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QUARTERLY PROGRESS REPORT NO. 1

CHARACTERISTIC SYSTEMS FOR ROTATIONAL FLOWS OF COMPRESSIBLE GAS

By

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Project 2201

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1. Work Plan

The following work was done on Project 2201 by the project supervisor, Mr. N. Coburn, prior to the actual start of the project in January, 1954. Most of this work was completed in the summer of 1953 (Abstract 660t, American Mathematical Society, 59, No. 6, 550, Nov. 1953). As will be noted, the desired basic relations for all compressible gases were completely established and many properties of simple waves in the plane rotational case of the isentropic flow of a polytropic gas were determined. Future plans for the project are as follows: (1) Mr. N. Coburn will complete the plane case during the summer of 1954; (2) Mr. J. Klein has started and will continue the study of simple waves in axial symmetric and conical flows; and (3) either a joint report or individual reports will be issued, depending on the wishes of the Ordnance Laboratory.

2. Work Done

First, the characteristic relations for the steady, rotational, three-dimensional, supersonic motion of a polytropic gas are expressed in intrinsic form. This means that the equations relating the rates of change of the magnitude of the velocity, q, and of the sound speed, c, with respect to displacements along the normal to the characteristic manifolds and along two independent directions in these manifolds and the curvature of these manifolds are determined. Next, canonical forms of these relations are found.

Finally, application is made to the case of plane isentropic rotational flows. It is shown that for the limiting case when the Mach number of the flow is one, the bicharacteristics are always a single family of radial straight lines (straight lines of fanlike type). For this type of rotational flow, the following conditions are satisfied: (1) the stream

lines are a family of circles which are orthogonal to the bicharacteristics; (2) the magnitude of the velocity and the sound speed are constant along a given stream line. In the case of general plane isentropic rotational flow, necessary and sufficient conditions for the existence of simple waves are obtained. These conditions consist of a system of two second-order ordinary differential equations for two dependent variables. Further, it is shown that neither the magnitude of the velocity nor the sound speed can be constant along a simple wave in plane isentropic rotational flow (in the plane isentropic irrotational case, both these quantities are constant along a simple wave). For $\gamma = 3$, where γ is the ratio of the specific heats of the gas, all possible plane isentropic simple waves are determined. In this case, the stream lines consist of concentric circles, the innermost circle being a "limiting line". The bicharacteristics are a family of lines tangent to the limiting line. These lines become radial in the limiting case of Mach number one and the limiting line reduces to the point from which these lines radiate. A similar class of simple waves exists for each γ, except $\gamma = 1$. For $\gamma = 5/3$, it is shown for this class of simple waves that the sound speed varies as the one-third power of the distance (along a simple wave) from the limiting line.

3. Work to be Done

A. Plane Case. A more general class of simple waves has been shown to exist. The properties of this class of waves will be studied in the future by Mr. N. Coburn.

B. Axial-Symmetric Case. Mr. J. Klein is determining relations for the desired curvatures. Solutions of the previously noted conical system and the Lame relations (the vanishing of the Reimann tensor) are to be determined.

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