THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
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HIGH ALTITUDE ENVIRONMENTAL STUDY
A TECHNIQUE FOR MEASURING UPPER ATMOSPHERIC WINDS

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Fig. A. missing

ORA Project 05911

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NOTATION

A_i	Coordinates of the rocket given by the trajectory.
k_x, k_y	Components of characteristic velocity of sound wave.
Δt	The increment of time spent by the sound in the unknown layer.
t_1	Flight time from lift-off to emittance of sound.
t_2	Total travel time of sound from source to array.
t_s	Time of sound ray intersection with the highest layer boundry.
T_i	Ambient temperature of the i^{th} layer.
τ	Total elapsed time from lift-off to detection of sound event at the array.
$\bar{V}_p$	Propagation velocity of sound wave.
V_{px}, V_{py}, V_{pz}	Components of propagation velocity.
$\bar{V}$	Velocity of sound.
V_x, V_y, V_z	Components of sound velocity.
W_x, W_y	Components of wind velocity.
X_1, Y_1, Z_1	Position of source in the coordinate system of the array.
X_s, Y_s, Z_s	Coordinates of ray intersection with the highest layer boundry.

Introduction

This report describes the research effort on Contract NAS8-11054 during the month of September. The Space Physics Laboratory has investigated the possibility of determining atmospheric winds from the exhaust noise of an ascending Saturn vehicle. The result of this study is a theoretical solution confirming the feasibility of this approach and providing the necessary equations to deduce the winds.

Discussion

The technique is analogous to the Rocket-Grenade experiment[5] in that the arrival angle of a sound generated in the atmosphere is measured at the ground. This angle of arrival is a function of the integrated effect of the winds and temperature along the sound path. By continously measuring this angle for an ascending rocket is it possible to determine a complete horizontal wind profile.

The approach differs from the Grenade experiment in that it must inherently determine the coordinates of the noise source. In the Grenade experiment these coordinates are found from an independent measurement. Thus, the measurement has a dual purpose - to determine the position of the noise source, then to solve for the winds.

The assumptions necessary for the approach described here are:

1. The ambient temperature profile is known. This information might be available from sounding rocket observations just prior to or following launching of the Saturn, or from on board measurements made during the flight.
2. The vertical component of wind is negligible compared with the local speed of sound. The ground level winds must be measured.

3. The exhaust noise is sufficiently loud to be detected from significant altitudes and exhibits a recognizable character. This assumption seems to be confirmed for the Saturn by reports of acoustical measurements[1,4].
4. The sound wave can be approximated by a plane wave at large distances from the source.

Discussion:

A ground-based, cross-shaped array of microphones continuously monitors the exhaust noise in order to determine the arrival angle. Time is measured at each microphone with respect to the same reference. Each recognizable noise event of the exhaust can be treated as a data point. Between two adjacent events, sufficiently close, the temperature and wind can be considered constant at their mean value for the interval. This then gives rise to a profile of the atmosphere as a stratified medium with values of the average temperature and wind in each layer.

Since the vertical winds are assumed negligible with respect to the velocity of sound and the temperature and wind are treated as constant in any layer, the segment of the sound ray in that layer is approximated by a straight line. The wavefront, assumed plane, is refracted at each layer interface in a way analogous to the refraction of light waves. This refraction is due to a change in the magnitude of velocity of sound between layers. Further refraction will occur if wind direction and magnitude are not identical across layer boundaries. Milne[2] has shown that the wavefront normal of the ray reaching the microphones remains parallel to the same vertical plane throughout its propagation. For a plane wave then, the characteristic velocities (apparent horizontal velocity components) of a specific sound ray are constant.

The first noise events recorded for calculation will occur in a region of known temperature and wind data from balloon measurements. Upon receiving the noise at the microphone array at a time τ from lift-off, the characteristic velocities k_x and k_y are determined and the ray is piece-wise traced by conventional methods until it intersects or comes within experimental error limits of the trajectory.

Calculations of this kind can conveniently represent an accuracy check on the ray tracing technique or on the measured data used for computation.

Ultimately the problem must be considered when the noise originates from an altitude above known wind data. The solution to this problem necessitates a unique approach incorporating time continuity equations in addition to the ray tracing techniques. A model is proposed in the following section.

Equations for Wind Determination:

This model presents the equations necessary to determine horizontal wind components in the layer under investigation. Ray tracing and characteristic velocity equations have been derived by Otterman[3] for the Grenade experiment.

From the trajectory, the rocket position versus time function is obtained.

$$A_i = f(t_1) \tag{1}$$

where t_1 represents flight time or time of emittance of a noise event measured from lift-off. The coordinates A_i are then transformed to (X_1, Y_1, Z_1) in the coordinate system at the microphone array. From the geometry of Figure A, the following time and velocity equations are apparent.

Time Relationships:

Time continuity equations

$$(\tau = t_1 + t_2) \tag{2}$$

$$(t_2 = t_s + \Delta t) \tag{3}$$

where Δt = the increment of time in the unknown layer.

t_2 = total time of sound propagation from (X_1, Y_1, Z_1) to the array.

Velocity relationships:

$$|\bar{V}_p| = \frac{\left[(X_1 - X_s)^2 + (Y_1 - Y_s)^2 + (Z_1 - Z_s)^2\right]^{\frac{1}{2}}}{\Delta t} \tag{4}$$

$$V_{pz} = \frac{Z_1 - Z_s}{\Delta t} \tag{5a}$$

$$V_{px} = \frac{X_1 - X_s}{\Delta t} \tag{5b}$$

$$V_{py} = \frac{Y_1 - Y_s}{\Delta t} \tag{5c}$$

where $\bar{V}_p$ = propagation velocity of the sound ray.

and V_{px}, V_{py}, V_{pz} are components of $\bar{V}_p$ in the x, y, z directions, respectively.

The propagation velocity is the total velocity of the wave front traveling along the straight line path from (X_1, Y_1, Z_1) to (X_s, Y_s, Z_s). It is composed of the velocity of sound and the wind and is, therefore, not in general normal to the wave front. Thus, equations 5a, 5b and 5c can be written as:

$$V_{pz} = V_z \tag{6a}$$

$$V_{px} = V_x + W_x \tag{6b}$$

$$V_{py} = V_y + W_x \tag{6c}$$

W_x, W_y are the components of wind in the layer under investigation for x and y direction, respectively. (W_z is neglected).

V_x, V_y, V_z are the velocity of sound components in the layer under investigation for the x, y and z directions, respectively.

For the unknown, but constant wind field in the layer between (X_1, Y_1, Z_1) and (X_s, Y_s, Z_s), there can be only one ray path from the trajectory that will satisfy the equations (2) through (7) and the condition that characteristic velocities are constant for a given ray. Otterman[3] has derived the following expressions for V_x and V_y. These equations exemplify this directional dependence of $\overline{V}$ in any layer, on the measured characteristic velocities at the array.

$$V_x = \frac{V_z^2}{k_x - V_{px} - V_{py}\frac{k_x}{k_y}} \tag{7a}$$

$$V_y = \frac{V_z^2}{k_y - V_{py} - V_{px}\frac{k_y}{k_x}} \tag{7b}$$

k_x and k_y are the characteristic velocities in the x and y directions, respectively.

The magnitude of the velocity of sound can be found from:

$$|\overline{V}| = (V_x^2 + V_y^2 + V_z^2)^{\frac{1}{2}} \tag{8}$$

Independent of all previous equations the magnitude of the velocity of sound in any layer can also be found to be:

$$|\overline{V}| \simeq 20.06 \sqrt{T_i} \quad \text{(m/sec)} \tag{9}$$

where T_i is expressed in degree K^o and is a known function of altitude.

From these nine equations, position (X_1, Y_1, Z_1, t_1) and average wind can be determined for the layer under investigation.

Procedure for Wind Determination:

A sound is heard at the time, $\mathcal{T}$, from lift-off at the microphone array. Characteristic velocities k_x and k_y are then determined by usual methods and the ray is retraced to the top of the level of known or calculated wind data. This point of intersection is (X_s, Y_s, Z_s, t_s). t_s is determined during the ray tracing calculation. The known values for a given ray are then $\mathcal{T}$, (X_s, Y_s, Z_s, t_s), k_x, k_y and T_i. The winds can be determined from the procedure outlined below:

1. Select a reasonable value for $A_i = (X_1, Y_1, Z_1)$ based upon velocity of sound and winds in lower layers.
2. Determine t_2 and Δt using equations (1), (2) and (3).
3. Using Δt and the position coordinates chosen, determine $\overline{V}_p$, V_{pz}, V_{px} and V_{py} from equations (4) through (5c).
4. These can be, in turn, used in equation (7) to obtain V_x and V_y.
5. From equation (8) and equation (9) separate values for the magnitude of the velocity of sound are calculated. If these values agree, the selected A_i is the true position of sound emittance. If they do not agree, an iteration process is carried out, until agreement is achieved within error limitations.

With the correct value for V_{px}, V_{py}, V_x and V_y, the horizontal wind components can then be found from equations (6b) and (6c).

Future Work

Work for the next reporting period will continue on an engineering study of the experiment. This study will include microphone selection and placement problems in addition to an investigation of over-all problems of instrumentation. This study will lead to a simple tentatively defined system which can be used to prove out the existing theory, hopefully during a Saturn flight.

It is also planned to continue theoretical investigation with the objective of eliminating the necessity of some of the assumptions, particularly the assumption of the plane nature of the wave and the omission of vertical winds.

Funds Remaining

As of 30 September 1963, $33,062,56 remains of the currently allotted funds.

Monthly Cost Breakdown

Prior Months Expenditures		$2,513.08
September		
Wages	$1,491.39	
Overhead	818.13	
Materials, Services	10.94	
Travel	118.90	
		2,439.36
Total Expenditures to Date		$4,952.44

References

1. Dorland, W. D. and Tedrick, R. N., "Results of Acoustical Survey of SA-2 Launch," MPT-TEST-62-5, August 20, 1962.
2. Milne, E. A., "Sound Waves in the Atmosphere," Phil. Mag. 42, 96, 1921.
3. Otterman, J., "A Simplified Method for Computing Upper-Atmosphere Temperature and Winds in the Rocket-Grenade Experiment," Univ. of Mich. Tech. Report 2387-40-T, Army Contract No. DA-36-039-SC-64657, June 1958.
4. Tedrick, R. N., and Thornton, C. C., "Results of the Far-Field Acoustical Survey of SA-4 Launch," MPT-TEST-63-5, May 24, 1963.
5. Stroud, W. G., Nordberg, W., and Walsh, J. R., "Temperatures and Winds between 30 and 80 KM," J. Geophys. Res., Vol. 1, No. 1, March 1956.