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AN EXPERIMENTAL FOLDED LOOP ANTIENNA

Technical Report No. 69

Department of Electrical Engineering
Electronic Defense Group

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ABSTRACT

From a mechanical standpoint, the folded loop antenna is essentially a half-wave folded dipole which has been bent into the shape of a letter C, except that the spacing of the parallel conductors is much greater than in the folded dipole and the method of coupling to the transmission is different. The radiation pattern and polarization are essentially the same as those of a simple loop. The radiation efficiency is high, and the half-power bandwidth is about 8 percent of the center frequency when connected to a 50-ohm resistive source. The center frequency can be adjusted by placing a variable capacitor across the open gap of the C, although this reduces the bandwidth. By removing the upper cross-beam support and the upper part of the mast, the bandwidth can be extended to about 15 percent of the center frequency. A study is being conducted to find a means for tuning the antenna by flexing or telescoping the loop in addition to varying the gap capacitance, in order to maintain a wide bandwidth as the center frequency is changed.

AN EXPERIMENTAL FOLDED LOOP ANTENNA

1. INTRODUCTION

The folded loop antenna may be described as being similar to an ordinary half-wave folded dipole which has been bent into the form of a letter C, except that the parallel conductors are more widely separated than in the usual folded dipole. Also, the method of coupling to the feed line is different. The construction of the folded loop is illustrated in Figure 1, which will be described in greater detail subsequently.

The folded loop antenna was chosen to fulfill certain requirements in a specific application. These requirements were (1) small size, (2) vertical polarization, (3) a simple pattern in azimuth, having one or more moderately deep nulls, (4) reasonably high radiation efficiency, (5) sufficient bandwidth so that matching or tuning adjustments are not critical, and (6) an impedance which is relatively independent caused by the presence of ground, personnel, motor vehicles, etc. With the exception of the efficiency and bandwidth requirements, most small-loop type antennas meet the above requirements adequately. To improve efficiency and bandwidth, either large conductors or smaller parallel conductors can be used to minimize inductance and ohmic resistance. The folded loop is designed on the parallel conductor principle. When the particular model shown in Fig. 1 connected to a 50-ohm source and when the capacitor plate (F) is set for minimum capacitance, the efficiency is in excess of 89 percent and the bandwidth is equal to 8 percent of the center frequency. When the mast (A), the upper cross-beam (C), and the

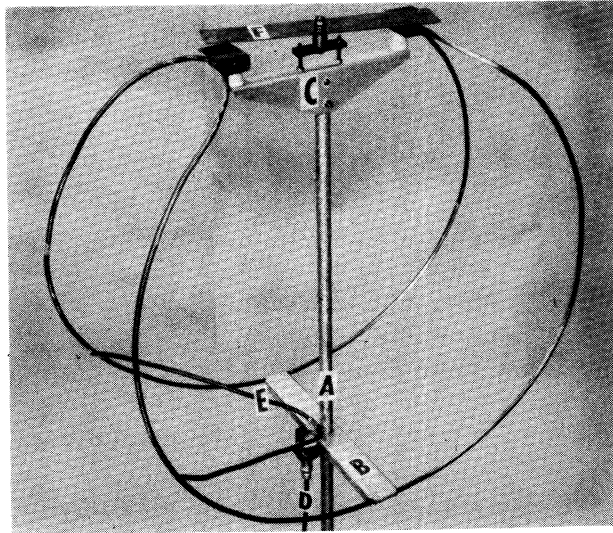


FIG. 1 FOLDED LOOP ANTENNA

(A) MAST; (B) LOWER CROSS-BEAM;
(C) UPPER CROSS-BEAM; (D) FEED LINE;
(E) FEED LOOP; (F) CAPACITOR ROTOR
PLATE.

capacitor plates are removed entirely, the Q of the loop is only about 14. In this stripped condition, and with the driving loop brought to resonance by the addition of a series capacitor, the bandwidth is increased to about 15 percent of the center frequency when connected to a 50-ohm resistive source.

2. PHYSICAL CHARACTERISTICS

The folded-loop antenna shown in Figure 1 is constructed as follows: A copper (or aluminum) rod or tube with a diameter of about $1/500$ wavelength is formed into a closed curve having the shape of a race-track with straight sides and rounded ends. The length of the major axis of this curve is about $1/2$ wavelength and the distance between the straight sides is about $1/15$ wavelength. The curve is then spread slightly so that the sides are no longer straight, but bowed out to a maximum separation at the center equal to about $1/10$ wavelength. Next, the antenna is wrapped around an imaginary circular cylinder having an axis parallel to the minor axis of the race-track and having a diameter of about $1/5$ wavelength. The resulting "folded loop" then has a spacing of about $1/10$ wavelength across the open gap of the C, a diameter of about $1/5$ wavelength, and a dimension of about $1/10$ wavelength in the direction normal to the loop plane.

The complete assembly is shown in Fig. 1. A vertical mast (A) supports two horizontal cross-beams (B) and (C). The folded-loop is supported at the bottom by the lower cross beam (B). This cross-beam, the mast, and the folded-loop are all electrically connected to each other. The top of the folded loop is supported by two porcelain stand-off insulators at the ends of the upper cross-beam (C). At each insulator, a capacitor stator plate is attached to the loop. The rotor plate (F) of the capacitor is electrically insulated from all other parts of the antenna, including the cross-beam (C) to which it is

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mechanically attached. The cross-beam and the stand-off insulators add a certain amount of stray fixed capacitance in parallel with the variable capacitance, and this decreases the tuning range in the same manner as in any tunable resonant circuit. The particular model of the folded-loop antenna pictured here has a center frequency which is variable from 75 to 100 mc/s. This frequency range is not related to the ultimate application of the antenna. It was selected to correspond to the frequency range of some of the test equipment available at the time of construction.

The U-shaped conductor (E) is called the feed loop. Its size and point of attachment to the folded loop are determined experimentally. The 50-ohm feed line (D) is connected to a coaxial connector, the outside conductor of which is connected to the center of the lower cross-beam (B) and the center conductor of which is connected to the center of the feed loop (E). An auxiliary stand-off insulator is used to support the center of the feed loop to avoid strain on the coaxial connector.

The model shown in Figure 1 is constructed of materials selected on the basis of availability at the time of assembly. The mast (A) is a galvanized steel pipe having an outside diameter of about $5/8$ inch. Since the electrical contribution of the mast to the antenna performance is small, the material of which it is constructed is unimportant, except for mechanical strength requirements. (Some other models have been constructed in various sizes using aluminum masts.) The lower cross-beam (B) is made of tinned copper, soldered to the loop. The folded loop itself is made of $3/16$ inch o.d. copper tubing. The feed loop (E) is made of the same tubing. The upper cross-beam (C) is made of aluminum. The stator plates and the rotor (F) of the variable capacitor are made of copper. The short bridge on which the rotor shaft is mounted is made of impregnated fibre insulating material. The standoff insulators supporting the ends

of the loop are porcelain. The connection to the coaxial feed line is made through a type BNC connector. Several other models (not shown) of the folded loop antenna have been constructed with a capacitor motion which is parallel to the mast axis rather than rotational about it.

3. RADIATION PATTERN

An ideal small loop exhibits a radiation pattern which, in the cross-section normal to the loop, consists of two tangent circles of equal size. The directivity of this pattern is 1.5 or 1.76 decibels.¹ The folded loop described here has essentially the same pattern, except that one of the circles is slightly larger than the other, as shown in Fig. 2. The directivity of the folded loop is estimated from the pattern to be 1.8 decibels. The pattern of Fig. 2 was obtained by using an auxiliary antenna of the loop type placed 5 wavelengths away and oriented so that the folded loop antenna under test was in the plane of the auxiliary loop. Another test with a vertical whip used as the auxiliary antenna yielded identical results. The pattern data were taken at 104.5 mc/s, with the capacitor plates removed, so that the antenna was resonant at the test frequency. The slightly larger lobe of the pattern occurs on the feed-loop side of the antenna. The depths of the two nulls were observed to be in excess of 30 decibels.

4. EFFICIENCY

The radiation efficiency was measured by the insertion-loss substitution method, corrected for directivity and impedance-mismatch effects. The folded loop under test and an auxiliary antenna of the loop type were mounted about 5 wavelengths apart in the same vertical plane, with centers a little less than 1/2

1. Kraus, J. D., "Electromagnetics," McGraw-Hill, 1953, pp. 502-4.

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210° 200° 190° 180° 170° 160° 150°
150° 160° 170°

Polar Co-ordinate
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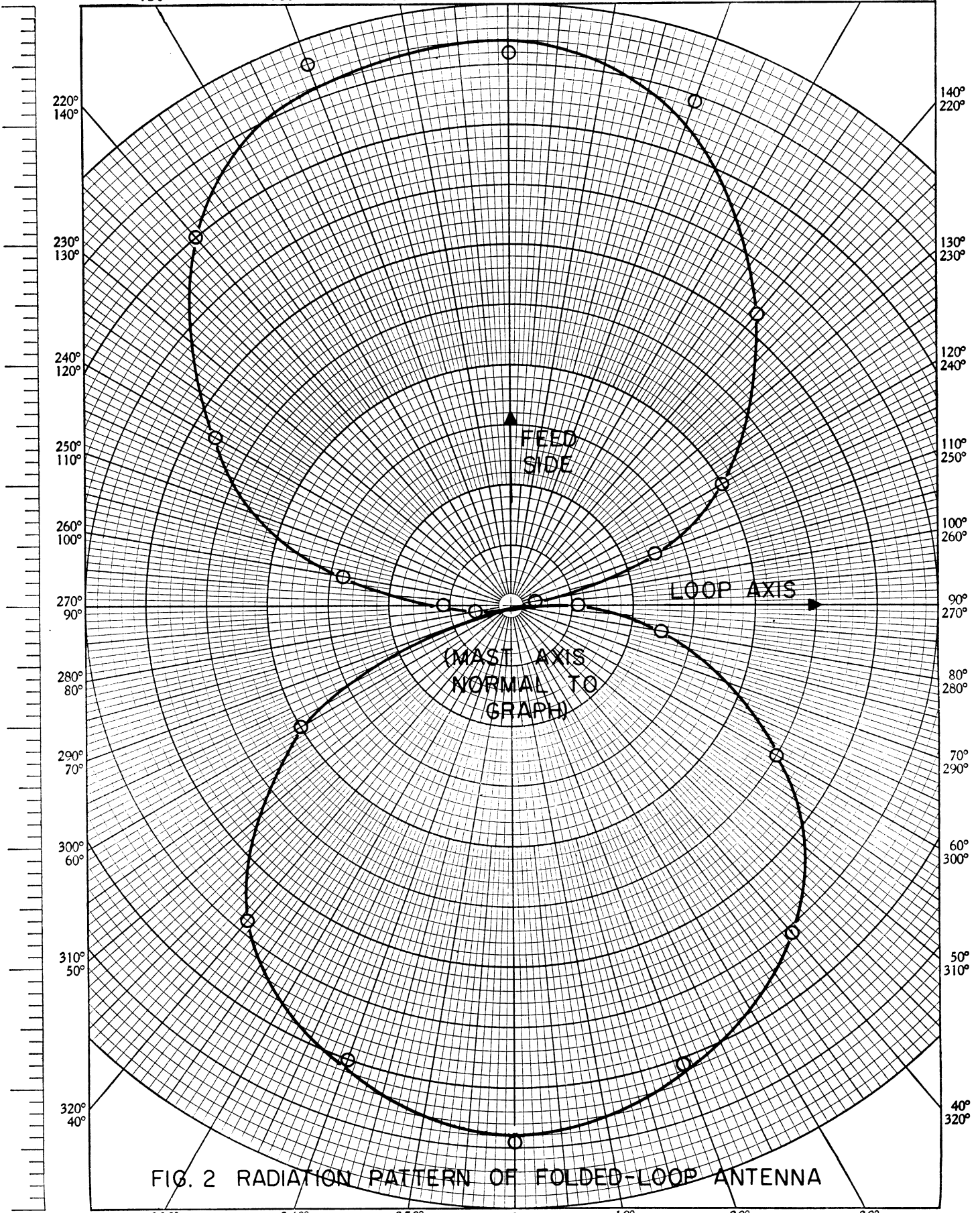


FIG. 2 RADIATION PATTERN OF FOLDED-LOOP ANTENNA

330° 30° 340° 20° 350° 10° 0 10° 350° 20° 340° 30° 330°
6

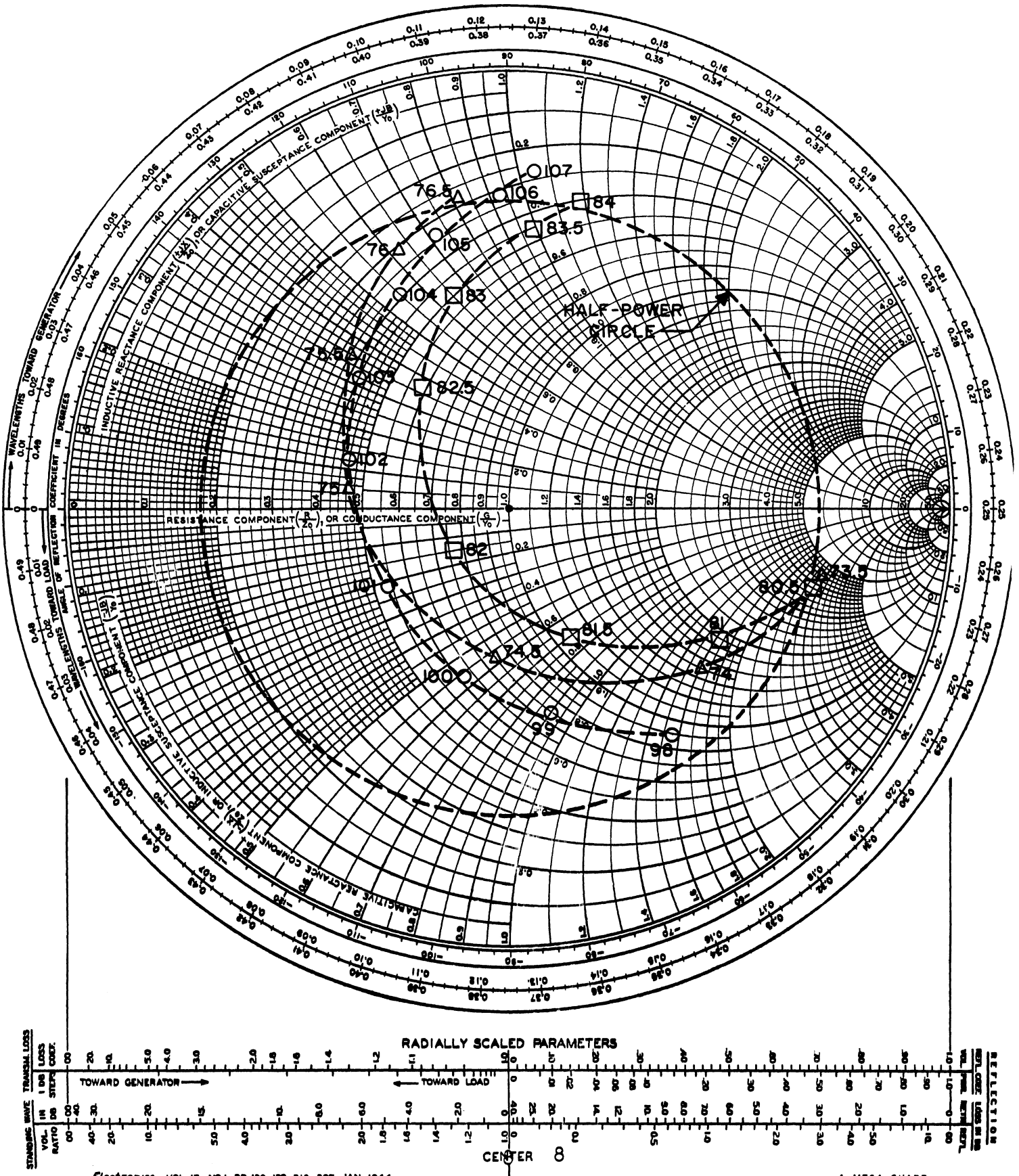
wavelength above a horizontal copper screen ground plane. The 5-wavelength spacing was sufficient to assure negligible contribution from induction field effects with the test antenna in the plane of the auxiliary loop. The spacing above the ground plane was small enough so that the reflected wave could be considered to add almost in phase with the direct wave. The signal generator, the receiving voltmeter, and the auxiliary loop were all matched to 50-ohm transmission lines. A correction was made for the known impedance mismatch of the antenna under test. The antenna capacitor was set at its minimum value, and the measurement was made at a frequency of 104.5 mc/s. From the pattern measurements, the directivities of the loops were estimated to be 1.8 decibels each. The insertion loss measurement yielded a result which was about 0.5 decibel greater than should theoretically be expected with lossless antennas, and the uncertainty of measurement was estimated to be plus or minus 0.5 decibel, so that the combined efficiency of the two antennas might be anywhere from 70 to 100 percent. If the two antennas are about alike in efficiency, as is believed, the value for each could lie anywhere between 89 and 100 percent.

5. IMPEDANCE PROPERTIES

The impedance properties of the folded-loop antenna pictured in Fig. 1 are shown in Fig. 3 for three different settings of the capacitor. With a 50-ohm source, the half-power bandwidths apparently vary from 4 percent to 8 percent of the center frequencies, being largest for the highest center frequency. The impedance curves appear to be approximately circular, as would be expected from theoretical considerations.

FIG. 3. IMPEDANCE OF FOLDED-LOOP ANTENNA FOR THREE DIFFERENT CAPACITOR SETTINGS. FREQUENCIES IN MEGACYCLES

IMPEDANCE OR ADMITTANCE COORDINATES



To understand how the loop operates and why the impedance curves have the circular shapes shown in Fig. 3, reference should be made to Fig. 4, which illustrates a loop made of sheet copper, supported by the coaxial feed line at the bottom, and held together at the top by a short piece of insulating material. This loop is one which was considered during the development of the folded-loop, but which was discarded because of mechanical considerations. Experimental evidence indicates that the folded-loop may be considered to be essentially the same as this sheet copper loop from an electrical standpoint. Geometrically, the folded-loop is nothing more than the sheet metal loop with the main body of the sheet metal removed, leaving only the edges, as can be seen by comparing Fig. 4 with Fig. 1.

In cross-section, either the folded loop of Fig. 1 or the sheet metal loop of Fig. 4 may be represented as shown at the left in Fig. 5. The essential electrical characteristics of this arrangement are indicated at the right in Fig. 5, where the conducting parts are represented as inductances and the loop gap is represented as a capacitance. The resistance is added to account for radiation and losses. The driving point terminal-pair is identified by the letters A and B, A corresponding to the outside conductor and B to the inside conductor of the coaxial connector.

The circuit of Fig. 5 may be arranged in the more conventional form shown in Fig. 6. Any mutual inductance which exists between the main antenna loop and the driving loop consisting of L_1 , L_2 , and the pair AB merely alters the sizes of the inductances, but does not basically increase the complexity of the circuit configurations. The impedance seen at the terminal pair AB can be written by inspection as

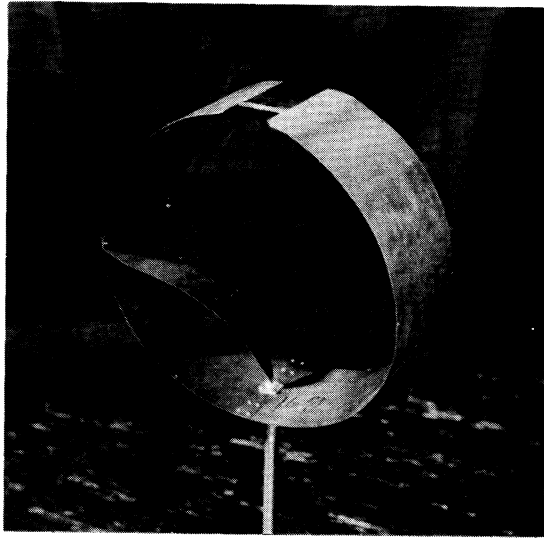


FIG. 4

SHEET COPPER LOOP ANTENNA FROM WHICH THE FOLDED-LOOP IDEA WAS DEVELOPED. THE COAXIAL FEED LINE ENTERS AT THE BOTTOM. THE LOOP ENDS ARE HELD APART AT THE TOP BY A PIECE OF INSULATING MATERIAL.

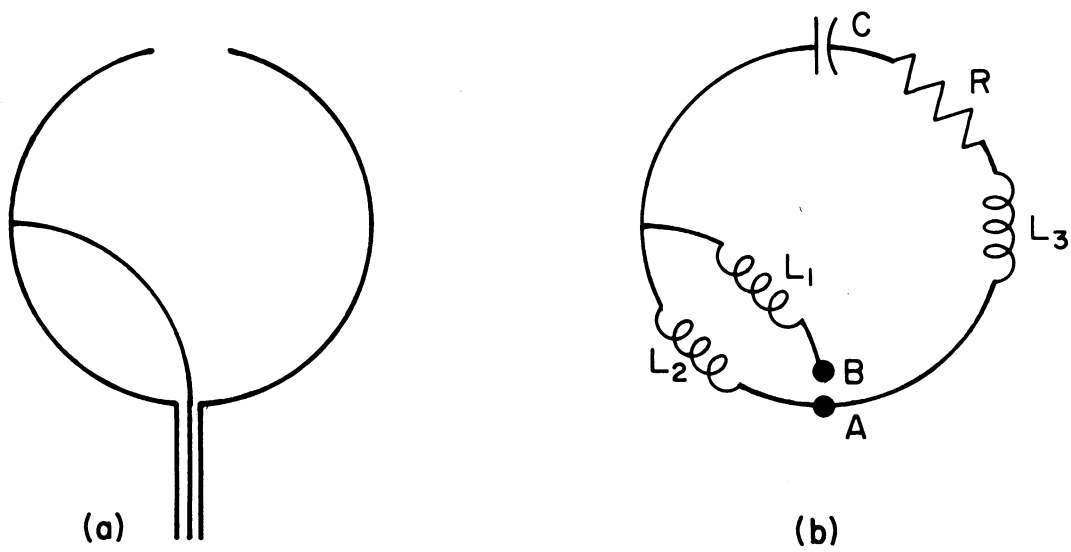


FIG. 5. EQUIVALENT CIRCUIT FOR EITHER THE SHEET — METAL LOOP OF FIG. 4 OR THE FOLDED LOOP OF FIG. 1.

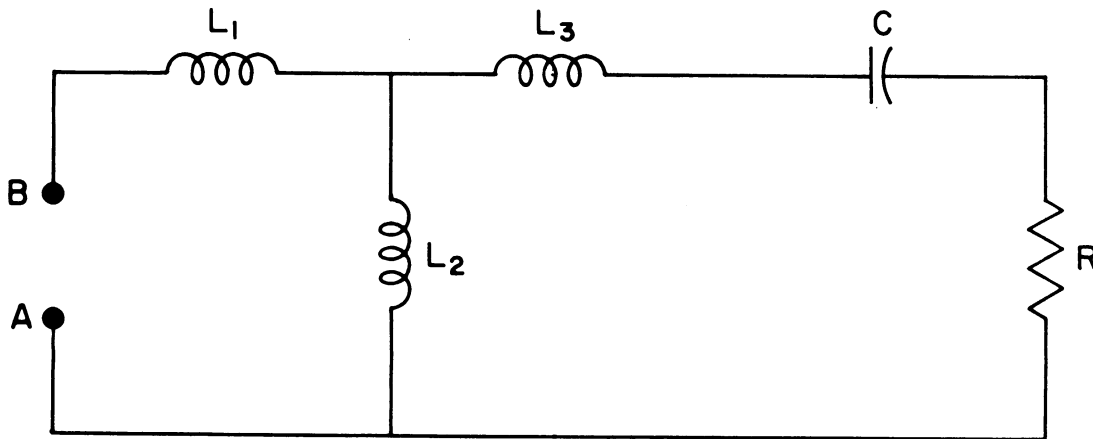


FIG. 6. THE CIRCUIT OF FIG. 5, ARRANGED IN CONVENTIONAL FORM.

$$Z_{AB} = j\omega L_1 + \frac{1}{\frac{1}{j\omega L_2} + \frac{1}{R + j\omega L_3 + \frac{1}{j\omega C}}} \quad (1)$$

which can be rearranged by algebraic manipulation into the form

$$Z_{AB} = j\omega(L_1 + L_2) + \frac{\omega^2 L_2^2 R}{R^2 + X^2} + j \frac{-\omega^2 L_2^2 X}{R^2 + X^2} \quad (2)$$

in which

$$X = \omega(L_2 + L_3) - \frac{1}{\omega C} \quad (3)$$

Equation 2 may be simplified by letting

$$R' = \frac{\omega^2 L_2^2 R}{R^2 + X^2} \quad \text{and} \quad X' = \frac{-\omega^2 L_2^2 X}{R^2 + X^2} \quad (4)$$

so that its new form is

$$Z_{AB} = j\omega(L_1 + L_2) + R' + jX' \quad (5)$$

It is not difficult to show that

$$[X']^2 + \left[R' - \frac{\omega^2 L_2^2}{2R} \right]^2 = \left[\frac{\omega^2 L_2^2}{2R} \right]^2 \quad (6)$$

from which it is evident that R' and X' vary in such a way that the locus of $R' + jX'$ in the Z_{AB} plane is approximately a circle tangent to the X axis at the origin, provided ω is restricted to a small range of variation. The diameter of this circle is evidently $\omega^2 L_2^2 / R$. Such a circle is shown in Fig. 7 as a dashed line. The addition of the term $j\omega(L_1 + L_2)$ to complete the graphical representation of (5) moves the circle upward to the position indicated by the solid line. Figure 8 shows the same two circles transformed into the reflection-coefficient plane (Smith chart). In addition, the straight dotted lines $X = \omega(L_1 + L_2)$ and $R = \omega^2 L_2^2 / R$ are transformed into the corresponding dotted circles in Fig. 8.

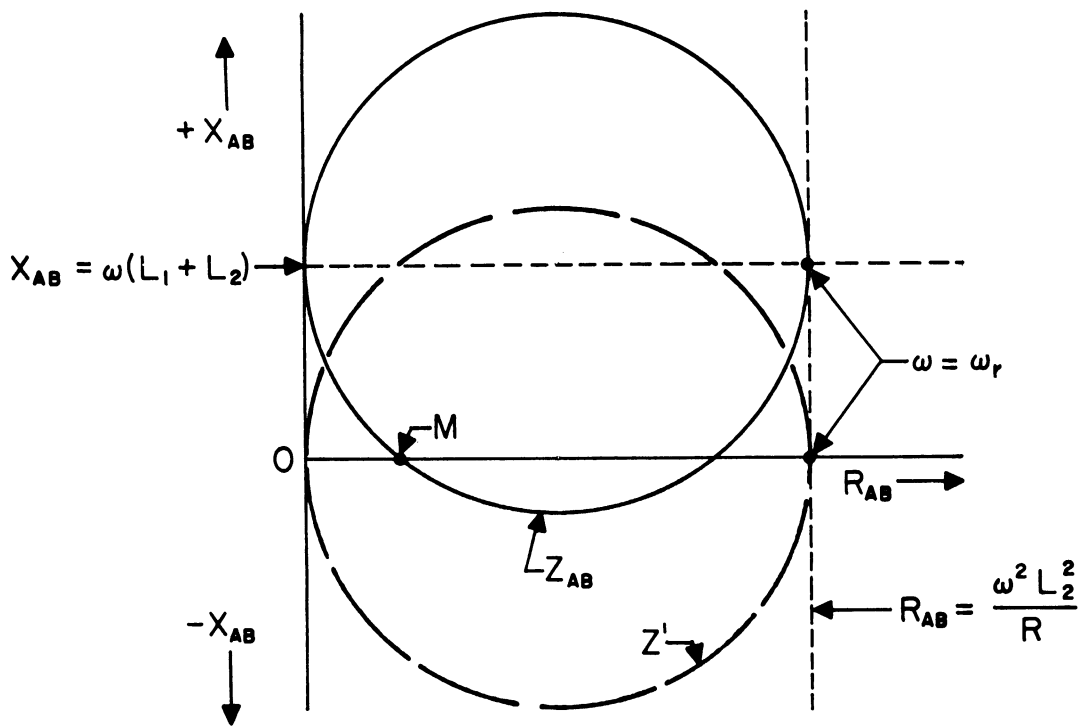


FIG. 7. LOCUS OF R' AND X' (DASHED CIRCLE), AND LOCUS OF TOTAL R_{AB} AND X_{AB} (SOLID CIRCLE), IN LINEAR CARTESIAN COORDINATES. (SEE EQUATIONS 5 AND 6)

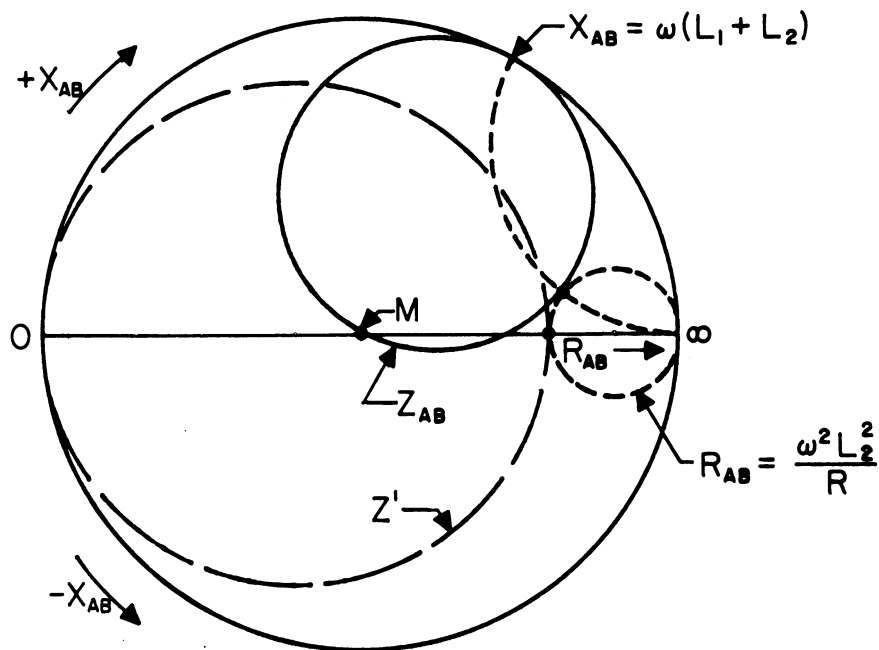


FIG. 8. TRANSFORMATION OF FIG. 7 INTO THE REFLECTION COEFFICIENT PLANE. (SMITH CHART)

The value of R' reaches a maximum and X' becomes zero at a value of ω for which $X = 0$, that is, at a value of ω for which the combination of L_2 , L_3 and C becomes resonant. If this resonant value of ω is denoted as ω_r , we have, from Eq 3,

$$\omega_r = \frac{1}{(L_2 + L_3)C} \quad (7)$$

This value is marked on each of the curves in Fig. 7 and Fig. 8. The value of ω increases with clockwise motion around the curves.

The fact that ω in Eq 6 is not a constant is not the only reason why the circles are distorted. Another reason is that R increases with about the fourth power of ω , as is the case with all small radiating loops. Additional distortion is found in the Z_{AB} circle because the term $j\omega(L_1 + L_2)$ in Eq 5 also varies with ω . However, if the Q of the resonant circuit consisting of L_1 , L_2 , C and R exceeds 10, as it does in the antennas considered here, the curves appear nearly circular to the eye upon casual inspection.

6. BANDWIDTH

The problem of designing broadband coupling networks to match the folded-loop antenna to a constant resistance source or load is being worked out currently. The problem is made difficult by the distributed nature of the antenna and by the variation of the radiation resistance with frequency. Pending the completion of the analytical study, the design procedure has been to cut and try various antenna dimensions to obtain a reasonably low value of Q , consistent with weight and ruggedness requirements, with the feeling that the lower the Q , the easier it should be to accomplish the broadband matching.

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The Q's of various loop configurations have been measured and a few of the results are given below. The approximate value of Q is obtained by finding the frequencies f_1 and f_2 at which the resistive component R' in (Eq 5 is equal to half of the maximum value it attains at the resonant frequency. The value of Q is then found from the formula

$$Q = \frac{f_r}{f_2 - f_1} \quad (8)$$

The curves of Fig. 3 obviously cannot be used for this purpose, because the data do not extend to cover the maximum resistance region. Even if they were extended, the maximum resistance point would fall in a region of the graph where a very small error in placement of a point would result in a large error in the determination of the maximum value of R' . This situation is remedied by decreasing the size of the coupling loop, moving the points of attachment closer to the lower cross-beam, and inserting a capacitor temporarily in series at terminal B. In this way the impedance circle can be swung around into the positions illustrated in Figs. 9, 10, 11, and 12.

Figure 9 shows one of these impedance circles, or Q-circles, for a loop (not folded) of about the same size as the sheet-metal loop in Fig. 4, but made instead of a single C-shaped piece of 3/16 inch o.d. copper tubing. The dotted line in Fig. 9 marks the half-maximum resistance value. The frequencies f_1 , f_2 and f_r appear to be 122.6, 126.5 and 124.6 mc/s, which when substituted in Eq 8, give a Q of 32.

Figure 10 is a similar Q-circle, in this case for a sheet-metal loop like that in Fig. 4, but made of a sheet strip only half as wide as that in Fig. 4.

The Q turns out to be about 22. Figure 11 is the Q-circle for the antenna shown

1. Terman and Pettitt, "Electronic Measurements," McGraw-Hill, 2nd ed., 1952, pp.180-183.
2. Cline, "Coupled Impedance Method of Q Measurement," University of Michigan, Engineering Research Institute, Electronic Defense Group Technical Memorandum No. 25, January 1956.

FIG. 9. Q-CIRCLE FOR A SIMPLE LOOP (NOT FOLDED), CONSISTING OF A SINGLE C-SHAPED PIECE OF 3/16 INCH O.D. COPPER TUBING, COUPLED TO THE FEED LINE IN THE MANNER INDICATED IN FIG. 5 (a).

IMPEDANCE OR ADMITTANCE COORDINATES

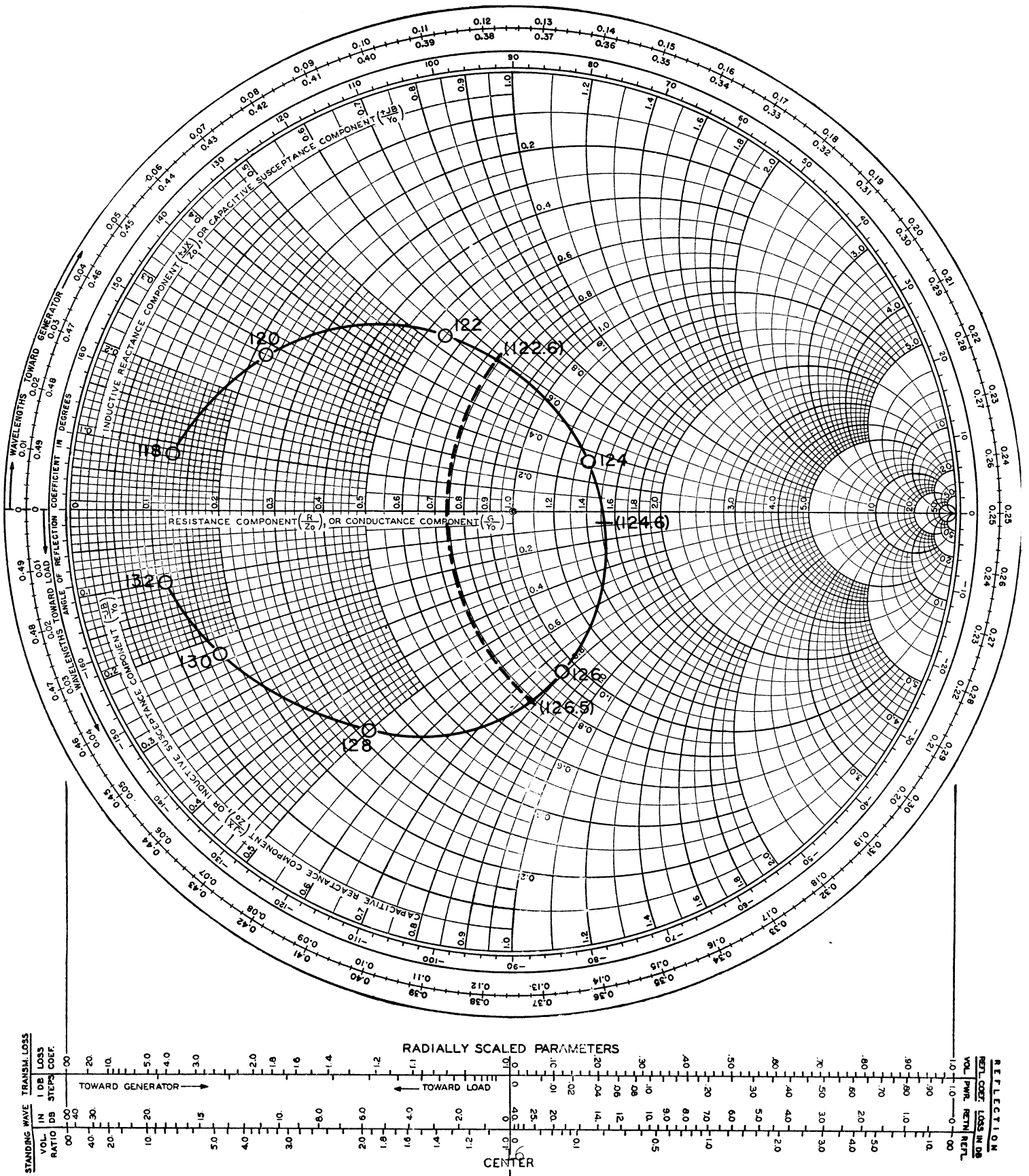


FIG. 10. Q-CIRCLE FOR A SHEET-METAL LOOP SIMILAR TO THAT ILLUSTRATED IN FIG. 4, BUT ONLY 3 INCHES WIDE INSTEAD OF 6 INCHES. FREQUENCIES IN MEGACYCLES.

IMPEDANCE OR ADMITTANCE COORDINATES

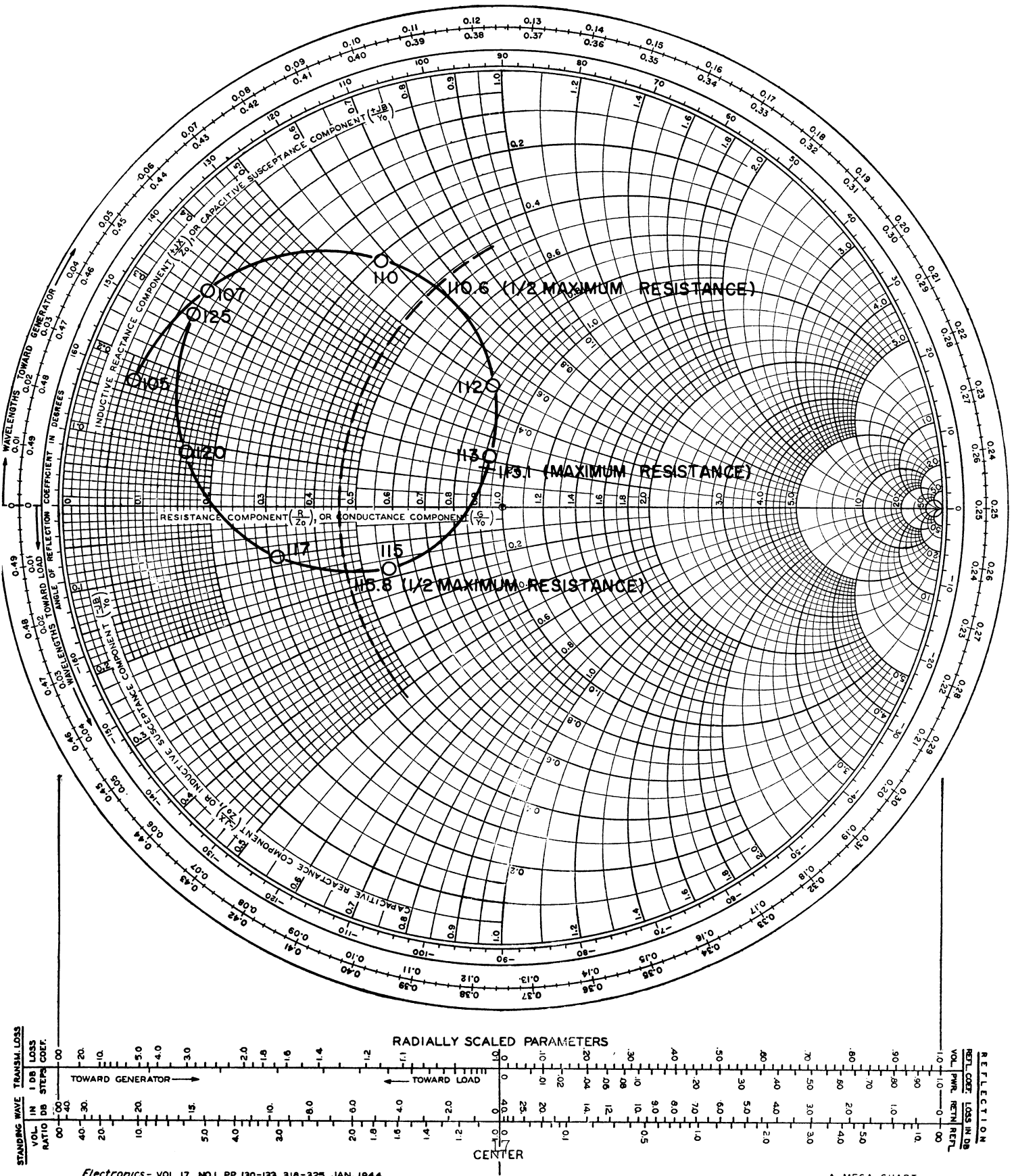


FIG. II. Q-CIRCLE FOR THE 6-INCH-WIDE SHEET METAL LOOP ILLUSTRATED IN FIG. 4. FREQUENCIES IN MEGACYCLES.

IMPEDANCE OR ADMITTANCE COORDINATES

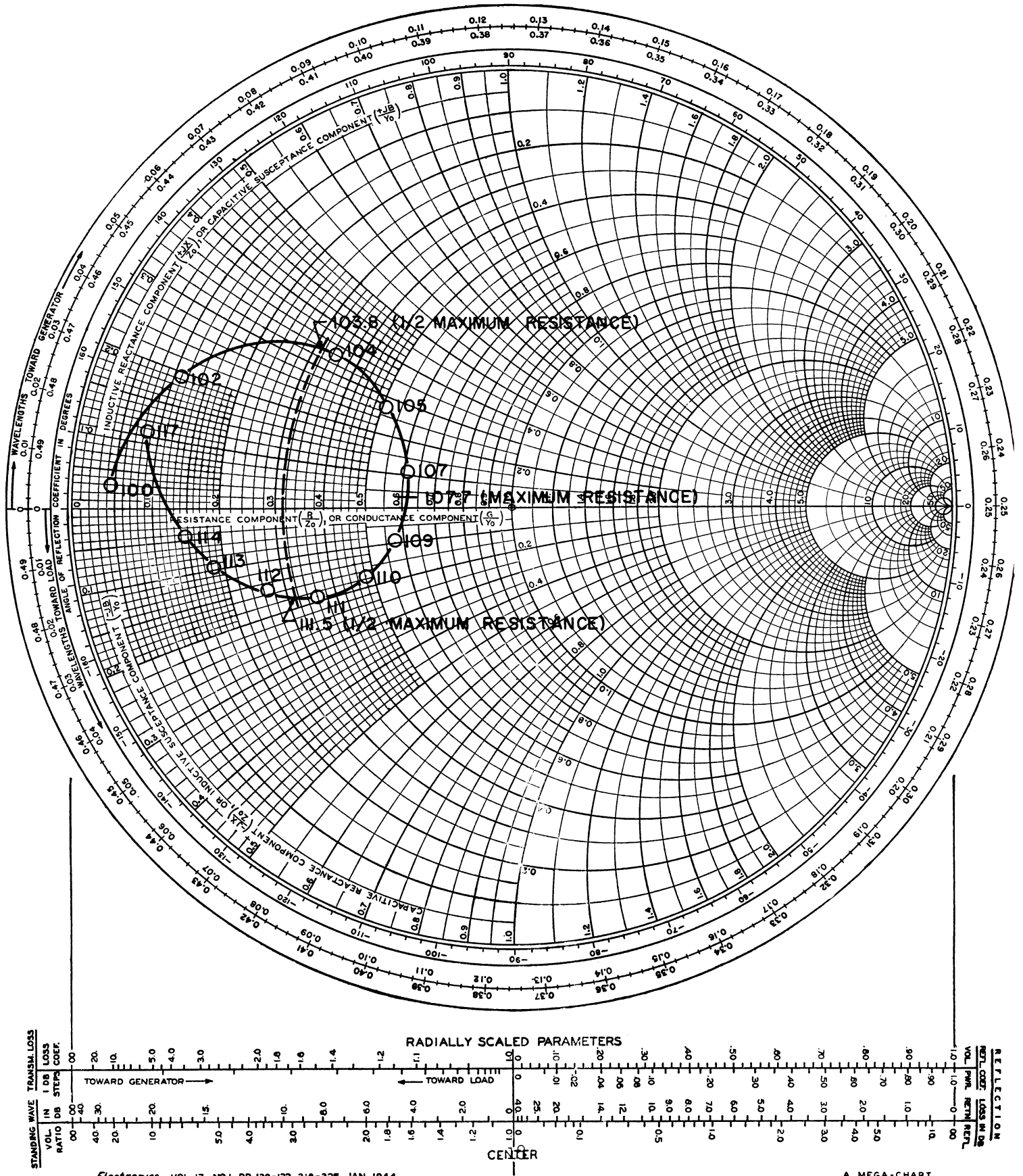
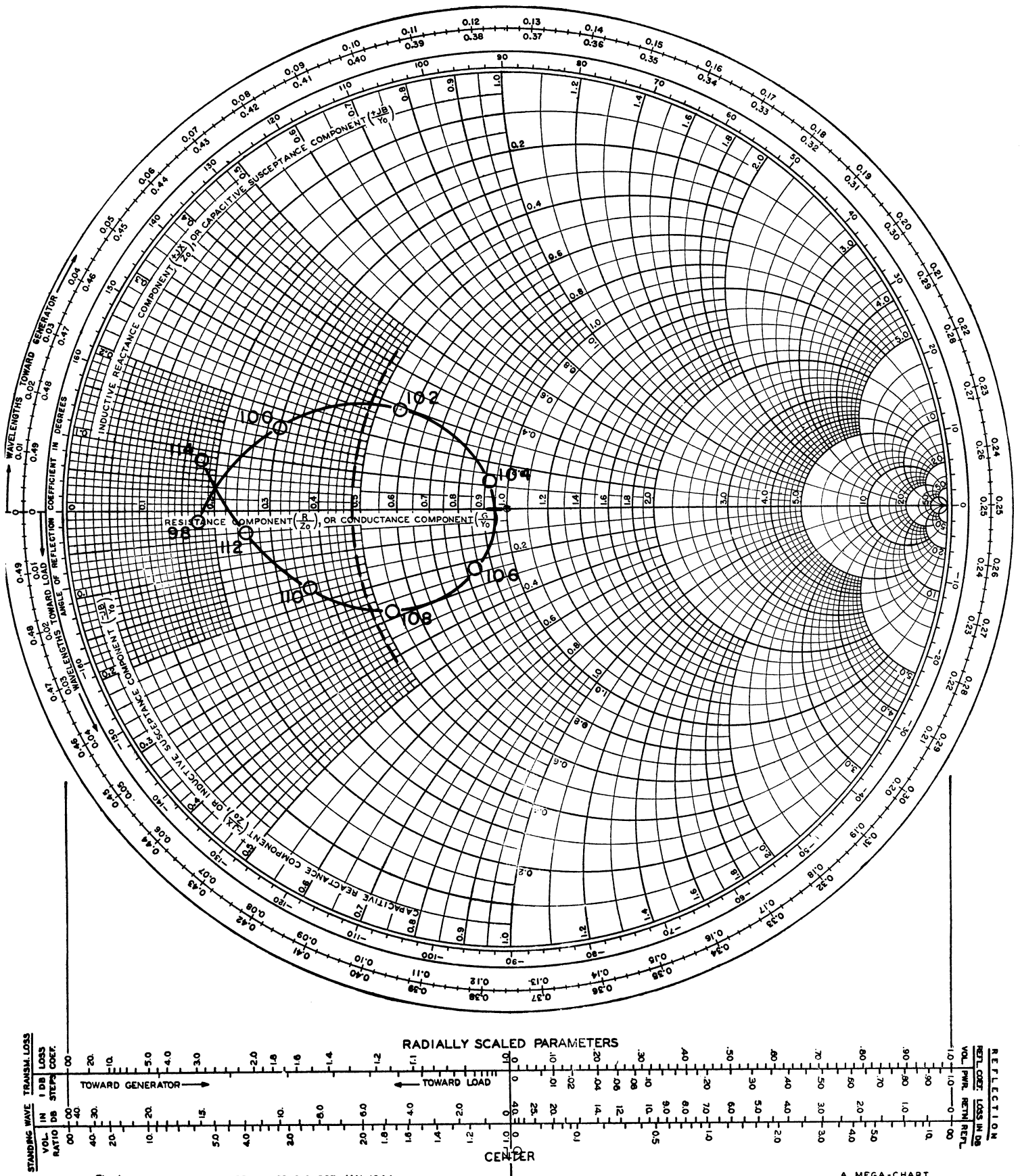


FIG. 12. Q-CIRCLE FOR THE FOLDED LOOP ILLUSTRATED IN FIG. 1, BUT WITH THE MAST, UPPER CROSS-BEAM, AND CAPACITOR REMOVED. FREQUENCIES IN MEGACYCLES.

IMPEDANCE OR ADMITTANCE COORDINATES



in Figure 4. Its Q is about 14. All three of the antennas just described have about the same proportion of diameter to end-gap spacing. Evidently the Q decreases as the loop conductor size increases. A copper screen antenna having the same dimensions as the sheet-metal loop in Fig. 4 was tried, with almost identical results. Next, the coarseness of the screen was increased to the point where there were only five circumferential wires, without significant change. The next step was the folded-loop of Fig. 1, minus the mast, upper cross-beam, and capacitor. Its Q , in this condition, also appears to be about 14. Addition of the cross-beam and capacitor increases the Q . This is to be expected from the analysis of the equivalent circuit of Fig. 6, and is borne out by experiment. In applications where the capacitor can be omitted, advantage can be taken of the greater bandwidth which results.

7. TUNING

The center frequency adjustment is now made by means of the variable capacitor across the gap, as described above. This results in a value of Q which increases as frequency decreases, which is undesirable. The reason for this behavior is that the radiation resistance increases with approximately the fourth power of ω , so that Q , which is equal to $\omega L/R$, decreases inversely as the cube of ω . Some attention is being given to a method by which L can be varied in such a way as to partially offset this undesirable change in Q . Although it is unlikely that it will be possible to vary L enough to completely compensate for the other factors, it may be possible, by designing the loop so that it can be bent or telescoped, to make significant progress in this direction.

8. CONCLUSIONS

The folded-loop antenna described here has radiation pattern and polarization properties similar to those of a simple loop, but because of its lower Q, it has impedance properties which are superior to those of a simple loop made of the same size conductor. It can be used in proximity to large objects with less capacitance detuning effects than is the case with a dipole antenna, and its maximum dimension of about $1/5$ wavelength is less than the $1/2$ wavelength dimension of the conventional dipole. An antenna of this type can be used at any frequency normally used for radio communication, provided its size corresponds to the frequency. If untuned, a half-power bandwidth of 15 percent of the center frequency can be obtained. Although loop antennas have the reputation (perhaps deserved in some cases) of having low efficiency, the efficiency of the antenna described here is in the vicinity of 90 percent or more.

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ERRATA SHEET

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2262-133-T

Page

- 1 Paragraph 2 - line 6, should read: relatively independent of proximity effects caused by the presence of ground, personnel, motor vehicles,
- 1 Paragraph 2 - line 11 should read: When the particular model shown in Figure 1 is connected
- 7 Line 14 - Change 70 to read 79.
- 12 Eq. 5 Change $Z_{AB} = j\omega(L_1 + L_2) + R' \times jX'$
to read $Z_{AB} = j\omega(L_1 + L_2) + R' + jX'$
- 14 Eq. 7 Change $\omega_r = \frac{1}{(L_2+L_3)C}$ to read $\omega_r = \sqrt{\frac{1}{(L_2+L_3)C}}$