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Annual Report

THE EFFECT OF ANISOTROPIC SCATTERING ON RADIATIVE TRANSFER

AFSWP

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FORWARD

This report was typed by Mrs. Marguerite Schaible. Dr. George C. Clark, now with the Continental Oil Company, James R. Street and Larry B. Evans assisted in the early phases of the research. The machine computations were carried out on the IBM-704 at the University of Michigan Computing Center.

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