0 \ \& \ z \leq x_* \ \text{or} \ x_* < z \leq x_1 \Rightarrow \text{the agent issues a note.} \quad (1.12)$$

Finally, when *type-1 agents meet*, the seller produces q_1 , the agreed amount of good, for the buyer. Clearly, $q_1 \leq q^*$, where q^* maximizes the trading surplus, $u(q) - kq$:

$$u'(q^*) = k, \quad (1.13)$$

and q_1 is maximal among those for which incentive compatibility constraints below are satisfied.

1.4. Fit the Fourth (Incentive compatibility constraints). Type-0 agents face no incentive compatibility constraints, since they make only individual decisions and do not collude, whereas colluding type-1 agents face three such constraints, when they carry a unit of money, and three similar constraints when they have no money. These constraints ensure that an agent finds it optimal on an individual level to obey the rules imposed by a "social planner" of 1-sector. If these constraints are violated, individually optimizing agents will defect, and the collusion of type-1 agents will become impossible.

Denote by V_s (V_m) the value of a type-1 agent without (with) a unit of money who is deprived of the privilege of issuing universally accepted notes. The first pair of incentive compatibility constraints is obvious and expresses the fact that the payoff to a type-1 agent, when she redeems a note, is not worse than the gain from the failure to do so:

$$V^1 \geq kq_N + V_s. \quad (1.14)$$

$$V_m^1 \geq kq_N + V_m. \quad (1.15)$$

Secondly, we need a pair of non-defection conditions for the inside exchange among type-1 agents, which states that there should be no positive gain from the failure to produce q_1 whenever required: $V^1 \geq kq_1 + V_s$ and $V_m^1 \geq kq_1 + V_m$. Hence,

$$kq_1 = \min\{kq^*, V^1 - V_s, V_m^1 - V_m\}, \quad (1.16)$$

where q^* solves (1.13).

The last group of constraints are conditions of non-defection from the assigned probability of note-issuing x_1 . If $x_1 = x_*$, an individually optimal probability, there is no incentive to deviate. If $x_1 < x_*$, and a type-1 agent issues a note in violation of this condition, she derives an additional instantaneous utility $u(q_0)$ but her continuation value becomes V_s (if she has no money) or V_m (if she carries a unit of money). Hence,

if $x_1 < x_*$, then

$$V^1 \geq u(q_0) + V_s, \quad V_m^1 \geq u(q_0) + V_m, \quad (1.17)$$

and if $x_1 > x_*$, then similarly,

$$V^1 \geq V_s, \quad V_m^1 \geq V_m. \quad (1.18)$$

Note that

$$0 \leq V_s \leq V_s^0, \quad (1.19)$$

since the marginal production cost for type-0 agents is 1, and for type-1 agents, it is $k \geq 1$; $V_s \geq 0$, since the autarky gives 0 for a defector.

When a defector with a unit of money trades, she extracts the same surplus as a type-0 agent with a unit of money, and after that the former has a continuation value V_m , and the latter $-V_m^0$. Hence,

$$V_m \leq V_m^0. \quad (1.20)$$

1.5. Fit the Fifth (Stationary equilibria). Consider a tuple

$$g = \{M_0, M_1, N, x_0, x_1, x_N, x_{m0}, x_m, x_{m1}, x_{m1}^+, q_0, q_1, q_N, q_{m0}, q_{m1}, V^1\}.$$

We call it a stationary equilibrium if and only if the following five conditions are met

(i) it is rational for an individual agent to behave in the same way as an average agent, therefore we may use the same letter to denote an individual variable and the corresponding aggregate variable;

(ii) probabilities $x_0, x_N, x_{m0}, x_m, x_{m1}, x_{m1}^+$ of spending a unit of a medium of exchange in a round of trade are either 0 or positive, and satisfy optimality conditions (1.2), (1.4), (1.6), (1.7), (1.8) and (1.10) in the case of note-issuing with probability below or the same as the individually optimal one; in the case of note-issuing with probability above

the individually optimal one, (1.8) is replaced with (1.11), and (1.10) becomes redundant since $x_{m1}^+ = 0$;

- (iii) $x_1 \in [0, 1]$, and $N, M_0, M_1, q_0, q_1, q_N, q_{m0}, q_{m1}, V^1$ are non-negative;
- (iv) all endogenous variables are time-independent.

In the next two Subsections, we will deduce from the stationarity condition balance equations (1.22), (1.23) and (1.24),

Bellman's equations (1.15)–(1.29),

and rewrite incentive compatibility constraints in the form (1.30)–(1.34);

- (v) type-1 agents choose x_1, q_1 and q_N in order to maximize their welfare

$$W = (1 - M_1)V^1 + M_1V_m^1 = (1 - M_1)V^1 + M_1(V^1 + kq_{m1}) = V^1 + M_1kq_{m1}.$$

1.6. Fit the Sixth (Balance equations). Denote by p_j a fraction of type- j agents ($j = 0, 1$), by N the proportion of type-0 agents carrying notes, and by p_0M_0 and p_1M_1 denote fractions of the aggregate money supply, M , which belong to type-0 and type-1 agents, respectively. Clearly,

$$p_0 + p_1 = 1, \tag{1.21}$$

and

$$p_0M_0 + p_1M_1 = M \tag{1.22}$$

(the money neither arrives nor disappear).

The remaining two balance equations state that in a steady state, the flow of notes being issued must equal to the flow of notes being destroyed, and the flow of money from 1-sector to 0-sector is equal to the flow in opposite direction.

In the case when the socially optimal probability of note-issuing is not higher than the individually optimal one,

the flow of notes being issued is equal to

$$p_0(1 - N - M_0)p_1\{(1 - M_1)x_1 + M_1(x_1 - x_{m1})\};$$

the flow of notes being destroyed is equal to $p_0Np_1x_N$;

the inflow of money into 1-sector is equal to $p_0M_0x_m p_1(1 - M_1)$;

the outflow of money from 1-sector is equal to

$$p_0(1 - M_0 - N)p_1M_1(x_{m1} + x_{m1}^+);$$

and the balance equations are

$$(1 - N - M_0)(x_1 - M_1x_{m1}) = Nx_N; \tag{1.23}$$

$$M_0x_m(1 - M_1) = (1 - M_0 - N)M_1(x_{m1} + x_{m1}^+). \tag{1.24}$$

(In deriving (1.23) and (1.24), we have used conditions $0 < M, p_0 > 0, p_1 > 0$, which ensure that the model is monetary and there are two sectors in the economy).

In the case when the assigned probability of note-issuing is higher than that at the individual level,

the flows of notes being issued and destroyed, and the inflow of money into 1-sector are given by the same formulas as above, but

the outflow of money from 1-sector is equal to

$$p_0(1 - M_0 - N)p_1M_1x_{m1}.$$

Thus, the balance equation for notes is the same equation (1.23) (with x_{m1} having a different meaning, though), and if we introduce $x_{m1}^+ = 0$ in the case of the note-issuing above the individually optimal level, then the balance equation for money is the same equation (1.24).

We use the same agreement about x_{m1}^+ in the next subsection.

1.7. Fit the Seventh (Bellman's equations for stationary equilibria). We assume that it is rational for an individual agent to behave in the same way as an average agent, and therefore we may use the same letter to denote an aggregate variable and the corresponding individual variable.

Let $\rho = r/\alpha$ be a normalized discount rate. Assume that the economy is in a steady state, and consider a type-0 seller. During a small time interval Δt , she meets someone with the probability $1 - e^{-\alpha\Delta t}$. Conditioned on a meeting having taken place,

1) with the probability p_0Nx_0 , she meets a type-0 note-holder, who likes her good. In the result of such a meeting, she produces, suffers a production cost q_0 , and becomes a note-holder with the value function V_N^0 . Thus, her net gain is $V_N^0 - V_s^0 - q_0$, and the expected net gain of such a meeting is

$$(1 - e^{-\alpha\Delta t})p_0Nx_0[V_N^0 - V_s^0 - q_0] = \alpha\Delta tp_0Nx_0[V_N^0 - V_s^0 - q_0] + o(\Delta t);$$

2) with the probability $p_0M_0x_{m0}$, she meets a type-0 money holder, who likes her good. In the result of such a meeting, she produces, suffers a production cost q_{m0} , and becomes a type-0 money-holder with the value function V_m^0 . Thus, her net gain is $V_m^0 - V_s^0 - q_{m0}$, and the expected net gain of such a meeting is

$$(1 - e^{-\alpha\Delta t})p_0M_0x_{m0}[V_m^0 - V_s^0 - q_{m0}] = \alpha\Delta tp_0M_0x_{m0}[V_m^0 - V_s^0 - q_{m0}] + o(\Delta t);$$

3) with the probability $p_1M_1(x_{m1} + x_{m1}^+)$, she meets a type-1 money holder, who likes her good and wants to pay with money. In the result of such a meeting, she produces, suffers a production cost q_{m0} , and becomes a type-0 money-holder with the value function V_m^0 . Thus, her net gain is $V_m^0 - V_s^0 - q_{m0}$, and the expected net gain of such a meeting is

$$(1 - e^{-\alpha\Delta t})p_1M_1(x_{m1} + x_{m1}^+)[V_m^0 - V_s^0 - q_{m0}] = \alpha\Delta tp_1M_1(x_{m1} + x_{m1}^+)[V_m^0 - V_s^0 - q_{m0}] + o(\Delta t);$$

4) with the probability $p_1(x_1 - M_1x_{m1})$, she meets a type-1 agent, who likes her good and issues a note in exchange. In the result of such a meeting, she produces, suffers a production cost q_0 , and becomes a note-holder with the value function V_N^0 . Thus, her net gain is $V_N^0 - V_s^0 - q_0$, and the expected net gain of such a meeting is

$$(1 - e^{-\alpha\Delta t})p_1(x_1 - M_1x_{m1})[V_N^0 - V_s^0 - q_0] = \alpha\Delta tp_1(x_1 - M_1x_{m1})[V_N^0 - V_s^0 - q_0] + o(\Delta t).$$

Finally, a type-0 seller may meet no buyer who wishes to buy her good; in this case, she remains in the same state, hence the expected net gain of such a meeting is 0.

Since agents discount the future at the rate r , the sum of all these expected gains must equal to

$$(e^{r\Delta t} - 1)V_s^0 = r\Delta tV_s^0 + o(\Delta t),$$

so that we obtain an equation

$$\begin{aligned} r\Delta tV_s^0 &= \alpha\Delta tp_0Nx_0[V_N^0 - V_s^0 - q_0] + \alpha\Delta tp_0M_0x_{m0}[V_m^0 - V_s^0 - q_{m0}] + \\ &+ \alpha\Delta tp_1M_1(x_{m1} + x_{m1}^+)[V_m^0 - V_s^0 - q_{m0}] + \alpha\Delta tp_1(x_1 - M_1x_{m1})[V_N^0 - V_s^0 - q_0] + o(\Delta t). \end{aligned}$$

By dividing by $\alpha\Delta t$, passing to the limit as $\Delta t \rightarrow +0$ and using $\rho = r/\alpha$, we obtain the first Bellman's equation:

$$\begin{aligned} \rho V_s^0 &= p_0Nx_0[V_N^0 - V_s^0 - q_0] + p_0M_0x_{m0}[V_m^0 - V_s^0 - q_{m0}] + \\ &+ p_1M_1(x_{m1} + x_{m1}^+)[V_m^0 - V_s^0 - q_{m0}] + p_1(x_1 - M_1x_{m1})[V_N^0 - V_s^0 - q_0]. \end{aligned}$$

Similarly, we consider agents of other types and obtain 4 more Bellman's equations:

$$\rho V_N^0 = p_0(1 - M_0 - N)x_0[u(q_0) - (V_N^0 - V_s^0)] + p_1x_N[u(q_N) - (V_N^0 - V_s^0)];$$

$$\rho V_m^0 = p_0(1 - M_0 - N)x_{m0}[u(q_{m0}) - (V_m^0 - V_s^0)] + p_1(1 - M_1)x_m[u(q_{m1}) - (V_m^0 - V_s^0)];$$

$$\begin{aligned} \rho V^1 &= p_0(1 - M_0 - N) \min\{x_1, x_*\}u(q_0) - p_0Nx_Nkq_N + \\ &+ p_0M_0x_m[V_m^1 - V^1 - kq_{m1}] + p_1x_*(u(q_1) - kq_1); \end{aligned}$$

$$\begin{aligned} \rho V_m^1 &= p_0(1 - M_0 - N)\{\min\{x_1, x_*\} - x_{m1}\}u(q_0) + \\ &+ (x_{m1} + x_{m1}^+)[u(q_{m0}) + V^1 - V_m^1] - p_0Nx_Nkq_N + p_1x_*(u(q_1) - kq_1). \end{aligned}$$

By using (1.1), (1.3) and (1.5), we can simplify the Bellman's equations:

$$V_s^0 = 0; \quad V_N^0 = q_0; \quad V_m^0 = q_{m0}; \tag{1.25}$$

$$\rho q_0 = p_0(1 - M_0 - N)x_0[u(q_0) - q_0] + p_1x_N[u(q_N) - q_0]; \tag{1.26}$$

$$\rho q_{m0} = p_0(1 - M_0 - N)x_{m0}[u(q_{m0}) - q_{m0}] + p_1(1 - M_1)x_m[u(q_{m1}) - q_{m0}]; \tag{1.27}$$

$$\rho V^1 = p_0(1 - M_0 - N) \min\{x_1, x_*\}u(q_0) - p_0Nx_Nkq_N + p_1x_*(u(q_1) - kq_1); \tag{1.28}$$

$$\begin{aligned} \rho kq_{m1} &= (\rho V_m^1 - \rho V^1) = \\ &= p_0(1 - M_0 - N)\{-x_{m1}u(q_0) + (x_{m1} + x_{m1}^+)[u(q_{m0}) - kq_{m1}]\}. \end{aligned} \tag{1.29}$$

By using (1.25), we can derive from (1.19) and (1.20) that $V_s = 0$ and $V_m = V_m^0 = q_{m0}$, and rewrite non-defection conditions (1.14)–(1.18) as follows:

$$V^1 \geq kq_N; \tag{1.30}$$

$$V^1 \geq kq_N + q_{m0} - kq_{m1}; \tag{1.31}$$

$$kq_1 = \min\{kq^*, V^1, V^1 + kq_{m1} - q_{m0}\}; \quad (1.32)$$

if $x_1 < x_*$, then

$$V^1 \geq u(q_0), \quad V^1 \geq u(q_0) + q_{m0} - kq_{m1}; \quad (1.33)$$

if $x_1 > x_*$, then

$$V^1 \geq 0, \quad V^1 \geq q_{m0} - kq_{m1}. \quad (1.34)$$

1.8. Fit the Eighth (Types of stationary equilibria and the Vanishing of the Rouble). It is natural to classify these equilibria according to the following criteria:

a) *Types of exchange*, which take place, the list of possible ones being

- monetary exchange inside 0-sector;
- monetary exchange between the two sectors;
- note exchange inside 0-sector;
- note exchange between the two sectors.

It is possible to show that it is optimal for type-1 agents to sustain a non-zero level of inside exchange (multilateral trade credit), therefore we consider only cases when $q_1 > 0$.

b) Which *probability of note-issuing* is used by 1-sector:

- below the individually optimal level;
- equal to the individually optimal level;
- above the individually optimal level.

The list of possible combinations of patterns of exchange and note-issuing rules is huge but we can make it much shorter if we rule out cases when there are no trades between sectors and the economy consists of two separate economies, each comprising agents of one type, and the case without note exchange between the two sectors. Such a pattern of exchange is possible if either there remain no notes in circulation at all and the economy is pure monetary or some notes remain in circulation in 0-sector and play the role of additional fiat money for 0-sector.

In terms of parameters of the model, these restrictions imply:

$$N + M_0 < 1, \quad N > 0 \quad (1.35)$$

(if these conditions fail, all type-0 agents have either notes or money and do not trade at all, or there is no notes in circulation);

$$x_1 + x_{m1} + x_{m1}^+ > 0 \quad (1.36)$$

(if this condition fails, all type-1 agents pay with neither notes (if $x_1 = 0$) nor money ($x_{m1} = x_{m1}^+ = 0$), and hence, there is no exchange between sectors);

$$x_N + x_m > 0 \quad (1.37)$$

(if this condition fails, all type-0 agents pay to type-1 agents with neither notes ($x_N = 0$) nor money ($x_m = 0$), and hence, there is no exchange between sectors).

Since we assume that there is note exchange between the two sectors, we must have $x_1 > 0$ or $x_N > 0$. The stationarity assumption and a balance equation (1.23) imply

$$x_1 > 0 \iff x_N > 0, \quad (1.38)$$

therefore we may impose either of conditions in (1.38), and then (1.36)–(1.37) are satisfied.

Finally, we assume that notes and money are not given away as gifts (at least, by someone) so that

$$q_0 + q_N > 0, \quad q_{m0} + q_{m1} > 0. \quad (1.39)$$

Now, we divide equilibria satisfying additional conditions (1.35)–(1.39) into two groups:

- *type I_M* -equilibria (without monetary exchange between sectors);
- *type II_M* -equilibria (with monetary exchange between sectors).

In Sections 2 and 3, we study type- I_M and type- II_M equilibria, respectively. An economy with only notes in circulation is studied in Boyarchenko and Levendorskii (1999), and an economy without notes in circulation and collusion among agents – in Boyarchenko (1999). Notice that if there is no optimal $x_1 > 0$ for type-1 agents, and their welfare grows as $x_1 \rightarrow 0$, then we may say that the optimal note-issuing threshold is 0, and we obtain a pure monetary economy with two groups of agents. We were able neither to prove that this possibility never realizes nor find an example of such equilibria.

For small h , there are no such equilibria, since for such h , type- II_M equilibria are inferior from the point of view of 1-sector. Thus, the Rouble vanishes from the majority of transactions, and here is an appropriate almost-quotation from *The Hunting of the Snark* by Lewis Carrol:

”It’s a *Stabilization!*” was the sound that first came to *our* ears,⁴
And seemed almost too good to be true.

.....

In the midst of the word *they were* trying to say,
In the midst of *their* laughter and glee,
Rouble had softly and suddenly vanished away –
For *an equilibrium was wrong*⁵, you see.

⁴The reader remembers Chernomyrdin, Chubais and Lifschitz going places and telling fairy tales about successes of the financial stabilization

⁵As usual, *the Snark was a Boojum*

2. STATIONARY EQUILIBRIA OF TYPE I_M : THE CASE OF NO MONETARY TRADES BETWEEN THE TWO SECTORS

2.1. Equilibria specification. Suppose, a (non-degenerate) equilibrium without monetary trades between the two sectors exist. Then, instead of M , a fraction $M_0 \in (0, 1)$ of type-0 agents with money plays the part of an exogenous parameter, and balance equations (1.22) and (1.24) become redundant. Further, in (1.23), we must set $M_1 x_{m1} = 0$, and solve (1.23) for N :

$$N = \frac{(1 - M_0)x_1}{x_1 + x_N}.$$

By introducing $\gamma = x_1/x_*$, we can write

$$N = \frac{(1 - M_0)\gamma}{1 + \gamma}. \quad (2.1)$$

Since we assume that there is trade between sectors, with notes as mean of exchange, it must be the case that $x_1 > 0$, and (1.38) gives $x_N > 0$, hence $x_N = x_*$, and from (1.7), we obtain

$$u(q_N) \geq q_0. \quad (2.2)$$

By using (1.39), (2.2), (1.2) and (1.26), we obtain

$$q_N > 0, \quad q_0 > 0. \quad (2.3)$$

Further, the absence of monetary trade between sectors is possible in one of the following cases:

1. In an equilibrium, prices are such that it is not optimal for type-0 agents to spend money when trading with type-1 agents, and for type-1 agents to pay with money at all. By (1.6) and (1.8)–(1.10), these assumptions imply

$$x_m = 0, \quad u(q_{m1}) < q_{m0}, \quad (2.4)$$

and

$$x_{m1} = x_{m1}^+ = 0, \quad u(q_{m0}) < kq_{m1}. \quad (2.5)$$

From $x_{m1} = x_{m1}^+ = 0$ and (1.29), it follows that $q_{m1} = 0$, which contradicts the inequality in (2.5), since $u(q_{m0}) \geq 0$. Thus, an equilibrium with properties (2.4)–(2.5) does not exist.

2. Equilibrium prices are such that it is not optimal for type-1 agents to spend money (hence, (2.5) holds), though money-holders of type-0 are willing to trade with type-1 agents. By (1.6), this implies $x_m = x_*$, $u(q_{m1}) \geq q_{m0}$. From $x_{m1} = x_{m1}^+ = 0$ and (1.29), it follows that $q_{m1} = 0$, hence $u(q_{m1}) \geq q_{m0}$ gives $q_{m0} = 0$, which contradicts an inequality in (2.5).

3. Equilibrium prices are such that it is not optimal for type-0 agents to spend money when trading with type-1 agents but type-1 agents are willing to spend money in some cases provided they have it. In a steady state, this pattern of exchange leads to all the money accumulated in 0-sector: $M_1 = 0$, $M_0 p_0 = M$. Since we rule out the autarky

for 0-sector, this case can be taken into consideration only if $M < p_0$ and hence, $M_0 = M/p_0 < 1$. Also, by (1.6) and (1.8)–(1.10), we must have that (2.1)–(2.4) hold, and

$$x_{m1} + x_{m1}^+ > 0, \quad u(q_{m0}) \geq q_{m1}. \quad (2.6)$$

In the sequel, we consider the case 3, with $M_0 \in (0, 1)$ fixed. Notice that we must assume that

$$q_{m1} < q_{m0}; \quad (2.7)$$

if $q_{m1} \geq q_{m0}$, there is no reason why type-0 money holders should not accept q_{m1} units of good from type-1 agents if they accept $q_{m0} \leq q_{m1}$ from type-0 sellers.

Set $h = \rho/x_*$, $B(\gamma) = p_0(1 - M_0)/(1 + \gamma)$, $\kappa_m = (x_{m1} + x_{m1}^+)/x_*$, $\gamma_{m1} = x_{m1}/x_*$. Using (1.2) and (2.1), we can rewrite (1.26)–(1.29) as

$$hq_0 = B(\gamma) \frac{x_0}{x_*} (u(q_0) - q_0) + p_1(u(q_N) - q_0); \quad (2.8)$$

$$hq_{m0} = B(\gamma) \frac{x_{m0}}{x_*} (u(q_{m0}) - q_{m0}); \quad (2.9)$$

$$hV^1 = B(\gamma) [\min\{1, \gamma\}u(q_0) - \gamma kq_N] + p_1(u(q_1) - kq_1); \quad (2.10)$$

$$hkq_{m1} = B(\gamma) \{ \kappa_m(u(q_{m0}) - kq_{m1}) - \gamma_{m1}u(q_0) \}. \quad (2.11)$$

Since $M_1 = 0$, the "social planner" of 1-sector maximizes V^1 . In order that the note issuing for 1-sector make sense, we need a participation constraint

$$\min\{1, \gamma\}u(q_0) - \gamma kq_N \geq 0. \quad (2.12)$$

2.2. Some simplification and additional characterization of type- I_M equilibria.

In Appendix, we prove that it is optimal for type-0 agents to use both media of exchange among themselves, and gains from monetary trade are positive:

$$x_{m0} = x_*, \quad u(q_{m0}) > q_{m0}; \quad (2.13)$$

$$x_0 = x_*, \quad u(q_0) \geq q_0. \quad (2.14)$$

By using (2.13) and (2.14), we simplify (2.8) and (2.9):

$$\left(1 + \frac{h}{B(\gamma)}\right) q_0 - u(q_0) = \frac{p_1}{B(\gamma)} (u(q_N) - q_0); \quad (2.15)$$

$$\left(1 + \frac{h}{B(\gamma)}\right) q_{m0} - u(q_{m0}) = 0. \quad (2.16)$$

Rewrite (2.11) in a form similar to (2.15)–(2.16):

$$\left(\frac{h}{B(\gamma)\kappa_m} + 1\right) kq_{m1} - u(q_{m0}) = -\frac{\gamma_{m1}}{\kappa_m} u(q_0). \quad (2.17)$$

The RHS in (2.15) is non-negative, since $u(q_N) \geq q_0$, and in (2.16), it is 0, whereas the LHS's have the same structure, therefore applying Lemma 4.3 in Appendix to (2.15) and (2.16), we see that

$$q_{m0} \leq q_0, \quad \text{and} \quad q_{m0} = q_0 \iff u(q_N) = q_0. \quad (2.18)$$

Suppose, that $\gamma_{m1} > 0$, which means that type-1 money holders would prefer to pay with the money, if they had it, even when they are permitted to issue notes. Then (1.8) and (2.18) imply

$$q_{m0} \geq q_0, \quad \text{and} \quad q_{m0} = q_0 \iff kq_{m1} = 0. \quad (2.19)$$

By comparing (2.18) and (2.19), we conclude that if $\gamma_{m1} > 0$, then $u(q_N) = q_0 = q_{m0}$, and $kq_{m1} = 0$, and then (2.17) gives $\gamma_{m1}/\kappa_m = 1$. From the definitions of x_{m1} and x_{m1}^+ , it follows that if $\gamma_{m1} > 0$, then $\kappa_m = 1$, therefore $\gamma_{m1} = 1$, too.

To sum up: either

$$\gamma = 1, \quad x_{m1} = x_*, \quad q_{m1} = 0, \quad u(q_N) = q_0 = q_{m0}, \quad (2.20)$$

where q_{m0} is a (unique) positive solution to an equation

$$\left(1 + \frac{2h}{p_0(1 - M_0)}\right) q = u(q), \quad (2.21)$$

or

$$\gamma \in (0, 1), \quad x_{m1} = 0, \quad \kappa_m = 1 - \gamma. \quad (2.22)$$

In Subsection 2.4, we show that if the trading friction, h , is small, then a choice (2.22) is non-optimal, and hence, an equilibrium is determined by (2.20)–(2.21) (or it does not exist at all).

It seems to be impossible to obtain analytical results for arbitrary h , since equilibria under consideration are solutions to a very complicated non-linear system in more than a dozen unknowns.

2.3. The case of small h and ineffective type-1 agents. If the economy is very active and the number of single coincidence meetings per unit of time is large, the *trading friction*, h , is small. This means that agents can be patient and wait for a better production opportunity to arrive. A type-0 buyer with a note will not be willing to trade with type-1 sellers, unless type-1 agents redeem for a note a sufficiently high amount q_N . This q_N may be too high a production level for type-1 agents, since they suffer higher production cost than type-0 agents, so that it would be optimal for them not to trade at all.

In this case, it remains for type-0 agents to trade among themselves only. The economy splits into two disjoint sectors, a case which we do not consider.

Theorem 2.1. *For $k > 1$ fixed, there exists $h_0 > 0$ such that if $h \in (0, h_0)$ then type- I_M equilibria do not exist.*

For proof, see Appendix.

Remark 2.1. In Boyarchenko and Levendorskii 1999, we have shown for a non-monetary economy with inside money, that if $k > 1$ is fixed and h and x_* are small and satisfy $h \gg x_*$ (i.e. agents are very picky), then there exists a non-degenerate equilibrium with properties: $u(q_N) = q_0$, and $\gamma \rightarrow +\infty$, $q_0 \rightarrow 0$ as $h \rightarrow 0$. In this paper, if we disregard

monetary constraints, a subsystem in (γ, q_0, q_N) has a solution with the same properties, but as we already saw, the very presence of money forces $\gamma \leq 1$, under assumption that there are no money flows between the two sectors. Thus, a possible type- I_M equilibrium with large γ must turn into a type- II_M equilibrium, which we will study in the next section.

Suppose, the economy is in a type- I_M steady state, and type-1 agents are less effective than type-0 ones. Theorem 2.1 and the corresponding result for type- II_M equilibria mean that if agents are not very picky, and the trading friction vanishes (due to the investment in infrastructure, say) but the ineffectiveness of type-1 agents remains the same, the economy loses stability. Thus,

if a policy maker willing to improve an economy invests in the trading infrastructure at the expense of the production sector, the economy loses stability and splits into two disjoint economies.

In Subsection 2.5, we will show that the stability will be lost even if the ineffectiveness of type-1 agents vanishes with the trading friction but slower.

2.4. The case of small h and effective type-1 agents. If $k = 1$, i.e. type-1 agents are as effective as type-0 agents, a problem described in the previous Subsection does not arise and an equilibrium with interacting sectors exists for arbitrary small $h > 0$. This result and the next one, for type-1 agents with small ineffectiveness $k - 1$, are similar to results for a non-monetary economy with inside money studied in Boyarchenko and Levendorskii 1999.

Theorem 2.2. *Let $k = 1$.*

Then there exists $h_0 > 0$ such that for $h \in (0, h_0)$

a) an equilibrium exists, and it is unique;

b) an equilibrium amount of good produced by type-0 agents for a note, q_0 , can be found as a unique positive solution to

$$hq = \frac{p_0(1 - M_0)}{2}[u(q) - q], \quad (2.23)$$

and they produce the same amount $q_{m0} = q_0$ for a unit of money;

c) an optimal choice of a note-issuing rule for 1-sector is $x_1 = x_$ (i.e. an individually optimal threshold), an equilibrium amount of notes in circulation is equal to $p_0(1 - M_0)/2$, and q_N , an optimal amount of good redeemed for a note, is determined from*

$$u(q_N) = q_0; \quad (2.24)$$

d) let q_ denote a unique positive solution to*

$$u(q) = q; \quad (2.25)$$

then

$$q^* < q_N < q_0 = q_{m0} < q_*; \quad (2.26)$$

e) as $h \rightarrow +0$, q_N and $q_0 = q_{m0}$ converge to q_ ;*

f) $q_{m1} = 0$, i.e. 1-sector would have produced nothing for a unit of money, if offered one.

For proof, see Appendix.

We see that

- type-1 agents exercise their monopoly power and redeem for a note less than type-0 agents produce for the same note to each other and to type-1 agents, and for a unit of money to each other;
- as the trading friction vanishes ($h \rightarrow 0$), so does the monopoly power (we measure it by $1 - q_0/q_N$);
- as the trading frictions vanishes ($h \rightarrow 0$), so do trading surpluses in trades of all types.

The next theorem shows that even when the trading friction is small, 1-sector may be ineffective but the ineffectiveness of type-1 agents (i.e. $k - 1$) must be small. It also gives a necessary and sufficient condition for an admissible level of ineffectiveness.

Theorem 2.3. *Let q_0 be a unique positive solution to (2.23).*

Then there exists $h_0 > 0$ such that for $h \in (0, h_0]$, a type- I_M equilibrium exists if and only if

$$(1 \leq) k \leq u(q_0)/u^{-1}(q_0). \quad (2.27)$$

The equilibrium is unique, and in this equilibrium,

q_0 and q_N are determined from (2.23) and (2.24), respectively,

$\gamma = 1$,

$q_{m0} = q_0$,

an equilibrium amount of notes in circulation is equal to $p_0(1 - M_0)/2$,

(2.26) hold, and

as $h \rightarrow +0$, q_N and $q_0 = q_{m0}$ converge to q_ .*

For proof, see Appendix.

Notice that $u(q_0)/u^{-1}(q_0) > 1$, hence equilibria described in Theorem 2.3 and satisfying $k > 1$ do exist.

If $k > 1$ but satisfies (2.27) then remarks made after Theorem 2.2 remain valid. In addition, if the trading friction and the ineffectiveness of 1-sector are small then the equilibrium quantities q_0 and q_N , and the welfare of a type-0 agent $V = (1 + M_0)q_0/2$ are independent of the level of ineffectiveness, $k - 1$, but the cost kq_N , which a type-1 agent suffers while redeeming a note, increases, and the value function of a type-1 agent, V^1 , decreases with k growing, the loss being proportional to the level of the ineffectiveness:

$$V^1 = h^{-1}p_1(u(q^*) - q^*) + \frac{q_*(1 + u'(q_*))}{u'(q_*)} - k_1 \left(p_1q^* + q_* \frac{p_0(1 - M_0)}{2} \right) + O(h), \quad (2.28)$$

where $k_1 = (k - 1)/h$.⁶

When the trading friction and the ineffectiveness of type-1 agents are small, only type-1 agents have to bear the cost of their ineffectiveness, their monopoly power notwithstanding,

⁶ $O(h)$ is the standard notation for any function $f(h)$, which decays as fast as h , as $h \rightarrow 0$: $|f(h)| \leq Ch$, where C is independent of h (and on other parameters f may depend on).

whereas all the characteristics of the economy observable by type-0 agents (namely, prices) are independent of the level of ineffectiveness.

2.5. Instability Index and Instability Threshold. By Theorem 2.3, $q_0 \rightarrow q_*$ as $h \rightarrow +0$, therefore from (2.27), we deduce

Theorem 2.4. a) Let

$$k_1(1 - M_0) < \frac{2(1 + u'(q_*))}{u'(q_*)p_0}. \quad (2.29)$$

Then there exists $h_0 > 0$ such that if $h \in (0, h_0]$ and $k \leq 1 + k_1h$, then a type- I_M equilibrium exist.

b) Let

$$k_1(1 - M_0) > \frac{2(1 + u'(q_*))}{u'(q_*)p_0}. \quad (2.30)$$

Then there exists $h_0 > 0$ such that if $h \in (0, h_0]$ and $k \geq 1 + k_1h$, then a type- I_M equilibrium does not exist.

Notice that an expression in the LHS in (2.29) and (2.30) is equal to a product of the level of the ineffectiveness, $k-1$, the inverse trading friction, h^{-1} , and $1 - M_0$, the defect of the money supply. We call this product *the instability index*. When the instability index crosses certain threshold, namely, the RHS in (2.29) and (2.30), a type- I_M equilibrium loses stability.

An economy with ineffective 1-sector can preserve stability if the money supply and/or trading friction increase.

2.6. Dependence on Money Supply. Suppose that the government gives an additional amount of money to some type-0 sellers. Then some of type-0 sellers become buyers, fractions p_0 and p_1 of agents of type-0 and type-1 do not change, but the fraction M of agents carrying money and the fraction $M_0 = M/p_0$ of type-0 agents carrying money increase. After some transition period, the economy arrives into the new steady state. If the increase of the money supply is not very large, the type of an equilibrium remains the same, and in the new steady state

1. the amount of notes $p_0(1 - M_0)/2$ in circulation decreases but the total amount of liquidity $p_0M_0 + p_0(1 - M_0)/2 = p_0(1 + M_0)/2$ increases: additional money squeeze out a part of notes but slowly: 2 units of money are needed to replace 1 note;

2. the instability index $k_1(1 - M_0)$ decreases, hence the economy moves away from a line where it can lose stability and split into two disjoint economies;

3. the weighted average of value functions of type-0 agents

$$\begin{aligned} V(M) &= (1 - N - M_0V_s^0 + NV_N^0 + M_0V_m^0) = \\ &= Nq_0 + M_0q_{m0} = p_0 \frac{1 + M_0}{2} q_* + O(h) = \\ &= \frac{p_0 + M}{2} q_* + O(h) \end{aligned}$$

increases, provided M is not large (an optimal amount M from the point of view of 0-sector can be obtained as a maximizer for

$$V(M_0) = \frac{p_0(1 + M_0)}{2} q_0(M_0),$$

where $q_0(M_0)$ is a unique positive solution to (2.23));

4. effective type-1 agents prefer to have no money in economy at all:

Theorem 2.5. *If the trading friction is small, and type-1 agents are effective: $k = 1$, then their welfare decreases with M growing.*

(proof in Appendix), and hence, the same is true if their ineffectiveness $k - 1$ is positive but small. In general, for a given $k > 1$, there exist a minimal positive level $M_* = M_*(k)$ such that if $M < M_*(k)$, the economy splits (M_* can be found from (2.27)), and an optimal level $M_{**}(k) \geq M_*(k)$, which maximizes the welfare of 1-sector. If k is not large, an optimal money supply from the point of view of 1-sector is smaller than an optimal money supply from the point of view of 0-sector, but for large k , it is the other way round.

3. STATIONARY EQUILIBRIA OF TYPE II_M : THE CASE OF TRADES OF ALL TYPES

3.1. Main Results. Due to more complex pattern of exchange, it is more difficult to obtain analytical results, and constructions become long. So, we prefer to describe the main results first, next discuss some numerical examples, and only after that give explicit formulations and proofs.

We consider the case of not very picky agents and not very large money supply; the trading friction is assumed to be small, and here are the main results:

1. When the trading friction is small, in a "good"⁷ type- II_M monetary equilibrium, 1-sector issues notes below the individually optimal level, and a type-1 money holder pays with a note whenever allowed to. In other words, for a type-1 agent, it is individually optimal to pay with notes always, but the "social planner" of 1-sector restricts note-issuing in order to maximize the welfare of the sector.

2. In a type- II_M equilibrium, a 1-sector accumulates more money per capita than 0-sector.

3. If the trading friction vanishes faster than the ineffectiveness of 1-sector, any economy with trade between sectors loses stability and splits.

4. Type- II_M equilibria are more fragile than type- I_M equilibria.

5. If the trading friction is very small, monetary trade between sectors may exist only if type-1 agents need an inflow of money for some exogenous (from the point of view of our model) purposes, for instance, to pay taxes; otherwise, it is optimal for them not to use money at all.

Of course, to include taxation, our model should be modified but its structure will remain essentially the same.

6. In the preceding Section, we have reduced the initial problem to a relatively simple equation (which admits an explicit solution in the case of the Cobb-Douglas utility function, so that it is possible to obtain analytic expressions for all endogenous variables); here we manage to reduce the initial problem to a maximization of a rather complicated function on an interval $(0, 1)$, subject to a non-strict inequality. From the computational point of view, to solve such a problem and calculate equilibrium values of endogeneous variables is easy, and the corresponding procedures are robust and do not require much time of a PC, but an analytical solution is impossible even for simplest utility functions.

3.2. Numerical Examples. We consider the Cobb-Douglas utility $u(q) = dq^\beta$, where $d > 0$ and $\beta \in (0, 1)$.

In the first example, we fix h and p_1 , and vary the ineffectiveness level and the money supply. In Fig.3.1, we plot the gain of 1-sector from trade with 0-sector. A plane in the left corner (zero gain) indicates a region where the economy splits into two disjoint ones. In the right corner, there is a region where a type- II_M equilibrium becomes superior from the point of view of 1-sector. The region is more clearly seen on Fig.3.2, where we plot the weighted value function of type-0 agents. A type- II_M equilibrium becomes optimal for type-0 agents only where the money supply is not small, and the ineffectiveness of

⁷If agents are very picky, i.e. they are too specialized in consumption, there may exist "bad" equilibria when too many notes circulate and the production level is too low

agents is small; with M increasing, an admissible level of ineffectiveness increases as well. It is interesting to note that for moderate levels of money supply, the change of the type of an equilibrium from I_M to II_M yields a sharp decline of the welfare for 0-sector, and only for fairly large values of M , a type- II_M equilibrium becomes superior to a type- I_M equilibrium from the point of view of type-0 agents. However, the next picture (Fig.3.3) shows that

for large values of M , it is optimal for type-0 agents to sever all contacts with 1-sector, and if they happen to continue to trade with 1-sector, it is due to their inability to collude and change the type of the equilibrium. If they could, an increase of the money supply would have made the economy to split; type-1 agents would have become worse off, but type-0 agents – better off.

In Fig.3.2, it is also clearly seen that

the same change of the money supply, at the same existing level, can lead to opposite effects: at small levels of ineffectiveness, an increase of M may cause the welfare of type-0 agents to drop, but at larger values – to jump.

Roughly speaking,

an increase of the money supply is good for type-0 agents, if the money supply is not already large, and type-1 agents are rather ineffective; otherwise, a decrease of the money supply may be better.

As far as ineffective agents are concerned, Fig.3.1 shows that

if the ineffectiveness is not large, the note-issuing sector can preserve its note-issuing role and hence remain alive⁸ even for low levels of money supply, but the larger the ineffectiveness, the higher level of money supply is needed to activate the ineffective sector and make it alive and well. This activation is welfare-improving for the effective sector, too.

This consideration can be reversed, of course:

if 1-sector is very ineffective, then even moderate decrease of money supply can make it effectively die, and a disappearance of an additional source of liquidity and some of trading partners from an economy causes the welfare of effective agents to drop.

In Fig. 3.4–3.7, we plot amounts of good produced (or redeemed) by different agents for a unit of money or a note. We see that apart of jumps, where the type of an equilibrium changes, an increase of money supply causes the quantities to decrease.

Very similar pictures obtain when we fix M and allow p_1 , the size of 1-sector, to change, and the same is true if we take $M = p_1 M^*$, where M^* is fixed. The lines of transition from one type of equilibrium to another one, and the surfaces are of essentially the same shape (only the locations change a little). We do not show the pictures here in order to save the space, but make some comments.

The first general remark is:

p_1 , the size of 1-sector, and M , the money supply, play essentially the same stabilizing role for 1-sector, so all conclusions made above remain valid if we replace M by $M + p_1$.

⁸In the sense: continue to trade with 0-sector; if we introduce the taxation into our model, an isolated sector will be unable to earn money to pay taxes and its agents will become bankrupt

In particular, there is "the strength in numbers effect" for the ineffective sector: when the sector is sufficiently large, it will survive at any level of the money supply.

The second remark concerns a more general version of our model, with agents of varying level of ineffectiveness:

there is a multiplicative effect of an increase of money supply: first, it allows for some of ineffective agents to get activated, and the resulting increase of the size of an alive part of 1-sector activates some of even less effective agents, and so on – till the point, where there is an essential difference in the effectiveness of "alive" and "dead" agents. This "snow-balling effect" can push an economy into the region, where the welfare of effective type-0 agents is much lower than that in an initial state with a bit lower level of money supply.

Once again, this consideration can be reversed:

if, for a given level of money supply and size of an alive part of 1-sector, the ineffectiveness of the latter is close to a critical point, then a small decrease of money supply can lead to a rapid contraction of 1-sector.

We may say that the ineffective sector behaves like a "black hole": a small one quickly evaporates, and a large one is getting larger and larger.

In the next example, we fix $p_1 = 0.3$, and $M = 0.2$, and vary h and k_1 . For small values of h , a transition line, which separates type- 0_M and type- I_M equilibria, is clearly seen. A type- II_M equilibrium becomes optimal only when h is not very small, and the ineffectiveness is small; the larger the h , the larger values of ineffectiveness become admissible.

In agreement with analytical results, we see that when h vanishes, q_{m0} in pure monetary equilibria and q_0, q_N, q_{m0} in type- I_M equilibria stabilize (to $q_* = 1$, of course), and gains from trade with 0-sector, for 1-sector, become a linear decreasing function in k_1 .

With h growing, value functions and amounts of good produced or redeemed decrease (once again, not at transition lines, where they can jump or drop.)

To end the discussion of numerical examples, we note that if h is not very small, then all conclusions made after the first example essentially remain valid if we replace M with h^{-1} or with d . In particular, a small change of the trading friction or consumption technology can produce "snow-balling effects" described above.

3.3. Specification of Type- II_M equilibria. Since we assume that there is trade between sectors, with notes as a medium of exchange, it must be the case that $x_1 > 0$, and (1.38) gives $x_N > 0$, hence $x_N = x_*$, and from (1.7), we obtain

$$x_N = x_*, \quad \text{and} \quad u(q_N) \geq q_0. \quad (3.1)$$

By using (1.39), (3.1), (1.2) and (1.26), we deduce

$$q_N > 0, \quad q_0 > 0. \quad (3.2)$$

Further, since we consider equilibria with money flows between sectors, it is necessary that $x_m > 0$ and $x_{m1} + x_{m1}^+ > 0$; then on the strength of (1.6),

$$x_m = x_*, \quad \text{and} \quad u(q_{m1}) \geq q_{m0}, \quad (3.3)$$

and from (1.8) and (1.10),

$$u(q_{m0}) \geq kq_{m1}. \quad (3.4)$$

We assume that $k \in [1, C_0]$, where $C_0 > 1$ is independent of other parameters; then (3.3) and (3.4) and properties of u imply that q_{m1} and q_{m0} are bounded and bounded away from 0: there exist $C, c > 0$ such that for any h and $k \in [1, C_0]$,

$$c \leq q_{m0} \leq C, \quad c \leq q_{m1} \leq C. \quad (3.5)$$

Lemma 3.1.

$$x_{m0} = x_*, \quad \text{and} \quad u(q_{m0}) \geq q_{m0}. \quad (3.6)$$

Proof. By (1.4), the statements in (3.6) are equivalent. Suppose, that $u(q_{m0}) < q_{m0}$. Then from (3.4), $q_{m0} > kq_{m1}$, and from (3.3), $u(q_{m1}) > u(q_{m0})$, which is equivalent to $q_{m1} > q_{m0}$. But $k \geq 1$ and $q_{m1} > 0$, which leads to a contradiction. \square

To simplify the study of type- II_M equilibria, we make two assumptions:

$$h/x_* \leq \epsilon, \quad (3.7)$$

where $\epsilon > 0$ is sufficiently small, i.e. agents are not very picky, and

$$M < p_1, \quad (3.8)$$

which means that a fraction of type-1 money holders is bounded away from 1: from (3.3) and (1.22),

$$M_1 \leq M/p_1 < 1. \quad (3.9)$$

As we will show, these two conditions exclude "bad" equilibria with $x_1 \sim 1$, when too many notes are being issued and equilibrium quantities are small (in Theorem 2.2 in Boyarchenko and Levendorskii 1999, we constructed such equilibria for an economy with only notes in circulation, and if $h/x_* \gg 1$, it is possible to construct type- II_M equilibria with similar properties.)

By using (3.4) and (1.24), we obtain

$$M_0 \rightarrow 0 \iff 1 - M_0 - N \rightarrow 0 \iff N \rightarrow 1, \quad (3.10)$$

and from (1.23), (3.4) and (3.5), it follows that

$$1 - M_0 - N \rightarrow 0 \iff \gamma \rightarrow +\infty,$$

where $\gamma = x_1/x_*$; to be more precise, there exist $C, c > 0$ such that

$$c/\gamma \leq 1 - N - M_0 \leq C/\gamma. \quad (3.11)$$

Lemma 3.2. *There exists $\epsilon_0 > 0$ such that if $\epsilon \in (0, \epsilon_0]$, then in any type- II_M equilibrium,*

$$x_0 = x_*, \quad \text{and} \quad u(q_0) \geq q_0. \quad (3.12)$$

Proof in Appendix.

Lemma 3.3. *There exists $\epsilon_0 > 0$ such that if (3.7) holds with $\epsilon \in (0, \epsilon_0]$, then in any type- II_M equilibrium, $\gamma \in (0, 1)$, and $x_{m1} = 0$.*

Proof in Appendix.

3.4. A "good" type-II_M equilibrium. Fix $\gamma \in (0, 1)$, divide (1.23) and (1.24) by x_* , using $x_N = x_m = x_*$, $x_1/x_* = \gamma$ and $(x_{m1} + x_{m1}^+)/x_* = 1 - \gamma$:

$$(1 - N - M_0)\gamma = N,$$

$$M_0(1 - M_1) = (1 - M_0 - N)M_1(1 - \gamma).$$

By using (1.22) and these two equalities, we find

$$M_1 = (M - p_0M_0)/p_1, \quad (3.13)$$

$$N = \frac{(1 - M_0)\gamma}{1 + \gamma}, \quad (1 - M_0 - N) = \frac{1 - M_0}{1 + \gamma}, \quad (3.14)$$

where $M_0 = M_0(\gamma)$ is a positive solution to an equation

$$2\gamma p_0 M_0^2 + M_0(1 + \gamma(p_1 - p_0 - 2M)) + M(\gamma - 1) = 0. \quad (3.15)$$

Lemma 3.4. a) For any $\gamma \in (0, 1)$, a positive root of an equation (3.15) exists, is unique, and satisfies $M_0 \in (0, M)$.

b) If $\gamma = 0$, (3.15) has one root $M_0 = M$.

c) $M_0 = M_0(\gamma)$ is continuous on $[0, 1)$.

Proof. a) Since $2\gamma p_0 > 0$ and $M(\gamma - 1) < 0$, one of solutions is positive, and the other – negative, and to prove that $M_0 < M$, it suffices to check that with $M_0 = M$, the LHS is positive. But this is straightforward.

b) Evident.

c) The continuity on $(0, 1)$ follows from the Implicit Function Theorem. To prove the continuity at $\gamma = 0$, we note that from (3.15), $M_0(\gamma) \rightarrow M$ as $\gamma \rightarrow 0$. \square

Now from (1.21) and (3.13) it follows that $M_1 > M$, i.e. in a "good" type-II_M equilibrium, 1-sector accumulates more money per capita than 0-sector.

For $\gamma \in [0, 1)$, set $B(\gamma) = p_0(1 - M_0(\gamma))/(1 + \gamma)$, and notice that since $M_0 \leq M < 1$, $B(\gamma)$ is bounded away from 0 uniformly in h . Having in mind that M_0 and M_1 are uniquely defined by a choice of γ , and using (3.14), we rewrite (1.26)–(1.29) as follows

$$hq_0 = B(\gamma)(u(q_0) - q_0) + p_1(u(q_N) - q_0); \quad (3.16)$$

$$hq_{m0} = B(\gamma)(u(q_{m0}) - q_{m0}) + p_1(1 - M_1(\gamma))(u(q_{m1}) - q_{m0}); \quad (3.17)$$

$$hV^1 = B(\gamma)\gamma(u(q_0) - kq_N) + p_1(u(q_1) - kq_1); \quad (3.18)$$

$$hkq_{m1} = B(\gamma)(1 - \gamma)(u(q_{m0}) - kq_{m1}). \quad (3.19)$$

Lemma 3.5. There exists $h_0 > 0$ such that for any $h \in (0, h_0]$, in a "good" type-II_M equilibrium,

a) $q_1 = q^*$;

b) incentive compatibility constraints (1.30)–(1.34) are satisfied;

c) a condition (1.8) is redundant, and

d)

$$u(q_N) = q_0. \quad (3.20)$$

Proof in Appendix.

Now, for a given $\gamma \in [0, 1)$, we can define step by step:

$M_0 = M_0(\gamma)$ from (3.15);

$M_1 = M_1(\gamma)$ from (3.13);

$q_0 = q_0(\gamma)$ from

$$hq_0 = B(\gamma)(u(q_0) - q_0) \quad (3.21)$$

(due to properties of u , a positive solution to (3.21) exists, and it is unique);

$q_N(\gamma) = u^{-1}(q_0(\gamma))$;

$q_{m1} = q_{m1}(\gamma, q_{m0})$ as a function of q_{m0} and γ :

$$q_{m1} = k^{-1} \left(1 + \frac{h}{B(\gamma)(1 - \gamma)} \right)^{-1} u(q_{m0}); \quad (3.22)$$

by substituting (3.22) into (3.17), we obtain an equation

$$hq_{m0} = B(\gamma)(u(q_{m0}) - q_{m0}) + p_1(1 - M_1(\gamma))(u(q_{m1}(\gamma, q_{m0})) - q_{m0}). \quad (3.23)$$

For γ fixed, (3.23) can be written in the form

$$Aq = v(q),$$

where A is a positive constant, and v satisfies the same conditions as u , namely, v is increasing, concave and satisfies the Inada conditions (the verification of all these properties is straightforward). Hence, (3.23) has a (unique) positive solution $q_{m0} = q_{m0}(\gamma)$, and after that, (3.22) defines q_{m1} as a function of γ .

We see that 1-sector maximizes a function

$$\Phi(\gamma) = h^{-1}B(\gamma)\gamma(u(q_0) - kq_N) + M_1(\gamma)kq_{m1}(\gamma),$$

on $(0, 1)$, subject to (3.3), (3.4) and (3.6); inequalities (1.35), (1.36), (1.39), (3.1), (3.2) and (3.12) are satisfied by construction. Since $q_{m0} > 0$, (3.22) gives $q_{m1} > 0$, and (3.19) implies (3.4). Thus, (3.4) is redundant. Further, let (3.6) be violated: $u(q_{m0}) < q_{m0}$. Then (3.22) gives $q_{m1} < q_{m0}$, and since u is increasing, $u(q_{m1}) < u(q_{m0}) < q_{m0}$, i.e. (3.3) is violated, too. Thus, a pair of constraints (3.3), (3.6) is equivalent to one constraint (3.3). Hence, the only remaining constraint is (3.3).

On the strength of (3.5) and (3.19), γ is bounded away from 1: $\gamma \leq c$, where $c < 1$ (in the next subsection, we will show that γ is bounded away from 1 uniformly in h, M, p_0, k : there exists $c \in (0, 1)$ such that $\gamma \leq c$, for all $h \in (0, 1], M < 1, p_0 \in (0, 1)$, and $k \geq 1$, in any type- II_M equilibrium). Hence, we may assume that 1-sector maximizes Φ on a set $J \setminus 0$, where

$$J = \{\gamma \in [0, c] \mid u(q_{m1}(\gamma)) \geq q_{m0}(\gamma)\}.$$

In order that the trade with 0-sector make sense, it is necessary that $\Phi(\gamma)$, the gain from trade of a type-1 agent with 0-sector, be non-negative. In the next subsection, we will show that if $M > 0$ and J is non-empty, then Φ is positive on J , hence there is no need to introduce an additional constraint.

Theorem 3.6. *For any $M_1 < 1$, there exist $C > 0$ and $h_0 > 0$ such that if $h \in (0, h_0]$ and $x_* \geq Ch$, then*

- a) *a type-II_M equilibrium exists if and only if $J \neq \emptyset$, and the maximum of Φ on J is attained at a point $\gamma > 0$;*
- b) *if conditions in a) are satisfied, then an optimal γ maximizes W on J , and optimal q_0, q_N , etc. are defined by this γ as described above;*
- c) *if conditions in a) are satisfied, then $q_0 = q_* + O(h)$, as $h \rightarrow 0$, and the same is true for q_N, q_{m0} , and q_{m1} .*

Proof. a) J is a subset of a compact $[0, c]$, which is defined by non-strict inequalities for continuous functions, hence J is compact. By continuity of $M_0 = M_0(\gamma)$ and u , and applying the Implicit Function Theorem to (3.21) and (3.23), we see that q_0, q_N, q_{m0}, q_{m1} are continuous, hence Φ is continuous, and the maximum of Φ on J exists if and only if J is non-empty. If the maximum is achieved at some $\gamma > 0$, it is an optimal choice for 1-sector, and if the maximum is attained only at 0, then an optimization problem on $J \setminus 0$ has no solution.

b) is evident, and c) will be proved in Appendix. □

Remark 3.1. For some parameters' values, J is empty. For instance, the following "nonexistence-of-type-II_M-equilibria" theorem holds:

Theorem 3.7. *Let $M < p_1$. There exist $C > 0$ and $h_0 > 0$ such that if $h \in (0, h_0]$, $x_* \geq Ch$, and $k \geq 1 + Ch$, then a type-II_M equilibrium does not exist.*

Proof in Appendix.

Corollary 3.8. *Let $M < p_1$. There exist $C > 0$ and $h_0 > 0$ such that if $h \in (0, h_0]$, $x_* \geq Ch$, and $k \geq 1 + Ch$, then no non-degenerate equilibria exist.*

Proof. In Section 2, we have shown that under these conditions, a type-I_M equilibrium does not exist as well, and in Boyarchenko and Levendorskiĭ (1999) and Boyarchenko (1999) the same has been shown for economies without money and notes, respectively. □

Thus,

If the trading friction vanishes faster than the ineffectiveness of 1-sector, then any economy with an ineffective 1-sector loses stability and splits into disjoint parts.

3.5. Some asymptotic analysis. Here we derive asymptotic formulas for equilibrium quantities as $h \rightarrow 0$, which allow us to find approximate conditions of existence of equilibria, approximate formulas for value functions, and compare type-II_M and type-I_M equilibria.

By Theorem 3.7, we may assume that $k = 1 + k_1h$, where $k_1 = O(1)$ as $h \rightarrow 0$; for simplicity, we assume that k_1 is a non-negative constant, and in the end, formulate existence (nonexistence) conditions in terms of k_1 .

Lemma 3.9. a) *As $h \rightarrow +0$,*

$$\Phi(\gamma) = \Phi_0(\gamma) + O(h), \tag{3.24}$$

where

$$\begin{aligned}\Phi_0(\gamma) &= q_* B(\gamma) \gamma \left(u'(q_*) q_{01} - \frac{q_{01}}{u'(q_*)} - k_1 \right) + M_1 q_* = \\ &= q_* \left(\gamma \frac{u'(q_*) + 1}{u'(q_*)} - k_1 \gamma B(\gamma) + M_1(\gamma) \right).\end{aligned}\quad (3.25)$$

b) There exists $C > 0$ such that for $\gamma \in J$,

$$\Phi(\gamma) \geq q_* \left(\frac{\gamma}{1-\gamma} + M \right) - Ch,$$

and hence, for small h , Φ is positive on J .

c) A set

$$J_0 = \left\{ \gamma \in [0, 1) \mid \frac{u'(q_*) + 1}{u'(q_*)} - \frac{k_1 p_0 (1 - M_0(\gamma))}{1 + \gamma} - \frac{1}{1 - \gamma} \geq 0 \right\}$$

is an approximation to J in the sense that the left (resp., right) ends of J and J_0 are at a distance of order $O(h)$, as $h \rightarrow +0$.

By using Φ_0 as an approximation to Φ , and J_0 as an approximation to J , we can simplify the study of type- II_M equilibria.

For instance, if a function

$$F(\gamma) = \frac{k_1 p_0 (1 - M_0(\gamma))}{1 + \gamma} + \frac{1}{1 - \gamma} \text{ is increasing on } [0, 1), \quad (3.26)$$

then $J_0 \setminus 0 \neq \emptyset$ if and only if F is less than or equal to $Q = (u'(q_*) + 1)/u'(q_*)$, in some right neighborhood of 0. Now the following specification of Theorem 3.6 is straightforward.

Theorem 3.10. a) Let (3.26) hold, and

$$k_1(1 - M) < \frac{1}{u'(q_*) p_0}. \quad (3.27)$$

Then there exists $h_0 > 0$ such that if $h \in (0, h_0]$ and $k \leq 1 + k_1 h$, then a type- II_M equilibrium exist.

b) Let (3.26) hold, and

$$k_1(1 - M) > \frac{1}{u'(q_*) p_0}. \quad (3.28)$$

Then there exists $h_0 > 0$ such that if $h \in (0, h_0]$ and $k \geq 1 + k_1 h$, then a type- II_M equilibrium does not exist.

Notice that an expression in the LHS in (3.27) and (3.28) is equal to a product of the level of the ineffectiveness, $k - 1$, the inverse trading friction, h^{-1} , and $1 - M$, the defect of the money supply, i.e. it can be interpreted as *the instability index* similar to the one introduced for type- I_M equilibria in Subsection 2.8; then the RHS in (3.27) and (3.28) is *the instability threshold*. We see that it is significantly lower than the instability threshold for type- I_M equilibria, $2(1 + u'(q_*))/u'(q_*) p_0$, hence type- II_M equilibria are more fragile than type- II_M equilibria.

3.6. Comparison of type- II_M and type- I_M equilibria.

Theorem 3.11. *There exists $h_0 > 0$ such that if $h \in (0, h_0]$ and a type- II_M equilibrium exists, then a type- I_M equilibrium exists as well, and the value function of type-1 agents in the latter is larger than the one in the former.*

Proof. It follows from (3.24)–(3.25) and (2.30), that the difference of the value function of type-1 agents in type- I_M equilibrium and the one in type- II_M equilibrium is equal to $\Psi_0 - \Phi_0(\gamma) + O(h)$, where $\Phi_0(\gamma)$ is given by (3.25), γ is an optimal threshold, of which we know that $\gamma \in J_0$, and

$$\Psi_0 = q_* \left(\frac{u'(q_*) + 1}{u'(q_*)} - k_1 \frac{p_0 - M}{2} \right).$$

We have

$$\Psi_0 - \Phi_0(\gamma) = q_* \left((1 - \gamma) \frac{u'(q_*) + 1}{u'(q_*)} + k_1 \gamma B(\gamma) - k_1 \frac{p_0 - M}{2} - M_1(\gamma) \right),$$

but $\gamma \in J_0$, therefore

$$(1 - \gamma) \frac{u'(q_*) + 1}{u'(q_*)} \geq k_1 B(\gamma)(1 - \gamma) + 1,$$

and $\Psi_0 - \Phi_0(\gamma) \geq q_* [k_1(B(\gamma) - (p_0 - M)/2) + 1 - M_1(\gamma)]$. But $B(\gamma) - (p_0 + M)/2 = p_0(1 - M_0(\gamma))/(1 + \gamma) - (p_0 - M)/2$ is positive for $\gamma \in (0, 1)$, since $p_0 < 1$ and $M_0(\gamma) < M$, and $1 - M_1(\gamma)$ is positive, since $M_1(\gamma) < 1$. \square

The reason for a type- II_M equilibrium to be inferior is a participation constraint (3.3), which must be satisfied if 1-sector is to get money from type-0 agents ever: to achieve the same level of welfare as in a type- I_M equilibrium, 1-sector must issue notes in large quantities, but then the relative value of holding money increases, type-0 agents start to require of type-1 agents too much good in exchange for a unit of money (as compared to an amount of good which type-1 agents get for a unit of money), so that it becomes non-optimal for type-1 agents to sell their good for money on such unfavorable conditions.

4. APPENDIX: TECHNICAL RESULTS FOR TYPE- I_M EQUILIBRIA

4.1. **Auxiliary results.** To simplify our problem, we need the following lemmas.

Lemma 4.1. *Let c be a positive constant, and functions $u, v : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ satisfy*

$$u \text{ is smooth, increasing and concave, and satisfies the Inada conditions.} \quad (4.1)$$

Then functions cu , $u + v$, $u \circ v$ and $q \mapsto u(cq)$ also satisfy (4.1).

Proof. Direct verification. □

Lemma 4.2. *If u satisfies (4.1), then for any $c > 0$, there exists $\bar{q} > 0$ such that*

$$u(q) \geq cq \iff q \in [0, \bar{q}]. \quad (4.2)$$

Proof. Evident. □

Lemma 4.3. *a) For any tuple $(x_0, x_1, q_N) \in \{0, x_*\} \times [0, 1] \times (0, +\infty)$, there exists a unique $q_0 = q_0(x_0, x_1, q_N) > 0$ satisfying (2.8) and (1.2);*

b) q_0 is a smooth function in (x_1, q_N) ;

c) q_0 is an increasing concave function in q_N , satisfying $\partial q_0 / \partial q_N \rightarrow 0$ as $q_N \rightarrow +\infty$;

d) $q_0 \rightarrow +\infty$ as $q_N \rightarrow +\infty$, but slower than q_N :

$$\lim_{q_N \rightarrow +\infty} \max_{x_0, x_1} q_0(x_0, x_1, q_N) / q_N = 0;$$

e) if $x_0 > 0$, then there exists $\epsilon > 0$ such that

$$q_0(x_0, x_1, q_N) \geq \epsilon, \quad \forall (x_1, q_N) \in [0, 1] \times (0, +\infty);$$

f) if $x_0 > 0$, then q_0 is decreasing in x_1 , on a set where $u(q_0) > q_0$.

Proof. a) Write (2.8) as $\phi(q_0) = u(q_N)$, where

$$\phi(q) = Aq - Bu(q), \quad A = (h/p_0 + 1 + B)k, \quad B = p_0(1 - M_0)x_0 / ((1 + \gamma)p_1x_*) \geq 0.$$

If $x_0 = 0$, then $q_0 = u(q_N)/A$ satisfies (4.1) by Lemma 4.1. If $x_0 > 0$, then $B > 0$, and $\phi'(q) = A - Bu'(q) < 0$ in a sufficiently small right vicinity of 0, due to $u'(+0) = +\infty$. Hence, $\phi(q) < 0$ in some right vicinity of 0. Since u decreases and $u'(+\infty) = 0$, ϕ' has the only root on $(0, +\infty)$, call it \hat{q} , and it is a simple root. Clearly, $\phi(\hat{q}) < 0$, $\phi(q) < 0$ on $(0, \hat{q})$, and ϕ increases on $(\hat{q}, +\infty)$. Moreover, $\phi(q) \rightarrow +\infty$ as $q \rightarrow +\infty$, hence q_0 , a solution to (2.8), is unique and satisfies $q_0(x_1, q_N) > \tilde{q}(x_1)$, where $\tilde{q}(x_1)$ is the root of $\phi(q) = 0$.

b) follows from the Implicit Function Theorem.

c) On $(\tilde{q}, +\infty)$, $\phi'(q) = A - Bu'(q) > 0$, and $\phi''(q) = -Bu''(q) > 0$. Hence, ϕ is convex on $(\tilde{q}, +\infty)$, and $\phi^{-1} : (0, +\infty) \rightarrow (\tilde{q}, +\infty)$ is increasing and concave. Hence, $q_0(q) = \phi^{-1}(u(q))$ also is.

The second statement is evident since $\phi'(q) \sim A$ for q large, and $u'(q_N) \rightarrow 0$ as $q_N \rightarrow +\infty$.

d) For large q and q_N , $\phi(q) = q + o(q)$, $u(q_N) = o(q_N)$, and d) follows.

e) Evidently, we can set $\epsilon = \min_{x_1} \tilde{q}(x_1)$; it is positive by the proof of part a).

f) By calculating the derivative of an implicit function given by (2.8), we obtain for $q_0 > \hat{q}$:

$$\frac{\partial q_0}{\partial x_1} = -\frac{p_0(1-M_0)x_0}{(x_1+x_*)^2 p_1} u(q_0)(A - Bu'(q_0))^{-1}.$$

It is negative on a set, where $u(q_0) > q_0$, since $A - Bu'(q_0) > 0$ there. \square

Lemma 4.4. *For all $M_0 < 1$, $x_1 \in [0, 1]$ and $h > 0$, a positive solution to (2.9) exists. It is unique and satisfies*

$$u(q_{m0}) > q_{m0}. \quad (4.3)$$

If $x_{m0} = 0$, then (2.9) has a unique solution $q_{m0} = 0$.

Proof. If $x_{m0} > 0$, then $q_{m0} > 0$ exists and is unique since u satisfies (4.1); (4.3) is immediate from (2.9) for $q_{m0} > 0$.

If $x_{m0} = 0$, then $q_{m0} = 0$ is evident from (2.9). \square

Proof of (2.13). Since $u(0) \geq 0$, (4.3) means that in all cases, $u(q_{m0}) \geq q_{m0}$, and then (2.13) follows from (1.4).

Notice that if $q_{m1} = 0$, then $q_{m0} > 0$ by (1.39), and if $q_{m1} > 0$, then $q_{m0} > 0$ by (4.1).

Proof of (2.14). Suppose, $x_0 = 0$, then by (1.2), $u(q_0) < q_0$, and by comparing with (2.2), $q_0 < q_N$. On the other hand, (2.12) imply $kq_N \leq u(q_0) < q_0$. Since $k \geq 1$ and $q_N > 0$, we have a contradiction, and $x_0 > 0$. By (1.2), (2.14) holds.

Denote by Q a set of tuples $\{q_0, q_N, q_{m0}, x_1, V^1\}$ satisfying all conditions in the definition of a type- I_M equilibrium, except for the requirement of maximization of V^1 .

Lemma 4.5. *There exists $C > 0$ such that for all $h > 0$ and $X \in Q$,*

$$q_0 = q_0(X) \leq C, \quad q_N = q_N(X) \leq C, \quad q_{m0} = q_{m0}(X) \leq C. \quad (4.4)$$

Proof. Suppose, there exists a sequence $(X_n)_{n \geq 1} \subset Q$ s.t. $q_0(X_n) \rightarrow +\infty$. Then for n large enough, Lemma 4.3 d) and (2.2) contradict each other. Hence, $q_0 \leq C$, therefore by (2.12), $q_N \leq C_1$.

Since u satisfies (4.1), $q_{m0} \leq C$ follows from (2.9) and Lemma 4.2. \square

Lemma 4.6. *There exists $h_0 > 0$ such that for $h \in (0, h_0]$,*

a) *the problem of maximization of V^1 is equivalent to the problem of maximization for a function*

$$\Phi(\gamma, q_0, q_N) = \frac{1}{1+\gamma} [\min\{1, \gamma\}u(q_0) - \gamma k q_N];$$

b) *incentive compatibility constraints (1.30)–(1.34) are redundant.*

Proof. If 1-sector chooses $q_1 = q^*$, we have $u(q^*) - kq^* > 0$, and (2.10) gives $V^1 \geq ch^{-1}$, where $c > 0$ is independent of $h > 0$. Now, for small h , (4.4) gives b) and proves that this choice of q_1 is really optimal. Since this q_1 is independent of x_1 and q_N , a) follows. \square

Notice that this proof is valid for type- II_M equilibria, too.

Lemma 4.7. *There exist $C, h_0 > 0$ s.t. if $h \in (0, h_0]$,*

$$q_* - Ch \leq q_0 \leq q_*, \quad q_* - Ch \leq q_N \leq q_*. \quad (4.5)$$

Proof. By (4.4), the LHS in (2.15) is $O(h)$, and $B(\gamma)$ is bounded away from 0 uniformly in h since γ is bounded. Hence, by using (2.2) and (2.14), we conclude from (2.15), that $0 \leq u(q_0) - q_0 \leq Ch$ and $0 \leq u(q_N) - q_0 \leq Ch$, where $C > 0$ is independent of h . Since $\gamma \in (0, 1]$, part e) of Lemma 4.3 shows that q_0 is bounded away from 0, hence $q_0 \rightarrow q_*$, and then $q_N \rightarrow q_*$, too. Since $u'(q_*) - 1 < 0$ and $u \in C^1$, we see that in a sufficiently small neighborhood of q_* , a function $q \mapsto u(q) - q$ is invertible, and the inverse, call it f , decreases and has a bounded derivative. By applying f to an estimate $Ch \geq u(q_0) - q_0 \geq 0$, and using the Lagrange formula, we obtain $-C_1h \leq q_0 \leq q_*$. After that we have $-C_2h + q_* \leq u(q_N) \leq q_*$. Since $u'(q_*) \neq 0$, the inverse u^{-1} has a bounded derivative in a sufficiently small neighborhood of q_* . We apply u^{-1} to the last inequality, use the Lagrange formula and finish the proof of (4.5). \square

4.2. Proofs of Main Theorems. *Proof of Theorem 2.1.* If $k > 1$ is fixed, $h \rightarrow +0$, and γ remains bounded, then (4.5) holds, and (2.12) implies $q_* + O(h) \geq kq_* + O(h)$. For small h , this is impossible.

Proofs of Theorems 2.2 and 2.3. By Lemma 4.7, q_0 and q_N are in small vicinity of q_* . For these q_0 and q_N , and $\gamma \leq C$, we prove two useful estimates. Since $u \in C^1((0, +\infty))$ and $u'(q_*) < 1$, we conclude from (4.5), that there exist $c > 0$ and $h_0 > 0$ s.t. if $h \in (0, h_0]$ then

$$u'(q_0) \leq 1 - c, \quad u'(q_N) \leq 1 - c. \quad (4.6)$$

Further, let ϕ be from the proof of Lemma 4.3:

$$\phi(q_0, \gamma) = [(p_1 + h)q_0 + p_0(1 - M_0)(q_0 - u(q_0))/(1 + \gamma)]/p_1.$$

Then

$$\frac{\partial \phi}{\partial q_0} = \frac{p_1 + h + p_0(1 - M_0)(1 - u'(q_0))/(1 + \gamma)}{p_1}.$$

Since γ is bounded, $1/(1 + \gamma)$ is bounded away from zero, and by using (4.6), we obtain

$$\frac{\partial \phi}{\partial q_0} \geq 1 + c, \quad (4.7)$$

where $c > 0$ is independent of $h \in (0, h_0]$, and $h_0 > 0$ is sufficiently small. By using (4.6) and (4.7), we obtain (for small h , as in all estimates below):

$$u'(q_0) \frac{\partial q_0}{\partial q_N} = u'(q_0) u'(q_N) \left(\frac{\partial \phi}{\partial q_0} \right)^{-1} < 1,$$

therefore

$$\frac{\partial \Phi}{\partial q_N} = \frac{1}{1 + \gamma} [\min\{1, \gamma\} u'(q_0) \frac{\partial q_0}{\partial q_N} - k\gamma] < \frac{\min\{1, \gamma\} - k\gamma}{1 + \gamma} < 0.$$

It follows that an optimal X cannot be an interior point, and a constraint (2.12) is non-binding in a sense that we may decrease q_N without violating (2.12). Since

$$\frac{\partial(u(q_0) - q_0)}{\partial q_N} = (u'(q_0) - 1) \frac{\partial q_0}{\partial q_N} < 0,$$

a constraint $u(q_0) \geq q_0$ is non-binding as well: we may decrease q_N if we are at this part of the boundary, and increase Φ . Finally,

$$\begin{aligned} \frac{\partial(u(q_N) - q_0)}{\partial q_N} &= u'(q_N) - \frac{\partial q_0}{\partial q_N} = \\ &= u'(q_N) - u'(q_N) \left(\frac{\partial \phi}{\partial q_0} \right)^{-1} = u'(q_0) \left(1 - \left(\frac{\partial \phi}{\partial q_0} \right)^{-1} \right) > 0, \end{aligned}$$

therefore the maximum of Φ is attained when $u(q_N) = q_0$, i.e. (2.24) hold. From (2.15), we obtain an equation for $q_0 = q_0(\gamma)$:

$$h q_0 = \frac{p_0(1 - M_0)}{1 + \gamma} [u(q_0) - q_0]. \quad (4.8)$$

Now we are going to show that $\gamma < 1$ cannot be an optimal choice. For $\gamma \in (0, 1]$, define

$$\chi(\gamma) = \Phi(\gamma, q_0, u^{-1}(q_0)) = \frac{\gamma}{1 + \gamma} [u(q_0) - k u^{-1}(q_0)],$$

where $q_0 = q_0(\gamma)$ is given as a (unique) positive solution to (4.8). By differentiating (4.8), we obtain

$$(h + p_0(1 - M_0)[1 - u'(q_0)]/(1 + \gamma))q'_0 = \frac{p_0(1 - M_0)}{(1 + \gamma)^2} [q_0 - u(q_0)].$$

Since $u \in C^1$, (4.5) implies that the RHS is $O(h)$, and by (4.6),

$$(h + p_0(1 - M_0)[1 - u'(q_0)]/(1 + \gamma)) \geq c,$$

where $c > 0$ is independent of $h \in (0, h_0]$. Hence,

$$q'_0 = O(h). \quad (4.9)$$

By (4.6), $1 - u'(q_0) > 0$, and since $q_0 - u(q_0) < 0$,

$$q'_0(\gamma) < 0. \quad (4.10)$$

We calculate

$$\chi'(\gamma) = \frac{1}{(1 + \gamma)^2} [u(q_0) - k u^{-1}(q_0)] + \frac{\gamma}{1 + \gamma} \left[u'(q_0) - \frac{k}{u'(u^{-1}(q_0))} \right] q'_0(\gamma).$$

Since $k \geq 1$, $u^{-1}(q_0) = q_* + O(h)$, and $u'(q_*) \in (0, 1)$, (4.6) implies, for small $h > 0$,

$$u'(q_0) - k/u'(u^{-1}(q_0)) < 0.$$

By (2.12), we consider only q_0 for which $u(q_0) - k u^{-1}(q_0) \geq 0$ holds, then (4.10) and the last estimate give $\chi'(\gamma) > 0$. Thus, if $\gamma < 1$, we may enlarge it and obtain a larger value of χ ; a condition $u(q_0) \geq q_0$ remains intact, due to (4.8).

Thus, in an equilibrium (if it exists at all) we must have $\gamma = 1$, then (4.8) reduces to (2.23), and (2.23) requires that (2.29) hold.

This proves statements in parts a) – c) of Theorem 2.2 and corresponding statements in part a) of Theorem 2.3 about q_0, q_N and γ . To prove that $q_{m0} = q_0$, note that under condition (2.23), equations (2.15) and (2.16) are identical, each possessing the only positive root.

Since u satisfies (4.1), and $u(q_0) > q_0$ by (2.23), we have $q_0 < q_*$, and after that $q_N < q_0$ follows from (2.1) and (4.1). By applying (4.1) once again, we obtain $q^* < q_*$, and then for small h , (4.5) gives $q^* < q_N < q_0$. Also, (4.5) implies that $q_0 \rightarrow q^*$ and $q_N \rightarrow q^*$ as $h \rightarrow 0$.

This proves parts d) and e) of Theorem 2.2 and corresponding statements in part a) of Theorem 2.3.

It remains to prove part b) of Theorem 2.3. If $u(q_0(1)) - ku^{-1}(q_0(1)) < 0$, then there is no equilibrium since (2.12) is violated for any $\gamma \in (0, 1]$: $u(q_0(\gamma)) - ku^{-1}(q_0(\gamma)) < 0$. To see this, consider curves $y = u(q)$ and $y = ku^{-1}(q)$. By our assumption, on $(q_0(1), +\infty)$, the latter is above the former, but $q_0(\gamma)$ increases as γ decreases.

Thus, Theorems 2.2 and 2.3 have been proved.

Finally, to prove (2.28), it suffices to calculate the first two terms of the asymptotics of $q_0 = q_0(h)$ as $h \rightarrow +0$ (from (2.23)), then from (2.24) – the first two terms of the asymptotics of $q_N = q_N(q_0(h))$, and substitute into (2.10). By differentiating (2.23), we find $q'_0(0) = 2q_*/(p(u'(q_*) - 1))$, where $p = p_0(1 - M_0)$, and then (2.24) gives $q'_N(0) = (u^{-1})'(q_*)q'_0(0)$. Hence,

$$\begin{aligned} & \frac{p_0(1 - M_0)}{1 + \gamma} [u(q_0) - kq_N] = \\ &= \frac{p}{2} [q_* + hu'(q_*) \frac{2q_*}{p(u'(q_*) - 1)} + O(h^2) - q_*(1 + k_1h) - h(u^{-1})'(q_*) \frac{2q_*}{p(u'(q_*) - 1)} - O(h^2)] = \\ &= hq_*(u'(q_*) - 1/u'(q_*))/ (u'(q_*) - 1) - hk_1q_*p/2 + O(h^2) = \\ &= hq_*(1 + u'(q_*))/u'(q_*) - hk_1q_*p/2 + O(h^2), \end{aligned}$$

and (2.28) follows.

Proof of Theorem 2.5. Set $\tilde{h} = 2h/(p_0(1 - M_0))$ and notice that

1) $q_0 = q_0(\tilde{h})$ is a unique positive solution to an equation $(1 + \tilde{h})q = u(q)$;

2) an optimization problem for 1-sector is equivalent to: on $\tilde{h} \in [0, \tilde{h}_0]$, where $\tilde{h}_0 > 0$ is small, maximize

$$\Psi(\tilde{h}) = \frac{1}{\tilde{h}} \chi(\tilde{h}),$$

where

$$\chi(\tilde{h}) = u(q_0(\tilde{h})) - u^{-1}(q_0(\tilde{h})) = q_0(\tilde{h}) - u^{-1}(q_0(\tilde{h})) + q_0(\tilde{h}).$$

By using equalities $q_0(0) = u^{-1}(q_0(0)) = q_*$ and the Implicit Function Theorem, we can find $\Psi(0) = 0$, and calculate explicitly $\chi'(0)$ and $\chi''(0)$; after that, using inequalities $u'(q_*) > 0$ and $u''(q_*) < 0$, we check that $\chi''(0) < 0$ (calculations are standard, lengthy and available on request). Now, by using the Taylor formula in the neighborhood of 0, we obtain

$$\Psi(\tilde{h}) = \chi'(0) + \tilde{h}\chi''(0)/2 + o(\tilde{h}).$$

It follows that Ψ decreases in a neighborhood of 0. When h is small, and M_0 is bounded away from 1, \tilde{h} is small as well, so the last formula is applicable. If h is fixed and M decreases, \tilde{h} decreases as well, hence $\Psi(\tilde{h}(M))$ increases.

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Proof of Lemma 3.2. By (1.2), both statements are equivalent.

From (3.7) and (3.11), it follows that $h/(1 - N - M_0) \rightarrow 0$ as $\epsilon \rightarrow 0$, and therefore (1.27) gives that $u(q_{m0}) - q_{m0} \rightarrow 0$ and $u(q_{m1}) - q_{m0} \rightarrow 0$ as $\epsilon \rightarrow 0$. If we take into account (3.5), then we obtain $q_{m0} \sim q_*$, $q_{m1} \sim q_*$ as $\epsilon \rightarrow 0$, where q_* is a unique positive root of an equation $u(q) = q$.

If $x_{m1} > 0$, then from (1.8), we conclude that $u(q_0) = o(1)$, and then $q_0 = o(u(q_0))$. Hence, $u(q_0) > q_0$.

Suppose, $x_{m1} = 0$ and $u(q_0) < q_0$; then

1) $x_0 = 0$, and from (1.26), $q_N = u^{-1}[(1 + h/p_1)^{-1}q_0]$, and

2) a subsystem (1.27), (1.29) uniquely defines q_{m1} and q_{m0} as functions of γ ; both are independent of q_N and q_0 . We conclude, that for γ fixed, 1-sector chooses q_N – equivalently, q_0 , – to maximize

$$\Phi(q_0) = u(q_0) - ku^{-1}[(1 + h/p_1)^{-1}q_0],$$

s.t. $u(q_0) < q_0$. But it is easy to see that where $u(q_0) < q_0$ and $\Phi(q_0) \geq 0$, Φ decreases, and $q \mapsto q - u(q)$ increases, hence neither q_0 with $u(q_0) < q_0$ can be optimal, and our assumption has lead to a contradiction.

Proof of Lemma 3.3. Since $\gamma \leq 1/x_*$ and (3.7) hold, we have, as $\epsilon \rightarrow 0$:

from (1.26) and (3.11):

$$u(q_0) - q_0 = O(\epsilon), \quad u(q_N) - q_0 = O(\epsilon); \quad (5.1)$$

from (1.27) and (3.11):

$$u(q_{m0}) - q_{m0} = O(\epsilon), \quad u(q_{m1}) - q_{m0} = O(\epsilon). \quad (5.2)$$

Consider separately two cases: $\gamma \geq 1$, and $\gamma \in (0, 1)$.

If $\gamma \geq 1$, then $x_{m1} = x_{m1} + x_{m1}^+ = x_*$, and from (1.29) and (3.11) we conclude that

$$kq_{m1} = u(q_{m0}) - u(q_0) + O(\epsilon). \quad (5.3)$$

If $q_0 \rightarrow 0$, then $u(q_0)/q_0 \rightarrow +\infty$, which contradicts (1.26) in view of (3.11). Hence, from (5.1), $q_0 = q_* + O(\epsilon)$, and $q_N = q_* + O(\epsilon)$. Similarly, (3.5) and (5.2) give $q_{m0} = q_* + O(\epsilon)$ and $q_{m1} = q_* + O(\epsilon)$. But then (5.3) implies $q_{m1} = O(\epsilon)$, a contradiction.

Hence, $\gamma \in (0, 1)$. For these γ , we have similarly to (5.1)–(5.3)

$$u(q_0) - q_0 = O(h), \quad u(q_N) - q_0 = O(h); \quad (5.4)$$

$$u(q_{m0}) - q_{m0} = O(h), \quad u(q_{m1}) - q_{m0} = O(h); \quad (5.5)$$

$$kq_{m1} = u(q_{m1}) - \gamma u(q_{m0}) + O(h). \quad (5.6)$$

(5.4)–(5.5) give

$$Q = q_* + O(h), \quad \forall Q \in \{q_0, q_N, q_{m0}, q_{m1}\}, \quad (5.7)$$

and since $k \geq 1$ and $\gamma < 1$, (5.6) and (5.7) are compatible only if $k - 1 = O(h)$ and $\gamma = O(h)$. If $x_{m0} > 0$, we can use (5.7) and rewrite (1.8) as

$$\gamma u(q_0) \geq u(q_0) + O(h);$$

from (5.7), we see that $O(h) \geq q_* + O(h)$. For sufficiently small h , this is impossible. Thus, $x_{m0} = 0$, and Lemma 3.3 has been proved.

Proof of Lemma 3.5. a) and b) If 1-sector chooses $q_1 = q^*$, we have $u(q^*) - kq^* > 0$, and (3.18) gives $V^1 \geq ch^{-1}$, where $c > 0$ is independent of $h > 0$. Now, for small h , (5.7) gives b) and proves that this choice of q_1 is really optimal. Since this q_1 is independent of x_1 and q_N , a) follows.

c) Since $x_{m1} = 0$, (1.8) reduces to $u(q_{m0}) - kq_{m1} < u(q_0)$, which holds due to (5.7).

d) The proof is the same as the first part of the proof of Theorems 2.2 and 2.3 in Subsection 4.2.

Lemma 3.5 has been proved.

Proof of Theorem 3.6 c). This is (5.7).

Proof of Theorem 3.7. From (5.7), there exist C_1 and $h_0 > 0$ such that for $h \in (0, h_0]$ and $\gamma \in [0, 1)$,

$$\Phi(\gamma) \leq C_1 + h^{-1}(1 - k)q_*.$$

If $C > C_1/q_*$ and $k \geq 1 + Ch$, $\Phi(\gamma) < 0$, hence it is not optimal to issue notes.

Proof of Lemma 3.9. On the strength of (5.7), we look for q_0, q_N, q_{m0} and q_{m1} in the form

$$q_0 = q_*(1 + q_{01}h + O(h^2)), \quad q_N = q_*(1 + q_{N1}h + O(h^2)), \quad (5.8)$$

$$q_{m0} = q_*(1 + q_{m01}h + O(h^2)), \quad q_{m1} = q_*(1 + q_{m11}h + O(h^2)). \quad (5.9)$$

By substituting (5.8) into (3.21), using the Taylor formula around $h = 0$, and equating terms of order h , we obtain

$$q_*/B(\gamma) + (1 - u'(q_*))q_*q_{01} = 0,$$

hence

$$q_{01} = 1/((u'(q_*) - 1)B(\gamma)). \quad (5.10)$$

By using (5.8) once again, we find

$$\begin{aligned} q_N &= u^{-1}(q_0) = q_* + (u^{-1})'(q_*)(q_0 - q_*) + O(h^2) = \\ &= q_*(1 + hq_{01}/u'(q_*) + O(h^2)). \end{aligned} \quad (5.11)$$

By using (5.9), we deduce from (3.22)

$$q_{m11} = -k_1 - \frac{1}{B(\gamma)(1 - \gamma)} + u'(q_*)q_{m01}. \quad (5.12)$$

Introduce the notation

$$D(\gamma) = \frac{p_1(1 - M_1)}{p_0(1 - M_0)}(1 + \gamma), \quad G(\gamma) = \frac{1}{B(\gamma)(1 - \gamma)}.$$

From (3.22) and (5.12), we derive

$$\begin{aligned} &(1 + D(\gamma) + hB(\gamma)^{-1})q_*(1 + hq_{m01}) + O(h^2) = \\ &= q_*(1 + hu'(q_*)q_{m01} + D(\gamma)(1 + hu'(q_*)(-k_1 - G(\gamma) + u'(q_*)q_{m01})) + O(h^2)). \end{aligned}$$

After simplification, we obtain

$$(u'(q_*) - 1)q_{m01}(1 + D(\gamma)(1 + u'(q_*))) = B(\gamma)^{-1} + D(\gamma)u'(q_*)(k_1 + G(\gamma)),$$

hence

$$q_{m01} = \frac{B(\gamma)^{-1} + D(\gamma)u'(q_*)(k_1 + G(\gamma))}{(u'(q_*) - 1)(1 + D(\gamma)(1 + u'(q_*)))} \quad (5.13)$$

By using (5.8)–(5.11), we deduce (2.24)–(2.25).

Similarly, from (5.12)–(5.13),

$$\begin{aligned} h^{-1}(u(q_{m1}) - q_{m0}) &= q_*(u'(q_*)q_{m11} - q_{m01}) + O(h) = \\ &= q_*[u'(q_*)(-k_1 - G(\gamma) + u'(q_*)q_{m01}) - q_{m01}] + O(h) = \\ &= q_*[(u'(q_*)^2 - 1)q_{m01} - u'(q_*)(k_1 + G(\gamma))] + O(h) = \\ &= \frac{q_*D(\gamma)u'(q_*)(k_1 + G(\gamma)) + B(\gamma)^{-1}}{1 + D(\gamma)(1 + u'(q_*))} \cdot (u'(q_*) + 1) - u'(q_*)(k_1 + G(\gamma)) + O(h) = \\ &= \frac{q_*u'(q_*)}{B(\gamma)(1 + D(\gamma)(1 + u'(q_*)))} \left[\frac{u'(q_*) + 1}{u'(q_*)} - (k_1 + G(\gamma))B(\gamma) \right] + O(h) = \\ &= \frac{q_*u'(q_*)}{B(\gamma)(1 + D(\gamma)(1 + u'(q_*)))} \left[\frac{u'(q_*) + 1}{u'(q_*)} - k_1B(\gamma) - \frac{1}{1 - \gamma} \right] + O(h), \end{aligned}$$

and parts b)–c) of Lemma follow.

POSSIBLE EXTENSIONS AND RAMIFICATIONS

By using our model as a benchmark, it is possible to introduce additional frictions and/or externalities.

For instance, it is straightforward to modify the model so that

- 1) an exogenous probability of a note to become worthless is given;
- 2) taxation in a form of exogenous probability of a unit of money to be taken away by a government agent is introduced;
- 3) ineffective agents may pay some cost of restructuring and become type-0 agents;
- 4) type-0 agents are more mobile, so that a Poisson rate of arrival of meetings "type-0–type-0" is larger than the one of meetings "type-0–type-1", and the latter is larger than the one of meetings "type-1–type-1";
- 5) a type-0 agent can invest in her ability to find trading partners, i.e. to pay for an increase of the Poisson rate of arrival of her trading partners.

The following possibilities are also feasible but more difficult.

It is very interesting to analyze how the economy react to some shocks, i.e. to consider a dynamic model for small fluctuations around a steady state found in this paper. Especially interesting would be to analyze the dependence of the welfare on the size of the note-issuing sector since it is a source of endogenous supply of liquidity in the economy.

It is also possible to consider a stochastic dynamical model (also for small fluctuations), when some exogenous factors, like the money supply, level of taxes and/or parameters characterizing u evolve stochastically, and analyze how this uncertainty affects the decision to restructure.

Finally, it would be very interesting to develop corresponding models with many levels of ineffectiveness; in the pure monetary case, it has been done in Boyarchenko (1999).

CONCLUSION

We have considered an economy where

1. There are two sectors: a sector of ineffective agents (1-sector) who can collude and issue universally accepted notes (inside money), and a sector of effective agents who cannot collude and issue universally accepted notes.
2. There may be genuine money (outside money) as well, and there may be none, so that only notes are circulating (we consider such economies in Boyarchenko and Levendorskiĭ (1999); in Boyarchenko (1999), pure monetary economy with heterogeneous agents is considered).
3. Even inside each sector, agents are heterogeneous in a sense that they have idiosyncratic tastes for goods, and they specialize in production, too.

We have introduced such characteristics of the trading process as the *trading friction*, and showed that essential properties of equilibria and the very existence of the equilibria strongly depend on the trading friction, the ineffectiveness of agents of 1-sector, the money supply and the degree of specialization in consumption.

We classified possible equilibria when the trading friction is small, and made a series of policy recommendations. In particular, we have shown that if the friction is small, and the agents are rather ineffective, then only equilibria with bad properties (too many notes

in circulation and too low level of production) are possible, if there remains the trade between sectors; if, in addition, agents are not very picky, then an equilibrium with the trade between sectors does not exist at all.

If agents are sufficiently effective and the trading friction is small then there may exist one or both of the following equilibria:

1. an equilibrium where there are no money flows between the two sectors because 1-sector finds it optimal to produce only zero quantities of good for money (type- I_M equilibrium), and
2. an equilibrium where both money and notes are used in trades between the two sectors, a special case being a pure monetary economy (type- II_M equilibrium).

We have shown that if the trading friction is very small, then type- II_M equilibria are inferior from the point of view of the welfare of 1-sector, hence it would not use money unless forced by some exogenous (from the point of view of the model) factor, like the necessity to pay taxes with money (to treat this situation consistently, our model should be modified). This agrees with previous findings by N.Wallace and others about superiority of inside money to outside money (see Introduction).

Nevertheless, if the trading friction is not very small, and type-1 agents are fairly effective, then a type- II_M equilibrium is superior for them.

We have introduced *the instability index* and *instability threshold* and showed that type- II_M equilibria are more fragile than type- I_M equilibria in the sense that the instability threshold for the former is lower than that for the latter.

Finally, we analyzed possible effects of an increase (decrease) of the money supply and showed that

1. At small levels of money supply, its increase can lead to a revitalization of a significant part of effectively dead ineffective type-1 agents, which improves the welfare of effective agents, and conversely, a small decrease of the money supply may result in a significant part of the economy to become isolated from the rest of it and a source of an additional liquidity to disappear, which leads to decrease of the welfare of effective agents.
2. Similar effects are observed when the trading friction decreases and increases, and/or the utility of consumption increases and decreases (due to some taste shock, say), respectively.
3. At moderately high levels of money supply, an increase of the money supply leads to an improvement of the welfare of rather ineffective 1-sector, but if it is effective, it would rather have no money in economy at all. For agents of 0-sector, the effect of the increase may be positive or negative, the last possibility realizing when a fairly effective part of 1-sector starts to use money, and not only notes, as they do at low levels of the money supply.
4. At high levels of money supply, the welfare of all agents decreases when the money supply grows.
5. Starting from not very high levels of the money supply, type-0 agents would be better off if they stop trade with the note-issuing sector at all, and live by themselves; however, in order to realize this possibility, they must be able to collude, which is not the case, – or a policy maker must effectively forbid the notes. An economy would continue to use notes, if left alone.

6. A decrease of the trading friction makes everybody better off – but only in a region where type-1 agents remain alive for 0-sector. When they effectively die, the welfare of 0-agents drops.

7. If 1-sector is effective and the trading friction is small, then an optimal money supply from the point of view of 1-sector is 0, and the same holds for small positive levels of the ineffectiveness, $k - 1$. If $k - 1$ is not small, there exists a positive level of money supply which is optimal from the point of view of 1-sector.

If the existing money supply is at or above an optimal level, there is a strong incentive for 1-sector to transfer new money out of the economy, should the new money arrive (by making an agreement to sell their goods for money though it is not optimal in a steady state; to treat this effect properly, one has to modify the model and make it a sort of a model of a small open economy.) This observation may provide an additional explanation for the capital flight. The more effective 1-sector, the lower an optimal amount of the money in the economy for this sector, hence the larger the incentive for the capital flight.

POLICY CONCLUSIONS

In an economy with rather ineffective 1-sector, at low levels of the money supply, a policy maker can effectively control the size of the ineffective sector by

decreasing the money supply and/or improving the infrastructure (i.e. making the trading friction smaller), if she wishes the ineffective agents to die (in the sense isolated from the rest of an economy; if we introduce the taxation into the model, they will become bankrupt), or

increasing the money supply and creating some artificial obstacles for effective trade (i.e. making the trading friction larger), if she wishes them to remain alive.

Notice that in the present model, where we do not consider a possibility of restructuring, the first course of actions leads to a steep drop of the welfare of effective agents, and the second one – to a jump.

By choosing her policy, a policy maker should be aware of a snow-balling effect: when a change of money supply causes the size of an alive part of an ineffective note-issuing sector to increase or decrease, the latter may continue to do so and increase (resp., decrease) to unforeseen, and, perhaps, undesirable levels.

A policy maker cannot make an economy to stop using notes by increasing the money supply to high levels: though effective type-0 agents would be better off if they stop trading with ineffective agents and using notes, they could not change the type of the equilibrium since they cannot collude.

So, if a policy maker wishes to get rid of notes, without causing large welfare losses, she must simultaneously increase the money supply and effectively forbid the usage of notes, by creating some artificial obstacles to the usage.

From the point of view of our model, the most natural thing to do is the simultaneous

- increase of the money supply;
- demonopolization of note-issuing large firms, like Gazprom and UES – notes of smaller firms are less acceptable, and
- heavy taxation of intermediaries involved in organizing of barter chains.

Finally, the investment in the energy sector (a core of the sector of colluding agents) at the expense of restructured enterprises of 0-sector may have a side-effect: an additional incentive for the capital flight.

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Fig.3.1. Gains from trade with 0-sector, for type-1 agents ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

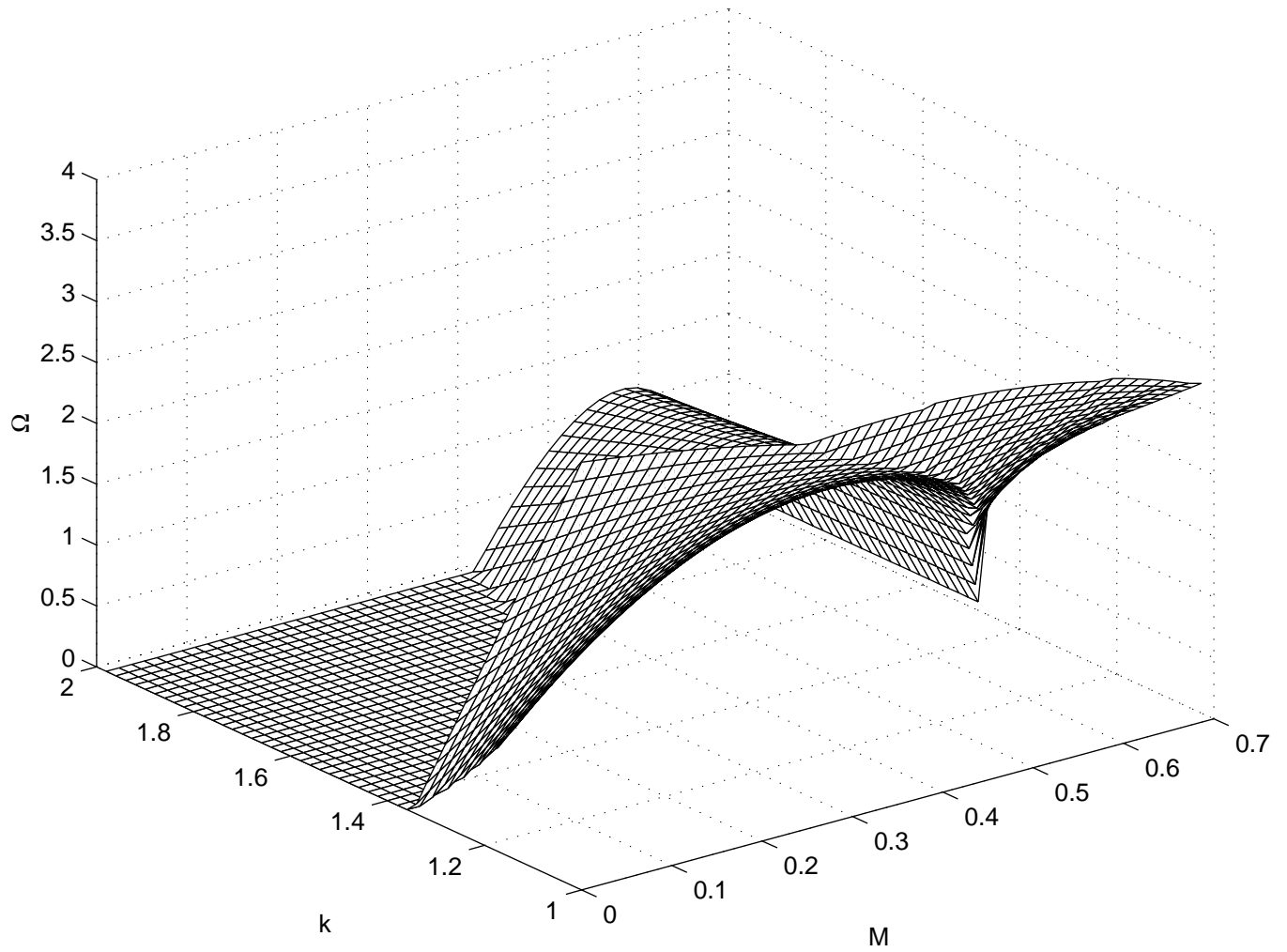


Fig.3.2. Weighted value function of type-0 agents ($h=0.025, d=1, \beta=0.3, p_1=0.3$)

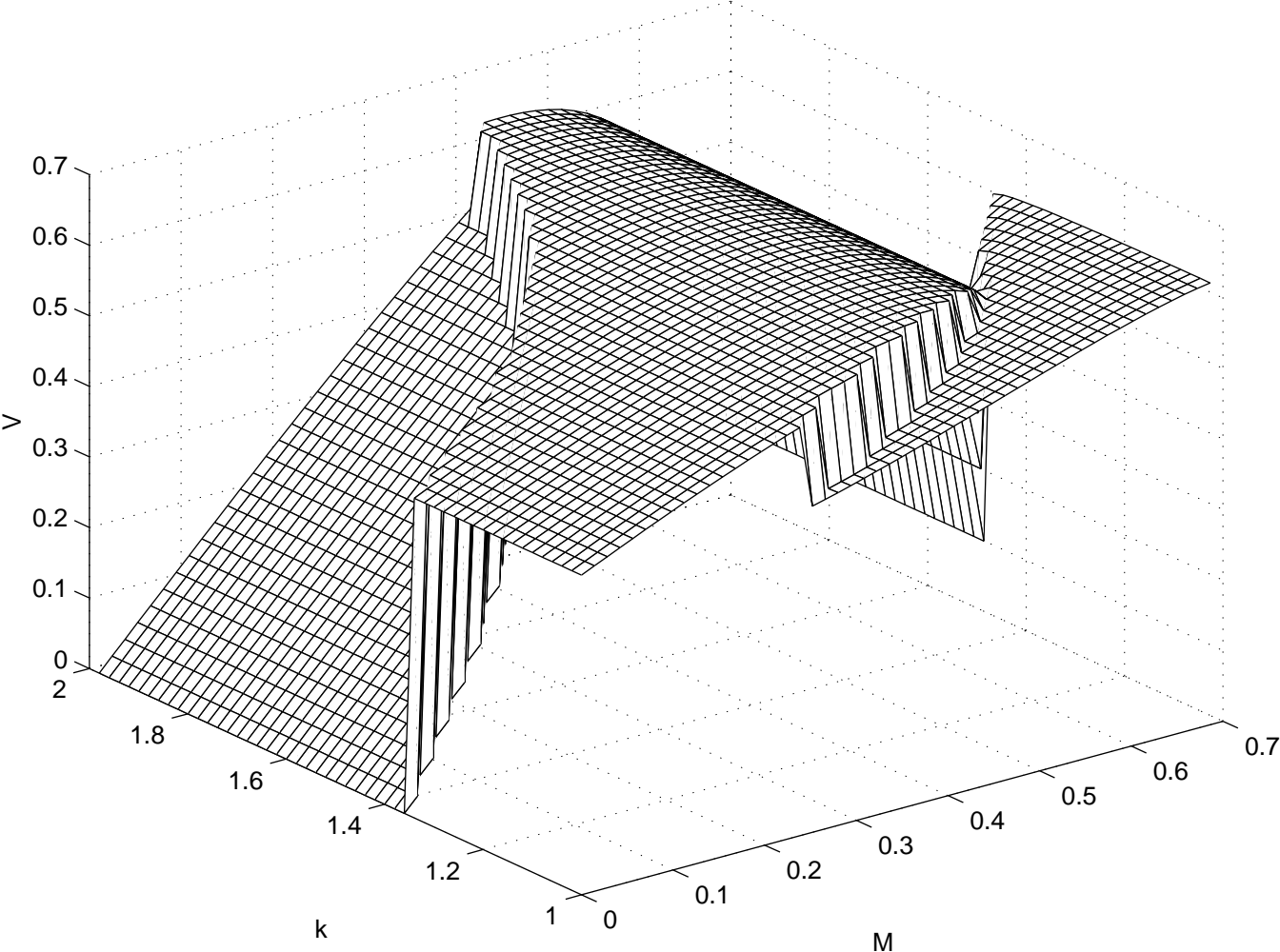


Fig.3.3. Gains from trade with 1-sector, for type-0 agents ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

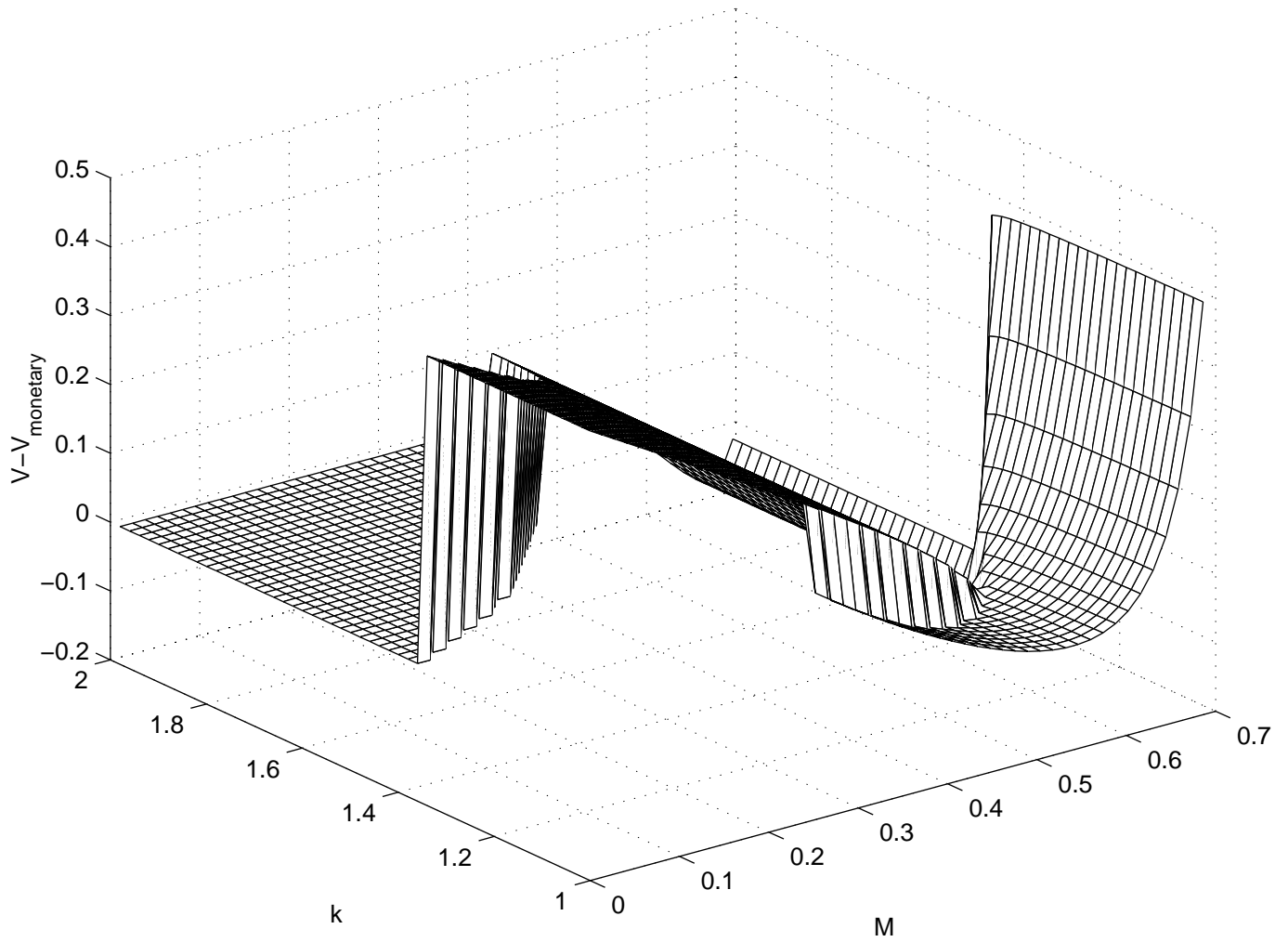


Fig.3.4. Amount produced by a type-0 seller for a note ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

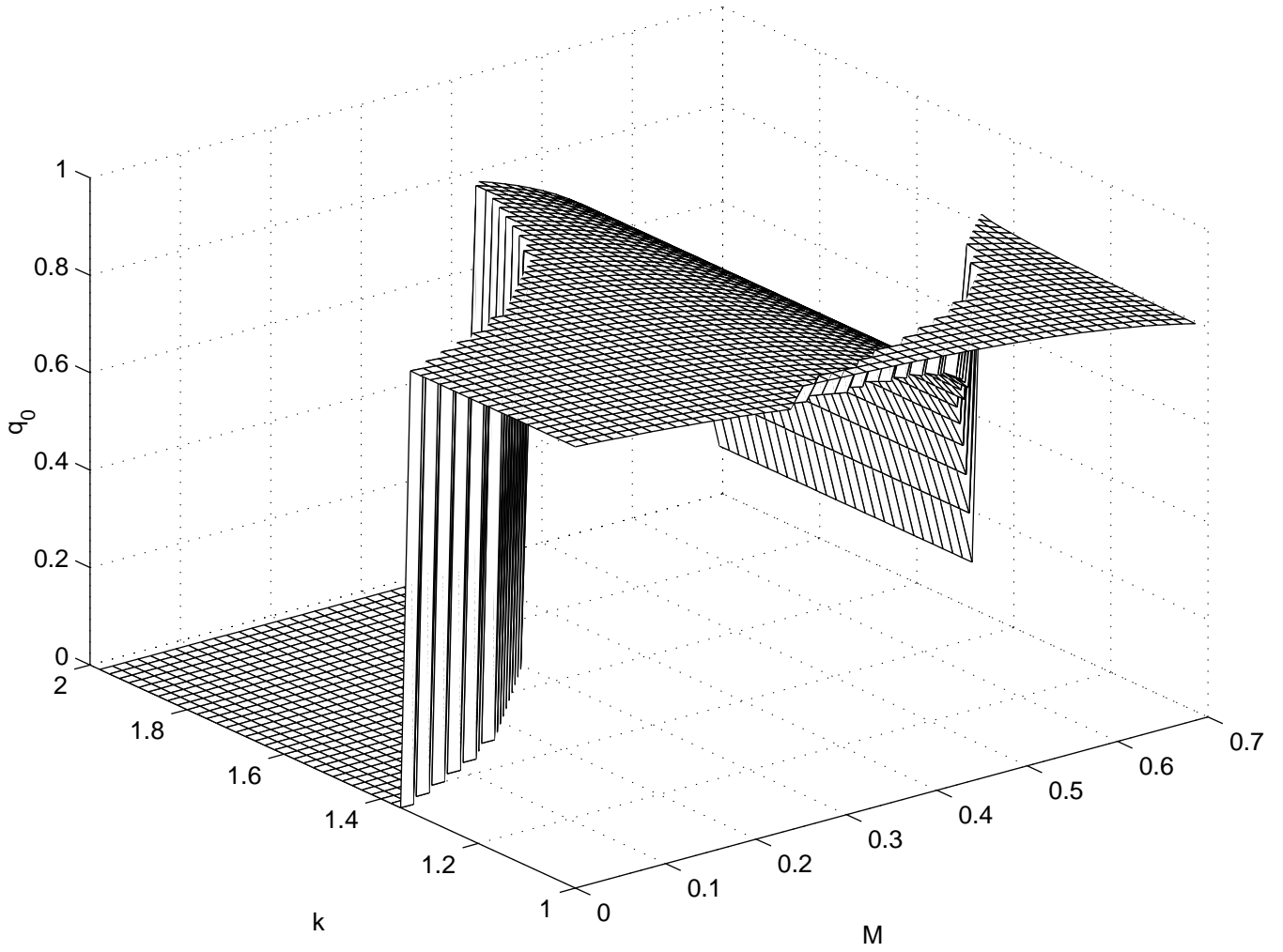


Fig.3.5. Amount redeemed for a note by a type-1 agent ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

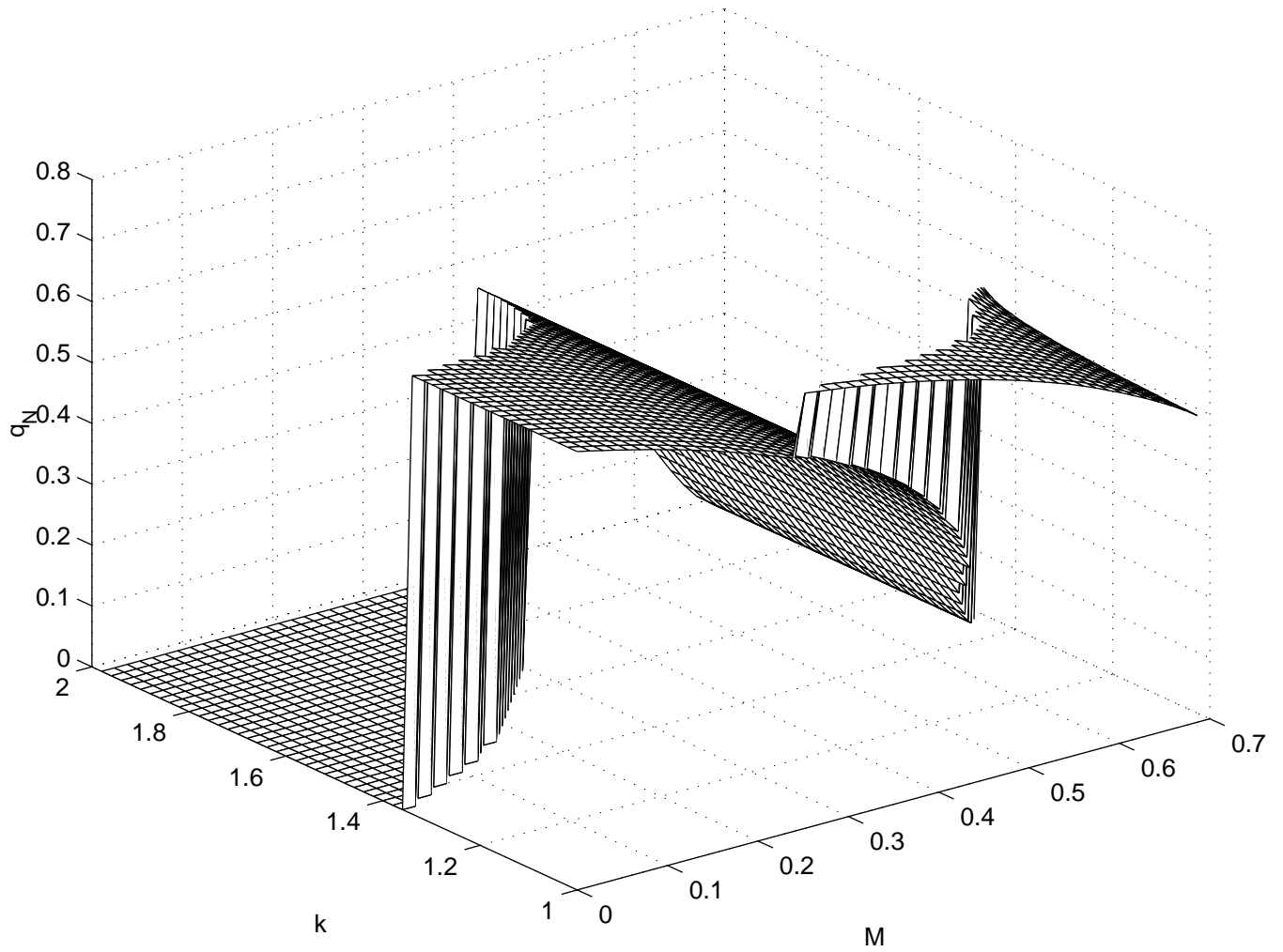


Fig.3.6. Amount produced by a type-0 seller for a unit of money ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

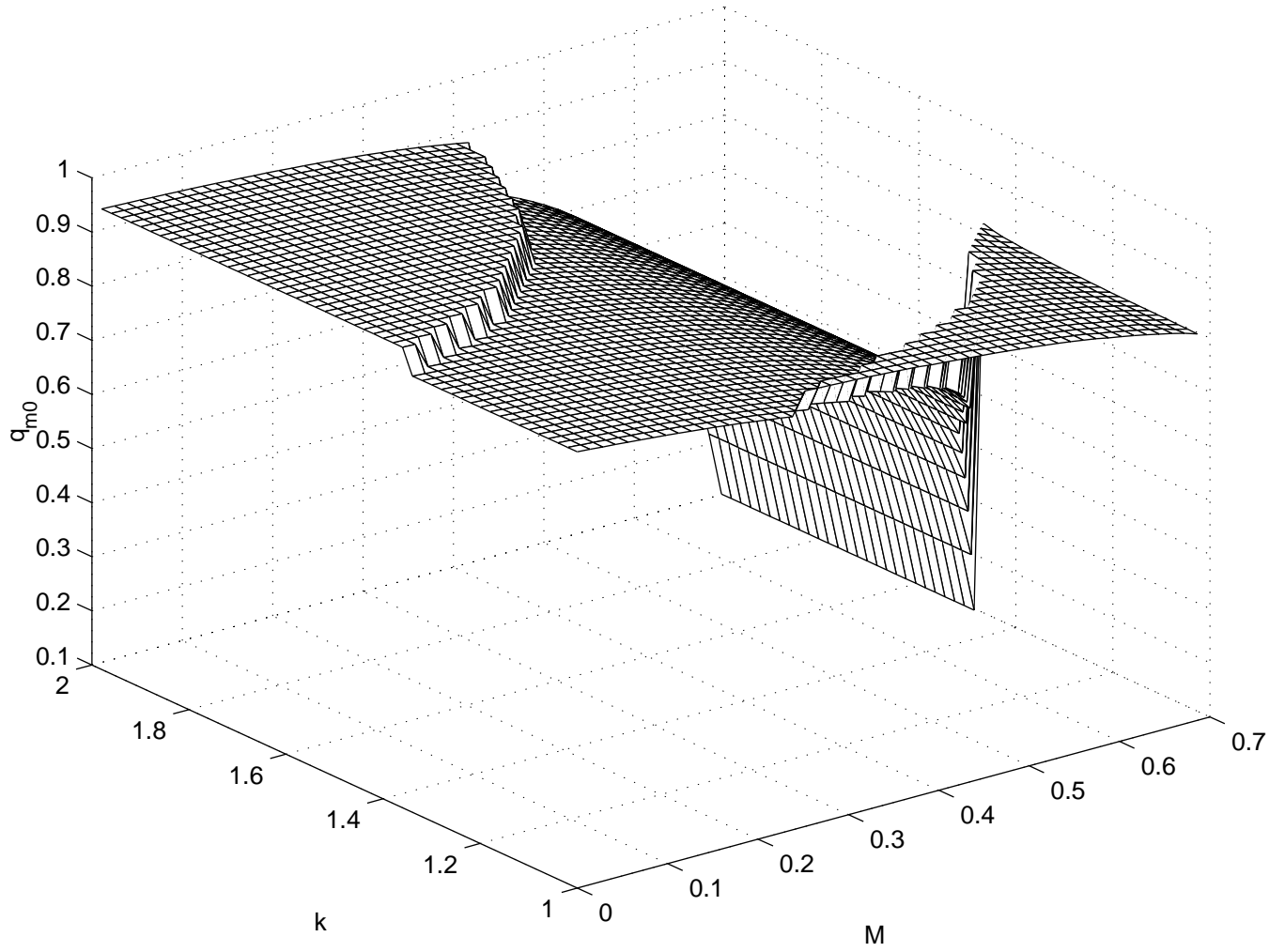


Fig.3.7. Amount produced by a type-1 seller for a unit of money ($h=0.025$, $d=1$, $\beta=0.3$, $p_1=0.3$)

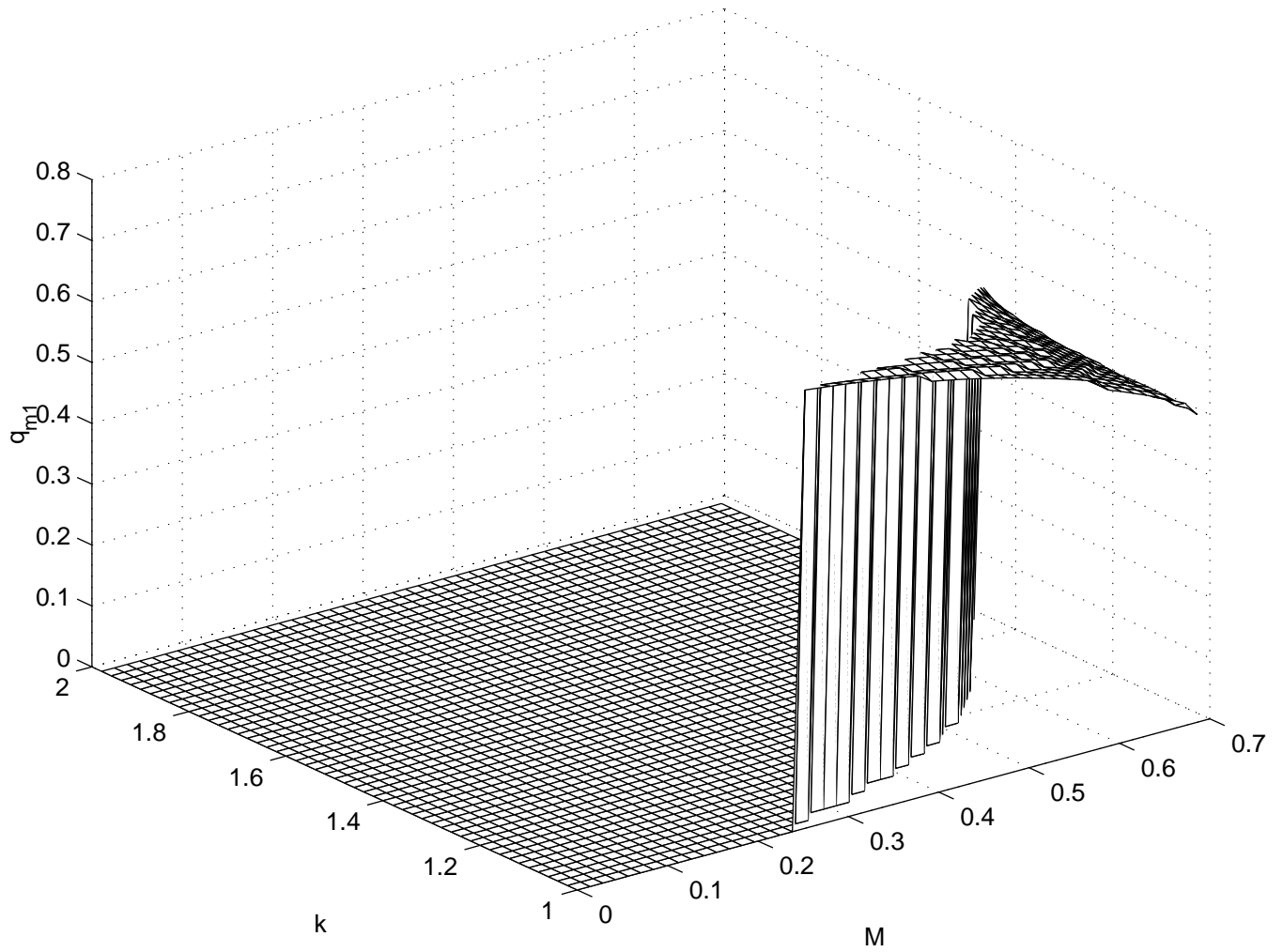


Fig.3.8. Gains from trade with 0-sector, for type-1 agents ($M=0.3, d=1, \beta=0.3, p_1=0.3$)

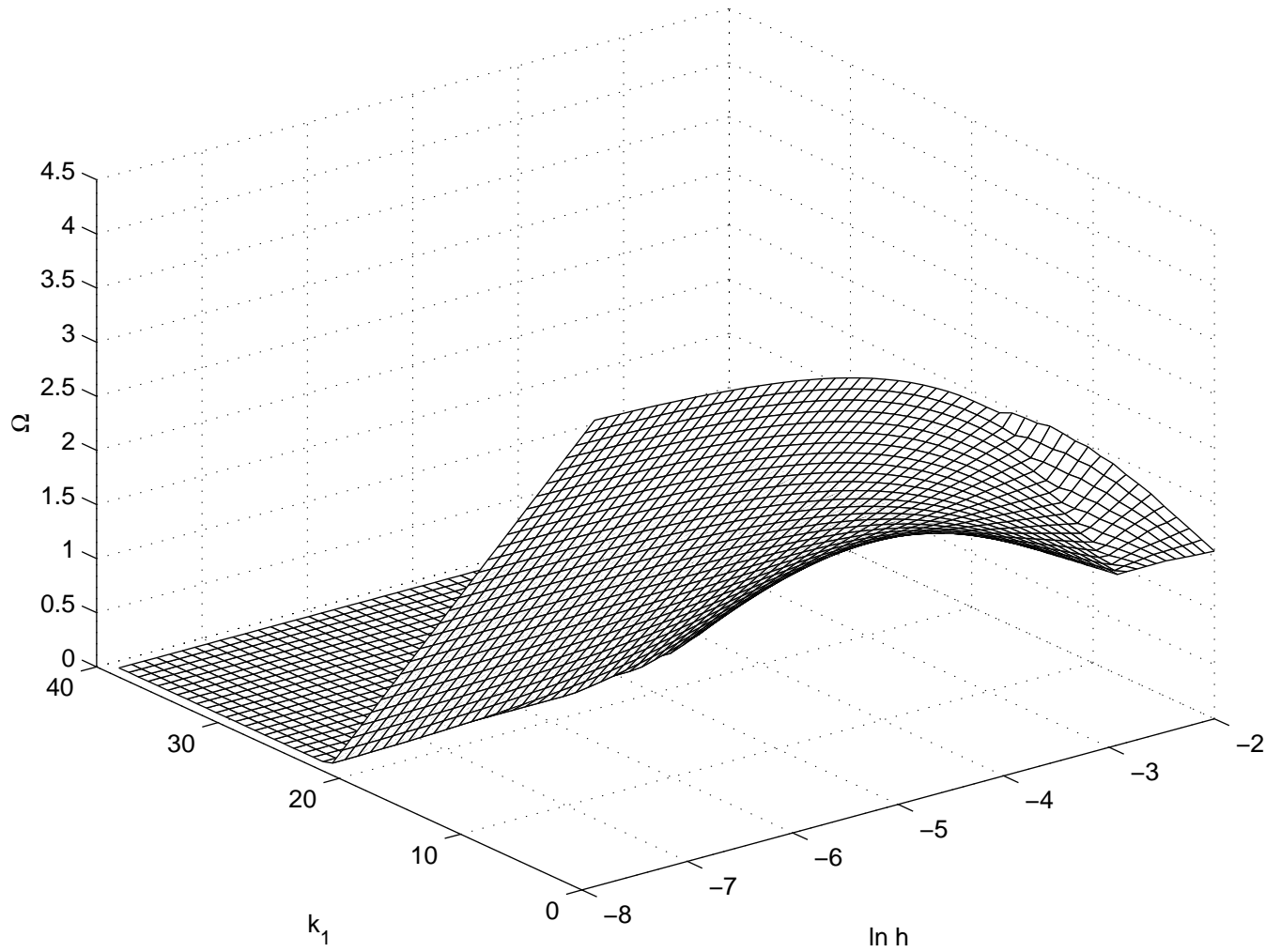


Fig.3.9. Weighted value function of type-0 agents ($M=0.3$, $d=1$, $\beta=0.3$, $p_1=0.3$)

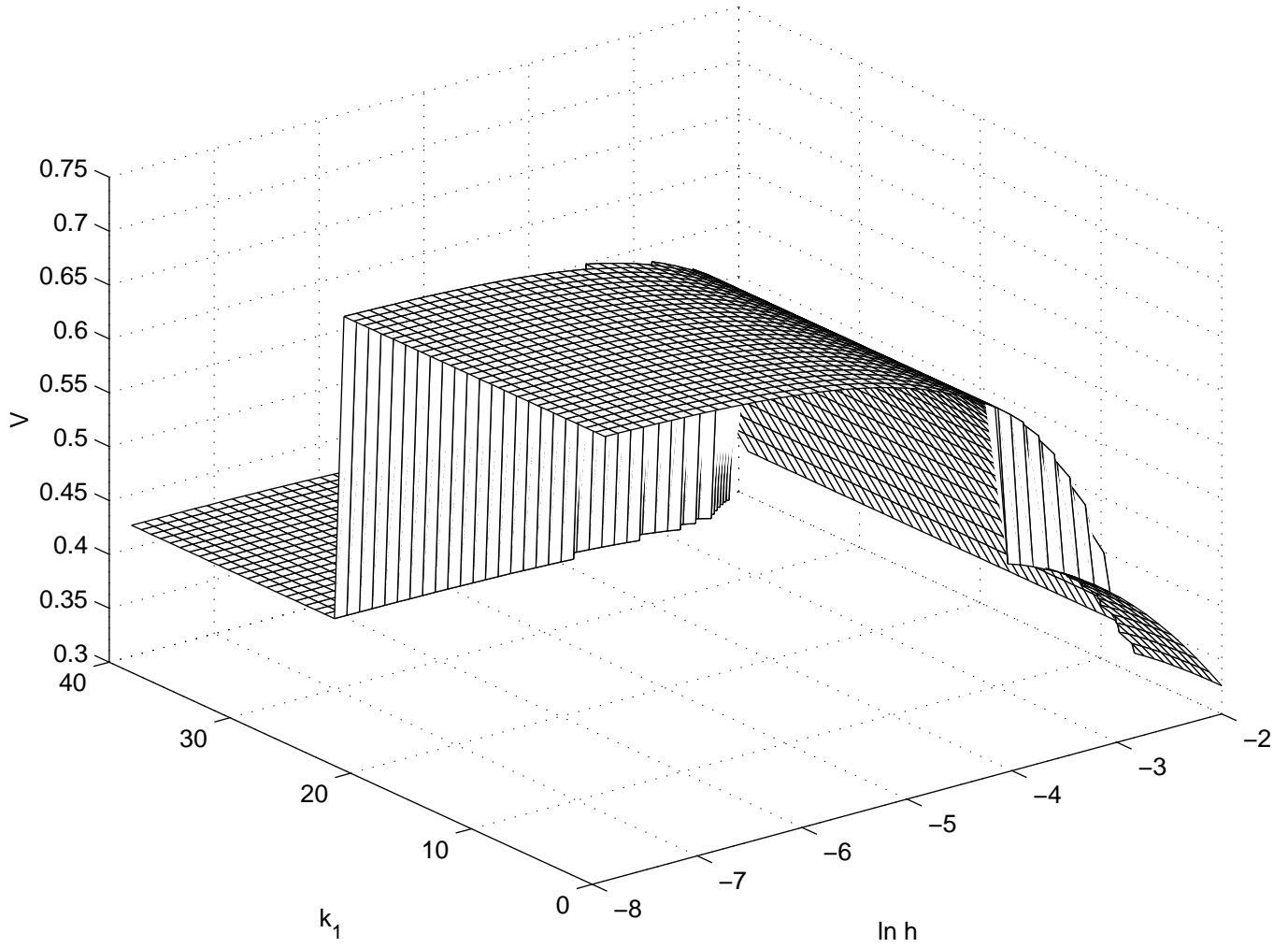


Fig.3.10. Gains from trade with 1-sector, for type-0 agents ($M=0.3, d=1, \beta=0.3, p_1=0.3$)

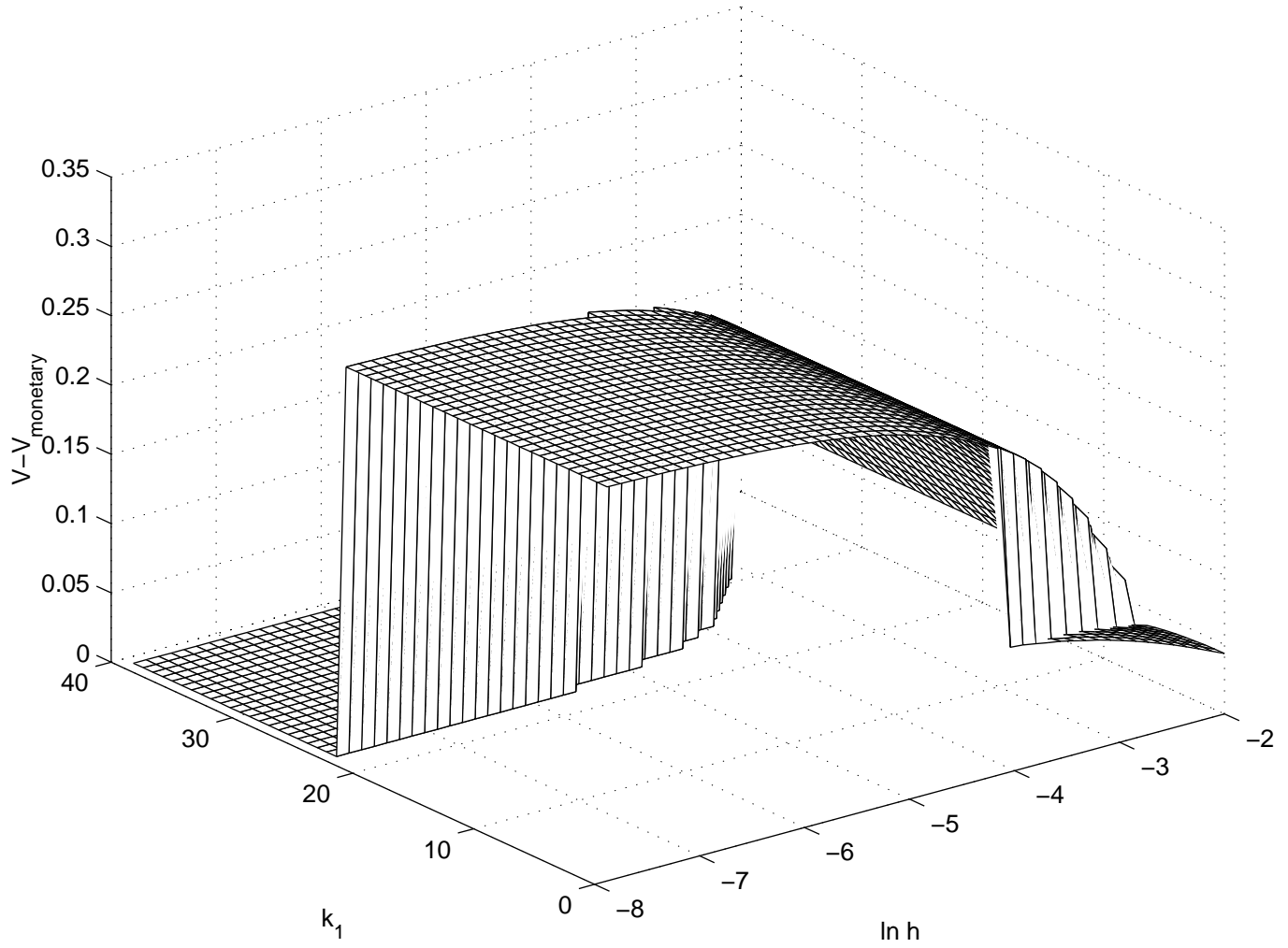


Fig.3.11. Amount produced by a type-0 seller for a note ($M=0.3, d=1, \beta=0.3, p_1=0.3$)

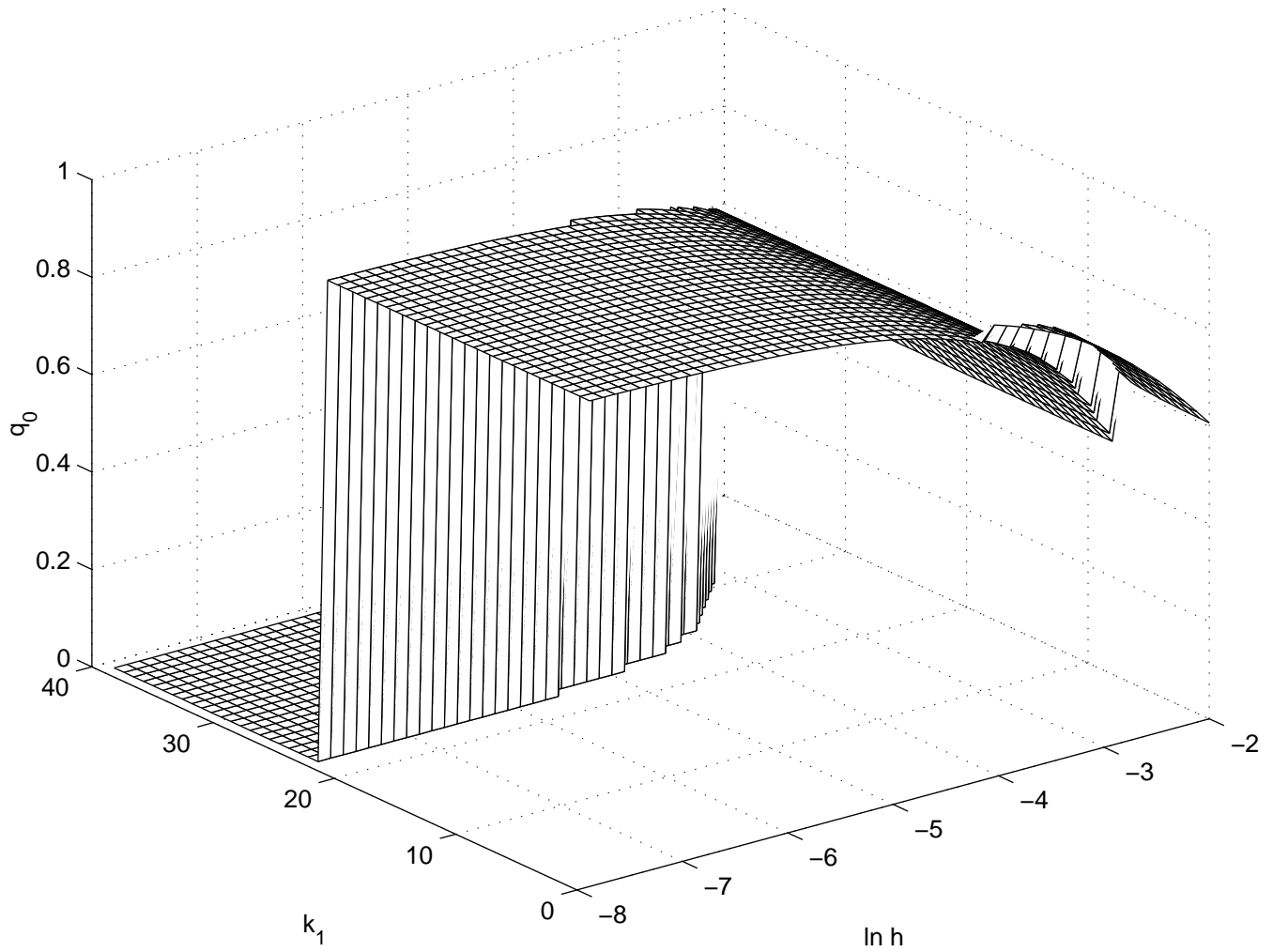


Fig.3.12. Amount redeemed for a note by a type-1 agent ($M=0.3$, $d=1$, $\beta=0.3$, $p_1=0.3$)

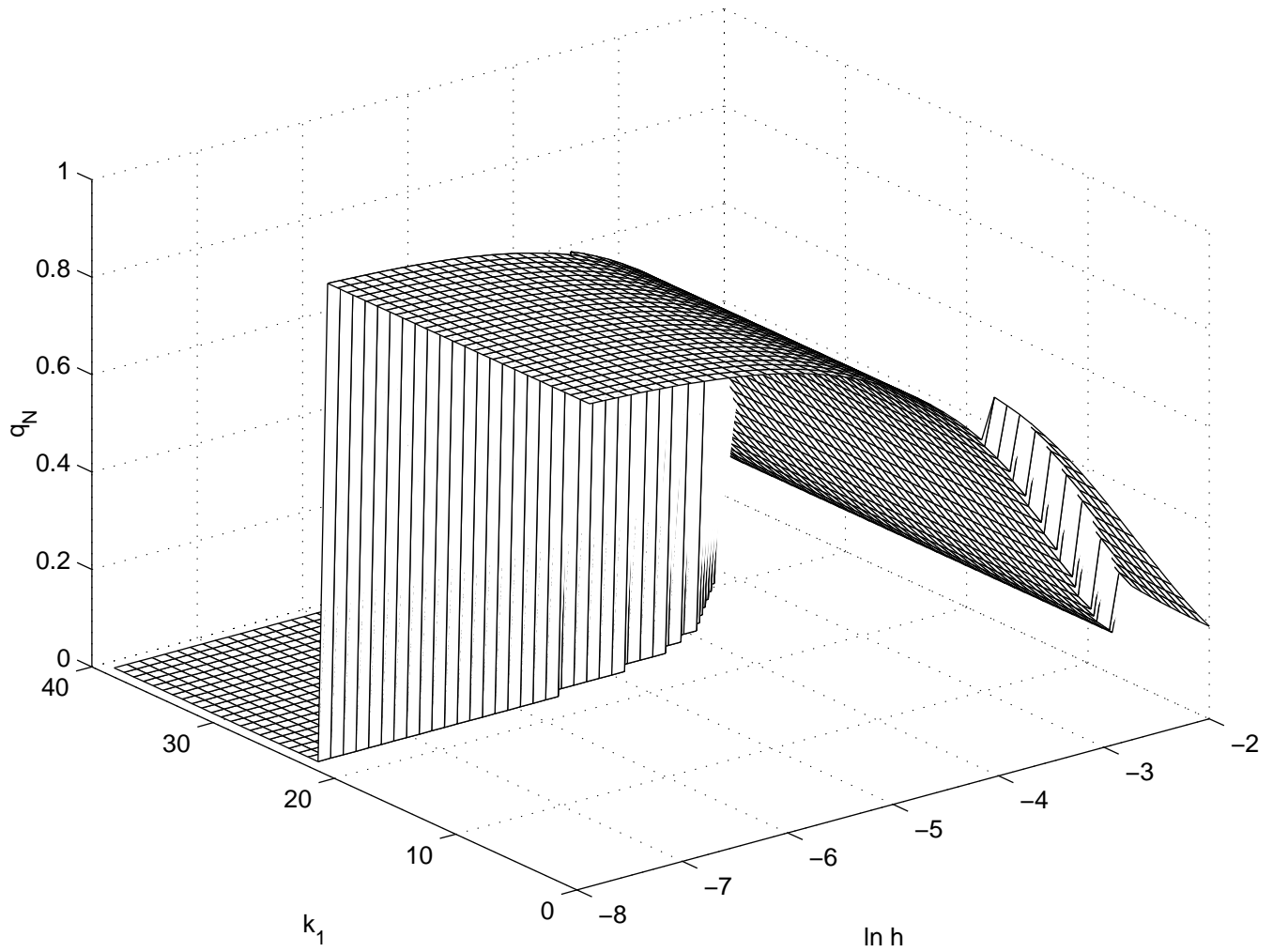


Fig.3.13. Amount produced by a type-0 seller for a unit of money ($M=0.3, d=1, \beta=0.3, p_1=0.3$)

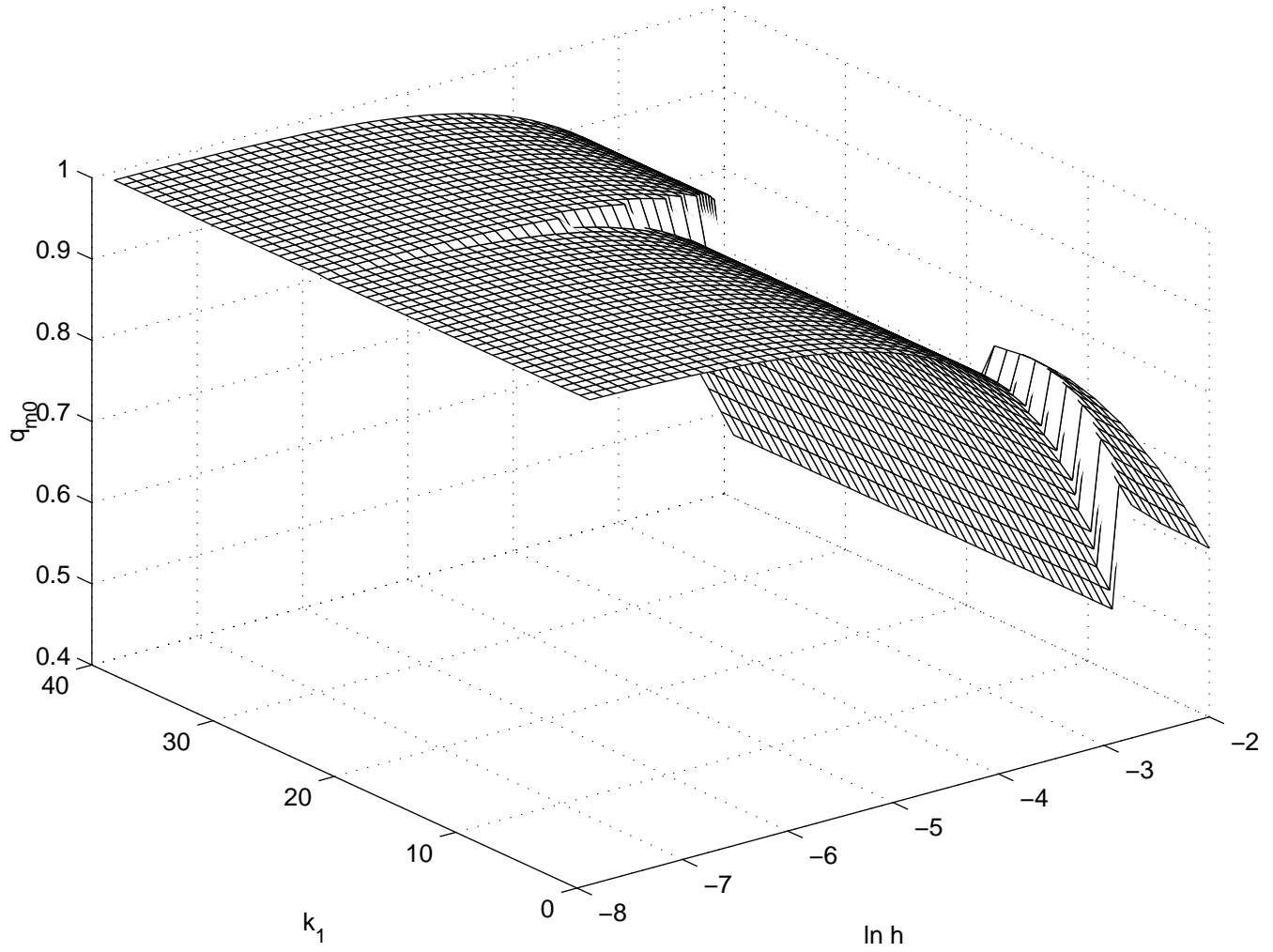
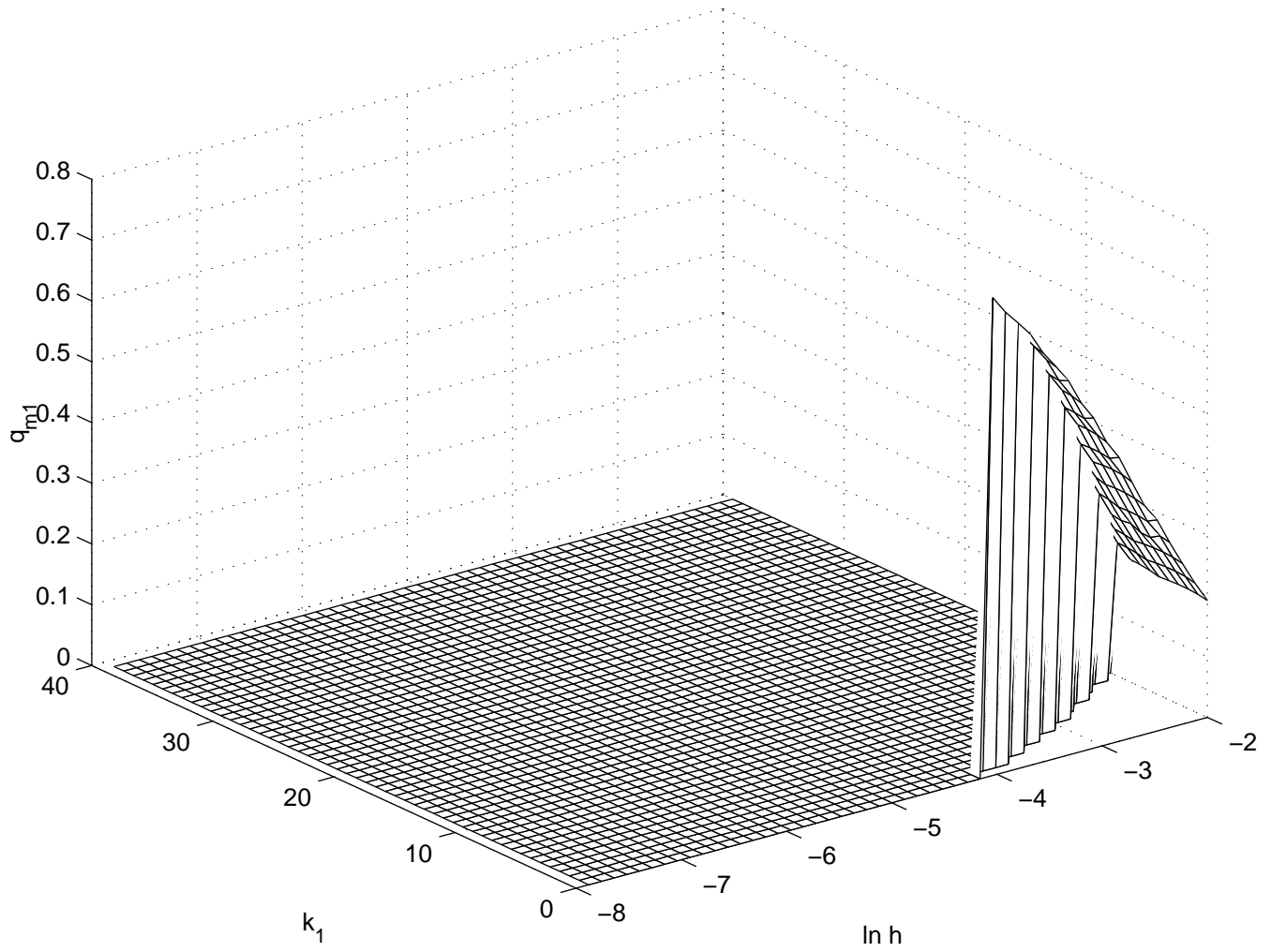


Fig.3.14. Amount produced by a type-1 seller for a unit of money ($M=0.3$, $d=1$, $\beta=0.3$, $p_1=0.3$)




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