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The Drilling Problem: A Stochastic Modeling and Control Example in Manufacturing

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Abstract

A machining economics problem is considered where feed speed selection and tool replacement policies are to be determined. A new stochastic model for tool wear, called a diffusion-threshold model, is proposed. This tool wear model allows the machining economics problem to be formulated as a stochastic optimal control problem incorporating measurement feedback of tool wear. A class of cost functionals is introduced and the optimal policy is described. An example problem based on actual data is worked out.

1. INTRODUCTION

An emphasis on improved productivity in manufacturing has initiated a surge of interest in all aspects of the manufacturing field. Ramifications of this interest are the reconsideration of traditional manufacturing practices and the development of new approaches to manufacturing problems. One of these new approaches is called intelligent manufacturing. Intelligent manufacturing recognizes the increased capability for acquiring and processing information on the factory floor, and makes use of this capability to improve the manufacturing process. Central to this approach is the question of how to effectively use the feedback of information in making decisions influencing the operation of the manufacturing system.

The intent of this paper is to describe one simple manufacturing example, called the drilling problem, and to show how one new approach to this problem can be formulated. Drilling is chosen for illustrative purposes and is not a restriction of the approach. The drilling problem incorporates elements of traditional machining economics, but in the context of intelligent manufacturing. This new approach to the problem allows a control theoretic view of the problem to be taken. This view recognizes the potential for enhancing the performance of systems through information feedback and on-line control strategies. The control theoretic view is made possible by the proposal of a new stochastic model for tool wear. The new model for tool wear incorporates several features that make the control theoretic view a natural one. This new model for tool wear is applied to the drilling problem, resulting in the formulation of a machining economics problem as an optimal control problem.

The optimal control problem makes reasonable assumptions about the availability of information in the manufacturing environment. Solution to the problem yields optimal tool replacement and feed speed selection strategies. Sufficient conditions for existence of optimal solutions are given for one restricted case. These conditions are satisfied by observed tool wear phenomena.

The drilling problem shows how an appropriate balance between low and high level detail, information feedback, and manufacturing performance criteria can be obtained in a control theoretic approach to manufacturing.

2. THE DRILLING PROBLEM

2.1. Problem Description

The drilling problem represents a simple version of a problem found in metal-removal manufacturing operations, but with some new extensions. The heart of the problem is the question of how to operate a machining system in the best way subject to the limitations of uncertain physical phenomena. In this simplified version, only a single machine carrying out a single operation will be considered. It is important not to divorce the problem entirely from the surrounding manufacturing complex, however. That is, the problem formulation should represent at once aspects of manufacturing and aspects of drilling.

Assume there is a metal drilling manufacturing operation. The machine is a single tool drill, repetitively carrying out a single identical task on parts as they arrive. The task involves the drilling of a single hole in each part. An unlimited supply of parts for drilling is available. When a part is completed, another part is immediately available for drilling.

As holes are drilled, the tool wears and is susceptible to breakage. It is assumed that there is a limit to the wear beyond which the tool is unacceptable. A broken tool is considered unacceptable. Evolution of tool wear necessitates the occasional replacement of the tool. Replacement of the tool involves some costs for both time and material.

The spindle speed and the feed speed of the drill are variable. The machining rate, i.e. the rate of metal removal, is proportional to the feed speed. The time required to complete an operation is inversely proportional to the machining rate. Assuming that the tool does not break, the time to complete a part is inversely proportional to the feed speed. See the Appendix for a description of relevant drilling terminology.

The dynamics of tool wear are not completely understood. Experiments have been made from which empirical formulas for tool wear have been derived. These formulas are known to be only approximate, and considerable variation in tool life is evident. Furthermore, the material to be drilled has some uncertain characteristics that also affect tool wear. The empirical formulas for tool life suggest that under the assumption of a fixed feed (the ratio of feed speed to spindle speed), the tool wear rate is an increasing function of the cutting speed, and an increasing function of the feed speed. In this paper increasing means monotonically increasing (non-decreasing).

The replacement of tools prior to breakage is feasible. If the tool breaks, it must be replaced before continuing. Furthermore, tool breakage may cause damage to the current part, possibly resulting in scrappage or necessitating rework. Costs are therefore associated with tool breakage. Tool breakage is an unplanned event, so replacement of the tool due to breakage may take longer (and thus cost more) than planned replacement.

The drilling problem is to determine the best policies for operating the drill. The required decisions are when to replace the tool, and the selection of spindle and feed speeds used to drill the parts. In this paper it is assumed that the feed is constant so that the feed speed and spindle speed are necessarily proportional. Therefore only the feed speed needs to be specified. For simplicity in later analysis, assume that the feed speed can be changed at the start of each part, but once chosen remains constant until the part is completed, or the drill breaks. The criteria for ranking policies is a function of the various economic considerations. Any of several different criteria may be reasonable.

The above description of the drilling problem is typical of problems of machining economics, and as such does not appreciably extend previous considerations of these problems. The on-line variability of the spindle and feed speeds is not usual, however. Similar problems can be found in many references, including [7], [10], [11], [13], [18], [19], [21], [23], [26], and [27].

The problem will now be extended. Suppose that occasional measurements of the extent of tool wear are made. Assume that these measurements can be made with reasonable accuracy, and may be taken as exact. Continuous measurements cannot be made while the tool is engaged in the part however. Measurements can only be made between parts, when the tool is disengaged. The method of wear measurement is not specified. This new information is to be incorporated into the decision policies.

The essential features of the drilling problem are:

(1) Mechanical Aspects: The drill is used in a fixed operation carried out repetitively on parts. A fixed volume of material is removed from each part. As metal is removed from the parts, the tool wears. The tool wear mechanism is only known empirically, and variation in tool life is evident. The wear rate is observed to be an increasing function of the feed speed for a fixed feed. As the tool wears, it eventually breaks or becomes unacceptable and must be replaced. Failure of the drill necessitates replacment, but can also cause damage to the part being machined.

(2) Economic/Manufacturing Aspects: The production of parts results in profit, and the profit rate is related to the production rate. The profit rate is assumed to be an increasing function of the feed speed. Tool replacement is costly in terms of time and material. Tool breakage is in general more costly than simple replacement, due to possible part damage and the unplanned replacement that results.

(3) Control Aspects: The objective is to determine policies for tool replacement and for feed speed selection. Tool replacement and changes in feed speed may only occur between parts, unless there is a tool breakage.

(4) Information Feedback: Tool wear can be measured when the tool is disengaged from the part, but not during the machining.

The specific use of a drilling operation in this problem is only to facilitate the presentation of the concepts and is not meant to imply a restriction of the ideas to that type of machining operation. In fact, the authors believe that the ideas presented in this paper are applicable to many other problems in manufacturing, including ones outside the realm of machining.

2.2. The Tool Wear Subproblem

The tool wear problem has been the subject of much study in its own right. Since the choice of models for tool wear greatly influences the drilling problem, it warrants a careful examination. There are at least two views of the problem that can be taken. The first is to explain on a physical basis the mechanisms of tool wear. The attempts in this area often lead to very complicated formulations for tool wear as a function of several variables. See [4] and [14] as examples. Unfortunately, these models are not complete, describing at best the normal evolution of some aspects of tool wear under restricted conditions. Since the manufacturing environment is uncertain, and the information requirements of these models is considerable, their utility in the machining economics problem is questionable.

The second view is that usually taken in machining economics. Precise description of tool wear phenomena is foresaken for empirical formulas that are easier to use, and that capture the essential character of tool life as a function of machining conditions. The empirical formulation of tool wear can be considered either deterministically or stochastically. Deterministic formulations are prevelant in the earlier literature, but more recently the stochastic nature of tool life has been recognized. The stochastic formulations usually assume that the deterministic formulas properly represent mean values of tool life. References [5], [12], [16], and [29] give various viewpoints of the tool wear problem. Also, the references on machining economics usually give some tool wear model.

The stochastic formulation brings with it another level of complexity. If tool life is a random variable, its distribution must be specified. Actually, a family of distributions parameterized by machining conditions must be specified. Many distributions have been proposed for tool life including exponential, normal, and lognormal. In general these distributions are arrived at from empirical considerations, and not from any physical basis. This is not an entirely satisfying situation. It should be mentioned that some authors have avoided this problem by parameterizing only the moments of the tool life (usually the mean and variance) and not specifying a distribution.

It is asserted that a new model of tool wear is required in order to accomodate the intelligent manufacturing approach. This new model must incorporate certain features. The stochastic behavior of tool life must be recognized. The influence of machining conditions must enter into the model in a clear way. The distributions should arise from physical considerations of the process. The results of the model should agree with observed behavior in a statistical sense. This leads to the development of the diffusion-threshold model for tool wear as considered in Sec. 4.

3. SURVEY OF RELATED WORK

Aspects of the drilling problem have been considered by many authors. The work of Taylor [27] is generally considered the first extensive treatment of tool wear and machining economics. One result of his work is the Taylor tool life formula relating tool life to cutting speed.

Much of the work after Taylor presumed that the tool life is deterministically related to the cutting speed and other machining parameters, using variations of the Taylor formula. Under the assumptions of a deterministic relation, simple calculus can be used to arrive at optimal machining parameters for any of several criteria. See also [6] and [11] for examples. Many refinements and extensions are possible, including the consideration of multiple machines, constraints on machining parameters due to finish and power requirements, and tool geometry. Usually these treatments assume that the tool is changed at the end of its life, as given by the tool life formula, and that tool breakage and scrappage do not occur..

More recent work in the area has recognized the stochastic aspects of tool life and the impact that tool life uncertainty has on machining operation productivity. In these works, tool life is viewed as a random variable whose distribution is parameterized by cutting speed and other factors. Some researchers have assumed particular distributions, while a few have only specified moments of the tool life random variable. See [5], [7], [16], and [23].

The machining economics problem, whatever the assumption about tool life, usually considers as performance measures production rate, profit, or time to produce a part. Various types of constraints have been proposed for inclusion in the problem. Several things are not usually considered, however. The concept of information feedback and on-line control is almost never considered. As a result, most machining optimization problems have assumed constant machining parameters. On-line variability of machining parameters with changes in machining parameters based on measurements and other information is not usually considered. Policies for tool change invariably rely on such information as number of parts machined, or change upon failure. Most models do not consider costs due to scrap and damage, or the status of other machines in the manufacturing system in their tool replacement policies. Most importantly, tool wear information is not incorporated into the policies.

Other authors have considered control theoretic approaches to different problems in manufacturing. The reference [9] reviews many types of manufacturing problems from a control perspective, and includes a survey of work in several areas of research.

4. DIFFUSION-THRESHOLD PROCESSES

4.1. Process Description

A stochastic process, called a diffusion-threshold process, is now introduced and proposed as a model for tool wear in the drilling problem. The use of this process will allow formulation of the drilling problem as a stochastic optimal control problem.

Let $\{W(\omega,t), \omega \in \Omega, t \in IR^+\}$ be a standard Brownian motion (i.e. a Wiener Process) on some probability space (Ω,F,P), where $IR^+ \triangleq [0,\infty)$. A standard Brownian motion is a real valued scalar process such that:

$$P[W(\omega,0) = 0] = 1 \tag{4.1a}$$

$$E[W(\omega,t)] = 0 \text{ for } t \in IR^+ \tag{4.1b}$$

$$E[W(\omega,s)W(\omega,t)] = \min(s,t) \text{ for } s,t \in IR^+ \tag{4.1c}$$

Without loss of generality, assume $W(\omega,t)$ to have continuous sample paths for all $\omega \in \Omega$. The explicit dependence of the process on ω is generally omitted: $W_t \triangleq W(\omega,t)$

Define a real-valued scalar stochastic process $\{X_t, t \in IR^+\}$ by

$$X_t = X_0 + \int_0^t b(\tau, X_\tau, u_\tau)d\tau + \int_0^t \sigma(\tau, X_\tau, u_\tau)dW_\tau \tag{4.2}$$

where the last integral is interpreted as an Ito integral. Under appropriate technical conditions on the functions $b(\cdot,\cdot,\cdot)$, $\sigma(\cdot,\cdot,\cdot)$, and $u_t(\cdot)$, the process $\{X_t\}$ is called a (controlled) diffusion. See [8], [15], [17], and [30] for much more on diffusions. The functions $b(\cdot,\cdot,\cdot)$ and $\sigma(\cdot,\cdot,\cdot)$ will be called the infinitesimal coefficients of the diffusion, with b called the drift coefficient and σ called the diffusion coefficient. The functions $u_t(\cdot)$ will be called the control functions, with values u_t. The important properties of diffusions are that they have continuous sample paths, and that they are Markovian. X_t can be thought of as the solution to the stochastic differential equation

$$dX_t = b(t, X_t, u_t)dt + \sigma(t, X_t, u_t)dW_t \tag{4.3}$$

with the first term describing the deterministic evolution and the second term an additive noise. However, this is not precise except when defined in terms of the above integral equation.

Our attention is restricted to a simpler class of diffusion processes by examining a subset of the admissible infinitesimal coefficients. Consider those diffusions where b and σ are independent of X and t:

$$X_t = X_0 + \int_0^t b(u_\tau)d\tau + \int_0^t \sigma(u_\tau)dW_\tau \tag{4.4}$$

where u and thus b and σ are piecewise constant, i.e. there exists a set of times $0=t_0<t_1< \cdots$ such that

$$u_\tau = u_k,\ t_k \leq \tau < t_{k+1} \tag{4.5a}$$

$$b(u_\tau) = b_k,\ t_k \leq \tau < t_{k+1} \tag{4.5b}$$

$$\sigma(u_\tau) = \sigma_k,\ t_k \leq \tau < t_{k+1} \tag{4.5c}$$

for $k = 0,1, \cdots$. Under these assumptions, $\{X_t\}$ is a Brownian motion with piecewise constant drift and piecewise linear variance.

Let A denote a threshold value, and let T_A denote the first hitting time for the threshold A by the process X_t. That is

$$T_A = \inf\{t \geq 0: X_t = A\} \tag{4.6}$$

Note that T_A is a random variable, called a stopping or Markov time, and is well defined because of sample path continuity. If the event $\{X_t = A\}$ does not occur, let $T_A = \infty$. When the process achieves A for the first time, the process is stopped and re-initialized to zero. The process continues to evolve until the next time it achieves A, when it is again re-initialized. Thus the process generates a sequence of independent cycles, with one cycle being the time from last re-initialization to the next hitting of threshold A.

The times $t_0, t_1, \cdots$ are referred to as decision times. The motivation for this terminology is that certain control actions will be allowed at these times. At each decision time, the following actions may be permitted:

(1) A change in the control value u_t

(2) A decision to stop the current process and re-initialize it to zero.

The diffusion-threshold process can be placed in a more general setting, where arbitrary discontinuous state changes are permitted. This type of controlled process is considered in [1], where many variations and examples are given. In particular, that reference considers examples in maintenance and quality control which are related to the problems discussed in this paper.

4.2. Diffusion-Threshold Model of Tool Wear

The diffusion-threshold process is now proposed as a model for tool wear in the drilling problem. The following analogy is made. Let X_t represent an aggregated wear variable for the tool. It is understood that all of the categories of tool wear are somehow represented by a single variable, and X_t therefore represents this wear at time t. The feed speed will be regarded as the control input u_t, with the drift coefficient $b(\cdot)$ being a positive increasing function of the feed speed. The drift denotes a local mean rate of wear. The diffusion coefficient is the square root of the local rate of change of the variance. Let the maximum allowable wear correspond to the threshold A. When the wear reaches the threshold, the tool is considered unacceptable and must be replaced by a new tool. The new tool is assumed to have zero wear. This is the re-initialization action. Likewise, the decision to replace the tool results in a re-initialization prior to reaching the threshold. The decision times will take place between parts; i.e., just prior to the start of a new part. At these decision times, information concerning drill wear becomes available. Assume that exact measurements of the drill wear, X_{t_k}, become available. Based upon this information, a decision is to be made whether or not to replace the drill, and what feed speed should be selected for the next part.

4.3. Properties of Diffusion-Threshold Models

Given the mathematical formulation of the previous section, probabilities associated with the random variable T_A can be defined. For the case of a general diffusion process with arbitrary drift and diffusion coefficients, calculating the distribution of T_A may be a very hard problem. See [2], [15], [20], [22], and [25] for discussions of threshold crossing problems. Reference [2] is a survey. However, when the class of diffusions under consideration is restricted to those with piecewise constant coefficients, a solution can be obtained.

The restricted problem is as follows. Given that $X_{t_k} = x_k$, and constant u so that both $b = b(u)$ and $\sigma = \sigma(u)$ are constant, calculate the probability of first crossing of the threshold in a future time interval. Define the conditional distribution function for the random variable T_A by:

$$Q_A(t \mid x_k ;u) \triangleq P\{ T_A \leq t \mid T_A > t_k , X_{t_k} = x_k \} \tag{4.7}$$

for $t \geq t_k$. Under the Markov property, this is equivalent to

$$P\{ T_A \leq t \mid X_{t_k} = x_k < A\} \tag{4.8}$$

Let the density function be denoted $q_A(t \mid x_k ;u)$ where

$$q_A(t \mid x_k ;u) = \frac{dQ_A}{dt}(t \mid x_k ;u) \tag{4.9}$$

The special cases $Q_A(t ;u) \triangleq Q_A(t \mid X_{t_0}=0 ;u)$ and $q_A(t ;u) \triangleq q_A(t \mid X_{t_0}=0 ;u)$ follow. In addition, the moments associated with the random variable T_A are to be calculated.

The result for $x_k < A$, $b \geq 0$, $\sigma > 0$ can be derived as:

$$q_A(t \mid x_k ;u) = \frac{A - x_k}{\sqrt{2\pi}\sigma(t - t_k)^{\frac{3}{2}}} \exp\left\{ \frac{-(A - x_k - b(t - t_k))^2}{2\sigma^2(t - t_k)} \right\} \tag{4.10}$$

See [2] and [20]. This result can also be obtained by Laplace transform of the integral equation in [2].

Assume that X_t is measured at discrete times $t_0, t_1, t_2, \cdots$. Using the above density, the conditional probability of hitting the threshold A before t_{k+1} given X_{t_k} can be calculated. By (4.10), calculate:

$$P\{ t_k < T_A \leq t_{k+1} \mid X_{t_k} = x_k < A ;u_k\} = Q_A(t_{k+1} \mid x_k ;u_k) =$$

$$\int_{t_k}^{t_{k+1}} \frac{A - x_k}{\sqrt{2\pi}\sigma_k(\tau - t_k)^{\frac{3}{2}}} \exp\left\{ \frac{-(A - x_k - b_k(\tau - t_k))^2}{2\sigma_k^2(\tau - t_k)} \right\} d\tau \tag{4.11}$$

This is the one step ahead threshold crossing probability. Equation (4.11) is the key formula for deriving many of the results of interest. This formula can be somewhat simplified by the following change of variables. Let $\alpha = A - x_k$,

$\eta = \tau - t_k$,

and $\Delta t_{k+1} = t_{k+1} - t_k$. Then

$$Q_A(t_{k+1} \mid x_k; u_k) = Q_\alpha(\Delta t_{k+1}; u_k) = \int_0^{\Delta t_{k+1}} \frac{\alpha}{\sqrt{2\pi}\sigma_k \eta^{\frac{3}{2}}} \exp\left\{ \frac{-(\alpha - b_k \eta)^2}{2\sigma_k^2 \eta} \right\} d\eta \quad (4.12)$$

Several properties of the random variable T_A and its probability density function $q_A(\cdot)$ can be determined. Assume the case $A > x_0$, and $b \geq 0$, $\sigma > 0$ both constant. Then

$$P\{T_A < \infty\} = 1 \quad (4.13)$$

$$\lim_{\eta \to 0^+} q_A(\eta \mid x_0; b) = 0 \quad (4.14)$$

Note that the first of these asserts that the probability of T_A never occurring is zero, even when the drift is zero. However, when $b = 0$, $E[T_A] = \infty$. When $b > 0$, T_A has finite moments of all order. The first four are:

$$E[\, T_A \,] = \frac{A}{b} \quad (4.15a)$$

$$var[T_A] = \frac{A\sigma^2}{b^3} \quad (4.15b)$$

$$\mu_3 = \frac{3A\sigma^4}{b^5} \quad (4.15c)$$

$$\mu_4 = \frac{15A\sigma^6}{b^7} + \frac{3A^2\sigma^4}{b^6} \quad (4.15d)$$

where μ_3 and μ_4 are the third and fourth central moments.

In the special case that α, b, σ are all positive constants as above, the resulting distribution for T_α in (4.12) has received some attention in the statistical literature. It is referred to as either the Inverse Gaussian or a Wald Distribution. See [3], [24], and [28]. Note that [24] also considers the Inverse Gaussian

Distribution as a tool life model, though in a different spirit than this paper.

4.4. Rapproachment with Taylor Tool Life Formula

For the drilling problem and other tool wear problems, a comparison can be made between the diffusion-threshold model and other more usual models. We will consider a comparison to the (simple) Taylor tool life formula [6]. The Taylor formula, in use for many years, is a strictly empirical formula for calculating tool life. As is shown in the appendix, for fixed feed the Taylor formula may be written as

$$uT^n = C_1 \tag{4.16}$$

where:

u $\triangleq$ the feed speed

T $\triangleq$ life of tool

n $\triangleq$ an empirical constant dependent upon the part material, tool type, and machine type

$C_1 \triangleq$ an empirical constant dependent upon the part material, tool type, and machine type

Assume that replacement is indicated by having reached a certain level of wear that is fixed for a given tool type. Let this level of wear be denoted by W and let the wear of the tool at time t be denoted w(t). Also, define the instantaneous feed speed by u(t). If the tool life given by the Taylor equation is interpreted as a mean value, a diffusion threshold model with the same expectation can be constructed. Let the drift coefficient be given by

$$b(u(t)) = \frac{W}{C_1^{\frac{1}{n}}} u(t)^{\frac{1}{n}} \tag{4.17}$$

so that

$$E[w(t)] = \int_0^t \frac{W}{C_1^{\frac{1}{n}}} u(\tau)^{\frac{1}{n}} d\tau \tag{4.18}$$

When the feed speed is fixed at constant value u, the expected value for the the tool life is computed as

$$E[T] = \frac{W}{b(u)} = \left(\frac{C_1}{u}\right)^{\frac{1}{n}} \tag{4.19}$$

in agreement with the Taylor tool life formula. No restrictions are placed on the diffusion coefficient σ by the Taylor formula.

5. AN OPTIMAL CONTROL PROBLEM

5.1. General Formulation

Consider the drilling problem previously described. Assume that pieces are being machined, and that the time to complete each piece depends on the selected feed speed. Furthermore, assume that measurements of tool wear can be made at the completion of each piece. At each measurement time, a decision can be made to adjust the feed speed, and/or change the drill. Each of these decisions has some cost associated with it. In addition, there is a penalty cost associated with breaking the drill. The optimization problem is to make these decisions in such a way as to minimize some cost functional.

The cost functional used must capture the important manufacturing concerns and allow for the additional information and control capability indicated.

In traditional formulations of machining economics problems there is no provision for feedback of tool wear information, nor are on-line changes in feed speed allowed. Furthermore, tool life is considered a deterministic function of feed speed. These limitations suggest that a cost functional different from those traditionally used should be introduced.

The class of cost functionals considered in this analysis are called one step. One step in this context means one part ahead. One step costs incorporate the profits earned and the costs incurred in producing the next part. The formulation of one step costs must be carefully considered. In particular they must be formulated so that behavior extremely detrimental to successive parts is not encouraged.

For this problem, define a control vector $\underline{u} \triangleq (u, v)$ where $u \in (0,\infty)$ is the feed speed and $v \in \{0,1\}$ is the replacement decision, where 0 corresponds to not replacing the tool and 1 corresponds to replacing the tool.

Define the following:

g(u) $\triangleq$ profit rate for selected feed speed u

$T_f(u) \triangleq$ processing time for a part corresponding to a constant feed speed u

R $\triangleq$ cost of a tool replacement, including tool and time costs

B $\triangleq$ cost of a tool breakage, not including replacement cost

$$M_{[t_0, t_1]} \triangleq \sup_{t_0 \le t \le t_1} \{X_t\}$$

$h(X_{t_k+T_f}, M_{[t_k,t_k+T_f]}, z_k) \triangleq$ tool utilization cost for processing a part

K $\triangleq$ overhead cost rate for machine in operation

Overhead costs incurred during tool replacement are assumed to be included in R. Note that a total cost of B+R is associated with a threshold crossing (breakage or unacceptable wear). A candidate function for h will be presented later. In the following formulation it is assumed that all profit for the current part is lost when a breakage occurs.

A cost functional can now be formulated:

$$Minimize\ J(\underline{u}, z_k) = (1-v)\cdot J_1(u, z_k) + v\cdot J_2(u, 0) \tag{5.1}$$

where

$$J_1(u, z_k) = E[\,-g(u)\cdot T_f\cdot 1_{[t_k+T_f < T_A]} + (B+R)\cdot 1_{[T_A \le t_k+T_f]} + h(X_{t_k+T_f}, M_{[t_k,t_k+T_f]}, z_k) + K\cdot(T_f \wedge (T_A - t_k)) \mid X(t_k)=z_k\,] \tag{5.2a}$$

is the expected one step cost if the tool is not replaced, and

$$J_2(u, 0) = E[\,-g(u)\cdot T_f\cdot 1_{[T_f < T_A]} + R + (B+R)\cdot 1_{[T_A \le T_f]} + h(X_{T_f}, M_{[0,T_f]}, 0) + K\cdot(T_f \wedge T_A) \mid X(0)=0\,] \tag{5.2b}$$

is the expected one step cost if the tool is replaced. Note that $(T_1 \wedge T_2) \triangleq min(T_1, T_2)$ and that z_k is the measured wear. The explicit use of 0 in J_2 is only a reminder of its conditioning on zero wear.

Before examining a special case of this problem, there remains the question of the reasonability of this cost functional. In response to the earlier comment, this cost functional contains terms which discourage initial detrimental behavior (high feed speeds). In particular, high feed speeds may be discouraged through the tool utilization cost term, and by the increased probability of a threshold crossing. A threshold crossing causes a complete loss of profit for the current part, and incurs the additional costs of replacement due to breakage (R+B).

5.2. Restricted Formulation

A restricted formulation of interest can be obtained under the following assumptions. Without loss of generality, assume that $t_k=0$. Let the threshold value A = W, the maximum tool wear. Let $b(u) = \beta u^m$, where $\beta>0$ and $m>1$ are constants. This form of the drift function is motivated by the Taylor tool life formula with $m = \frac{1}{n}$ and $\beta = \frac{W}{C_1^m}$. Assume that σ is constant. Let $g(u) = g \cdot u$, where $g>0$ is a constant, and $T_f(u)=\frac{V}{u}$ where $V>0$ is the depth of the hole to be drilled in each part. In this case, $g(u) \cdot T_f(u) = gV = G$, the profit earned per completed part. Let

$$h = D \cdot (X_{T_f} - x_0) \cdot 1_{[M_{T_f} < A]} + D \cdot (A - x_0) \cdot 1_{[M_{T_f} \geq A]} \tag{5.3}$$

That is, the tool utilization is proportional to the amount of tool consumed, with a maximum tool utilization given by $A-x_0$, the remaining tool utility. The constant D is the tool utilization cost rate. Further note the equivalence of the random variables:

$$1_{[M_{T_f} < A]} = 1_{[T_f < T_A]}$$

The restricted formulation becomes

$$\mathit{Minimize}\ J(\underline{u}, x_0) = (1-v) \cdot J_1(u, x_0) + v \cdot J_2(u, 0) \tag{5.4}$$

where

$$J_1(u, z_0) = E[\, -G \cdot 1_{[T_f < T_A]} + (B + R) \cdot 1_{[T_A \le T_f]} + D \cdot (X_{T_f} - z_0) \cdot 1_{[T_f < T_A]} + \tag{5.5a}$$

$$D \cdot (A - z_0) \cdot 1_{[T_A \le T_f]} + K \cdot (T_f \wedge T_A) \mid X(0) = z_0 \,]$$

and

$$J_2(u, 0) = E[\, -G \cdot 1_{[T_f < T_A]} + R + (B + R) \cdot 1_{[T_A \le T_f]} + D \cdot X_{T_f} \cdot 1_{[T_f < T_A]} + \tag{5.5b}$$

$$D \cdot A \cdot 1_{[T_A \le T_f]} + K \cdot (T_f \wedge T_A) \mid X(0) = 0 \,]$$

and

$$T_f(u) = \frac{V}{u}$$

The term $(T_f \wedge T_A)$ can be alternatively expressed as

$$(T_f \wedge T_A) = T_f \cdot 1_{[T_f < T_A]} + T_A \cdot 1_{[T_A \le T_f]} \tag{5.6}$$

As a consequence of this restricted formulation, the following functions become relevant.

$$s_A(u \mid z_0) = E[\, 1_{[T_A \le \frac{V}{u}]} \mid z_0 \,] = P[\, T_A \le \frac{V}{u} \mid z_0 \,] \tag{5.7}$$

$$r_A(u \mid z_0) = E[\, (X_{\frac{V}{u}} - z_0) \cdot 1_{[\frac{V}{u} < T_A]} \mid z_0 \,] \tag{5.8}$$

$$z_A(u \mid z_0) = E[\, T_A \cdot 1_{[T_A \le \frac{V}{u}]} \mid z_0 \,] \tag{5.9}$$

The function $s_A(u \mid z)$ is the conditional probability of hitting the threshold before part completion. The function $r_A(u \mid z)$ is the conditional expected change in wear for the tool for those sample paths that do not have a threshold crossing prior to finishing the part, weighted by the probability of a threshold crossing not occurring. The function $z_A(u \mid z)$ is the conditional expected time of the threshold crossing for those sample paths that do have a threshold crossing prior to part completion, weighted by the probability of a threshold crossing occurring. It can be shown that the functions s_A, r_A, and z_A can each be expressed in the following integral forms.

$$s_A(u \mid x) = \int_0^{\frac{V}{u}} f(A-x, u, \eta)d\eta \tag{5.10}$$

$$z_A(u \mid x) = \int_0^{\frac{V}{u}} \eta f(A-x, u, \eta)d\eta \tag{5.11}$$

where

$$f(\alpha, u, \eta) = \frac{\alpha}{\sqrt{2\pi}\sigma\eta^{\frac{3}{2}}} \exp\left\{\frac{-(\alpha - \beta u^m \eta)^2}{2\sigma^2\eta}\right\} \tag{5.12}$$

and

$$r_A(u \mid x) = \int_{-\infty}^{A} \eta p(A-x, u, \eta)d\eta \tag{5.13}$$

where

$$p(\alpha, u, \eta) = \frac{\sqrt{u}}{\sqrt{2\pi V}\sigma} \exp\left\{\frac{-u(\eta - \beta V u^{m-1})^2}{2\sigma^2 V}\right\}\cdot\left[1 - \exp\left\{\frac{-u\alpha(2\alpha - \eta)}{\sigma^2 V}\right\}\right] \tag{5.14}$$

Since A and x only appear as a difference, it is convenient to introduce $\alpha = A - x$, $x < A$. Note that f is the threshold crossing density and that p is a Gaussian density times a weighting function.

The functions J_1 and J_2 can be expressed in terms of these functions as follows.

$$J_1(u, x_0) = (B + R + D\cdot(A - x_0))\cdot s_A(u \mid x_0) + (K\cdot\frac{V}{u} - G)\cdot(1 - s_A(u \mid x_0)) + D\cdot r_A(u \mid x_0) + K\cdot z_A(u \mid x_0) \tag{5.15a}$$

$$J_2(u, 0) = (B + R + D\cdot A)\cdot s_A(u \mid 0) + R + (K\cdot\frac{V}{u} - G)\cdot(1 - s_A(u \mid 0)) + D\cdot r_A(u \mid 0) + K\cdot z_A(u \mid 0) \tag{5.15b}$$

Clearly, the existence of minima is tied to the properties of the functions s_A, r_A, z_A and to the choice of parameters in the cost functional. Under the assumption of $m>1$ and strictly positive cost parameters either of the following two conditions is sufficient for the existence of a minimum.

(1) Suppose there exists u' such that $J_1(u', x_0)<0$ or $J_2(u', 0)<0$. Then $J(\underline{u}, x_0)$ has a minimum on $(0,\infty)\times\{0,1\}$.

(2) Suppose there exists u_m such that $u\leq u_m$ necessarily. Then $J(\underline{u}, x_0)$ has a minimum on $(0,u_m]\times\{0,1\}$ for $0\leq x_0<A$.

Note that condition 1 says that if the operation can be performed profitably for a given wear value a minimum exists. Condition 2 says that under practical constraints on the feed speed a minimum exists for all wear values in $[0,A)$. Other more complicated sufficient conditions are also available. In general, if the piece profit is sufficiently high or the overhead rate is sufficiently low, a minimum exists.

The assumption $m>1$ is satisfied according to Taylor tool life data for metal machining with all commonly used tools. Reference [6] gives typical values of n in the range 0.1 to 0.4, resulting in 2.5 to 10.0 as a range of values for m.

5.3. Optimal Drilling Policies

The simple way in which the control variable v enters the problem allows the optimal policy to be given in terms of the functions J_1 and J_2. The optimal one step policy is of the following form (assuming existence of all minima). Let x be the measured tool wear. Denote

$$J_1^*(x) \triangleq \min_u J_1(u,x) \quad u_1^*(x) \triangleq arg \min_u J_1(u,x)$$

$$J_2^* \triangleq \min_u J_2(u,0) \quad u_2^* \triangleq arg \min_u J_2(u,0)$$

The minimizing values of $J_1(\cdot,x)$ and $J_2(\cdot,0)$ are $u_1^*(x)$ and u_2^* respectively. Let $v^*(x)$ be the optimal replacement decision. The optimal one step policy is given by:

(i) if $J_1^*(x) < J_2^*$, continue with the current tool at feed speed $u_1^*(x)$. $v^*(x)$=0.

(ii) if $J_1^*(x) > J_2^*$, replace the tool and continue at feed speed u_2^*. $v^*(x)$=1.

(iii) if the tool breaks, replace the tool and continue at feed speed u_2^*.

The values $u_1^*(x)$ and u_2^* must be computed numerically even in the restricted formulation. There is no known closed form solution. Depending upon the parameters, J_1 and J_2 may exhibit large flat valleys, and so gradient methods may not always be appropriate. As will be seen in an example, reasonable suboptimal policies may exist for an interval of feed speeds. Therefore, it may be possible to maintain constant feed speed for a range of wear measurement values with negligible degradation in performance. This will become evident later.

Implementation of the policy may be carried out in the following way. J_2^* and u_2^* are constants and may be computed in advance. Initially, assume that a new tool is in place. The feed speed for the first part is u_2^*. For each part thereafter, a wear measurement x is made, and $J_1^*(x)$ and $u_1^*(x)$ must be computed numerically based on the wear measurement. A table lookup would be a reasonable alternative. Recall that under the assumption of fixed feed, each selection of the feed speed necessitates a corresponding selection of the spindle speed as well.

The optimal policy given previously is now used to determine the proper action for the next part.

The above implementation can be extended to the realistic situation of (stochastically) time-varying costs and profit rates. In this case, $J_1^*(x)$, J_2^*, $u_1^*(x)$, and u_2^* must all be computed for each part on the basis of the current cost functional parameters and wear measurement. It is assumed that the cost functional parameters are supplied by some higher level control and/or authority. In this situation, the higher level authority could encourage or discourage tool replacement and higher feed speeds through manipulation of the costs and profit rate. In particular, tool replacement could be vetoed, in which case the next part would be machined at feed speed $u_1^*(x)$. Also, tool replacement could be strongly encouraged by a sufficient reduction in R. Rationale for manipulating tool replacement policies and feed speed selection could include tool availability, service availability, the status of other machines and components in the manufacturing system, part and supplies inventory levels and part demand. This manipulation represents an indirect feedback of other information in the manufacturing system to the drill controller.

In actual practice, the selected feed speed will necessarily be constrained by other considerations: surface finish, spindle and drive power, and machine, tool, and part characteristics.

In summary, the optimal feedback policy can be implemented in a manufacturing setting, provided sufficient computational capability is present, and tool wear measurement feedback is available. At the least, the policy incorporates tool wear measurement in the decision process. By allowing time variable cost and profit parameters, indirect feedback of other information about the manufacturing system can be introduced into the policy. The controller might be implemented in a local processor at the machine site.

5.4. Numerical Example

An example optimal control problem has been worked out based upon actual drilling and manufacturing data. The restricted formulation is used for the cost functional. All results have been computed numerically. The results have been summarized in graphical form. Operating regions of feed speed for different wear values are presented. As will be seen, this provides more insight than simple plots of u* vs. wear.

The drilling operation parameters and data have been taken from [31]. They are as follows:

Part Material:	4340 Alloy Steel 341 BHN
Drill:	M2 HSS Twist Standard Point 0.25 in x 4.00 in 29 ° Helix 118 ° Point Angle 7 ° Lip Relief
Operation:	0.5 in through hole heavy oil lubrication
Feed:	0.002 in/rev
Tool life:	0.015 in end point wear

The tool life is assumed to agree with the Taylor tool life formula in the mean. This formula is given by (4.16). The coefficients n and C_1 were determined from tool life data provided in [31] for $u \in [2.1, 3.0]$ (in/min). A least square fit of the data resulted in

$$C_1 = 3.966$$

$$n = 0.194$$

Recall that these coefficients are derived under the assumption of constant feed. The spindle speed is presumed to vary in proportion to u so as to maintain constant feed. The drift function in the diffusion threshold model is given by $b(u) = \beta u^m$ with

$$m = \frac{1}{n} \approx 5.2$$

$$\beta = \frac{W}{C_1^m} \approx 1.2 \times 10^{-5}$$

These values were used in the drift function.

No information concerning the variance of the tool life was provided. This necessitated estimation of σ based upon reasonable assumptions. Assume first a nominal feed speed of 2.6 in/min. This is approximately mid-range for the data available. For this feed speed, the mean tool life is about 8.7 min. Assume that the standard deviation for tool life at this nominal speed is about 20% of the mean; this corresponds to a value of about 1.7 min. From (4.15b), σ can be computed:

$$\sigma = \left[\frac{b^3}{A} \, var(T_A) \right]^{\frac{1}{2}} \approx 0.001$$

Clearly, it would be desirable to have information concerning tool life variance.

Such information could be used to compute σ and also to test the assumption of constant σ.

Summarizing, the diffusion threshold parameters used for this problem are given by:

$$\begin{aligned} A &= 0.015 \\ \beta &= 1.2 \times 10^{-5} \\ m &= 5.2 \\ \sigma &= 0.001 \end{aligned}$$

The functions s_A, r_A, and z_A in the cost functional can be computed using the above parameters independently of the economic considerations. Wear value z_0 was taken from the set { 0.0, 0.003, 0.006, 0.009, 0.012, 0.014, 0.0145 } and u was taken from [0.5, 7.5] at intervals of 0.2. A Romberg integration method was used.

The economic parameters in the cost functional were based on the following assumptions. Assume an overhead cost rate of 30.00 (dollars/hour) = 0.50 (dollars/min). Let the cost of a drill be 1.00 dollar. Assume that the time to change a drill under normal conditions is 0.5 min with an additional time of 0.5 min under the assumption of tool breakage. Assume a tool utilization cost rate based on the normal replacement cost. The parameters are thus given by:

$$\begin{aligned} K &= 0.50 \text{ (dollars/min)} \\ R &= 1.00 + 0.5 \times 0.5 = 1.25 \text{ (dollars)} \\ B &= 0.5 \times 0.5 = 0.25 \text{ (dollars)} \\ D &= 1.25/0.015 = 83.33 \text{ (dollars/in)} \end{aligned}$$

In order to determine the effect of part worth on the policy, G was allowed to take on the values 1.00, 10.00, and 100.00 dollars. For each fixed G, the cost functionals were computed and the operating policy was determined. Note that tabulation of the functions s_A, r_A, and z_A allows easy investigation of the effects of varying the economic parameters.

The functions $J_1(u, x_0)$ were computed for each of the wear values, for each fixed G. The function $J_2(u, 0)$ is easily computed from the zero wear $J_1(u, 0)$ function. For each G, the family of associated curves was thus determined. For each wear value, near optimal and suboptimal feed speed operating regions were determined. The wear value threshold for tool replacement and a critical upper boundary indicating the economic break even point were also determined. These results are presented in Figs. 5.1-5.3. The near optimal region indicates a cost within 1% of optimal, and the suboptimal region indicates a cost within 10% of optimal.

Examination of the graphs reveals several interesting properties. When a tool has little or no wear, the near optimal operating regions are relatively wide, indicating a relatively large interval from which feed speed selection can be made with near optimal performance. As the tool wear increases to the high wear region, however, the near optimal region narrows appreciably. In general, it would seem that the choice of feed speed is more critical towards the end of the tool's life than it is for new and little worn tools. What is perhaps most surprising is the observation that in some cases constant feed speed can be used throughout the tool's life while maintaining near optimal performance.

Two effects of piece profit (G) on the policy are readily apparent. First, as G increases the width of the near optimal region increases for most wear values.

Furthermore, as G increases tool replacement becomes more conservative; i.e., replacement of the tool occurs at lower wear levels. In particular, for sufficiently profitable parts and cheap tools, one could argue for tool replacement after very little wear (i.e., every part). Conversely, for cheap tools and cheap parts, replacement is close to the wear threshold. This is reminiscent of a "run until it breaks" policy. However, the region of operation for small G is generally narrower, so even a run until breakage replacement policy must be coupled with carefully chosen feed speeds. This seems to agree with the observation that manufacturing products with low profit margins requires more careful control of the operation in order to maintain (reasonable) profitability.

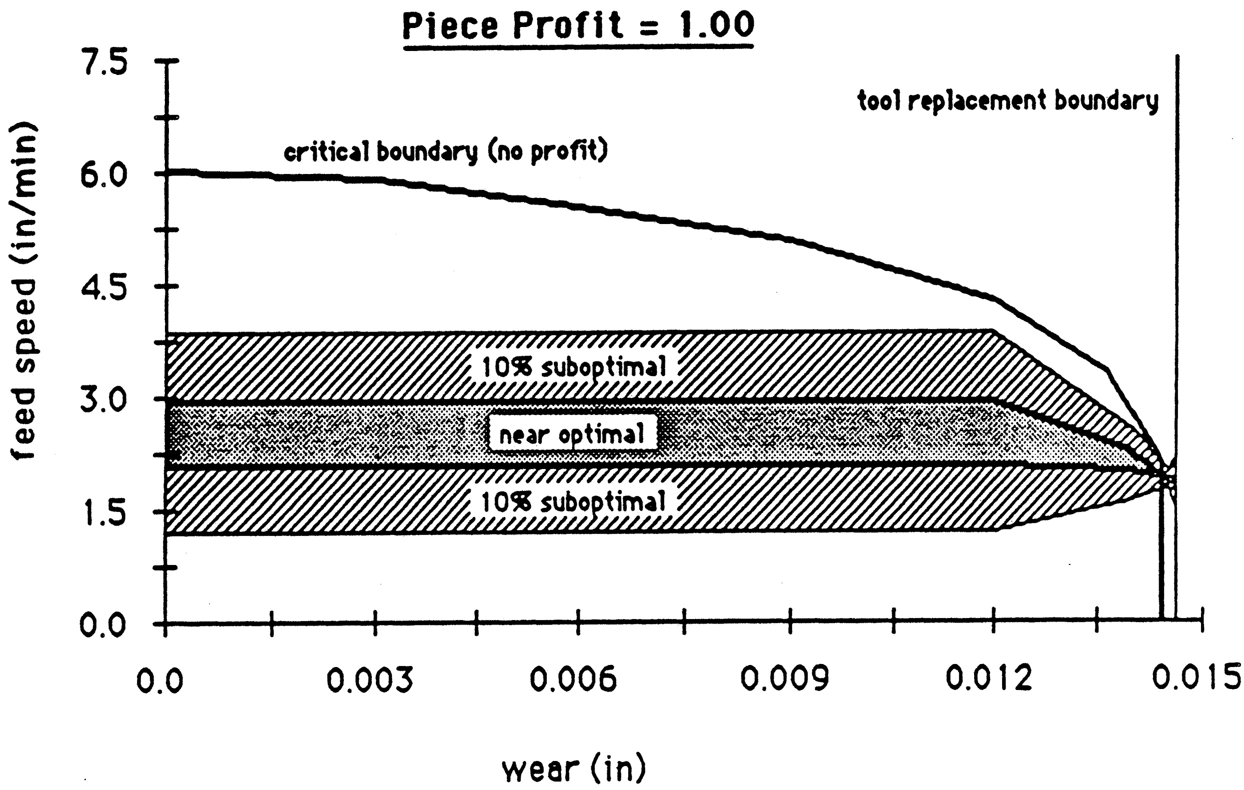
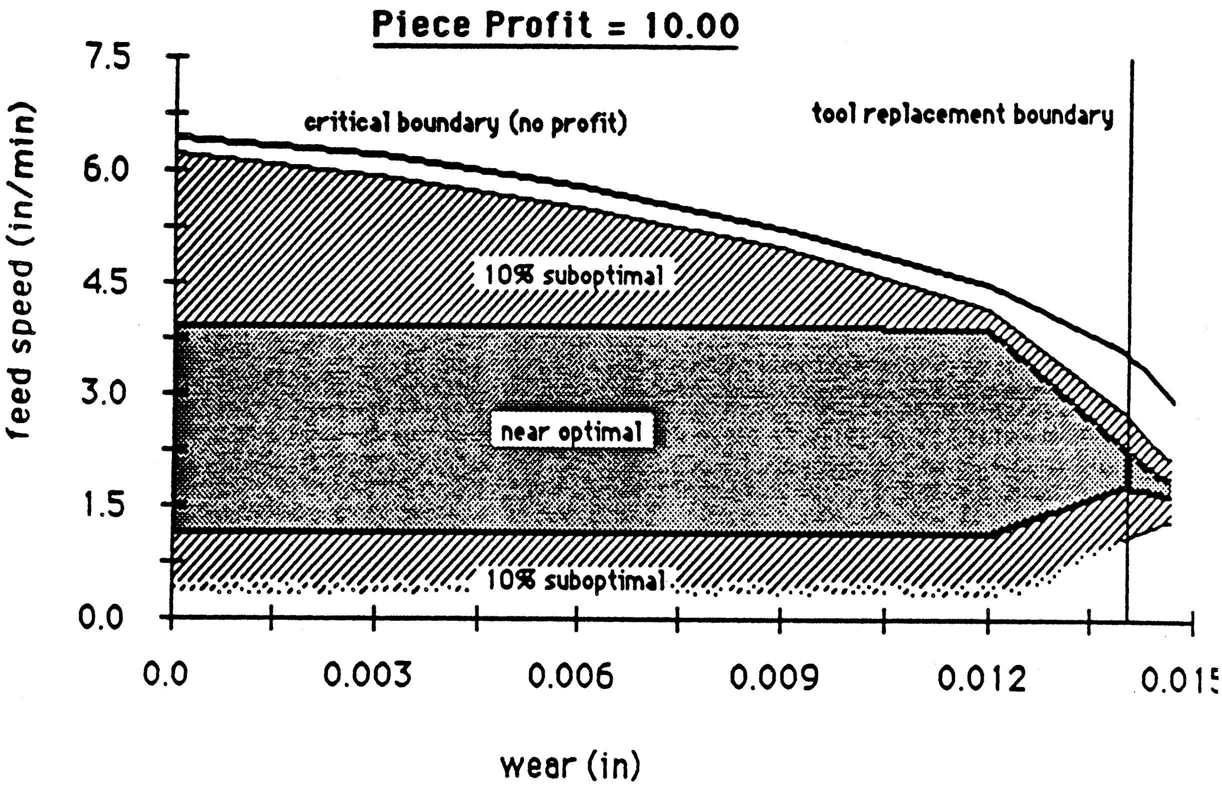

Piece Profit = 1.00
7.5
6.0
4.5
3.0
1.5
0.0
feed speed (in/min)
tool replacement boundary
critical boundary (no profit)
10% suboptimal
near optimal
10% suboptimal
0.0
0.003
0.006
0.009
0.012
0.015
wear (in)

Fig. 5.1 G = 1



Piece Profit = 10.00
critical boundary (no profit)
tool replacement boundary
10% suboptimal
near optimal
10% suboptimal
feed speed (in/min)
7.5
6.0
4.5
3.0
1.5
0.0
0.0
0.003
0.006
0.009
0.012
0.015
wear (in)

Fig. 5.2 G = 10

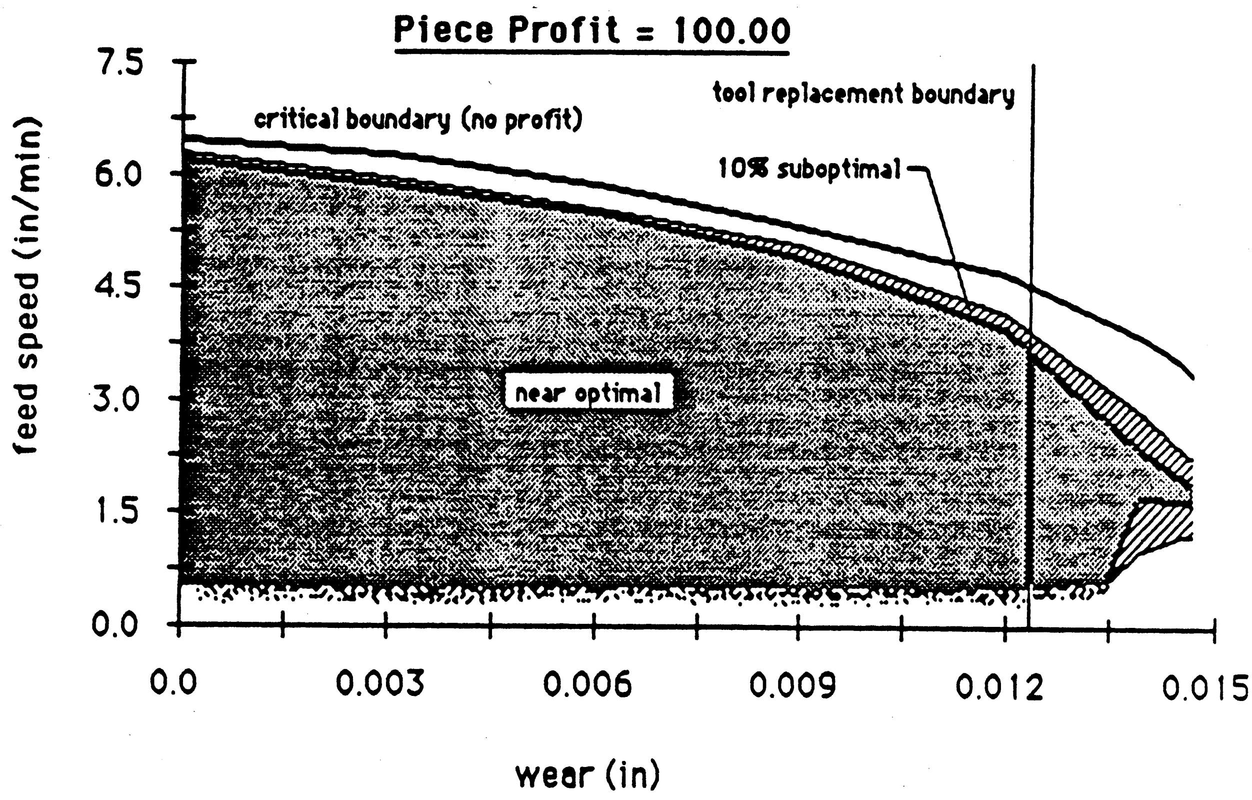

Piece Profit = 100.00
7.5
6.0
4.5
3.0
1.5
0.0
feed speed (in/min)
critical boundary (no profit)
tool replacement boundary
10% suboptimal
near optimal
0.0
0.003
0.006
0.009
0.012
0.015
wear (in)

Fig. 5.3 G = 100

6. MODEL EXTENSIONS

The drilling problem as proposed here can be extended in many ways. Some possibilities include:

(1) The information used in formulating the optimal control problem could be changed. The measurements of X_t may be available only every n decision times, or delayed, or the measurements might be noisy. The measurement of X_t might be costly.

(2) The indicated optimization problem was one-step ahead. This could be extended to multistep decision processes or optimal open loop feed speed selection and replacement policies. These may require the use of dynamic programming approaches, in general, for solution.

(3) Other types of machining operations could be considered, in some cases without modification of the model or optimization formulation. Other manufacturing problems might be approached by alternate application and interpretation of the diffusion-threshold model.

(4) The model could be extended to multi-tooled machines and multiple machine operations. The former is not usually considered in machining economics problems, but the later has been considered in the traditional ways by many researchers. Multiple manufacturing operations might be modeled as a network of diffusion-threshold processes or as a single aggregate diffusion-threshold process.

(5) The model could be extended to include independent control of the spindle and feed speeds subject to constraints. The tool wear would be a function of both speeds, but the processing time for an operation would be a function of the feed speed only. Comparison to more general tool life formulas (e.g., extended Taylor formulas) could be made.

7. SUMMARY

A problem in manufacturing called the drilling problem has been formulated. The drilling problem combines elements of traditional machining economics problems and elements of intelligent manufacturing. A drilling operation is carried out on arriving parts. The drill wears and is susceptible to breakage. Drill wear measurements are available at discrete times. The problem is to determine tool replacement policies and to select feed speeds.

A new stochastic model for tool wear, called a diffusion-threshold model, is introduced. This model is compared to a Taylor tool life formula. Conditions are given so that the two models agree in the mean.

The diffusion-threshold model for tool wear allows a stochastic optimal control formulation of the drilling problem. One step costs, which look one part ahead, are introduced in the optimal control problem as one type of optimization criteria.

A restricted formulation of the drilling problem is considered in detail. Sufficient conditions for the existence of optimal one step policies are given. These conditions are met in actual machining problems. The associated optimal policies are developed in terms of the given problem data.

A numerical example using actual tool and manufacturing data has been worked out. Feed speed operating regions and tool replacement boundaries are presented graphically. The results indicate that feed speed selection is more critical when the tool wear is high.

The control theoretic approach used in the drilling problem shows how stochastic modeling can be applied to a manufacturing problem and how information feedback can be accomodated in on-line control for intelligent manufacturing systems.

APPENDIX

This appendix summarizes relevant terms used in drilling and in the paper. These terms are typical of what is found in machining data handbooks. U.S. customary units and typical metric units are indicated.

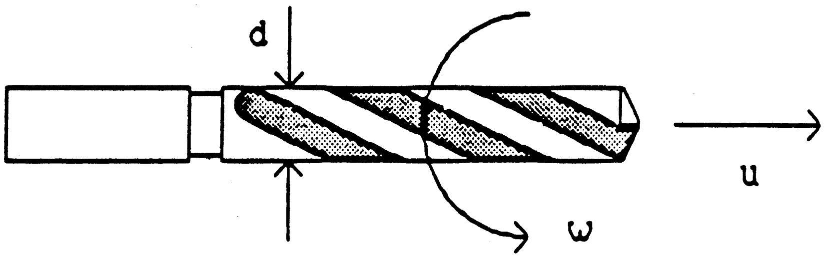


Fig. A.1

d $\triangleq$ drill diameter (in or mm)

$\omega \triangleq$ spindle speed (rpm)

u $\triangleq$ feed speed (in/min or mm/min)

V $\triangleq$ cutting speed - the tool/material relative velocity at the outer tool edge (surface feet per minute (sfm) or m/min)

f $\triangleq$ feed - the amount of linear travel of the tool per revolution (in/rev or mm/rev)

In U.S. customary units the cutting speed is given by

$$V = \frac{\pi d \omega}{12} \quad (sfm) \tag{A.1}$$

The feed is given by

$$f = \frac{u}{\omega} \tag{A.2}$$

Note that

$$V = \frac{\pi d u}{12 f} \tag{A.3}$$

so that only two of the variables u, f, and V are independent. Machining data guides often give recommended values for f and V.

The simple Taylor tool life formula for fixed feed is given by [6]:

$$VT^n = C \tag{A.4}$$

which can be expressed as (4.16):

$$uT^n = \frac{12 f C}{\pi d} = C_1 \tag{A.5}$$

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