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A STUDY OF TROPOSPHERIC SCATTER PROPAGATION

by

C. M. Chu, A. W. Wren, J. J. LaRue
E. K. Miller and D. T. Politis

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Keeve M. Siegel

Keeve M. Siegel
Project Director

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TABLE OF CONTENTS

TABLE OF CONTENTS

(continued)

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

a	One-half of the length of the chord joining the transmitter and receiver
A_R	Aperture of receiving antenna
A_T	Aperture of transmitting antenna
$A(t)$	A function of time varying slowly with respect to the carrier frequency
b	Horizontal dimension of layer in a direction parallel to chord joining transmitter and receiver (see Fig. II-4)
$c(\underline{r})$	The space correlation function of the velocity field
c	Horizontal dimension of layer measured in direction perpendicular to chord joining transmitter and receiver (Fig. II-4)
D	Distance between transmitter and receiver along the earth's surface (Fig. II-1)
dv, dv_i, dv_j	Differential element of volume
$d\underline{a}$	Differential element of surface area
$D(\hat{n}, \hat{n}')$	Normalized distribution function of scattered power
$\mathcal{E}_o$	Dielectric constant of free space
$\mathcal{E}_m(\underline{r})$	Mean value of the dielectric constant in excess of free space value
$\underline{E}$	Total electric field intensity vector
$\underline{E}_o$	Electric field intensity vector of incident plane wave
$\underline{E}_s$	Electric field intensity vector of scattered field
$\hat{e}$	A unit vector defining the direction of the electric field intensity $\underline{E}_o$ in the incident plane wave
$E(k)$	The spectrum of energy of the turbulent medium

$\mathcal{E}$	The rate of energy input to the atmosphere
$e_1(t)$	A complex variable denoting a system input function
$e_2(t)$	A complex variable denoting a system output function
$E_1(\omega)$	Fourier transform of the system input function $e_1(t)$
F	A symbol used for convenience of notation (eq. (3.13))
$f_1(t)$	The slowly varying (with respect to carrier) amplitude of an amplitude modulated signal
$f_2(t)$	The detected output of an amplitude modulated signal
$F(\hat{n})$	Power pattern of an antenna
G_T	Gain of transmitting antenna
$g(k'^{2/3}, \tau)$	A universal function defined by Silverman (Ref. 5)
$\underline{H}$	Total magnetic field intensity vector
$\underline{H}_o$	Magnetic field intensity vector of incident plane wave
$\underline{H}_s$	Magnetic field intensity vector of scattered field
$H(t, \omega)$	System function (eq. (3.1))
k	Propagation constant $\frac{2\pi}{\lambda}$
k'	A symbol used for convenience of notation (eq. (3.10))
$\underline{k}$	A vector defined by the velocity $u(x)$ at the point $\underline{x}$ and the velocity $u(\underline{x}+\underline{r})$ at the point $\underline{x}+\underline{r}$
K	The magnitude of the change of the gradient of the dielectric constant at a discontinuity in the gradient
k_o	The wave number separating the inertial range from the input range
k	The wave number $\frac{\pi}{L}$ associated with eddies of characteristic length L.

k_s	The wave number separating the inertial range from the dissipation range
L	The characteristic length of an eddy or the scale of oscillations at the wave number k
L_s	The characteristic length of an eddy or scale of oscillations at the wave number k_s
L_o	The characteristic length of an eddy or scale of oscillations at the wave number k_o
$L(\tau, \omega, \hat{n}_2, \hat{n}_1)$	The space time correlation function of the complex amplitude of the electric field
L_1	That part of $L(\tau, \omega, \hat{n}_2, \hat{n}_1)$ due to space inhomogeneity
L_2	That part of $L(\tau, \omega, \hat{n}_2, \hat{n}_1)$ due to time fluctuation
$\hat{n}, \hat{n}_1$	Unit vectors from the transmitter to the point P in space
$\hat{n}', \hat{n}_2$	Unit vectors from the point P in space to the receiver
N_i	The number of discontinuities of the i-th kind per unit volume in the gradient of the mean value of the dielectric constant
N_α	The expected number of times a random function of time will cross a line of magnitude α per unit time
O	The origin of a coordinate system which is within the intersection of the transmitting and receiving antenna beams
P_R	Mean power received
P_{FS}	Mean power that would be received over a line-of-sight path of length D
P_T	Mean power transmitted
$p(\omega, \hat{n}', t)$	The magnitude of the Poynting vector
$p(\underline{u})$	Probability density function of the velocity $\underline{u}$

$p(\omega, \hat{n}, \underline{r}, t)$	Energy in the frequency range ω to $\omega + d\omega$ per unit time intercepted by a unit area from a unit solid angle in the direction $\hat{n}$
$q(\omega, \hat{n}, \underline{r}, t)$	Source term in the transport equation (eq. (4.2))
q	A symbol used for convenience of notation in layer theory (eq. (2.42))
R_1	The distance from the transmitter to the origin O of a coordinate system located in the common volume of the transmitting and receiving antenna beams.
R_2	The distance from the receiver to the origin O of a coordinate system located in the common volume of the transmitting and receiving antenna beams.
$R(\underline{r}_i - \underline{r}_j)$	Normalized space covariance of $\Delta \mathcal{E}$
$\underline{r}_o, \underline{r}_i, \underline{r}_j$	Position vectors with origin at O
$\underline{r}$	A position vector with origin at the point $\underline{x}$
$R(\underline{r}_i - \underline{r}_j, \tau)$	The normalized space and time correlation function of $\Delta \mathcal{E}$
r	Distance to any point in space from dv_j
$S_1(\omega)$	The spectral density of the input $e_1(t)$
$S_2(\omega)$	The spectral density of the output $e_2(t)$
$S(k)$	The spectral density of $R(\rho)$
$S(\underline{k}, \tau)$	A spectral density in the variable $\underline{k}$ and a correlation function in the variable τ of the correlation function $R(\underline{r}_i - \underline{r}_j, \tau)$ or the spatial semi-transform of $R(\underline{r}_i - \underline{r}_j, \tau)$
$\underline{u}(x)$	The velocity of the atmosphere at the point $\underline{x}$
$\underline{u}_j$	The velocity of the atmosphere in a differential volume dv_j
$\underline{u}_s$	The value of the velocity $\underline{u}$ in a small volume V_s
V_s	A volume small enough such that the velocity of the atmosphere within the volume may be considered a constant

$\underline{x}$	A position vector in space
$\Gamma(\omega,\omega',\hat{n}_2,\hat{n}_1)$	The generalized cross section per unit volume or the spectral density transport function. Physically, the spectral density in the angular frequency range ω to $\omega+d\omega$ scattered into a unit solid angle in the direction $\hat{n}_2$ by a unit volume of scattering medium from a plane wave propagating in the direction $\hat{n}_1$
θ	The angle between the unit vectors $\hat{n}_1$ and $\hat{n}_2$
$\Delta\mathcal{E}(r,t)$	The variation of the dielectric constant about its mean value, a random function of position and time
$\delta(\underline{r})$	Dirac delta function
Δ	The angle between the chord joining the transmitter and receiver and the lower edge of antenna beam
ν	Kinematic viscosity of the atmosphere
ρ	The magnitude of $\underline{r}_i-\underline{r}_j$
$\rho(\tau)$	The correlation function of the complex amplitude of the received signal
$\sigma(\hat{n}_2,\hat{n}_1)$	Power scattered by a unit volume of atmosphere from a plane wave of unit intensity in the direction $\hat{n}_1$, into a unit solid angle in the direction $\hat{n}_2$
σ_u	The variance of the magnitude of the velocity $\underline{u}$
$\psi_a(\tau)$	Correlation function of the complex amplitude of the received signal
$\psi_2(\tau)$	Correlation function of the output $e_2(t)$
$\psi_H(\tau,\omega)$	Correlation function of the system function $H(t,\omega)$

ABSTRACT

Various theories proposed to explain the mean received power from a tropospheric scatter circuit are reviewed. A derivation is given which shows that both fluctuation scattering and inhomogeneity scattering can contribute to the received signal.

The concept of scattering cross section is extended by the introduction of a transfer function of the spectral density and it is proposed that this be used to specify the scattering properties of the medium. The relation between the fading rate and the transform of the transfer function is given.

The theory of multiple scattering is outlined and some well known results are given.

A summary of some experimental results appearing in the literature is given and a statistical analysis of these data is carried out to determine a frequency and distance dependence for the received power.

I. Introduction

This report contains the results of a basic study of tropospheric propagation. In general, the theories which have been proposed in the past deal separately with the mean received power and the statistics of the received signal. In addition, these separate theories of the mean received power and the statistics of the received signal often ignore either the fluctuation of the dielectric constant about the mean or space inhomogeneities in the mean dielectric constant. Our results suggest that at least in principle a completely general formalization of scatter propagation can be made.

Section II of the report discusses various theories that have been proposed to explain the mean received power. These theories are classified by types (i.e., fluctuation scattering and inhomogeneity scattering). A derivation of the cross section is given which shows that both types of scattering can contribute to the received signal.

While the characterization of the atmosphere by a scattering cross section is useful in obtaining the mean received power, this method is not at all adequate for predicting the distortion or fluctuation of the received signal. In Section III, the concept of a scattering cross section is extended by the introduction of the transfer function of the spectral density $\Gamma(\omega, \omega', \hat{n}_2, \hat{n}_1)$ and its transform $L(\tau, \omega, \hat{n}_2, \hat{n}_1)$. It is proposed that the transfer function of the spectral density $\Gamma(\omega, \omega', \hat{n}_2, \hat{n}_1)$ should be used to specify the scattering properties of the medium. In Section 3.2, the relation between the fading rate and the transform of the transfer function of the spectral density is given. It is shown that for the restricted case of fluctuation scattering, the fading rate reduces to the relation of Booker and Gordon. The distortion of the received signal is discussed in Section 3.3 for the single scattering case.

The theory of multiple scattering is outlined in Section IV and some generally known results are given.

Some of the experimental data which has been published in recent years has been collected and tabulated in Appendix B. To have some basis for comparison between theory and experiment, a statistical analysis of the tabulated data is carried out in Appendix C to determine the distance and frequency dependence of the mean received power and the signal fading rate.

II. The Mean Received Power

2.1 Classification by Scattering Mechanism

The problem of tropospheric propagation beyond the line of sight is a controversial subject which has received considerable attention in recent years. In general, the problem involves the propagation of electromagnetic waves in a heterogeneous medium whose properties are described statistically. If we assume that, within a certain interval of time, the medium is stationary (in the statistical sense), the characteristics of the medium can be expressed approximately by the mean dielectric constant and the mean square deviation of the dielectric constant from the mean. Both parameters should be functions of the space variables.

Since the signal levels encountered in beyond-the-horizon tropospheric propagation cannot be accounted for by diffraction theory (with earth curvature correction), one is naturally looking for some mechanism that would scatter energy out of the primary beam. The present theories may be classified according to the mechanism used in accounting for the energy scattered out of the primary beam. With this type of classification the theories may be divided between the following categories:

2.1 a Fluctuation Scattering

Theories in this category attribute the signal received beyond the horizon to the fluctuation of the dielectric constant $\Delta\mathcal{E}$. Although this type of fluctuation has been studied by Einstein (Ref. 1) in connection with light scattering, the application of the principle to tropospheric propagation was first introduced by Booker and Gordon (Ref. 2). The works of Staras (Ref. 3), Wheelon (Ref. 4), Silverman (Ref. 5), Villars and Weisskopf (Ref. 6), etc. essentially employ the same principle, except different (but more accurate) models of the fluctuation are assumed.

2.1 b Scattering Due to Space Inhomogeneity

These theories assert that the space variation of the mean dielectric constant is responsible for the beyond-the-line-of-sight propagation. The works of Carroll and Ring (Ref. 7) investigate the effects of certain types of inhomogeneity, but the results seem to be too complicated to be of general use. Friis, Crawford and Hogg (Ref. 8), in noticing the discontinuities in the slope of the variation of dielectric constant with height, suggested the layer theory, by treating the discontinuities in the gradient of dielectric constant as distributed reflecting layers. Their model seems to be supported by some recent experimental results (Ref. 9).

2.2 The Scattering Cross Section

2.2 a Scattering Cross Section for a Heterogeneous Medium

In general, insofar as the median signal strength is concerned, the macroscopic properties of the atmosphere can be characterized by the scattering cross section, $\sigma(\hat{n}_2, \hat{n}_1)$, defined as the power scattered by a unit volume of atmosphere from a plane wave of unit intensity in the direction $\hat{n}_1$, into a unit solid angle in the direction $\hat{n}_2$ (Ref. 2). In terms of this parameter, the mean received power can be expressed simply as the integration over the common volume between the transmitter and receiver beam patterns. Referring to Figure II-1, it can easily be shown that*

$$\frac{P_R}{P_{FS}} = D^2 \iiint_{\substack{\text{common} \\ \text{volume}}} dv \, \frac{\sigma(\hat{n}_2, \hat{n}_1)}{R_1^2 R_2^2} \tag{2.1}$$

where P_R is the mean power received beyond the line of sight and P_{FS} is the power that would be received in free space over a line-of-sight path of length D.

Since most of the experimental data can be expressed in the form P_R/P_{FS}, the existing theories essentially concern themselves with the variation of this quantity with distance and frequency. The distance dependence in general can be made anything one pleases by assuming different space variations of σ**. The frequency variation, however, is fixed in the form of σ and may serve as a criterion for determining the validity of the theories. The dependence of the scattering cross section on the fluctuation of the dielectric constant has been derived from Maxwell's field equations by various authors (Refs. 3, 11, 12). In

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* Eq. (2.1) assumes only single scattering; see Section IV for the multiple scattering formulation.

** See, for instance, References 8 and 10.

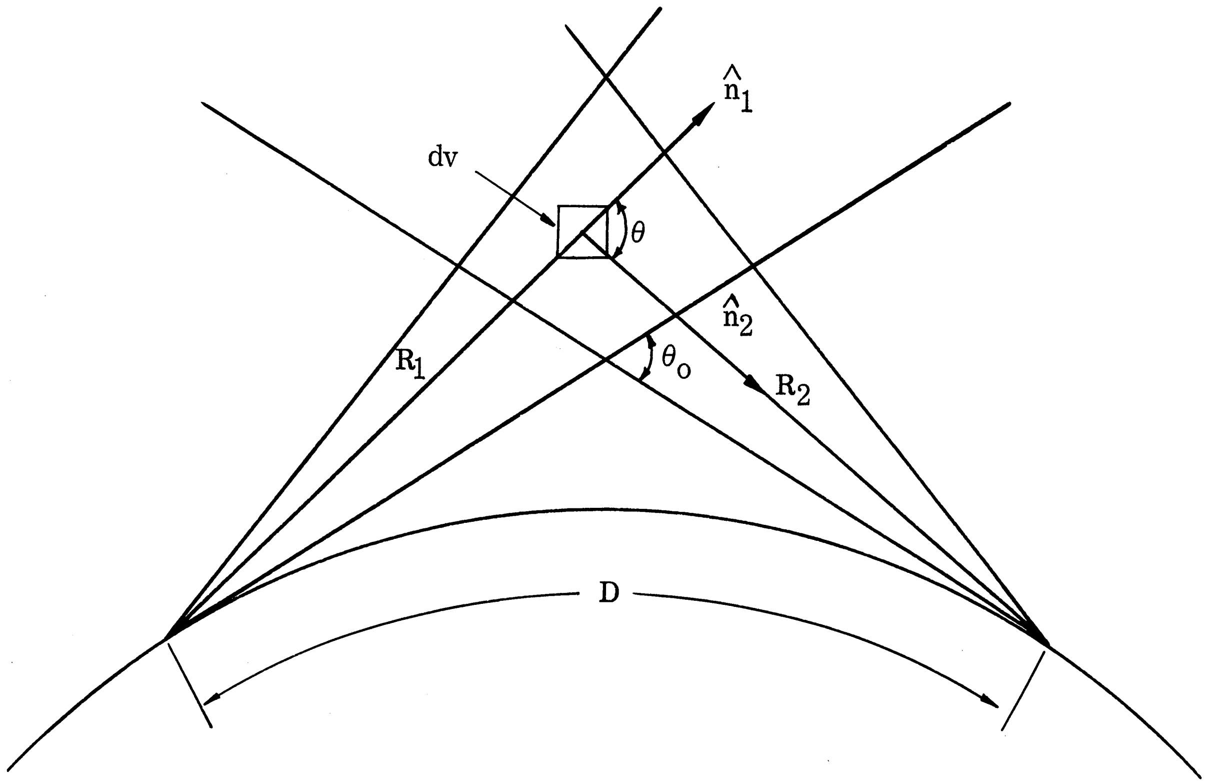

Figure II-1: Geometry for Scattering Cross Section

all these derivations, the deviation of dielectric constant from the free-space value $\mathcal{E}_o$ is assumed to be locally stationary in space. It has been pointed out by Silverman (Ref. 5) that in practice, only the deviation from the mean value of the dielectric constant can be considered as locally stationary in space. It is really the space variation of the mean value of the dielectric constant that prompts the suggestion of the Bell Telephone Laboratories layer theory. We shall therefore start from Maxwell's equations and formally show the two types of scattering mechanism.

Maxwell's equation for a monochromatic wave of angular frequency ω may be written as*

$$\begin{aligned} \nabla \cdot \mathcal{E} \underline{E} &= 0 \\ \nabla \times \underline{E} &= j\omega\mu_o \underline{H} \\ \nabla \cdot \underline{H} &= 0 \\ \nabla \times \underline{H} &= -j\omega \mathcal{E} \underline{E} \ . \end{aligned} \tag{2.2}$$

For an atmosphere whose dielectric constant is locally stationary in time (an assumption which is good for an interval of a few minutes), the dielectric constant may be written as**:

$$\mathcal{E} = \mathcal{E}_o + \mathcal{E}_m(\underline{r}) + \Delta\mathcal{E}(\underline{r}, t)$$

where $\mathcal{E}_m(\underline{r})$ is the mean value of dielectric constant minus the free-space value while $\Delta\mathcal{E}(\underline{r}, t)$ is the variation from the mean.

The statistics of $\Delta\mathcal{E}(\underline{r}, t)$ may be stated as follows:

a) $$\langle \Delta\mathcal{E}(\underline{r}, t) \rangle = 0 \tag{2.3}$$

b) $$\langle \Delta\mathcal{E}(\underline{r}_i, t)\, \Delta\mathcal{E}(\underline{r}_j, t) \rangle = \langle (\Delta\mathcal{E})^2 \rangle \; R(\underline{r}_i - \underline{r}_j) \ , \tag{2.4}$$

where $\langle (\Delta\mathcal{E})^2 \rangle$ is the mean square fluctuation while $R(\underline{r}_i - \underline{r}_j)$ is the normalized space covariance of $\Delta\mathcal{E}$.

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* Two assumptions were made in writing down eq. (2.2): 1) the value of $\mathcal{E}$ is assumed to be changing slowly with respect to the angular frequency ω and 2) the Doppler shift due to the motion of air is neglected.

** Note that $\Delta\mathcal{E}(\underline{r}, t)$ is the deviation of the dielectric constant from the mean, not the incremental dielectric constant from the free-space value.

c) The covariance may sometimes be transformed into a spectrum function defined by:

$$S(\underline{k}) = \iiint R(\underline{\rho})\, e^{i\underline{k}\cdot\underline{\rho}}\, dv \quad . \tag{2.5}$$

To study the effect of the atmosphere on the propagation of waves, let us consider how an incident plane wave,

$$\underline{E}_o = \hat{e}\, e^{ik\hat{n}_1\cdot\underline{r}} \tag{2.6}$$

is perturbed by passing through the medium (see Fig. II-2).

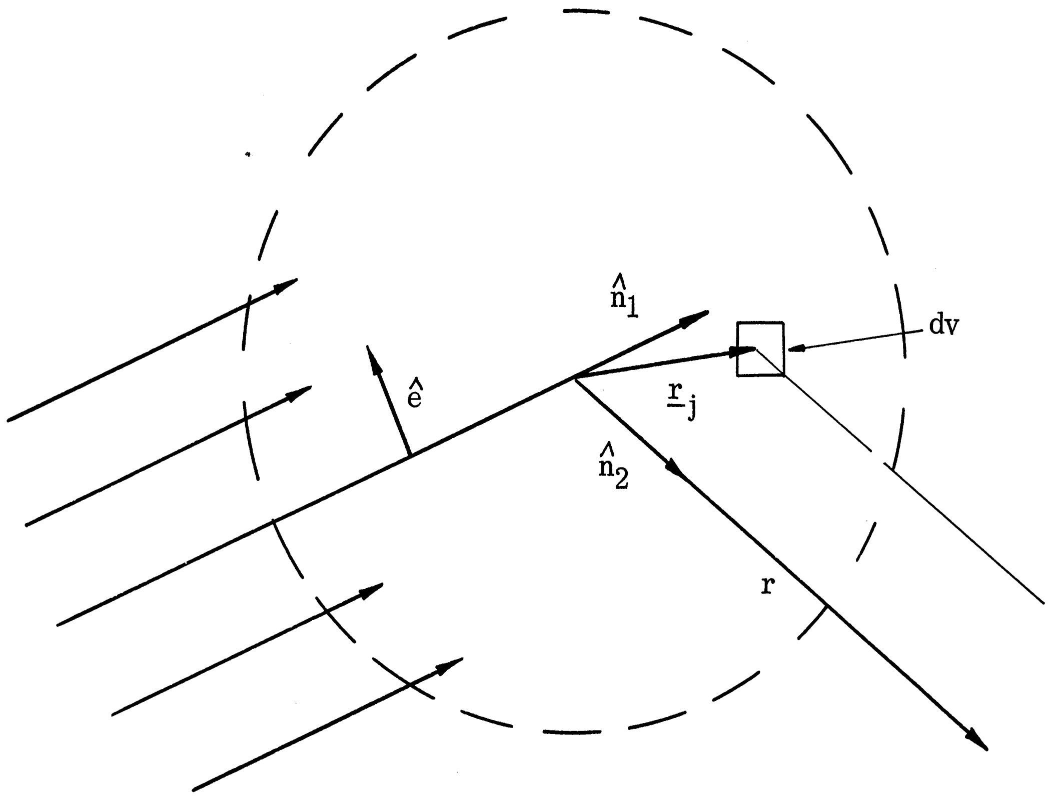

Figure II-2: Scattering of a Plane Wave

If one writes the solution of (2.2) in the form

$$\underline{E} = \underline{E}_o + \underline{E}_s$$

$$\underline{H} = \underline{H}_o + \underline{H}_s \quad ,$$

the equations for the perturbed or scattered field are*:

$$\mathcal{E}_o \nabla \cdot \underline{E}_s + \underline{E}_o \cdot \nabla \left[\mathcal{E}_m(\underline{r}) + \Delta \mathcal{E}(\underline{r}, t) \right] = 0$$

$$\nabla \cdot \underline{H}_s = 0$$

$$\nabla \times \underline{E}_s = j\omega\mu_o \underline{H}_s \qquad (2.7)$$

$$\nabla \times \underline{H}_s = -j\omega \, \mathcal{E}_o \underline{E}_s - j\omega \underline{E}_o \left(\mathcal{E}_m(\underline{r}) + \Delta \mathcal{E}(\underline{r}, t) \right) .$$

Elimination of $\underline{H}$ from equation (2.7) yields the equation for $\underline{E}_s$:

$$\nabla \times \nabla \times \underline{E}_s - k^2 \underline{E}_s = \frac{k^2 \underline{E}_o}{\mathcal{E}_o} \left(\mathcal{E}_m(\underline{r}) + \Delta \mathcal{E}(\underline{r}, t) \right) . \qquad (2.8)$$

The solution of eq. (2.8) subjected to the condition (2.7) is well known (Ref. 11). Referring to Figure II-2, for a volume element dv_j located at $\underline{r}_j$, the far-zone scattered field in the direction $\hat{n}_2$ is given by:

$$d\underline{E}_s = \frac{\hat{n}_2 \times (\hat{n}_2 \times \hat{e}) k^2}{4\pi r \mathcal{E}_o} \, e^{ik(\hat{n}_1 - \hat{n}_2) \cdot \underline{r}_j} \left[\mathcal{E}_m(\underline{r}_j) + \Delta \mathcal{E}(\underline{r}_j, t) \right] e^{ikr} \, dv_j .$$

The total scattered field is therefore,

$$\underline{E}_s = e^{ikr} \frac{\hat{n}_2 \times (\hat{n}_2 \times \hat{e}) k^2}{4\pi r \mathcal{E}_o} \int_{\text{volume}} e^{ik(\hat{n}_1 - \hat{n}_2) \cdot \underline{r}_j} \left[\mathcal{E}_m(\underline{r}_j) + \Delta \mathcal{E}(\underline{r}_j, t) \right] dv_j . \qquad (2.9)$$

The scattered power in any direction $\hat{n}_2$, per unit incident power is therefore,

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*The higher order terms $\left[(\mathcal{E}_m + \Delta \mathcal{E}) \underline{E}_s \right]$ are neglected in equations (2.7).

$$\left|\underline{E}_s\right|^2 = \frac{k^4}{(4\pi r)^2\, \mathcal{E}_o^2} \left| \hat{n}_2 \times (\hat{n}_2 \times \hat{e}) \right|^2$$

$$\int dv_i \int dv_j \; e^{ik(\hat{n}_1 - \hat{n}_2)\cdot \underline{r}_j} \; e^{-ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}_i} \qquad (2.10)$$

$$\left(\mathcal{E}_m(\underline{r}_j) + \Delta\, \mathcal{E}(\underline{r}_j, t)\right) \left(\mathcal{E}_m(\underline{r}_i) + \Delta\, \mathcal{E}(\underline{r}_i, t)\right) \; .$$

The expected (or average) power scattered in the direction $\hat{n}_2$, per unit incident power, is therefore

$$\left\langle \left|\underline{E}_s\right|^2 \right\rangle = \frac{k^4}{(4\pi r)^2\, \mathcal{E}_o^2} \left| \hat{n}_2 \times (\hat{n}_2 \times \hat{e}) \right|^2 \qquad (2.11)$$

$$\left\{ \int dv_i \int dv_j \; e^{ik(\hat{n}_1-\hat{n}_2)\cdot\left[\underline{r}_j - \underline{r}_i\right]} \left\langle \Delta \mathcal{E}(\underline{r}_i, t)\, \Delta \mathcal{E}(\underline{r}_j, t) \right\rangle \right.$$

$$\left. + \left| \int dv_i \; e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}_i} \; \mathcal{E}_m(\underline{r}_i) \right|^2 \right\} .$$

It is evident from eq. (2.11) that the scattered field can be considered as arising from two sources: 1) the fluctuation of $\Delta\mathcal{E}$ and 2) the variation of mean value of the dielectric constant. A discussion of the two types of scattering processes has been given by A.J. Siegert and H. Goldstein (Ref. 14).

2.2 b Scattering Cross Section Due to Dielectric Fluctuations

The derivation of the scattering cross section due to dielectric fluctuation is obtained from the first term of eq. (2.11). If one takes a small volume within which $\Delta\mathcal{E}$ is stationary both in space and time, then by eq. (2.4) and eq. (2.5), one has:

$$\int dv_i \int dv_j \; e^{ik(\hat{n}_1-\hat{n}_2)\cdot(\underline{r}_j-\underline{r}_i)} \left\langle \left(\Delta\mathcal{E}(\underline{r}_i)\right)^2 \right\rangle R(\underline{r}_i-\underline{r}_j)$$

$$= \int dv_i \left\langle \left(\Delta\mathcal{E}(\underline{r}_i)\right)^2 \right\rangle S\left[k(\hat{n}_1-\hat{n}_2)\right] \quad .$$

The scattering cross section due to dielectric fluctuation is therefore

$$\sigma(\hat{n}_2, \hat{n}_1) = \frac{k^4}{16\pi^2} \left\langle \left(\frac{\Delta\mathcal{E}(\underline{r}_i)}{\mathcal{E}_o}\right)^2 \right\rangle S\left[k(\hat{n}_1-\hat{n}_2)\right] \left|\hat{n}_2 \times (\hat{n}_2 \times \hat{e})\right|^2 . \quad (2.12)$$

2.2 c Scattering Cross Section Due to Discontinuities in the Gradient of the Dielectric Constant

The second term of eq. (2.11) is actually the basis of the layer theory. It is evident that if $\mathcal{E}_m(\underline{r})$ is constant, no power will be scattered away from the main beam. If the rate of change of $\mathcal{E}_m(\underline{r})$ or grad $\mathcal{E}_m(\underline{r})$ is uniform, then the average direction of the beam can be obtained by modifying the radius of curvature of the earth. It is actually the sudden changes of grad $\mathcal{E}_m(\underline{r})$ that cause the scattering of power away from the main beam. These sudden changes in grad $\mathcal{E}_m(\underline{r})$, incidentally, are not predicted by homogeneous turbulence theory.

In general, the discontinuities in grad $\mathcal{E}_m(\underline{r})$ are in the vertical direction and have been shown to behave like reflecting layers (Ref. 8). To show formally the effect of such discontinuities, let us carry out a partial integration of the second term of eq. (2.11) as follows:

$$\int dv\, e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}}\, \mathcal{E}_m(\underline{r}) = -\frac{1}{k^2|\hat{n}_1-\hat{n}_2|^2}\int dv\, \mathcal{E}_m(\underline{r})\nabla^2\left(e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}}\right)$$

$$= -\frac{1}{k^2|\hat{n}_1-\hat{n}_2|^2}\left[\int dv\, e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}}\,\nabla^2 \mathcal{E}_m(\underline{r}) \right. \tag{2.13}$$

$$\left. + \oint\!\!\oint e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}}\,\nabla\mathcal{E}_m(\underline{r})\cdot \underline{da} + \oint\!\!\oint \mathcal{E}_m(\underline{r})\,\nabla e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}}\cdot \underline{da}\right].$$

Now, if $\nabla\mathcal{E}_m(\underline{r})$ has some discontinuity at $\underline{r}_o$, then $\nabla^2 \mathcal{E}_m(\underline{r})$ has a singular point at $\underline{r}_o$. One may consider the contribution of this singularity by enclosing a small volume around $\underline{r}_o$ and letting the volume shrink to zero. Thus, the surface integral vanishes, and one obtains the contribution at this singularity as:

$$-\frac{1}{k^2|\hat{n}_1-\hat{n}_2|^2}\, K\, e^{ik(\hat{n}_1-\hat{n}_2)\cdot \underline{r}_o} .$$

K represents the "amount" of discontinuity at $\underline{r}_o$. Hence the contributions of such singularities to the scattered power is,

$$\left|\underline{E}_s\right|^2 = \frac{1}{|\hat{n}_1-\hat{n}_2|^4}\,\frac{1}{(4\pi)^2\mathcal{E}_o^2 r^2}\, K^2\, \left|\hat{n}_2 x(\hat{n}_2 x\hat{e})\right|^2 . \tag{2.14}$$

If the discontinuities are randomly distributed in space, then the contribution due to such discontinuities is:

$$\sigma(\hat{n}_2, \hat{n}_1) = \frac{1}{\left|\hat{n}_1 - \hat{n}_2\right|^4} \frac{1}{(4\pi \mathcal{E}_o)^2} \sum_i K_i^2 N_i \left[\hat{n}_2 \times (\hat{n}_2 \times \hat{e})\right]^2, \quad (2.15)$$

where N_i is the number of discontinuities of i^{th} kind per unit volume. Equation (2.15) is only used to show the contribution of space heterogeneity to the mean value of dielectric constant. Actually, the drastic changes of $\mathcal{E}_m$ are usually in the vertical direction, and the singularities are one dimensional. A better approximate treatment is probably the one presented by Schelkunoff (Ref. 14).

It is to be noted that the expression for $\sigma(\hat{n}_2, \hat{n}_1)$ usually involves the factor

$$\left|\hat{n}_1 - \hat{n}_2\right| = 2 \sin \frac{\theta}{2} \quad (2.16)$$

where θ, the angle between $\hat{n}_1$ and $\hat{n}_2$, can be used as a measure of the distance between the transmitter and the receiver (see Fig. II-1).

2.3 Various Methods for Calculating the Scattering Cross Section

2.3 a Classification of the Methods

The various methods used for calculating the scattering cross section may be separated into three categories; (1) Correlation Function Models, (2) Turbulence Models, and (3) Space Models. In the first method one assumes some correlation function for $\Delta\mathcal{E}$ and calculates σ by use of equation (2.12).

The second method applies dimensional analysis to turbulence theory to obtain the spectrum of the velocity field. Then from the spectrum of the velocity field, a reasonable spectrum for the scalar dielectric fluctuations may be deduced. The space model method refers mainly to the space variation of the mean dielectric constant. By postulating some representative forms for the variation of the dielectric constant in space, the scattered (or reflected) power can be computed.

2.3 b Correlation Function Models

As indicated above, this method is very straightforward in that the use of equation (2.12) will produce a cross section when an appropriate correlation function is used. Different authors have proposed various forms for the correlation function of the incremental dielectric constant $\Delta\mathcal{E}$. To illustrate the various forms which may be produced we summarize a few of the functions which have been proposed in Table II-1. Since we are interested in the near-forward direction scattering, the effect of polarization, or the term $|\hat{n}_2 \times \hat{n}_2 \times \hat{e})|^2$ is omitted in Table II-1.

2.3 c Turbulence Models

All turbulence models assume that the fluctuation of the dielectric constant is caused by the turbulent motion of the atmosphere. This turbulent motion of the atmosphere is further assumed to be locally homogeneous and isotropic.

Ref. No.	Correlation Function	Scattering Cross Section $\sigma(\hat{n}_1 - \hat{n}_2)$
2, 3	$R(\rho) = e^{-\frac{\rho}{L_o}}$ (Exponential Correlation)	$\frac{1}{2\pi}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4 L_o^3}{\left[1+(2kL_o \sin\frac{\theta}{2})^2\right]^2}$
3	$R(\rho) = e^{-\frac{\rho^2}{L_o^2}}$ (Gaussian Correlation)	$\frac{1}{16\sqrt{\pi}}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle k^4 L_o^3\, e^{-k^2 \sin^2\frac{\theta}{2} L_o^2}$
15	$R(\rho) = \frac{\rho}{L_o} K_1\left(\frac{\rho}{L_o}\right)$ (Bessel Correlation)	$\frac{3}{8}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4 L_o^3}{\left[1+(2kL_o \sin\frac{\theta}{2})^2\right]^{5/2}}$
4	$R(\rho) = (1 - \frac{L_s}{L_o})^{-1}\left[e^{-\frac{\rho}{L_o}} - \frac{L_s}{L_o} e^{-\frac{\rho}{L_s}}\right]$ (Modified Exponential Correlation)	$\frac{1}{2\pi}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4}{1-\frac{L_s}{L_o}}\left[\frac{L_o^3}{\left[1+(2kL_o \sin\frac{\theta}{2})^2\right]^2} - \frac{L_s^4/L_o}{\left[1+(2kL_s \sin\frac{\theta}{2})^2\right]^2}\right]$
3	$R(\rho) = \frac{1}{\left[1+\frac{\rho^2}{L_o^2}\right]^2}$ (Cauchy Correlation	$\frac{1}{16}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle k^4 L_o^3\, e^{-2k L_o \sin\theta/2}$
3	$R(\rho) = \left[1 - \frac{\rho}{L_o}\right]^2, \ \rho \le L_o$; $= 0, \ \rho > L_o$	$\frac{1}{64\pi L_o \sin^4\frac{\theta}{2}}\left\langle\left(\frac{\Delta\mathcal{E}(r)}{\mathcal{E}_o}\right)^2\right\rangle\left[4 - \frac{3\sin(2kL_o \sin\frac{\theta}{2})}{k L_o \sin\frac{\theta}{2}} + 2\cos(2kL_o \sin\frac{\theta}{2})\right]$

TABLE II-1: SUMMARY OF CORRELATION FUNCTIONS AND CORRESPONDING CROSS SECTIONS

The spectrum of the velocity field for homogeneous, isotropic turbulence has been discussed in detail by Batchelor (Ref. 16). If one assumes a system which is locally stationary and homogeneous, the correlation between the velocity at two points ($\underline{x}$) and ($\underline{x} + \underline{r}$) can be defined as

$$\left\langle \underline{u}(\underline{x}) \cdot \underline{u}(\underline{x}+\underline{r}) \right\rangle = \left\langle u^2 \right\rangle \quad C(\underline{r}) \quad . \tag{2.17}$$

One immediately notes that, except for the factor ρ (density), $\frac{1}{2} \left\langle u^2 \right\rangle C(o)$ is the energy contained in the turbulence.

For the isotropic case, of course

$$C(\underline{r}) = C(r) \quad . \tag{2.18}$$

The Fourier transform of C(r) is found to be:

$$S(k) = \int_0^{+\infty} C(r)\, e^{i\underline{k} \cdot \underline{r}}\, d\underline{r} = \frac{4\pi}{k^2} \int_0^{\infty} kr\, C(r) \sin kr\, dr \quad . \tag{2.19}$$

One usually defines the spectrum of energy as:

$$E(k) = \frac{k^2}{4\pi^2} \left\langle u^2 \right\rangle S(k) = \frac{1}{\pi} \left\langle u^2 \right\rangle \int_0^{\infty} kr\, C(r) \sin kr\, dr \quad . \tag{2.20}$$

Inversion of (2.20) yields

$$\left\langle u^2 \right\rangle \quad C(r) = 2 \int_0^{\infty} E(k) \, \frac{\sin kr}{kr} \, dk \quad . \tag{2.21}$$

Hence,

$$\int_0^{\infty} E(k)\, dk = \frac{1}{2} \left\langle u^2 \right\rangle \quad . \tag{2.22}$$

Equation (2.22) may be interpreted as follows. The variation of $\underline{u}$ may be considered as an infinite number of oscillations in space with different wave numbers k, and E(k) dk is the contribution to the energy from the oscillations with wave numbers between k and k+dk. The length $L = \pi/k$ may be considered as the "length" or scale of oscillations at wave number k. The oscillations at large wave numbers (i.e., small wavelength) are damped by viscosity, while the oscillations at small wave numbers experience negligible damping. However, the energy in an oscillation at a small wave number k is gradually transferred by inertia to larger wave numbers and eventually dissipated. Thus, for a sustained turbulent motion, one may picture the fluid receiving energy continuously from some small wave numbers $k < k_o$ (input range) which is transferred through a range of wave numbers $(k_o < k < k_s)$ to large wave numbers $k > k_s$ (dissipation range) where the energy is dissipated. If k_o and k_s are widely separated, an equilibrium condition will exist in this range $k_o < k < k_s$ (inertial range), and the spectrum E(k) must be determined by two factors, the kinematic viscosity ν (dimensions L^2/T) and the rate of the energy input $\mathcal{E}$ (dimensions L^2/T^3). Dimensional arguments can now be used to obtain an expression for E(k)

$$E(k) = a\, \mathcal{E}^{2/3}\, k^{-5/3}, \quad k_o < k < k_s \quad . \tag{2.23}$$

The constant a can be evaluated approximately through the use of equation (2.22) to give

$$a = \frac{2}{3}\left(\frac{1}{2}\langle u^2\rangle\right) k_o^{2/3} \quad \mathcal{E}^{-2/3} . \tag{2.24}$$

Substituting (2.24) in (2.23) gives the expression for E(k) valid in the range $k_o < k < k_s$

$$E(k) = \frac{2}{3}\left(\frac{1}{2}\langle u^2\rangle\right) k_o^{2/3} \quad k^{-5/3} . \tag{2.25}$$

By introducing the concept of an eddy viscosity, Heisenberg (Ref. 17) has derived the following expression for E(k) valid for k equal to or greater than k_s

$$E(k) \approx \frac{1}{2} <u^2> \left(\frac{8}{9c} k_o\right)^{2/3} \cdot k^{-5/3} \left[1 + \frac{8\nu^3}{3\mathcal{E}c^2} k^4\right]^{-4/3} ,$$

$$k \geqq k_s . \tag{2.26}$$

From this equation it can be seen that the approximate expression for k_s is

$$k_s = \left[\frac{3\mathcal{E}c^2}{8\nu^3}\right]^{1/4} = .525 \left(\frac{\mathcal{E}}{\nu^3}\right)^{1/4} \tag{2.27}$$

where an experimental value for c (c = 0.45) has been used (Ref. 18).

In terms of k_s, eq. (2.26) may be rewritten as

$$E(k) = \frac{2}{3}\left(\frac{1}{2}\langle u^2\rangle\right)\left(\sqrt{\frac{8}{3}}\,\frac{k_o}{c}\right)^{2/3} \frac{k^{-5/3}}{\left[1 + \frac{k^4}{k_s^4}\right]^{4/3}} . \tag{2.28}$$

The extension of the spectrum E(k) for values of k equal to or less than k_o has never been carried out analytically. According to homogeneous turbulence theory, $E(k) \sim k^4$, as k approaches zero. However, the large-scale turbulences (k very small) are never isotropic, so the k^4 criterion may not be valid. Megaw, (Ref. 18) using a phenomenological approach states that since the scattering cross section for large wavelengths should be of the order of k^4 (Rayleigh scattering law), S(k) must be independent of k (from eq. (2.12)) and therefore $E(k) \sim k^2$ for k less than or near k_o. He thus proposed the complete form of the spectrum as

$$E(k) \cong .8 \left[\frac{1}{2} \langle u^2 \rangle\right] \frac{1}{k_o} \frac{\left(\frac{k}{k_o}\right)^2}{\left[1+\left(\frac{k}{k_o}\right)^2\right]^{11/6}} \frac{1}{\left[1+\left(\frac{k}{k_s}\right)^4\right]^{4/3}} \tag{2.29}$$

where k_o is defined by

$$k_o = \frac{8}{\sqrt{3}} \frac{\mathcal{E}}{(\langle u^2 \rangle)^{3/2}} . \tag{2.30}$$

The fluctuation of the dielectric constant due to temperature and moisture fluctuation caused by the velocity fluctuation in turbulence, may be approximately assumed to have the same statistical structure as the velocity field itself. Thus, apart from a constant factor, we may assume that the spectrum for $\Delta \mathcal{E}(r)$ is

$$S(k) \sim .8 \; 4\pi^2 \frac{1}{k_o} \frac{\left(\frac{k}{k_o}\right)^2}{\left[1+\left(\frac{k}{k_o}\right)^2\right]^{11/6}} \frac{1}{\left(1+\left(\frac{k}{k_s}\right)^4\right)^{4/3}} \tag{2.31}$$

yielding for the cross section*

$$\sigma\left(\frac{2\pi}{\lambda}, \theta\right) \sim \frac{.8}{4\pi} \frac{\langle \Delta^2 \mathcal{E} \rangle}{\mathcal{E}_o^2} \left(\frac{2\pi}{\lambda}\right)^4 \frac{1}{k_o^3} \frac{1}{\left[1+\left(\frac{2\pi\theta}{\lambda k_o}\right)^2\right]^{11/6}} \frac{1}{\left(1+\left(\frac{2\pi\theta}{\lambda k_s}\right)^4\right)^{4/3}} . \tag{2.32}$$

---

* Note that $k\left|\hat{n}_1 - \hat{n}_2\right| = \frac{4\pi}{\lambda} \sin\frac{\theta}{2} \sim \frac{2\pi}{\lambda}\theta$ for near-forward scattering.

The predicted frequency and angular dependence therefore are:

a) For $\frac{2\pi}{\lambda}\theta < k_o \ll k_s$ (2.33)

$$\sigma \sim \lambda^{-4}$$

b) For $k_s \gg \frac{2\pi}{\lambda}\theta \gg k_o$ (2.34)

$$\sigma \sim \lambda^{-1/3}\ \theta^{-11/3}$$

c) For $\frac{2\pi}{\lambda}\theta \gg k_s$ (2.35)

$$\sigma \sim \lambda^{5}\,\theta^{-9} \quad .$$

The assumption that the spectrum of the dielectric fluctuations is the same as the spectrum of the velocity field is supported by the work of Bolgiano (Ref. 19) who has shown that for isotropic turbulence the spectrum of any scalar field caused by turbulent velocity should be nearly the same as the spectrum E(k), at least in the inertial range. Thus the relation $\sigma \sim \lambda^{-1/3}$ (eq. 2.34) is the best prediction from turbulence theory in the inertial range. Experimental data, unfortunately seem to support a $\sigma \sim \lambda$ dependence (Ref. 20). A statistical analysis of a variety of experimental data that we have collected, also seems to support the λ-dependence of the scattering cross section*.

One necessary check on the apparent deviation of experiment from theory, of course, is to be sure that for all experimental data the factor $\frac{4\pi}{\lambda}\sin\frac{\theta}{2}$ does lie within the range between k_o and k_s (or $\frac{\lambda}{4\sin\theta/2}$, lies within

* Appendix C of this report.

the range $L_s = \frac{\pi}{k_s}$ and $L_o = \frac{\pi}{k_o}$). To accomplish this, the chart given in Figure II-3 has been prepared. The details of the construction of the chart are given in Appendix A. In essence, corresponding to the frequency and distance used in each experiment, one can calculate approximately $L = \frac{\lambda}{4 \sin \theta / 2}$, or the size of the eddy responsible for the scattering process, and the approximate height at which the scattering takes place. Using Megaw's values of (Ref. 18) L_o and L_s at various altitudes, two limiting lines marked L_s, L_o, shown in Figure II-3 are also obtained. All the experimental data can then be represented by individual points on this chart according to their distance and frequency. All but two of these points lie between the two limiting lines, indicating that, indeed, the inertial spectrum of turbulence should be used.

In order to explain the λ-law of scattering by turbulence, Villars and Weisskopf (Refs. 6 and 21) introduced the "Mixing in Gradient" model, by postulating that the fluctuation in the dielectric constant (or density) in a turbulent atmosphere should be separated into two parts: 1) the fluctuation due to homogeneous turbulence and 2) the fluctuation of density due to the mean gradient of density caused by turbulence mixing. Starting from Bernoulli's equation the fluctuation of density can be represented by *

$$\rho \sim \langle (\Delta u)^2 \rangle ,$$

they argue that for dielectric constant fluctuations due to density fluctuations

$$E(k) \sim k^{-13/3} \qquad (2.36)$$

---

* $\sim$ means proportional to

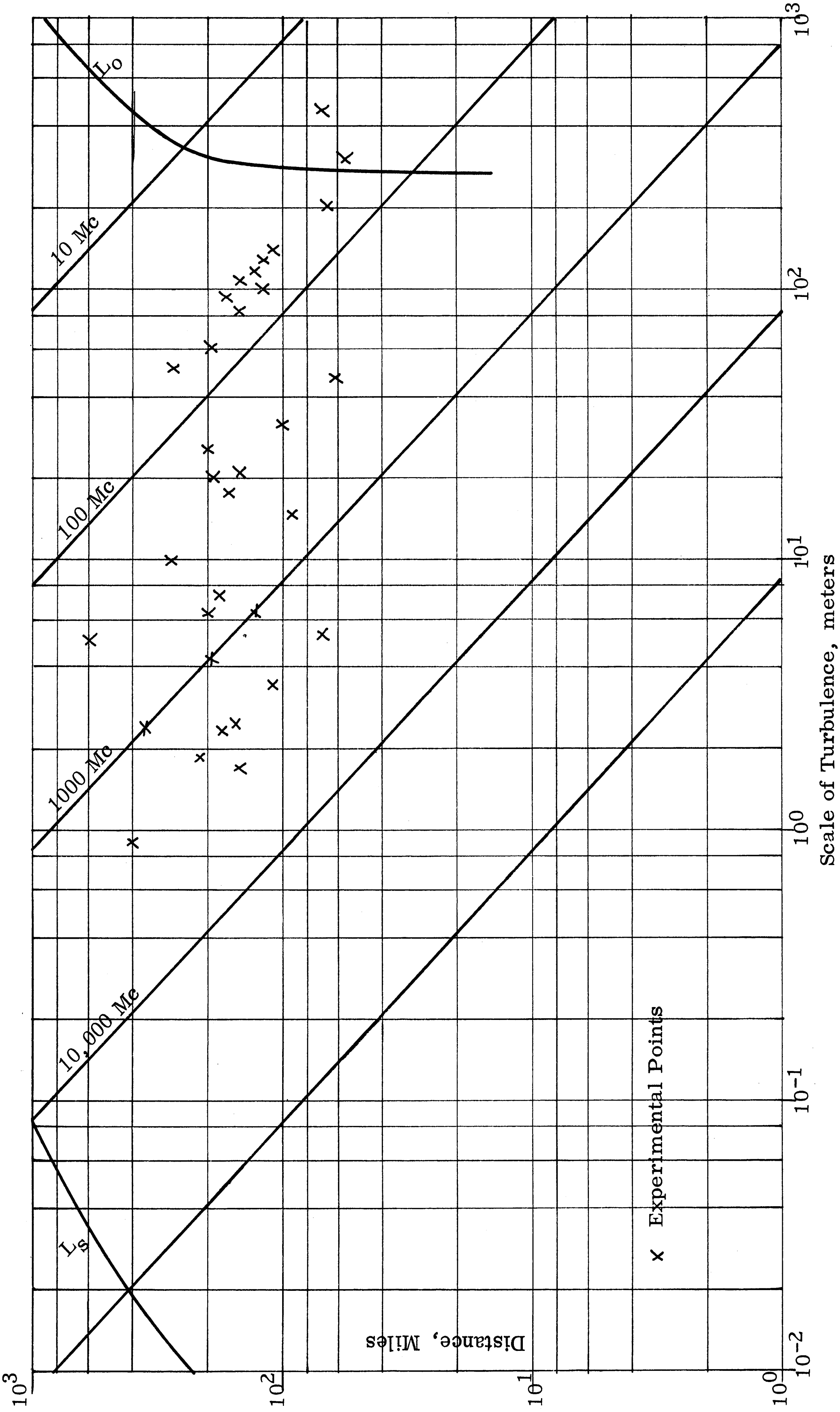

Figure II-3: Effective Scatter Length (L) Versus Distance and Frequency which results in a σ

$$\sigma \sim \lambda^{1/3} \theta^{-13/3} . \tag{2.37}$$

However, for mixing in gradient they obtain

$$\sigma \sim \lambda \theta^{-5} . \tag{2.38}$$

This approach, however, has been justifiably criticized (Ref. 19), since it appears to be physically unsound. In our opinion, not only the gradient but the existence of any space inhomogeneity in the mean dielectric constant (or density) should certainly appear to be a source of turbulence mixing, and therefore, it would modify to some extent the spectrum of the dielectric fluctuation. This modification may not be significant insofar as the turbulence structure is concerned. On the other hand insofar as scattering of electromagnetic waves is concerned, certain types of space inhomogeneity, (such as the discontinuity in the gradient of mean dielectric constant shown in eq. (2.13)) could be of great importance. Since a rigorous mathematical treatment on space inhomogeneities has never been successfully carried out, the phenomenological approach in the form of "layer theory" proposed by Friis, Crawford and Hogg (Ref. 8) is certainly the best available in the literature.

2.3 d Space Models

In this category we include theories which assume some fixed configuration for the variation of the dielectric constant. The best example of this approach is the BTL layer theory (Ref. 8). One might also include in this category the Normal

Mode theory since it involves an assumption of some fixed variation of the dielectric constant.

The BTL theory postulates the existence of certain area-extensive discontinuities in the gradient of the dielectric constant. The postulation of such discontinuities was prompted by airborne refractometer measurements which show that relatively sharp variations in the gradient of the refractive index exist in both the horizontal and the vertical planes. Although the existence of discontinuities in the gradient of the dielectric constant seems to be well established, knowledge of the extent, geometrical shape and number of extended discontinuities is small. Because of this lack of knowledge the BTL group postulates an atmosphere composed of many layers of limited extent and arbitrary aspect. The theory then considers rectangular layers with dimensions c and b (see Fig. II-4) and treats three cases, large layers, small layers and intermediate layers. The power received from layers of these sizes is found to be as follows:

Large layers, $b > \sqrt{2a\lambda}/\Delta$ and $c > \sqrt{2a\lambda}$

$$P_R = P_T \frac{A_T A_R}{4\lambda^2 a^2} q^2 \tag{2.39}$$

Small layers, $b < \sqrt{2a\lambda}/\Delta$ $c < \sqrt{2a\lambda}$

$$P_R = P_T \frac{A_T A_R}{\lambda^4 a^4} c^2 (b\,\Delta)^2 q^2 \tag{2.40}$$

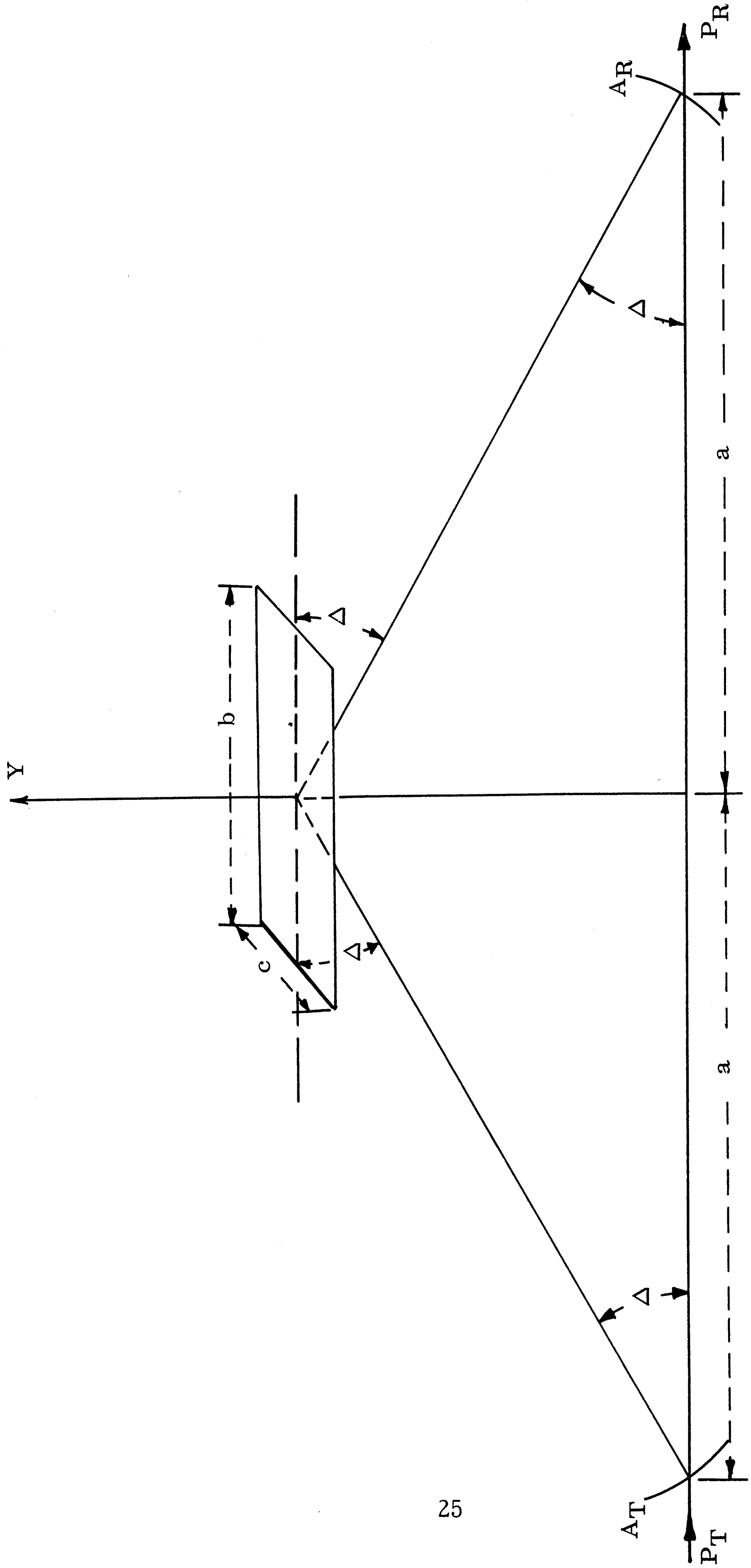

Figure II-4: Reflection by a Layer

Intermediate layers, $c = b, \quad \sqrt{2a\lambda} < b < \sqrt{2a\lambda}/\Delta$

$$P_R = P_T \frac{A_T A_R}{2\lambda^3 a^3} (b\Delta)^2 q^2 . \qquad (2.41)$$

The work of Schelkunoff (Ref. 14) is used to show that a reasonable value for q is

$$q = K\lambda/16\pi \Delta^3 . \qquad (2.42)$$

To compare with the cross section $\sigma(\hat{n}_2, \hat{n}_1)$ defined previously we note that in eq. (2.1) R_1 and R_2 are approximately equal to D/2 so that we may write

$$\frac{P_R}{P_{FS}} = \frac{4}{D^2} \iiint dv \ \sigma(\hat{n}_2, \hat{n}_1) . \qquad (2.43)$$

We may now put eqs. (2.39), (2.40) and (2.41) in the form of (2.43) by noting

$$P_{FS} = P_T \frac{A_T A_R}{4\lambda^2 a^2}$$

$$a = D/2$$

$$\Delta \approx \sin\theta/2 .$$

When this is done we find for

<u>Large layers</u>

$$\frac{P_R}{P_{FS}} = \frac{4}{D^2} \iiint dv \ \frac{K^2 \lambda^2 D^2}{4(16\pi)^2 \sin^6 \frac{\theta}{2}}$$

yielding

$$\sigma(\hat{n}_1, \hat{n}_2) \sim \frac{K^2 \lambda^2 D^2}{4(16\pi)^2 \sin^6 \frac{\theta}{2}} ; \qquad (2.44)$$

Small layers

$$\frac{P_R}{P_{FS}} = \frac{4}{D^2} \iiint dv \, \frac{4c^2 b^2 K^2}{(16\pi)^2 \sin^4 \frac{\theta}{2}}$$

$$\sigma(\hat{n}_1, \hat{n}_2) \sim \frac{4c^2 b^2 K^2}{(16\pi)^2 \sin^4 \frac{\theta}{2}} \quad ; \qquad (2.45)$$

Intermediate layers

$$\frac{P_R}{P_{FS}} = \frac{4}{D^2} \iiint dv \, \frac{b^2 K^2 \lambda D}{(16\pi)^2 \sin^4 \frac{\theta}{2}}$$

$$\sigma(\hat{n}_1, \hat{n}_2) \sim \frac{b^2 K^2 \lambda D}{(16\pi)^2 \sin^4 \frac{\theta}{2}} \quad . \qquad (2.46)$$

It can be seen that we have a far-zone approximation only for the case of the small layers. For the intermediate and large layers we may consider that we are in the quasi-Fresnel or Fresnel region respectively. It would appear that the wavelength dependence generated in the cross section σ is determined by the assumption of Fresnel, quasi-Fresnel or far-zone approximations.

Although the layer theory admits the possibility of layers of all three sizes, the calculations are based on layers of intermediate size. The results of this theory, therefore, have the proper wavelength dependence

$$\sigma \sim \lambda \ .$$

2.4 Integration of the Scattering Cross Section

The limits of integration in eq. (2.1) are determined by the beamwidths of the transmitting and receiving antennas. However, if the scattered power decreases more rapidly due to the scattering cross section σ than it does due to the radiation pattern of the antennas, the volume of integration is essentially limited by the scattering cross section. Thus in the evaluation of the integral two cases arise

1. Volume limited by beamwidth
2. Volume limited by scattering cross section.

In the first case two expressions, exhibiting different wavelength dependence, may be obtained for the ratio of scattered power to free-space power, depending on whether the gain or aperture of the antennas are held constant as the wavelength is changed.

In the second case, since the scattered power depends on the cross section σ and not on the radiation pattern of the antennas, the wavelength dependence of the ratio of scattered power to free-space power will be the same as that of the differential cross section.

Table II-2 summarizes the wavelength and distance dependence for some of the integrations appearing in the literature. In all cases the term $\left| n_2 \times (n_2 \times e) \right|^2$ has been neglected due to near-forward scattering and identical transmitting and receiving antennas are assumed. The expressions from Reference 2 have been modified by replacing the rectangular beam cross section by a square beam cross section. In appropriate cases two expressions for the power ratio have been given, $(P/P_{FS})_G$ for antennas of constant gain and $(P/P_{FS})_A$ for antennas of constant aperture.

The geometry used in evaluating the integrals is shown in Figure II-5 with the distance D as defined in Figure II-1. It should be noted in Figure II-5 that the intersection of R_1 and R_2 can be any point above the tangent planes.

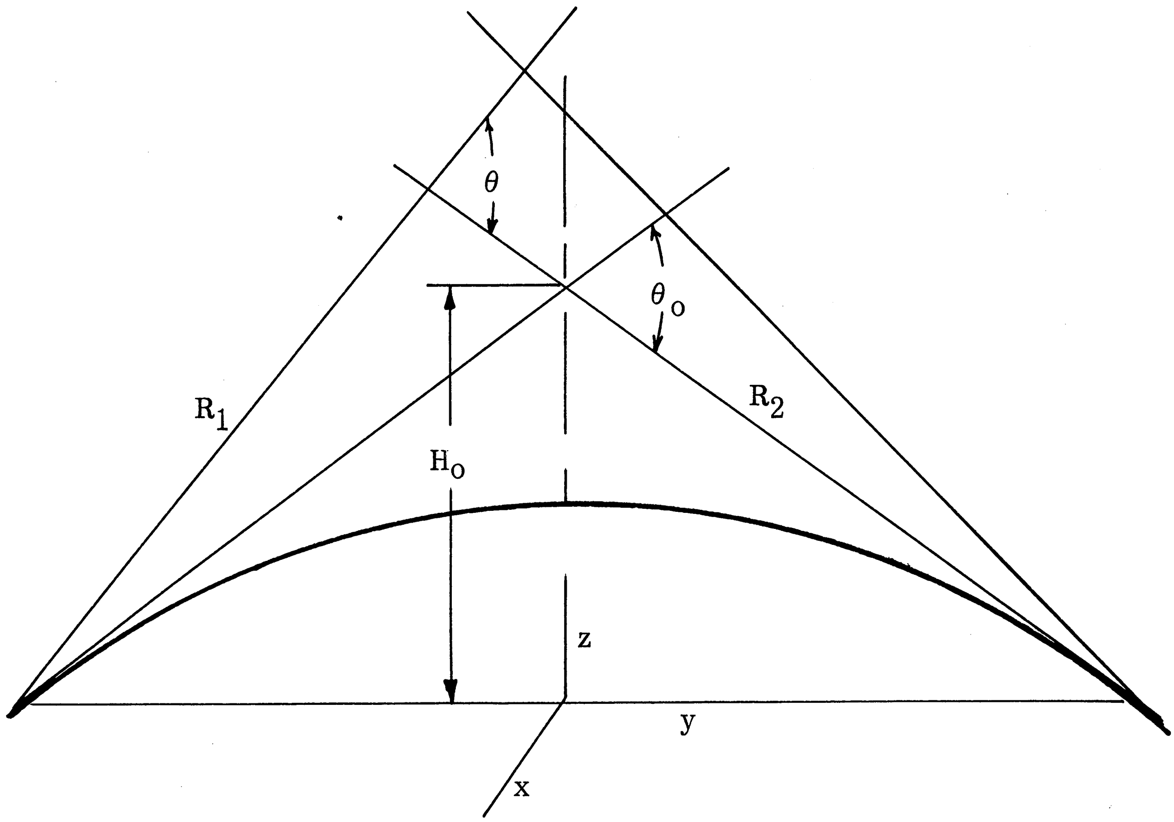

Figure II-5: Geometry for the Evaluation of Equation 2.1

Ref. No.	Cross Section	Approximation	Ratio of Power Received to Free-Space Power
2	$\dfrac{\left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2 \right\rangle k^4 L_o^3}{2\pi\left[1+\left(2kL_o \sin\frac{\theta}{2}\right)^2\right]^2}$	1. Forward scatter ($\theta = 0$) 2. Receiver in main scattering beam 3. Flat earth 4. Square beam cross section 5. Common volume limited by beamwidth 6. Dimension of volume constant in X direction	$\left(\frac{P_R}{P_{FS}}\right)_G = \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle (2\pi L_o)^3 \frac{\alpha^2 D}{\lambda^4}$ $\left(\frac{P_R}{P_{FS}}\right)_A = \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle \frac{(2\pi L_o)^3}{A} \frac{D}{\lambda^4}$
2	$\dfrac{\left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2 \right\rangle k^4 L_o^3}{2\pi\left[1+\left(2kL_o \sin\frac{\theta}{2}\right)^2\right]^2}$	1. Forward scatter ($\theta = 0$) 2. Common volume limited by beamwidth 3. $R_1 = R_2 = D/2$ 4. Square beam cross section 5. Dimension of volume constant in X direction	$\left(\frac{P_R}{P_{FS}}\right)_G = \frac{1}{2} \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle (2\pi L_o)^3 R \frac{\alpha^3}{\lambda^4}$ $\left(\frac{P_R}{P_{FS}}\right)_A = \frac{1}{2} \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle (2\pi L_o)^3 \frac{R}{A^{3/2}} \frac{1}{\lambda}$
2	$\dfrac{\left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2 \right\rangle k^4 L_o^3}{2\pi\left[1+\left(2kL_o \sin\frac{\theta}{2}\right)^2\right]^2}$	1. $1 > \theta = D/R$ 2. $L_o > 1/k\theta$ 3. Flat earth 4. $R_1 = R_2 = D/2$ 5. Common volume limited by beamwidth 6. Square beam cross section 7. Dimension of volume constant in X direction	$\left(\frac{P_R}{P_{FS}}\right)_G = \frac{1}{4\pi L_o} \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle R^5 \frac{\alpha^3}{D^4}$ $\left(\frac{P_R}{P_{FS}}\right)_A = \frac{1}{4\pi L_o} \left\langle \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)^2 \right\rangle \frac{R^5}{A^{3/2}} \frac{\lambda^3}{D^4}$

TABLE II-2

Ref. No.	Cross Section	Approximation	Ratio of Power Received to Free-Space Power
4	$\frac{1}{2\pi}\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4}{\left(1-\frac{L_s}{L_o}\right)} \left[\frac{L_o^3}{\left[1+\left(2kL_o\sin\frac{\theta}{2}\right)^2\right]^2} - \frac{L_s^4/L_o}{\left[1+\left(2kL_s\sin\frac{\theta}{2}\right)^2\right]^2}\right]$	1. $L_o > 1/k\theta$ 2. $1 > \theta = D/R$ 3. $L_s > 1/k\theta$ 4. Common volume limited by beamwidth 5. $R_1 = R_2 = D/2$ 6. Dimension of volume constant in X direction	$\frac{1}{16\pi^2}\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle \frac{R^4}{L_s^2}\frac{\lambda^2}{D}$
22	$\frac{1}{16\pi^2} < \left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2 >$ $\frac{8\pi L_v R_1^4 R_2^4}{L_o^2 D^4\left[x^2+\left(\frac{L_v}{L_o}h\right)^2\right]^2}$	1. $1 > \theta \simeq D/R$ 2. $L > 1/k\theta$ where L is the smaller of L_o and L_v 3. Anisotropic turbulence 4. σ independent of λ 5. Volume limited by cross section 6. $\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle = C_2 h^{-2}$ 7. $R_1 = R_2 = D/2$	$\frac{P_R}{P_{FS}} = \frac{61 L_o R^3 C_2}{L_v^2 D^5}$

TABLE II-2 (continued)

Ref. No.	Cross Section	Approximation	Ratio of Power Received to Free-Space Power
10	$\frac{1}{2\pi}\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4 L_o^3}{\left[1+(2kL_o\sin\frac{\theta}{2})^2\right]^2}$	1. $L_o > 1/k\theta$ 2. Scattered power decrease with height (4th power or greater) 3. $\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle = C_n h_i^{-n}$ 4. $R_1 = R_2 = D/2$ 5. Dimension of volume constant in X direction	$.215\ \frac{kC_0}{2\pi L_o}\frac{R^2}{D}$ $n = 0$ $.627\ \frac{kC_1}{2\pi L_o}\frac{R^3}{D^3}$ $n = 1$ $2.45\ \frac{kC_2}{2\pi L_o}\frac{R^4}{D^5}$ $n = 2$ $11.3\ \frac{kC_3}{2\pi L_o}\frac{R^5}{D^7}$ $n = 3$
4	$\frac{1}{2\pi}\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle \frac{k^4}{(1-\frac{L_s}{L_o})}\left[\frac{L_o^3}{\left[1+(2kL_o\sin\frac{\theta}{2})^2\right]^2} - \frac{L_s^4/L_o}{\left[1+(2kL_s\sin\frac{\theta}{2})^2\right]^2}\right]$	1. $L_o > 1/k\theta$ 2. $1 > \theta = D/R$ 3. $L_s < 1/k\theta$ 4. Common volume limited by beamwidth 5. $R_1 = R_2 = D/2$ 6. Dimension of volume constant in X direction	$\frac{1}{4}\left\langle\left(\frac{\Delta\mathcal{E}}{\mathcal{E}_o}\right)^2\right\rangle \frac{R^2}{L_o D}$

TABLE II-2 (continued)

III. Fluctuation of Received Signal

3.1 Extended Definition of the Scattering Cross Section

If one imagines tropospheric propagation as the transmission of information from a transmitter to a receiver, it is obvious that the use of the scattering cross section as discussed above (Section 2.2) is not sufficient. In fact it is well known that when a constant amplitude, single frequency is transmitted, the received signal has characteristics of varying amplitude known as fading, or what amounts to the same thing, the spectrum of the received energy is broader. The main causes of fading are:

(1) the motion of the transmission medium which causes a doppler shift in frequency,

(2) the differential time delay due to different propagation paths between the transmitter and receiver.

To formulate this problem, we shall use the concept of transmission through a network with random parameters proposed by Zadeh* (Ref. 23). For a time varying network, one may introduce a system function $H(t, \omega)$, defined as

$$H(t, \omega) = \left. \frac{e_2(t)}{e_1(t)} \right|_{e_1(t) = e^{-i\omega t}} \tag{3.1}$$

Hence if the complex form of the input is

$$e_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_1(\omega) e^{-i\omega t} d\omega ,$$

* Equations (3.1) to (3.6) follow the arguments of Zadeh, Ref. 23.

the output is given by

$$e_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(t, \omega)\, E_1(\omega)\, e^{-i\omega t}\, d\omega \tag{3.2}$$

From eq. (3.2) it can easily be shown that the auto-correlation function of $e_2(t)$ can be expressed in terms of the spectral density $S_1(\omega)$ of $e_1(t)$ as:

$$\psi_2(\tau) = \langle e_2^*(t)\, e_2(t+\tau) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_1(\omega)\, \psi_H(\tau, \omega)\, e^{-i\omega\tau} d\omega \tag{3.3}$$

where $\psi_H(\tau, \omega)$ is the correlation function of the system function, defined by,

$$\psi_H(\tau, \omega) = \langle H^*(t, \omega)\, H(t+\tau, \omega) \rangle \tag{3.4}$$

Taking the Fourier transform of eq. (3.3), yields the spectrum of the output:

$$S_2(\omega) = \int_{-\infty}^{\infty} \Gamma(\omega, \omega')\, S_1(\omega')\, d\omega' \tag{3.5}$$

where

$$\Gamma(\omega, \omega') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_H(\tau, \omega')\, e^{i(\omega - \omega')\tau}\, d\tau \tag{3.6}$$

Equation (3.5) is the relation that we are looking for. It gives the spectrum of the output in terms of the input spectrum.

The above concept certainly fits nicely in the problem of tropospheric scattering, provided that one can find the appropriate system function. But due to the relatively large time of wave propagation from the transmitter to receiver, the system function for the whole tropospheric link is hard to find. One can, however, generalize the concept of the scattering cross section given in Section II to include the effect of doppler shift. This is equivalent to finding the average system function for small volumes of the atmosphere. Referring to Figure III-1, if one assumes that each part of the scattering medium dv_j has some velocity $\underline{u}_j$, then the electric field strength at the receiver (using the far zone approximation) can be derived approximately by the following steps:

(a) If the transmitting frequency is ω, then the field strength at the scattering volume dv_j is,

$$\hat{e} E_o e^{-i\omega t} e^{i\frac{\omega}{c}R_1} e^{i\frac{\omega}{c}\underline{r}_j \cdot \hat{n}_1} \tag{3.7}$$

(b) Due to the velocity $\underline{u}_j$, the induced polarization at dv_j is,

$$\hat{e} E_o e^{-i\omega(1-\hat{n}_1 \cdot \frac{\underline{u}_j}{c})t} e^{i\frac{\omega}{c}R_1} e^{i\frac{\omega}{c}\underline{r}_j \cdot \hat{n}_1} \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j, t)\right] \tag{3.8}$$

where c is the velocity of light.

(c) The field strength at the receiver due to this induced polarization is

$$d\underline{E}_s = \frac{\hat{n}_2 \times (\hat{n}_2 \times \hat{e})}{4\pi R_2 \mathcal{E}_o} E_o \left(\frac{\omega}{c}\right)^2 e^{i\frac{\omega}{c}(R_1+R_2)} e^{-i\omega t} \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j, t)\right]$$
$$\left\{e^{i\underline{k}' \cdot \underline{u}_j t} e^{i\underline{k}' \cdot \underline{r}_j} e^{-i\underline{k}' \cdot \frac{\underline{u}_j}{c}(R_2 - \underline{r}_j \cdot \hat{n}_2)}\right\} dv_j \tag{3.9}$$

where, for simplicity, we write

$$\underline{k}' = \frac{\omega}{c}(\hat{n}_1 - \hat{n}_2) . \tag{3.10}$$

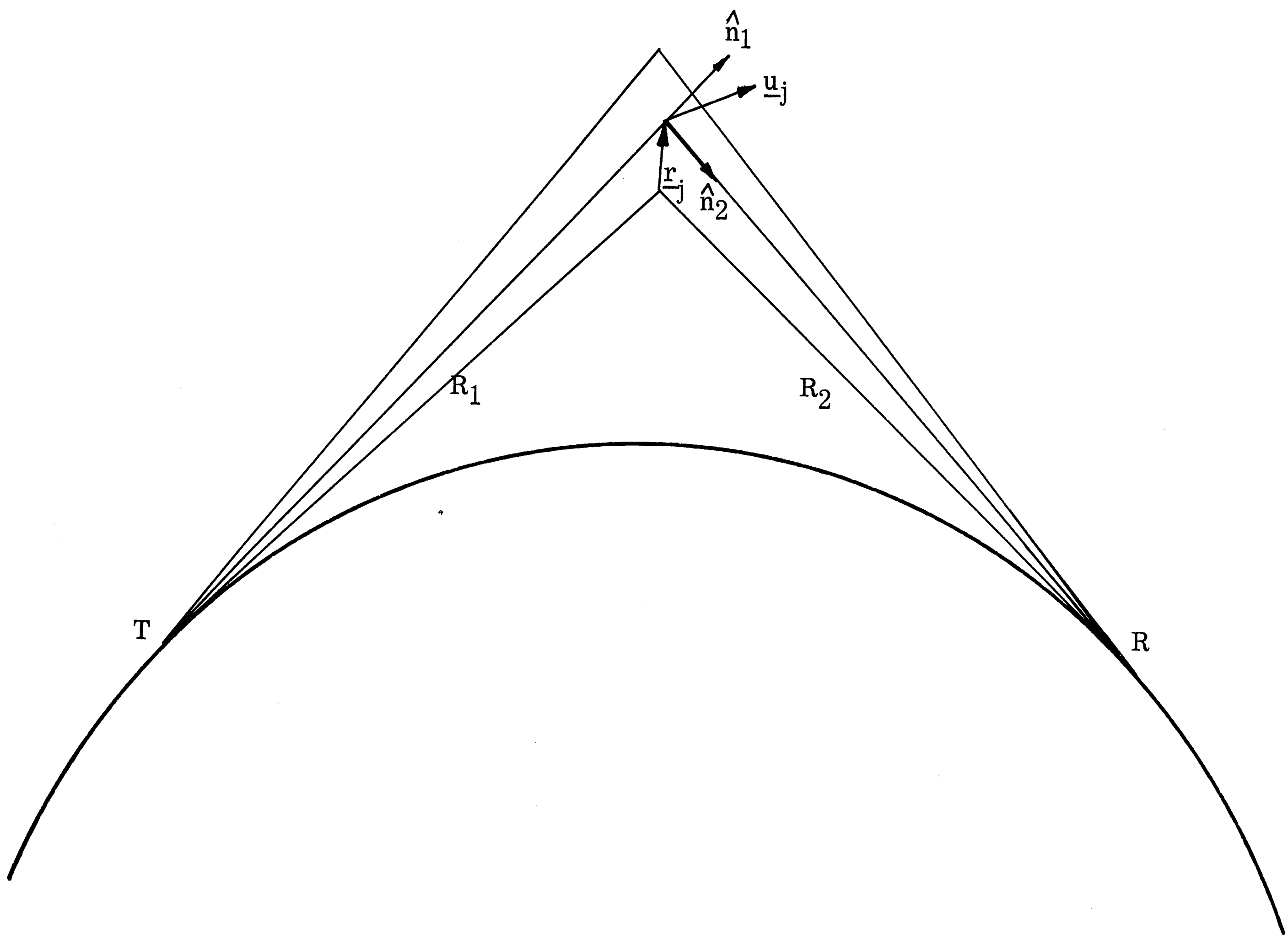

Figure III-1: Geometry for Extended Definition of Scattering Cross Section

Therefore,

$$E_s(t) = \left(\frac{\omega}{c}\right)^2 \frac{\hat{n}_2 \times (\hat{n}_2 \times \hat{e})}{4\pi R_2 \mathcal{E}_o} E_o e^{i\frac{\omega}{c}(R_1+R_2)} e^{-i\omega t} \int dv_j \, e^{i\underline{k}' \cdot \underline{u}_j t}$$

$$\cdot \, e^{i\underline{k}' \cdot \underline{r}_j} e^{-i\underline{k}' \cdot \frac{\underline{u}_j}{c}(R_2 - \underline{r}_j \cdot \hat{n}_2)} \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j, t)\right] \quad (3.11)$$

The explanation for the three exponential factors inside the integral is obvious. The first factor is the effect of doppler shift, the second, the differential path delay from the reference point and the third is the increment of phase delay due to doppler shift.

From eq. (3.1) one can obtain the system function by taking the ratio of the field strength at the receiver to the assumed input to the medium (i.e., $E_o e^{-i\omega t}$)

$$H(t,\omega) \cong \left(\frac{\omega}{c}\right)^2 \frac{1}{4\pi \mathcal{E}_o R_2} e^{i\frac{\omega}{c}(R_1+R_2)} \int dv_j \, e^{i\underline{k}' \cdot \underline{u}_j t} e^{i\underline{k}' \cdot \underline{r}_j}$$

$$\cdot \, e^{-i\underline{k}' \cdot \frac{\underline{u}_j}{c}(R_2 - \underline{r}_j \cdot \hat{n}_2)} \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j, t)\right] \quad (3.12)$$

In eq. (3.12), we assume near forward scattering, so that $\hat{n}_2 \times (\hat{n}_2 \times \hat{e})$ is approximately in the direction of $\hat{e}$. In principle now, at least, one can find

$$\psi_H(\tau,\omega) = \langle H^*(t,\omega) H(t+\tau,\omega) \rangle \, .$$

On account of the $\frac{1}{R_2}$ dependence of $H(t,\omega)$ in eq. (3.12) one may define, as in the derivation of the scattering cross section, the function $L(t,\omega,\hat{n}_2,\hat{n}_1)$, to characterize the medium, as the value of $\psi_H(\tau,\omega)$ per unit volume per unit solid angle. Formally this may be expressed as

$$L(\tau,\omega,\hat{n}_2,\hat{n}_1) = \left(\frac{\omega}{c}\right)^4 \frac{1}{(4\pi\mathcal{E}_o)^2} \frac{1}{V} \int_V dv_j \int_V dv_i \, F$$

$$\left\{ \left\langle \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j,t)\right] \left[\mathcal{E}_m(\underline{r}_i) + \Delta\mathcal{E}(\underline{r}_i,t+\tau)\right] \right\rangle \right\} \tag{3.13}$$

where for simplicity, we denote

$$F = \left\langle e^{i\underline{k}'\cdot(\underline{u}_i-\underline{u}_j)t} \right\rangle e^{-i\underline{k}'(\underline{r}_j-\underline{r}_i)} \, e^{i\underline{k}'\cdot\underline{u}_i\tau}$$

$$\cdot \, e^{i\underline{k}'\cdot(\underline{u}_i-\underline{u}_j)\frac{R_2}{c}} \, e^{i\frac{\hat{n}_2}{c}\cdot\left[\underline{r}_i(\underline{k}'\cdot\underline{u}_i) - \underline{r}_j(\underline{k}'\cdot\underline{u}_j)\right]}$$

and V is some appropriate small volume in which this average is meaningful. In eq. (3.13) one has to interpret the average as the ensemble average, hence:

$$\left\langle \left[\mathcal{E}_m(\underline{r}_j) + \Delta\mathcal{E}(\underline{r}_j,t)\right] \left[\mathcal{E}_m(\underline{r}_i) + \Delta\mathcal{E}(\underline{r}_i,t+\tau)\right] \right\rangle$$

$$= \mathcal{E}_m(\underline{r}_j)\mathcal{E}_m(\underline{r}_i) + \langle(\Delta\mathcal{E})^2\rangle R(\underline{r}_j-\underline{r}_i,\tau) \tag{3.14}$$

where $R(\underline{r}_j - \underline{r}_i, \tau)$ is the space and time correlation of the fluctuation of $\Delta\mathcal{E}$, which is assumed to be stationary both in space and time.

It is quite evident from eq. (3.13) that, as in the case of the scattering cross section, the contributions to $L(\tau, \omega, \hat{n}_2, \hat{n}_1)$ are really due to two factors. For the space inhomogeneity, one has

$$L_2 = \left(\frac{\omega}{c}\right)^4 \frac{1}{(4\pi \mathcal{E}_o)^2} \frac{1}{V} \int_V dv_j \int dv_i \, \mathcal{E}_m(\underline{r}_i) \mathcal{E}_m(\underline{r}_j) F \qquad (3.15)$$

while for time fluctuation, one has:

$$L_1 = \left(\frac{\omega}{c}\right)^4 \frac{1}{(4\pi \mathcal{E}_o)^2} \frac{1}{V} \int_V dv_j \int_V dv_i \left\langle (\Delta \mathcal{E})^2 \right\rangle R(\underline{r}_j - \underline{r}_i, \tau) F \, . \qquad (3.16)$$

From the value of $L(\tau, \omega, \hat{n}_2, \hat{n}_1)$ one can, following eq. (3.6) obtain its transform, namely:

$$\Gamma(\omega, \omega', \hat{n}_2, \hat{n}_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} L(\tau, \omega', \hat{n}_2, \hat{n}_1) e^{i(\omega - \omega')\tau} \, d\tau \qquad (3.17)$$

This function, defined in such a manner, may be called the generalized cross section per unit volume or the spectral density transport function*. The physical interpretation of this function is rather obvious, and can be stated as: the spectral density in the angular frequency range ω to $\omega + d\omega$ scattered into a unit solid angle in the direction $\hat{n}_2$ by a unit volume of scattering medium from a plane wave propagating in the direction $\hat{n}_1$.

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* This is essentially the same form given by Bugnolo, (Ref. 24), derived in a different way.

Without any information about the space distribution of the velocity $\underline{u}$ and the mean dielectric constant, it is hard to proceed any further. For the component L_1, due to fluctuation, we shall make some approximations and simplify the expressions further. However, for the component L_2 due to space inhomogeneity, such approximations may not be reasonable, and a further investigation should be carried out. It may be pointed out, though, that the function L_2 is actually related to the fading rate (see next subsection), and some of the features of the fading rate that cannot be accounted for by fluctuation theory can, at least qualitatively, be explained by layer theory as has been done in Reference 9.

To simplify eq. (3.16), one makes the assumption* that $\underline{u}$ is varying slowly within regions of space and time where $R(\underline{r}_j - \underline{r}_i, \tau)$ is significant. Thus the volume integral V may be divided into small volumes V_s within which $\underline{u}_s$ is substantially constant. Assume that there are n such volumes in V, then eq. (3.16) may be reduced to:

$$L_1(\tau, \omega, \hat{n}_2, \hat{n}_1) = \left(\frac{\omega}{c}\right)^4 \frac{1}{(4\pi \epsilon_0)^2} \frac{1}{V} \sum_{s=1}^{n} V_s \, e^{i\underline{k}' \cdot \underline{u}_s \tau}$$

$$\langle (\Delta \epsilon)^2 \rangle S(\underline{k}', \tau) \qquad (3.18)$$

where

$$S(\underline{k}, \tau) = \iiint e^{i\underline{k} \cdot \underline{r}} R(\underline{r}, \tau) \, dv$$

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* This is essentially the assumption made by Silverman, see Ref. 25.

is the spatial semi-transform of $R(r,\tau)$. It is to be noted that in deriving eq. (3.18) one also makes the reasonable approximation that:

$$\underline{k}' - \hat{n}_2(\underline{k}' \cdot \frac{\underline{u}_j}{c}) \cong \underline{k}' \ .$$

In eq. (3.18) we recognize that

$$\frac{1}{V} \sum_{s=1}^{n} V_s \, e^{-i\underline{k}' \cdot \underline{u}_s \tau}$$

is the spatial average of a function of velocity, namely $e^{-i\underline{k}' \cdot \underline{u}\tau}$. Thus if the distribution of velocity in space within some reasonable time is stationary, we invoke the ergodic hypothesis and define a probability density function $p(\underline{u})$, such that

$$\frac{1}{V} \sum_{s=1}^{n} V_s \, e^{-i\underline{k}' \cdot \frac{\underline{u}_s}{c}} \cong \int e^{-i\underline{k}' \cdot \frac{\underline{u}}{c}\tau} p(\underline{u})\, d\underline{u} \ . \qquad (3.19)$$

Equation (3.18) therefore reduces to:

$$L_1(\tau,\omega,\hat{n}_2,\hat{n}_1) = (\frac{\omega}{c})^4 \frac{1}{(4\pi)^2} \langle (\frac{\Delta \varepsilon}{\varepsilon_o})^2 \rangle S(\underline{k}',\tau) \int e^{-i\underline{k}' \cdot \underline{u}\tau} p(\underline{u})\, d\underline{u} \ . \qquad (3.20)$$

For different velocity distributions, one can have different forms of $L_1(\tau,\omega,\hat{n}_2,\hat{n}_1)$, and hence $\Gamma(\omega,\omega',\hat{n}_2,\hat{n}_1)$.

Due to the limitation of time, we were not able to investigate the effect of the velocity distribution on the scattering properties of the medium. Instead we shall quote the following models that have been postulated:

(a) If $\underline{u} = 0$, (i.e., neglect the velocity) one finds that

$$L_1(\tau, \omega, \hat{n}_2, \hat{n}_1) = \frac{\omega^4}{c^4} \frac{1}{(4\pi)^2} \left\langle \left(\frac{\Delta \mathcal{E}}{\mathcal{E}_o} \right)^2 \right\rangle S(\underline{k}', \tau). \tag{3.21}$$

Now, if one makes the further assumption that

$$S(\underline{k}', \tau) \cong S(\underline{k}', 0) = S(\underline{k}') \tag{3.22}$$

which means that the time scale in which the fluctuations of dielectric constant are significantly correlated, is small compared with that of the signal variation

$$\begin{aligned} \Gamma(\omega, \omega', \hat{n}_2, \hat{n}_1) &= \frac{\omega^4}{c^4} \frac{1}{(4\pi)^2} \left\langle \left(\frac{\Delta \mathcal{E}}{\mathcal{E}_o} \right)^2 \right\rangle S(\underline{k}') \delta(\omega - \omega') \\ &= \sigma(\hat{n}_2, \hat{n}_1) \delta(\omega - \omega') \end{aligned} \tag{3.23}$$

which is essentially the scattering cross section given in Section II.

(b) Assume that the approximation given by eq. (3.22) holds, but the velocity is uniform, i.e., $\underline{u} = \underline{u}_o$, then,

$$L_1(\tau, \omega, \hat{n}_2, \hat{n}_1) = \left(\frac{\omega}{c} \right)^4 \frac{1}{(4\pi)^2} \left(\frac{\Delta \mathcal{E}}{\mathcal{E}_o} \right)^2 S(\underline{k}') \, e^{i\tau(\omega + \underline{k}' \cdot \underline{u}_o)} \ .$$

Therefore,

$$\Gamma(\omega, \omega', \hat{n}_2, \hat{n}_1) = \sigma(\hat{n}_2, \hat{n}_1) \, S(\omega - \omega' - \underline{k}' \cdot \underline{u}_o) \ . \tag{3.24}$$

This is the form used by Bugnolo (Ref. 24).

(c) If $S(\underline{k}',\tau) = S(\underline{k}')$ and assume that the velocity is isotropically Guassian distributed*:

$$p(\underline{u}) = \frac{1}{(2\pi)^{3/2}\,\sigma_u^3} \exp\left(-\frac{u^2}{2\sigma_u^2}\right)$$

where σ_u is the variance of $\underline{u}$, then

$$\int e^{-i\underline{k}'\cdot\underline{u}}\, p(\underline{u})\, d\underline{u} = \exp\left(-\frac{k'^2\sigma_u^2\tau^2}{2}\right) \qquad (3.25)$$

and

$$L_1(\tau,\omega,\hat{n}_2,\hat{n}_1) \doteq \frac{\omega^4}{c^4}\,\frac{1}{(4\pi)^2} \left\langle \left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)^2 \right\rangle S(\underline{k}')\exp\left(-\frac{k'^2\sigma_u^2}{2}\right) . \qquad (3.26)$$

Equation (3.26) is used in the calculation of the fading rate given in the next section.

3.2 The Fading Rate

The general criterion for the rate of fading is expressed in terms of the expected number of crossings of the mean per second, known as the rate of zero crossing (Ref. 26). Approximately, this rate is related to the correlation function of the quantity of interest. A brief account of the derivation of such a relation is given as follows:

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* Actually, the results are true if the component of the velocity in the direction of $\underline{k}'$ is Gaussian distributed.

(a) If y(t) is a stationary random variable and y'(t) is its time derivative, one can form the joint probability

$$f(\alpha,\beta)\, d\alpha\, d\beta = \text{Probability that } \begin{matrix} \alpha < y < a + d\alpha \\ \beta < y' < \beta + d\beta \, . \end{matrix} \qquad (3.27)$$

(b) For values of y and y' within the above range, the time necessary for y to change its value from α to $\alpha + d\alpha$ is,

$$\frac{d\alpha}{|\beta|} \, .$$

Therefore, the expected number of crossings at the level $y = \alpha$, due to y' within the above range is

$$N_\alpha = \int_{-\infty}^{\infty} f(\alpha,\beta) \left|\beta\right| d\beta \, . \qquad (3.28)$$

(c) The number of zero crossings is,

$$N_0 = \int_{-\infty}^{\infty} f(0,\beta) \left|\beta\right| d\beta \qquad (3.29)$$

(d) The problem of finding the joint probability $f(\alpha,\beta)$ is a difficult one. Therefore, as an approximation, we assume that both y and y' are Gaussian distributed, and their joint probability depends only on the covariance matrix of the two variables. Now, it can be easily shown that

$$\left.\begin{aligned} \langle y(t)\, y(t)\rangle &= \langle y^2\rangle \\ \langle y(t)\, y'(t)\rangle &= 0 \\ \langle y'(t)\, y'(t+\tau)\rangle &= -\langle y^2\rangle\, C''(\tau) \end{aligned}\right\} \tag{3.30}$$

where $C(\tau)$ is the correlation function of y. Therefore

$$f(0,\beta) = \frac{1}{2\pi\langle y^2\rangle\sqrt{-C''(0)}} \quad \frac{\beta^2}{e^{-2\langle y^2\rangle\left[-C''(0)\right]}} \; . \tag{3.31}$$

By carrying out the integration in eq. (3.29) we obtain

$$N_0 = \frac{1}{\pi}\sqrt{-C''(0)} \; . \tag{3.32}$$

When applied to tropospheric scattering, eq. (3.13) gives the correlation (except for a proportionality constant) of the complex amplitude of the electric field at any point. The correlation of the (fluctuation of the power from the mean) envelope $C(\tau)$ is related to the correlation of the complex amplitude by (Ref. 27).

$$C(\tau) \cong \frac{\pi}{4(4-\pi)}\, \rho^2(\tau) \; . \tag{3.33}$$

Hence

$$C''(0) \cong \frac{\pi}{2(4-\pi)}\, \rho''(0) \tag{3.34}$$

because

$$\begin{aligned} \rho(0) &= 1 \\ \rho'(0) &= 0 \end{aligned} \tag{3.35}$$

Hence, if $L(\mathcal{T},\omega,\hat{n}_2,\hat{n}_1)$ is known, the fading of the power received, at least, for a narrow beam antenna is:

$$N_0 = \frac{1}{\pi}\sqrt{\frac{\pi}{2(4-\pi)}}\sqrt{-\frac{L''(0,\omega,\hat{n}_1,\hat{n}_2)}{L(0,\omega,\hat{n}_1,\hat{n}_2)}} \quad . \tag{3.36}$$

It is evident now that if the spectral density transport function $\Gamma(\omega',\omega,\hat{n}_1,\hat{n}_2)$ is known, then $L(\mathcal{T},\omega',\hat{n}_1,\hat{n}_2)$ can be calculated, hence the fading rate can be calculated. Unfortunately, the part of L , due to space inhomogeneity, L_2 has never been calculated. If one considers the effect of L_1 alone, then it can easily be seen that, for the model postulated in Section 2.1 c , eq. (3.26) yields:

$$\frac{L_1''(0,\omega,\hat{n}_1,\hat{n}_2)}{L_1(0,\omega,\hat{n}_1,\hat{n}_2)} = -k^2\sigma_u^2 = -\left(2\frac{\sigma_u\omega}{c}\sin\frac{\theta}{2}\right)^2 . \tag{3.37}$$

Thus the fading rate is

$$N_0 = \frac{1}{\pi}\sqrt{\frac{\pi}{2(4-\pi)}}\;\frac{2\sigma_u\omega}{c}\sin\frac{\theta}{2} \quad . \tag{3.38}$$

Equation (3.38) is the relation initially given by Booker and Gordon (Ref. 2). The simple relation that the fading rate is proportional to frequency does not check with experiment very well*. Due to the complicated nature of the function L_2 not much has been done as of this date, to determine its contribution to the fading rate. In terms of space inhomogeneity, the failure of the ratio of the

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* See Ref. 31 of the experimental references.

fading rate to follow the ratio of frequency is qualitatively explained by layer theory as the "breaking up" of layers so that a single layer for a large wavelength behaves like many layers at shorter wavelengths. Using turbulence theory Silverman (Ref. 5) tried to estimate the effect of τ in the factor $S(\underline{k}',\tau)$ in eq. (3.20). By dimensional arguments on the decay of turbulent field, he inferred that:

$$S(\underline{k}',\tau) = g(k'^{2/3},\tau) \tag{3.39}$$

is a universal function. Hence

$$\frac{L''(0,\omega,\hat{n}_2,\hat{n}_1)}{L(0,\omega,\hat{n}_2,\hat{n}_1)} = g''(0)\left[k'^{4/3} + k'^2\sigma_u^2\right] \tag{3.40}$$

so that

$$N_o = \frac{1}{\pi}\sqrt{\frac{\pi}{2(4-\pi)}}\sqrt{(-g''(0))\left[\left(\frac{2\omega}{c}\sin\frac{\theta}{2}\right)^{4/3} + \left(2\frac{\sigma_u\omega}{c}\sin\frac{\theta}{2}\right)^2\right]} \tag{3.41}$$

in which $g''(0)$ is negative from the properties of turbulence.

3.3 Distortion of the Received Signal

It is shown in the last section that the spectral transport function $\Gamma(\omega,\omega',\hat{n},\hat{n}')$ satisfactorily specifies the scattering property of the medium. From this function the relation between the transmitted and received power can be obtained by a single integration if the effect of higher order scattering can be neglected. In this section, we shall consider such "common volume integration".

If the information transmitted is such that the power density of the transmitted signal is spread over a small range about the carrier frequency and is varying slowly with respect to the carrier frequency, we may write the power density of the source as $S_1(\omega, t)$.

Let the origin 0 be a point within the intersection of the beams of the transmitting and receiving antennas (see Fig. III-2). The poynting's vector at any point $\underline{r}_j$ is therefore in the direction $\hat{n}'$ (a function of $\underline{r}_j$) and its magnitude is equal to

$$p(\omega, \hat{n}', t) = \frac{G_T}{R_1^2} \; S_1\left(\omega, t - \frac{R_1 + \hat{n}' \cdot \underline{r}_j}{c}\right) \tag{3.42}$$

where G_T is the gain of the transmitting antenna and the term $\frac{R_1 + \hat{n}' \cdot \underline{r}_j}{c}$ accounts for the time delay. The power per unit solid angle scattered by a small volume dv_j into the direction of the receiver, or direction $\hat{n}$ (also a function of $\underline{r}_j$), at the position of the receiver is:

$$p(\omega, \hat{n}, t) = \frac{G_T}{R_1^2} \, dv_j \int d\omega' \; S_1\left(\omega', t - \frac{R_1 + R_2 + (\hat{n}' - \hat{n}) \cdot \underline{r}_j}{c}\right) \Gamma(\omega, \omega', \hat{n}, \hat{n}'). \tag{3.43}$$

If the aperture of the receiving antenna is A_R , then the spectrum of the received signal is

$$S_2(\omega, t) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \int dv_j \int d\omega' \; S_1\left(\omega', t - \frac{R_1 + R_2 + (\hat{n}' - \hat{n}) \cdot \underline{r}_j}{c}\right) \Gamma(\omega, \omega', n, n') \, . \tag{3.44}$$

As indicated in eq. (3.44), the spectrum of the received power differs from that of S_1 for the following reasons:

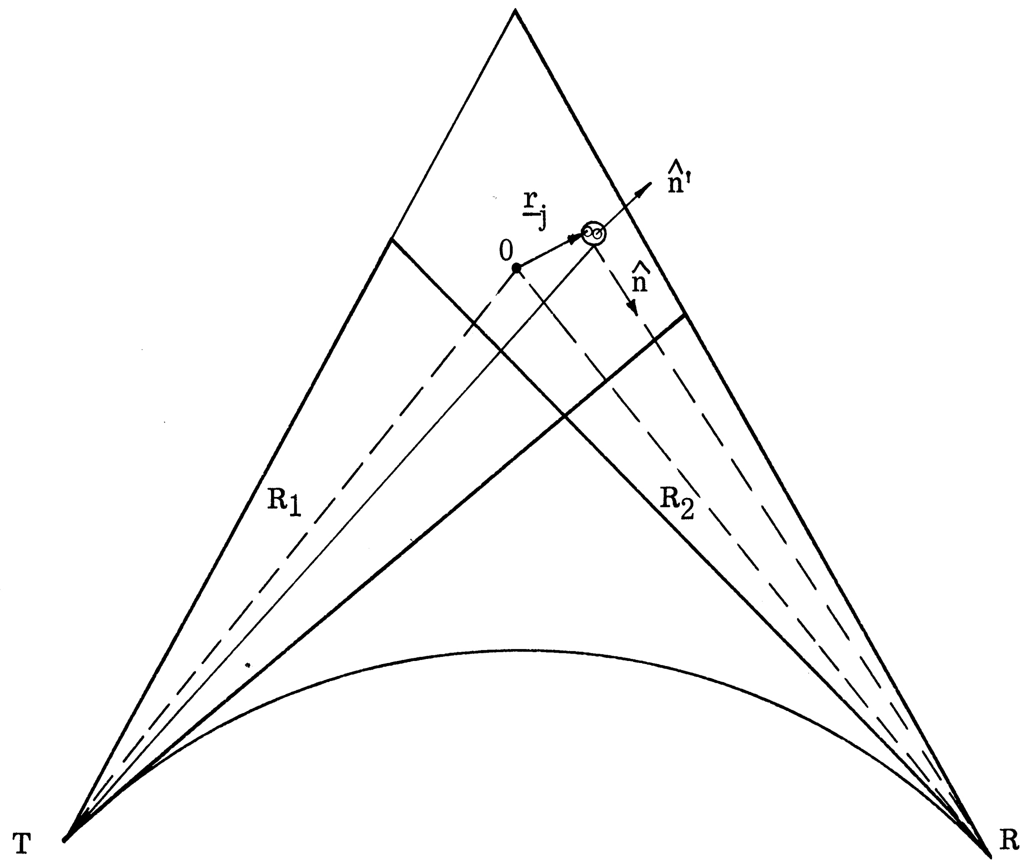

Figure III-2: Geometry for Deriving Distortion of Received Signal

(a) shift of frequency due to the factor $\Gamma(\omega, \omega', \hat{n}, \hat{n}')$

(b) $\hat{n}, \hat{n}'$ varies with $\underline{r}_j$, which should be second order effect

(c) the different path delay, $\frac{(\hat{n}' - \hat{n})}{c} \cdot \underline{r}_j$, which is usually the major source of distortion

(d) any additional space variation of Γ one may please to postulate.

A simple illustrative example of the use of eq. (3.44) can be given as follows:

(a) For the fading problem discussed in Section 3.2, one has

$$S_1(\omega, t) = \delta(\omega - \omega_c)$$

where ω_c is the transmitted frequency. Therefore

$$S_2(\omega) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \int dv_j \Gamma(\omega, \omega_c, \hat{n}, \hat{n}') \ .$$

For a narrow beam,

$$S_2(\omega) \cong \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} V \Gamma(\omega, \omega_c, \hat{n}, \hat{n}') \qquad (3.45)$$

where V is the common volume. The correlation of the complex amplitude of the received signal can be approximately obtained by the following argument:

If $$e(t) = A(t) e^{-i\omega_c t}$$

where A(t) is a slowly varying function, then,

$$\psi(\tau) = \psi_a(\tau) e^{-i\omega_c \tau}$$

where $\psi_a(\tau)$ is the correlation of the complex amplitude.

Now, from eq. (3.45), one has

$$\psi_2(\tau) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} V \frac{1}{2\pi} \int \Gamma(\omega, \omega_c, \hat{n}, \hat{n}') e^{-i\omega\tau} d\omega$$

$$= \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} V e^{-i\omega_c \tau} L(\tau, \omega_c, \hat{n}, \hat{n}')$$

by virtue of eq. (3.17), therefore the correlation of the complex amplitude is,

$$\psi_a(\tau) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} V L(\tau, \omega_c, \hat{n}, \hat{n}') . \tag{3.46}$$

(b) The effect of antenna apertures on the fading rate can therefore simply be obtained from

$$\psi_a(\tau) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \int dv_j \, L(\tau, \omega_c, \hat{n}, \hat{n}') . \tag{3.47}$$

From this, the fading rate can easily be calculated.

(c) For an amplitude modulated signal, let

$$e_1(t) = f_1(t) e^{-i\omega_c t}$$

where $f_1(t)$ is a function containing information varying slowly with respect to the carrier. Therefore,

$$S_1(\omega, t) \cong f_1^2(t) \delta(\omega - \omega_c) .$$

This yields from eq. (3.45), the spectrum of the received signal,

$$S_2(\omega, t) \cong \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \int dv_j f_1^2 \left(t - \frac{R_1 + R_2}{c} - \frac{(\hat{n}' - \hat{n}) \cdot \underline{r}_j}{c}\right) \Gamma(\omega, \omega_c, \hat{n}, \hat{n}') \quad . \tag{3.48}$$

For simplicity, if one uses the model for Γ given in eq. (3.23),

$$\Gamma(\omega, \omega', \hat{n}, \hat{n}') = \sigma(\hat{n}, \hat{n}') \delta(\omega - \omega')$$

then,

$$S_2(\omega, t) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \delta(\omega - \omega_c) \int dv_j f_1^2 \left(t - \frac{R_1 + R_2}{c} - \frac{(\hat{n}' - \hat{n}) \cdot \underline{r}_j}{c}\right) \sigma(\hat{n}, \hat{n}') \, .$$

Thus, the output may be represented as:

$$S_2(\omega, t) \cong f_2^2 \left(t - \frac{R_1 + R_2}{c}\right) \delta(\omega - \omega_c)$$

where

$$f_2^2(t) = \frac{G_T}{R_1^2} \frac{A_R}{R_2^2} \int dv_j \, f_1^2 \left(t - \frac{\hat{n}' - \hat{n}}{c} \cdot \underline{r}_j\right) \sigma(\hat{n}, \hat{n}') \quad . \tag{3.49}$$

IV. Theory of Multiple Scattering

4.1 General

Although the single scattering approach described in the previous sections is the generally accepted approach to tropospheric scattering, the more fundamental approach to the problem is that of multiple scattering. The general formulation of multiple scattering in the microscopic sense has been discussed in detail by Lax (Refs. 28 and 29). In theory, if one knows the scattering property of individual scatterers (molecules, for example) and the distribution of scatterers in space, one can arrive at a set of integral equations, from whose solution the average description of the propagation through the medium can be obtained. Such a treatment, when applied to a homogeneous, isotropic medium in statistical equilibrium, is essentially, the derivation of the dielectric constant $\mathcal{E}$ (or the index of refraction) of the medium (Ref. 30).

Due to the lack of exact statistical knowledge of the atmosphere, such microscopic treatment cannot be very fruitful. However, it is reasonable to assume that in a small enough region within a small enough time interval, the medium can be treated as nearly isotropic and homogeneous and characterized by a measurable macroscopic parameter $\mathcal{E}$, which is a function of space and time. From this macroscopic parameter, one can, by means of eq. (3.13) and (3.17) specify the scattering property of the medium by the parameter Γ. In the macroscopic sense, therefore, we again have the multiple scattering problem, where the individual scatterers are small volumes each with scattering property Γ. Before formulating the multiple scattering in this macroscopic sense, it seems to be proper to consider phenomenologically what approximations were involved in the single scattering picture discussed in the last section.

(a) Attenuation. Since the scattering cross section given in eq. (2.12) is the fraction of power scattered from the direction $\hat{n}_1$ to $\hat{n}_2$, it is evident that the power density from the primary beam is continually decreased by scattering as it travels through the medium, and is therefore "attenuated". This attenuation is not accounted for by a single scattering treatment.

(b) Distortion and Noise. The received signal in a single scattering treatment is distorted by multipath delay. This is the lower bound for the distortion. Actually, power will be scattered into the receiving antenna which has been scattered once, twice, etc. Some rescattered power will invariably be received after such a long time delay that it can be regarded as an increase of noise.

It is obvious that both effects will be more pronounced when the transmission distance is large. Hence, if it is desired to increase the range of tropospheric propagation, the multiple scattering formulation should be investigated.

4.2 The Transport Equation

The problem of multiple scattering occurs in the transport of energy or mass through a medium of scatters, and is a familiar problem in astrophysics

(Ref. 31) and neutron diffusion (Ref. 32). The main equation governing such phenomena is the well-known transport equation. The same equation can also be approximately applied to the case of tropospheric scattering if we accept the following two statements:

(a) For propagation in the near-forward direction, the effect of polarization can be neglected.

(b) If the information contained in the wave is in the form of frequency or amplitude variation and is a slowly varying factor as compared with the carrier frequency, the mean transport parameter $\Gamma(\omega, \omega', \hat{n}, \hat{n}')$ can be treated as a constant.

The pertinent quantity to be investigated in (b) is the so-called specific intensity* $p(\omega, \hat{n}, \underline{r}, t)d\omega$, defined as the energy in the spectral range $(\omega, \omega + d\omega)$, per unit time intercepted by a unit area from a unit solid angle in the direction $\hat{n}$. It is evident that for an antenna of aperture A_R with a power pattern $F(\hat{n})$ located at $\underline{r}$, the received power in the spectral range $(\omega, \omega + d\omega)$ at any time t is:

$$d\omega \int_{\substack{\text{over the} \\ \text{power pattern} \\ \text{of antenna}}} p(\omega, \hat{n}, \underline{r}, t)\, A_R F(\hat{n})\, d\Omega_n . \tag{4.1}$$

The differential equation for the specific intensity $p(\omega, \hat{n}, \underline{r}, t)$ can be obtained from the energy balance in a small volume of unit area and unit length in

* We use the definition given by Ref. 31.

the direction $\hat{n}$. Taking into account the finite velocity of propagation c, the equation for $p(\omega,\hat{n},\underline{r},t)$ reads:

$$\frac{1}{c}\frac{\partial}{\partial t}\, p(\omega,\hat{n},\underline{r},t) \qquad \text{(change due to time variation of p)}$$

$$+\nabla \cdot \hat{n}\, p(\omega,\hat{n},\underline{r},t) \qquad \text{(change due to transport)}$$

$$+p(\omega,\hat{n},\underline{r},t)\int d\omega' \int_{\Omega_{n'}} \Gamma(\omega,\omega',\hat{n},\hat{n}')\, d\,\Omega_{n'} \qquad \text{(power lost due to scattering)}$$

$$=\int d\omega' \int p(\omega',\hat{n}',\underline{r},t)\,\Gamma(\omega,\omega',\hat{n},\hat{n}')\, d\,\Omega_{n'} \qquad \text{(power scattered \underline{into} the direction } \hat{n}\text{)}$$

$$+\, q(\omega,\hat{n},\underline{r},t) \qquad \text{(source term)} \qquad (4.2)$$

where q is the source function. In our problem, this is related to the energy radiated from the transmitter. In most cases, $\Gamma(\omega,\omega',\hat{n},\hat{n}')$ depends only on the angle between $\hat{n}$ and $\hat{n}'$, therefore, the integral

$$\int d\omega' \int_{\Omega_{n'}} \Gamma(\omega,\omega',\hat{n},\hat{n}')\, d\Omega_{n'} = Q(\omega) \qquad (4.3)$$

is a function of ω and is called the scattering cross section per unit volume of the medium.

4.3 Discussion of the Transport Equation

From eq. (4.1), it is evident that the tropospheric scattering problem can be obtained from the solution of eq. (4.2) if a reasonable form of Γ can be assumed.

During the past half century, this equation has been investigated in various branches of physics. As far as we know, only a few special cases have been rigorously solved. It is clear that investigations of this important equation of mathematical physics will continue until its solutions and the physical context of the solutions are obtained and understood.

Some of the results of previous investigations of the transport equation are briefly stated as follows:

(a) In principle, by using a Laplace transform, the time dependent case can be reduced to the steady-state case (time independent).

(b) For the steady-state case, with no exchange of frequency, $\Gamma(\omega,\omega',\hat{n},\hat{n}')$ can be reduced to

$$\Gamma(\omega,\omega',\hat{n},\hat{n}') = Q(\omega)\, D(\hat{n},\hat{n}')$$

where $D(\hat{n},\hat{n}')$ is the distribution function of the scattered power with

$$\int D(n,n')\, d\,\Omega_{n'} \equiv 1$$

The resulting transport equation reduces to the more familiar form:

$$\nabla \cdot \hat{n}\, p(\omega, \hat{n}, \underline{r}) + Q(\omega)\, p(\omega, \hat{n}, \underline{r}) =$$

$$= Q(\omega) \int p(\omega, \hat{n}', \underline{r})\, D(\hat{n}, \hat{n}')\, d\,\Omega_{n'} + q(\omega, \hat{n}, \underline{r}) \quad .$$

(c) For isotropic scattering, i.e., $D(\hat{n},\hat{n}') = 1/4\pi$, in an infinite medium, the solution of eq. (4.2) can be expressed rigorously by using a Green's function (Ref. 32). But this solution is of little use in the application of tropospheric propagation where $D(\hat{n}, \hat{n}')$ is known to be very peaked in the forward direction.

(d) If the $\underline{r}$ dependence of eq. (4.2) can be expressed in terms of one space variable, then one has the following one-dimensional problem:

$$\cos\theta \frac{\partial}{\partial x} p(\omega, \hat{n}, x) + Q(\omega)\, p(\omega, \hat{n}, x) =$$

$$= Q(\omega) \int p(\omega, \hat{n}, x)\, D(\hat{n}, \hat{n}')\, d\Omega_{n'} + q(\omega, \hat{n}, x) \quad .$$

Multiple scattering problems of this type have been investigated most thoroughly. The classical Milne problem (isotropic scattering) is a familiar problem of this type. The Milne solution has been extended by Chandrasekar (Ref. 31) to the reflection and transmission of parallel beams through a medium of finite thickness.

(e) Bugnolo (Ref. 24) recently attempted the solution of a parallel, monochromatic beam incident normally into a half space characterized by the scattering parameter Γ of the form given by eq. (3.24). Although only an approximate solution, his result yields to a first approximation the frequency deviation and the distribution of power in space which should offer some qualitative insight into practical problems.

(f) A familiar, but tedious method of solving eq. (4.2) is by a Neumann series. That is, by assuming

$$p(\omega, \hat{n}, \underline{r}, t) = p_0 + p_1 + p_2 + \ldots .$$

and let

$$\frac{1}{c}\frac{\partial}{\partial t} p_0 + \nabla \cdot \hat{n}\, p_0 + Q(\omega)\, p_0 = q$$

$$\frac{1}{c}\frac{\partial}{\partial t} p_1 + \nabla \cdot \hat{n} p_1 + Q(\omega) p_1 = \int d\omega' \int p_0(\omega', \hat{n}', \underline{r}) \Gamma(\omega, \omega', \hat{n}, \hat{n}') d\Omega_{n'}$$

$$\frac{1}{c}\frac{\partial}{\partial t} p_2 + \nabla \cdot \hat{n} p_2 + Q(\omega)\, p_2 + \int d\omega' \int p_1(\omega', \hat{n}', \underline{r}) \Gamma(\omega, \omega', \hat{n}, \hat{n}')\, d\Omega_{n'}$$

etc.

Physically, p_0, p_1, p_2 ....etc., can be interpreted as the primary beam, once scattered, twice scattered energy, etc. For certain cases (see next paragraph), the series converges quite rapidly, and can be used. It should be noted that the "common volume" integration given in Section 3.3 is essentially the calculation of p_1 by assuming p_0 is approximately q except for a time delay factor.

(g) On the convergence of the above series, the rule of thumb is that at any distance x from source, the contribution of p_n is most if

$$x \cong \frac{n}{Q(\omega)} .$$

This is exactly true if the scattered power is all in the forward direction. An approximate deduction of such a statement is given by H. Theissing (Ref. 33) using his "one flux model". Recently Bugnolo *, using probability concept, arrived at the criterion

$$x \cong \frac{1.7}{Q(\omega)}$$

* See Ref. 24.

when the twice-scattered radiation becomes important. His value shows that at 10 Kmc this critical distance is approximately 300 Km (assume $\ell_o = 100$ m. $\left\langle (\Delta \mathcal{E})^2 \right\rangle = 10^{-12}$).

(h) Although the power resulting from high order scattering may be of small order of magnitude, this rescattered energy, nevertheless, would arrive at the receiving antenna at some later time, and should be calculated and considered as the increased noise of the transmission system.

V. Conclusion

The mean power received in a beyond-the-horizon transmission path can be explained in terms of either the fluctuation of the dielectric constant about the mean value of the dielectric constant or discontinuities in the gradient of the dielectric constant or both.

To distinguish between the two types of scattering, measurement of the wavelength dependence of the scattering cross section is suggested as a possible approach.

An extended definition of the scattering cross section can be made in terms of the spectral density transfer function. It is concluded that knowledge of this function will permit calculation of some of the important statistical characteristics of the received signal such as fading rate and distortion.

From previous work in multiple scattering, it is concluded that further work in this area is desirable especially for the case of long distance microwave links.

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APPENDIX A

Calculation of Size of the Effective Scattering Eddy (L_{eff}) Vs. Distance for Various Frequencies

By using the relation

$$k = \frac{\pi}{L} = \frac{4\pi}{\lambda} \sin\frac{\theta}{2} \tag{A.1}$$

one can determine the characteristic length L of the eddies responsible for the scattered energy received at a given distance D and wavelength λ. If we assume the standard 4/3 earth we find

$$\sin\frac{\theta}{2} \approx \frac{3}{8}\frac{D}{R} \tag{A.2}$$

where D is the distance between the transmitter and the receiver and R is the earth's radius. Substituting (A.2) in (A.1) one obtains

$$L_{eff} = \frac{2}{3}\frac{R\lambda}{D} \tag{A.3}$$

For a given frequency (wavelength λ) we may plot the effective length L_{eff} vs. distance D. It should be noted that we are computing the effective length at the lower point of the intersection of the transmitter and receiver beams.

Using Megaw's figures (Ref. 18) for the scale of turbulence where the transition between the dissipation range and the inertial range occurs one can determine L_s as a function of altitude if we make the assumption that L_s varies linearly with height. We assume therefore that

$$L_s = ah + c \tag{A.4}$$

where h is altitude and a and c are constants. Megaw gives

$$L_s = .8 \text{ cm} \qquad \text{at 1 kilometer}$$

$$L_s = 2.8 \text{ cm} \qquad \text{at 10 kilometers}$$

From these points we determine

$$L_s(\text{cm}) = \frac{2}{9}\, h(\text{Km}) + .58 \qquad (A.5)$$

At the lower intersection of the beams we find

$$\frac{D}{2} = \sqrt{2h} \qquad (A.6)$$

where D is in statute miles and h is in feet. Using (A. 5) and (A. 6) then we may plot the transition point L_s as a function of distance D. The same process may be repeated for the transition between the inertial range and the input range.

The results are shown in Figure II-3. The various values of L_{eff} for the experimental data in Appendix B are plotted on the resulting graph.

APPENDIX B

A Summary of Tropospheric Scatter Data

A resume of a number of reports and papers containing tropospheric experimental data is included here. The summary will be divided into eight categories: (1) Frequency and distance dependence of the received signal, (2) Fading characteristics, (3) Meteorological relationships, (4) Diversity measurements, (5) Polarization measurements, (6) Beam widening measurements, (7) Height-gain relationship, and (8) Aperture-to-medium coupling loss. In each case a general summary of all the relevant material obtained is followed by a more detailed summary of each reference.

1. Frequency and distance dependence. Appendix C of this report contains the results of a statistical examination of the variation of signal level with frequency and distance. It is of interest to note that some investigators represent the relationship of the signal decrease (referred to free space or to the transmitted power) with distance as a linear one varying from 0.09 to about 0.18 db/statute mile, while two sources report a logarithmic decrease of 18 db/octave. One paper also indicated that the signal level, referred to free space, decreases at 3 db per octave for increasing frequency, although it is mentioned that this figure should perhaps be revised downward to 6 db/octave on the basis of further data (Ref. 13*).

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* For convenience, all of the experimental references are listed separately under "References for Appendices B and C".

Note: Unless otherwise specified, the antenna gains given in this summary are the plane wave gains with respect to an isotropic radiator.

TABLE B-1

Frequency and Distance Dependence

Reference	Range (miles)	Frequency (Mc)	Loss (db)	
1	to 500	220	0.13 db/st mi	TL
33	94-188	400	0.19 db/st mi	TL
33	188-350	400	0.15 db/st mi	TL
33	350-618	400	0.12 db/st mi	TL
8	to 325	40-4,000	18 db/octave	FS
11	to 600	385.5	0.14 db/st mi	FS
13	----	----	18 db/octave	FS
18	to 400	3,000	0.18 db/st mi	FS

TL = Total Transmission Loss
FS = Loss with Respect to Free Space

2. Fading characteristics. The fading characteristics of the tropospheric signal may be subdivided into (a) fading rate, (b) fading depth, (c) diurnal variation in signal level, (d) yearly variation in signal level, and (e) the distribution of the amplitude of the signal envelope.

(a) Fading rate - This parameter is examined in Appendix C as a function of the transmission frequency and distance. We might conclude from the experimental results that (1) the fading rate follows a diurnal pattern with a maximum rate occurring during the afternoon hours and a minimum rate during the night-time hours, and (2) the fading rate will increase as the median signal level decreases.

(b) Fading depth - The 10-90 per cent fading depth appears to have a maximum value of about 20 db. Conflicting results are reported concerning the fading depth variation with the median signal level. See for example, References 2, 6, 9, and 28.

(c) Diurnal variation - The variation of the hourly median signal level within the day is reported to be anywhere from "insignificant" to as great as 40 db. In all cases where such a variation is reported, however, there is agreement that the maximum signal is observed during the night-time hours or at sunrise, while the minimum will occur during the afternoon hours.

(d) Yearly variation - It is fairly well established that the monthly median signal level will have a maximum value during the summer months and a minimum value in the winter. The reported differences between the highest and lowest monthly median signal levels range from 3 to 25 db, with a median value of about 10 db.

(e) Distribution - The signal level distribution on a short time basis is generally found to be Rayleigh for sample periods of perhaps less than ten minutes, while for longer periods the distribution will approach the Gaussian (or normal) distribution. Two papers reported finding a Rayleigh-distributed signal for periods up to two hours in length (Refs. 11 and 26).

A detailed summary is given in Table B-2 below.

TABLE B-2

Fading Characteristics

Ref.	FADING					EXPERIMENTAL		
	Type	Rate/sec	Depth (db)	Diurnal (db)	Yearly (db)	f (MC)	Path (miles)	Remarks (FR = Fading Rate)
1		0.05			25	220	200-400	The FR doubled as the signal decreased by 6 db.
2	R	1.2		12.8	9	3480	173	The maximum signal occurred between 2000 and 0600 hrs. The FR was found to be correlated with the wind.
3						1750	75	The signal fading depth was greater in the summer.
4	R				10	468	100	
5			8.5, 1-99% 14.5 " 29.0 " 48 " 23 "	12-15 5-10	6.5 9.5 6.5 18.0 13.5	230 230 230 230 230 100	49.3 70.2 96.6 141 226 70.2	The diurnal variation was greater in August than it was in February.
6								The signal had a maximum value at midnight and sunrise, and a minimum value in the afternoon. The seasonal effects were less pronounced on the longer paths. The fading depth (10-90%) was greater in the summer.
7			23, 1-99% 23 43 28 52 53 59			534.75 93.1 93.1 45.1 474 92.9 92.9	33.0 40.5 67.2 70.1 70.1 127.3 150.4	
8	R, 5-10 min G, longer periods	0.1-10				535	60-325	
9	R		15		19.5 13	505 4090	150 150	The FR was 8 times faster at 4090 Mc than at 505 Mc. Smaller fading depths (50-99% level) were found to occur with higher signals.
10	R, 1 min G, 16 min	1.5	10-13, 50-90%		11	400	184	
11	R, to 110 min		13.4, 10-90%		8	386	to 500	No correlation was found between the FR and the distance of transmission.
12	R	0.8-1.5			11-14	2720	215	No significant diurnal variation was found. The FR had adiurnal variation with a maximum occurrence at 1500 hrs, and a minimum between 0000 and 0900 hrs. The FR was correlated with the surface wind speed.
14		0.6-1.1	5-10 max to min			960	130-350	
15				*25-40	20-25	80	85	The diurnal variation was less in the winter. No diurnal variation occurred at the signal level exceeded 99% of the time.
16	R, short periods G, long time distribution					145 and 435	50-300	
19	R to G with increasing time intervals	2	12-14, 10-90%			3000	116-177	The FR was found to be proportional to the distance. The fading depth was found to be independent of the momentary signal median if this level was not to different from the long term median.
21	R and MR					9370	50-200	
22	R to G					204	195	
23				9	3-10	87.5	124	The diurnal variation was found to be greater in the winter.
24	R	1 cpm	5-13, 10-90%			100	30-200	The longer paths produced signals with greater maximum to minimum fading depths. No correlation was found between wind speed and the FR.
26	R up to 2 hrs; G also observed				8	238	240	
27						238	230	The FR increased for decreasing signal levels.
28		0.5	10, 10-90%	8	5	858	98-200	The FR increased and the fading depth decreased as the signal level decreased.
31		0.8-1.8 2.1-3.8 3.2-5				1250 3330 9325	46.3 46.3 46.3	The FR's are for 1000 and 1800 hrs respectively. Some diurnal variation was found which seemed to increase with increasing frequency. No correlation was found between the wind and FR.
32	R, 1 min	1.50 3.75				417 2290	188 188	
33			15 18 8 4		16 10 8 15 13 19	400 400 400 400 22 906 3670	94 188 350 618 160 213 188	
34		2.7 1.1	15, 10-90% 14 "			1046 1046	226 162	The recording circuit time constant was found to affect the signal level.
36		0.2-4.4	10-13			2720	215	The FR decreased as the antenna aperture was increased.
37	R	0.2 2.3	9-13, 10-90%		15 15	460 4110	171 171	The fading depth was 9 db for an 8' antenna and 13 db for a 60' antenna. The FR decreased as the antenna aperture was increased. A Rayleigh distribution was obtained 65% of the time at 460 Mc for 97% of the time at 4110 Mc. The FR exhibited a strong diurnal variation at 460 Mc.

R = Rayleigh; G = Gaussian; MR = Modified Rayleigh

3. Meteorological relationships. Generally it is found that (a) a correlation exists between moisture in the air and the median signal level, (b) the fading of the signal is correlated with wind speed and weather fronts or path inhomogeneities, and (c) the signal level is correlated with the refractive index or its gradient.

Ref. No.	Remarks
1	The presence of fog seemed to have no effect upon the signal level (220 Mc, 200 to 400 miles).
2	Fog and mist were found to enhance the signal level while rain depressed it. A correlation of -0.56 was found between N* and the transmission loss, and a correlation of 0.45 was found between the signal fading rate and the normal wind velocity as measured at the path midpoint and at an altitude of 1500 meters. The presence of fronts in the path produced higher signal levels and a greater diurnal variation (3,480 Mc, 173 miles).
3	Fluctuations in the signal strength were found to increase with the increase of path inhomogeneities, i.e., wind (1750 Mc, 75 miles).
5	A correlation was found to exist between the N profile and the fading of the signal,(1.046 Mc, 70 miles).
8	During hurricane conditions, the signal level was found to increase by about 35-40 db. The received power level was relatively unaffected by the weather or by variation in frequency (40 to 4,000 Mc, 60 to 325 miles).
9	The presence of fog enhanced the signal level by about 6-10 db, while snow depressed the signal level by about 4-6 db (505 and 4,090 Mc, 150 miles).
11	Winds greater than 20 knots were found to be accompanied by marked increases in the fading rate (386 Mc, distances up to 500 miles).
12	Higher signals were found to be accompanied by hot moist air, and low signals were accompanied by cold dry air (2720 Mc, 215 miles).

* $N = (n - 1) \times 10^6$ where n is the refractive index.

13 A correlation was found between the dew point temperatures and the signal levels.

15 The N profile was found to be correlated with the hourly median signal level, (80 Mc, 100 miles).

17 A correlation was found between the path loss and the refractive index, and the gradient of the refractive index for various paths.

19 The loss in field strength referred to free space was found to have a 0.55 correlation with the dew point temperature, and a 0.62 correlation with $(N - \bar{N})$, (3,000 Mc, 120 to 180 miles).

21 No correlation was found between the terminal weather and the signal variation. Refractive-index data showed that there was a tendency for higher signal levels to occur when the effective earth's radius was smaller than standard (9,370 Mc, 50 to 200 miles).

23 The fading rate was found to be greater in the winter over sea paths, and greater in the summer over land paths. Signals with little or no fading were found to occur most often at night, and least often in the morning. The diurnal variation in the signal level was found to be greatest in the winter. Cyclonic conditions were found to cause larger deviations from the average signal level than quiet conditions (87.5 to 100 Mc, 115 to 275 miles).

24 No correlation was found between the wind velocity and the fading rate. Low pressure conditions were found to produce more rapid fading than high pressure conditions. The fading rate was greater at 1400 hours than at 0700 hours, with the lowest median signal level occurring at 1400 hours (100 Mc, 30 to 200 miles).

25 Fog and mist were found to enhance the signal level, while rain and snow depressed it. The gradient of the refractive index was found to be correlated with the signal level.

28 Fog and mist were found to enhance the signal while rain or snow depressed it. The gradient of the refractive index measured over the first 5500 meters of the atmosphere at the path center was found to be correlated with the mean signal level. The fading rate was found to be correlated with the wind velocity measured at the path center (858 Mc, 98 to 200 miles).

31 No correlation was found between the wind speed and the fading rate (1,000 to 10,000 Mc, 46.3 miles).

37 The received power was found to be closely correlated with the temperature and refractive index measured at the path midpoint (460, 4,110 Mc, 171 miles).

4. Diversity measurements. Several types of diversity schemes have been considered, but to the present time, space diversity has probably been investigated most thoroughly. Space diversity results depend upon the spacing of the antennas and the characteristics of the signal being received; the correlation of the signal received by two spaced antennas will decrease with increasing fading rate of the received signal and thus improve the gain due to the diversity. There is some evidence that for the same correlation, the vertical spacing does not have to be as great as the horizontal spacing for antennas spaced transverse to the path.

Ref. No.	Remarks
2	Two antennas, spaced horizontally transverse to the great circle transmission path, and separated by 60 wavelengths, produced a gain of 4.8 db (over the signal received by a single antenna) at the signal level exceeded 90 per cent of the time. This gain increase corresponds to a correlation coefficient of 0.35 (3,480 Mc, 173 miles).
4	The median value for the correlation coefficient between fields for a dual-feed parabola at the receiver site was 0.9 (468 Mc, 289 miles).
9	Two 28-foot paraboloidal antennas spaced horizontally transverse to the transmission path and separated by 200 feet yielded percentile signal levels 4 to 16 db higher than signals received without diversity, with a median difference of 10.3 db (505, 4,090 Mc, 150 miles).
10	The correlation between signals received by two corner reflectors, one deflecting horizontally and the other vertically, was found to

vary grossly over hourly periods, but indicated that the correlation decreased with decreasing signal level (425 Mc, 161 miles).

24 The correlation between signals received by antennas spaced horizontally transverse to the transmission path decreased with increasing antenna separation, but increased with decreasing signal fading rate. The correlation was 0.5 for signals with 1 to 5 second fading periods and 1.0 for fading periods over ten minutes for antennas separated by 45 meters. Corresponding values for the correlation for antennas separated by 370 meters are 0.0 to 0.1 and 0.9 (100 Mc, 30 to 200 miles).

28 The correlation between signals received by antennas spaced horizontally transverse to the transmission path fell to 0.2 at about 60 feet separation, and for spacing vertically transverse to the path fell to 0.2 at about 40 feet. The cross correlation between the signals received by the two antennas had a maximum value for 1 to 2 seconds time difference (858 Mc, 200 miles).

35 Angular diversity reception by a 28-foot antenna for beams separated by 0.8 degrees produced an improvement of 4 to 6 db over a Rayleigh distribution having a mean equal to the geometric mean of the two signals received. The improvement for a beam separation of 1.2 degrees was about 8 db over a Rayleigh distribution obtained by normalizing the two received signals to equal median values. These improvements were for the signal level exceeded 90 per cent of the time, and for dual selection diversity. Frequency sweep experiments showed that the correlation coefficient fell to zero between 3 and 5 Mc separation for a single transmitting and receiving antenna system (2290 Mc, 188 miles).

36 The correlation between the signals received by 4-foot antennas spaced horizontally transverse to the transmission path was found to fall to 0.2 for a separation of 12 feet. The time shift required for maximum correlation between the signals was 0.2 to 0.7 seconds for a spacing of 20 feet (2,720 Mc, 215 miles).

37 The improvement realized for a twin-feed diversity scheme was found to decrease as the fading rate of the received signal decreased. Antenna beams separated by 0.75 degrees in the vertical direction were found to yield the predicted improvement in all cases (4,110 Mc). For antenna beams separated by 2.33 degrees in the horizontal direction, the diversity improvement was effective about 20 per cent of the time at 460 Mc and for most cases at 4110 Mc.

38 Space diversity reception was found to be effective at an antenna separation of 20 wavelengths due to the low correlation between the signals at the two antennas (134, 432 Mc, 120 miles).

5. Polarization tests. The concensus of opinion regarding the cross-polarized component of the signal seems to be that this component will be about 12-30 db less than the signal received with the correct polarization, and may be undetectable. Simultaneous reception of the vertical and horizontal components of the incident signal for transmission at 45 degrees polarization indicates near synchronous fading (Ref. 10).

Ref. No.	Remarks
8	No horizontal component could be found above the noise when transmitting a vertically polarized signal (3,700 Mc, 22 to 285 miles).
10	For transmission of either vertically or horizontally polarized signals, the cross-polarized component was found to be 12 to 29 db below that received with the transmitted polarization. Transmission with a polarization of 45 degrees and reception with two antennas, one polarized horizontally and the other vertically, showed that the correlation between the two signals was about 0.6.
18	The vertical component of the received signal was found to be down 20 db or more with transmission of a horizontally polarized signal (3,000 Mc).
26	No practical results were observed with a change in polarization (238 Mc).
28	The horizontal component of the received signal was found to be down 30 db with transmission of a vertically polarized signal (858 Mc).

6. Beam widening measurements. Beam widening experiments are usually conducted by rotating the transmitting and/or receiving antennas in azimuth

or elevation and comparing the resulting gain diagram with the original antenna radiation pattern, or with the product of the transmitting and receiving antenna gain diagrams. The difference between expected and calculated values is deduced to be due to the tropospheric scattering mechanism. The results of these tests seem to indicate a beam widening of 1 or 2 degrees at most.

Ref. No.	Remarks
1	No beam widening was observed when using a 27 db, 4.0 degree beamwidth antenna (220 Mc, 200 miles). Only the transmitting antenna was rotated.
8	The antenna beams were not widened by more than 1 or 2 degrees in elevation or azimuth when using a transmitting antenna of about 23 db gain and a receiving antenna of about 18 db (459 Mc, 183 and 290 miles). A signal was received over a 30 degree azimuthal angle with an antenna which had a normal beam of 5 degrees during hurricane conditions (3,700 Mc).
10	Simultaneous rotation in azimuth of both the transmitting and receiving antennas, each with a gain of 47.5 db, produced scatter patterns 1.2 to 1.7 degrees wider than the multiplied free space pattern at the -12 db points (3,670 Mc, 188 miles).
28	Simultaneous rotation in both azimuth and elevation of the 35 db transmitting antenna and the 23 db receiving antenna produced a scatter pattern 0.5 to 1.0 degrees wider than the multiplied free space pattern at the -12 db points (858 Mc).
33	Simultaneous azimuthal rotation of the transmitting and receiving antennas, each with a 0.65 degree half-power beamwidth, produced a scatter pattern about 1 degree wider than the multiplied free space pattern at the -12 db points. The same results were obtained for simultaneous rotation in elevation (3,670 Mc, 188 miles).
37	Scanning tests indicate that the beam is broadened in both elevation and azimuth by a factor of about 3 on the average. A 60-foot antenna with a free-space pattern 0.5 degrees wide at the -10 db points was used (4,110 Mc, 171 miles).

7. Height-gain relationship. Little data is available concerning the variation of the signal level with height of the transmitting and receiving antennas. The results obtained are given below.

Ref. No.	Remarks
1	No variation of signal strength with altitude was deducible from aircraft tests conducted at 500 to 15,000 feet altitude (220 Mc).
28	At a distance of 98 miles a well-marked maximum was observed in moving the receiving antenna from the ground level to a height of 15 to 20 feet, and the signal then decreased by 3 db as the height was increased to 40 feet. At a distance of 200 miles, well-marked maxima and minima were observed to a height of 30 feet, but little coherence was observed between the various tests, above this height (858 Mc).
31	With the transmitting antenna at a height of 194 feet, the receiving antenna was lowered from 194 feet to 4 feet and the signal received above 130 feet was found to follow a diffracted field pattern, while below 130 feet the signal fluctuated about at approximately 50 db transmission loss. When the transmitting antenna was placed at 4 feet, the received signal varied about a level of 70 db loss over the whole receiving antenna height range (9,375 Mc, 46.3 miles).

8. Aperture-to-medium coupling loss. This is the phenomenon which is observed when antennas are found to yield gains less than their plane-wave values. The point beyond which the full gain is not realized occurs between 30 and 40 db, and the loss increases as the gain is increased above this point.

Ref. No.	Remarks
2	The gain of an 8-foot parabolic antenna with a plane wave gain of 36.5 db was reduced by 1.2 db with horizontal polarization and by 1.6 db with vertical polarization when used on a tropospheric path (3,480 Mc, 173 miles).

8 The substitution of a half-wave dipole for an antenna with a plane wave gain of about 18 db indicated that the plane wave gain was realized (459 Mc, 183 and 290 miles).

9 The gain reduction for a 31 db antenna was found to be 1 db (505 Mc), and the gain reduction for a 46 db antenna was found to be 2 to 4.5 db (4,090 Mc).

10 The plane wave gains of 12 to 28.6 db antennas were realized at the receiver over a tropospheric scatter path (425 Mc, 161 miles). The measured gain ratio for a combination of a 28-foot and 5-foot antenna at the receiver was found to be 9.4 to 11.5 db compared with the plane wave gain ratio of 18.2 db (3,670 Mc, 188 miles).

31 The measured gain ratios of various combinations of 4-foot and 1-foot antennas at the transmitting and receiving sites were found to vary between 5 to 7.1 db compared with the plane wave gain ratio of 12 db (9,375 Mc, 46.3 miles).

33 Measurements using two 60-foot and two 28-foot antenna systems showed that the ratio of observed signal level for the two systems equalled the ratio of the plane wave gains of the antennas (400 Mc, 618 miles).

35 Various combinations of antennas were used over a 188 mile path at a frequency of 2,290 Mc and the results showed that the realized gain is equal to the product of the plane wave gains of the transmitting and receiving antennas up to a gain product of about 35 db, and for larger gain products the realized gain is less than the products of the plane wave gains, decreasing in a linear fashion to the point where the realized gain of 82 db is obtained for a gain product of 90 db.

36 Four-foot antennas were used in parallel combinations with as many as 6 antennas being used together, and the realized gain for the combination of 6 antennas in parallel ranged to as much as 5 db below the expected gain. The realized gains seemed to follow a diurnal variation with the maximum gains being measured during the morning hours (2,720 Mc, 215 miles).

37 The ratio of the median signal levels received by a 60-foot and an 8-foot antenna over a tropospheric path was 5.7 db compared with a plane wave gain ratio of 17.0 db. The difference between median signal levels received by the two antennas decreased with decreasing signal level (4,110 Mc, 171 miles).

APPENDIX C

A Statistical Evaluation of Scatter Parameters

In this appendix an effort has been made to use the available experimental data on tropospheric scatter propagation to obtain by statistical methods a formula for the dependence of the transmitted power on distance and wavelength. It is also shown how one could use the same method to obtain the loss in effective antenna gain and study the dependence of short-time fading on distance and frequency.

1. Distance and frequency dependence. In the past ten years considerable progress has been made in understanding the phenomenon of tropospheric scatter propagation. However, none of the several theories proposed has been adequately in agreement with the results of the experiments performed.

So far it has been established beyond question that the received power is a function of the transmitter-to-receiver distance d, the wavelength λ of the transmitted wave and the gain of the antennas involved. The degree of that dependence though is still a point of discussion. To be more specific it is generally agreed that as a first approximation one can write that

$$\frac{P_R}{P_{FS}} = k\, d^m \lambda^n \qquad (C.1)$$

where

P_R = received power

P_{FS} = free-space power

d = distance

λ = wavelength

and k is a factor covering all other experimental parameters. However, the suggested values of the exponents of d and λ, m and n, in the above-mentioned theories vary from 0.3 to 1.5 for n and from -5 to -8 for m*. This difference results from the fact that for a theoretical calculation of m and n a knowledge of the variation of the dielectric constant with elevation is required. But actual measurements are difficult to obtain, hence the usual practice is to assume a variation and proceed to calculate m and n. It is obvious then that the obtained values depend on the assumption made, rendering the whole calculation a flair of guessed estimation.

It is the purpose of this discussion to show that m and n can be calculated by statistical methods from the available experimental data without any assumptions or other restrictions being necessary.

Repeating formula (C.1) again, we have that

$$\frac{P_R}{P_{FS}} = k\, d^m \lambda^n \quad . \tag{C.1}$$

Taking the logarithm of this we obtain

$$10 \log \left(\frac{P_R}{P_{FS}}\right) = 10 \log k + 10\, m \log d + 10\, n \log \lambda$$

or

$$F = K + mD + n\Lambda \tag{C.2}$$

*See October 1955 issue of IRE Proc.

where

$$F = \text{free space loss in db}$$
$$K = 10 \log k$$
$$D = 10 \log d$$
$$\Lambda = 10 \log \lambda$$

and logarithms are to the base 10. Now eq. (C. 2) is the equation of a plane in a cartesian space, where F is plotted versus D and Λ . Theoretically, all experimental measurements should yield points on that plane, but actually we know that the plane represents the expected value of the measured free space loss. It is clear then that if the free space loss in db as obtained from experiments is plotted against D and Λ, a plane may be passed through these points such that the deviation between the experimentally obtained values and the values calculated from (C. 2) is a minimum. The values of m and n can be obtained as follows:

Let us form the expression,

$$\sum_{i=1}^{N} (F_i - F_e)^2 = \sum_{i=1}^{N} (F_i - K - mD_i - n\Lambda_i)^2 \qquad \text{(C. 3)}$$

where F_i is the experimental free space loss, F_e is the expected loss from eq. (C. 2), D_i and Λ_i are the appropriate values from the i^{th} experiment and N is the total number of observations. If we partially differentiate (C. 3) with respect to K, m and n, and set these expressions equal to zero we will then have three equations in three unknowns, which when solved will yield the values for K, m and n which minimize (C. 3). These equations are:

$$\left.\begin{aligned}
\sum_1^N F_i &= NK + m \sum_1^N D_i + n \sum_1^N \Lambda_i \\
\sum_1^N F_i D_i &= K \sum_1^N D_i + m \sum_1^N D_i^2 + n \sum_1^N \Lambda_i D_i \\
\sum_1^N F_i \Lambda_i &= K \sum_1^N \Lambda_i + m \sum_1^N D_i \Lambda_i + n \sum_1^N \Lambda_i^2
\end{aligned}\right\} . \qquad (C.4)$$

If we perform the indicated summations and divide through by N to find the average values, (C. 4) will become:

$$\begin{aligned}
\overline{F} &= \hat{K} + \hat{m}\overline{D} + \hat{n}\overline{\Lambda} \\
\overline{FD} &= \hat{K}\overline{D} + \hat{m}\,\overline{D^2} + \hat{n}\ \overline{\Lambda D} \\
\overline{F\Lambda} &= \hat{K}\overline{\Lambda} + \hat{m}\,\overline{D\Lambda} + \hat{n}\,\overline{\Lambda^2}
\end{aligned} \qquad (C.5)$$

where ($\wedge$) indicates the value for best fit and ($\overline{}$) indicates average values.

Solving for $\hat{K}$ in the first equation and replacing in the second and third equations of (C. 5) we obtain,

$$\begin{aligned}
\overline{FD} - \overline{F}\,\overline{D} &= \hat{m}(\overline{D^2} - \overline{D}^2) + \hat{n}(\overline{D\Lambda} - \overline{D}\,\overline{\Lambda}) \\
\overline{F\Lambda} - \overline{F}\,\overline{\Lambda} &= \hat{m}(\overline{\Lambda D} - \overline{\Lambda}\,\overline{D}) + \hat{n}(\overline{\Lambda^2} - \overline{\Lambda}^2) \quad .
\end{aligned} \qquad (C.6)$$

Now let

$$\overline{FD} - \overline{F}\,\overline{D} = Q_1$$
$$\overline{F\Lambda} - \overline{F}\,\overline{\Lambda} = Q_2$$

and

$$\overline{D^2} - \overline{D}^2 = S_{11} \qquad \overline{D\wedge} - \overline{D}\,\overline{\wedge} = S_{12}$$

$$\overline{\wedge D} - \overline{\wedge}\,\overline{D} = S_{21} \qquad \overline{\wedge^2} - \overline{\wedge}^2 = S_{22} \quad .$$

Then one can write (C.6) in matrix form:

$$S\hat{b} = Q$$

where

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \qquad \hat{b} = \begin{pmatrix} \hat{m} \\ \hat{n} \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} .$$

Let $\mathcal{S}$ be the determinant of S, $\mathcal{S} \neq 0$. Then S^{-1} exists and

$$\hat{b} = S^{-1}Q = CQ \tag{C.7}$$

where

$$C = S^{-1} = \begin{pmatrix} \frac{S_{22}}{\mathcal{S}} & - \frac{S_{21}}{\mathcal{S}} \\ - \frac{S_{12}}{\mathcal{S}} & \frac{S_{11}}{\mathcal{S}} \end{pmatrix}$$

or expanding (C.7):

$$\hat{m} = C_{11}Q_1 + C_{12}Q_2 \tag{C.8}$$

$$\hat{n} = C_{21}Q_1 + C_{22}Q_2 \tag{C.9}$$

where

$$C_{ij} = (-1)^{1+j} \frac{S_{ji}}{\mathcal{S}} .$$

Equations (C.8) and (C.9) are the desired values for m and n.

One can easily find the variance* in $\hat{m}$ and $\hat{n}$ from:

$$V(\hat{m}) = C_{11}\,\sigma^2(F)$$

$$V(\hat{n}) = C_{22}\,\sigma^2(F)$$

where

$$\sigma^2(F) = \left[\frac{\overline{F^2} - \overline{F}^2 - \hat{m}Q_1 - \hat{n}Q_2}{N-2}\right] .$$

When the above calculations are performed on the experimental data collected (see Table C-1), the following numerical results are obtained:

$S_{11} = 5.00$

$S_{12} = S_{21} = -3.438$

$S_{22} = 35.68$

$\mathcal{S} = 166.58$

$C_{11} = 0.2142$

$\overline{\Lambda} = 18.69$

$C_{12} = C_{21} = 0.0206$

$C_{22} = 0.0300$

$Q_1 = -38.39$

$Q_2 = 60.02$

$\sigma^2(F) = 1.1148$

$\overline{D} = 21.84$

so that

$$\hat{m} = -6.99$$

$$\hat{n} = 1.01$$

$$\hat{K} = 66.66$$

$$k = 4.64 \times 10^6$$

$$V(\hat{m}) = 0.2388$$

$$V(\hat{n}) = 0.03344$$

$$\sigma(\hat{m}) = 0.4887$$

$$\sigma(\hat{n}) = 0.1829$$

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*See C. R. Rao, Advanced Statistical Methods in Biometric Research, John Wiley Publ. Co.

TABLE C-1

Transmission Loss (With Respect to Free Space) Data:

Below, in tabular form, appear the data from which the values for m, n and r were obtained.

Reference	Wavelength	Distance	Loss	G Transmitter	G Receiver
	cm	mi	db	db	db
1	136.36	200	-61	27	15.5*
4	64.10	289	-99	33	34
7	322.23	67.2	-44	-	-
7	665.19	70.1	-35	-	-
7	10.71	70.1	-50	-	-
7	63.29	70.1	-32	-	-
7	322.92	127.3	-62	-	-
7	322.92	150.4	-63	-	-
7	53.60	165.5	-76	-	-
7	143.03	189.0	-77	-	-
7	324.32	55.5	-30	-	-
7	324.32	119.0	-52	-	-
7	324.32	129.0	-64	-	-
7	324.32	133.0	-50	-	-
7	324.32	142.0	-50	-	-
7	324.32	162.0	-57	-	-
7	324.32	189.0	-65	-	-
7	324.32	229.0	-61	-	-
9	59.41	150	-68	31	31
9	7.33	150	-83	46	46
11	75.00	100	-51	31*	25*
11	75.00	600	-116	31*	25*
12	11.03	215	-79	-	-
14	31.25	132	-86+	33*	36
14	31.25	197	-87+	33*	36
14	31.25	350	-94+	33*	36
18	10.00	400	-95	36*	29
19	10.00	118	-67	37	33*
19	10.00	180	-72	37	33*
19	10.00	161	-86	37	33*
28	34.96	98	-54	35	14.6
28	34.96	187	-68	35	14.6
28	34.96	200	-73	35	14.6

* Mean of values given

+ Calculated from transmission loss

and finally assuming a normal distribution and for 95 percent certainty:

$$\hat{m} = -6.99 \pm 0.96$$

$$\hat{n} = 1.01 \pm 0.36$$

or for 50 percent certainty:

$$\hat{m} = -6.99 \pm 0.33$$

$$\hat{n} = 1.01 \pm 0.12 \quad .$$

On the basis, therefore, of these data the formula for the received power referred to free space is:

$$\frac{P_R}{P_{FS}} = 4.64 \times 10^6 \frac{\lambda}{d^7} \qquad \text{(c.10)}$$

where λ is in cm and d in miles. This expression permits the calculation of the received power when the transmitted power, frequency and the transmitter-to-receiver distance are known. One can also find the variance of F_e from:

$$\sigma^2\left[\hat{F}_e(\hat{m}, \hat{n}, \hat{K})\right] = \sigma^2(F)\left[\frac{1}{N} + (D-\bar{D})^2 C_{11} + (\Lambda - \bar{\Lambda})^2 C_{22} + 2(D-\bar{D})(\Lambda - \bar{\Lambda}) C_{12}\right]$$

where $\hat{F}_e = 10 \log_{10} \left(\frac{P_R}{P_{FS}}\right)$, as calculated from (C.10).

The values in eq. (C.10) were obtained from the data of experiments available in the current literature. These experiments were performed by different experimenters in places scattered around the world, some over water, some over flat land, some over rough terrain. Almost all the data supplied are the averages from long-period observations including both the cold and warm months of the year. The random nature of these data therefore make eq. (C.10) an excellent expression for yearly median level calculations.

If the proper data were available using the method described above, equations for transmission over water or flat land could have been obtained that might fit these situations with closer tolerances.

One more remark here is in order. It would have been more accurate, and therefore desirable, to express the ratio of received to free-space power as a function of the scattering angle θ rather than the distance d between transmitter and receiver. Unfortunately, insufficient data on θ are available.

However, it must be pointed out that in some of the experiments foreground obstruction is mentioned, which usually implies a larger scattering angle than necessary for the actual path distance. Hence, were effective distances available, an exponent less than 7 could be expected.

2. Effective aperture variation. It has been found that in tropospheric propagation the plane wave antenna gain is not fully realized. The obtained gain is found to fall off rapidly for a plane wave gain of 30 - 40 db and higher.

Although the available data (Table C-1) are not enough to permit conclusions from a statistical analysis approach, we will go here through such an investigation mainly to indicate how it would be done and the result will be interpreted to indicate the order of magnitude. Let us write then that as a first approximation,

$$\frac{P_R}{P_{FS}} = k_1 d^m \lambda^n a_e^r \qquad (C.11)$$

where a_e is the combined effective plane wave aperture area, which may be calculated from the plane wave gain from

$$g = \left(\frac{4\pi}{\lambda^2}\right)^2 a_T \epsilon_T a_R \epsilon_R = \left(\frac{4\pi}{\lambda^2}\right)^2 a_e \qquad (C.12)$$

where a_T and a_R are the actual antenna areas, and ϵ_T and ϵ_R are the antenna efficiencies. Then $a_e = (a_{Te}\, a_{Re})$ and $g = g_T g_R$. Then taking logarithms and multiplying by 10 gives:

$$F = K_1 + mD + n\Lambda + rA$$

where $A = 10(\log a_e)$ and the other symbols are defined in eq. (C.2). Now by extending the previously described method to four dimensions and using the available data to obtain numerical answers, we find that:

$$\hat{r} = -0.8 \qquad \hat{K}_1 = 124.63$$
$$\hat{m} = 7 \qquad k_1 = 2.90 \times 10^{12}$$
$$\hat{n} = 2 \qquad .$$

Our calculations have been carried out for d in miles, and antenna areas in cm^2. The results indicate that the effective scatter area is the 0.8 power of the combined effective plane wave areas, that is $a_{es} = (a_{Te}\, a_{Re})^{0.8}$. This constitutes a reduction of the effective plane wave area, a fact which is supported by experiment.

It must be noted here that the value obtained for $\hat{n}$ differs from the value calculated in part one of this report. Since λ and a are independent, this is against theoretical expectations. However, the covariance of a and λ as calculated from the data of the few experiments available indicates an appreciable correlation between λ and a and this fact is reflected in the value of the exponent of λ.

As it was mentioned previously, eq. (C.11) is a first approximation. If a plot is made of modified free space loss, $F - \hat{m}D - \hat{n}\Lambda$, versus antenna area from the data of Table C-2, a curve with a drooping characteristic is obtained (see Fig. C-1). An equation that will give a better approximation to this curve is:

$$f(a) = \frac{1}{k_1 a}\left(1 - e^{-k_1 a}\right) + k_2 a^n e^{-ma} \qquad (C.13)$$

where k_1, k_2, m and n are constants. However, this equation is much harder for statistical manipulation and will not be considered at this time.

3. Fading rate characteristics. In a similar manner now one could perform an investigation to establish the dependence of fading rate on distance and frequency. Here again the available data (Table C-3) are scant and the numerical results of necessity will be unreliable. Keeping this in mind we will proceed then with the analysis.

Let the fading rate be described by:

$$FR = c\, f^p d^q \qquad (C.14)$$

f = frequency in cycles/sec

d = distance in miles

and c is a parameter independent of f and d. Taking logarithms we have:

$$\log_{10} FR = \log_{10} c + p \log_{10} f + q \log_{10} d \ .$$

The best values for p and q then will be obtained from the plane of best fit to the experimental points in the cartesian space of $\log_{10} FR$, $\log_{10} f$ and $\log_{10} d$.

TABLE C-2

Modified Free Space Loss and Combined Antenna Area

(Data for Figure C-1)

$$a_e = g_T g_R \left(\frac{\lambda^2}{4\pi} \right)^2$$

Reference	$\frac{a_e}{m^4}$	$\frac{FSL - \hat{m}D - \hat{n}\Lambda}{db}$
1	3.91×10^2	+78.7
4	1.95×10^3	+55.2
9	1.25×10^3	+66.6
9	1.25×10^3	+56
9	2.9×10^2	+60.7
11	1.18×10^3	70.1
11	1.18×10^3	63.5
14	4.81×10^2	47.5
14	4.81×10^2	58.6
14	4.81×10^2	69.1
18	2.25×10^0	77.1
19	6.34×10^0	75.8
19	6.34×10^0	58.3
19	6.34×10^0	68.0
28	8.67×10^0	69.9
28	8.67×10^0	75.6
28	8.67×10^0	72.6
37	6.38×10^0	74.7

$\hat{m} = -7.0$

$\hat{n} = 1.0$

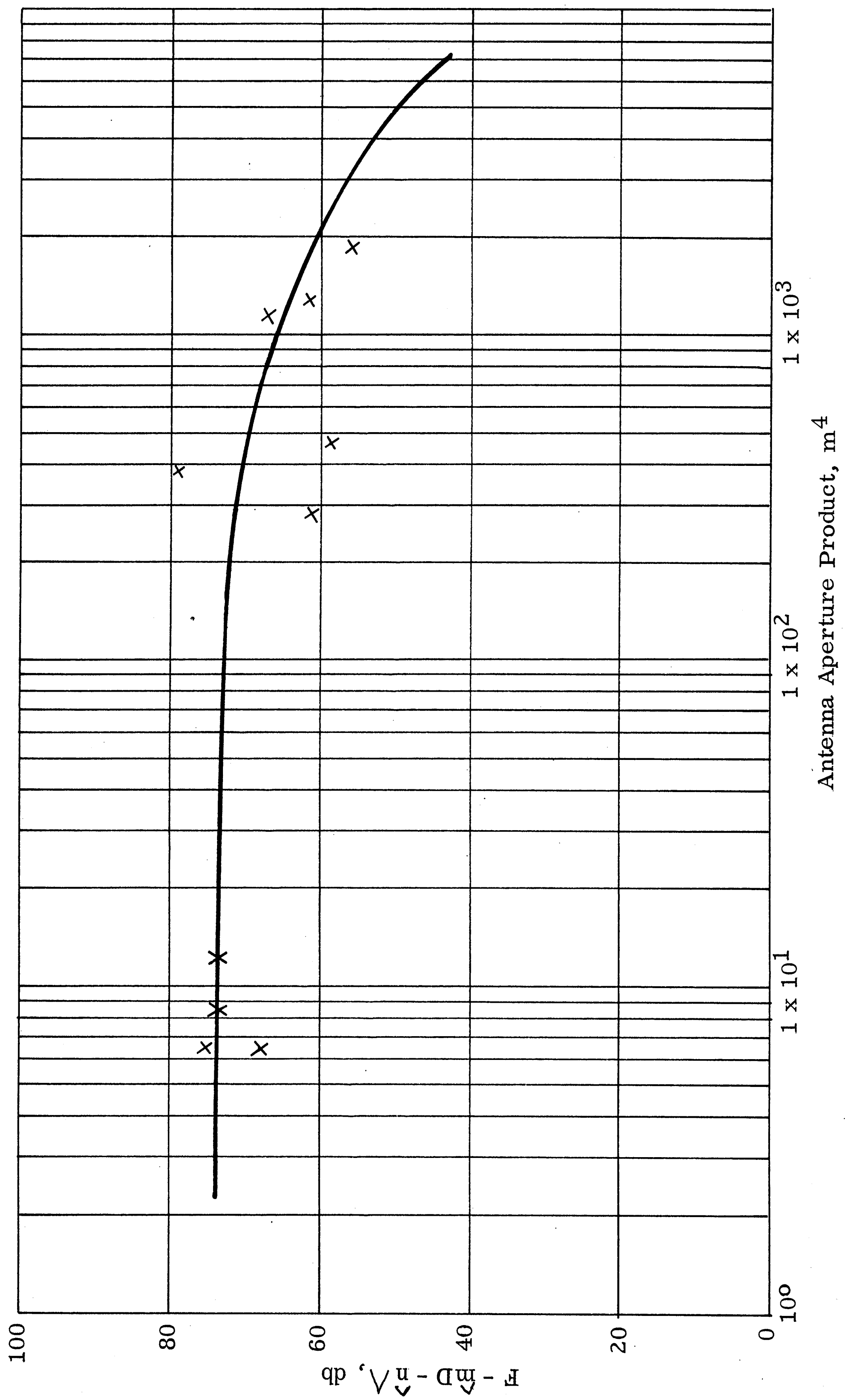

Figure C-1: Modified Free Space Loss

TABLE C-3

Fading Rate Data

(Data for Fading Rate Calculations)

Reference	Fading Rate FR c/s	f Mc	d mi	V - Calculated from Booker-Gordon mph
2	1.2	3480	173	5.3
10	1.5	3995	184	54.1
12	1.15*	2720	215	5.2
14	0.85*	960	242*	9.7
19	2.0	3000	146*	12.1
28	0.5	858	149*	10.4
31	1.3*	1250	46.3	59.6
31	2.95*	3330	46.3	50.7
31	3.25	9325	46.3	20.0
34	1.11*	1046	162	17.4
34	2.69*	1046	226.5	30.1

* Mean of given range

Using the same notation as in Part One:

$$Q_1 = \overline{(FR)\,D} - \overline{(FR)}\,\overline{D} = -0.0296$$

$$Q_2 = \overline{(FR)\,F} - \overline{(FR)}\,\overline{F} = 0.0447$$

$$S_{11} = \overline{D^2} - \overline{D}^2 = 0.0757 \qquad C_{11} = 17.641$$

$$S_{12} = S_{21} = \overline{DF} - \overline{D}\,\overline{F} = -0.0508 \qquad C_{12} = C_{21} = 7.449$$

$$S_{22} = \overline{F^2} - \overline{F}^2 = 0.1351 \qquad C_{22} = 9.891$$

$$\sigma(q) = 0.276$$

$$\sigma(p) = 0.206$$

$$\log_{10} c = C = 0.1486 \qquad c = 1.41 \quad \text{and} \quad FR = 1.41\,\frac{f^{0.22}}{d^{0.19}} .$$

When the calculations are carried out (see page 92) we find that for 95 percent certainty,

$$\hat{q} = -0.19 \pm 0.54$$
$$\hat{p} = 0.22 \pm 0.40$$
$$C = 0.14855$$
$$c = 1.41$$

and for 50 percent certainty,

$$\hat{q} = -0.19 \pm 0.19$$
$$\hat{p} = 0.22 \pm 0.14 \quad .$$

Obviously the tolerance range is too large to allow any conclusions, even for 50 percent certainty.

For a better investigation of the fading rate, data on the wind velocity in the scatter volume must also be available since undoubtedly the wind velocity in the scatter volume has a bearing on the fading rate*. In that case, then eq. (C.14) must be written so as to include a velocity term and a four-dimensional analysis carried out. There is some indication that correlation exists between fading rate and antenna aperture (see Refs. 47 and 48). To account for this, an aperture term might also be introduced.

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*The expression for fading rate given by Booker and Gordon in "Radio Scattering in the Troposphere", IRE Proc., April 1950, is:

$$FR = \frac{2vf}{c} \sin \frac{\theta}{2} \simeq \frac{vfd}{ca}$$

where d = distance; c = speed of light; f = frequency; θ = scattering angle; v = vertical wind velocity; a = radius of the earth

and the approximation is for small $\frac{\theta}{2}$, so that $\sin \frac{\theta}{2} \cong \frac{d}{2a}$. Using the above formula and the data available, we obtain velocities from 5 to 60 mph.

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