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A General Numerical Model for Evaluating Size Limit Regulations with Application to Michigan Bluegill (Lepomis macrochirus Rafinesque)

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Fisheries Research Report No. 1897

September 30, 1981

MICHIGAN DEPARTMENT OF NATURAL RESOURCES
FISHERIES DIVISION

Fisheries Research Report No. 1897

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A GENERAL NUMERICAL MODEL FOR EVALUATING SIZE LIMIT REGULATIONS WITH APPLICATION TO MICHIGAN BLUEGILL (*LEPOMIS MACROCHIRUS* RAFINESQUE)*

By Kelley David Smith

*This is a reprint of a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Fisheries, in the School of Natural Resources, The University of Michigan, 1981.

ACKNOWLEDGEMENTS

I would first like to thank Dr. Richard L. Patterson not only for chairing my thesis committee, but also for advising me throughout my entire Master's program. His eagerness and willingness to help in any matter went far and above any necessary duties. With his generous help, my years at the University of Michigan have been a wonderful experience. I would also like to thank Dr. Alvin L. Jensen for his critical review of this manuscript. His expertise in the field of fisheries modeling was extremely helpful during the development of this research.

I also want to express my thanks to the employees of the Institute for Fisheries Research who gave of their time willingly. Dr. William C. Latta reviewed the thesis and was very helpful in the data collection and analysis. Mercer H. Patriarche, Percy W. Laarman, James C. Schneider, and Roger N. Lockwood also gave much of their time and knowledge in the analysis of the biological data published for bluegills. James R. Ryckman helped in some of the statistical analysis as did Dr. Gary W. Fowler from the School of Natural Resources at the University of Michigan.

Dr Latta obtained funding for the project from the Michigan Department of Natural Resources, which was greatly appreciated.

I am also indebted to Richard D. Clark, Jr. who allowed me to work on and modify his trout model. His willingness to help and the long hours he devoted to discussing and interpreting some of the theoretical aspects of the model were greatly appreciated and very useful in the development of this study. Without his generosity and inspiration, this study may never have been undertaken.

Finally, I would like to thank my mother and father, Hadley and Maureen Smith for their love and understanding during my time here at the University. Without their help, this reality may have forever remained a dream.

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ABSTRACT

A general numerical model was developed which can be used to study the biological response of a fishery to a variety of size limit restrictions. Effects of minimum, maximum (inverted), or slot size limits, or a catch-and-release regulation can be studied using this model. Fishing and hooking mortality are adjustable for simulating effects of different gear types and restrictions. Density-dependent growth can be used and seasonal fluctuations of growth may be assessed. It is also possible to model seasonal fluctuation in fishing mortality, including shifts in season length or time periods.

Effects of a 7.0 inch maximum size limit (*i.e.*, all fish 7.0 inches and longer have to be returned to the water) were analyzed for slow, average, and fast growing bluegill (*Lepomis macrochirus* Rafinesque) populations in Michigan. Density-dependent mortality was used to estimate the number of fry surviving to age I. Density-dependent growth was simulated using a relationship between number of fry produced and total initial mean length.

Equations were developed to simulate the processes of mortality (natural, fishing, and hooking), growth, and recruitment. Number of fish in a population, number harvested, number caught and released, number lost to hooking mortality, and number of natural deaths were calculated using these equations and length-frequency information. Yield was calculated for harvested and caught and released fish using a length-weight regression.

Model simulations demonstrated that a 7.0 inch maximum size limit restriction was not effective in controlling bluegill populations. Variable and constant recruitment were modeled separately, and in neither case did the size limit regulation increase the number of bluegills 7.0 inches and larger, nor did mean length at each age change appreciably as compared to the same populations simulated under a 5.0 inch minimum size limit.

The greatest impact was observed in the fast growing population (using constant recruitment). Equilibrium numbers of 7.0 inch plus bluegills increased from 589 fish under a 5.0 inch minimum to 724 fish under the 7.0 inch maximum restriction, at a fishing mortality rate (m) of 0.40 for both. Total catch remained about equal - 2,038 and 2,089 fish per year for a 5.0 inch minimum and a 7.0 inch maximum size limit respectively, but legal harvest dropped from 1,530 under the existing conditions to 1,332 fish per year for the special regulation. As the conditional fishing mortality rate decreased, the population characteristics became virtually equal for the 5.0 inch minimum and the 7.0 inch maximum restrictions.

INTRODUCTION

Management techniques developed for the purpose of controlling and improving stunted or slow growing bluegill (Lepomis macrochirus Rafinesque) populations are many and often controversial. Techniques such as partial or complete poisoning and restocking of lakes, poisoning of spawning beds, encouragement of predatory fishes, or lowering of water levels to expose fingerlings to predation have either failed or only worked for short periods of time (Snow et al. 1960; Hooper et al. 1964; Beyerle and Williams 1967 and 1972; Schneider 1973b; Becker 1976; Beyerle 1977; Novinger and Legler 1978). These methods were usually discontinued because funds needed for reapplication or use on a widescale basis were not available.

New types of management techniques, which are not limited by money or manpower, have become the focus of recent studies. Regulations aimed at controlling a particular species have been imposed on lakes and streams. Minimum, maximum (inverted), or slot size limits, catch-and-release, and gear restrictions are being used to optimize survival, growth rate, and ultimately the yield of a fishery (Patriarche 1968; Schneider 1973a and 1978; Clark et al. 1979 and 1980).

Optimum yield can be defined as total harvest, "trophy" harvest, or total number of fish caught and released, depending on anglers' preference. Because fishermen define "quality fishing" in a number of ways, many sociological problems arise when implementing such complex regulations (Gulland 1968; Anderson 1975; Weithman and Anderson 1978). Although angler reaction and behavior can cause any fishery restriction to fall short of expected goals, biological response of the population is also of importance. Management techniques should not be employed until a careful theoretical analysis of the biological response by the population has been made using a variety of possible environmental conditions. With the widespread use of computers, numerical modeling is growing as a feasible and useful tool in fishery management (Clark and Lackey 1976; Clark et al. 1977). Biological response of fish populations to size limit regulations may be simulated and studied in detail. This allows selection and use of the best size restriction for achieving management objectives, hopefully eliminating most of the field oriented trial and error process.

The purpose of this study was twofold. The major objective was to develop a general numerical model which would aid in assessing the biological response of fish populations to a variety of size limit regulations. This model was patterned after one developed for trout fisheries by Clark et al. (1980). Five basic improvements were made on Clark's model to make it more applicable for warmwater inland lake fisheries (e.g., bass, bluegill, and similar species). These modifications included the use of density-dependent growth, a normal distribution of lengths at each age, a more explicit definition of the annual conditional hooking mortality rate, a breakdown of the annual conditional fishing and hooking mortality rates into a probability of capture and a probability of death given capture, and finally, distributing the different types of losses that are interacting in a fish population as a percentage of the competing mortality rates. The advantages of these improvements will be discussed as each is developed.

The second step of this research was to apply the model to a study of Michigan bluegill. Computer simulation was used to evaluate the effects of a 7.0 inch maximum limit on slow, average, and fast growing bluegill populations in Michigan. A FORTRAN computer program, Size Limit Regulation Analysis (S.L.R.A.), was written to perform the model simulations.

MODEL ASSUMPTIONS

Perfect simulation of real world processes is neither feasible nor possible. All numerical models must be built using basic assumptions as the starting foundation. If the assumptions are good approximations of the situation being simulated, then model performance is usually adequate. If too many assumptions are made or many are unrealistic, model results can be disastrous. The assumptions used to build this model are discussed in the following sections.

Mortality

Mortality includes many subdivisions which may be treated separately. Natural mortality at each age was assumed to remain constant from one year to the next. It was distributed by percentage throughout each year. Survival of fry was assumed to be density-dependent. A regression equation relating number of eggs to survival rate was used to estimate the number of fry reaching age I.

Fishing and hooking mortality were developed as two independent sources of death. Fishing mortality was applied only to legal fish in the population ("legal" refers to fish that can be harvested, and "illegal" to fish that must be released). The annual conditional fishing mortality rate (m) was constant for all age and size groups from one year to the next. This rate was divided into two components, a probability of capture (p') and a probability of death given capture (d'). The latter was assumed to be constant for all legal fish regardless of age or size.

Only illegal fish were susceptible to hooking mortality. The annual conditional hooking mortality rate (h) was constant for all ages and size groups from one year to the next, and all illegal fish caught were released. This assumption excluded the effects of poaching which were considered to be negligible. Although this could lead to erroneous results, especially for certain species or size limits, such an assumption was necessary because the amount of poaching is basically unknown.

Hooking mortality was divided into two components, a probability of capture (p'') and a probability of death after being caught and released (d''). The latter was assumed to be constant for all illegal fish regardless of age or size.

Growth

A basic assumption used in many fishery models is that growth remains constant in the population (Patriarche 1968; Clark *et al.* 1980). However, constant growth is not always observed in fish populations. Changes in growth are caused by density-dependent factors (Goodyear 1980).

Observed changes in mean length of a population over time may be simulated if the actual growth rate in a population remains constant but the initial mean length of a cohort varies with changes in density of fry (Gerking 1967;

Goodyear 1980). Therefore, although each new cohort experiences the same rate of size increase, their initial mean length may be different than that of other previous cohorts. This gives different lengths at each age than previously observed for the population.

The assumption of density-dependent growth was used in developing this model. The actual growth rate was constant for all cohorts, while their initial mean length depended on the number of fry hatching. Use of an expression developed empirically by Ford (1933) and a density-dependent regression relating number of fry to length allowed changes in growth over time to be a function of initial length rather than changes in the actual growth rate.

Recruitment

Many assumptions must be made in any fishery model when dealing with a process as complex as recruitment. Recruitment is a function of number of spawning adults, number of eggs produced, survivorship of fry, and growth (Goodyear 1980). Both variable and constant recruitment were studied in separate simulation runs.

A major problem observed in many fisheries is year class dominance caused by many density related factors (*e.g.*, food or available spawning areas). With variable recruitment, it is possible to model and study this phenomenon. By using density-dependent relationships for fry survival, number of eggs produced, and growth, variable recruitment was incorporated as a density-dependent function. Two basic assumptions were necessary to perform this task. First, the sex ratio was assumed to be 1:1 when calculating the number of females in each age-length group. Second, spawning was allowed during only one time period each year. Although many warmwater lake species may spawn two or more times during a summer season, a spawning "peak" is usually observed. It was assumed that the majority of spawning activity was accomplished during this period.

Constant recruitment was also simulated using these same basic assumptions. Although the number of fry surviving to age I remained constant, the number of eggs produced and the actual fry survival were still predicted each year to allow use of the density-dependent growth function.

MODEL DEVELOPMENT

A general numerical model based on a modification of Ricker's (1975) method was developed to simulate the processes of natural mortality, fishing mortality, hooking mortality, growth, and recruitment. This model may be used to study minimum, maximum (inverted), or slot size limits, or catch-and-release regulations by subdividing a population into length groups of illegal and legal size fish. Model predictions include estimates by age and length groups of number and weight of legal fish harvested and illegal fish caught and released, hooking deaths, and natural deaths. Such a breakdown of population dynamics is more useful in evaluating size limit restrictions on sport fisheries than is one of the older models (*e.g.*, surplus production model) which estimate total weight of harvested fish.

Model Variables

The variables used in a simulation model may be categorized into three major groups. The important input, state, and output variables used in this model are summarized in the following section. A flow diagram showing input strategy and output objectives is found in Figure 1.

Input:

A) Size limit regulation.

B) Fishing pressure (seasonal distribution).

C) Density-dependent growth relationship.

D) Variable (constant) recruitment.

<u>State</u> <u>Variables</u>:

A) Nondynamic state variables.

  1) Natural, fishing, and hooking mortality rates.

  2) Seasonal distribution of mortality (natural, fishing, and hooking) and growth.

B) Dynamic state variables.

  1) Population numbers (weight).

  2) Total catch and total catch-and-release (numbers and weight).

  3) Harvest and hooking deaths (numbers and weight).

  4) Natural deaths (numbers).

  5) Mean length and standard deviation by age group.

<u>Output</u>:

A) Population structure.

  1) By age group.

  2) By length-frequency distribution.

B) Yield.

  1) Total catch and harvest.

  2) Catch-and-release and hooking deaths.

C) Mean length.

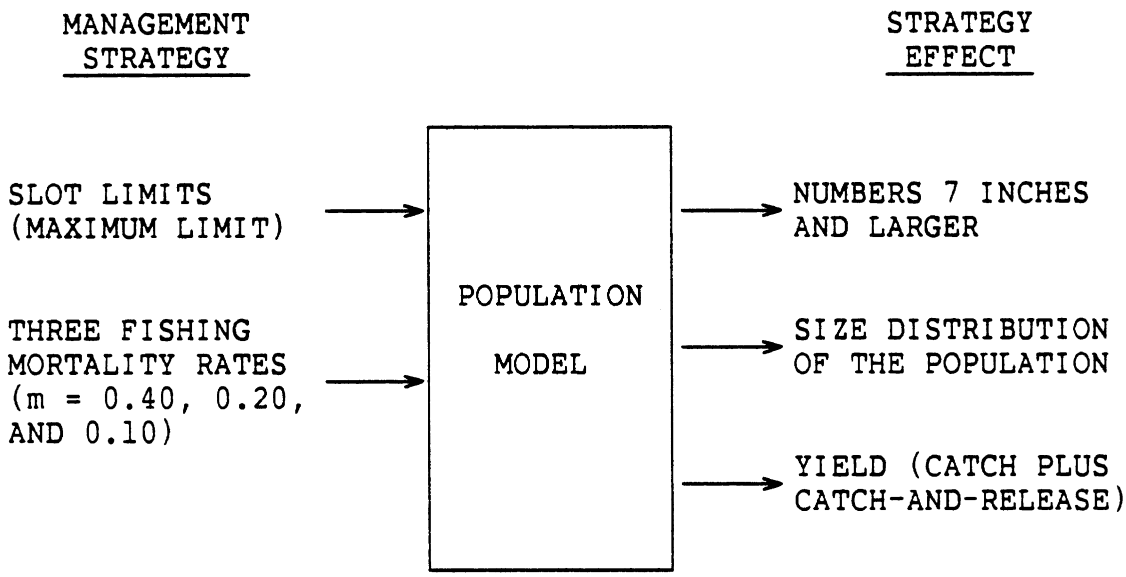


Figure 1. Flow diagram depicting input strategy and output objectives to be analyzed using the population model for warmwater inland lake fisheries.

Mortality

Ricker (1975) showed a method for separating mortality (Z) operating in a fish population into two major components, instantaneous natural mortality (M) and instantaneous fishing mortality (F). Relationships between these parameters is described by the following equations:

Instantaneous total mortality rate:

$$Z = F + M \qquad (1)$$

Annual expectation of death:

$$A = 1 - e^{-Zt} \qquad (2)$$

Annual survival rate:

$$S = e^{-Zt} \qquad (3)$$

Conditional mortality rates may be defined as:

Annual conditional natural mortality rate:

$$n = 1 - e^{-Mt} \qquad (4)$$

Annual conditional fishing mortality rate:

$$m = 1 - e^{-Ft} = p'd' \qquad (5)$$

where:

p' - probability of capture for legal fish.

d' - probability of harvest after being caught (constant over time and age).

The use of the variable d' is an improvement of the model developed by Clark et al. (1980). Clark's method assumed that all legal size fish caught were harvested. The model in this paper relaxes that assumption and allows appropriate adjustment of the total harvest if necessary. This may be useful in fisheries where fishermen voluntarily release many of the fish they catch, regardless of size or the regulations currently in effect.

By combining equations (4) and (5), a new expression for the annual expectation of death (A) may be written as a sum of the conditional rates:

$$A = 1 - e^{-Zt} = m + n - mn \qquad (6)$$

A conditional rate is the fraction of the original population which would have been killed by a certain type of mortality if no other source of death was operating in the population. Because both natural and fishing mortality compete for the same fish, the interaction term in equation (6) must be subtracted to prevent a fish from being counted as lost to more than one type of mortality.

Using this same logic, hooking mortality may be added as a third component of total mortality by redefining equations (1) and (6) and including a conditional hooking mortality rate:

Instantaneous total mortality rate:

$$Z = F + M + H \tag{7}$$

where:

H - instantaneous hooking mortality.

Annual conditional hooking mortality rate:

$$h = 1 - e^{-Ht} = p''d'' \tag{8}$$

where:

p'' - probability of capture for illegal fish.

d'' - probability of a fish dying after being caught and released (constant over time and age).

This explicit definition of the annual conditional hooking mortality rate was not used by Clark et al. (1980). They used the same variable to describe the conditional fishing mortality rate for legal fish and the catch rate of illegal fish. The probability of capture (p'') used in this model allows hooking mortality to be defined in a way that is consistent with the definitions of the natural (n) and fishing (m) mortality rates. This explicit definiton is also advantageous in that separate probabilities of capture (and separate probabilities of death given capture) may be assigned to both legal and illegal size fish.

Using equations (4), (5), and (8), equation (6) can be redefined as:

$$A = 1 - e^{-Zt} = m + n + h - mn - mh - nh + mnh \tag{9}$$

Including the interaction terms in equation (9), although they may be insignificant, is necessary from both an analytical and a biological standpoint. First, the terms are theoretically necessary to satisfy the numerical relationships between parameters. Second, these terms adjust for the combined effect of the three conditional mortality rates. Since these rates are acting concurrently, some fish already lost to one source of mortality may also be counted as lost to another. The Venn diagram in Figure 2 depicts this problem. Thus, from a biological viewpoint, the interactions are necessary to prevent losing the same fish to more than one type of mortality.

A change in the number of fish in the population from one time period to the next is described by the equation (Ricker 1975):

$$N_{t+1} = N_t - N_t A_1 \tag{10}$$

Because of the assumptions made concerning the different types of mortality, it is reasonable to subdivide a cohort into five length groups as (Clark _et al._ 1980):

$$N_t = P_t + R_t + T_t + U_t + V_t \tag{11}$$

where:

Interval P_t represents fish affected only by natural mortality.

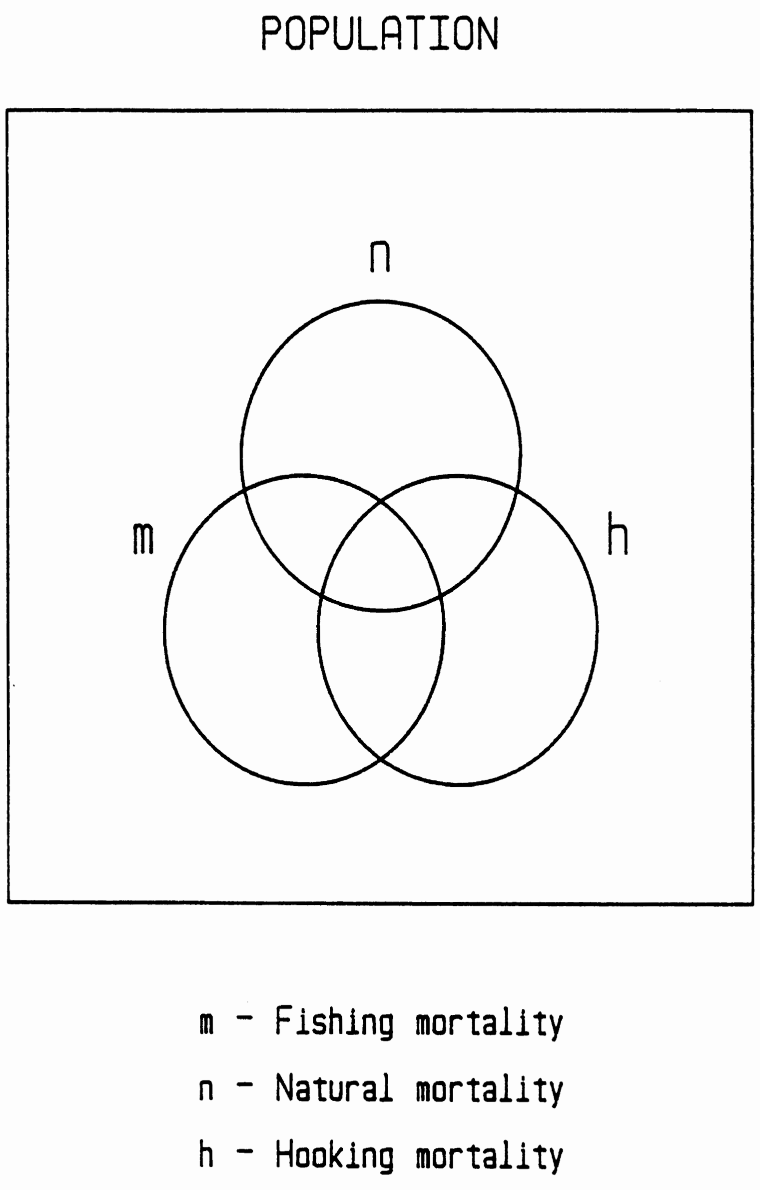


Figure 2. A graphical representation depicting the interactions between competing processes of loss acting in a fish population.

Intervals R_t and U_t represent fish of illegal size.

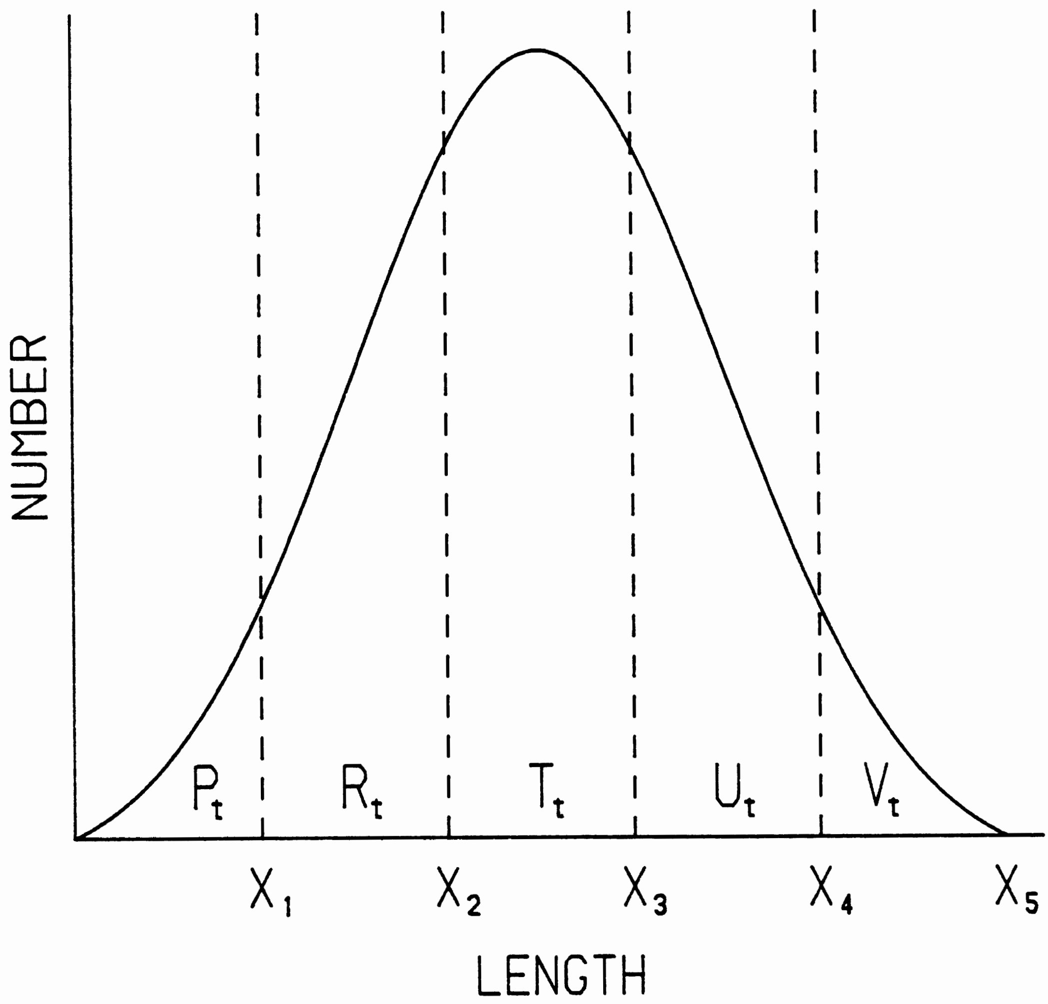
Intervals T_t and V_t represent fish of legal (harvestable) size.

This subdivision (Figure 3) allows simulation of slot limits directly, and other limits with slight modification.

By combining equations (9), (10), and (11), a change in the number of fish from one time period to the next becomes:

$$N_{t+1} = N_t - N_t n_1 - T_t m_1 - V_t m_1 - R_t h_1 - U_t h_1 + T_t m_1 n_1 + V_t m_1 n_1 + R_t n_1 h_1 + U_t n_1 h_1 - 0 m_1 h_1 + 0 m_1 n_1 h_1 \qquad (12)$$

Note that the interaction terms $m_1 h_1$ and $m_1 n_1 h_1$ do not apply to any part of the cohort. This is necessary to conform to the assumptions concerning mortality. There can be no interaction between fishing and hooking mortality because they apply to two independent groups of the cohort as depicted by the Venn diagram in Figure 4. The interactions shown in Figure 4 are not between the conditional fishing and hooking mortality rates explicitly, but between the respective probabilities of capture as necessitated by the variables defined above. This implies that a fish must be caught before it can die from harvest or hooking.



P_t - Uncatchable range

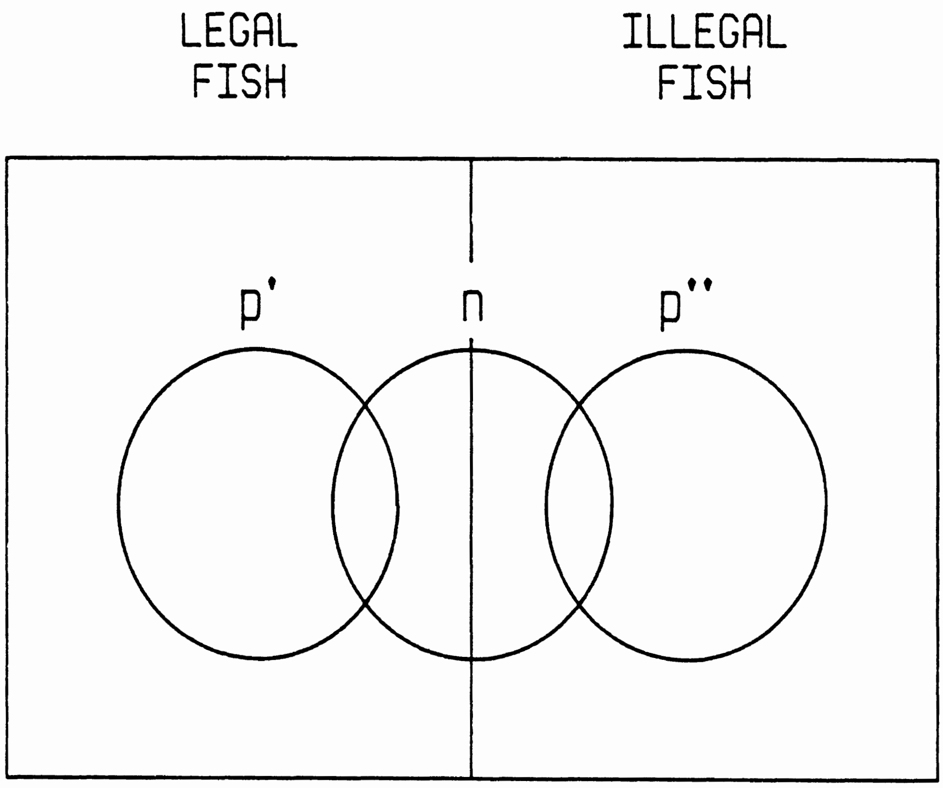
R_t - First illegal range

T_t - First legal range

U_t - Second illegal range

V_t - Second legal range

Figure 3. Hypothetical length-frequency distribution of a fish cohort depicting a slot size limit regulation.



p' - Probability of capture for legal fish

n - Natural mortality

p'' - Probability of capture for illegal fish

Figure 4. A graphical representation depicting the interactions between competing processes of removal acting in a fish population which has been divided into two independent groups.

A further breakdown of N_{t+1} may be written as (Clark *et al.* 1980):

$$N_{t+1} = N_t - C_t - D_t - H_t \qquad (13)$$

where:

C_t - number of legal fish harvested.

D_t - number of fish dying natural deaths.

H_t - number of fish lost due to hooking mortality.

Each of the three losses in equation (13) may be expressed as a combination of terms from equation (12) as:

Legal Catch:

$$C_t = T_t m_1 + V_t m_1 - \{n_1/(p'_1 + n_1)\}T_t m_1 n_1 - \{n_1/(p'_1 + n_1)\}V_t m_1 n_1 \qquad (14)$$

Natural Deaths:

$$D_t = N_t n_1 - \{p''_1/(n_1 + p''_1)\}R_t n_1 h_1 - \{p''_1/(n_1 + p''_1)\}U_t n_1 h_1 - \{p'_1/(p'_1 + n_1)\}T_t m_1 n_1 - \{p'_1/(p'_1 + n_1)\}V_t m_1 n_1 \qquad (15)$$

Hooking Deaths:

$$H_t = R_t h_1 + U_t h_1 - \{n_1/(n_1 + p''_1)\}R_t n_1 h_1 - \{n_1/(n_1 + p''_1)\}U_t n_1 h_1 \qquad (16)$$

The interaction terms which apply to two sources of mortality have been distributed using a ratio of the mortality rate to the total combined effect as a weighting factor. This seems reasonable as the number of fish involved in an interaction term should be divided between the two sources of death according to the respective sizes of the mortality rates in question. This is a more accurate division than used by Clark et al. (1980), who divided the effects of the interaction terms equally.

Two final quantities, numbers of legal and illegal fish caught and released that lived, may be calculated as:

$$JL_t = T_t p'_1 + V_t p'_1 - \{n_1/(p'_1 + n_1)\}T_t n_1 p'_1 - \{n_1/(p'_1 + n_1)\}V_t n_1 p'_1 = C_t/d' \qquad (17)$$

$$JI_t = R_t p''_1 + U_t p''_1 - \{n_1/(n_1 + p''_1)\}R_t n_1 p''_1 - \{n_1/(n_1 + p''_1)\}U_t n_1 p''_1 = H_t/d'' \qquad (18)$$

Growth

The numerical model of mortality developed in the previous section requires the approximation of a length-frequency distribution for a cohort to determine the number of fish in each of the areas defined in Figure 3. Any unimodal distribution which adequately describes the length distribution may be used for this purpose. Clark and Lackey (1976) demonstrated a method for approximating a length-frequency distribution using a three parameter Weibull probability density function. Clark et al. (1980) used this method in estimating length-frequency distributions for trout. Another distribution that may be used for this purpose is the normal (Jones 1958; Ricker 1969), with the probability density function:

$$f(x) = (1/\sqrt{2\pi\sigma})e^{-(x - \mu)^2/2\sigma^2} \qquad -\infty < x < +\infty \qquad (19)$$

where:

x - random variable (length).

μ - mean of the distribution.

σ - standard deviation of the distribution.

The cumulative distribution function is:

$$g(x) = (1/\sqrt{2\pi\sigma})\int_{-\infty}^{x} e^{-(x - \mu)^2/2\sigma^2}\,dx \qquad (20)$$

This distribution is very versatile as any normal distribution may be standardized using the transformation:

$$Z = (X - \mu)/\sigma \qquad (21)$$

giving:

$$P(X \le x) = P(\{X - \mu\}/\sigma \le \{x - \mu\}/\sigma) = P(Z \le \{x - \mu\}/\sigma) \qquad (22)$$

thus:

$$g(x) = \int_{-\infty}^{x} f(x)\, dx = P(Z \le \{x - \mu\}/\sigma) \tag{23}$$

If the mean and standard deviation of the distribution are known, g(x) may be evaluated directly from a standard normal table.

Using equation (23), the number of fish in each area defined in Figure 3 may be expressed as:

$$P_t = N_t g(x_1) \tag{24}$$

$$R_t = N_t\{g(x_2) - g(x_1)\} \tag{25}$$

$$T_t = N_t\{g(x_3) - g(x_2)\} \tag{26}$$

$$U_t = N_t\{g(x_4) - g(x_3)\} \tag{27}$$

$$V_t = N_t\{1 - g(x_4)\} \tag{28}$$

Using equation (23), g(x) may be defined for each length interval in Figure 3:

$$g(x_1) = \Theta_1 = P(X \le x_1) = P(Z \le \{x_1 - \mu_{i,t}\}/\sigma_{i,t}) \tag{29}$$

$$g(x_2) = \Theta_2 = P(X \le x_2) = P(Z \le \{x_2 - \mu_{i,t}\}/\sigma_{i,t}) \tag{30}$$

$$g(x_3) = \Theta_3 = P(X \le x_3) = P(Z \le \{x_3 - \mu_{i,t}\}/\sigma_{i,t}) \tag{31}$$

$$g(x_4) = \Theta_4 = P(X \le x_4) = P(Z \le \{x_4 - \mu_{i,t}\}/\sigma_{i,t}) \tag{32}$$

where:

subscript "i" denotes the age of a cohort.

$\mu_{i,t} = \bar{l}_{i,t}$ - mean length at time "t".

Substituting respective values of g(x) from equations (29) through (32) into equations (24) through (28) gives:

$$P_{i,t} = N_{i,t}\Theta_1 \qquad (33)$$

$$R_{i,t} = N_{i,t}(\Theta_2 - \Theta_1) \qquad (34)$$

$$T_{i,t} = N_{i,t}(\Theta_3 - \Theta_2) \qquad (35)$$

$$U_{i,t} = N_{i,t}(\Theta_4 - \Theta_3) \qquad (36)$$

$$V_{i,t} = N_{i,t}(1 - \Theta_4) \qquad (37)$$

To simulate a fishery, the length distributions for each cohort must be moved through time as the fish grow larger. The fraction (G_t) of annual growth experienced in some small time interval (1.0/t') was used to move distributions through time, with:

$$\sum_{t=1}^{t'} G_t = 1.0 \qquad (38)$$

where:

t' - number of time intervals in one year.

The use of G_t allows modeling of seasonal patterns in growth (and thus in recruitment of fish into and out of legal ranges) to be simulated more accurately within a year.

Changes in mean length for a given cohort within a year were expressed as:

$$\bar{l}_{i,t+1} = \bar{l}_{i,t} + (G_t)(\bar{l}_{i+1,l} - \bar{l}_{i,l}) \qquad (39)$$

Clark et al. (1980) used constant growth in their trout model. However, this assumption is not realistic for inland lake fisheries where growth may be affected by density, food availability, and other environmental factors.

The assumption of density-dependent growth used in this model necessitated the use of a function to relate the mean length at age "i+1" to the mean length at age "i", while keeping the mean annual growth rate constant. Such an equation was developed by Ford (1933) of the form:

$$\bar{l}_{i+1,l} = L_\infty(1.0 - k) + k\bar{l}_{i,l} \qquad (40)$$

where:

- k - Ford's growth coefficient.
- L_∞ - mean asymptotic length.
- $\bar{l}_{i,l}$ - mean length at age "i".

The parameters "k" and "L_∞" may be estimated from actual data using a computer program (VONB) developed by Allen (1966 and 1967).

By keeping "k" and "L_∞" constant for all cohorts, it can be seen from equation (40) that the mean length at any age "i" depends on the initial length at age I. Thus, it is possible to observe different lengths at some age "i" for each new cohort (depending on $\bar{l}_{1,1}$) even though the actual growth rate remains unchanged.

A regression relating mean length of fry to number of fry was used to estimate $\bar{l}_{1,1}$ for each new cohort. The form of the equation used was:

$$\bar{l}_{1,1} = a + b\log_e(FRY) \qquad (41)$$

where:

a - intercept.

b - slope.

$\bar{l}_{1,1}$ - mean length in time period 1 (age 1).

FRY - number of fry produced.

The mean length estimated for fry from equation (41) was used as the mean length of age-I fish at the start of each year. Ford's equation (40) was used to calculate mean lengths for all other ages using the initial mean length estimated by equation (41).

A ratio of mean length to standard deviation for a previous cohort at each age was used to calulate the standard deviations of new cohorts at each age. The relationship used was:

$$\sigma_{q',i,1} = (\bar{l}_{q',i,1})(\sigma_{q,i,1})/(\bar{l}_{q,i,1}) \qquad q'>q \qquad (42)$$

where:

q - denotes each cohort.

Thus, the ratio of mean length to standard deviation for all cohorts is the same at each age "i".

The reasoning used to develop equation (42) was based on the regression used in equation (41). As the number of fry increases, the initial mean length of the cohort decreases. A decrease in length should cause a corresponding drop in the standard deviation. With increasing density, competition for food and space would be spread out more evenly, thus decreasing the chances for any fish to gain or lose a competitive edge (Goodyear 1980). Therefore, length and associated variation should both decrease. This is often observed in lakes containing stunted populations which have very large numbers of fish of roughly the same length.

The opposite holds true if the number of fry decreases. Mean length and associated deviation increase as fish have a greater chance to gain or lose a competitive edge depending on their ability to survive.

Standard deviations of length were moved through time analogous to the method used for mean lengths. Using equations (38) and (39), this was expressed as:

$$\sigma_{i,t+1} = \sigma_{i,t} + (G_t)(\sigma_{i+1,1} - \sigma_{i,1}) \qquad (43)$$

Equation (43) allows changes in standard deviation through time to be proportional to corresponding changes in mean length.

Recruitment

Recruitment of fish depends upon the number of young produced during the spawning season and their ability to survive to harvestable length. The total number of eggs produced in one season was calculated as (Clark *et al.* 1980):

$$EGGS = \sum_{i=1}^{x} \sum_{j=1}^{y} (FEM_{ij})(FMAT_{ij})(EC_j) \qquad (44)$$

where:

- x - number of age groups.
- y - number of length groups.
- FEM_{ij} - number of females in each age-length group.
- $FMAT_{ij}$ - percent females mature in each age-length group.
- EC_j - mean egg content of females in each length group.

Mean egg content was determined using a regression relating length to number of eggs of the form:

$$EC_j = a + bl_j \qquad (45)$$

where:

a - intercept.

b - slope.

l_j - length group.

When the total number of eggs was calculated from equation (44), a stock-recruitment curve (Ricker 1975) relating number of eggs produced and number of fry was used to estimate the number of fry surviving to age I. The form of the equation used was:

$$S_F = a(EGGS)e^{-b(EGGS)} \tag{46}$$

where:

a - intercept.

b - slope.

S_F - number of fry surviving to age I.

The number of fry calculated from equation (46) was used as the initial number of age-I fish at the start of the next year.

Combining Mortality, Growth, and Recruitment

The numerical model thus far developed allows the population processes of interest to be described as single equations.

Population Numbers:

Changes in number of fish for each age group was expressed by combining equations (12) and (33) through (37):

$$N_{i,t+1} = N_{i,t}(1.0 - n_{i,1})(1.0 - m_{i,1}\{1.0 - \theta_2 + \theta_3 - \theta_4\} + h_{i,1}\{\theta_1 - \theta_2 + \theta_3 - \theta_4\}) \quad (47)$$

Catch:

Total catch of legal fish was calculated using equations (17), (35), and (37):

$$JL_{i,t} = N_{i,t}p'_{i,1}(1.0 - \{n_{i,1}/(p'_{i,1} + n_{i,1})\}n_{i,1})(1.0 - \theta_2 + \theta_3 - \theta_4) \quad (48)$$

Catch-and-release of illegal size fish was defined using equations (18), (33), (34), and (36):

$$JI_{i,t} = N_{i,t}p''_{i,1}(\{n_{i,1}/(n_{i,1} + p''_{i,1})\}n_{i,1} - 1.0)(\theta_1 - \theta_2 + \theta_3 - \theta_4) \quad (49)$$

Harvestable catch (legal size fish) was expressed as a combination of equations (14), (35), and (37):

$$C_{i,t} = N_{i,t}m_{i,1}(1.0 - \{n_{i,1}/(p'_{i,1} + n_{i,1})\}n_{i,1})(1.0 - \theta_2 + \theta_3 - \theta_4) \quad (50)$$

Hooking Deaths:

The number of fish lost to hooking mortality was expressed using equations (16), (33), (34), and (36):

$$H_{i,t} = N_{i,t}h_{i,1}(\{n_{i,1}/(n_{i,1} + p''_{i,1})\}n_{i,1} - 1.0)$$
$$(\theta_1 - \theta_2 + \theta_3 - \theta_4) \quad (51)$$

Natural Deaths:

Number of fish lost to natural deaths was defined as a combination of equations (15) and (33) through (37):

$$D_{i,t} = N_{i,t}n_{i,1}(1.0 + \{p''_{i,1}/(n_{i,1} + p''_{i,1})\}h_{i,1}(\theta_1 -$$
$$\theta_2 + \theta_3 - \theta_4) - \{p'_{i,1}/(p'_{i,1} + n_{i,1})\}m_{i,1}$$
$$(1.0 - \theta_2 + \theta_3 - \theta_4)) \quad (52)$$

Yield:

Yield in weight may also be calculated using the model equations developed for estimating catch. Length and weight for fish of a given age were related using the regression (Ricker 1975):

$$\log_e(\bar{w}_i) = a + b\log_e(\bar{l}_i) \quad (53)$$

where:

a - intercept.

b - slope.

$\bar{w}_i$ - mean weight of fish at age "i".

$\bar{l}_i$ - mean length of fish at age "i".

The catch equation (14), which represents a sum of the total harvest in two length classes (i.e., intervals $T_{i,t}$ and $V_{i,t}$ in Figure 3), may be used to calculate yield in weight as:

$$Y_{i,t} = \bar{w}_T T_{i,t} m_{i,1} + \bar{w}_V V_{i,t} m_{i,1} - \{n_{i,1}/ (p'_{i,1} + n_{i,1})\} \bar{w}_T T_{i,t} m_{i,1} n_{i,1} - \{n_{i,1}/(p'_{i,1} + n_{i,1})\} \bar{w}_V V_{i,t} m_{i,1} n_{i,1} \quad (54)$$

where:

$\bar{w}_T$ - mean weight of a fish in interval $T_{i,t}$.

$\bar{w}_V$ - mean weight of a fish in interval $V_{i,t}$.

Mean weight in each interval was calculated using equation (53) and the corresponding mean lengths in each interval ($\bar{l}_T$ and $\bar{l}_V$). The mean lengths were calculated as:

$$\bar{l}_T = \int_{x_2}^{x_3} x f(x)\ dx/(\theta_3 - \theta_2) \quad (55)$$

$$\bar{l}_V = \int_{x_4}^{1.0} x f(x)\ dx/(1.0 - \theta_4) \quad (56)$$

where:

$f(x)$ is the normal probability density function.

Using equations (35), (37), and (53) through (56), harvested yield was expressed as:

$$Y_{i,t} = N_{i,t} m_{i,1} (1.0 - \{n_{i,1}/(p'_{i,1} + n_{i,1})\} n_{i,1})$$
$$(e^{\phi_4} - e^{\phi_2}\{\theta_2\} + e^{\phi_2}\{\theta_3\} - e^{\phi_4}\{\theta_4\}) \qquad (57)$$

where:

$$\phi_2 = a + b\log_e(\bar{l}_T)$$
$$\phi_4 = a + b\log_e(\bar{l}_V)$$

Similarly, using equation (18), yield in weight of illegal fish caught and released was expressed as:

$$YJ_{i,t} = \bar{W}_R R_{i,t} p''_{i,1} + \bar{W}_U U_{i,t} p''_{i,1} - \{n_{i,1}/$$
$$(n_{i,1} + p''_{i,1})\} \bar{W}_R R_{i,t} n_{i,1} p''_{i,1} -$$
$$\{n_{i,1}/(n_{i,1} + p''_{i,1})\} \bar{W}_U U_{i,t} n_{i,1} p''_{i,1} \qquad (58)$$

where:

$\bar{W}_R$ - mean weight of a fish in interval $R_{i,t}$.
$\bar{W}_U$ - mean weight of a fish in interval $U_{i,t}$.

Mean lengths were calculated as:

$$\bar{I}_R = \int_{x_1}^{x_2} xf(x)\ dx/(\Theta_2 - \Theta_1) \qquad (59)$$

$$\bar{I}_U = \int_{x_3}^{x_4} xf(x)\ dx/(\Theta_4 - \Theta_3) \qquad (60)$$

Combining equations (33), (34), (36), (53), and (58) through (60) gave a final form for the yield in caught and released fish:

$$YJ_{i,t} = N_{i,t}p''_{i,1}(\{n_{i,1}/(n_{i,1} + p''_{i,1})\}n_{i,1} - 1.0)$$

$$(e^{\Phi_1}\{\Theta_1\} - e^{\Phi_1}\{\Theta_2\} + e^{\Phi_3}\{\Theta_3\} - e^{\Phi_3}\{\Theta_4\}) \qquad (61)$$

where:

$\Phi_1 = a + b\log_e(\bar{I}_R)$

$\Phi_3 = a + b\log_e(\bar{I}_U)$

MODEL APPLICATION

The model presented above was coded into a FORTRAN program (S.L.R.A.) and used to simulate population responses by Michigan bluegill to a 7.0 inch maximum limit. Necessary input data were collected from a variety of sources and are summarized in this section.

Population Characteristics

Bluegill populations were classified into three specific groups to cover a wide range of the existing conditions found in Michigan. Slow, medium, and fast growing populations were defined according to the mean length of a cohort at a given age. The stratification used is summarized in Table 1.

Table 1. Strata (mean total length in inches) used to classify slow, medium, and fast growing bluegill populations in Michigan.

Age	Slow	Medium	Fast
I	≤2.8	2.9 - 3.9	≥4.0
II	≤3.8	3.9 - 4.9	≥5.0
III	≤4.9	5.0 - 6.0	≥6.1
IV	≤5.8	5.9 - 6.9	≥7.0
V	≤6.4	6.5 - 7.5	≥7.6
VI	≤6.9	7.0 - 8.0	≥8.1
VII	≤7.3	7.4 - 8.4	≥8.5
VIII	≤8.0	8.1 - 9.1	≥9.2

Data published by Laarman (1963) were grouped using the above classes, and mean length with associated standard deviation and mean weight were calculated for each population at each age (Table 2).

Annual conditional natural mortality rates for each population were obtained from data published by Schneider (1973b) for Mill Lake in Washtenaw County, Michigan. Natural mortality was assumed to be size- rather than age-specific (Table 3). Plots of mortality against mean length from Schneider's data served as a guideline for determining the rates used in this study. These rates were allowed to increase as growth increased and resulted in a natural mortality rate of 55% per year for bluegills larger than 6.0 inches (Schneider 1973b).

The initial population size was chosen to be 10,000 fish and a lake size of 100 acres was used for all three populations. The initial age structures were calculated using the esimated natural mortality rates to determine the number of fish at each age (Table 3).

Ford's growth equation (40) was fit to the length data in Table 2 for each population using a computer program (VONB) developed by Allen (1966 and 1967). The calculated coefficients are presented in Table 4.

The regression equation (53) relating mean weight to mean length was fit for each population using least squares. A single regression was desired for all three populations, but tests for equal slopes and equal intercepts were both found to be highly significant ($p<0.001$). Therefore separate regression equations were used for the slow,

Table 2. Estimated mean total length in inches ($\bar{l}$), standard deviation of length (s), and mean weight in pounds ($\bar{w}$) of slow, medium, and fast growing bluegill populations in Michigan.

Age	Slow			Medium			Fast		
	$\bar{l}$	s	$\bar{w}$	$\bar{l}$	s	$\bar{w}$	$\bar{l}$	s	$\bar{w}$
I	2.5	0.237	0.009	3.5	0.274	0.027	4.5	0.576	0.065
II	3.5	0.275	0.024	4.4	0.330	0.054	5.7	0.615	0.132
III	4.4	0.403	0.050	5.4	0.336	0.106	6.7	0.697	0.218
IV	5.1	0.464	0.080	6.4	0.306	0.175	7.5	0.573	0.306
V	5.6	0.496	0.113	6.9	0.314	0.222	8.1	0.582	0.401
VI	6.1	0.519	0.148	7.4	0.243	0.273	8.9	0.593	0.542
VII	6.7	0.581	0.190	7.8	0.326	0.323	9.3	0.447	0.668
VIII	7.3	0.689	0.244	8.7	0.391	0.438	9.7	0.463	0.776

Table 3. Initial population (assuming a 100 acre lake and 100 fish per acre) and associated annual conditional natural mortality rates (n) for slow, medium, and fast growing bluegill populations in Michigan.

Age	Slow		Medium		Fast	
	Fish per acre	n	Fish per acre	n	Fish per acre	n
I	59.04	0.78	40.01	0.44	49.15	0.53
II	12.99	0.32	22.41	0.24	23.10	0.36
III	8.83	0.23	17.03	0.28	14.78	0.41
IV	6.80	0.24	12.26	0.51	8.72	0.62
V	5.17	0.28	6.01	0.68	3.31	0.75
VI	3.72	0.36	1.92	0.83	0.83	0.88
VII	2.38	0.55	0.33	0.91	0.10	0.95
VIII	1.07	0.81	0.03	0.98	0.01	0.98
Total	100.0		100.0		100.0	

medium, and fast growing populations. The regression coefficients are summarized in Table 4.

Table 4. Parameter values (and coefficients of determination) estimated for Ford's growth equation and the length-weight relationship for slow, medium, and fast growing bluegill populations in Michigan.

	Growth		
	k	L_∞	R^2
Slow	0.8861	10.6659	0.99383
Medium	0.8793	11.9500	0.98866
Fast	0.8428	11.9720	0.99830
	Length-weight		
	Slope (b)	Intercept (a)	R^2
Slow	3.1275	-7.6057	0.99922
Medium	3.0815	-7.4670	0.99972
Fast	3.2118	-7.6063	0.99783

Recruitment

Three additional assumptions were necessary to incorporate variable recruitment into the model. First, spawning activity was assumed to peak in the third week of June. Choice of this week was based on studies by Carbine (1939) who reported that spawning peaked in late June, Karvelis (1952) who reported mid-June, and Snow *et al.* (1960), Breder and Rosen (1966), and Becker (1976) who reported early to mid-June ranges. This week also corresponded to "t_0" in the model and it was at this time that fish were moved to the next age group. This was done not only because spawning was assumed to occur at this time, but also because the mean lengths reported by Laarman (1963) were considered early to mid-summer estimates. Second, a sex ratio of 1:1 was assumed for all ages capable of spawning (Beckman 1946; Fabian 1954; Parker 1958). Third, it was necessary to assume some minimum length for mature females. Ulrey et al. (1938) reported a minimum of 5.2 inches in Indiana lakes, Mayhew (1956) showed 4.3 inches in an Iowa lake, Snow et al. (1960) found the minimum to be 4.5 inches in Wisconsin lakes, and Scott and Crossman (1973) reported 5.4 inches for Canadian lakes. An average of these estimates was used in this study. A value of 4.8 inches was chosen as the minimum length of mature females.

A regression relating the mean egg content of females (EC) to mean length in inches ($\bar{l}$) was reported by Latta and Merna (1976). This equation was:

$$EC = -50{,}154.78 + 10{,}697.64(\bar{l})$$

This equation shows that the average 4.8 inch female bluegill would contain 1,194 eggs and an average 6.0 inch female would have 14,031 eggs. These figures correspond well with estimates reported by Ulrey et al. (1938), Mayhew (1956), and Snow et al. (1960). This equation also agrees well with the choice of a 4.8 inch minimum length for mature females. Equation (44) was then used to calculate the total number of eggs produced (EGGS) for the year.

In this model, fish were considered fry from the time of hatching until the following second week in June when they recruited into the age-I group. To use the assumption of density-dependent survival of fry, it was necessary to relate the number of eggs produced to number of fry surviving. Such a relationship for bluegills was developed by Latta and Merna (1976). A stock-recruitment curve (equation (46)) was fit to their observed data of the form:

$$S_F = 0.74647(\text{EGGS})e^{-(1.776 \times 10^{-6})(\text{EGGS})}$$

This curve (Figure 5) results in a maximum number of fry surviving (154,000 per acre) when the egg production is about 563,000 per acre. Because data were lacking beyond this peak area of the curve, an assumption was made that the curve to the right of the peak should decrease. This implies that as egg production increases beyond 563,000 per acre, the number of fry decreases. This seems reasonable because, as density increases, available spawning sites may be used up causing many fish to spawn in areas unsuitable for successful hatching, or suppression of spawning altogether (Snow et al. 1960). Swingle and Smith (1943) reported that at high densities bluegills will eat many or all of their own eggs which further supports this assumption.

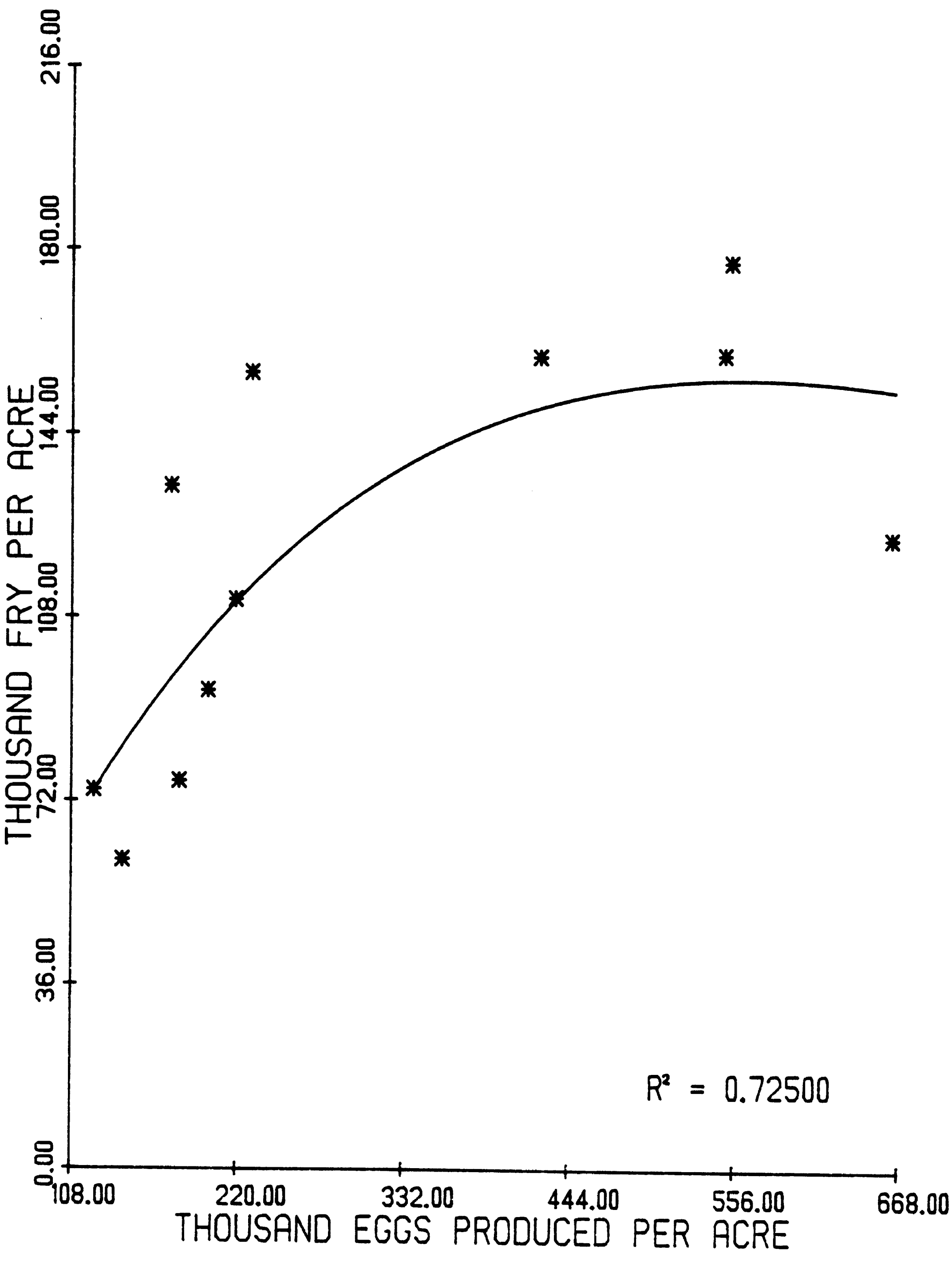


Figure 5. Fit of a stock-recruitment curve to observed fry densities.

Another regression developed by Latta and Merna (1976) related fry density to mean length of fry. Use of this regression allowed density-dependent growth to be incorporated into the model. Equation (41) was fit to these data giving:

$$\bar{l}_{1,1} = 4.4396 - 0.28716(\log_e(FRY))$$

The resulting curve (Figure 6) shows decreasing length with increasing density. This phenomenon was also reported by Karvelis (1952), Anderson (1959), and Novinger and Legler (1978). Goodyear (1980) also supported such a relationship in his comprehensive report on compensation in fish populations.

The regressions developed by Latta and Merna (1976) were obtained from experiments done in ponds at the Saline Fisheries Research Station in Michigan. These regressions needed to be adjusted for use with the slow, medium, and fast growing populations previously described for two reasons. First, the predicted number of fry surviving and their estimated mean length were probably higher in the ponds than would be found in natural lakes. This would be caused by the fact that no predation existed in the ponds, except from the few older bluegills stocked to produced the fry. However, the effect of their predation was assumed negligible. Second, the populations studied at Saline showed some characteristics seen in slow growing

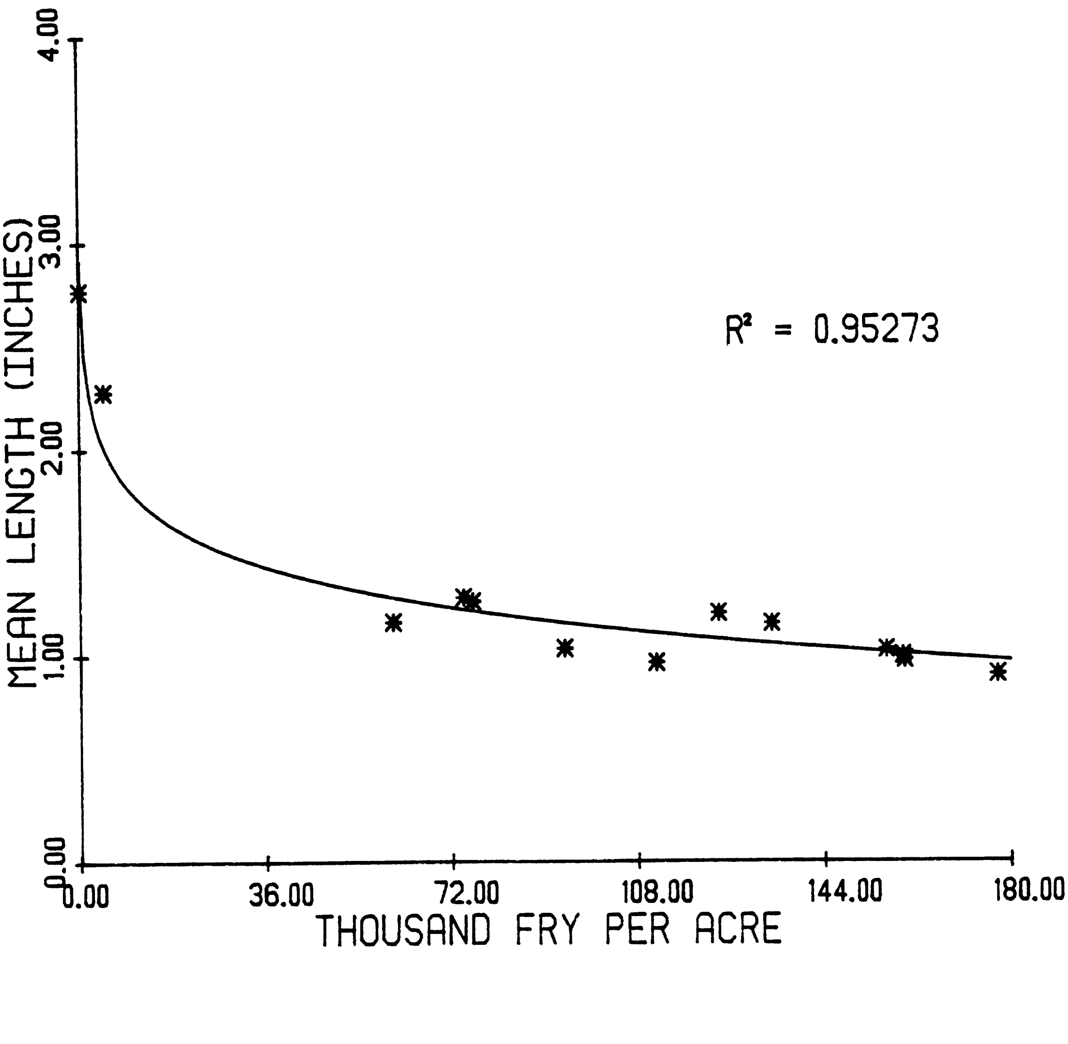


Figure 6. Fit of a length-density regression equation to observed mean lengths of fry.

populations. The regressions predicted survival and initial length better for the slow growing model populations than for the other two. However, these regressions were adjusted for use with each of the three populations.

Another problem existed in that the regressions predicted number of fry per acre and mean length occurring in the fall. Since the model "t_0" was the third week in June, mortality rates were assigned for fry from the fall to the second week in June of 0.99915033, 0.99968773, and 0.99968186 in slow, medium, and fast growing populations respectively. Although these may seem high, it must be remembered that the predicted estimates of fry per acre in the fall were also high due to lack of predation. These rates were chosen so that the populations to be simulated were in an equilibrium state when no fishing was allowed. Because fry dynamics were not modeled in any detail, the only important value was number of fry recruiting into the age-I group. The initial lengths predicted in the fall were also increased by one season of growth to keep them in phase with the model "year".

The constant mortality rate of fry from the fall to the following spring did not allow the population to compensate for any change in the number of large bluegills (7.0 inches plus), the conditional fishing mortality rate, or the size of previous year classes. Thus, population characteristics were also simulated using constant recruitment each year. The number of fry entering the age-I group each year was set equal to the number of age-I fish found in the unexploited populations, 5,904, 4,001, and 4,915 fish for slow, medium, and fast growing populations respectively (Table 3). However, the actual number of fry produced each season was calculated using the fecundity relationships so that the density-dependent growth relationship could still be used in the model.

Fishing Mortality

Three annual conditional fishing mortality rates (m) were chosen for use in this model. The values picked were 0.40, 0.20, and 0.10, each rate being applied to all three populations of bluegills during separate simulation runs. A consideration in determining what fishing rates to choose was that special regulations have often caused fishing pressure to drop on those lakes included in the regulations (Schneider and Lockwood 1979). Therefore, an upper level of 0.40 was chosen and then reduced substantially to cover a fairly wide range of possible fishing mortality rates.

The probability of death (harvest) given capture for legal fish (d') was set equal to 1.0, meaning that all legal fish caught were harvested. Although this may not be true especially for a 7.0 inch maximum limit on bluegill, Bennett (1962) showed that most fishermen would keep bluegills that were 5.0 inches or larger. Thus, a lower bound on the legal (harvestable) range was set at 5.0 inches, supporting the assumption that all legal fish caught would be harvested.

Because the probability of harvest given capture was set to 1.0, the capture probability for legal fish (p') was equal in magnitude to the conditional fishing rates chosen, depending on which was being used in the simulation process.

Hooking Mortality

An annual conditonal hooking mortality rate (h) was chosen corresponding to each fishing rate. The probability of death given capture for illegal fish (d'') was set equal to 0.20, and the probability of capture of illegal fish (p'') was set equal to the probability of capture for legal fish (p'). This resulted in hooking mortality rates of 0.08, 0.04, and 0.02 for fishing rates of 0.40, 0.20, and 0.10 respectively (h = p''d'').

The assumption of equal catchability for legal and illegal fish seems reasonable because, as Snow et al. (1960) and Becker (1976) stated in their studies, "bluegills are always ready and willing to take a hook". The assumption of a probability of death given capture for illegal fish equal to 0.20 also seems reasonable since it applies to bluegills over 7.0 inches with no restrictions on the types of fishing gear or bait used. Many bluegills often swallow hooks when fishermen are still-fishing with live bait (the most popular way to fish for bluegill) and will probably die when released. Although no experiments have been done on hooking mortality for bluegill, this figure is probably representative according to P. W. Laarman (personal communication).

Seasonal Distribution of Natural Mortality, Growth, and Fishing Mortality

The numerical model developed here allows a simulated year to be broken down into as many discrete time periods as seems reasonable. Since the dynamics of fish populations are all continuous processes, it follows that as the number of discrete time periods within a year increases (i.e., as $t' \rightarrow \infty$), better estimates of the population characteristics will be calculated. Ricker (1975) pointed out that intervals as small as a day were probably unnecessary and that any period less than a day was unreasonable because diurnal fluctuations in predation, etc., would invalidate many results unless a calculus of finite differences was employed in the model.

A discrete time period of one week was assumed to give accurate results and any smaller interval would not be justified by the increase in precision of the population estimates. Based upon this reasoning, time intervals of one week ($t'=52$) were used. Each month was assigned a specific number of weeks (Table 5), allowing natural and fishing mortality rates and growth to be spread throughout a year.

Patriarche (1968) published seasonal natural mortality rates for two lakes in Michigan. He assigned 7% to the spring (May 1 to June 7), 81% to the summer (June 7 to September 1), 12% to the fall period (September 2 to

Table 5. The number of weeks allotted to each month, and monthly percentage distribution of natural mortality (%n), growth (%g), and fishing mortality (%m).

Month	Number of weeks	%n	%g	%m
Jan.	5	0.0	0.0	4.0
Feb.	4	0.0	0.0	4.0
Mar.	5	0.0	0.0	1.0
Apr.	4	0.0	0.0	1.0
May	4	5.6	16.0	9.0
June	4	21.6	29.0	20.0
July	5	33.8	25.0	26.0
Aug.	4	27.0	12.0	21.0
Sep.	4	3.7	14.0	11.0
Oct.	5	4.6	4.0	1.0
Nov.	4	3.7	0.0	1.0
Dec.	4	0.0	0.0	1.0

December 1), and no natural mortality during the winter (December 2 to April 30). Schneider (1973a) used this same type of distribution in his model for Mill Lake in Washtenaw County, Michigan. The percent of mortality occurring in each month was computed as a ratio of the number of weeks in each month to the total number of weeks attributed to each season (Table 5).

Anderson (1959) showed a monthly percentage breakdown of total annual growth in his study of Third Sister Lake in Washtenaw County, Michigan. Both field data and laboratory experiments performed by Anderson gave similar results. Karvelis (1952) and Fabian (1954) reported approximately the same distribution for Ford Lake in Otsego County, Michigan as did Snow <u>et al.</u> (1960) for Wisconsin lakes. Schneider

(1973a) also used a similar pattern of growth in his Mill Lake model. Anderson's figures were used in this model (Table 5).

The distribution of fishing mortality was calculated from angler census data collected on Bear Lake (1952-1953) in Hillsdale County, Michigan (Schneider and Lockwood, 1979). Fishing pressure on this lake of 117 acres in size was assumed to be representative of pressure received on small lakes in the 100 acre range as used in this study. On the basis of these data, 62% of the total fishing mortality was assigned to the summer months (June 24 to September 15), 6% to the fall (September 16 to December 1), 10% to the winter (December 2 to March 30), and 22% to the spring (April 1 to June 23). This pattern was slightly modified (as recommended by M. H. Patriarche and P. W. Laarman, personal communication) and a final distribution of percent fishing mortality was estimated to be (same seasonal dates): 57.5% in the summer, 7.5% in the fall, 10% in the winter, and 25% in the spring. Christensen (1953) showed a distribution of percent fishing mortality for six lakes in Michigan (averaged over a five year period) that was very similar to this latter distribution. These seasonal values were then spread over the corresponding months (Table 5) using the same method as described earlier for natural mortality.

The monthly percentages of natural mortality, growth, and fishing mortality were spread uniformly over the weeks in each month. Each week was given an equal amount of the total value for the month by dividing the number of weeks in a month into the monthly percent estimate.

RESULTS AND DISCUSSION

The characteristics of slow, medium, and fast growing bluegill populations were simulated using the S.L.R.A. computer program. Simulation was continued until the populations reached equilibrium for the fishing mortality rate being used. The initial populations were also subjected to a 5.0 inch minimum size limit regulation, used as a control to determine the impact of the 7.0 inch maximum restriction. Although no regulation was in effect for bluegills, a 5.0 inch minimum was assumed to be representative of conditions found in Michigan at this time. Two separate sets of output were generated, one using variable recruitment and the other constant recruitment.

Results Using Variable Recruitment

Annual fishery statistics of the three bluegill populations at equilibrium (assuming variable recruitment) are summarized in Tables 6-17. Number of fish (per 100 acres) and attained mean length in inches at each age are found in Tables 6, 10, and 14 for slow, medium, and fast growing bluegill populations respectively. These results show no change in length (at a given fishing mortality rate) between a 5.0 inch minimum and a 7.0 inch maximum regulation, and a large decrease in the total number of fish as the fishing mortality rate increases, especially in the slow growing population. This suggests that the density-dependent recruitment relationships used in the simulation were not adequate in describing the processes of fry survival and recruitment into the age-I group. This could be attributed to many factors including the assumptions of only one spawning period per year and/or a 4.8 inch minimum length of mature females. Both of these assumptions could cause the number of eggs produced to be far less than seen in the field under the same conditions. Multiple spawning periods were not modeled because of the lack of data concerning this phenomenon and the complexity involved in simulating such a process. The effect of a 4.8 inch minimum length of mature females was not as important in the medium or fast growing populations where this length was attained by age I or II. However, in the slow growing population, this length was not reached until age III or IV and the spawning stock had been greatly depleted by then because of fishing. This caused the large reduction in total number of fish as the fishing mortality rate increased.

It was hoped that the use of a density-dependent growth relationship would offset these problems. However, the regression used to predict the mean length of fry was not sensitive enough to changes in the density of fry. Although the mean length at each age increased as density decreased, the change in length was not enough to offset the loss of fish over 4.8 inches.

Another important factor was the assumption of a constant mortality of fry from the fall to the following spring. This assumption did not allow any compensation by the population for changes in the number of large bluegills (7.0 inches plus), the density of the previous year class, or the fishing rate. Better feedback mechanisms are necessary to model this compensation in any greater detail.

The fishery statistics are recorded by length group in Tables 7-9 for slow growing, Tables 11-13 for medium growing, and Tables 15-17 for fast growing populations. The largest impact of the special regulation was seen in the fast growing population at a fishing mortality rate of 0.40 (Table 15). Numbers of 7.0 inch plus fish increased from 398 under existing conditions to 590 using the special regulation. Total catch was greater under the 7.0 inch maximum restriction (1,617 fish versus 1,363 fish), while harvest was essentially equal (1,024 and 1,020 fish per year for a 5.0 inch minimum and a 7.0 inch maximum respectively). The special regulation had little effect on controlling the slow and medium growing populations and the fishery statistics were virtually the same for the 5.0 inch minimum and 7.0 inch maximum restrictions. Changing from a 5.0 inch minimum to a 7.0 inch inverted regulation had little effect because less than 5% of the fish in these populations were over 7.0 inches (i.e., the same fish were still harvested). Any change between the existing conditions and the 7.0 inch maximum in all three populations diminished as the fishing mortality rate decreased.

Table 6. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with variable recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a slow growing bluegill population in Michigan.

Age	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	721	3.0	751	3.0	2,546	2.7	2,670	2.7	4,156	2.6	4,252	2.6
II	163	4.0	175	4.0	571	3.7	599	3.7	922	3.6	946	3.6
III	104	4.9	115	4.9	388	4.6	401	4.6	622	4.5	637	4.5
IV	62	5.6	68	5.6	275	5.3	285	5.3	465	5.2	475	5.2
V	33	6.1	38	6.0	180	5.8	186	5.8	330	5.7	337	5.7
VI	18	6.4	19	6.4	109	6.3	114	6.3	219	6.2	224	6.2
VII	8	7.0	9	7.0	59	6.9	64	6.9	129	6.8	134	6.8
VIII	2	7.5	4	7.5	22	7.4	27	7.4	54	7.3	57	7.3
Total	1,111		1,179		4,150		4,346		6,897		7,062	

Table 7. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	361	0	0.0	0	0.0	0	0.0	0
7 inch maximum	376	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	592	0	0.0	98	4.2	98	4.2	20
7 inch maximum	627	0	0.0	104	4.6	104	4.6	21
				5.0- to 7.0-inch fish				
5 inch minimum	148	62	7.7	0	0.0	62	7.7	0
7 inch maximum	165	69	8.3	0	0.0	69	8.3	0
				Over 7.0-inch fish				
5 inch minimum	10	4	0.7	0	0.0	4	0.7	0
7 inch maximum	11	0	0.0	3	0.9	3	0.9	1
				All fish				
5 inch minimum	1,111	66	8.4	98	4.2	164	12.6	20
7 inch maximum	1,179	69	8.3	107	5.5	176	13.8	22

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 8. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	2,244	0	0.0	0	0.0	0	0.0	0
7 inch maximum	2,352	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	1,280	0	0.0	146	6.5	146	6.5	29
7 inch maximum	1,337	0	0.0	153	6.9	153	6.9	31
				5.0- to 7.0-inch fish				
5 inch minimum	573	110	13.7	0	0.0	110	13.7	0
7 inch maximum	596	115	13.8	0	0.0	115	13.8	0
				Over 7.0-inch fish				
5 inch minimum	53	10	2.0	0	0.0	10	2.0	0
7 inch maximum	61	0	0.0	10	2.6	10	2.6	2
				All fish				
5 inch minimum	4,150	120	15.7	146	6.5	266	22.2	29
7 inch maximum	4,346	115	13.8	163	9.5	278	23.3	33

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 9. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
1.0- to 2.9-inch fish								
5 inch minimum	3,957	0	0.0	0	0.0	0	0.0	0
7 inch maximum	4,047	0	0.0	0	0.0	0	0.0	0
3.0- to 4.9-inch fish								
5 inch minimum	1,860	0	0.0	114	5.2	114	5.2	23
7 inch maximum	1,907	0	0.0	117	5.4	117	5.4	24
5.0- to 7.0-inch fish								
5 inch minimum	981	89	11.3	0	0.0	89	11.3	0
7 inch maximum	1,005	92	11.2	0	0.0	92	11.2	0
Over 7.0-inch fish								
5 inch minimum	99	9	1.8	0	0.0	9	1.8	0
7 inch maximum	103	0	0.0	9	2.2	9	2.2	2
All fish								
5 inch minimum	6,897	98	13.1	114	5.2	212	18.3	23
7 inch maximum	7,062	92	11.2	126	7.6	218	18.8	26

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 10. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with variable recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a medium growing bluegill population in Michigan.

Age	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	1,076	3.9	1,229	3.9	2,506	3.6	2,606	3.6	3,195	3.5	3,244	3.5
II	569	4.8	639	4.8	1,363	4.5	1,416	4.5	1,766	4.4	1,795	4.4
III	315	5.8	349	5.8	933	5.5	966	5.5	1,286	5.4	1,305	5.4
IV	145	6.7	165	6.7	552	6.5	571	6.5	845	6.4	857	6.4
V	46	7.1	60	7.1	222	7.0	237	7.0	377	6.9	387	6.9
VI	10	7.6	17	7.6	59	7.5	71	7.5	110	7.4	119	7.4
VII	1	8.0	3	8.0	8	7.9	12	7.9	17	7.8	20	7.8
VIII	0	...	0	...	1	8.8	1	8.8	1	8.7	2	8.7
Total	2,162		2,462		5,644		5,880		7,597		7,729	

Table 11. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
1.0- to 2.9-inch fish								
5 inch minimum	1	0	0.0	0	0.0	0	0.0	0
7 inch maximum	2	0	0.0	0	0.0	0	0.0	0
3.0- to 4.9-inch fish								
5 inch minimum	1,483	0	0.0	330	17.4	330	17.4	66
7 inch maximum	1,686	0	0.0	376	19.8	376	19.8	75
5.0- to 7.0-inch fish								
5 inch minimum	613	275	36.9	0	0.0	275	36.9	0
7 inch maximum	688	311	41.1	0	0.0	311	41.1	0
Over 7.0-inch fish								
5 inch minimum	65	24	5.5	0	0.0	24	5.5	0
7 inch maximum	86	0	0.0	27	7.1	27	7.1	5
All fish								
5 inch minimum	2,162	299	42.4	330	17.4	629	59.8	66
7 inch maximum	2,462	311	41.1	403	26.9	714	68.0	80

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 12. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	41	0	0.0	0	0.0	0	0.0	0
7 inch maximum	42	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	3,801	0	0.0	426	20.8	426	20.8	85
7 inch maximum	3,951	0	0.0	442	21.7	442	21.7	89
				5.0- to 7.0-inch fish				
5 inch minimum	1,595	309	43.1	0	0.0	309	43.1	0
7 inch maximum	1,655	322	44.0	0	0.0	322	44.0	0
				Over 7.0-inch fish				
5 inch minimum	207	35	8.4	0	0.0	35	8.4	0
7 inch maximum	232	0	0.0	36	9.6	36	9.6	7
				All fish				
5 inch minimum	5,644	344	51.5	426	20.8	770	72.3	85
7 inch maximum	5,880	322	44.0	478	31.3	800	75.3	96

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 13. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	109	0	0.0	0	0.0	0	0.0	0
7 inch maximum	110	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	4,942	0	0.0	274	13.2	274	13.2	54
7 inch maximum	5,020	0	0.0	278	13.4	278	13.4	56
				5.0- to 7.0-inch fish				
5 inch minimum	2,261	203	28.6	0	0.0	203	28.6	0
7 inch maximum	2,298	206	28.6	0	0.0	206	28.6	0
				Over 7.0-inch fish				
5 inch minimum	285	23	5.4	0	0.0	23	5.4	0
7 inch maximum	301	0	0.0	23	6.0	23	6.0	4
				All fish				
5 inch minimum	7,597	226	34.0	274	13.2	500	47.2	54
7 inch maximum	7,729	206	28.6	301	19.4	507	48.0	60

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 14. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with variable recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a fast growing bluegill population in Michigan.

Age	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	3,281	4.7	3,837	4.7	4,443	4.5	4,597	4.5	4,791	4.5	4,830	4.5
II	1,123	5.8	1,314	5.8	1,841	5.7	1,906	5.7	2,121	5.7	2,138	5.7
III	457	6.8	564	6.8	966	6.7	1,019	6.7	1,236	6.7	1,257	6.7
IV	169	7.6	263	7.6	463	7.5	536	7.5	661	7.5	702	7.5
V	40	8.2	90	8.2	143	8.1	193	8.1	228	8.1	260	8.1
VI	6	9.0	21	9.0	29	8.9	46	8.9	52	8.9	64	8.9
VII	0	...	2	9.4	3	9.3	5	9.3	6	9.3	8	9.3
VIII	0	...	0	...	0	...	0	...	0	...	0	...
Total	5,076		6,091		7,888		8,302		9,095		9,259	

Table 15. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a fast growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
1.0- to 2.9-inch fish								
5 inch minimum	8	0	0.0	0	0.0	0	0.0	0
7 inch maximum	9	0	0.0	0	0.0	0	0.0	0
3.0- to 4.9-inch fish								
5 inch minimum	2,374	0	0.0	339	21.7	339	21.7	68
7 inch maximum	2,777	0	0.0	397	27.6	397	27.6	79
5.0- to 7.0-inch fish								
5 inch minimum	2,296	860	131.4	0	0.0	860	131.4	0
7 inch maximum	2,715	1,020	147.2	0	0.0	1,020	147.2	0
Over 7.0-inch fish								
5 inch minimum	398	164	46.7	0	0.0	164	46.7	0
7 inch maximum	590	0	0.0	200	68.9	200	68.9	39
All fish								
5 inch minimum	5,076	1,024	178.1	339	21.7	1,363	199.8	68
7 inch maximum	6,091	1,020	147.2	597	96.5	1,617	243.7	118

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 16. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a fast growing bluegill population in Michigan.

	Simulated fishery statistics*							
Fishing regulation	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	20	0	0.0	0	0.0	0	0.0	0
7 inch maximum	21	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	3,808	0	0.0	289	18.1	289	18.1	57
7 inch maximum	3,941	0	0.0	299	20.2	299	20.2	60
				5.0- to 7.0-inch fish				
5 inch minimum	3,160	577	91.7	0	0.0	577	91.7	0
7 inch maximum	3,296	602	88.8	0	0.0	602	88.8	0
				Over 7.0-inch fish				
5 inch minimum	900	160	47.7	0	0.0	160	47.7	0
7 inch maximum	1,044	0	0.0	170	59.1	170	59.1	34
				All fish				
5 inch minimum	7,888	737	139.4	289	18.1	1,026	157.5	57
7 inch maximum	8,302	602	88.8	469	79.3	1,071	168.1	94

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 17. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming variable recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a fast growing bluegill population in Michigan.

	Simulated fishery statistics*							
Fishing regulation	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	22	0	0.0	0	0.0	0	0.0	0
7 inch maximum	21	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	4,125	0	0.0	156	9.8	156	9.8	31
7 inch maximum	4,159	0	0.0	157	10.8	157	10.8	31
				5.0- to 7.0-inch fish				
5 inch minimum	3,685	320	52.0	0	0.0	320	52.0	0
7 inch maximum	3,730	325	48.4	0	0.0	325	48.4	0
				Over 7.0-inch fish				
5 inch maximum	1,263	106	32.4	0	0.0	106	32.4	0
7 inch maximum	1,349	0	0.0	109	37.9	109	37.9	22
				All fish				
5 inch minimum	9,095	426	84.4	156	9.8	582	94.2	31
7 inch maximum	9,259	325	48.4	266	48.7	591	97.1	53

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Results Using Constant Recruitment

Annual fishery statistics of the three bluegill populations at equilibrium (assuming constant recruitment) are summarized in Tables 18-29. Number of fish (per 100 acres) and attained mean length in inches at each age are found in Tables 18, 22, and 26 for slow, medium, and fast growing populations respectively. As in the results using variable recruitment, no change in length (at a given fishing rate) was observed between a 5.0 inch minimum and a 7.0 inch maximum. However, because constant recruitment allows some compensation in the recruitment of age-I fish, the total number of fish in the population is much higher than seen in the earlier results for variable recruitment. The population structures seem much more realistic and are no longer greatly altered by the choice of a fishing mortality rate. The largest impact from using constant recruitment was observed in the slow growing population at a fishing mortality rate of 0.40. Total number increased from 1,111 fish (assuming variable recruitment) to 8,880 fish (assuming constant recruitment). This was expected because of the slow growth and the 4.8 inch minimum length of mature females. This impact decreases from medium to fast growth, but a significant change in total numbers was still observed between the variable and constant recruitment estimates.

Although the assumption of constant recruitment did increase total numbers, the use of a 7.0 inch maximum regulation was still not effective in controlling the bluegill populations. Yield was much greater over that seen in the previous results, but this was expected because of the increase in total number of fish in the populations. Simulated fishery statistics by length group are found in Tables 19-21 for slow growing, Tables 23-25 for medium growing, and Tables 27-29 for fast growing bluegill populations. The greatest change between existing conditions and the special regulation was again seen in the fast growing population using a conditional fishing mortality rate of 0.40 (Table 27). Number of fish over 7.0 inches increased from 589 (5.0 inch minimum) to 724 fish for a 7.0 inch maximum regulation. Harvest decreased slightly from 1,530 to 1,332 fish per year when the special regulation was applied. Total catch was virtually the same, 2,038 under existing conditions and 2,089 fish per year using a 7.0 inch maximum restriction. The statistics for the slow and medium growing populations showed much smaller changes at a fishing mortality rate of 0.40 when the special regulation was applied, than observed for the fast growing population. The number of fish over 7.0 inches, harvest, and total catch became essentially equal for a 5.0 inch minimum and a 7.0 inch maximum with a decrease in the fishing mortality rate, as observed in the simulation results using variable recruitment.

Table 18. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with constant recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a slow growing bluegill population in Michigan.

	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
Age	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	5,904	2.7	5,904	2.7	5,904	2.6	5,904	2.6	5,904	2.6	5,904	2.6
II	1,281	3.7	1,281	3.7	1,293	3.6	1,293	3.6	1,296	3.6	1,296	3.6
III	803	4.6	803	4.6	848	4.5	848	4.5	866	4.5	866	4.5
IV	483	5.3	483	5.3	593	5.2	593	5.2	637	5.2	637	5.2
V	243	5.8	243	5.8	379	5.7	379	5.7	446	5.7	446	5.7
VI	110	6.3	112	6.3	223	6.2	224	6.2	292	6.2	293	6.2
VII	44	6.9	49	6.9	116	6.8	120	6.8	170	6.8	173	6.8
VIII	12	7.4	17	7.4	42	7.3	48	7.3	69	7.3	74	7.3
Total	8,880		8,892		9,398		9,409		9,680		9,689	

Table 19. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	5,202	0	0.0	0	0.0	0	0.0	0
7 inch maximum	5,202	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	2,793	0	0.0	639	28.1	639	28.1	127
7 inch maximum	2,793	0	0.0	639	28.3	639	28.3	127
				5.0- to 7.0-inch fish				
5 inch minimum	844	372	43.5	0	0.0	372	43.5	0
7 inch maximum	850	374	43.0	0	0.0	374	43.0	0
				Over 7.0-inch fish				
5 inch minimum	41	17	3.4	0	0.0	17	3.4	0
7 inch maximum	47	0	0.0	17	4.1	17	4.1	3
				All fish				
5 inch minimum	8,880	389	46.9	639	28.1	1,028	75.0	127
7 inch maximum	8,892	374	43.0	656	32.4	1,030	75.4	130

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 20. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	5,619	0	0.0	0	0.0	0	0.0	0
7 inch maximum	5,619	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	2,564	0	0.0	317	14.2	317	14.2	64
7 inch maximum	2,564	0	0.0	317	14.4	317	14.4	64
				5.0- to 7.0-inch fish				
5 inch minimum	1,128	220	26.9	0	0.0	220	26.9	0
7 inch maximum	1,133	221	26.2	0	0.0	221	26.2	0
				Over 7.0-inch fish				
5 inch minimum	87	16	3.2	0	0.0	16	3.2	0
7 inch maximum	93	0	0.0	16	4.0	16	4.0	3
				All fish				
5 inch minimum	9,398	236	30.1	317	14.2	553	44.3	64
7 inch maximum	9,409	221	26.2	333	18.4	554	44.6	67

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 21. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a slow growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	5,619	0	0.0	0	0.0	0	0.0	0
7 inch maximum	5,619	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	2,604	0	0.0	160	7.2	160	7.2	32
7 inch maximum	2,604	0	0.0	160	7.3	160	7.3	32
				5.0- to 7.0-inch fish				
5 inch minimum	1,328	122	15.4	0	0.0	122	15.4	0
7 inch maximum	1,331	123	14.9	0	0.0	123	14.9	0
				Over 7.0-inch fish				
5 inch minimum	129	11	2.3	0	0.0	11	2.3	0
7 inch maximum	135	0	0.0	11	2.9	11	2.9	2
				All fish				
5 inch minimum	9,680	133	17.7	160	7.2	293	24.9	32
7 inch maximum	9,689	123	14.9	171	10.2	294	25.1	34

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 22. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with constant recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a medium growing bluegill population in Michigan.

	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
Age	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	4,001	3.6	4,001	3.6	4,001	3.5	4,001	3.5	4,001	3.5	4,001	3.5
II	2,083	4.5	2,083	4.5	2,168	4.4	2,168	4.4	2,204	4.4	2,204	4.4
III	1,224	5.5	1,224	5.5	1,492	5.4	1,492	5.4	1,597	5.4	1,597	5.4
IV	551	6.5	552	6.5	876	6.4	876	6.4	1,044	6.4	1,044	6.4
V	168	7.0	183	7.0	349	6.9	357	6.9	464	6.9	469	6.9
VI	33	7.5	49	7.5	91	7.4	104	7.4	135	7.4	143	7.4
VII	4	7.9	8	7.9	13	7.8	17	7.8	21	7.8	24	7.8
VIII	0	...	1	8.8	1	8.7	2	8.7	2	8.7	2	8.7
Total	8,064		8,101		8,991		9,017		9,468		9,484	

Table 23. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	67	0	0.0	0	0.0	0	0.0	0
7 inch maximum	67	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	5,962	0	0.0	1,348	65.5	1,348	65.5	270
7 inch maximum	5,962	0	0.0	1,348	65.5	1,348	65.5	270
				5.0- to 7.0-inch fish				
5 inch minimum	1,886	846	111.9	0	0.0	846	111.9	0
7 inch maximum	1,895	850	111.1	0	0.0	850	111.1	0
				Over 7.0-inch fish				
5 inch minimum	149	58	13.6	0	0.0	58	13.6	0
7 inch maximum	177	0	0.0	57	15.0	57	15.0	11
				All fish				
5 inch minimum	8,064	904	125.5	1,348	65.5	2,252	191.0	270
7 inch maximum	8,101	850	111.1	1,405	80.5	2,255	191.6	281

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 24. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	136	0	0.0	0	0.0	0	0.0	0
7 inch maximum	136	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	6,132	0	0.0	683	32.7	683	32.7	137
7 inch maximum	6,132	0	0.0	683	32.8	683	32.8	137
				5.0- to 7.0-inch fish				
5 inch minimum	2,470	470	64.9	0	0.0	470	64.9	0
7 inch maximum	2,475	471	64.1	0	0.0	471	64.1	0
				Over 7.0-inch fish				
5 inch minimum	253	43	10.1	0	0.0	43	10.1	0
7 inch maximum	274	0	0.0	42	11.1	42	11.1	8
				All fish				
5 inch minimum	8,991	513	75.0	683	32.7	1,196	107.7	137
7 inch maximum	9,017	471	64.1	725	43.9	1,196	108.0	145

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 25. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a medium growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
1.0- to 2.9-inch fish								
5 inch minimum	136	0	0.0	0	0.0	0	0.0	0
7 inch maximum	136	0	0.0	0	0.0	0	0.0	0
3.0- to 4.9-inch fish								
5 inch minimum	6,179	0	0.0	342	16.5	342	16.5	69
7 inch maximum	6,179	0	0.0	342	16.5	342	16.5	69
5.0- to 7.0-inch fish								
5 inch minimum	2,801	252	35.5	0	0.0	252	35.5	0
7 inch maximum	2,805	252	34.9	0	0.0	252	34.9	0
Over 7.0-inch fish								
5 inch minimum	352	28	6.7	0	0.0	28	6.7	0
7 inch maximum	364	0	0.0	28	7.3	28	7.3	5
All fish								
5 inch minimum	9,468	280	42.2	342	16.5	622	58.7	69
7 inch maximum	9,484	252	34.9	370	23.8	622	58.7	74

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 26. Predicted equilibrium number and attained mean length ($\bar{l}$) by age group with constant recruitment for two size limit regulations and three annual conditional fishing mortality rates (m) for a fast growing bluegill population in Michigan.

	m = 0.40				m = 0.20				m = 0.10			
	5" minimum		7" maximum		5" minimum		7" maximum		5" minimum		7" maximum	
Age	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$	Number	$\bar{l}$
I	4,915	4.7	4,915	4.7	4,915	4.5	4,915	4.5	4,915	4.5	4,915	4.5
II	1,678	5.8	1,741	5.7	2,037	5.7	2,037	5.7	2,176	5.7	2,176	5.7
III	678	6.8	720	6.8	1,068	6.7	1,089	6.7	1,268	6.7	1,279	6.7
IV	249	7.6	337	7.5	512	7.5	573	7.5	678	7.5	714	7.5
V	59	8.2	115	8.2	158	8.1	206	8.1	234	8.1	264	8.1
VI	9	9.0	26	8.9	32	8.9	50	8.9	53	8.9	65	8.9
VII	1	9.4	3	9.4	3	9.3	6	9.3	6	9.3	8	9.3
VIII	0	...	0	...	0	...	0	...	0	...	0	...
Total	7,589		7,857		8,725		8,876		9,330		9,421	

Table 27. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.40 for a fast growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
1.0- to 2.9-inch fish								
5 inch minimum	12	0	0.0	0	0.0	0	0.0	0
7 inch maximum	12	0	0.0	0	0.0	0	0.0	0
3.0- to 4.9-inch fish								
5 inch minimum	3,557	0	0.0	508	32.5	508	32.5	101
7 inch maximum	3,610	0	0.0	514	35.9	514	35.9	103
5.0- to 7.0-inch fish								
5 inch minimum	3,431	1,286	196.3	0	0.0	1,286	196.3	0
7 inch maximum	3,511	1,332	190.8	0	0.0	1,332	190.8	0
Over 7.0-inch fish								
5 inch minimum	589	244	69.2	0	0.0	244	69.2	0
7 inch maximum	724	0	0.0	243	82.8	243	82.8	49
All fish								
5 inch minimum	7,589	1,530	265.5	508	32.5	2,038	298.0	101
7 inch maximum	7,857	1,332	190.8	757	118.7	2,089	309.5	152

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 28. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.20 for a fast growing bluegill population in Michigan.

Fishing regulation	Simulated fishery statistics*							
	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	23	0	0.0	0	0.0	0	0.0	0
7 inch maximum	23	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	4,213	0	0.0	320	20.0	320	20.0	64
7 inch maximum	4,213	0	0.0	320	21.6	320	21.6	64
				5.0- to 7.0-inch fish				
5 inch minimum	3,496	638	101.4	0	0.0	638	101.4	0
7 inch maximum	3,523	644	94.9	0	0.0	644	94.9	0
				Over 7.0-inch fish				
5 inch minimum	993	177	52.7	0	0.0	177	52.7	0
7 inch maximum	1,117	0	0.0	183	63.2	183	63.2	37
				All fish				
5 inch minimum	8,725	815	154.1	320	20.0	1,135	174.1	64
7 inch maximum	8,876	644	94.9	503	84.8	1,147	179.7	101

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

Table 29. Predicted equilibrium levels of density, catch, and yield by length group for two size limit regulations, assuming constant recruitment and an annual conditional fishing mortality rate (m) of 0.10 for a fast growing bluegill population in Michigan.

	Simulated fishery statistics*							
Fishing regulation	N	C	Y	J	YJ	C + J	Y + YJ	H
				1.0- to 2.9-inch fish				
5 inch minimum	23	0	0.0	0	0.0	0	0.0	0
7 inch maximum	23	0	0.0	0	0.0	0	0.0	0
				3.0- to 4.9-inch fish				
5 inch minimum	4,232	0	0.0	161	10.1	161	10.1	33
7 inch maximum	4,233	0	0.0	161	11.0	161	11.0	33
				5.0- to 7.0-inch fish				
5 inch minimum	3,781	329	53.3	0	0.0	329	53.3	0
7 inch maximum	3,796	330	49.2	0	0.0	330	49.2	0
				Over 7.0-inch fish				
5 inch minimum	1,294	108	33.2	0	0.0	108	33.2	0
7 inch maximum	1,369	0	0.0	111	38.5	111	38.5	22
				All fish				
5 inch minimum	9,330	437	86.5	161	10.1	598	96.6	33
7 inch maximum	9,421	330	49.2	272	49.5	602	98.7	55

*N = number of fish in the population; C = annual number of legal-size bluegills harvested; Y = annual yield in pounds of harvest; J = annual number of illegal-size bluegills caught and released; YJ = annual yield in pounds of illegal-size bluegills caught and released; H = annual number of illegal-size bluegills dying from hooking mortality.

SUMMARY

The simulations using the model developed in this report demonstrated two important points. First, a 7.0 inch maximum size limit regulation can not improve and control bluegill populations. The best results were obtained for a population showing characteristics of fast growth, and which was subjected to heavy fishing mortality even after application of the special regulation. However, these results depicted only a nominal improvement of the population. Any gains realized by restricting the size of harvestable bluegills could be easily offset or even reversed by poaching or any natural disaster.

Further analysis reveals that a closed season on harvest in May, June, and some or all of July might be beneficial (along with the special size restriction) for two reasons. One, 55% of the annual fishing mortality occurs during these months. Large bluegills are very susceptible to angling at this time because they come into shallow water to spawn. Closing the fishing season during these months would protect the larger fish and possibly cut down on poaching. Second, 70% of the annual growth occurs during this three month period. A closed season would allow many more of the fish to recruit into the illegal size range (i.e., over 7.0 inches) and thus not be harvested. This would increase the number of large fish and possibly make the 7.0 inch special regulation much more effective as a management strategy for improving poor bluegill populations.

Hooking mortality was essentially insignificant using 0.20 as the probability of a hooking death for a fish caught and released. This implies that special restrictions on gear (e.g., artificial lures only) would not help improve the populations and are thus unnecessary if the probability of a hooking death is 0.20 or less. However, if the probability of a hooking death is much higher than 0.20, gear restrictions may be necessary to prevent large numbers of fish from dying after being caught and released.

The second point demonstrated by this model is the need for further research regarding bluegill spawning, survival of fry, and recruitment into the age-I group. It is necessary to determine the factors, dependent and/or independent, which influence these processes and allow the population to compensate for environmental and man-made stresses. The assumptions in the model of variable recruitment, one spawning period per year, a 4.8 inch minimum length of mature females, and a constant mortality of fry from the fall to the following spring were not sufficient to produce a bluegill population with realistic characteristics. Constant recruitment was then assumed, and although this did improve the population characteristics, it was still somewhat inadequate in producing the population dynamics associated with spawning and recruitment. A further density-dependent feedback mechanism may improve model performance. One hypothesis is cannibalism by large bluegills, illustrated in Figure 7. A variety of curves

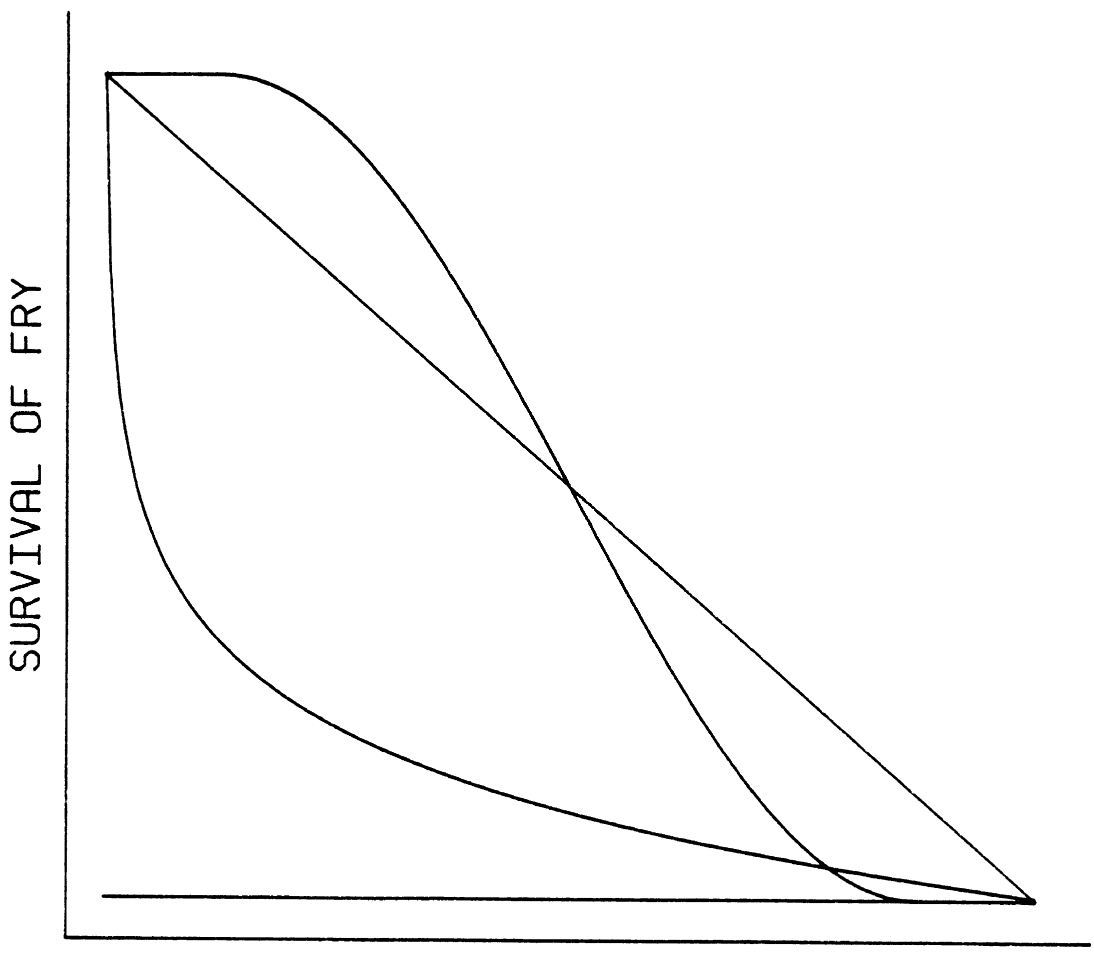


Figure 7. Possible hypothetical functions relating fry survival to the density of 7.0 inch and larger bluegills.

relating fry survival to the number of bluegills 7.0 inches and larger is depicted on this graph. The low horizontal line is the assumption used in this model along with the assumption of variable recruitment (i.e., high constant mortality regardless of the density of large fish). However, one of the other curves may be more realistic, the use of which might possibly enhance model performance depending on the sensitivity of the relationship.

The model presented in this paper is very useful as a management tool. Its general applicability makes possible the simulation of a wide variety of size limit regulations, gear and season restrictions, and studies of seasonal growth and natural mortality. It may be applied to any inland warmwater fishery if the necessary data are available for accurate simulation. Application of this model to Michigan bluegill populations showed that more research is still needed before a successful solution for managing bluegill populations can be determined. It is much easier to find a cure if the symptoms are known rather than to use over and over a trial and error process.

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