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INERTIAL PROPERTIES OF COMMERCIAL VEHICLES

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DESCRIPTIVE PARAMETERS USED IN ANALYZING THE BRAKING  
AND HANDLING OF HEAVY TRUCKS

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## 1.0 INTRODUCTION

Computerized models for analyzing the braking and handling performance of commercial vehicles require two types of input information. First, a physical description of the vehicle to be studied is needed. Second, control inputs must be specified for the type of maneuver to be simulated. This volume deals with obtaining the parametric information necessary for describing the inertial properties of heavy vehicles.

The inertial properties of commercial vehicles can be difficult to determine because of the sizes and weights of these vehicles. Nevertheless, HSRI has measured certain of these inertial properties for a rather large number of vehicles. This document presents the methods used and the results obtained by HSRI in an effort to assist simulation users in describing vehicles of their interest.

Hence, the purposes of this report are to (1) provide example data and (2) describe devices that can be used to measure inertial properties. This information is presented in the sections that follow. The HSRI Pitch Plane Inertial Properties Facility is described in the next section of this report. In addition, Section 2.0 gives pitch plane data for over thirty heavy vehicles. Section 3.0 discusses additional inertial measurement techniques and provides data in all three planes for a limited number of heavy vehicles. The last section (Section 4.0) describes measurements of the inertial properties of vehicle components such as axles and wheels.

## 2.0 THE PITCH PLANE INERTIAL TEST FACILITY

The heart of the HSRI inertial properties measurement capability is the Pitch Plane Inertial Test Facility. This is a permanently installed facility which allows for the measurement of total vehicle inertial properties in the pitch plane, i.e., longitudinal and vertical center of gravity position and the polar moment of inertia in pitch.

The facility, shown in Figure 1, is best described as a "swing" on which the vehicle is mounted for testing. Center-of-gravity positions are measured using static moment balance techniques. Moment of inertia is measured by treating the vehicle as a compound pendulum. Following a physical description of the facility in Section 2.1, a simplified mathematical explanation of these test methods will be given in Section 2.2. Section 2.3 contains a listing of data gathered on the facility.

### 2.1 The Facility

The Pitch Plane Inertial Test Facility is pictured in Figure 1. The facility can normally accept two- or three-axle vehicles with a gross test weight of up to 25,000 lb and with no axle located more than 12 feet (longitudinally) from the vehicle center of gravity. (Special modifications can be made to accept some vehicles outside of these specifications.)

The facility is basically a swing-like fixture upon which the test vehicle rests and which, in turn, is supported by knife-edge pivots such that the swing and vehicle may rotate freely about a lateral axis. More specifically, the facility is composed of:

- 1) Static frame
- 2) Two lift and tilt mechanisms
- 3) Two knife-edge pivot assemblies
- 4) Two side members
- 5) Two (or three) cross members for use with two- (or three-) axle trucks
- 6) Instrumentation

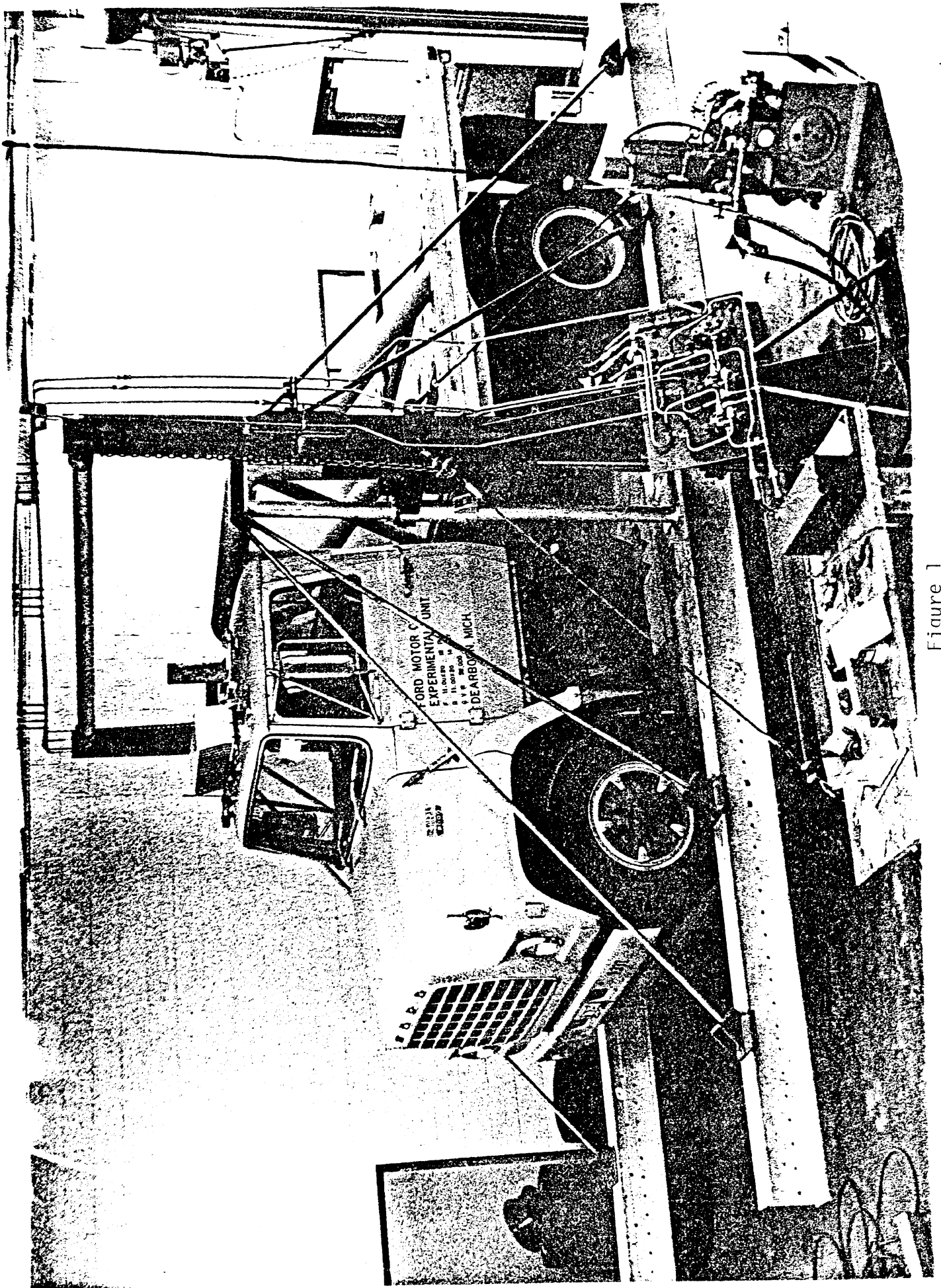


Figure 1

Referring to Figure 1, the static frame is fixed in the ground and provides vertical guides for the hydraulically powered lifting mechanism. The lifting mechanism supports the swing and vehicle on two knife-edge bearings, which are secured in the upright portion of the side members. The elevation of the knife-edge assemblies is adjustable over virtually the full height of the side members. (This feature leads to enhanced measurement accuracy, as explained in Section 2.2.)

The side members are each composed of a 25-foot aluminum I-beam located longitudinally beside the vehicle, the center upright structure, and four bracing rods. The cross members span the distance between the side members, resting on the lower, inside flange of the side member I-beams. The webs of these beams are perforated along their entire length allowing the cross members to be securely bolted in place anywhere along the length of the beams. The test vehicle rests on its tires on top of the cross members.

During c.g. position testing, the hydraulically motivated tilting mechanisms apply an external torque to the swing through a pair of strain-gauged load cells. The static angular position of the swing, with and without applied torque, is measured using an inclinometer placed on the upper surface of both side member I-beams. Tilt angles used do not exceed 8 degrees. It has been found that at this level of tilt, vertical tire and suspension deflections for heavy vehicles are insignificant, and so artificial constraints are rarely used. However, small, albeit significant, longitudinal deflections of the vehicle on its tires and of the swing members can and do occur during tilting. Longitudinal constraints are used to reduce the magnitude of these deflections and dial gauges are employed to measure those that do occur.

For moment of inertia measurements, the tilting mechanisms are decoupled from the swing. The swing is allowed to oscillate freely (from an initial deflection of less than 5 degrees) and the total period of 50 cycles is measured.

Additional geometric measurements which locate the various adjustable members of the swing, as well as the position of the vehicle on the swing, are made.

The data are reduced by computer program and are presented as shown in Figure 2. The "estimated error" values are determined by a linear error analysis explained in the following section and represent the maximum (plus or minus) error expected for the given measurement. Notice that the vertical position of the c.g. is referenced to an easily defined point in the sprung mass, thus avoiding the need to specify tire radius in the test condition. (Vertical c.g. position with respect to ground is given only for convenience.)

## 2.2 Test Techniques: A Mathematical Explanation

To provide for clarity in the following discussions, the inertial properties of the swing itself have been ignored. In the actual treatment of test data, the combined properties of the vehicle and swing are determined and the known properties of the swing are then "subtracted" to determine vehicle properties.

Figure 3 illustrates a generalized c.g. test arrangement. The upper portion of the figure illustrates the test vehicle resting on the swing with no external torque applied. As is generally the case, the vehicle c.g. is not directly aligned with the centerline of the swing, and so this static condition involves a non-zero angular position of the swing,  $\theta_0$  (which has a negative value as shown in the figure). In the lower portion of the figure, the system is shown at static equilibrium with a known, external torque applied to the swing. The swing has assumed the angular position,  $\theta_T$  (after moving through the differential rotation,  $\theta_T - \theta_0$ ). During a c.g. test series, measurements of  $\theta_0$ ,  $T$ ,  $\theta_T$ , and  $W$  are taken (as mentioned previously) and calculations of the following nature are made to determine c.g. position:

The equation for the summation of moments about the pivot point, for the lower figure, is

$$T = W \ell_4 \quad (1)$$

and from the geometry of the lower figure

$$\ell_4 / \ell_T = \sin(\theta_T - \theta_0) \quad (2)$$



HSRI PITCH PLANE INERTIAL  
PROPERTIES TEST

EST NO. KH-2

DATE 10-26-78 TIME \_\_\_\_\_ OPERATOR D Brown

I. VEHICLE ID

MANUFACTURER AMC WHEELBASE 109  
MODEL NO. \_\_\_\_\_ SERIAL NO. \_\_\_\_\_

II. BODY ID

MANUFACTURER \_\_\_\_\_ DESCRIPTION FWH DR.  
MODEL NO. EACLR  
SERIAL NO. \_\_\_\_\_

III. VEHICLE CONDITION

FRS Full LOADING EMT

IV. RESULTS

	TEST			
	1	2	3	AVG
C.G. POSITION (INCHES)				
HPT OF FRONT AXLE	43.52	43.55	43.53	43.54
EXPECTED ERROR	0.167	0.170	0.168	
STANDARD DEVIATION				0.012
HEIGHT ABOVE VEHICLE REFERENCE*	11.50	11.27	11.78	11.52
EXPECTED ERROR	0.489	0.442	0.448	
STANDARD DEVIATION				0.206
HEIGHT ABOVE GROUND				22.64
PITCH MOMENT OF INERTIA ABOUT CG (IN-LB-SEC**2)	23308.	23326.	22973.	23208.
EXPECTED ERROR	1873.	1874.	1860.	
STANDARD DEVIATION				162.
WEIGHT (LBS)	3448.			

\* REFERENCE POINT IS FRAME RAIL AT REAR OF T. BRACK.

Figure 2

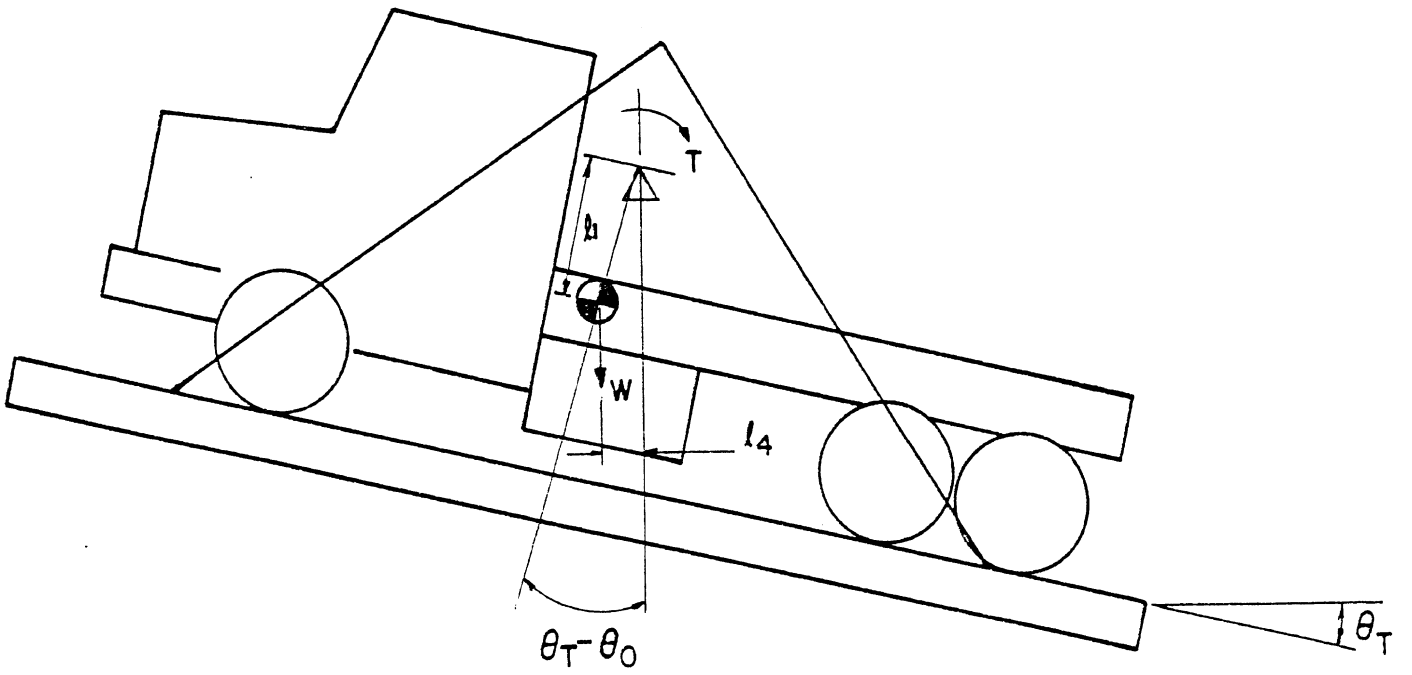
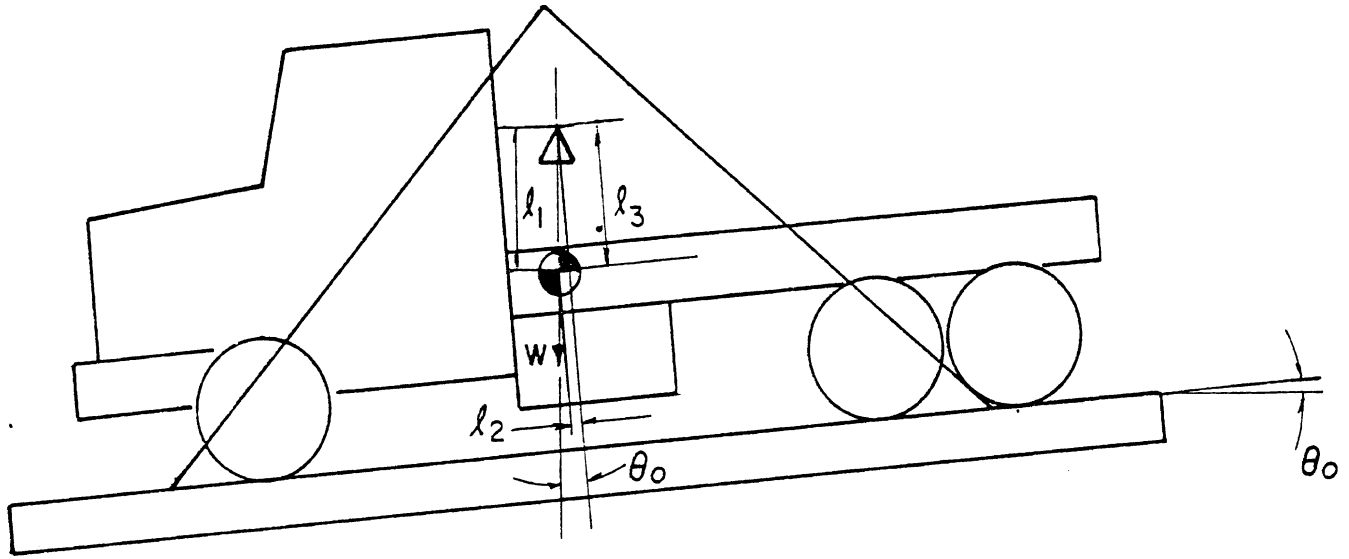


Figure 3

Combining (1) and (2) yields the expression for  $\ell_1$ , viz.:

$$\ell_1 = \frac{T}{W \sin (\theta_T - \theta_0)} \quad (3)$$

From the geometry of the upper figure

$$\ell_3 = \ell_1 \cos \theta_0 \quad (4)$$

$$\ell_2 = \ell_1 \sin \theta_0 \quad (5)$$

These values,  $\ell_2$  and  $\ell_3$ , are the fundamental experimental measurements which, when combined with straightforward geometric measurements locating the vehicle on the swing, lead to the determination of c.g. height and longitudinal position.

As mentioned above, the actual data reduction process is more complex. A number of geometric measurements are made to determine the relative position of the various adjustable members of the facility. These are used to determine the inertial properties of the swing assembly in its specific test condition. These properties are ultimately removed from the measured properties to determine the vehicle properties. Also, since the vehicle rests on the swing on its own tires and since the swing structure is, of course, somewhat flexible, a small longitudinal motion of the masses occurs (in an axis system with its origin at the pivot point and which rotates with the swing) between the "tilted" and "non-tilted" test conditions. Experimental results are very sensitive to this shift in position, and so these motions are measured (with dial gauges) and are treated appropriately in the calculations.

To determine the pitch moment of inertia, the vehicle is supported as shown in the upper portion of Figure 3 and is allowed to oscillate freely, through small angles, as a compound pendulum. The period of oscillation (P) is measured. Again, ignoring swing properties, the pitch moment of inertia ( $I_y$ ) can be calculated from the equation

$$I_y = \frac{W \ell_1 P^2}{4\pi^2} - \frac{W \ell_1^2}{g} \quad (6)$$

where "g" is the gravitational constant and  $\ell_1$  has been determined from the c.g. test procedure. (Equation (6) derives from the linear analysis of a compound pendulum.)

As mentioned previously, the vertical position of the knife-blade pivot is adjustable relative to the vehicle in order that the geometry of the test set-up may be arranged to enhance measurement accuracy. The following presentation, although greatly simplified relative to the actual calculations, serves to illustrate the principles involved.

If we assume that the vehicle is precisely located on the swing so that  $\ell_2 = 0$ , then  $\ell_3 = \ell_1$ , then Equation (3) serves as an exact formulation of the primary c.g. height measurement. That is:

$$\ell_3 = \frac{T}{W \sin \theta_T}$$

where we have also assumed  $\theta_0 = 0$  for simplicity. A first-order estimate of the measurement error on  $\ell_3$ , that is,  $\Delta \ell_3$ , is given by

$$\Delta \ell_3 = \frac{\partial \ell_3}{\partial T} \Delta T + \frac{\partial \ell_3}{\partial W} \Delta W + \frac{\partial \ell_3}{\partial \theta_T} \Delta \theta_T \quad (7)$$

where  $\partial \ell_3 / \partial T$ ,  $\partial \ell_3 / \partial W$ , and  $\partial \ell_3 / \partial \theta$  are the partial derivatives of  $\ell_3$  with respect to T, W, and  $\theta$ , respectively, and  $\Delta T$ ,  $\Delta W$ , and  $\Delta \theta$  are the measurement errors on T, W, and  $\theta_T$ , respectively. Calculating the partial derivatives and substituting them into Equation (7) yields:

$$\Delta \ell_3 = \left[ \frac{1}{W \sin \theta_T} \right] \Delta T + \left[ - \frac{T}{W^2 \sin \theta_T} \right] \Delta W + \left[ - \frac{2 T^2}{W \sin \theta_T \sin (2\theta_T)} \right] \Delta \theta_T \quad (8)$$

To reduce experimental error, it is generally desirable to reduce the absolute value of each of the partial derivatives (the bracketed terms). Equation (8) shows that

- 1) The absolute value of each of the partial derivatives is reduced by increasing the absolute value of  $\theta_T$ .

- 2) The absolute value of two of the partial derivatives is reduced by reducing the absolute value of T.

In practice, Point (1) can be used by employing the maximum tilt angle available with the facility. Point (2) can be used by adjusting the knife-edge pivot point to provide the shortest  $\lambda_3$  length which is practical. That is, the value of T required to produce a given tilt angle ( $\theta_T$ ) becomes smaller for smaller values of  $\lambda_3$ .

In theory, if the knife-edge position was adjusted so that it aligned directly with the c.g., the affected partials would become zero, eliminating these sources of error. In practice, we have found that it is best to maintain a minimum value of 6-8 inches on  $\lambda_3$ . Smaller values produce such a low level of system stability that the static condition is difficult to attain due to normally present air currents, etc.

The same analysis technique provides a first-order error estimation for pitch moment of inertia as follows:

$$\Delta I = \left[ \frac{I}{W} \right] \Delta W + \left[ \frac{I}{\lambda_3} - \frac{W \lambda_3}{g} \right] \Delta \lambda_3 + \left[ \frac{W \lambda_3^{1/2}}{\pi} \sqrt{\frac{\lambda_3^2}{g} + \frac{I}{W}} \right] \Delta P \quad (9)$$

where the bracketed quantities are the appropriate partial derivatives and the "Δ" terms are experimental measurement errors. Equation (9) shows that:

- 1)  $\frac{\partial I}{\partial W}$  is beyond the experimenter's control
- 2)  $\frac{\partial I}{\partial \lambda_3}$  is minimized (at zero) when
 
$$\lambda_3 = \left( \frac{I g}{W} \right)^{1/2} \quad (10)$$
 (i.e., when  $\lambda_3$  equals the radius of gyration)
- 3)  $\frac{\partial I}{\partial P}$  is minimized (also at zero) when  $\lambda_3 = 0$

Points (2) and (3) imply that a compromise is in order when choosing  $\lambda_3$  for moment of inertia testing. Equation (10) produces values on the order of 1/2 of the wheelbase for typical trucks. For the HSRI facility, analysis has shown that values of  $\lambda_3$  on the order of 25 inches produce minimized error estimates.

Although the major sources of error have been included here, the actual error estimate calculations used in data reduction are far more complex. Terms are included that represent potential errors deriving from errors in a host of swing inertial and geometric properties which may be "built in," as well as the specific measurement errors deriving from each test sequence. The "estimated errors" which appear on the data sheet, as shown previously in Figure 2, are calculated according to the form:

$$\left[ \sum \left( \frac{\partial M}{\partial x_i} \Delta x_i \right)^2 \right]^{1/2} \quad i = 1, 2, \dots$$

where

M is the measurement of interest

$\partial M / \partial x_i$  is the partial derivative of M with respect to parameter  $x_i$

$\Delta x_i$  is an estimate of the accuracy of parameter  $x_i$

i varies sufficiently to include virtually all parameters used in the calculation of M.

In addition to calculating the "estimated errors," each test procedure is repeated three times and a standard deviation of the three results of each measurement is computed. Commonly, standard deviations are an order of magnitude smaller than the "estimated errors" indicating that test repeatability errors are small relative to our estimates of potential parametric errors (i.e., the  $\Delta x_i$  quantities used).

### 2.3 Pitch Plane Data

The following pages contain a tabular presentation reviewing the vehicles measured and measurements made on the Pitch Plane Facility.

An additional numeric, the normalized pitch radius of gyration, is also presented for trucks and tractors. This numeric is the radius of gyration in pitch divided by the wheelbase, viz.:

Normalized Radius of Gyration  $\equiv$

$$\left[ \frac{I_y \times g}{W} \right]^{1/2} / WB$$

where

$I_y$  is the pitch moment of inertia, in-lb-sec<sup>2</sup>

$g$  is the gravitational constant, 386 in/sec<sup>2</sup>

$W$  is the weight, lbs

$WB$  is the wheelbase, in

For single rear-axle trucks, except for one (short wheelbase) outlier, these parameters range from .46 to .53 with an average value of .48 and a standard deviation of .027. For dual rear-axle vehicles, the parameter averages .50 with a standard deviation of .013. These results suggest that wheelbase and weight may be used to produce reasonably good estimates of pitch moment of inertia.

TYPE OF VEHICLE: Single Rear-Axle Trucks (Tractors)

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec <sup>2</sup> /rad	Normalized Radius of Gyration in
GMC 6500 V-8	Dump Empty Springs Compressed	11,920	125		68.06	35.43	131,634	.52
GMC T3060 6500 V-8	Iron Blocks in Body Axle. Loads - 8700-F; 17,800-R	26,325	125		84.12	64.12	259,832	.49
GM Astro 95	Dump Empty Springs Compressed	15,749	143		57.37	38.83	176,556	.46
GM Astro 95	Dump Empty Springs Free	15,749	143		57.39	39.56	177,109	.46
GMC 6500 V-8	Dump Empty Springs Free	11,525	125		67.82	36.64	129,654	.53
GM 9500 Astro 95	Dump Roll Bar	14,415	139		69.76	36.24	159,824	.47
GMC 6500 V-8		10,770	109		47.68	35.42	123,358	.61
Ford 9000	Empty Flat Short Bed	17,850	137		67.43	37.61	187,166	.46
Ford		10,828	156		83.18	37.84	145,327	.46
Ford 9000	Empty Bed Roll Bar	16,314	138		72.35	38.99	220,044	.52
Ford 9000	Sleeper Cab	13,861	134.5		49.6	37.21	138,510	.46
Freightliner	Load Rack	17,194	190		82.11	37.61	341,362	.46



TYPE OF VEHICLE: Dual Rear-Axle Trucks (Tractors)

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec <sup>2</sup> /rad	Normalized Radius of Gyration in
GMC 9500	Diesel Springs Compressed	17,277	146	50	82.27	34.15	235,750	.50
GMC Astro 95		19,250	151.5	55.5	80.22	35.68	284,134	.50
GMC 9500 Diesel	Dump Empty	25,945	146.6	50	97.83	50.53	362,949	.50
GMC Astro 95	5th Wheel Empty	17,389	150.75	49	68.58	32.64	241,479	.49
GMC	High Box Partial Filled (Sand)	25,945	147	50	97.80	50.5	372,714	.51
Ford 9000	Flat Bed Roll Bar	20,975	144.5	50.12	77.76	35.95	302,995	.52
Ford A73 LNT 9000	Flat Bed Roll Bar	25,215	170	55	97.7	39.79	491,364	.51
White Freight	Empty	21,255	175	72	88.25	36.37	383,829	.48
International Harvester	Bare Frame	14,761	143	49	65.58	44.52	176,762	.48
TYPE OF VEHICLE: Four-Wheel Drive								
Ford Bronco	Ranger with Spare Tire & Top	3,793	92.5		39.59	27.19	21,212	
Ford Bronco	Modified Roll Bar - 2 Full Gas Cans - No Spare	3,673	92.5		41.17	28.62	24,963	

TYPE OF VEHICLE: Four-Wheel Drive

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec <sup>2</sup> /rad	Normalized Radius of Gyration in
AMC Jeep CJ-5	No Spare	2,852	83		37.92	26.45	10,455	
AMC Jeep CJ-7	No Spare	2,756	93.5		40.99	24.8	11,853	
Chevrolet Blazer K-5 1974	With Spare	5,005	107		51.16	27.14	39,628	
AMC Jeep CJ-5	With Spare 8 Cyl.	3,895	83		39.92	24.12	14,710	
AMC Eagle	2-Door Empty	3,448	109		43.54	22.64	23,202	

TYPE OF VEHICLE: Passenger Cars

GM Nova	4-Door Empty	3,773	111		48.68	19.03	31,269	
AMC Pacer	2-Door Empty	3,275	104		43.23	21.38	20,275	
AMC Concord	2-Door Empty	3,244	108		45.89	21.14	23,731	
AMC Spirit	4-Door Empty	3,125	96.5		38.9	21.00	18,005	

TYPE OF VEHICLE: School Bus

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec <sup>2</sup> /rad	Normalized Radius of Gyration in
Ford		25,625	242		74.95 Forward of Rear Axle	47.74	801,654	
Ford		14,738	260		94.19	43.80	688,748	

TYPE OF VEHICLE: Travel Trailers

Airstream	31' Travel	5,545	At Support Position 169	33.125	42.6 Ahead of Rear Axle	42.38	123,352	
Holiday Rambler	31' Travel #1 Stock	6,950	At Support Position 167.5	39.8	73.8 Ahead of Rear Axle	49.9	253,119	
Holiday Rambler	31' Travel #2 Loaded with Sand Bags	4,540	At Support Position 169	32.5	33.6 Ahead of Rear Axle	35.07	87,584	
Holiday Rambler	31' Travel #2 Stripped	3,193	At Support Position 169	32	40.67 Ahead of Rear Axle	38.13	78,969	
Prowler Fleetwood H	Standard Equip.	3,238	At Support Position 139	30	30.54 Ahead of Rear Axle	38.9	40,758	
Starcraft 6	Empty Stan. Equip.	1,479	At Support Position 174.9	30	15.46	28.11	7,858	

TYPE OF VEHICLE: Equipment Trailers

Name	Vehicle Load Distribution	Test Weight lbs	Wheelbase in	Tandem Spacing in	Longitudinal Position Aft of Ft. Axle in	C.G. Hgt. Above Ground in	Pitch Moment of Inertia in-lb-sec <sup>2</sup> /rad	Normalized Radius of Gyration in
Donahue Farm Trailer	3-Axle	5,961		36	34.75	34.86	79,229	
Ford Backhoe 555	With Backhoe & Front Loader		6'8"		65.32	24.8	262,130	
Ford Backhoe 555	Without Backhoe With Front Loader		6'8"		27.56	35.7	70,136	

### 3.0 ADDITIONAL VEHICLE INERTIAL MEASUREMENTS

HSRI uses two additional arrangements to conduct moment of inertia measurements of heavy vehicles. These test set-ups are both much more temporary in nature than the facility of Section 2.0.

First is a swing facility designed to perform inertial measurements in either roll or pitch. The facility is shown in these two modes, respectively, in Figures 4 and 5. The test technique and data handling methods for this device are analogous to those describing moment of inertia testing in Section 2.0.

The second test method, used for yaw moment of inertia testing is shown in Figures 6 and 7. With the suspensions constrained to their static positions by cables, the vehicle is primarily supported at a pivot point located slightly aft of the vehicle c.g. This leaves a small portion of the vehicle weight (a few hundred pounds) to be supported by the front wheels. Under each of the front wheels are placed two steel plates separated by a number of ball bearings. Thus, the front wheels are free to move about on a horizontal plane. A grounded coil spring is attached at right angles to the vehicle frame some distance from the pivot point. With this arrangement, a small oscillation in yaw may be introduced and the period of oscillation,  $\tau$ , determined. Using the notation of Figure 6, yaw moment of inertia of the vehicle,  $I_{ZZ}$ , may be determined using Equation (11).

$$I_{ZZ} = \frac{K_s \ell^2 \tau^2}{4\pi^2} - \frac{W}{g} \ell \text{ cg}^2 \quad (11)$$

The supporting pivot is illustrated in Figure 8. Unrestrained yaw motion results from the use of a hydrostatic bearing. Vehicle weight is supported by a single 3/4-inch ball atop the bearing in order to limit pitch and roll moments passed to the bearing. The test arrangement is shown in Figure 9.

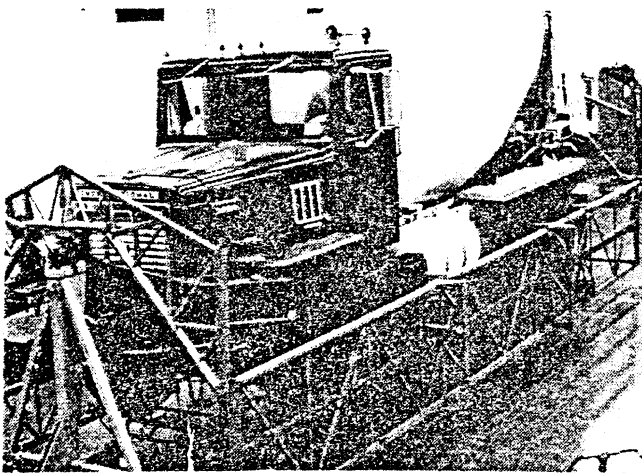


Figure 4

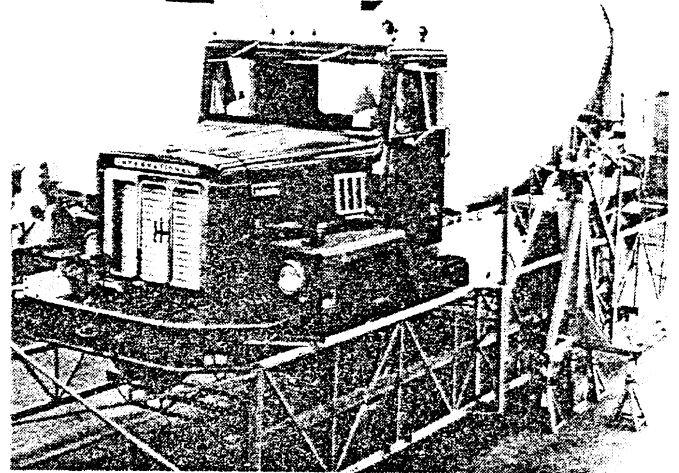


Figure 5

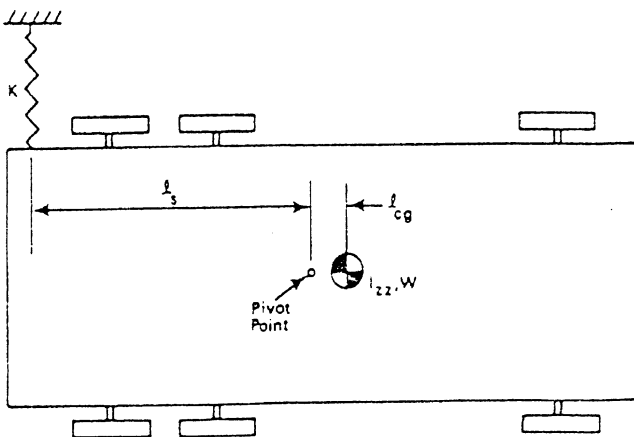


Figure 6

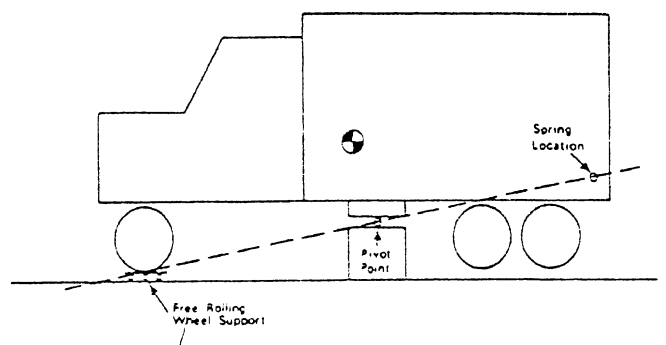


Figure 7

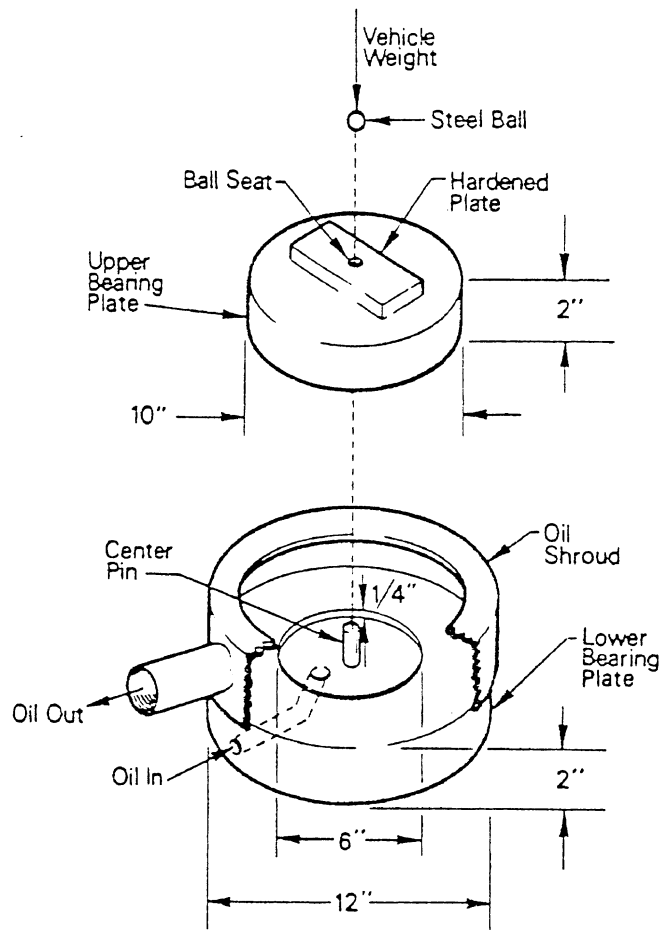


Figure 8

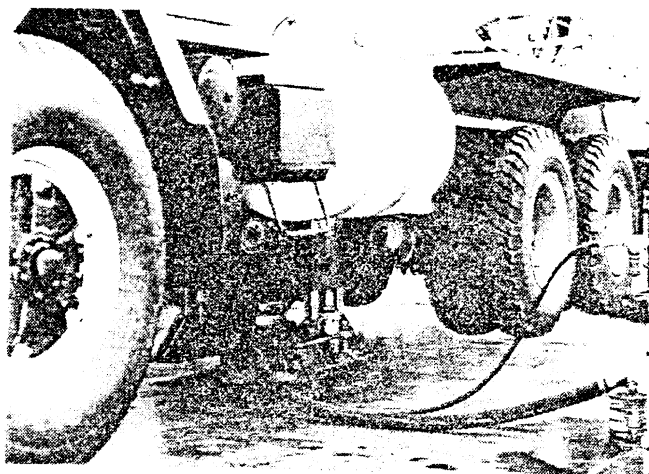


Figure 9

Under certain conditions, unwanted oscillations tend to appear during yaw inertia testing. A tendency for the vehicle to oscillate slightly in roll was noted. As it is supported during testing, the vehicle may roll about an axis passing through the ball bearing at the pivot point and the front tire contact point. (The front suspension is effectively rigid due to the constraining cables.) This axis is shown by the dashed line in Figure 7. To minimize the excitation of roll oscillations, the coil spring was anchored to the vehicle as close to this roll axis as possible. Furthermore, the spring constant,  $K$ , and the length,  $l_s$ , were chosen such that the natural yaw frequency of the system was considerably different from the roll frequency, thus reducing the tendency for yaw oscillations to excite roll oscillations.

An additional mode of oscillation was observed during yaw inertia tests. The construction of commercial vehicles typically results in considerable torsional compliance of the frame. Consequently, some vehicles showed a tendency to oscillate in a twisting manner along the length of their frames. This problem was effectively reduced by locating the spring near the horizontal centerline of the frame rails, thus reducing the moment resulting from the spring force which was passed into the frame.

Results for vehicles tested in roll and/or yaw are given in Table 2.



Table 4

Vehicle Description	Weight lbs	Wheelbase in	C.G. Height Above Ground in	C.G. Aft of Front Axle in	Moment of Inertia, in-lb-sec <sup>2</sup>		
					I <sub>x</sub>	I <sub>y</sub>	I <sub>z</sub>
Packer Truck: HEIL Model #A615E850 GMC 8500 V-6	18,000	150	55 1/4	131	71,400	476,800	453,500
Concrete Mixer REX, 9-yd. drum I.H. F5070-6x4	23,600	182	58 1/3	103 1/2	95,500	505,000	536,500
Medium Duty Van Truck GM CE 56703	9,380	168	44 1/4	98 1/4	61,800	296,800	308,300
Motor Home Chevrolet (PA 31442)	9,395	157	44.3	91 1/4	33,639	205,900	
Straight Truck No Body Diamond Reo 6x4	13,740	189	31 1/4	85 1/4	28,000	283,200	
Tractor White 6x4	14,270	142	39 3/4	66 1/2	36,000	178,760	

#### 4.0 VEHICLE COMPONENT MEASUREMENTS

Moments of inertia of vehicle components, particularly the roll inertia of unsprung masses and the polar inertia of tire/wheel/drum assemblies, have been measured for two heavy vehicles.

The test technique used is illustrated in Figure 10. As shown in this figure, the component of interest was suspended on a three-cable, multifilar pendulum. A small rotational oscillation about the vertical axis was introduced and the period determined.

The equation for calculating the moment of inertia,  $I$ , about the c.g. of the assemblies is

$$I = \frac{Wr^2\tau^2}{4\pi^2\lambda} + \frac{W_b r^2}{4\pi^2\lambda} (\tau^2 - \tau_0^2) \quad (12)$$

where

$W$  = test weight of the assembly

$W_b$  = weight of the supporting platform

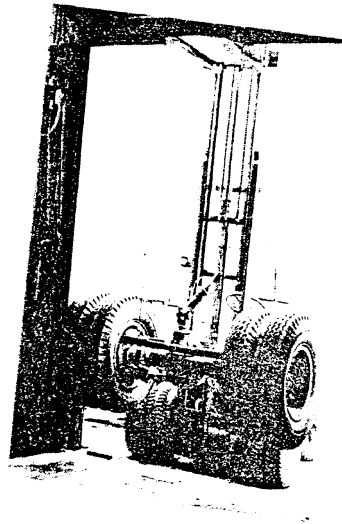
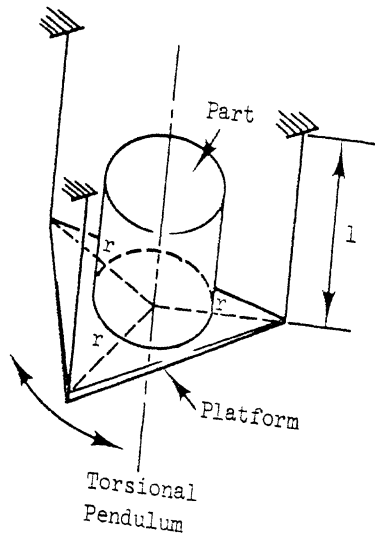
$\lambda$  = length of the supporting cables

$r$  = horizontal distance from center of platform to supporting cables

$\tau$  = period of oscillation of platform plus assembly

$\tau_0$  = period of oscillation of platform only.

Data gathered in this manner is presented in Table 3.



Pendulum Suspended by Means of Lift Truck

Figure 10

Table 3

Polar Moment of Inertia of Wheel/Tire/Drum Assemblies:

<u>Description</u>	<u>Inertia (in-lb-sec<sup>2</sup>)</u>
18 x 22.5 tire, spoke wheel, 15" drum	326
10 x 20 duals, spoke wheels, 15" drum	410
10 x 20 tire, spoke wheels, 15" drum	206
10 x 20 duals, spoke wheels, 16.5" drum	462

Roll Moment of Inertia, Unsprung Mass Assembly:

Front unsprung mass, 10 x 20 tires, spoke wheels, 15" drum; wt: 1321 lb	3719
Drive axle assembly; 10 x 20 duals, 15" drum, spoke wheels; wt: 2330 lb	4458
Trailer axle assembly; 10 x 20 duals, 15" drum, spoke wheels; wt: 1520 lb	4100