THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Mechanical Engineering

Final Report

# RESIDUAL STRESSES IN HARDENED BEARING RACES

L. V. Colwell
L. J. Quackenbush

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

## ABSTRACT

The failure of anti-friction bearing races usually results from fatigue induced by cyclical ball loads. Bearing life may be significantly increased if residual compressive stresses can be developed in surface regions which are normally subjected to tensile stresses in service.

The results of this investigation indicate that average compressive residual stresses of the order of 30,000 to 40,000 psi may be developed initially in the first few thousandths of an inch beneath the outer surface of a heat-treated 52100 bearing race. When the race has been completely finished to size after hardening and tempering, those regions in which compressive residual stresses are developed by hardening have been removed, so that no beneficial residual stresses remain to influence and increase bearing life.

# INTRODUCTION

The failure of the balls and races of anti-fraction bearings is usually due to fatigue. The cyclical nature of the rotating loads and the development of Hertzian stresses ultimately lead to a spalling of either or both of the elements of the bearing. Under such conditions anything which can induce significant residual compressive stresses in the loaded surfaces and slightly below them may greatly improve the service life of a bearing. Two possibilities for developing favorable residual compressive stresses lie either in the hardening of the balls and races or in some machining operating such as grinding. In either case such residual stresses must lie far enough beneath the surface not to be removed when the balls or races are finished to size. They must lie in regions where tensile stresses develop under normal loading. In the particular case at hand, an investigation was made to determine the nature and magnitude of whatever stresses might be developed in the outside diameter of an outer race of a ball bearing as a result of hardening the race.

## I. SUMMARY AND INTERPRETATION OF RESULTS

This report is divided into three major sections. Section I includes a summary and interpretation of the results of the investigation. Section II presents a detailed summary of the various test data and their correlation. Section III includes details of the setups and the procedures by which the data were taken and analyzed.

The results of these investigations lead to the conclusion that the hardening of 52100 steel outer bearing races induces residual compressive stresses in the outer layers. These compressive stresses may be on the order of perhaps as high as 50,000 psi maximum if a generalized analysis of data is made. Figure 1 shows a plot of what the distribution of such stresses might be. However, these stresses are only a few thousandths of an inch (0.001-0.004 in.) below the surface, and these outer layers are removed by the subsequent grinding necessary to finish the outside diameters to the proper dimension. As a consequence, any beneficial residual stresses that might be developed by hardening are eliminated, and only residual stresses arising from the grinding operations remain.

Table I is a compilation of the calculated residual stresses and their

TABLE I

CALCULATED RESIDUAL STRESSES AND THEIR DEPTHS BELOW THE SURFACE

| Group No. | Metal Ground Off, O.D., In. | Average Stress, Psi |
|---|---|---|
| 1 | 0.00177 | 13,100 Tension |
| 2 | 0.00462 | 38,000 Compr. |
| 3 | 0.00723 | 21,800 Tension |
| 4 | 0.00973 | 7,100 Tension |
| 5 | 0.01223 | 2,180 Tension |
| 6 | 0.01500 | 10,900 Compr. |
| 7 | 0.00000 | 21,800 Tension |

depths from the hardened surface. These stresses have been calculated by treating one quarter of the bearing race as a simple cantilever and using the analysis shown in Fig. 4 of Section III. They should be considered as

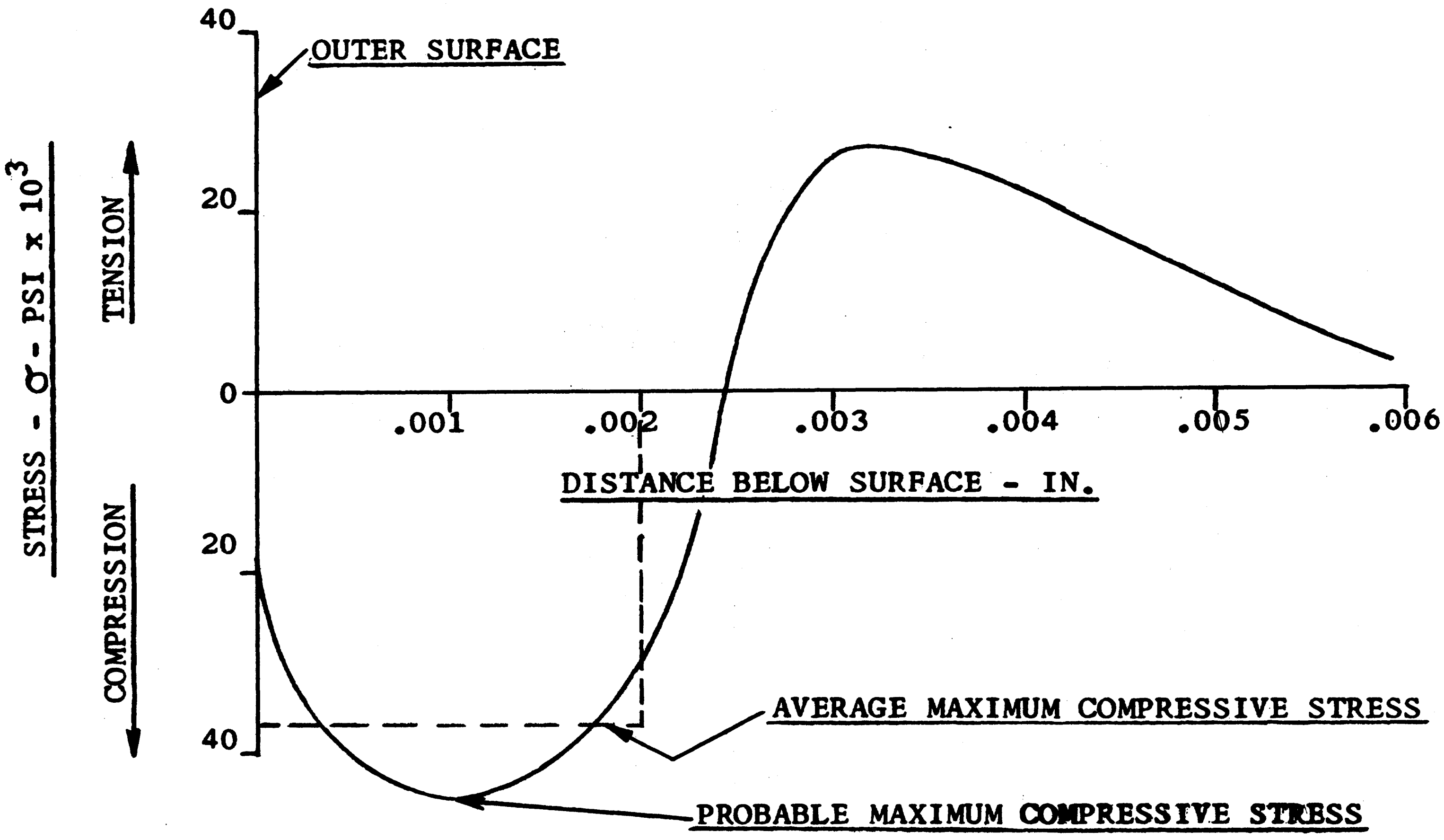


Fig. 1. Probable stress distribution in hardened outer race.

average stresses based on the average change in diameter of fifteen specimens after an average of 0.001 in. of metal was ground from the outside diameter of each of six groups in one or more passes. In the case of Group 1, which had a nominal 0.001 in. ground off the outside diameter but did not clean up because its out-of-roundness was greater than 0.001 in., the calculated values are somewhat questionable and may be low.

A plot of the change in outside diameter of the bearing race when cut versus the amount of metal removed from the outside diameter, as shown in Fig. 2, will give some idea of what the nature and levels of stress are in the layers removed by grinding. The nature or direction of the stress in a particular layer, whether tension or compression, can be determined by what effect its removal has upon the change in diameter of the race when the race has been cut after the layer has been removed. If the diameter were unchanged, then the layer would have had no stress in it. If the diameter increase is less than the diameter increase due to the removal of the prior layer, then the stress in the removed layer would be tensile. If the diameter increase is greater than the diameter increase due to the removal of the prior layer, then the stress in the removed layer is compressive. Thus, in the races considered, the layers between 0.002 and 0.003 in. and those between 0.005 and 0.006 in. are in compression, when results in an increase in the diameter when they are removed by grinding and their restraining influence is removed.

Figure 1 is a plot of the probable stress distribution as a function of its depth beneath the surface of the hardened race. It represents more nearly the idealized situation that could be expected rather than what was actually determined. The failure of the races in Group 1 to clean up when initially ground and the subsequent effect on the change in diameter when the races was cut tended to shift the stress distribution to the right as shown by Fig. 2. In addition the effect of any residual stresses on the inside diameter as a result of heat treatment were neglected since the primary concern was a determination of stresses beneath the outer surface. Finally, so long as the general nature and magnitude of the stresses just beneath the outer surface had been established, it was deemed unnecessary to further refine their exact distribution.

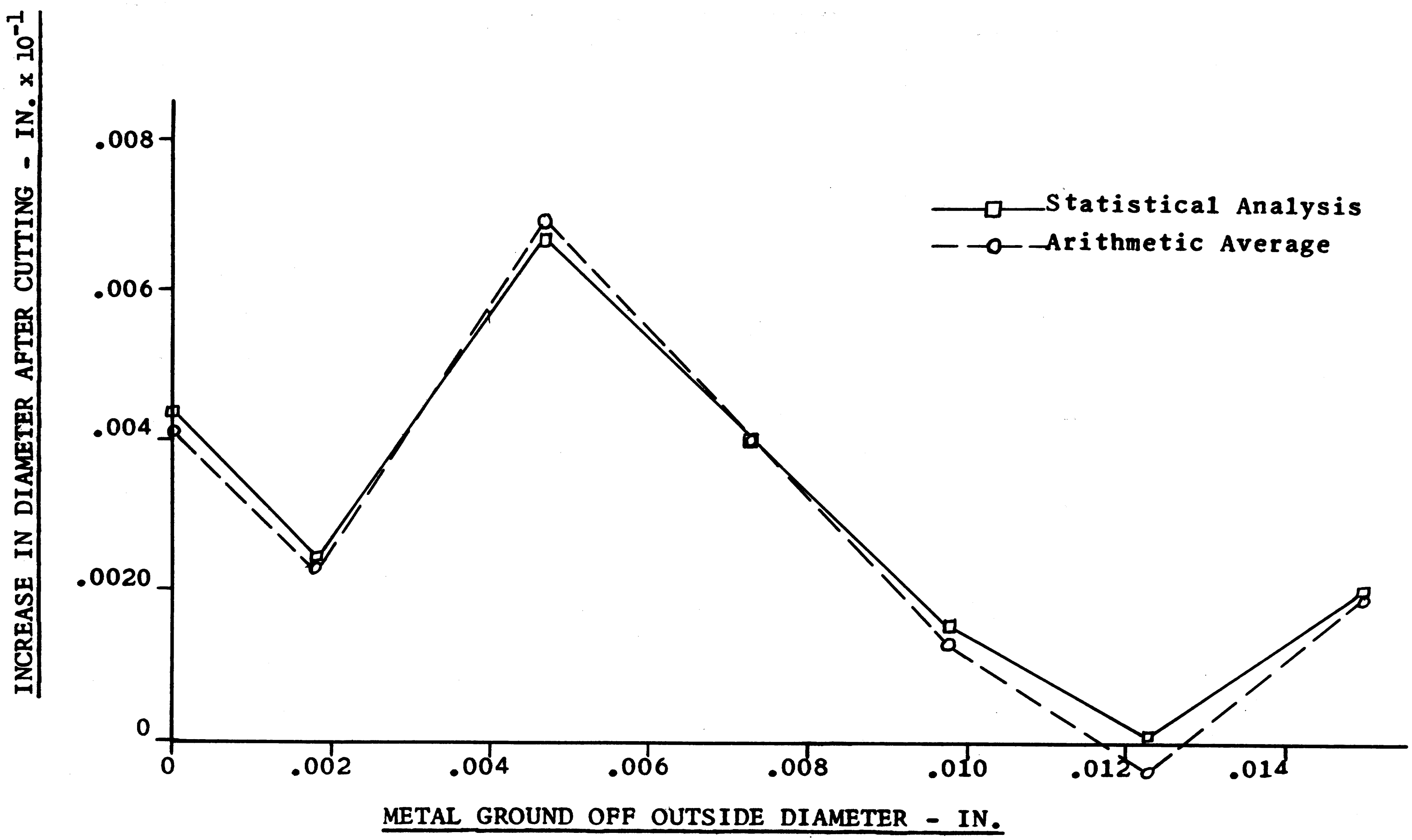


Fig. 2. Diameter change vs. metal removed for hardened outer race.

## II. RESULTS OF DETAILED INSPECTION

Tabulated on the following pages are the inspection data on the six groups, of fifteen outer races each, used to establish the diameter changes that resulted from hardening, grinding, and cutting.

### SPECIMEN NUMBERS ARRANGED BY INCREASING SIZE AND GRINDING GROUPS

| Group 1 Spec. No. | Group 2 Spec. No. | Group 3 Spec. No. | Group 4 Spec. No. | Group 5 Spec. No. | Group 6 Spec. No. | Group 7 Spec. No. |
|---|---|---|---|---|---|---|
| 29 | 17 | 68 | 85 | 50 | 78 | 36 |
| 114 | 18 | 81 | 38 | 48 | 28 | 70 |
| 31 | 96 | 26 | 118 | 25 | 77 | 45 |
| 97 | 99 | 80 | 61 | 13 | 104 | 57 |
| 33 | 30 | 94 | 10 | 86 | 11 | 59 |
| 117 | 55 | 115 | 119 | 111 | 106 | 34 |
| 2 | 41 | 32 | 101 | 79 | 23 | 20 |
| 54 | 116 | 22 | 1 | 40 | 95 | 14 |
| 73 | 92 | 4 | 75 | 82 | 12 | 8 |
| 47 | 5 | 64 | 91 | 112 | 6 | 103 |
| 88 | 93 | 42 | 56 | 49 | 44 | 108 |
| 35 | 120 | 19 | 51 | 52 | 63 | 110 |
| 3 | 39 | 105 | 37 | 66 | 21 | 46 |
| 72 | 43 | 69 | 102 | 71 | 53 | 87 |
| 9 | 24 | 62 | 58 | 65 | 7 | 84 |

Note:

Group 1: 0.002 in. to be removed from the diameter in one step.
Group 2: 0.004 in. to be removed from the diameter in two equal steps.
Group 3: 0.006 in. to be removed from the diameter in three equal steps.
Group 4: 0.008 in. to be removed from the diameter in four equal steps.
Group 5: 0.010 in. to be removed from the diameter in five equal steps.
Group 6: 0.012 in. to be removed from the diameter in six equal steps.
Group 7: Unground; as received after hardening.

TEST GROUP DIAMETERS BEFORE GRINDING

Average Diameter = 2.84825 in.

| Specimen No. | Diameter, in. | Deviation of Diameter from Average | Specimen No. | Diameter, in. | Deviation of Diameter from Average |
|---|---|---|---|---|---|
| 1 | 2.84835 | + 10 | 41 | 2.84705 | -120 |
| 2 | 2.84627 | -198 | 42 | 2.84783 | - 42 |
| 3 | 2.84655 | -170 | 43 | 2.84746 | - 79 |
| 4 | 2.84770 | - 55 | 44 | 2.84982 | +157 |
| 5 | 2.84731 | - 94 | 45 | 2.85075 | +250 |
| 6 | 2.84979 | +154 | 46 | 2.85150 | +325 |
| 7 | 2.84997 | +172 | 47 | 2.84637 | -193 |
| 8 | 2.85030 | +205 | 48 | 2.84852 | + 27 |
| 9 | 2.84672 | -153 | 49 | 2.84897 | + 72 |
| 10 | 2.84832 | + 7 | 50 | 2.84852 | + 27 |
| 11 | 2.84959 | +134 | 51 | 2.84844 | + 19 |
| 12 | 2.84979 | +154 | 52 | 2.84901 | + 76 |
| 13 | 2.84858 | + 33 | 53 | 2.84992 | +167 |
| 14 | 2.85076 | +251 | 54 | 2.84628 | -197 |
| 15 | 2.85173 | +348 | 55 | 2.84700 | -125 |
| 16 | 2.84532 | -293 | 56 | 2.84843 | + 18 |
| 17 | 2.84675 | -150 | 57 | 2.85078 | +253 |
| 18 | 2.84675 | -150 | 58 | 2.84850 | + 25 |
| 19 | 2.84786 | -139 | 59 | 2.85150 | +325 |
| 20 | 2.85016 | +191 | 60 | 2.85197 | +372 |
| 21 | 2.84984 | +159 | 61 | 2.84830 | + 5 |
| 22 | 2.84768 | - 57 | 62 | 2.84808 | - 17 |
| 23 | 2.84965 | +140 | 63 | 2.84984 | +159 |
| 24 | 2.84751 | - 74 | 64 | 2.84773 | + 52 |
| 25 | 2.84856 | + 31 | 65 | 2.84904 | + 77 |
| 26 | 2.84759 | - 66 | 66 | 2.84903 | + 76 |
| 27 | 2.84321 | -504 | 67 | 2.85146 | +321 |
| 28 | 2.84932 | +107 | 68 | 2.84758 | - 67 |
| 29 | 2.84566 | -259 | 69 | 2.84804 | - 21 |
| 30 | 2.84692 | -133 | 70 | 2.85038 | +213 |
| 31 | 2.84584 | -141 | 71 | 2.84903 | + 78 |
| 32 | 2.84764 | - 61 | 72 | 2.84668 | -157 |
| 33 | 2.84606 | -219 | 73 | 2.84630 | -195 |
| 34 | 2.85032 | +207 | 74 | 2.84475 | -350 |
| 35 | 2.84645 | -180 | 75 | 2.84838 | + 13 |
| 36 | 2.85062 | +237 | 76 | 2.85190 | +365 |
| 37 | 2.84844 | + 19 | 77 | 2.84955 | +130 |
| 38 | 2.84818 | - 7 | 78 | 2.84922 | + 97 |
| 39 | 2.84743 | - 82 | 79 | 2.84874 | + 49 |
| 40 | 2.84884 | + 59 | 80 | 2.84762 | - 63 |

Concluded

| Specimen No. | Diameter, in. | Deviation of Diameter from Average | Specimen No. | Diameter, in. | Deviation of Diameter from Average |
|---|---|---|---|---|---|
| 81 | 2.84759 | - 66 | 101 | 2.84834 | + 9 |
| 82 | 2.84896 | + 71 | 102 | 2.84847 | + 22 |
| 83 | 2.85259 | +434 | 103 | 2.85057 | +232 |
| 84 | 2.85109 | +284 | 104 | 2.84958 | +133 |
| 85 | 2.84812 | - 13 | 105 | 2.84791 | - 34 |
| 86 | 2.84863 | + 38 | 106 | 2.84960 | +135 |
| 87 | 2.85027 | +202 | 107 | 2.85010 | +185 |
| 88 | 2.84641 | -184 | 108 | 2.85025 | +200 |
| 89 | 2.85082 | +257 | 109 | 2.85350 | +525 |
| 90 | 2.85393 | +568 | 110 | 2.85012 | +187 |
| 91 | 2,84839 | + 14 | 111 | 2.84868 | + 23 |
| 92 | 2.84713 | -112 | 112 | 2.84897 | + 72 |
| 93 | 2.84733 | - 92 | 113 | 2.85175 | +350 |
| 94 | 2.84763 | - 62 | 114 | 2.84580 | -245 |
| 95 | 2.84966 | +141 | 115 | 2.84764 | - 61 |
| 96 | 2.84687 | -138 | 116 | 2.84709 | -116 |
| 97 | 2.84591 | -234 | 117 | 2.84624 | -201 |
| 98 | 2.85293 | +468 | 118 | 2.84822 | - 3 |
| 99 | 2.84688 | -137 | 119 | 2.84833 | + 8 |
| 100 | 2.85351 | +526 | 120 | 2.84735 | - 90 |

MAXIMUM DIAMETER MEASUREMENTS

| Specimen No. | Maximum Diameter | Diameter at 90° to Maximum Diameter | Difference x $10^{-5}$ |
|---|---|---|---|
| | | Group 1 | |
| 29 | 2.84566 | 2.84241 | +325 |
| 114 | 2.84580 | 2.84545 | + 35 |
| 31 | 2.84584 | 2.84493 | + 91 |
| 97 | 2.84591 | 2.84599 | - 8 |
| 33 | 2.84606 | 2.84287 | +319 |
| 117 | 2.84624 | 2.84242 | +382 |
| 2 | 2.84627 | 2.84064 | +563 |
| 54 | 2.84628 | 2.84298 | +330 |
| 73 | 2.84630 | 2.84332 | +298 |
| 47 | 2.84637 | 2.84352 | +285 |
| 88 | 2.84641 | 2.84515 | +126 |
| 35 | 2.84645 | 2.84383 | +262 |
| 3 | 2.84655 | 2.84376 | +279 |
| 72 | 2.84668 | 2.84167 | +501 |
| 9 | 2.84672 | 2.84632 | + 40 |
| | | Average | 3828 |

| Specimen No. | Maximum Diameter | Diameter at 90° to Maximum Diameter | Difference x $10^{-5}$ |
|---|---|---|---|
| | | Group 6 | |
| 78 | 2.84922 | 2.84496 | +426 |
| 28 | 2.84932 | 2.84542 | +390 |
| 77 | 2.84955 | 2.84342 | +613 |
| 104 | 2.84958 | 2.84415 | +543 |
| 11 | 2.84959 | 2.84520 | +439 |
| 106 | 2.84960 | 2.84762 | +198 |
| 23 | 2.84965 | 2.84360 | +605 |
| 95 | 2.84966 | 2.84430 | +536 |
| 12 | 2.84979 | 2.84790 | +189 |
| 6 | 2.84979 | 2.84584 | +397 |
| 44 | 2.84982 | 2.84528 | +454 |
| 63 | 2.84984 | 2.84439 | +545 |
| 21 | 2.84984 | 2.84321 | +663 |
| 53 | 2.84992 | 2.84282 | +710 |
| 7 | 2.84997 | 2.84066 | +931 |
| | | Average | 7637 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER GRINDING

| Specimen No. | Average Diameter Before Grinding (in.) | Average Diameter After Grinding (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 1 | |
| 29 | 2.84566 | 2.84444 | 122 |
| 114 | 2.84580 | 2.84438 | 142 |
| 31 | 2.84584 | 2.84450 | 134 |
| 97 | 2.84591 | 2.84455 | 146 |
| 33 | 2.84606 | 2.84418 | 188 |
| 117 | 2.84624 | 2.84458 | 166 |
| 2 | 2.84627 | 2.84448 | 179 |
| 54 | 2.84628 | 2.84456 | 172 |
| 73 | 2.84630 | 2.84451 | 179 |
| 47 | 2.84637 | 2.84454 | 183 |
| 88 | 2.84641 | 2.84459 | 182 |
| 35 | 2.84645 | 2.84446 | 199 |
| 3 | 2.84655 | 2.84448 | 207 |
| 72 | 2.84668 | 2.84429 | 239 |
| 9 | 2.84672 | 2.84454 | 218 |
| **Average** | 2.84624 | 2.84447 | 177 |

| Specimen No. | Average Diameter Before Grinding (in.) | Average Diameter After Grinding (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 2 | |
| 17 | 2.84675 | 2.84250 | 425 |
| 18 | 2.84675 | 2.84295 | 430 |
| 96 | 2.84687 | 2.84251 | 436 |
| 99 | 2.84688 | 2.85255 | 433 |
| 30 | 2.84692 | 2.84254 | 438 |
| 55 | 2.84700 | 2.84255 | 445 |
| 41 | 2.84705 | 2.84255 | 450 |
| 116 | 2.84709 | 2.84256 | 453 |
| 92 | 2.84713 | 2.84263 | 450 |
| 5 | 2.84731 | 2.84247 | 484 |
| 93 | 2.84733 | 2.84247 | 486 |
| 120 | 2.84735 | 2.84246 | 489 |
| 39 | 2.84743 | 2.84250 | 493 |
| 43 | 2.84746 | 2.84243 | 503 |
| 24 | 2.84751 | 2.84237 | 514 |
| **Average** | 2.84712 | 2.84250 | 462 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER GRINDING

| Specimen No. | Average Diameter Before Grinding (in.) | Average Diameter After Grinding (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 3 | |
| 68 | 2.84758 | 2.84051 | 707 |
| 81 | 2.84759 | 2.84048 | 711 |
| 26 | 2.84759 | 2.84053 | 706 |
| 80 | 2.84762 | 2.84049 | 713 |
| 94 | 2.84763 | 2.84053 | 710 |
| 115 | 2.84764 | 2.84053 | 711 |
| 32 | 2.84764 | 2.84054 | 710 |
| 22 | 2.84768 | 2.84053 | 715 |
| 4 | 2.84770 | 2.84053 | 717 |
| 64 | 2.84773 | 2.84060 | 713 |
| 42 | 2.84783 | 2.84053 | 730 |
| 19 | 2.84786 | 2.84052 | 734 |
| 105 | 2.84791 | 2.84047 | 744 |
| 69 | 2.84804 | 2.84047 | 757 |
| 62 | 2.84808 | 2.84036 | 772 |
| **Average** | 2.84774 | 2.84051 | 723 |

| | | | |
|---|---|---|---|
| | | Group 4 | |
| 85 | 2.84812 | 2.83864 | 948 |
| 38 | 2.84818 | 2.83865 | 953 |
| 118 | 2.84822 | 2.83859 | 963 |
| 61 | 2.84830 | 2.83868 | 962 |
| 10 | 2.84832 | 2.83867 | 965 |
| 119 | 2.84833 | 2.83868 | 965 |
| 101 | 2.84834 | | |
| 1 | 2.84835 | 2.83874 | 961 |
| 75 | 2.84838 | 2.83862 | 976 |
| 91 | 2.84839 | 2.83863 | 976 |
| 56 | 2.84843 | 2.83865 | 978 |
| 51 | 2.84844 | 2.83864 | 980 |
| 37 | 2.84844 | 2.83853 | 991 |
| 102 | 2.84847 | 2.83848 | 999 |
| 58 | 2.84850 | 2.83847 | 1003 |
| **Average** | 2.84835 | 2.83862 | 973 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER GRINDING

| Specimen No. | Average Diameter Before Grinding (in.) | Average Diameter After Grinding (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 5 | |
| 50 | 2.84852 | 2.83657 | 1195 |
| 48 | 2.84852 | 2.83659 | 1193 |
| 25 | 2.84856 | 2.83682 | 1174 |
| 13 | 2.84858 | 2.83659 | 1199 |
| 86 | 2.84863 | 2.83663 | 1200 |
| 111 | 2.84868 | 2.83660 | 1208 |
| 79 | 2.84874 | 2.83658 | 1216 |
| 40 | 2.84884 | 2.83654 | 1230 |
| 82 | 2.84896 | 2.83662 | 1234 |
| 112 | 2.84897 | 2.83663 | 1234 |
| 49 | 2.84897 | 2.83657 | 1240 |
| 52 | 2.84901 | 2.83649 | 1252 |
| 66 | 2.84903 | 2.83646 | 1257 |
| 71 | 2.84903 | 2.83641 | 1252 |
| 65 | 2.84904 | 2.83638 | 1266 |
| Average | 2.84881 | 2.83657 | 1223 |

| Specimen No. | Average Diameter Before Grinding (in.) | Average Diameter After Grinding (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 6 | |
| 78 | 2.84922 | 2.83455 | 1467 |
| 28 | 2.84932 | 2.83461 | 1471 |
| 77 | 2.84955 | 2.83463 | 1492 |
| 104 | 2.84958 | 2.83463 | 1495 |
| 11 | 2.84959 | 2.83470 | 1489 |
| 106 | 2.84960 | 2.83475 | 1485 |
| 23 | 2.84965 | 2.83473 | 1492 |
| 95 | 2.84966 | 2.83474 | 1492 |
| 12 | 2.84979 | 2.83473 | 1506 |
| 6 | 2.84979 | 2.83473 | 1506 |
| 44 | 2.84982 | 2.83476 | 1506 |
| 63 | 2.84984 | 2.83473 | 1511 |
| 21 | 2.84984 | 2.83470 | 1514 |
| 53 | 2.84992 | 2.83471 | 1521 |
| 7 | 2.84997 | 2.83471 | 1526 |
| Average | 2.84962 | 2.83469 | 1500 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER CUTTING

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 1 | |
| 29 | 2.84444 | 2.84489 | +45 |
| 114 | 2.84438 | 2.84405 | -33 |
| 31 | 2.84450 | 2.84470 | +20 |
| 97 | 2.84455 | 2.84484 | +29 |
| 33 | 2.84418 | 2.84455 | +37 |
| 117 | 2.84458 | 2.84517 | +59 |
| 2 | 2.84448 | 2.84477 | +29 |
| 54 | 2.84456 | 2.84459 | +03 |
| 73 | 2.84451 | 2.84534 | +83 |
| 47 | 2.84454 | 2.84521 | +67 |
| 88 | 2.84459 | 2.84514 | +55 |
| 35 | 2.84446 | 2.84438 | -08 |
| 3 | 2.84448 | 2.84463 | +15 |
| 72 | 2.84429 | 2.84433 | +04 |
| 9 | 2.84454 | 2.84391 | -63 |
| Average | 2.84447 | 2.84470 | +23 |

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 2 | |
| 17 | 2.84250 | 2.84328 | +78 |
| 18 | 2.84245 | 2.84316 | +71 |
| 96 | 2.84251 | 2.84268 | +17 |
| 99 | 2.84255 | 2.84332 | +77 |
| 30 | 2.84254 | 2.84344 | +90 |
| 55 | 2.84255 | 2.84350 | +95 |
| 41 | 2.84255 | 2.84264 | +09 |
| 116 | 2.84256 | 2.84337 | +81 |
| 92 | 2.84263 | 2.84330 | +67 |
| 5 | 2.84247 | 2.84330 | +83 |
| 93 | 2.84247 | 2.84278 | +31 |
| 120 | 2.84246 | 2.84355 | +99 |
| 39 | 2.84250 | 2.84335 | +85 |
| 43 | 2.84243 | 2.84326 | +83 |
| 24 | 2.84237 | 2.84301 | +64 |
| Average | 2.84250 | 2.84320 | +69 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER CUTTING

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 3 | |
| 68 | 2.84051 | 2.84091 | +50 |
| 81 | 2.84048 | 2.84170 | +122 |
| 26 | 2.84053 | 2.84059 | +06 |
| 80 | 2.84049 | 2.84105 | +56 |
| 94 | 2.84053 | 2.84076 | +23 |
| 115 | 2.84053 | 2.84088 | +35 |
| 32 | 2.84054 | 2.84109 | +55 |
| 22 | 2.84053 | 2.84099 | +46 |
| 4 | 2.84053 | 2.84121 | +68 |
| 64 | 2.84060 | 2.84059 | -01 |
| 42 | 2.84053 | 2.84087 | +34 |
| 19 | 2.84052 | 2.84044 | -08 |
| 105 | 2.84047 | 2.84110 | +63 |
| 69 | 2.84047 | 2.84102 | +55 |
| 62 | 2.84036 | 2.84023 | -13 |
| Average | 2.84051 | 2.84090 | +40 |

| | | | |
|---|---|---|---|
| | | Group 4 | |
| 85 | 2.83864 | 2.83897 | +33 |
| 38 | 2.83865 | 2.83789 | -76 |
| 118 | 2.83859 | 2.83868 | +09 |
| 61 | 2.83868 | 2.83807 | -61 |
| 10 | 2.83867 | 2.84004 | +137 |
| 119 | 2.83868 | 2.83869 | +01 |
| 101 | | | |
| 1 | 2.83874 | 2.83861 | -13 |
| 75 | 2.83862 | 2.83827 | -35 |
| 91 | 2.83863 | 2.83829 | -34 |
| 56 | 2.83865 | 2.83946 | +81 |
| 51 | 2.83864 | 2.83887 | +23 |
| 37 | 2.83853 | 2.83891 | +38 |
| 102 | 2.83848 | 2.83870 | +22 |
| 58 | 2.83847 | 2.83911 | +64 |
| Average | 2.83862 | 2.83875 | +13 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER CUTTING

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 5 | |
| 50 | 2.83657 | 2.83662 | +05 |
| 48 | 2.83659 | 2.83607 | -52 |
| 25 | 2.83682 | 2.83713 | +31 |
| 13 | 2.83659 | 2.83729 | +70 |
| 86 | 2.83663 | 2.83607 | -56 |
| 111 | 2.83660 | 2.83710 | +50 |
| 79 | 2.83658 | 2.83721 | +63 |
| 40 | 2.83654 | 2.83661 | +07 |
| 82 | 2.83662 | 2.83621 | -41 |
| 112 | 2.83663 | 2.83600 | -63 |
| 49 | 2.83657 | 2.83604 | -53 |
| 52 | 2.83649 | 2.83683 | +34 |
| 66 | 2.83646 | 2.83579 | -67 |
| 71 | 2.83641 | 2.83640 | -01 |
| 65 | 2.83638 | 2.83651 | +13 |
| Average | 2.83657 | 2.83653 | -04 |

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 6 | |
| 78 | 2.83455 | 2.83518 | +63 |
| 28 | 2.83461 | 2.83491 | +30 |
| 77 | 2.83463 | 2.83471 | +08 |
| 104 | 2.83463 | 2.83503 | +40 |
| 11 | 2.83470 | 2.83467 | -03 |
| 106 | 2.83475 | 2.83571 | +96 |
| 23 | 2.83473 | 2.83478 | +05 |
| 95 | 2.83474 | 2.83479 | +05 |
| 12 | 2.83473 | 2.83522 | +49 |
| 6 | 2.83473 | 2.83454 | -19 |
| 44 | 2.83476 | 2.83483 | +07 |
| 63 | 2.83473 | 2.83448 | -25 |
| 21 | 2.83470 | 2.83436 | -34 |
| 53 | 2.83471 | 2.83470 | -01 |
| 7 | 2.83471 | 2.83532 | +61 |
| Average | 2.83469 | 2.83488 | +19 |

AVERAGE DIAMETER MEASUREMENTS BEFORE AND AFTER CUTTING

| Specimen No. | Average Diameter Before Cutting (in.) | Average Diameter After Cutting (in.) | Difference x $10^{-5}$ in. |
|---|---|---|---|
| | | Group 7 | |
| 36 | 2.85062 | 2.85097 | +35 |
| 70 | 2.85038 | 2.85018 | -20 |
| 45 | 2.85075 | 2.85126 | +51 |
| 57 | 2.85078 | 2.85160 | +82 |
| 59 | 2.85150 | 2.85145 | -05 |
| 34 | 2.85032 | 2.85069 | +37 |
| 20 | 2.85016 | 2.85038 | +22 |
| 14 | 2.85076 | 2.85070 | -06 |
| 8 | 2.85030 | 2.85132 | +102 |
| 103 | 2.85057 | 2.85077 | +20 |
| 108 | 2.85025 | 2.85072 | +47 |
| 110 | 2.85012 | 2.85065 | +53 |
| 46 | 2.85150 | 2.85177 | +27 |
| 87 | 2.85027 | 2.85080 | +53 |
| 84 | 2.85109 | 2.85233 | +124 |
| Average | 2.85062 | 2.85104 | +41 |

GROUP 1

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 29 | 2.84442 | 2.84445 | 2.84446 | 2.84444 | 04 |
| 114 | 2.84440 | 2.84438 | 2.84438 | 2.84438 | 02 |
| 31 | 2.84454 | 2.84449 | 2.84448 | 2.84450 | 06 |
| 97 | 2.84455 | 2.84453 | 2.84457 | 2.84455 | 04 |
| 33 | 2.84415 | 2.84417 | 2.84423 | 2.84418 | 08 |
| 117 | 2.84455 | 2.84459 | 2.84465 | 2.84458 | 10 |
| 2 | 2.84450 | 2.84447 | 2.84446 | 2.84448 | 04 |
| 54 | 2.84458 | 2.84455 | 2.84455 | 2.84456 | 03 |
| 73 | 2.84450 | 2.84450 | 2.84454 | 2.84451 | 04 |
| 47 | 2.84456 | 2.84452 | 2.84454 | 2.84454 | 04 |
| 88 | 2.84458 | 2.84460 | 2.84459 | 2.84459 | 02 |
| 35 | 2.84445 | 2.84446 | 2.84446 | 2.84446 | 01 |
| 3 | 2.84450 | 2.84445 | 2.84450 | 2.84448 | 05 |
| 72 | 2.84430 | 2.84428 | 2.84430 | 2.84429 | 02 |
| 9 | 2.84455 | 2.84453 | 2.84455 | 2.84454 | 02 |

GROUP 1

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

(Diameter at 90° to Maximum Diameter)

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 29 | 2.84263 | 2.84264 | 2.84261 | 2.84263 | 03 |
| 114 | 2.84443 | 2.84442 | 2.84449 | 2.84445 | 07 |
| 31 | 2.84431 | 2.84433 | 2.84435 | 2.84433 | 04 |
| 97 | 2.84455 | 2.84454 | 2.84449 | 2.84453 | 06 |
| 33 | 2.84260 | 2.84335 | 2.84360 | 2.84352 | 25 |
| 117 | 2.84259 | 2.84273 | 2.84272 | 2.84268 | 14 |
| 2 | 2.84130 | 2.84135 | 2.84132 | 2.84132 | 05 |
| 54 | 2.84302 | 2.84305 | 2.84304 | 2.84304 | 03 |
| 73 | 2.84367 | 2.84363 | 2.84370 | 2.84366 | 07 |
| 47 | 2.84373 | 2.84374 | 2.84372 | 2.84373 | 02 |
| 88 | 2.84453 | 2.84452 | 2.84457 | 2.84454 | 05 |
| 35 | 2.84375 | 2.84375 | 2.84372 | 2.84374 | 03 |
| 3 | 2.84379 | 2.84374 | 2.84362 | 2.84371 | 17 |
| 72 | 2.84220 | 2.84223 | 2.84223 | 2.84222 | 03 |
| 9 | 2.84463 | 2.84465 | 2.84470 | 2.84466 | 07 |

GROUP 1

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 29 | 2.84490 | 2.84489 | 2.84488 | 2.84489 | 02 |
| 114 | 2.84403 | 2.84405 | 2.84406 | 2.84405 | 03 |
| 31 | 2.84468 | 2.84472 | 2.84470 | 2.84470 | 04 |
| 97 | 2.84487 | 2.84481 | 2.84484 | 2.84484 | 06 |
| 33 | 2.84457 | 2.84454 | 2.84455 | 2.84455 | 03 |
| 117 | 2.84514 | 2.84518 | 2.84518 | 2.84517 | 04 |
| 2 | 2.84477 | 2.84480 | 2.84475 | 2.84477 | 05 |
| 54 | 2.84460 | 2.84462 | 2.84456 | 2.84459 | 06 |
| 73 | 2.84531 | 2.84536 | 2.84535 | 2.84534 | 05 |
| 47 | 2.84519 | 2.84524 | 2.84520 | 2.84521 | 05 |
| 88 | 2.84512 | 2.84516 | 2.84513 | 2.84514 | 04 |
| 35 | 2.84442 | 2.84437 | 2.84435 | 2.84438 | 07 |
| 3 | 2.84461 | 2.84464 | 2.84463 | 2.84463 | 03 |
| 72 | 2.84433 | 2.84432 | 2.84435 | 2.84433 | 03 |
| 9 | 2.84392 | 2.84392 | 2.84390 | 2.84391 | 02 |

GROUP 2

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 17 | 2.84244 | 2.84251 | 2.84254 | 2.84250 | 10 |
| 18 | 2.84243 | 2.84245 | 2.84245 | 2.84245 | 02 |
| 96 | 2.84248 | 2.84250 | 2.84254 | 2.84251 | 06 |
| 99 | 2.84253 | 2.84255 | 2.84257 | 2.85255 | 04 |
| 30 | 2.84252 | 2.84254 | 2.84256 | 2.84254 | 04 |
| 55 | 2.84255 | 2.84256 | 2.84255 | 2.84255 | 01 |
| 41 | 2.84255 | 2.84255 | 2.84257 | 2.84255 | 02 |
| 116 | 2.84252 | 2.84257 | 2.84258 | 2.84256 | 06 |
| 92 | 2.84260 | 2.84263 | 2.84265 | 2.84263 | 05 |
| 5 | 2.84244 | 2.84249 | 2.84248 | 2.84247 | 05 |
| 93 | 2.84243 | 2.84250 | 2.84247 | 2.84247 | 07 |
| 120 | 2.84246 | 2.84246 | 2.84247 | 2.84246 | 01 |
| 39 | 2.84250 | 2.84251 | 2.84250 | 2.84250 | 01 |
| 43 | 2.84240 | 2.84244 | 2.84244 | 2.84243 | 04 |
| 24 | 2.84235 | 2.84240 | 2.84236 | 2.84237 | 05 |

GROUP 2

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 17 | 2.84328 | 2.84327 | 2.84328 | 2.84328 | 01 |
| 18 | 2.84315 | 2.84316 | 2.84317 | 2.84316 | 02 |
| 96 | 2.84268 | 2.84267 | 2.84269 | 2.84268 | 02 |
| 99 | 2.84330 | 2.84333 | 2.84332 | 2.84332 | 03 |
| 30 | 2.84343 | 2.84347 | 2.84343 | 2.84344 | 04 |
| 55 | 2.84349 | 2.84350 | 2.84350 | 2.84350 | 01 |
| 41 | 2.84264 | 2.84265 | 2.84264 | 2.84264 | 01 |
| 116 | 2.84338 | 2.84337 | 2.84336 | 2.84337 | 02 |
| 92 | 2.84328 | 2.84330 | 2.84332 | 2.84330 | 04 |
| 5 | 2.84300 | 2.84300 | 2.84300 | 2.84300 | 00 |
| 93 | 2.84277 | 2.84278 | 2.84280 | 2.84278 | 03 |
| 120 | 2.84357 | 2.84354 | 2.84353 | 2.84355 | 04 |
| 39 | 2.84337 | 2.84335 | 2.84335 | 2.84335 | 02 |
| 43 | 2.84324 | 2.84326 | 2.84327 | 2.84326 | 03 |
| 24 | 2.84300 | 2.84301 | 2.84302 | 2.84301 | 02 |

## GROUP 3

### MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 68 | 2.84050 | 2.84053 | 2.84050 | 2.84051 | 03 |
| 81 | 2.84047 | 2.84048 | 2.84048 | 2.84048 | 01 |
| 26 | 2.84050 | 2.84055 | 2.84054 | 2.84053 | 05 |
| 80 | 2.84048 | 2.84049 | 2.84049 | 2.84049 | 01 |
| 94 | 2.84054 | 2.84053 | 2.84053 | 2.84053 | 01 |
| 115 | 2.84053 | 2.84055 | 2.84052 | 2.84053 | 03 |
| 32 | 2.84053 | 2.84055 | 2.8454 | 2.84054 | 02 |
| 22 | 2.84053 | 2.84052 | 2.84054 | 2.84053 | 02 |
| 4 | 2.84053 | 2.84053 | 2.84053 | 2.84053 | 00 |
| 64 | 2.84060 | 2.84060 | 2.84060 | 2.84060 | 00 |
| 42 | 2.84054 | 2.84053 | 2.84051 | 2.84053 | 03 |
| 19 | 2.84052 | 2.84052 | 2.84052 | 2.84052 | 00 |
| 105 | 2.84047 | 2.84048 | 2.84046 | 2.84047 | 02 |
| 69 | 2.84048 | 2.84047 | 2.84047 | 2.84047 | 01 |
| 62 | 2.84037 | 2.84035 | 2.84035 | 2.84036 | 02 |

GROUP 3

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 68 | 2.84091 | 2.84092 | 2.84090 | 2.84091 | 02 |
| 81 | 2.84170 | 2.84172 | 2.84169 | 2.84170 | 03 |
| 26 | 2.84056 | 2.84060 | 2.84060 | 2.84059 | 04 |
| 80 | 2.84104 | 2.84105 | 2.84105 | 2.84105 | 01 |
| 94 | 2.84076 | 2.84077 | 2.84079 | 2.84076 | 03 |
| 115 | 2.84089 | 2.84088 | 2.84086 | 2.84088 | 03 |
| 32 | 2.84110 | 2.84110 | 2.84108 | 2.84109 | 02 |
| 22 | 2.84100 | 2.84098 | 2.84099 | 2.84099 | 02 |
| 4 | 2.84121 | 2.84121 | 2.84121 | 2.84121 | 00 |
| 64 | 2.84058 | 2.84060 | 2.84060 | 2.84059 | 02 |
| 42 | 2.84087 | 2.84088 | 2.84087 | 2.84087 | 01 |
| 19 | 2.84042 | 2.84045 | 2.84045 | 2.84044 | 03 |
| 105 | 2.84110 | 2.84112 | 2.84108 | 2.84110 | 04 |
| 69 | 2.84102 | 2.84102 | 2.84101 | 2.84102 | 01 |
| 62 | 2,84019 | 2.84024 | 2.84027 | 2.84023 | 08 |

GROUP 4

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 85 | 2.83867 | 2.83862 | 2.83863 | 2.83864 | 05 |
| 38 | 2.83867 | 2.83865 | 2.83862 | 2.83865 | 05 |
| 118 | 2.83861 | 2.83858 | 2.83859 | 2.83859 | 03 |
| 61 | 2.83872 | 2.83866 | 2.83867 | 2.83868 | 06 |
| 10 | 2.83868 | 2.83867 | 2.83866 | 2.83867 | 02 |
| 119 | 2.83870 | 2.83868 | 2.83866 | 2.83868 | 04 |
| 101 | See Note Below | | | | |
| 1 | 2.83875 | 2.83875 | 2.83871 | 2.83874 | 04 |
| 75 | 2.83863 | 2.83862 | 2.83860 | 2.83862 | 03 |
| 91 | 2.83864 | 2.93963 | 2.83862 | 2.83863 | 02 |
| 56 | 2.83867 | 2.83865 | 2.83865 | 2.83865 | 02 |
| 51 | 2.83866 | 2.83864 | 2.83862 | 2.83864 | 04 |
| 37 | 2.83855 | 2.83852 | 2.83852 | 2.83853 | 03 |
| 102 | 2.83850 | 2.83847 | 2.83847 | 2.83848 | 03 |
| 58 | 2.83848 | 2.83847 | 2.83847 | 2.83847 | 01 |
| 107 | 2.83867 | 2.83865 | 2.83864 | 2.83865 | 03 |

Note:

Specimen No. 101 not found in this group as listed in letter July 22, 1965.

Specimen No. 107 in group instead.

GROUP 4

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 85 | 2.83898 | 2.83897 | 2.83896 | 2.83897 | 02 |
| 38 | 2.83787 | 2.83790 | 2.83789 | 2.83789 | 03 |
| 118 | 2.83867 | 2.83868 | 2.83868 | 2.83868 | 01 |
| 61 | 2.83805 | 2.83808 | 2.83807 | 2.83807 | 03 |
| 10 | 2.84003 | 2.84005 | 2.84005 | 2.84004 | 02 |
| 119 | 2.83875 | 2.83865 | 2.83866 | 2.83869 | 10 |
| 107 | 2.83882 | 2.83878 | 2.83878 | 2.83879 | 04 |
| 1 | 2.83860 | 2.83862 | 2.83862 | 2.83861 | 02 |
| 75 | 2.83827 | 2.83827 | 2.83826 | 2.83827 | 01 |
| 91 | 2.83829 | 2.83830 | 2.83829 | 2.83829 | 01 |
| 56 | 2.83944 | 2.83950 | 2.83944 | 2.83946 | 06 |
| 51 | 2.83884 | 2.83886 | 2.83890 | 2.83887 | 06 |
| 37 | 2.83890 | 2.83892 | 2.83890 | 2.83891 | 02 |
| 102 | 2.83869 | 2.83871 | 2.3870 | 2.83870 | 02 |
| 58 | 2.83912 | 2.83911 | 2.83911 | 2.83911 | 01 |

GROUP 5

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 50 | 2.83660 | 2.83656 | 2.83655 | 2.83657 | 05 |
| 48 | 2.83660 | 2.83659 | 2.83657 | 2.83659 | 03 |
| 25 | 2.83685 | 2.83680 | 2.83682 | 2.83682 | 05 |
| 13 | 2.83662 | 2.83658 | 2.83658 | 2.83659 | 04 |
| 86 | 2.83665 | 2.83662 | 2.83663 | 2.83663 | 03 |
| 111 | 2.83660 | 2.83660 | 2.83661 | 2.83660 | 01 |
| 79 | 2.83661 | 2.83658 | 2.83657 | 2.83658 | 04 |
| 40 | 2.83655 | 2.83653 | 2.83653 | 2.83654 | 02 |
| 82 | 2.83662 | 2.83662 | 2.83662 | 2.83662 | 00 |
| 112 | 2.83663 | 2.83665 | 2.83660 | 2.83663 | 05 |
| 49 | 2.83657 | 2.83656 | 2.83658 | 2.83657 | 02 |
| 52 | 2.83652 | 2.83648 | 2.83647 | 2.83649 | 05 |
| 66 | 2.83648 | 2.83645 | 2.83645 | 2.83646 | 03 |
| 71 | 2.83642 | 2.83640 | 2.83640 | 2.83641 | 02 |
| 65 | 2.83639 | 2.83637 | 2.83637 | 2.83638 | 02 |

GROUP 5

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 50 | 2.83663 | 2.83662 | 2.83660 | 2.83662 | 03 |
| 48 | 2.83607 | 2.83608 | 2.83606 | 2.83607 | 02 |
| 25 | 2.83705 | 2.83718 | 2.83716 | 2.83713 | 13 |
| 13 | 2.83728 | 2.83729 | 2.83730 | 2.83729 | 02 |
| 86 | 2.83607 | 2.83607 | 2.83608 | 2.83607 | 01 |
| 111 | 2.83710 | 2.83709 | 2.83710 | 2.83710 | 01 |
| 79 | 2.83723 | 2.83720 | 2.83720 | 2.83721 | 03 |
| 40 | 2.83662 | 2.83661 | 2.83660 | 2.83661 | 02 |
| 82 | 2.83621 | 2.83621 | 2.83622 | 2.83621 | 01 |
| 112 | 2.83600 | 2.83600 | 2.83600 | 2.83600 | 00 |
| 49 | 2.83603 | 2.83602 | 2.83606 | 2.83604 | 04 |
| 52 | 2.83683 | 2.83684 | 2.83683 | 2.83683 | 01 |
| 66 | 2.83580 | 2.83579 | 2.83579 | 2.83579 | 01 |
| 71 | 2.83639 | 2.83640 | 2.83640 | 2.83640 | 01 |
| 65 | 2.83652 | 2.83651 | 2.83651 | 2.83651 | 01 |

GROUP 6

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 78 | 2.83457 | 2.83452 | 2.83457 | 2.83455 | 05 |
| 28 | 2.83463 | 2.83463 | 2.83458 | 2.83461 | 05 |
| 77 | 2.83465 | 2.83463 | 2.83462 | 2.83463 | 03 |
| 104 | 2.83465 | 2.83463 | 2.83464 | 2.83463 | 02 |
| 11 | 2.83470 | 2.83470 | 2.83469 | 2.83470 | 01 |
| 106 | 2.83477 | 2.83474 | 2.83474 | 2.83475 | 03 |
| 23 | 2.83475 | 2.83472 | 2.83471 | 2.83473 | 04 |
| 95 | 2.83476 | 2.83473 | 2.83474 | 2.83474 | 03 |
| 12 | 2.83475 | 2.83472 | 2.83472 | 2.83473 | 03 |
| 6 | 2.83475 | 2.83471 | 2.83473 | 2.83473 | 04 |
| 44 | 2.83478 | 2.83478 | 2.83472 | 2.83476 | 06 |
| 63 | 2.83474 | 2.83472 | 2.83473 | 2.83473 | 02 |
| 21 | 2.83472 | 2.83467 | 2.83470 | 2.83470 | 05 |
| 53 | 2.83473 | 2.83471 | 2.83469 | 2.83471 | 04 |
| 7 | 2.83473 | 2.83470 | 2.83470 | 2.83471 | 03 |

GROUP 6

MAXIMUM DIAMETER MEASUREMENTS AFTER GRINDING

(Dia. at 90° To Max. Dia.)

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 78 | 2.83453 | 2.83451 | 2.83453 | 2.83452 | 02 |
| 28 | 2.83455 | 2.83456 | 2.83458 | 2.83456 | 03 |
| 77 | 2.83455 | 2.83458 | 2.83461 | 2.83458 | 06 |
| 104 | 2.83464 | 2.83464 | 2.83464 | 2.83464 | 00 |
| 11 | 2.83465 | 2.83465 | 2.83471 | 2.83467 | 06 |
| 106 | 2.83473 | 2.83472 | 2.83476 | 2.83474 | 05 |
| 23 | 2.83475 | 2.83475 | 2.83473 | 2.83474 | 02 |
| 95 | 2.83469 | 2.83470 | 2.83470 | 2.83470 | 01 |
| 12 | 2.83470 | 2.83470 | 2.83475 | 2.83472 | 05 |
| 6 | 2.83473 | 2.83475 | 2.83476 | 2.83475 | 03 |
| 44 | 2.83471 | 2.83477 | 2.83473 | 2.83474 | 06 |
| 63 | 2.83472 | 2.83472 | 2.83472 | 2.83472 | 00 |
| 21 | 2.83468 | 2.83469 | 2.83469 | 2.83469 | 01 |
| 53 | 2.83465 | 2.83468 | 2.83470 | 2.83468 | 05 |
| 7 | 2.83472 | 2.83470 | 2.83468 | 2.83470 | 04 |

GROUP 6

MAXIMUM DIAMETER MEASUREMENTS AFTER CUTTING

| Specimen No. | 1st Read. (in.) | 2nd Read. (in.) | 3rd Read. (in.) | Average (in.) | Dispersion .00001 in. |
|---|---|---|---|---|---|
| 78 | 2.83518 | 2.83520 | 2.83516 | 2.83518 | 04 |
| 28 | 2.83493 | 2.83491 | 2.83489 | 2.83491 | 04 |
| 77 | 2.83472 | 2.83472 | 2.83470 | 2.83471 | 02 |
| 104 | 2.83503 | 2.83503 | 2.83502 | 2.83503 | 01 |
| 11 | 2.83470 | 2.83467 | 2.83465 | 2.83467 | 05 |
| 106 | 2.83572 | 2.83571 | 2.83571 | 2.83571 | 01 |
| 23 | 2.83478 | 2.83478 | 2.83477 | 2.83478 | 01 |
| 95 | 2.83480 | 2.83478 | 2.83478 | 2.83479 | 02 |
| 12 | 2.83523 | 2.83522 | 2.83522 | 2.83522 | 01 |
| 6 | 2.83455 | 2.83454 | 2.83453 | 2.83454 | 02 |
| 44 | 2.83484 | 2.83482 | 2.83482 | 2.83483 | 02 |
| 63 | 2.83446 | 2.83449 | 2.83448 | 2.83448 | 03 |
| 21 | 2.83437 | 2.83434 | 2.83436 | 2.83436 | 03 |
| 53 | 2.83471 | 2.83469 | 2.83470 | 2.83470 | 02 |
| 7 | 2.83532 | 2.83532 | 2.83532 | 2.83532 | 00 |

## III. ANALYTICAL PROCEDURES

If residual stresses are present in hardened bearing races as a result of hardening treatments, then, since such stresses are elastic, reducing those stresses to zero should result in a corresponding strain from which the nature and magnitude of the residual stresses can be calculated. The problem, then, is to determine the strain when hardened races relax elastically. Therefore, the approach used was to measure accurately the outside diameters of bearing races before and after removing a predetermined amount and then to measure the diameter change after cutting the race, as shown in Fig. 3, and allowing it to relax. This approach enables the principal stress, that tangent to the bearing, to be calculated in the manner of a cantilever beam and ignores the small stresses in the other two directions.

There was considerable variation in the outside diameter of an initial sample of 20 bearings, which was inspected and centerless ground in groups of five each to remove nothing, 0.002 in., 0.004 in., and 0.006 in. from the outside diameter after hardening. Because of such variations, it was decided that a larger sample must be used and that the races must be as close to the same size as possible and uniformly subjected to essentially the same processes. As a result a group of 110 bearings, hardened but unground, were inspected and grouped into lots of 15 each with a minimum size variation within each group. Group 1 was centerless ground to remove nominally 0.002 in. from the outside diameter in one pass. Group 2 was centerless ground to remove 0.002 in. from the outside diameter in two equal passes. Group 3 was centerless ground to remove 0.006 in. from the outside diameter in three equal passes. Group 4 was ground to remove 0.008 in. in four equal passes. Group 5 was ground to remove 0.010 in. in five equal passes. Group 6 was ground to remove 0.012 in. in six equal passes. Group 7 had nothing done to it and represented the as-hardened condition.

After the grinding, the bearing diameters were again measured at the same point, after which they were cut at right angles to the measured diameter. An abrasive cut-off machine was used, with an emulsion as coolant. The diameter at right angles to the cut was again measured to determine the elastic strain that resulted from the release of the residual stresses.

### MEASURING TECHNIQUES

The outside diameter of the bearing race was measured by use of a Sheffield Comparator, Style 5000, Serial No. 1068RS, with a least reading of 25 millionths of an in. (0.000025 in.). It was set to the appropriate dimension by use of gage blocks, Johansson Standard Set 1-A, Serial No. 4253.

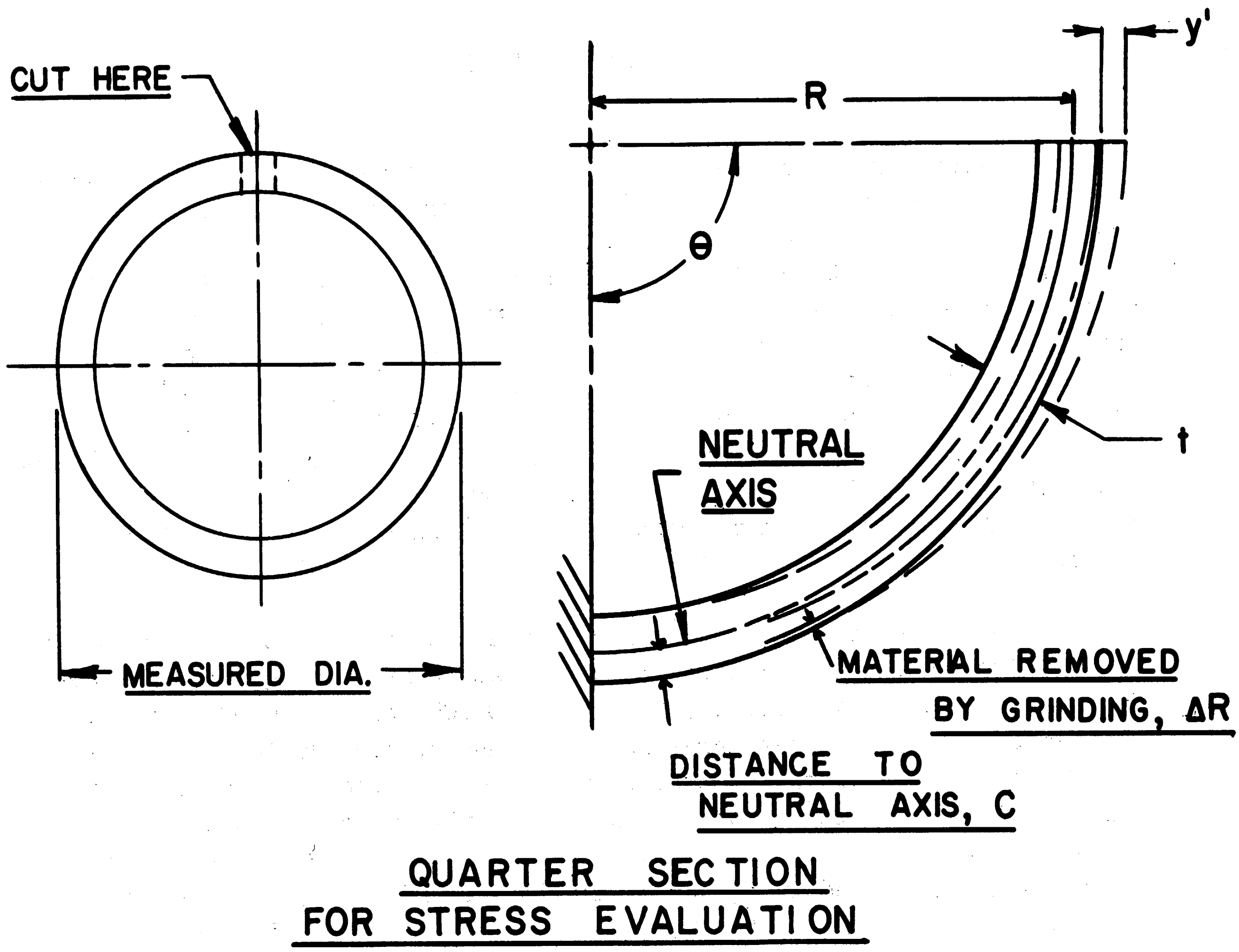


Fig. 3. Method of cutting bearing race.

**The outer race has been ground on the outside diameter, with an amount ΔR removed. It is then cut at right angles to the measured dia. and measured again. The stress is computed by treating one quarter of the race as a simple cantilever beam for which one half the change in diameter becomes the deflection as a result of the internal stresses. For the exact analytical relations, see the rest of the report.**

Each race was measured to determine its maximum diameter, which was then marked and used for future measurements. Each race was measured three times by the same person under essentially the same conditions. The average of the three readings was used as the diameter.

A statistical analysis of the measurement data was made to verify the validity of using the arithmetic average as the representative diameter. The originally measured diameters followed a Gaussian distribution, but this was eliminated when they were grouped into more nearly uniform sized lots of fifteen each. This changed the distribution to a modified log-normal type, which permitted the use of a Weibull distribution since one could assume that the sample size was homogeneous and that identical treatments had been given to each specimen within a group. A Weibull distribution follows the general equation

$$f(x) = b\left(\frac{X-X_0}{\theta}\right)^{b-1} e^{-\left(\frac{X-X_0}{\theta}\right)^{b}}$$

which can be modified to give a straight line when plotted on Weibull graph paper with diameter size and frequency of occurrence used as coordinates. After the data were plotted, the 50% readings were taken as the statistical mean. In every case the difference between the statistical mean and the arithmetic average was insignificant, which justifies the use of the arithmetic average as the measured diameter.

STRESS CALCULATIONS

Figure 4 shows the analysis of the stress in the outer part of the races as a result of the relaxation due to cutting the race. One quarter of the race is treated as a simply supported cantilever beam which has deflected an amount, y', equal to half the change in diameter of the cut race. Practically, the average change in diameter or deflection of each group can be ascribed to the removal of a stressed layer of metal 0.001 in. thick by each grinding operation. Thus:

$$y' = \int_a^b \frac{M}{EI} X \cdot dx$$

$$= \int_a^b \frac{M}{EI} (R\theta)(Rd\theta)$$

$$= \int_0^{\pi/2} \frac{M}{EI} R^2\theta \, d\theta$$

POSITIONING APPARATUS FOR CUT-OFF GRINDING

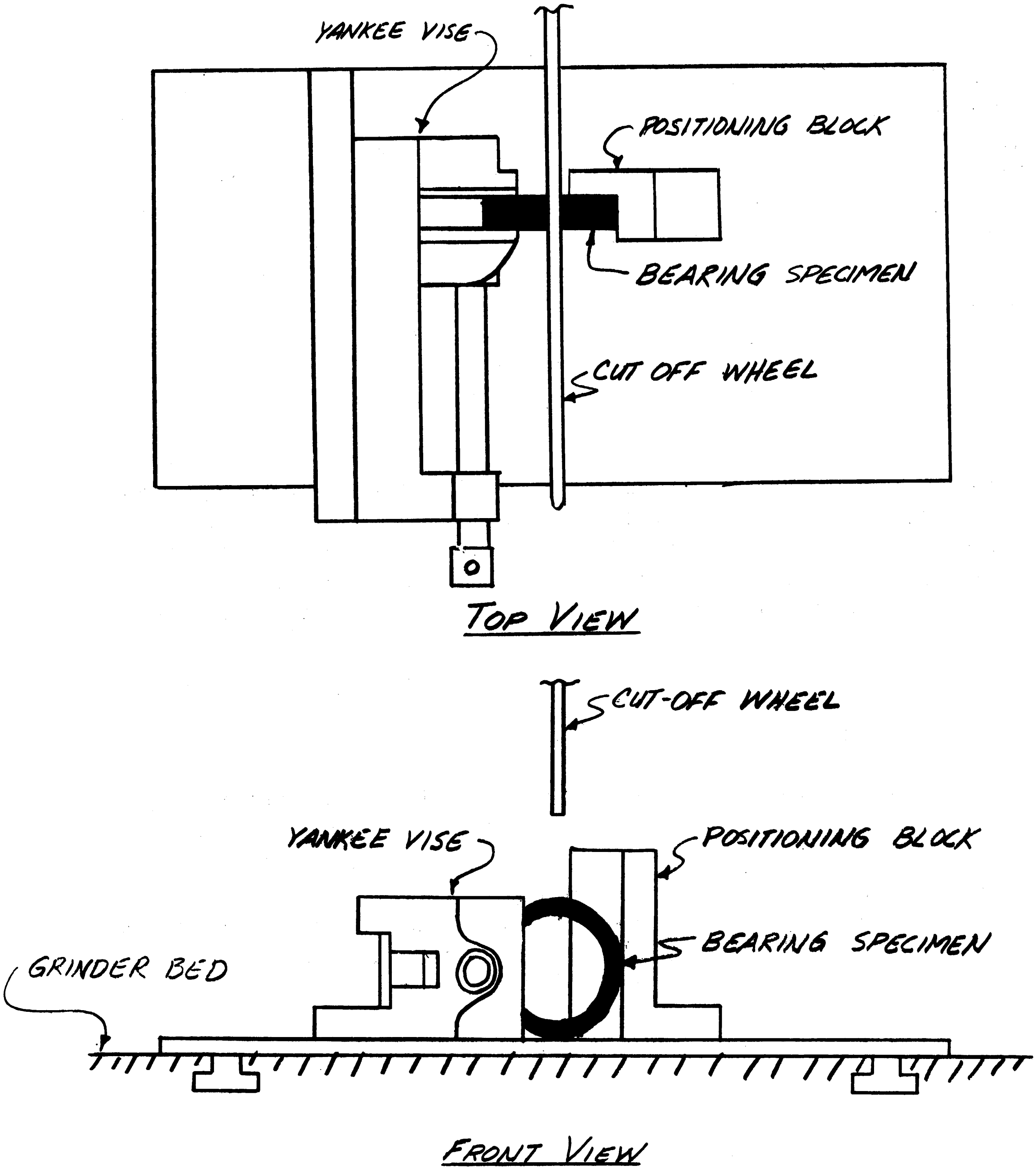


Fig. 4. Residual stress analysis for cut hardened outer race.

$$= \frac{MR^2}{EI} \left(\frac{\theta}{2}\right)^2$$

$$= \frac{MR^2}{EI} \times \frac{\pi^2}{8}$$

where

$M$ = bending moment
$X = R\theta$
$dx = Rd\theta$
$\theta = \pi/2$ radians
$E$ = modulus of elasticity
$I$ = moment of inertia

For the particular situation shown in Fig. 4 (see also Fig. 5),

$$M = \sigma \cdot w \cdot \Delta R \cdot c$$

where

$\sigma$ = stress, unknown
$w$ = width across which $\sigma$ acts, 0.68 in.
$\Delta R$ = thickness of material removed in grinding, on which $\sigma$ acts
$c$ = distance from neutral axis to outer fiber, 0.10 in.

Substituting in the equation above gives

$$y' = \frac{\pi^2 R^2}{8EI} \times \sigma\, w\, \Delta R\, c$$

$$\therefore \sigma = \frac{8EIy'}{\pi^2 R^2 w\, \Delta R\, c}$$

For the race used,

$$E = 30 \times 10^6, \text{ and}$$

$$I = \frac{wt^3}{12}, \text{ where } t = 2c.$$

$$\therefore \sigma = \frac{8}{\pi^2 R^2} \cdot E \cdot \frac{wt^3}{12} \cdot \frac{y'}{w \cdot \Delta R \cdot \frac{t}{2}}$$

$$= \frac{4Et^2 y'}{3\pi^2 R^2 \Delta R}$$

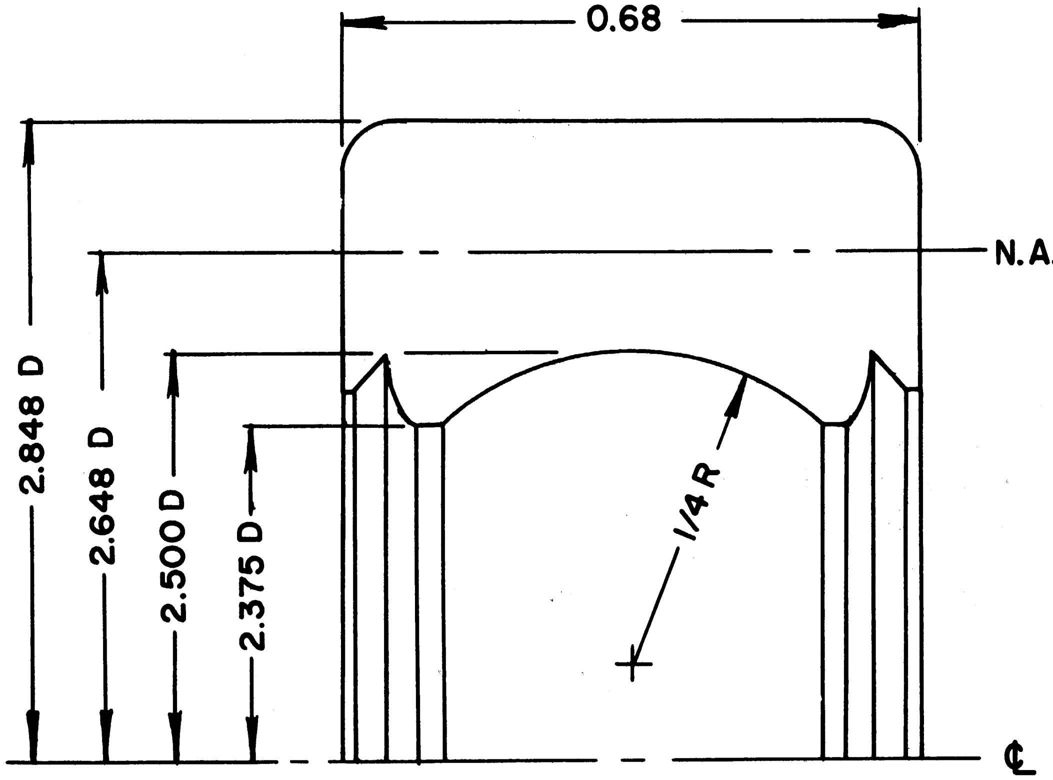


Fig. 5. Outer bearing race.

207 Size Outer Race, Conrad Type
AISI 52100 Steel, Vacuum Deoxidized
Nominal Finished Outside Diameter, 2.830 In.
Nominal Inside Diameter, 2.515 In.

$$\therefore \sigma_n = \frac{4 \times 30 \times 10^6 (.2)^2 y_n'}{3\pi^2 (1.22)^2 (.001)}$$

$$= 10.9 \times 10^7 y_n'$$

where

$R = 1.22$ in.
$t = 0.2$ in.
$w = .68$ in.
$\Delta R = 0.001$ in.

$\therefore$ For:

Group 1, $y' = + 1.2 \times 10^{-4}$ in.; $\sigma_1 = 10.9 \times 1.2 \times 10^{-4} \times 10^7$
$= 13,100$ psi

Group 2, $y' = + 2.0 \times 10^{-4}$ in.; $\sigma_2 = 10.9 \times 3.5 \times 10^{-4} \times 10^7$
$= 38,000$ psi

Group 3, $y' = + 2.0 \times 10^{-4}$ in.; $\sigma_3 = 10.9 \times 2.0 \times 10^{-4} \times 10^7$
$= 21,800$ psi

Group 4, $y' = + 0.65 \times 10^{-4}$ in.; $\sigma_4 = 10.9 \times .65 \times 10^{-4} \times 10^7$
$= 7,100$ psi

Group 5, $y' = - 0.2 \times 10^{-4}$ in.; $\sigma_5 = 10.9 \times 0.2 \times 10^{-4} \times 10^7$
$= 2,180$ psi

Group 6, $y' = + 1.0 \times 10^{-4}$ in.; $\sigma_6 = 10.9 \times 1.0 \times 10^{-4} \times 10^7$
$= 10,900$ psi

Group 7, $y' = + 2.0 \times 10^{-4}$ in.; $\sigma_7 = 10.9 \times 2.0 \times 10^{-4} \times 10^7$
$= 21,800$ psi

These calculated stress values represent only the average for an average increment of 0.001 in. metal removal. Actual differences in the parameters, as opposed to the nominal values used, may result in stresses perhaps 50% or more above the calculated values.

## HEAT TREATMENT OF AISI 52100 STEEL

Procedure used at Bearings Corporation of America:

1. Heat at 1540°F ± 5°F for 30 to 40 minutes at heat with a dewpoint of 30-35°F for the endothermic atmosphere in the furnace.

2. Oil quench in oil of 130°-150°F.

3. Check for hardness; must be Rc 64-66 as quenched.

4. Wash parts.

5. Temper for 2 to 3 hours at 375°F ± 5°F.

6. Final hardness to be Rc 60-64. Grain size should be no larger than 8.