ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

STRESS-STRAIN RELATIONS IN PLASTICITY AND RELATED TOPICS

TECHNICAL REPORT NO. 4

AN EXPERIMENT ON CIRCULAR PLATES IN THE PLASTIC RANGE

By

R. M. COOPER

G. A. SHIFRIN

Project 2027

OFFICE OF ORDNANCE RESEARCH, U.S. ARMY
CONTRACT DA-20-018-ORD-12099
PROJECT NO. TB 20001(234), DA PROJECT 599-01-004

March, 1954

ACKNOWLEDGMENTS

The authors wish to acknowledge the suggestions and helpful criticism of Professor P. M. Naghdi, under whose supervision the work was performed. Special thanks are due to Mr. J. C. Rowley who contributed many ideas and designed the major part of the instrumentation.

ABSTRACT

Nine simply-supported mild-steel circular plates were loaded well into the plastic range under concentric uniformly- distributed load. Deflection, strain, and slope data were obtained as functions of load. Comparison is made with the predicted limit loads of Hopkins and Prager.

AN EXPERIMENT ON CIRCULAR PLATES IN THE PLASTIC RANGE

INTRODUCTION

The recent work of Hopkins and Prager[1] suggests several experiments on the load-carrying capacities of symmetrically loaded and supported circular plates. The comparative ease with which the "simply-supported" edge condition may be achieved affords an excellent opportunity for verification of the theory of limit design as applied to circular plates, at least in a special case. Aside from this verification, the experimental results in themselves may prove to be of interest as an extension of the test data for plates with large deflection.[2,3,4]

In this paper, experimental results are reported for nine mild-steel circular-plate specimens of various radii and thicknesses. The plates were "simply supported" and taken well into the plastic range under the action of a concentric and uniformly-distributed load over a small area of the plate. Load-deflection data, the strain behavior near the supported edge, as well as slope measurements by optical means (on two plates) were obtained in the course of the tests. Several stress-strain curves in tension were also obtained for the plate materials. The term "simply supported" as used above is meant to describe the condition of zero transverse displacement and zero radial bending moment around the edge of the plate.

In determining the load-carrying capacities for axisymmetric plates in reference 1, the following were assumed: rigid-plastic material, small deflections, negligible membrane action and transverse shear deformation, the Tresca yield condition, and the associated flow rule. It was further assumed that the state of stress along a normal to the middle surface is given by two fixed diametrically opposite points on the Tresca yield hexagon, depending on which side of the middle surface the general point is located. Then, it was shown that the load at the flow limit for a simply-supported circular plate under a concentric distributed load over a small area of the plate is given by

$$P = \frac{6\pi M_o}{3 - 2(a/R)} , \qquad (1)$$

where R is the plate radius, a is the radius of the distributed load, and $M_o = \sigma_o h^2$, σ_o being the yield limit in simple tension and 2h the plate thickness.

In these experiments some tests were terminated when the von Mises (J_2) condition was satisfied at the edge of the plate. Other tests were terminated to prevent possible damage to test equipment, while three plates which were not instrumented were loaded to failure.

SPECIMENS AND TEST EQUIPMENT

Three types of circular-plate specimens made of mild steel were tested. These types were of various radii and thicknesses as given in Table I. Tensile specimens were also cut from the same stock as each type of plate.

TABLE I

Type	No.	2R Diameter, inches	2h Average Thickness, inches	2h/R
I	1	10 5/16	.048	.010
	2	10 5/16	.060	.012
	7	10 5/16	.060	.012
II	3	10 5/16	.105	.021
	4	10 5/16	.104	.021
	8	10 5/16	.105	.021
III	5	17 5/16	.104	.012
	6	17 5/16	.104	.012
	9	17 5/16	.104	.012

All plates were supported on one of two rings made from pipe stock. These rings had edges machined square with the center line of the pipe and in addition for each of the two rings, one edge was beveled to give support diameters of 10 and 17 inches, respectively.

The load was applied to the plate specimens through a load rod and a 0.6-inch diameter x 1/8-inch thick rubber pad, which was centered under the loading rod by a cylindrical sleeve that permitted the pad to project about 1/32 inch beyond the sleeve edge. The loading rod was gripped in the compression head of a 10,000 lb Riehle Universal Testing Machine for plate types I and II, and in a 120,000 lb Riehle Universal Testing Machine for type III. As load was applied, the rubber was forced out around the edge of the sleeve so that the 0.3-inch radius was maintained within reasonable limits.

Central deflection of each plate was measured by means of a 0.001-inch Federal dial gage mounted under the plate inside the supporting ring. For the type-I and type-II plates additional Federal dial gages were also positioned at three different radii over the top surface of the plate and a 0.0001-inch Ames dial gage was located at a point on the 4-inch radius to aid in obtaining accurate deflection data at the outer annulus. For type-III plates six other 0.001-inch Federal dial gages were positioned at different radii under the plate and the 0.0001-inch Ames dial and a depth micrometer were located above the plate. The Ames gage was located at a point on the 7.5-inch radius and the depth micrometer at the 0.875-inch radius. Also used in the testing of type-III plates was an optical system of small mirrors arranged over the annulus of one inch width nearest to the support, together with the necessary telescopic equipment.

Two 0.0005-inch Federal dial gages were mounted at the edge of the plate in such a manner as to measure the "pull-in" along a diameter for one of the type-III plates.

In Fig. 1 the loading bar over the plate, the two "pull-in" measuring dial gages, the Ames dial gage 1 inch away from the support, and the arrangement of the mirrors for the optical system can be seen.

Type A-7 SR4 electric strain gages were attached to the plates with orientations along radial and circumferential directions. Because of the approximately two-dimensional character of the strain field, corrections to the strain data were subsequently made using the formula and factors given in reference 5. Single strain-gage "sets" composed of a radially and a circumferentially oriented strain gage were located at two diametrically opposite positions over the support. Two "double sets" with similarly oriented gages on both top and bottom surfaces of the plates were located 1/2 inch away from the support and ninety degrees away from the single sets, as can be seen in Fig. 2a. Individual strain-gage readings were made on a Baldwin-Southwark Type L Strain Indicator.

The tensile specimens were tested in the 120,000 lb Riehle Universal Testing Machine. Strains were measured with the aid of two Huggenberger mechanical strain gages having nominal gage lengths of four cm mounted on the narrow edges of the specimen.

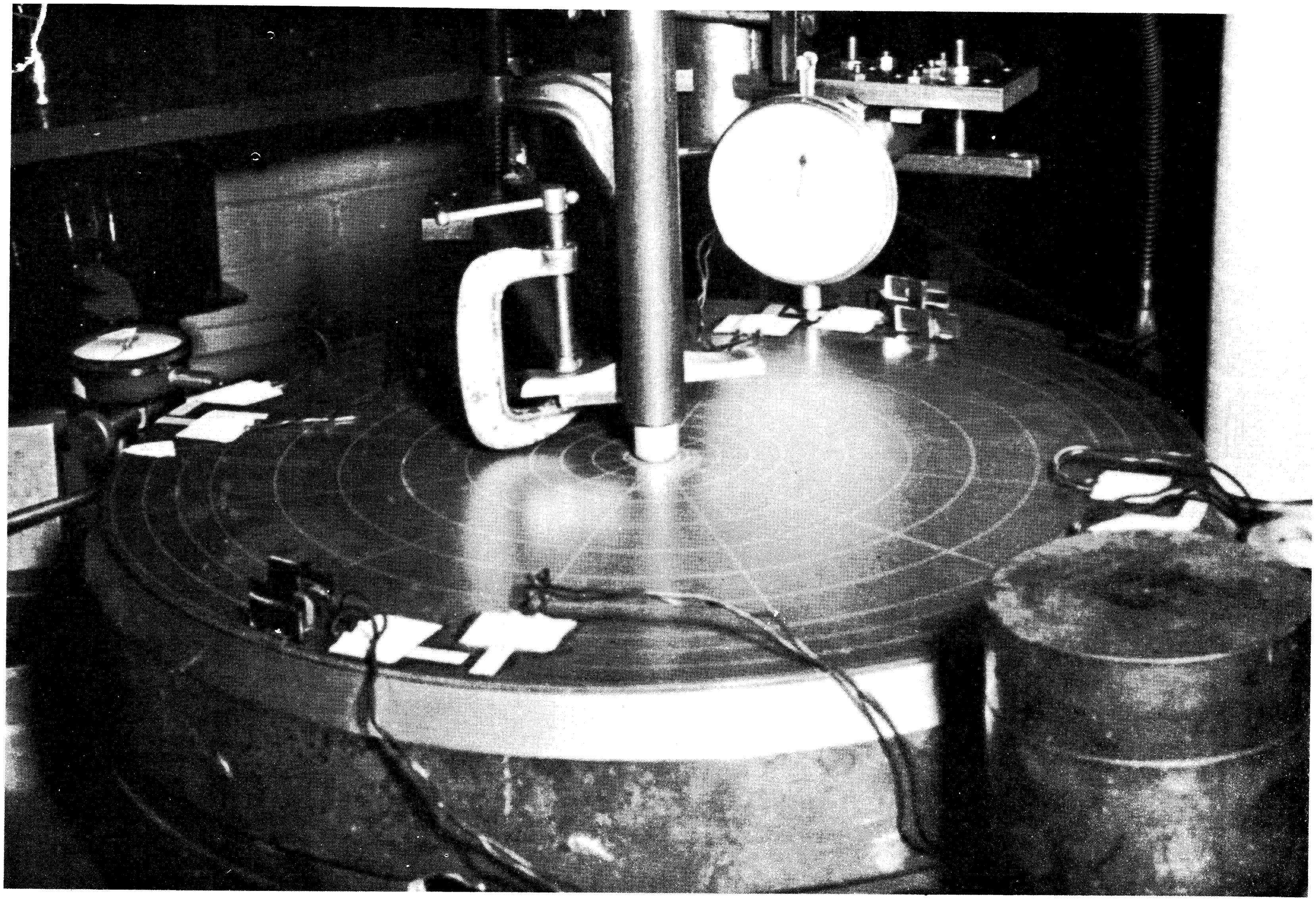

Fig. 1. Arrangements of loading bar and plate on support ring. Ames dial gage, pull-in gage, and mirrors may also be seen. Depth micrometer is hidden behind loading bar.

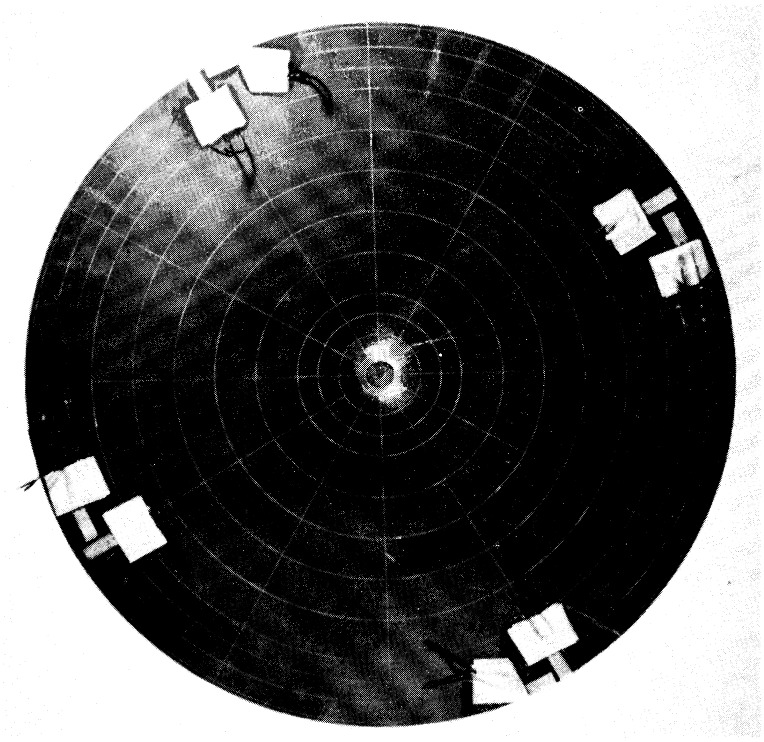

Fig. 2a. Top view of type-III plate. Radial cracks in mill scale may be seen at the edge over upper half of plate.

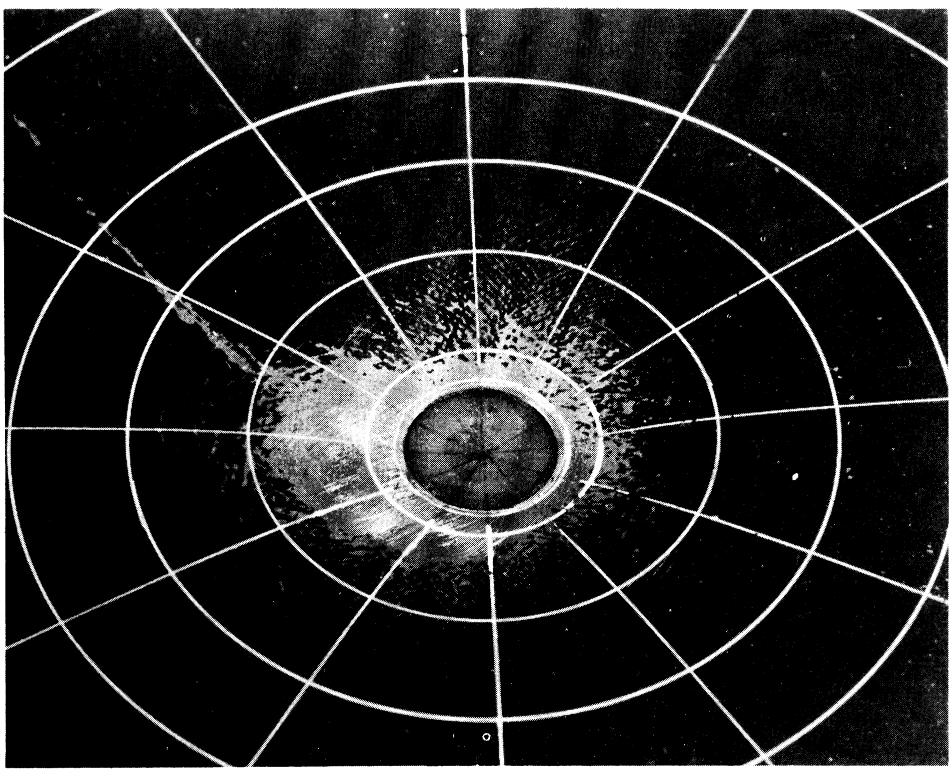

Fig. 2b. Oblique view of herring-bone crack pattern in mill scale around dimple at plate center.

EXPERIMENTAL RESULTS

An uninstrumented plate of each type was subjected to load at a low constant deflection rate and the "maximum load" (beyond which the load indicating pointer "fell off") was recorded. These plates failed with maximum loads recorded as 2400, 6700, and 4500 lb for types I, II, and III, respectively. A typical failure with its interesting mill-scale crack pattern is shown in Fig. 3a. For all three types of plates the cracks formed initally near the center, then numerous cracks formed at the supported edge and propagated toward the center. Wrinkles formed initally in the vicinity of the support and propagated toward the center.

The test results for the instrumented plates presented in Fig. 4 through Fig. 12 are as follows: typical stress-strain curves in tension, Fig. 4; deflection profiles for a type-III plate at several loads, Fig. 5; variation with load of central deflection for all three types of plates, Fig. 6; slope at several radii for type-III plates using optical data, Fig. 7; total radial strain (top surface and 1/2 inch away from the support) and corresponding membrane and bending strain components, Fig. 8a and 8b; total circumferential strain (top surface and 1/2 inch away from the support) and corresponding membrane and bending strain components, Fig. 9a and 9b; circumferential extreme-fibre bending strain obtained from optical data and from strain-gage data for a type-III plate, Fig. 10; total radial strain at the top surface over the support, Fig. 11; and total circumferential strain at the top surface over the support, Fig. 12.

The interior dial gages used for type-III plate tests were not located along the same radius, thus the plate profiles shown in Fig. 5 assume symmetry of plate deflection.

The curves in Figs. 8b and 9b for the top surface radial and circumferential bending strains were computed by assuming the relation

$$\varepsilon_{bend} = \frac{\varepsilon_{bottom} - \varepsilon_{top}}{2} ,$$

where total strains are employed on the right hand side. Similarly, the membrane strains were assumed to be given by

$$\varepsilon_{membrane} = \frac{\varepsilon_{bottom} + \varepsilon_{top}}{2} .$$

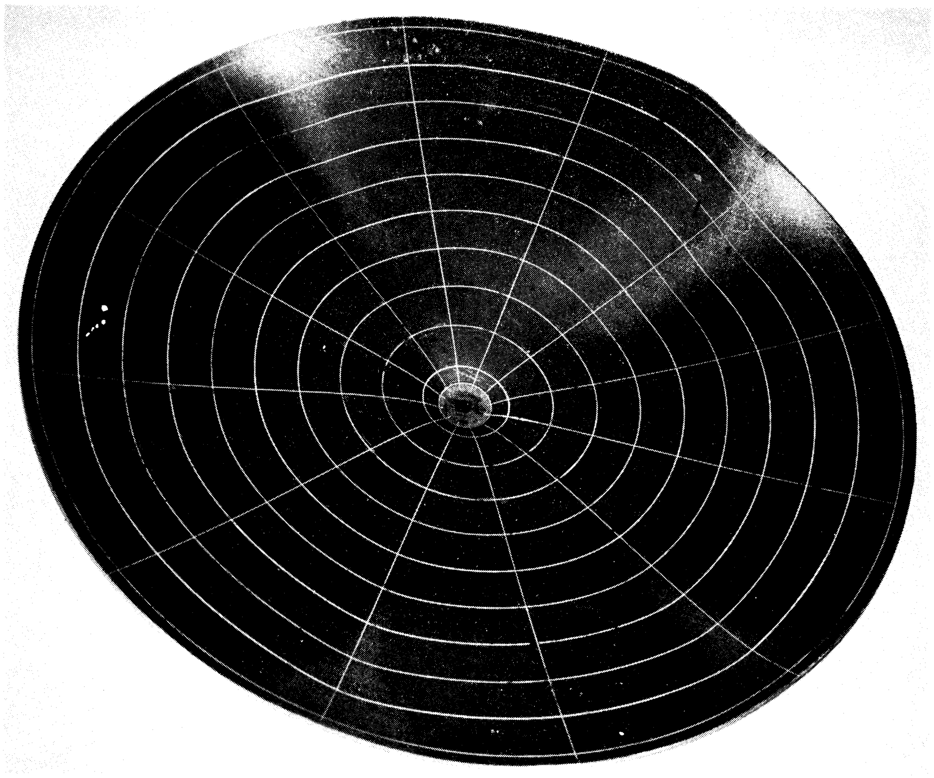

Fig. 3a. Type-I plate which failed in buckling.

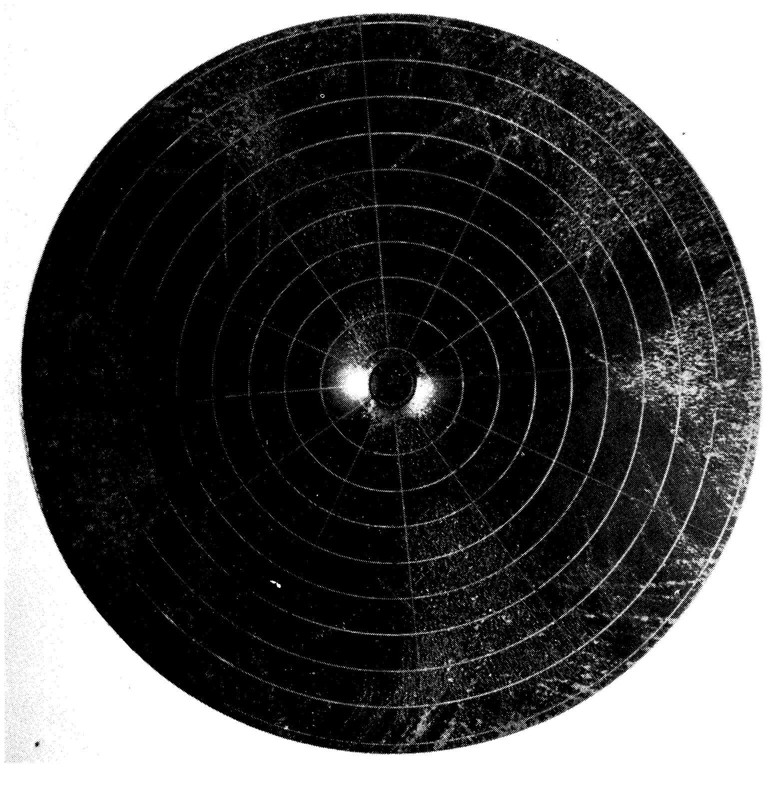

Fig. 3b. Type-II plate showing crack pattern in mill scale.

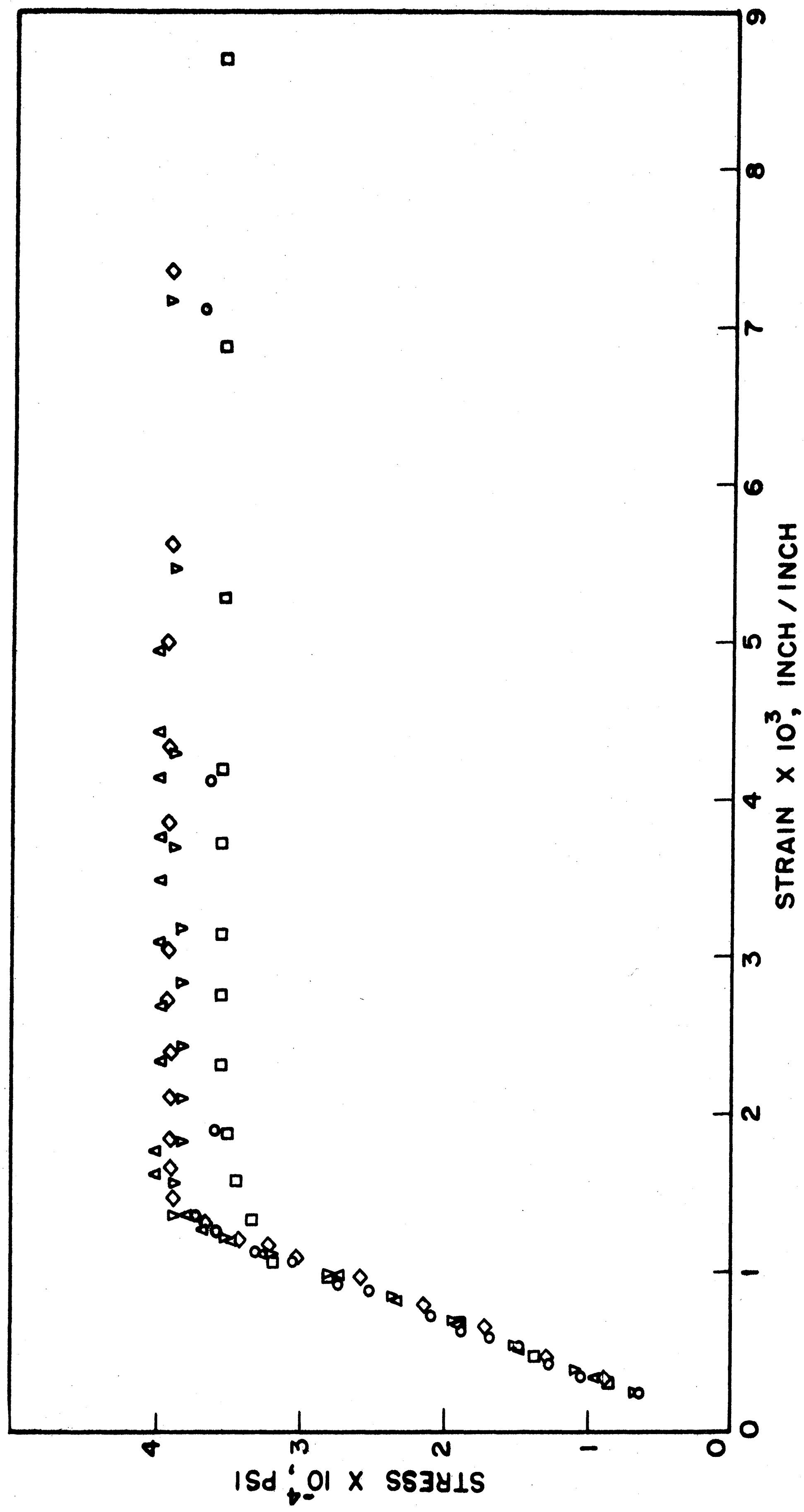

Fig. 4. Stress-strain curves in simple tension for the plate materials.

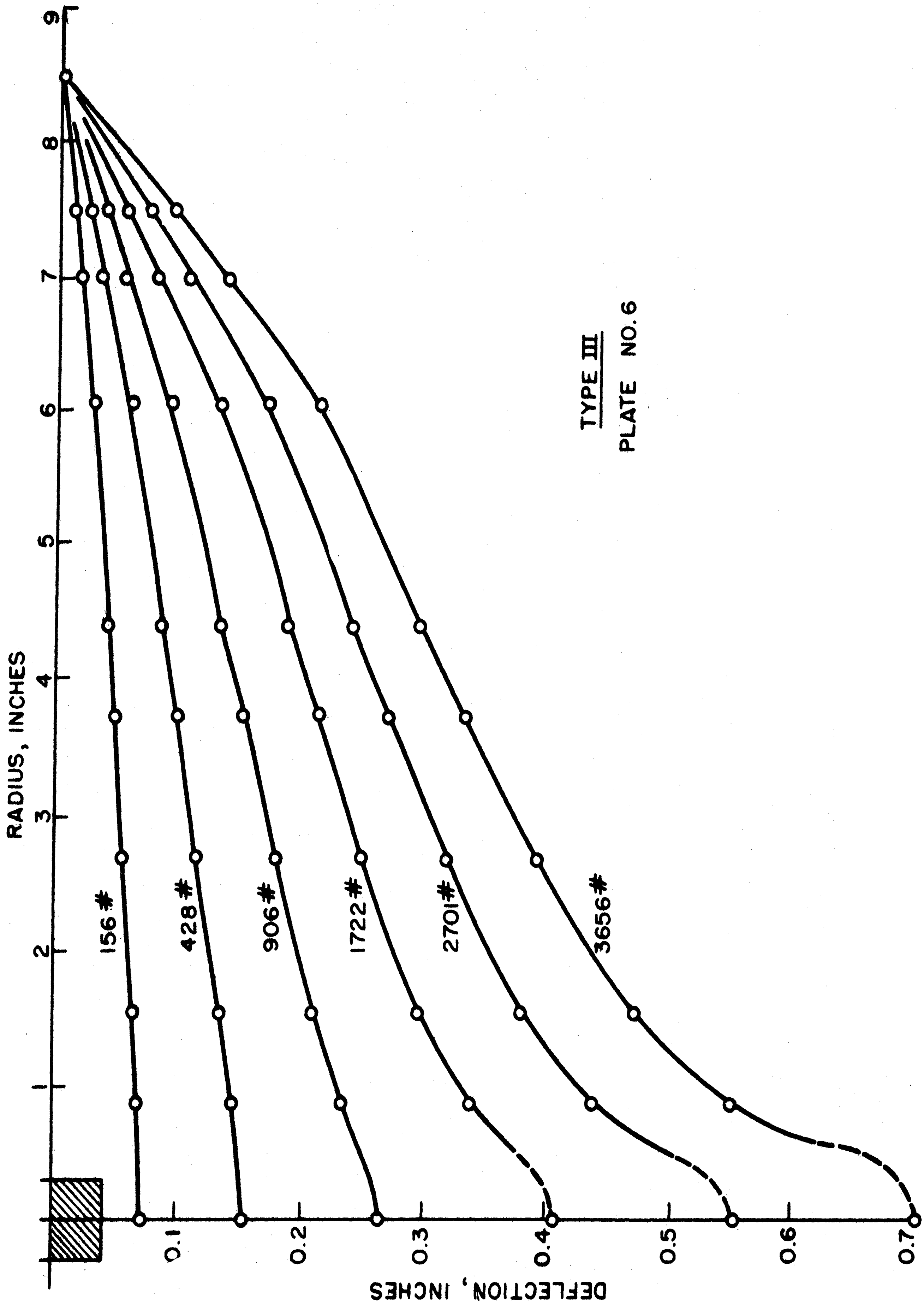

Fig. 5. Deflection profiles of a type-III plate.

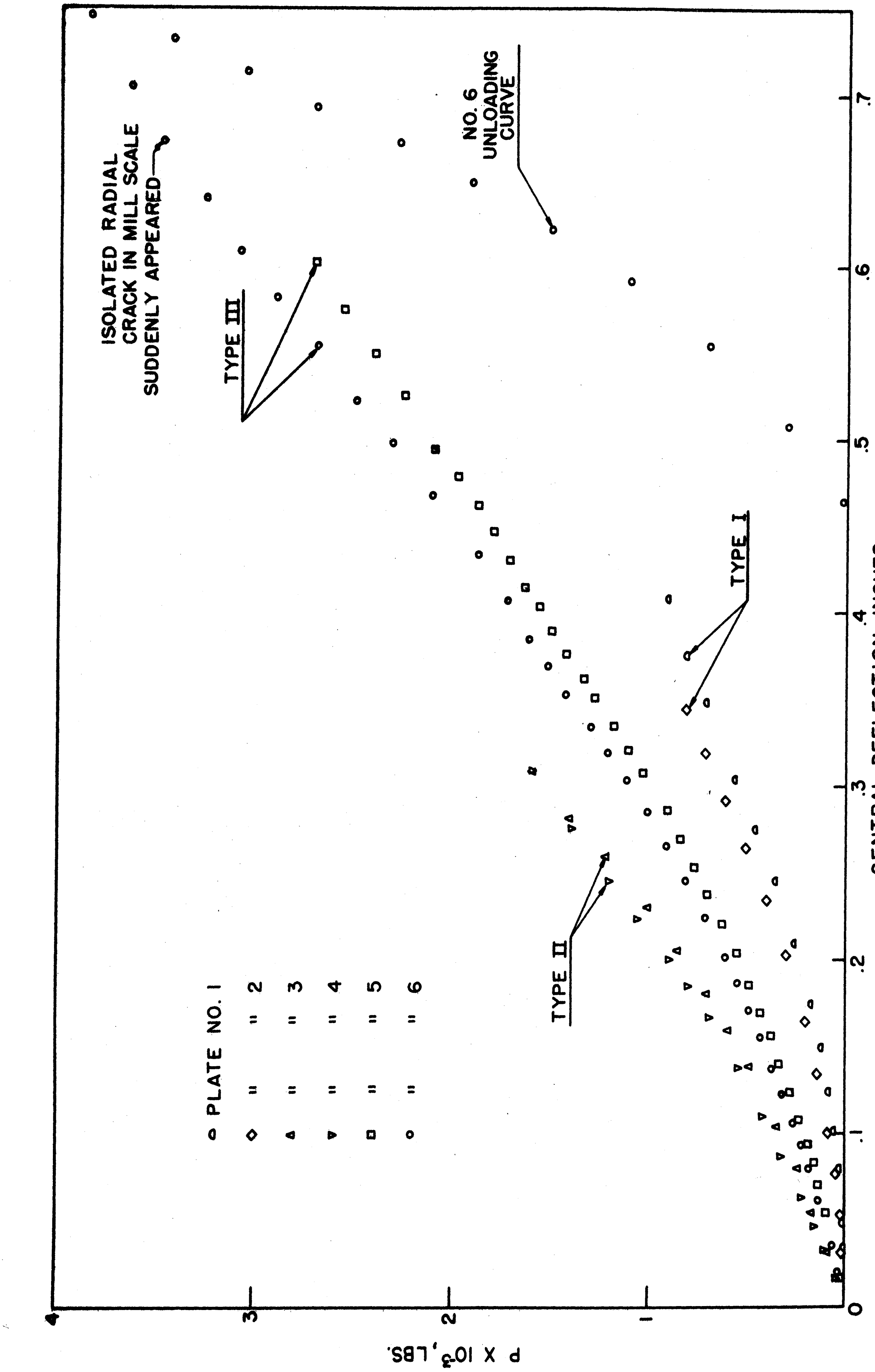

Fig. 6. Central deflection variation with load for all three types of plates.

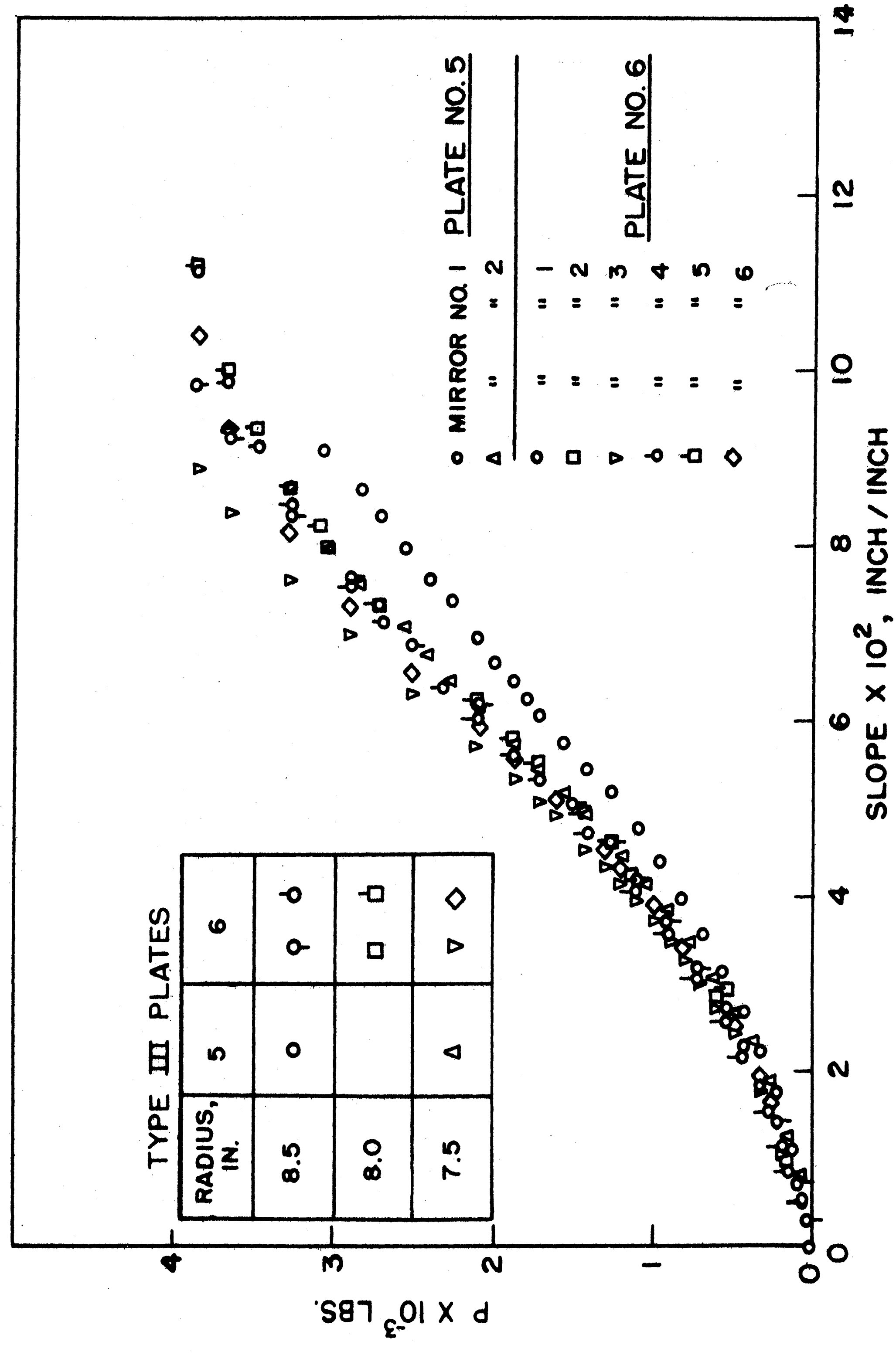

Fig. 7. Radial slopes from optical data, type-III plates.

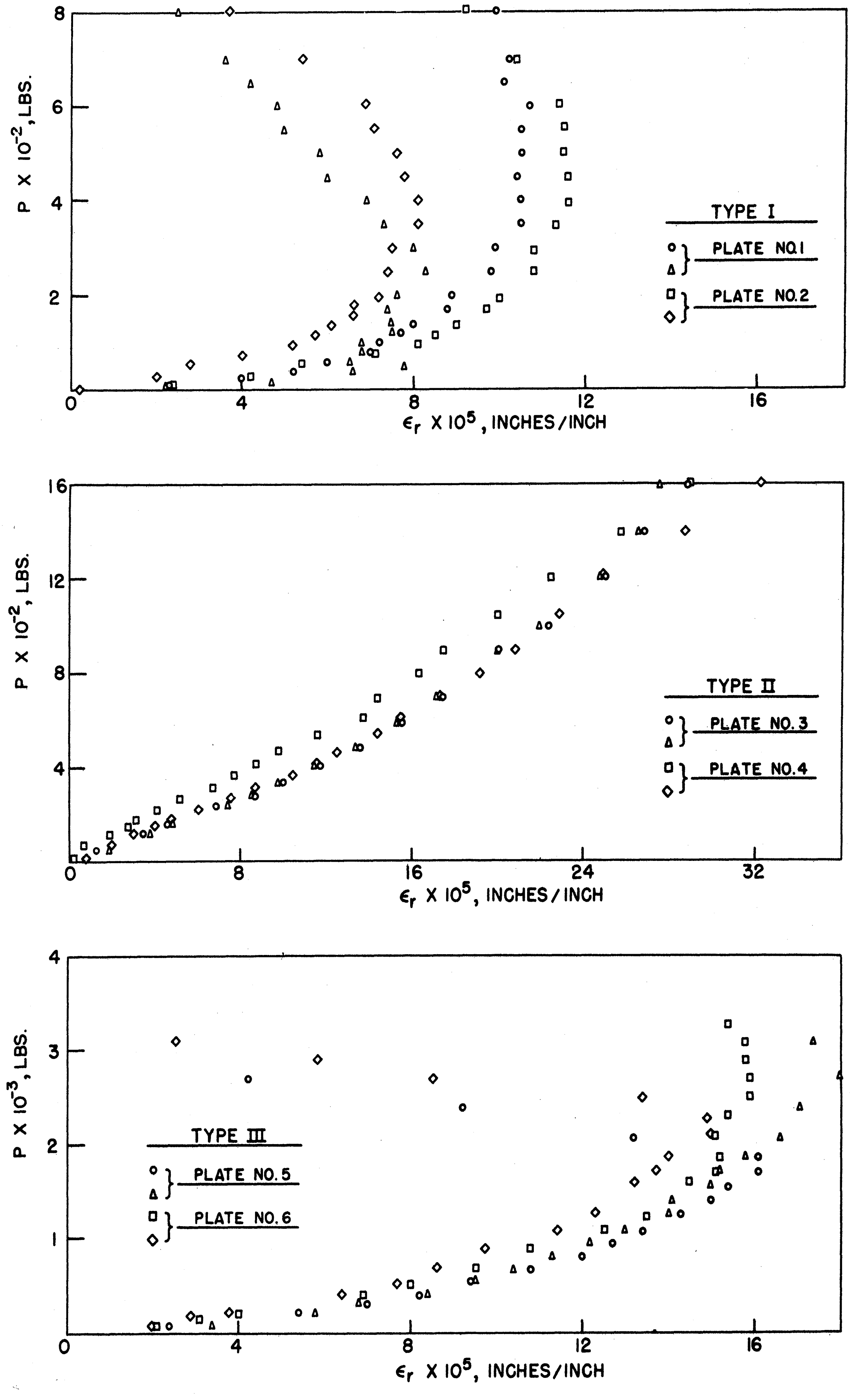

Fig. 8a. Top surface radial strain at station 1/2 inch away from support. Positive strain is tensile.

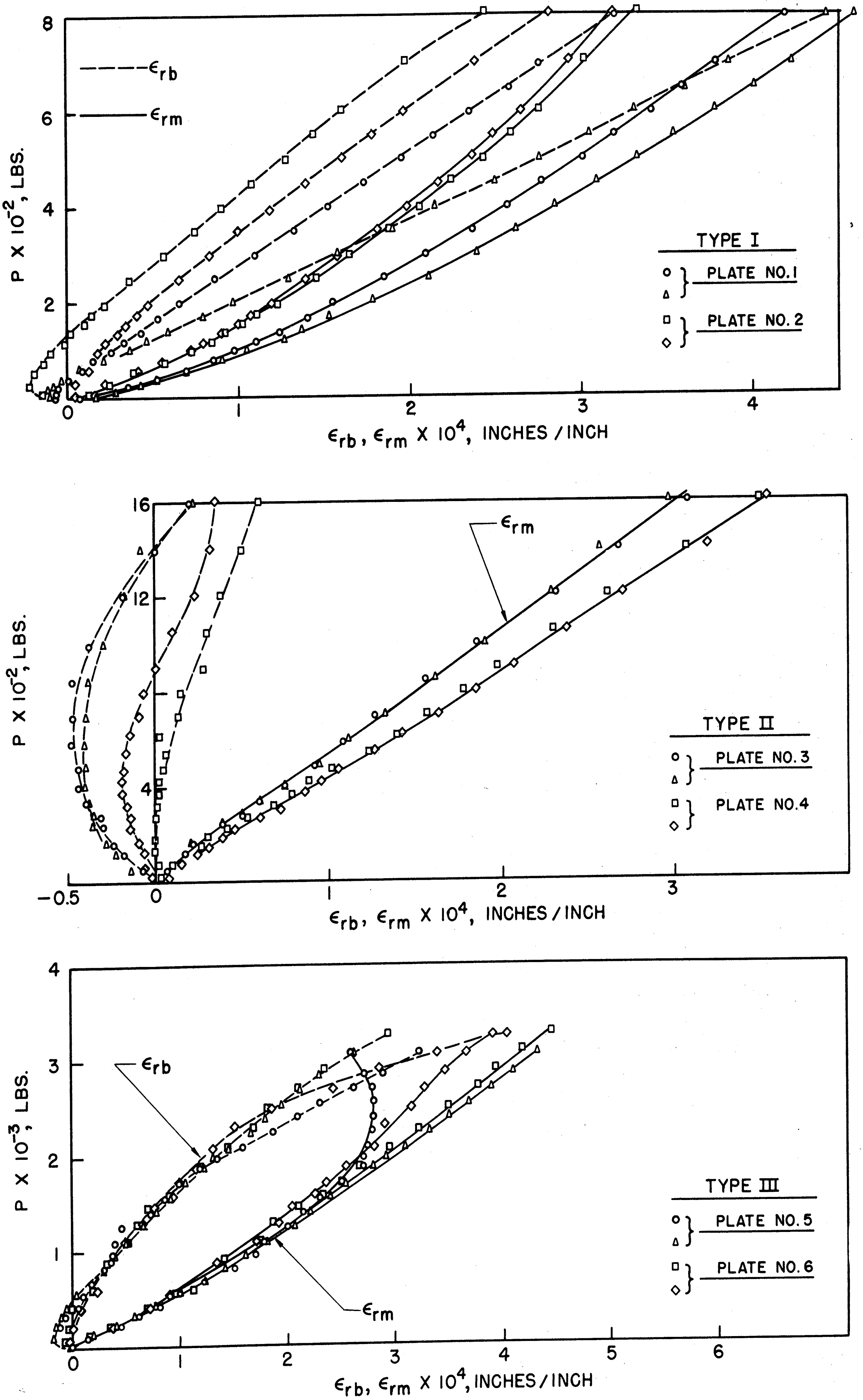

Fig. 8b Top surface radial membrane and bending strains 1/2 inch away from support. Positive membrane strains is tensile; positive bending strain is compressive.

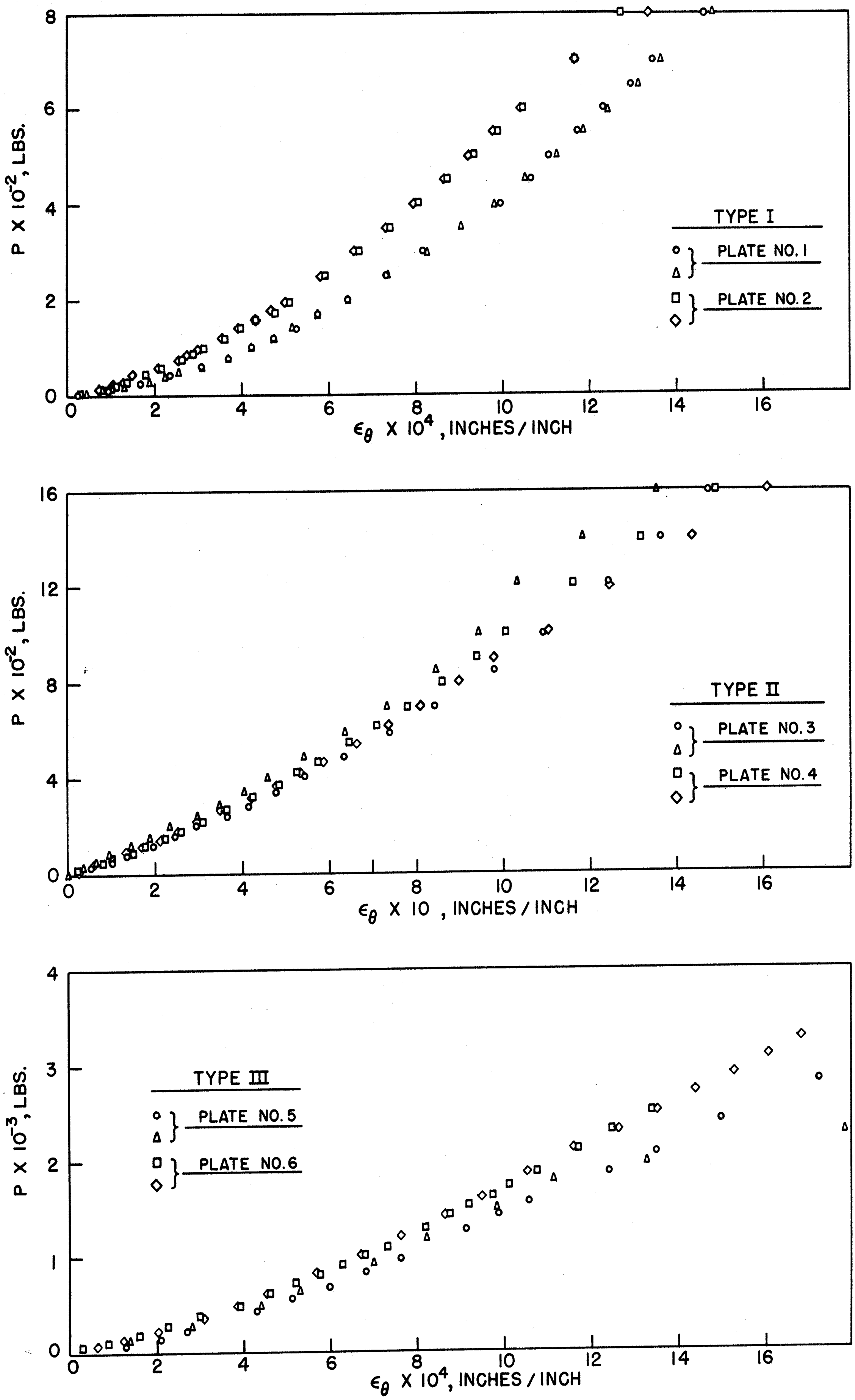

Fig. 9a. Top surface circumferential strain at station 1/2 inch away from support. Positive strain is compressive.

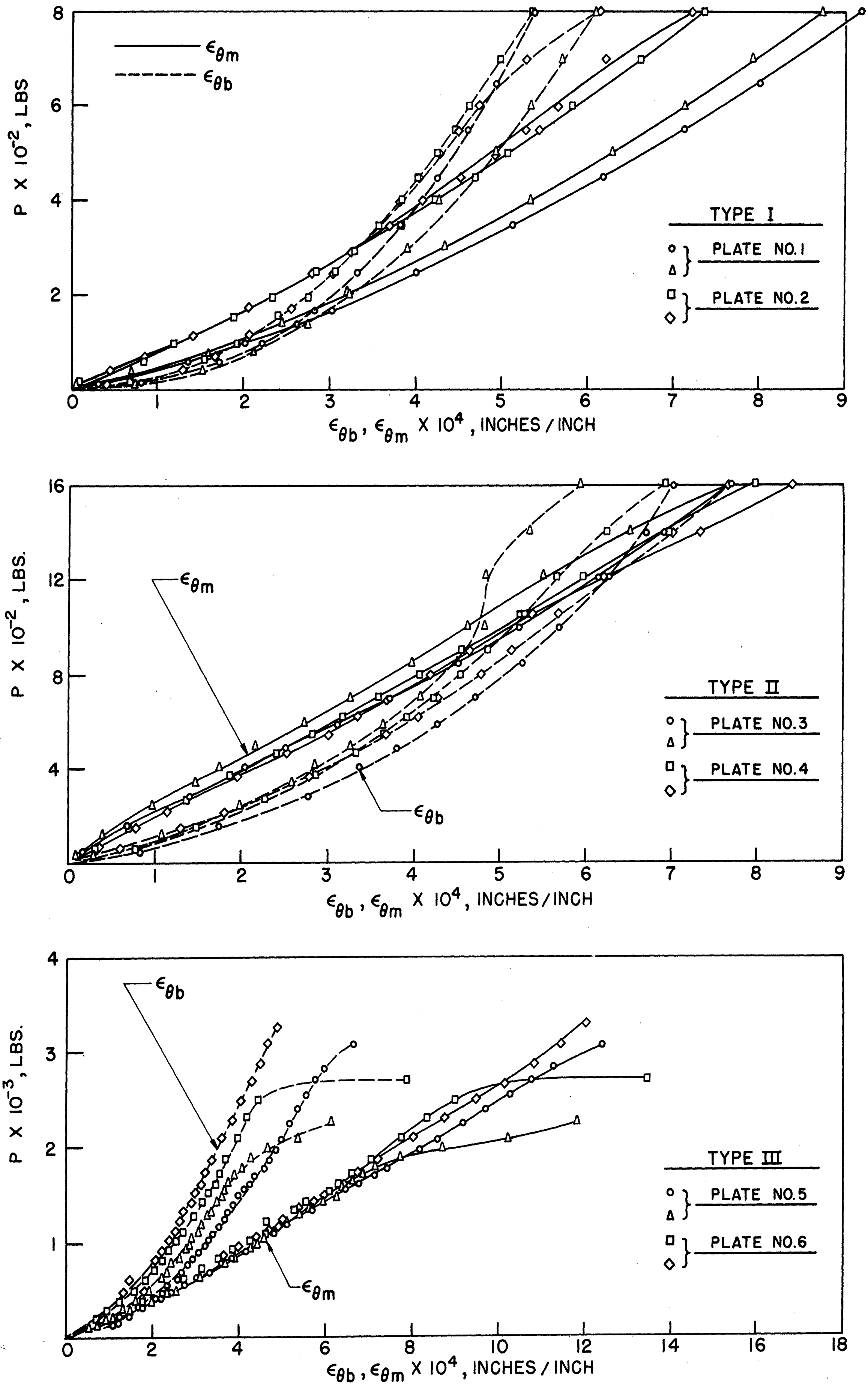

Fig. 9b. Top surface circumferential membrane and bending strains 1/2 inch away from support. Positive strain is compressive.

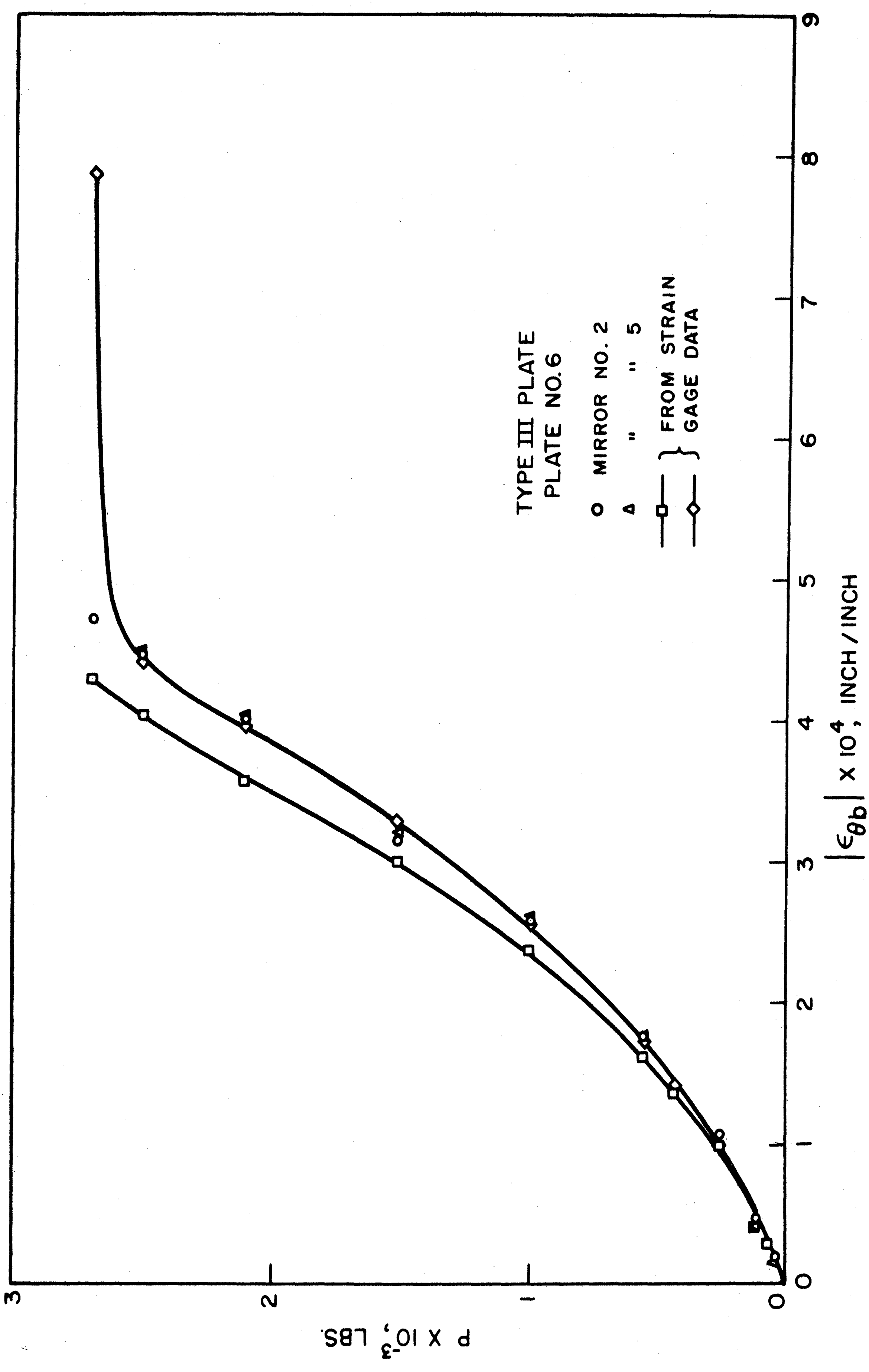

Fig. 10. Comparison of strain gage and optical measurements of circumferential bending strains at 8-inch radius for type-III plates.

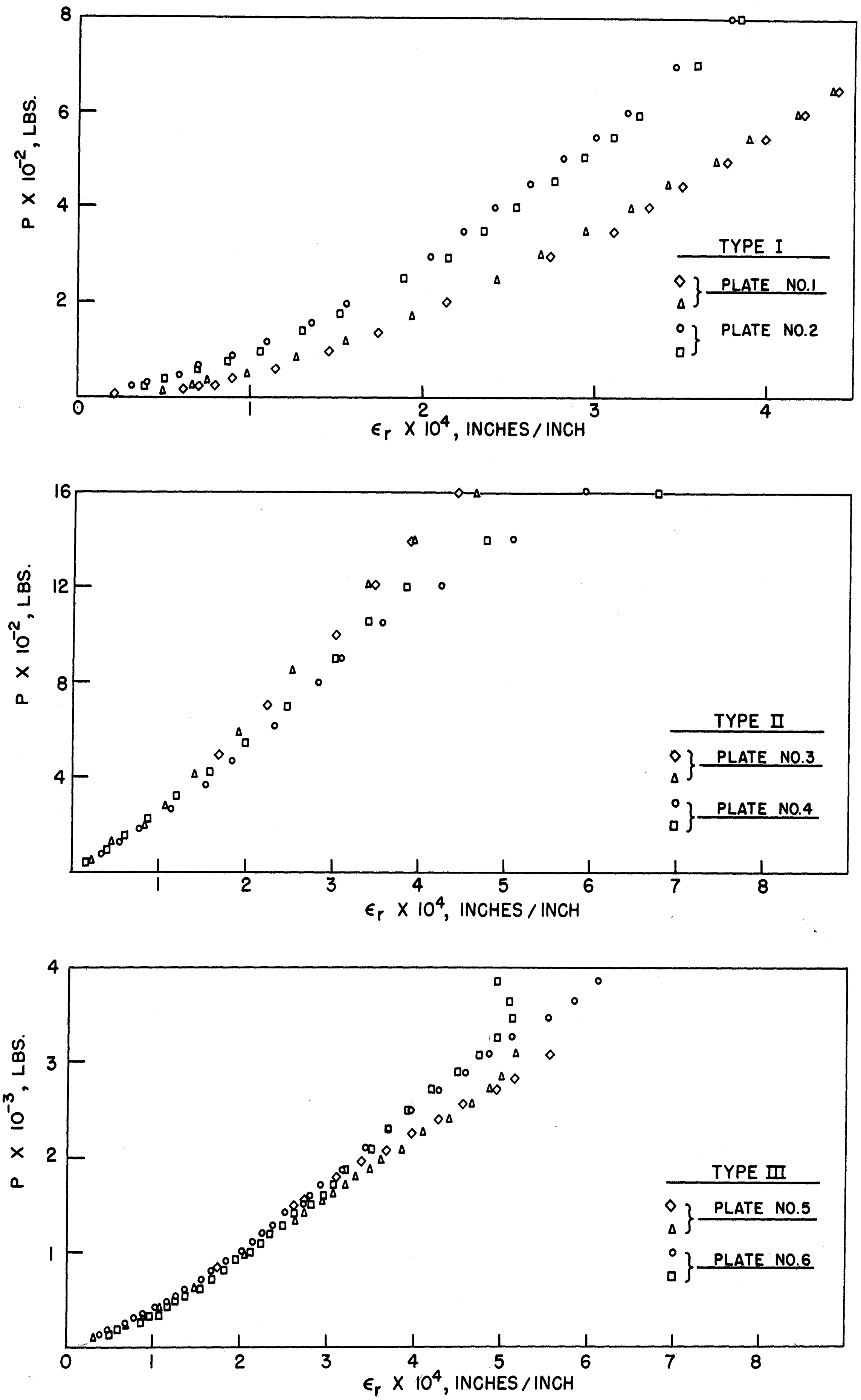

Fig. 11. Total radial strains at the top surface over the support. Positive strain is tensile.

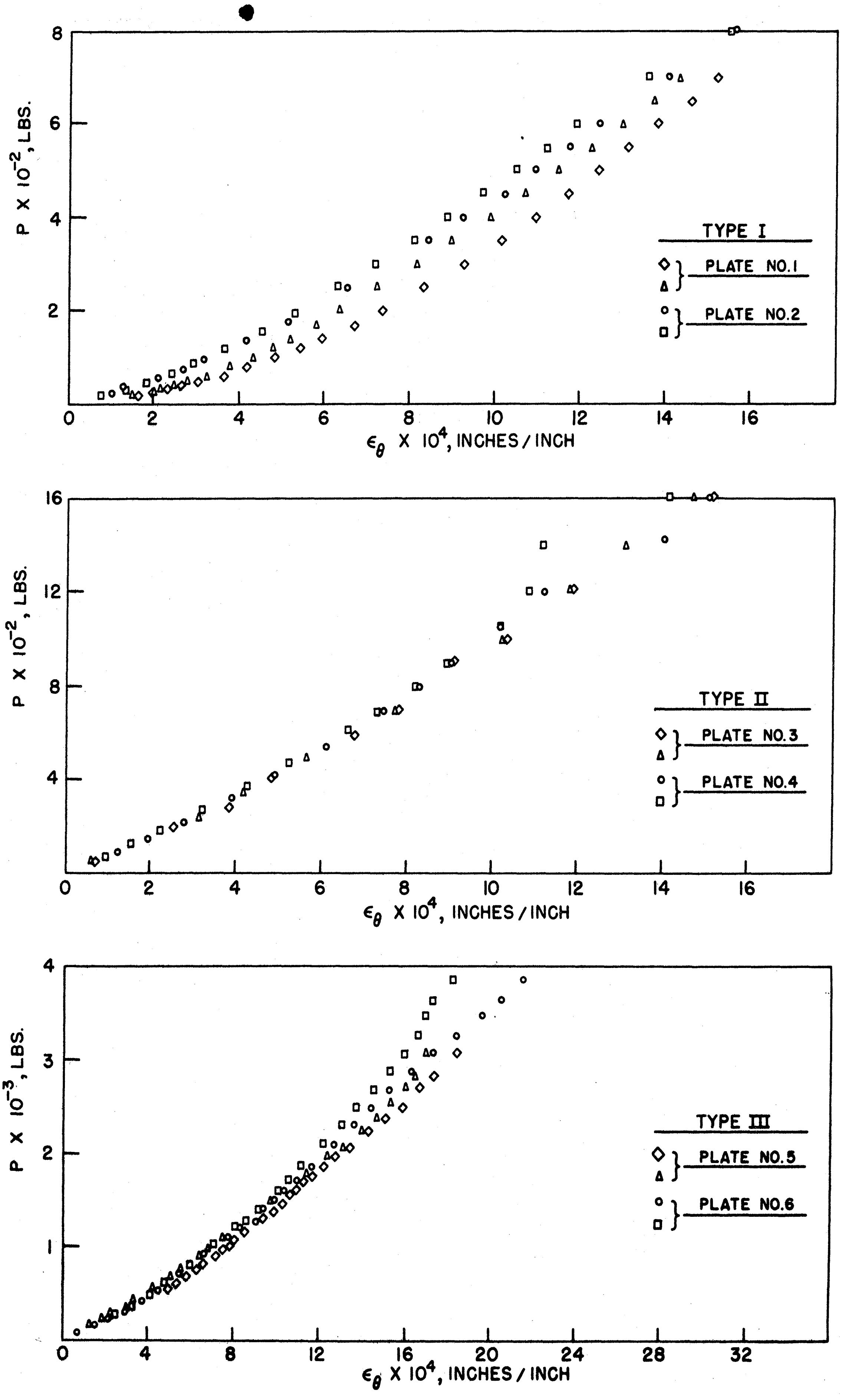

Fig. 12. Total circumferential strain at the top surface over the support. Positive strains are compressive.

This assumption implies a linear variation of bending strain and a constant membrane strain over the plate thickness at a given radius. The convention adopted for bending strains is in agreement with the convention for a positive bending moment used in reference 1. A positive numerical value for the bending strain indicates a compressive strain at the top surface.

The extreme-fibre circumferential bending strains shown in Fig. 10, as from optical data, were computed by using the slope data for the same radius as that at which the circumferential strain gages were located. The circumferential bending strain is assumed to be given at the top surface by

$$\varepsilon_{\theta b} = \frac{h}{r} \frac{dw}{dr} , \tag{2}$$

where r is the radius and dw/dr is the slope taken from the appropriate data of Fig. 7.

The sudden deviation of one of the curves of Fig. 10 is due to the initiation of a radial crack in the mill scale directly under the strain-gage "set". The plate from which the data was taken is shown in Fig. 2, where the radial mill-scale cracks may be seen.

The "pull-in" gages used as mentioned earlier indicate an average motion of 0.013 inch toward the center of the plate at a load of 3862 lb for a type-III plate.

DISCUSSION

Theoretical and experimentally-determined loads are given in Table II. With reference to Fig. 6 it appears that the theoretical loads as predicted in Equation 1 are not verified. It is possible that solution 1 could be verified for thicker plates, although shear deformation effects become progressively more important with increasing thickness.

For the plates tested it was found that membrane strains occur with the same order of magnitude as the bending strains, even at low loads, (see Figs. 8b and 9b). In view of the presence of these membrane strains, it may be remarked that the von Mises (J_2) yield condition is satisfied by the top surface strains considerably earlier than by the bottom surface strains at the same radius.

The comparison demonstrated in Fig. 10 between circumferential bending strains as computed from optical data and as obtained by strain-gage data appears to justify the assumption of a linear distribution of strain across the thickness of the plate and also is an indication of the reliability of the test data. Apparently the strain-gage data is reliable and is characteristic of the plate as a whole until the onset of localized yield effects. Although a similar comparison in the radial direction was not made, the optical data indicates the same trend of the bending strains as that obtained from the strain-gage data.

TABLE II

		Load from Ref. 1, lb	Load when $J_2=\sigma_0^2$ at Top Edge, lb	Highest Test Load, lb	Typical Maximum Load, lb	Reason for Termination of Tests
I	1	144	600	900	2400 (Plate 7)	J_2 satisfied at outer edge
	2	222	700	804		
II	3	675	1420	1600	6700 (Plate 8)	J_2 satisfied at outer edge
	4	667	1450	1605		
III	5	656	2075	3082	4500 (Plate 9)	Possible damage to test equipment
	6	659	2200	3862		

It is noted from Fig. 9b that the initial behavior of radial bending strains is not as expected. However, the plates as tested were not flat and required some initial load before coming completely down on the supporting ring. It is possible that this initial radial strain behavior could be due to machining and the strains produced by the initial load on the plate.

One of the interesting results of the tests was the appearance of cracks in the mill scale of the plates. Such cracks were particularly prominent on those plates tested to failure. For the type-I plate mill-scale cracks and the initiation of wrinkles in the plate appeared simultaneously. Failure eventually occurred due to buckling. For type-II and type-III plates the mill-scale crack pattern was well developed before wrinkles formed. The loading bar punched through the center of the type-II plate, whereas the type-III plate failed by buckling. After the formation of the dimple at the center of all the plates (accompanied for type-II and type-III plates by a characteristic, though limited, center crack pattern) mill-scale cracks appeared in the 1- or 2-inch annulus at the supported edge and propagated toward the center.

It thus appears that the plastic flow does not spread out from the center alone, but rather that there is first a progression from the center and then a new plastic region moves from the outer annulus toward the more slowly growing center region.

This behavior can be correlated with the fact that the von Mises (J_2) yield condition is satisfied at the outer edge on the top surface before it is satisfied 1/2 inch nearer the center. While the J_2 yield condition is not necessarily the exact condition for the plate material, it is a guide for the presence of the plastic state and thus may be considered as further indication of the suggested spreading of the plastic regions.

REFERENCES

1. H. G. Hopkins, and W. Prager, "On the Load Carrying Capacities of Circular Plates", Mech. and Phys. of Solids, 2, No. 1, 1-13, October, 1953.

2. W. Ramberg, A. E. McPherson, and S. Levy, "Normal-Pressure Tests of Rectangular Plates", N.A.C.A. TR 748 (1942).

3. J. H. Lamble, and J. P. Chandhary, "Support Reactions, Stresses, and Deflections for Plates Subjected to Uniform Transverse Loading", to appear in Quarterly transactions of the Institution of Naval Architects.

4. P. M. Naghdi, "Bending of Elasto Plastic Circular Plates with Large Deflections", Mech., 19, No. 3., 293-300 (1952).

5. Baumberger and Hines, "Practical Reduction Formulae for Strain Gages in Two Dimensional Stress Fields", Amer. Soc. Exper. Stress Anal. Proc. II No. 1 113 (1944).